Technical Note BN-443 March 1966

GENERALIZED LANDAU KINETIC EQUATION DESCRIBING THE TRANSVERSE DIFFUSION OF A NONGYROTROPIC PLASMA COLUMN*

by

Michael J. Haggerty

University of Maryland
College Park, Maryland

*This work was supported in part by the National Aeronautics & Space Administration under Grant NsG 695.

ABSTRACT

Some further preliminary results are reported on the application of the plasma kinetic theory of Prigogine and Balescu to a plasma column in a uniform magnetic field. At the cost of ignoring collective effects, the assumption of initial gyrotropy is removed. A relatively simple kinetic equation for the one particle distribution function is obtained. The resulting augmented kinetic theory is expected to describe a greater variety of diffusion phenomena. A multicomponent plasma is considered, but the effect of the macroscopic transverse electric field (due to charge separation) on the local interactions has not yet been calculated. The plasma is assumed to be homogeneous in the direction of the magnetic field, but an arbitrary inhomogeneity across the field is allowed. A collision integral due to Eleonskiĭ, Zyryanov, and Silin can be reproduced as a special case. Contact is also made with the recent work by Sundaresan on the homogeneous nongyrotropic plasma. A method of properly including collective effects is suggested.

INTRODUCTION AND SUMMARY

The following material is intended as a supplement to that in a previous report[1], to be referred to as "IVa", on the kinetic theory of diffusion of a plasma column across a constant uniform magnetic field. That work consisted of an extension of the theory of Prigogine, Balescu, and co-workers; see for example reference 2. The problem of deriving a kinetic equation (analogous to the well-known Balescu-Lenard-Guernsey equation) was reduced to that of solving a two-dimensional Fredholm integral equation with a complicated kernel. No explicit restriction was made concerning the amplitude or length scale of the inhomogeneity across the magnetic field.

The usefulness of this result was somewhat limited. Even if the solution of the Fredholm equation could be obtained in closed form, the resulting kinetic equation would be extremely lengthy and complicated, because of the collective terms associated with the long-range Coulomb interaction. These terms are made more complicated than usual by the transverse inhomogeneity. Furthermore, a strong initial condition was introduced; namely that the distribution of velocity components perpendicular to the magnetic field be isotropic. A correspondingly strong theorem was derived, namely that the condition is preserved in time within the approximations made. However, the assumption was obviously a restriction on the types of transport phenomena that the theory can describe. _If_ across any interface element tangential to the magnetic field, there were as many particles of a given speed going in one direction as in the opposite direction (which is not quite equivalent to the assumption), then the diffusion would necessarily be similar to that described by Fick's Law. (In fact, some currents were allowed, because the "gyrotropy" is for a fixed guiding center rather than for a local region. But this consideration tends to make the assumption less plausible rather than more plausible.)

The main content of this report is a derivation of a single short kinetic equation for a one particle distribution function having an arbitrary anisotropy in velocity space. The only restriction on the initial distribution (apart from the usual initial conditions of the Prigogine theory) is that it be homogeneous in the direction of the magnetic field. The "gyrotropic" initial condition is avoided. The result is analogous to the Boltzmann equation for weakly coupled ionized gases, also known as the Landau equation. It is obtained by ignoring the complicated collective terms. The long-range divergence difficulty could be eliminated either by introducing one of the collective factors found for other models, or by introducing some simple cutoff procedure, as is done for the Landau equation.

In concurrent and independent research, Sundaresan[3] has developed the Bogoliubov theory for a homogeneous nongyrotropic plasma including collective effects. In the concluding discussion, we make some provisional remarks on the comparison of our result for the homogeneous case with Sundaresan's result, when his collective factors are ignored. In addition, it is shown that for the gyrotropic plasma with an inhomogeneity in only one direction, our result reduces to the result of Eleonskiĭ, Zyryanov, and Silin[4] when their collective factors are ignored. (We regret that our previous report IVa misquoted their result.) It is hoped that future reports will contain more detailed comparisons of the various results reported in the literature, as well as descriptions of the quantitative and qualitative properties of the kinetic equations.

We should not rule out the possibility of properly including collective effects for the present model; that is, of constructing a simple kinetic equation of the Balescu-Lenard-Guernsey type for a non-gyrotropic plasma having an arbitrary transverse inhomogeneity. At the time of writing, however, such a construction appeared to present some difficulties. One possible approach is mentioned in the concluding discussion.

Our derivation contains another generalization, namely to a plasma consisting of more than one species of particles. However, we still do not have a kinetic theory of transverse ambipolar diffusion. Different species may have different diffusion rates. The resulting charge separation would produce growing macroscopic electric fields. The effect of such electric fields on the local interactions responsible for the diffusion has been ignored.

It should be emphasized that when leaving out the collective terms, we may be omitting very important effects. For example, it is very likely that a study of the generalized dielectric function $[\delta(\underset{\sim}{q} - \underset{\sim}{q}') - K(\underset{\sim}{q}, \underset{\sim}{q}')]$ found in Section IVa.7 will give conditions of stability and instability on the distribution of guiding centers as well as on the velocity distribution. Thus in a sense we have the possibility of finding macroscopic instabilities from microinstability theory.

CALCULATION

Only the differences between this derivation and that in IVa are present here. Those interested in further details should refer to IVa, and to works referred to there. We will call attention to a few changes in notation, but otherwise the symbols have the same meaning as before. The main change is the replacement of some of the distinguishing labels α, h, u, ... by 1, 2, 3, ... Numerical subscripts are more convenient when the diagram technique is not being emphasized. (The symbol σ is used here to distinguish different species.) Another change is the use of Ω for gyrofrequencies, which allows us to use ω, ω_j, ... for certain important variables similar to those previously denoted by G, G_j, ... It is hoped that these and other minor changes will result in a net reduction of confusion.

We consider a multicomponent plasma with N_σ particles of the σ-th species, having charge e_σ and mass m_σ. The plasma is neutral in the sense that

$$\sum_\sigma N_\sigma e_\sigma = 0 \quad .$$

The limit $\{N \to \infty,\ \Lambda \to \infty,\ N/\Lambda$ finite, Σ arbitrary$\}$ will be considered, where $N = \sum_\sigma N_\sigma$. The canonical momentum of particle number j is written as

$$m_j \underline{v}_j + (e_j/c)\ \underline{A}\ (\underline{x}_j) \ ,$$

where m_j and e_j are abbreviations for m_{σ_j} and e_{σ_j} . Its gyrofrequency is denoted by Ω_j ;

$$\Omega_j \equiv e_j \, B/m_j c \quad ,$$

where B is a constant magnetic flux density. As before, we often think of Ω_j and e_j as positive, to avoid the concept of negative frequencies. This mnemonic device will not affect the final results.

Again, the classical non-relativistic approximation is made. Radiation and surface effects are ignored. The plasma is assumed to be initially homogeneous in the direction $\hat{z} \equiv B/B$, but is allowed to have an arbitrary inhomogeneity in the directions $\hat{x}$ and $\hat{y}$. The time scale is assumed to be much larger than $\omega_{p\sigma}{}^{-1}$ and $|\,\Omega_\sigma{}^{-1}\,|$, where

$$\omega_{p\sigma} \equiv (4\pi e_\sigma{}^2 N_\sigma/m_\sigma \Lambda\Sigma\,)^{1/2} \quad .$$

The Hamiltonian of the system is

$$H = H_o + \lambda \sum_{i<j} V_{ij} \quad , \tag{1}$$

where $H_o = \sum_j \frac{1}{2} m_j v_j{}^2$ is the Hamiltonian excluding the interparticle Coulomb interaction (but including the magnetic field, which does not contribute to the kinetic energies of the particles), and λ is a coupling parameter to be set equal to unity after the dominant terms of the perturbation series have been selected. The potential V_{ij} is defined as follows:

$$V_{ij} = V_{ij}{}^{int} + W_{ij} \quad , \tag{2a}$$

$$V_{ij}^{int}(\underline{x}_{ij}) = (e_i e_j / |\underline{x}_{ij}|) \exp(-\kappa|\underline{x}_{ij}|)$$

$$= (2\pi/\Lambda) \sum_{k_{\|}} \iint d\underline{\ell}_{\perp}\, \hat{V}_{ij}^{int}([k_{\|}^2 + \ell_{\perp}^2]^{1/2})$$

$$\cdot \exp(ik_{\|} z_{ij}) \exp(i\underline{\ell}_{\perp} \cdot \underline{x}_{ij\perp}), \tag{2b}$$

$$\hat{V}_{ij}^{int}([k_{\|}^2 + \ell_{\perp}^2]^{1/2}) = e_i e_j / 2\pi^2 (k_{\|}^2 + \ell_{\perp}^2 + \kappa^2) \;; \tag{2c}$$

$$W_{ij}(\underline{x}_{ij}) = - \int_{-\Lambda/2}^{\Lambda/2} \frac{dz_j}{\Lambda}\, V_{ij}^{int}(\underline{x}_{ij}) \;; \tag{3}$$

where $k_{\|}$ is summed over values of the form $(2\pi/\Lambda) \cdot$ (integer), and κ is a cut-off parameter which must be held non-zero when collective effects are ignored. The symbol $\underline{x}_{ij}$ is an abbreviation for $\underline{x}_i - \underline{x}_j$. We sometimes write k, $\underline{\ell}$ in place of $k_{\|}$, $\underline{\ell}_{\perp}$. Combining eqs. (2) and (3), we get

$$V_{ij}(\underline{x}_{ij}) = (2\pi/\Lambda) \sum_k \int d\underline{\ell}$$

$$\cdot \hat{V}_{ij}([k^2 + \ell^2]^{1/2}) \exp(ikz_{ij}) \exp(i\underline{\ell} \cdot \underline{x}_{ij\perp}) \;, \tag{4a}$$

$$\hat{V}_{ij}([k^2 + \ell^2]^{1/2}) = \begin{cases} e_i e_j / 2\pi^2 (k^2 + \ell^2 + \kappa^2) \;, & k \neq 0 \\ 0 \;, & k = 0 \end{cases}$$

$$= \frac{e_i e_j [1 - \delta^{Kr}(k)]}{2\pi^2 [k^2 + \ell^2 + \kappa^2]} \;. \tag{4b}$$

The introduction of the potential W_{ij} is a device intended to eliminate the macroscopic self-consistent electric field. Mathematically speaking, it eliminates the matrix elements of δL_{ij} which are diagonal with respect to the k_j's. The collisionless "drift" terms of the type $(\underline{E} \times \underline{B} / B^2) \cdot \partial f/\partial \underline{x}$ could be added to the resulting equations, but cross-effects between local interactions and the macroscopic transverse electric field are ignored.

A physical picture of the effect of W_{ij} can be constructed if desired. It is analogous to the concept of a background charge whose local density somehow varies in time with the plasma density. Associated with each particle j one can imagine a rigid massless line charge in the z direction with charge density $- e_j/\Lambda$, which pierces the particle (cf. eq. (3)). The line charges are carried along with their particles without reacting upon them, and without interacting among themselves. However, the motion of each particle is influenced by the presence of the line charges tied to the other particles.

We now write down the formal solution of the Liouville equation. It is almost the same as that given in Section IVa, 3. If $\rho'_{\{k\}\{n\}}(t)$ is defined by the equation

$$f_N(t) = \sum_{\{k\}} \sum_{\{n\}} \rho'_{\{k\}\{n\}}(t) \exp(i \sum_j [k_j z_j - n_j \theta_j]), \tag{5}$$

then

$$\rho'_{\{k^{(o)}\}\{n^{(o)}\}}(t) = \frac{i}{2\pi} \oint_C d\zeta \exp(- i\zeta t) \sum_{q=0}^{\infty} [(-\lambda)^q$$

$$\cdot \left[\sum_j (k_j^{(o)} v_{j\|} + n_j^{(o)} \Omega_j) - \zeta \right]^{-1} \prod_{\mu=1}^{q} \Big\{ \sum_{\{k^{(\mu)}\}} \sum_{\{n^{(\mu)}\}} \sum_{i_\mu < h_\mu}$$

$$\cdot \left\langle \{k_j^{(\mu-1)}\} \{n_j^{(\mu-1)}\} \right| \delta L_{i_\mu h_\mu} \left| \{k_j^{(\mu)}\} \{n_j^{(\mu)}\} \right\rangle$$

$$\cdot \left[\sum_j (k_j^{(\mu)} v_{j\|} + n_j^{(\mu)} \Omega_j) - \zeta \right]^{-1} \Big\} \quad \rho'_{\{k^{(q)}\}\{n^{(q)}\}}(0)] \, , \tag{6}$$

where

$$\left\langle \{k_j\} \{n_j\} \right| \delta L_{12} \left| \{k'_j\} \{n'_j\} \right\rangle$$

$$= \left[\prod_{j=3}^{N} \delta^{Kr}(k_j - k'_j) \, \delta^{Kr}(n_j - n'_j) \right]$$

$$\cdot \; \delta^{Kr}(k_1 + k_2 - k'_1 - k'_2) \, [1 - \delta^{Kr}(k_1 - k'_1)]$$

$$\cdot \int_o^\infty \ell d\ell \int_o^{2\pi} d\beta \left[\frac{- e_1 e_2 \exp(i[n_1 + n_2 - n'_1 - n'_2]\beta)}{\pi\Lambda([k_1 - k'_1]^2 + \ell^2 + \kappa^2)} \right]$$

$$\cdot \Bigg\{ \Bigg[(k_1 - k'_1) \left(\frac{1}{m_1} \frac{\partial}{\partial v_{1\|}} - \frac{1}{m_2} \frac{\partial}{\partial v_{2\|}} \right)$$

$$- \frac{n'_1 \Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} - \frac{n'_2 \Omega_2}{m_2 v_{2\perp}} \frac{\partial}{\partial v_{2\perp}}$$

$$- \left(\underline{\ell} \times \left[\frac{1}{m_1\Omega_1} \frac{\partial}{\partial \underline{Q}_1} - \frac{1}{m_2\Omega_2} \frac{\partial}{\partial \underline{Q}_2} \right] \right)_{\parallel} \Bigg]$$

$$\cdot \left[J_{n_1 - n'_1} \left(\frac{\ell v_{1\perp}}{\Omega_1} \right) J_{n'_2 - n_2} \left(\frac{\ell v_{2\perp}}{\Omega_2} \right) \right]$$

$$+ \left[J_{n_1 - n'_1} \left(\frac{\ell v_{1\perp}}{\Omega_1} \right) J_{n'_2 - n_2} \left(\frac{\ell v_{2\perp}}{\Omega_2} \right) \right]$$

$$\left. \cdot \left[\frac{n_1\Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} + \frac{n_2\Omega_2}{m_2 v_{2\perp}} \frac{\partial}{\partial v_{2\perp}} \right] \right\} \exp(i\underline{\ell} \cdot \underline{Q}_{12}) \ . \qquad (7)$$

The reduced distribution functions are easily defined for a multispecies plasma. In specifying the expected number of r-tuples of particles, one must be careful to distinguish between species. Thus factors like

$$\prod_{\sigma} (N_\sigma/N)^{r_\sigma}$$

will appear, where r_σ is the number of times σ appears in the set $\sigma_1, \ldots , \sigma_r$. These, however, cause no problem. One obtains, for example,

$$U_2(1,2) = (N_{\sigma_1}N_{\sigma_2} / \Lambda^2)^{-1} f_{2,2}(1,2) - f_{o,1}(1)f_{o,1}(2)$$

$$= \frac{2\pi}{\Lambda} \sum_{k(\neq 0)} \sum_{n_1 n_2} \mathcal{V}_{N-2} \, \rho^{1\ 2\ 1\ 2}_{k\ -k\ n_1\ n_2}(t) \qquad (8)$$

$$\cdot \exp(-in_1\theta_1 - in_2\theta_2) \exp(ikz_{12}) \ ,$$

since $f_{1,1}$ is independent of z_1. Here,

$$\mathcal{Y}_{N-r} \equiv \int_{r+1} \int_{r+2} \cdots \int_N$$

$$\equiv \int d^{N-r}\underline{Q}_j \; d^{N-r}v_{j\parallel} \; d^{N-r}v_{j\perp} [\prod_{j=r+1}^{N} 2\pi m_j^{3} v_{j\perp}] \;, \tag{9}$$

$$f_{o,1}(1) \equiv f_{o,1}(\underline{Q}_1, \underline{v}_1, \sigma_1, t) \;,$$

and

$$\rho^{1\ 2\ 1\ 2}_{k\ -k\ n_1\ n_2} \equiv (\Lambda\ /\ 2\pi)\ \Lambda^N\ \rho'^{1\ 2\ 1\ 2}_{k\ -k\ n_1\ n_2} \;.$$

Initial conditions are then specified as in Section IVa . 4, with the exception of the condition of gyrotropy (IVa.4.6). Its removal does not affect the general diagram perturbation theory to any great extent, provided one assumes as always that very large values of the n's are irrelevant in some sense. One should also remember that $\mathcal{Y}_{N-r}\, \rho_{\{k\}\{n\}}$ is irrelevant unless $n_j = 0$, $j > r$.

Concerning the λ-dependence of the initial distributions, one point may be worth noting. We assume that

$$\rho'_{\{o\}\{n\}}(0) \propto \lambda^{o} \rho'_{\{o\}\{o\}}(0) \;,$$

$$\rho'^{1\ 2}_{k\ -k\{n\}}(0) \propto \lambda^{1} \rho'_{\{o\}\{o\}}(0) \;, \text{ etc.} \tag{10}$$

These assumptions are slightly stronger than the analogous assumption (IVa. 5.1) for the gyrotropic case, because the equilibrium distributions give no guide as to the order of magnitude of the nongyrotropic components.

We write

$$f(1) \equiv f_{1,1}(1) = (N_1 / \Lambda) f_{o,1}(1). \tag{11}$$

The contribution to $f(1,t)$ from a cycle diagram is as follows:

$$\begin{aligned} f(1) = \ldots + \frac{N_1}{\Lambda} \sum_{n_1} \exp(-\, i n_1 \theta_1) \sum_{\sigma_2} \\ \cdot \sum_{n_1'} \sum_{n_2'} \mathcal{Y}_{N-1} \frac{i}{2\pi} \oint_C d\zeta \exp(-\, i\zeta t) \\ \cdot (-\lambda)^2 (n_1 \Omega_1 - \zeta)^{-1} \frac{2\pi}{\Lambda} \sum_{k(\neq 0)} \langle n_1 | 12 | \rangle \\ \cdot i(\omega_{12} + n_1 \Omega_1 - \zeta)^{-1} \langle | 21 | n_1' n_2' \rangle \\ \cdot i(n_1' \Omega_1 + n_2' \Omega_2 - \zeta)^{-1} \rho_{\{o\} n_1' n_2'}(0) , \end{aligned} \tag{12}$$

where

$$\begin{aligned} \omega_{12} \equiv \omega_1 - \omega_2 , \\ \omega_1 \equiv k\, v_{1\parallel} + \nu_1 \Omega_1 , \end{aligned} \tag{13}$$

and the matrix elements are operators similar to $\langle 0 | 12 | \rangle$ and $\langle | 21 | 0 \rangle$:

$$\langle n_1 \,|1\,2|\rangle = \sum_{\nu_1} \sum_{\nu_2} \frac{N_2}{i} \langle n_1^{(o)} = n_1 \ , \ \{o\}' \,|\delta L_{21}$$

$$|k_1^{(1)} = k \ , \ k_2^{(1)} = -k \ , \ n_1^{(1)} = n_1 + \nu_1 \ ,$$

$$n_2^{(1)} = -\nu_2 \ , \ \{0\}' \rangle$$ (with certain terms that vanish under $\int_2$ omitted)

$$= \sum_{\nu_1} \sum_{\nu_2} \int_0^\infty \ell' d\ell' \int_0^{2\pi} d\beta' \ \frac{e_1 e_2 N_2 \exp(i\nu_{21}\beta')}{i\pi\Lambda(k^2 + \ell'^2 + \kappa^2)}$$

$$\cdot \left\{ \left[\frac{k}{m_1} \frac{\partial}{\partial v_{1||}} + \frac{\nu_1 \Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} - \left(\frac{\underline{\ell}'}{m_1 \Omega_1} \times \frac{\partial}{\partial \underline{Q}_1} \right)_{||} \right] \right.$$

$$\cdot \left[J_{\nu_1}\left(\frac{\ell' v_{1\perp}}{\Omega_1}\right) J_{\nu_2}\left(\frac{\ell' v_{2\perp}}{\Omega_2}\right) \right] + \frac{\ell' n_1}{m_1 v_{1\perp}} J'_{\nu_1}\left(\frac{\ell' v_{1\perp}}{\Omega_1}\right)$$

$$\left. \cdot \ J_{\nu_2}\left(\frac{\ell' v_{2\perp}}{\Omega_2}\right) \right\} \exp(i\underline{\ell}' \cdot \underline{Q}_{21}) \ , \tag{14}$$

$$\langle |21|\, n_1' n_2' \rangle = \frac{\Lambda}{2\pi i} \langle k_1^{(1)} = k, \ \ k_2^{(1)} = -k, \ \ n_1^{(1)} = n_1 + \nu_1,$$

$$n_2^{(1)} = -\nu_2 \ , \ \{0\}' \ | \ \delta L_{12} \ | \ n_1^{(2)} = n_1' \ ,$$

$$n_2^{(2)} = n_2' \ , \ \{0\}' \rangle$$

$$= \int_0^\infty \ell'' d\ell'' \int_0^{2\pi} d\beta'' \ \ \frac{-e_1 e_2 \exp(i\nu_{12}\,\beta'')}{2i\pi^2(k^2 + \ell''^2 + \kappa^2)}$$

$$\cdot \exp(i[n_1 - n_1' - n_2']\beta'') \exp(i\underline{\ell}'' \cdot \underline{Q}_{12})$$

$$\cdot \left\{ \left[J_{\nu_1 + n_1 - n_1'}\left(\frac{\ell'' v_{1\perp}}{\Omega_1}\right) J_{\nu_2 + n_2'}\left(\frac{\ell'' v_{2\perp}}{\Omega_2}\right) \right] \right.$$

$$\cdot \left[\frac{k}{m_1}\frac{\partial}{\partial v_{1\|}} - \frac{k}{m_2}\frac{\partial}{\partial v_{2\|}} + \frac{(\nu_1 + n_1 - n_1')\Omega_1}{m_1 v_{1\perp}}\frac{\partial}{\partial v_{1\perp}} \right.$$

$$\left. - \frac{(\nu_2 + n_2')\Omega_2}{m_2 v_{2\perp}} \frac{\partial}{\partial v_{2\perp}} - \left(\underline{\ell}'' \times \left[\frac{1}{m_1\Omega_1}\frac{\partial}{\partial \underline{Q}_1} - \frac{1}{m_2\Omega_2}\frac{\partial}{\partial \underline{Q}_2}\right]\right)_{\|} \right]$$

$$- \frac{\ell'' n_1'}{m_1 v_{1\perp}} J'_{\nu_1 + n_1 - n_1'}\left(\frac{\ell'' v_{1\perp}}{\Omega_1}\right) J_{\nu_2 + n_2'}\left(\frac{\ell'' v_{2\perp}}{\Omega_2}\right)$$

$$\left. - \frac{\ell'' n_2'}{m_2 v_{2\perp}} J_{\nu_1 + n_1 - n_1'}\left(\frac{\ell'' v_{1\perp}}{\Omega_1}\right) J'_{\nu_2 + n_2'}\left(\frac{\ell'' v_{2\perp}}{\Omega_2}\right) \right\} . \tag{15}$$

It should be noted that in writing these expressions we did not use the fact that $n_3, n_4, \ldots = 0$; the relations $n_3^{(o)} = n_3^{(1)}$, etc. were sufficient.

In the asymptotic limit $t \gg (\kappa \bar{v})^{-1}$, $\omega_{p\sigma}^{-1}$, $(\kappa_{corr}\ \bar{v})^{-1}$, $|\Omega_\sigma^{-1}|$, it is seen that eq. (12) contains cyclotron resonances, which are strongly dependent on the ratios $\Omega_\sigma / \Omega_{\sigma'}$. We assume as before that large values of the n's are irrelevant, and make the following simplifying assumption, which is violated if the ratios are close to but different from ratios of small whole numbers:

$$t \gg \left| n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2 \right|^{-1} \quad \text{for all}$$

small values of the n's for which $n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2 \neq 0$. (16)

(In principle, the results are easily extended to cover situations in which assumption (16) is not satisfied.) Then we have the resonance condition that the terms of eq. (12) for which $n_1'\Omega_1 + n_2'\Omega_2 = n_1\Omega_1$ give the highest power of t in the asymptotic limit. Important special cases are:

(i) $\Omega_{\sigma_1} = \Omega_{\sigma_2}$, so that $n_1' + n_2' = n_1$,

(ii) $\Omega_{\sigma_1} / \Omega_{\sigma_2}$ irrational, so that $n_1' = n_1$, $n_2' = 0$,

(iii) $\Omega_{\sigma_1} / \Omega_{\sigma_2}$ ratio of small whole numbers, leading to cyclotron resonances.

Upon integration with respect to the variable $\zeta' = \zeta - n_1\Omega_1$, eq. (12) reduces to

$$\begin{aligned} f(1) &= [O(\lambda^o t^o) + O(\lambda^1 t^o) + \dots] \\ &+ [\, \lambda^2 t \left\{ \sum_{n_1} \sum_{\sigma_2} \sum_{n_1'} \sum_{n_2'} \delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2) \right. \\ &\cdot \exp(-\, i n_1 [\theta_1 + \Omega_1 t]) \, \mathcal{Y}_{N-1} \, \frac{N_1}{\Lambda} \int dk \\ &\cdot \left. \langle n_1 | 12 | \rangle \, \delta_-(\omega_{12}) \, \langle | 2\, 1 | \, n_1' n_2' \rangle \rho_{\{o\} n_1' n_2'}(0) \right\} + O(\lambda^3 t) + \dots] \\ &+ [O(\lambda^4 t^2) + O(\lambda^5 t^2) + \dots] + \dots , \end{aligned} \tag{17}$$

where in the ordering we disregard the factors t in the exponents. In the weak coupling approximation, we neglect $O(\lambda^1 t^o)$, $O(\lambda^3 t^1)$, $O(\lambda^5 t^2)$, etc., compared with $\lambda^o t^o$, $\lambda^2 t^1$, $\lambda^4 t^2$, ... respectively. Therefore

$$\frac{\partial f(1)}{\partial t} - \Omega_1 \frac{\partial f(1)}{\partial \theta_1} = \lambda^2 \sum_{n_1} \sum_{\sigma_2} \sum_{n_1'} \sum_{n_2'} \delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2)$$

$$\cdot \exp(- in_1[\theta_1 + \Omega_1 t]) \frac{N_1}{\Lambda} \int dk \int_2$$

$$\cdot \left\langle n_1 | 1\,2 | \right\rangle \delta_-(\omega_{12}) \left\langle | 2\,1 | n_1' n_2' \right\rangle$$

$$\cdot \int_o^{2\pi} \int_o^{2\pi} \frac{d\theta_1'}{2\pi} \frac{d\theta_2'}{2\pi} \exp(in_1'\theta_1') \exp(in_2'\theta_2')$$

$$\cdot \quad f_{o,2}(1,\, 2;\, \theta_1',\, \theta_2';\, 0) + O(\lambda^4 t) + \ldots \quad . \tag{18}$$

There is no problem in generalizing the analysis which shows that the expression $O(\lambda^4 t) + \ldots$ can be dropped if $f_{o,2}(1,\, 2;\, 0)$ is replaced by a product of single particle distributions evaluated at time t. In this case, the replacement is

$$\frac{\Lambda^2}{N_1 N_2} \; f(1,\, t) \exp(in_1'\Omega_1 t) f(2,t) \exp(in_2'\Omega_2 t) \; ,$$

so that the oscillating factors cancel. After setting $\lambda = 1$, we write the final result as follows:

$$f(1) = f(\underline{Q}_1, \underline{v}_1, \sigma_1, t) = \sum_{n_1=-\infty}^{\infty} \hat{f}(\underline{Q}_1, v_{1\|}, v_{1\perp}, n_1, \sigma_1, t)$$

$$\cdot \exp(-\, i n_1 \theta_1) \ , \tag{19}$$

$$\frac{\partial \hat{f}(1, n_1)}{\partial t} + i n_1 \Omega_1 \hat{f}(1, n_1) = \sum_{\sigma_2} \sum_{n_1'} \sum_{n_2'} \delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2) \frac{\Lambda}{N_2}$$

$$\cdot \int dk \int_2 \left\langle n_1 | 1\,2 | \right\rangle \delta_-(\omega_{1\,2}) \left\langle | 2\,1 | \, n_1' n_2' \right\rangle \hat{f}(1, n_1')\hat{f}(2, n_2') \ . \tag{20}$$

Written out in detail, eq. (20) takes the form

$$\frac{\partial \hat{f}(\underline{Q}_1, v_{1\|}, v_{1\perp}, n_1, \sigma_1, t)}{\partial t} + i n_1 \Omega_1 \hat{f}(\underline{Q}_1, v_{1\|}, v_{1\perp}, n_1, \sigma_1, t)$$

$$= \sum_{\sigma_2} \frac{e_1^2 e_2^2}{2\pi^2} \sum_{n_1'} \sum_{n_2'} \delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2)$$

$$\cdot \int dk_\| \, m_2^3 \int_\Sigma d\underline{Q}_2 \int_{-\infty}^{\infty} dv_{2\|} \int_0^{\infty} 2\pi v_{2\perp} dv_{2\perp} \sum_{\nu_1} \sum_{\nu_2}$$

$$\cdot \int_0^{\infty} \ell' d\ell' \int_0^{2\pi} d\beta' \, \frac{\exp(i\nu_{21}\beta')}{k_\|^2 + \ell'^2 + \kappa^2} \left\{ \left[\frac{k_\|}{m_1} \frac{\partial}{\partial v_{1\|}} \right. \right.$$

$$\left. + \frac{\nu_1 \Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} - \left(\frac{\underline{\ell}'}{m_1 \Omega_1} \times \frac{\partial}{\partial \underline{Q}_1} \right)_\| \right] J_{\nu_1}\left(\frac{\ell' v_{1\perp}}{\Omega_1} \right) J_{\nu_2}\left(\frac{\ell' v_{2\perp}}{\Omega_2} \right)$$

$$\left. + \frac{\ell' n_1}{m_1 v_{1\perp}} J'_{\nu_1}\left(\frac{\ell' v_{1\perp}}{\Omega_1} \right) J_{\nu_2}\left(\frac{\ell' v_{2\perp}}{\Omega_2} \right) \right\} \exp(i\underline{\ell}' \cdot \underline{Q}_{21})$$

$$\cdot\ \pi^{-1}\ \delta_{-}(k_{\|}[v_{1\|} - v_{2\|}] + \nu_1\Omega_1 - \nu_2\Omega_2) \int_0^\infty \ell'' d\ell'' \int_0^{2\pi} d\beta''$$

$$\cdot\ \frac{\exp(i\nu_{12}\beta'')}{k_{\|}^2 + \ell''^2 + \kappa^2} \exp(i[n_1 - n_1' - n_2']\beta'') \exp(i\underline{\ell}'' \cdot \underline{Q}_{12})$$

$$\cdot \left\{ J_{\nu_1 + n_1 - n_1'} \left(\frac{\ell'' v_{1\perp}}{\Omega_1}\right) J_{\nu_2 + n_2'} \left(\frac{\ell'' v_{2\perp}}{\Omega_2}\right) \left[\frac{k_{\|}}{m_1} \frac{\partial}{\partial v_{1\|}}\right.\right.$$

$$- \frac{k_{\|}}{m_2} \frac{\partial}{\partial v_{2\|}} + \frac{(\nu_1 + n_1 - n_1')\Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}}$$

$$- \frac{(\nu_2 + n_2')\Omega_2}{m_2 v_{2\perp}} \frac{\partial}{\partial v_{2\perp}} - \left(\underline{\ell}'' \times \left[\frac{1}{m_1\Omega_1} \frac{\partial}{\partial \underline{Q}_1}\right.\right.$$

$$\left.\left.\left. - \frac{1}{m_2\Omega_2} \frac{\partial}{\partial \underline{Q}_2}\right]\right)_{\|}\right] - \frac{\ell'' n_1'}{m_1 v_{1\perp}} J'_{\nu_1 + n_1 - n_1'} \left(\frac{\ell'' v_{1\perp}}{\Omega_1}\right)$$

$$\left. \cdot\ J_{\nu_2 + n_2'} \left(\frac{\ell'' v_{2\perp}}{\Omega_2}\right) - \frac{\ell'' n_2'}{m_2 v_{2\perp}} J_{\nu_1 + n_1 - n_1'} \left(\frac{\ell'' v_{1\perp}}{\Omega_1}\right) J'_{\nu_2 + n_2'} \left(\frac{\ell'' v_{2\perp}}{\Omega_2}\right) \right\}$$

$$\cdot\ \hat{f}(\underline{Q}_1, v_{1\|}, v_{1\perp}, n_1', \sigma_1, t)\ \hat{f}(\underline{Q}_2, v_{2\|}, v_{2\perp}, n_2', \sigma_2, t)\ , \tag{21}$$

where

$$\pi^{-1}\ \delta_{-}(\omega) \equiv \delta(\omega) - \frac{i}{\pi} \mathcal{P}\left(\frac{1}{\omega}\right)\ .$$

DISCUSSION

The kinetic equation (21), while not properly taking into account collective effects, nevertheless is a generalization of previous results, because it includes both transverse-diffusion terms and nongyrotropic terms. The appearance of the cyclotron-resonance factor is a new feature.

Our discussion here is limited to the comparison of special cases of eq. (21) with previously known kinetic equations[1-4], with all collective factors replaced by cutoff parameters. Discussions of the present state of the kinetic theory of transverse diffusion will be found in IVa and in papers referred to there.

The collision integral due to Eleonskiĭ, Zyryanov, and Silin[4] was mentioned in Section IVa. 8. Their result was for a gyrotropic plasma with an inhomogeneity in the $\hat{y}$ direction only ("slab geometry"). In writing the noncollective version of their result (17) in the form (IVa.8.1), we set their collective factor A_o^{-1} equal to $(k_x^2 + k_y^2 + k_z^2)^{-1}$. In fact, it should be set equal to $(k_x^2 + k_y^2 + k_z^2)^{-1}\ \delta(k_y - k_y')$, in which case (IVa.8.1) reduces to our result (IVa.8.2). For the multispecies plasma, we may obtain their result as a special case of our eqs. (19) to (21), putting $\kappa = 0$ and leaving the range of integration of the wave numbers indefinite:

$$\frac{\partial f(Q_{1y}, v_{1||}, v_{1\perp}, \sigma_1, t)}{\partial t}$$

$$= \sum_{\sigma_2} \frac{e_1^2 e_2^2}{\pi} \int dk_{||} \int d\ell_x \int dQ_{2y}\, m_2^3 \int_{-\infty}^{\infty} dv_{2||}$$

$$\cdot \int_0^\infty 2\pi v_{2\perp} dv_{2\perp} \sum_{\nu_1} \sum_{\nu_2} \left[\frac{k_\perp}{m_1} \frac{\partial}{\partial v_{1\|}} + \frac{\nu_1 \Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} \right.$$

$$\left. - \frac{\ell_x}{m_1 \Omega_1} \frac{\partial}{\partial Q_{1y}} \right] \delta(k_\| [v_{1\|} - v_{2\|}] + \nu_1 \Omega_1 - \nu_2 \Omega_2)$$

$$\cdot \left| \int d\ell_y \frac{\exp(i\nu_{21}\beta)}{k_\|^2 + \ell^2} J_{\nu_1}\left(\frac{\ell v_{1\perp}}{\Omega_1}\right) J_{\nu_2}\left(\frac{\ell v_{2\perp}}{\Omega_2}\right) \right.$$

$$\cdot \left. \exp(i\ell_y Q_{21y}) \right|^2 \left[\frac{k_\|}{m_1} \frac{\partial}{\partial v_{1\|}} - \frac{k_\|}{m_2} \frac{\partial}{\partial v_{2\|}} \right.$$

$$\left. + \frac{\nu_1 \Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} - \frac{\nu_2 \Omega_2}{m_2 v_{2\perp}} \frac{\partial}{\partial v_{2\perp}} - \frac{\ell_x}{m_1 \Omega_1} \frac{\partial}{\partial Q_{1y}} + \frac{\ell_x}{m_2 \Omega_2} \frac{\partial}{\partial Q_{2y}} \right]$$

$$\cdot f(Q_{1y}, v_{1\|}, v_{1\perp}, \sigma_1, t)\, f(Q_{2y}, v_{2\|}, v_{2\perp}, \sigma_2, t) \,, \tag{22}$$

where $\ell^2 = \ell_x{}^2 + \ell_y{}^2$, $\beta = \arctan(\ell_y/\ell_x) = \arg(\ell_x + i\ell_y)$. The derivation of (22) from (21) is apparent upon writing $\left| \int d\ell_y \ldots \right|^2$ in the form $\left\{ \int d\ell_y{}' \ldots \right\} \left\{ \int d\ell_y{}'' \ldots \right\}^*$. Since this factor is real, we were able to drop the Principal Value term from the δ_- function.

The assumption of slab geometry is expected to be a good first approximation in many situations for which the magnetic field may be taken to be uniform, and the plasma to be homogeneous in the $\hat{z}$ direction. The model is meant to describe a local region of a large plasma boundary layer; the boundary may be arbitrarily sharp. Therefore eq. (22) is of more than academic interest. The presence of the Dirac delta function is expected to simplify considerably the study of the equation, since such a delta function is essential for the usual proofs of total energy conservation, the H-theorem, etc. Much work remains to be done in this direction.

Let us now leave the diffusion problem, and study eq. (21) for the homogeneous case, in order to make contact with the results of Sundaresan[3]. When $\hat{f}(1, n_1)$ is independent of $\underline{Q}_1$, eq. (21) reduces to

$$\frac{\partial \hat{f}(1, n_1)}{\partial t} + i n_1 \Omega_1 \hat{f}(1, n_1)$$

$$= \sum_{\sigma_2} 2 e_1^2 e_2^2 \sum_{n_1'} \sum_{n_2'} \delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2)$$

$$\cdot \delta^{Kr}(n_1 - n_1' - n_2') \int dk_{||} \int 2\pi k_\perp dk_\perp \, k^{-4}$$

$$\cdot m_2^3 \int_{-\infty}^{\infty} dv_{2||} \int_0^{\infty} 2\pi v_{2\perp} dv_{2\perp} \sum_{\nu_1} \sum_{\nu_2}$$

$$\cdot \left[\frac{k_{||}}{m_1} \frac{\partial}{\partial v_{1||}} + \frac{\nu_1 \Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} + \frac{n_1 k_1}{m_1 v_{1\perp}} \frac{J'_{\nu_1}(k_\perp v_{1\perp}/\Omega_1)}{J_{\nu_1}(k_\perp v_{1\perp}/\Omega_1)} \right]$$

$$\cdot J_{\nu_1}\left(\frac{k_\perp v_{1\perp}}{\Omega_1}\right) J_{\nu_2}\left(\frac{k_\perp v_{2\perp}}{\Omega_2}\right) \pi^{-1} \delta_-(k_{||}[v_{1||} - v_{2||}] + \nu_1\Omega_1 - \nu_2\Omega_2)$$

$$\cdot J_{\nu_1 + n_1 - n_1'}\left(\frac{k_\perp v_{1\perp}}{\Omega_1}\right) J_{\nu_2 + n_2'}\left(\frac{k_\perp v_{2\perp}}{\Omega_2}\right) \left[\frac{k_{||}}{m_1} \frac{\partial}{\partial v_{1||}} \right.$$

$$+ \frac{(\nu_1 + n_1 - n_1')\Omega_1}{m_1 v_{1\perp}} \frac{\partial}{\partial v_{1\perp}} - \frac{n_1' k_\perp}{m_1 v_{1\perp}} \frac{J'_{\nu_1 + n_1 - n_1'}(k_\perp v_{1\perp}/\Omega_1)}{J_{\nu_1 + n_1 - n_1'}(k_\perp v_{1\perp}/\Omega_1)}$$

$$\left. - \frac{k_\parallel}{m_2} \frac{\partial}{\partial v_{2\parallel}} - \frac{(\nu_2 + n_2')\Omega_2}{m_2 v_{2\perp}} \frac{\partial}{\partial v_{2\perp}} - \frac{n_2' k_\perp}{m_2 v_{2\perp}} \frac{J'_{\nu_2 + n_2'}(k_\perp v_{2\perp}/\Omega_2)}{J_{\nu_2 + n_2'}(k_\perp v_{2\perp}/\Omega_2)} \right]$$

$$\cdot \hat{f}(1, n_1') \hat{f}(2, n_2') \quad , \tag{23}$$

if κ is set equal to zero. We have assumed that $t \gg |\Omega_\sigma|^{-1}$ and that $t \gg |\Omega_\sigma - \Omega_{\sigma'}|^{-1}$ when $\Omega_\sigma \neq \Omega_{\sigma'}$ (cf. eq. (16)). Equation (23) is to be compared with Sundaresan's result (38), (39), (30), (24), when his collective factor ρ is set equal to k^2 and $J_r^2(N_1)L_r$ is set equal to q_r/k^2 .

While the results are very similar, it is apparent that they disagree in a number of ways, most of which can be attributed to trivial errors. One source of disagreement seems to be somewhat more interesting than the others; namely, that our factor $\delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2)$ appears to be missing from Sundaresan's result. It is thought that this difference might represent a deviation of the Prigogine theory for asymptotically large times from the Bogoliubov theory with the adiabatic assumption $f_2(t) = f_2(\ldots \mid f_1(t))$, for the homogeneous nongyrotropic plasma.

At the time of writing, a clear understanding of this matter did not exist, but was imminent. The deviation apparently does not represent as important a difference between the two theories as might be expected at first sight. Slight changes in the assumptions and methods may completely reconcile the results.

Before discussing this further, we mention specifically one of the trivial errors; namely the absence of the other Kronecker delta function

$$\delta^{Kr}(n_1 - n_1' - n_2') \equiv \int_0^{2\pi} \frac{d\beta}{2\pi} \exp(in_1\beta)$$

$$\cdot \exp(-\, in_1'\beta) \exp(-\, in_2'\beta) \ .$$

The method by which the error can be corrected suggests a possible way of making the "ring approximation" tractable for the nongyrotropic plasma, and deriving by the Prigogine method an equation analogous to the Balescu-Lenard-Guernsey equation, for the homogeneous plasma and possibly also for the diffusing plasma. The direction of attack is especially transparent for the single species homogeneous plasma, and the result should agree with a corrected version of Sundaresan's result. Of course, one would first have to justify the use of the "ring approximation", by some simple modification of the weak coupling approximation. If the assumption $t \gg |\Omega|^{-1}$ is not made, the justification may be more difficult than for models previously treated.

The factors $\{\exp(in\beta)\}$ are absent from Sundaresan's results because he chose a coordinate system for which $\beta = 0$. This procedure breaks down when $\underline{k}$ is not a fixed vector. One is not always working under the integral sign $\int d\underline{k}$, because it is necessary to make a Fourier transformation of the distribution function with respect to θ.

The correction would consist of multiplying each single particle distribution by a factor of the form $\exp(in\beta)$, and including the β integral. For the collision integral of the Landau type, the result is an extra factor $\delta^{Kr}(n_1 - n_1{}' - n_2{}')$; for Sundaresan's equation, it would be more complicated.

The difficulty in summing the ring diagrams for the nongyrotropic plasma stems from a factor $\delta^{Kr}(n_1\Omega_1 - n_1{}'\Omega_1 - n_2{}'\Omega_2 - n_3{}'\Omega_3 - \ldots - n'_{P+1}\Omega_{P+1})$ analogous to $\delta^{Kr}(n_1\Omega_1 - n_1{}'\Omega_1 - n_2{}'\Omega_2)$. The integer n'_{P+1} appearing in $\langle\, |P{+}1\ R|\ n'_R\, n'_{P+1} \rangle$ depends on $n_1, n_1{}', \ldots, n'_R, \ldots, n'_P$, so that recurrence relations of the usual type are difficult to obtain. But the treatment of $\delta^{Kr}(n_1 - n_1{}' - n_2{}')$ is very suggestive. One might write

$$\delta^{Kr}(n_1\Omega_1 - n_1{}'\Omega_1 - \ldots - n'_{P+1}\Omega_{P+1})$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \frac{d\tau}{T}\ \exp(-\,in_1\Omega_1\tau)\ \exp(in'_1\Omega_1\tau)$$

$$\cdots\ \exp(in'_{P+1}\Omega_{P+1}\,\tau)\ ,$$

and place the factors $\{\exp(-\,in\Omega_\sigma\tau)\}$ next to the factors $\{\exp(in\beta)\}$, effectively replacing β by $\beta-\Omega_\sigma\tau$. The integration over τ (and β) would then be postponed until after the diagrams are summed. One would not, however, get back the simple Kronecker delta functions; instead, the collective factor would be more complicated than for the gyrotropic plasma.

For the single species homogeneous plasma, the introduction of τ is unnecessary. The factors $\{\exp(in\beta)\}$ are sufficient.

We now make some provisional remarks concerning our factor $\delta^{Kr}(n_1\Omega_1 - n_1'\Omega_1 - n_2'\Omega_2)$ in eq. (23), and its absence in the non-collective analogue of Sundaresan's equation. At the time of writing, the origin of the discrepancy remained in some doubt, and will therefore not be discussed in detail here.

For a detailed examination of this point, it is evident that one must carefully check the τ integrations in Sundaresan's work, and his treatment of the oscillating time-dependent factors. We note that his equation (24) contains $\delta_+(r;s)$, instead of the factor $\delta_+(r-n; s\mp m)$ which would be expected from our result. Also, the factors analogous to our factors $\exp(-in_1\Omega_1 t)\exp(in_1'\Omega_1 t)\exp(in_2'\Omega_2 t)$ (cf. eq. (18) ff.) do not appear in his final results.

It is possible even in the Prigogine theory to base the calculation on the first equation of the BBGKY hierarchy. One could retain the first order contributions to the two particle distribution which depend on t through $f(\underline{v}, t)$, possibly also including oscillating factors when ring diagrams (collective terms) are taken into account. Contributions dependent on initial correlations would be neglected. In such a theory, it is somewhat difficult to see how the cyclotron resonance condition could appear. One can readily understand the restriction $n_1' + n_2' = n_1$, which occurs because the $\underline{k}$ integration results in cancellations due to invariances with respect to rotations of the x-y coordinates. It would appear at first sight that no other dependence relation between n_1', n_2' and n_1 should exist, because the selection of variables n_1', n_2' for the contribution of $\hat{f}(1, n_1')\hat{f}(2, n_2')$ to $\hat{f}_2(1, 2; \nu_1 + n_1, -\nu_2)$ should depend on $\nu_1 + n_1$ rather than n_1 alone.

The discrepancy disappears if it is remembered the functions $\hat{f}$ oscillate rapidly in time. The slowly varying quantity is $\exp(in_1\Omega_1 t)\ \hat{f}(1,n_1)$. If we multiply eq. (23) by the factor $\exp(in_1\Omega_1 t)$, we see that the dominant contribution to $\partial(\exp(in_1\Omega_1 t)\hat{f}(1,t))/\partial t$ occurs when $\exp(in_1\Omega_1 t) = \exp(in_1'\Omega_1 t)\ \exp(in_2'\Omega_2 t)$, so that the right side will not contain rapidly oscillating factors which tend to average to zero. The precise justification rests on the above calculation; it can also be made plausible from an argument based on some "coarse-graining" of the time variable.

It is therefore likely that the noncollective version of the two theories are essentially in agreement when our simplifying assumption (16) is made. This may no longer be true when collective factors are included, but a detailed study has not yet been completed.

Combining the two Kronecker delta functions in eq. (23), we obtain the restriction

$$n_2'(\Omega_2 - \Omega_1) = 0 .$$

Thus for a homogeneous nongyrotropic plasma relaxing with a time scale $t >> |\Omega_\sigma|^{-1},\ |\Omega_\sigma'|^{-1},\ |\Omega_\sigma - \Omega_\sigma'|^{-1}$, the Landau collision integral predicted by the Prigogine theory contains a "selection rule" which eliminates all interactions between the nongyrotropic parts of the velocity distributions of species σ and σ' with different gyrofrequencies. Such interactions are, however, expected to be present when collective factors are included.

ACKNOWLEDGMENTS

My interest in the Landau collision integral for nongyrotropic plasmas was stimulated by the unpublished results of Dr. P. Schram.

I wish to thank Professor D.A. Tidman for his interest and support, and Dr. R. Goldman for helpful discussions.

The research was sponsored by the National Aeronautics and Space Administration under Grant NsG 695.

REFERENCES

1. M.J. Haggerty, On the Kinetic Theory of Diffusion of a Plasma Column across a Magnetic Field, Research Report No. PIBMRI - 1289 - 65, Polytechnic Institute of Brooklyn, 10 August 1965 (unpublished), referred to as "IVa". (See also Bull. Am. Phys. Soc. 11, 123(1966).)

2. R. Balescu, Statistical Mechanics of Charged Particles (Interscience, New York, 1963).

3. M.K. Sundaresan, Can. J. Phys. 44, 247(1966).

4. V.M. Eleonskiĭ, P.S. Zyryanov, and V.P. Silin, Zh. Eksperim. i Teor. Fiz. 42, 896(1962); English transl.: Soviet Phys. JETP 15, 619(1962).