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ON THE ORIGIN OF STRUCTURE IN THE UNIVERSE*

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SUMMARY

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Cosmology is confronted with the problem of explaining how large-scale structures originated in the universe. Within the framework of conventional theory two hypotheses are possible. In the primordial structure hypothesis structural differentiation of a rudimentary form is inlaid within the universe from its earliest moments, whereas in the instability hypothesis structure evolves naturally from small initial disturbances. It is known that according to linearized gravitational theory, small disturbances in an expanding universe grow extremely slowly. This theory is outlined and furthermore it is shown that large structures are unlikely to form as the result of thermal instabilities. Thus, the instability hypothesis is in serious difficulty and it is proposed that we should re-examine the primordial structure hypothesis. One possibility is that differentiation into structural domains is a natural state of matter at very high densities during the earliest stage of expansion of the universe.

Carter

1. DISCUSSION

The main emphasis in cosmology is on models of an idealized universe containing a uniform and perfect fluid. Confronting cosmology, however, is the task of bridging the gap between the featureless models and the physical universe with its structural differentiations. It is this aspect of cosmology that we shall discuss.

The astrophysicist studies, among other things, the evolution of galactic structure and the condensation of stars out of gas clouds of irregular density and motion. He investigates configurations of matter whose mean density is large in comparison with the mean density of the universe, and his investigations frequently begin where those of the cosmologist end. The onus is on the cosmologist to present a convincing account of why in the first place a differentiated universe exists, and how it is possible that there are regions of relatively large density favorable to the formation of galaxies and stars.

A rational account of the origin of celestial structure involves the laws of physics and the initial conditions. At present there are two outstanding hypotheses concerning the initial conditions: the <u>primordial structure hypothesis</u> and the <u>instability hypothesis</u>.

The primordial structure hypothesis presupposes that structural differentiation, most likely in a rudimentary form, originates with the universe. This hypothesis is as old as mythology and conceives that an inherent structural design is an indispensable part of the universe. In the course of time the differentiation is enhanced and the structure develops complex detail in accordance with the known laws of physics. We shall show that it now appears necessary to reconsider this hypothesis in the light of modern knowledge.

At the other extreme, the instability hypothesis dismisses the idea of special initial conditions and declares that the laws of physics are fully capable of explaining the origin of structure. This hypothesis, which is as young as the theory of gravitation, states that the universe is unstable against small random perturbations. It possesses several attractive features: the smallness of the disturbances allows us to work initially with a linear theory; also it refrains from pleading for special ad hoc boundary conditions and thus conforms with the scientific spirit of stressing the importance of physical processes.

Two requirements must be satisfied before the instability hypothesis can be fully accepted. The first requirement is that structure must emerge out of amorphous initial conditions and possess the correct morphology. Clearly, in a normal mode analysis all possible wavelengths must not grow at the same rate. On the other hand we do not want a universe containing objects the size of tennis balls nor a universe broken into only a few large fragments. We do not observe any pronounced macroscopic anistropy and must therefore deduce that wavelengths of cosmic dimensions are relatively quiescent. Thus only a limited range of time-growing wavelengths is required to lay down the foundations of galactic and stellar structure. The time dependence of the various wavelengths must also tell us whether structure develops by fragmentation or clustering, or by a combination of both processes. If the longer wavelengths race ahead and lead the field then protogalaxies or even larger distributions of matter first form and provide an environment of enhanced density in which subsequent fragmentation can occur[1]. But if the short wavelengths take the lead, then small scale condensations first form and by subsequent interactions cluster together into larger and larger systems[2].

The second requirement is that the rate of growth of the perturbations must be adequate. If ρ is the density and $\delta\rho$ the perturbation in density, then the characteristic growth time of the contrast density $\mu = \delta\rho/\rho$ must be short compared with the age t of the universe. In other words

$$\frac{t}{\mu} \frac{d\mu}{dt} \gg 1. \qquad (1)$$

Let us suppose that by some means or other we obtain an equation of the kind

$$\mu = \mu_o (t/t_o)^m, \tag{2}$$

for a given wavelength, where μ_o is the initial amplitude of the fluctuation at time t_o. Then according to equation (1) we require $m \gg 1$. Now the instability hypothesis must explain how structure is created and therefore it cannot assume that the required structure pre-exists in the initial conditions at reduced amplitude. If this is the case then we are back to the primordial structure hypothesis and equation (2) accounts for no more than mere enhancement. The initial conditions are therefore structureless and consist only of random fluctuations. Statistical fluctuations of large numbers of particles are exceedingly small; for N particles

$$\mu_o \sim N^{-1/2}, \tag{3}$$

and for a galactic mass consisting of hydrogen $\mu_o \sim 10^{-35}$. The density of galaxies is several orders of magnitude greater than the mean density of the universe and therefore eventually $\mu \gg 1$. Whatever reasonable value is assumed for t/t_o in equation (2) it is obvious that m is an outlandish number. As an example, if $t/t_o = 10^{10}$ then $m > 25$. More modest values of m will mean that there is enhancement but not outright instability. For a clear case of instability we require exponential growth:

$$\mu = \mu_0 \exp(t/\tau), \tag{4}$$

and for a galactic mass the e-folding time is $\tau < t/100$ and can be as long as several millions of years.

We turn now without further ado to a more detailed examination of the instability hypothesis. We shall show that gravitational instability fails both the requirements that have been mentioned above. Without specifying any particular physical mechanism it is found that extreme thermal instability yields a barely adequate growth rate for the formation of galactic masses. This result is interesting but can scarcely be accepted until a physical basis is found for the occurrence of such processes over large ranges of density. Finally, we consider very briefly the primordial structure hypothesis and suggest that it should be updated and reformulated into a more acceptable proposition.

2. GRAVITATIONAL INSTABILITY

In its unperturbed state we assume that the universe is isotropic and homogeneous and use the line element

$$ds^2 = dt^2 - \frac{R^2}{c^2(1+\frac{1}{4}\kappa r^2)^2}(dr^2 + r^2 d\Omega^2), \quad (5)$$

$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, where r, θ, ϕ are comoving spherical coordinates and $\kappa = 0, \pm 1$ is the curvature constant; also, that the cosmic fluid density ρ is uniform and the pressure p is isotropic. If variations in pressure and density are related by the expression

$$\delta p = (\nu-1)c^2\delta\rho, \quad (6)$$

then for constant ν

$$p = (\nu-1)c^2\rho, \quad (7)$$

and $1 \le \nu \le \frac{4}{3}$. The energy-momentum tensor is

$$T^i_j = \nu\rho c^2 g_{kj} u^i u^k - \delta^i_j(\nu-1)c^2\rho, \quad (8)$$

where u^i is the fluid four-velocity, and in comoving coordinates

$$T^o_o = \rho c^2,\ T^1_1 = T^2_2 = T^3_3 = -(\nu-1)c^2\rho. \quad (9)$$

Using Einstein's equation

$$R^i_j - \tfrac{1}{2}\delta^i_j R^m_m + \delta^i_j \Lambda = \frac{8\pi G}{c^2} T^i_j, \tag{10}$$

where Λ is the cosmological term, we obtain the equations

$$\dot{R}^2 = \tfrac{1}{3}(8\pi G\rho + \Lambda)R^2 - \varkappa c^2, \tag{11}$$

$$2R\ddot{R} = \nu\Lambda R^2 - (3\nu-2)(\dot{R}^2 + \varkappa c^2), \tag{12}$$

and dots denote time derivatives. Let

$$\frac{dt}{d\chi} = \frac{2}{3\nu-2}\ \frac{R}{c}, \tag{13}$$

$$\beta_\nu = 8\pi G\rho R^{3\nu}/3c^2, \tag{14}$$

where β_ν is constant, and it follows that

$$\begin{aligned} \varkappa = 0: &\quad R = (\beta_\nu\, \chi^2)^{1/(3\nu-2)}, \\ \varkappa = +1: &\quad R = (\beta_\nu \sin^2 \chi)^{1/(3\nu-2)}, \\ \varkappa = -1: &\quad R = (\beta_\nu \sinh^2 \chi)^{1/(3\nu-2)}. \end{aligned} \tag{15}$$

We now consider perturbations in the cosmic fluid that are accompanied by small departures from the Robertson-Walker line element (5). The metric tensor g_{jk} becomes $g_{jk} + h_{jk}$, where h_{jk} and its derivatives are assumed to be small. It can be shown[3] that to a first order the contracted Riemann Christoffel tensor is given by*

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* A more detailed treatment is given in reference 3. The present paper supplements and extends the discussion on the origin of structure found in this reference.

$$g^{ik}(h^m_{m;jk} - h^m_{k;jm} - h^m_{j;km}) + g^{mn}h^i_{j;mn} = 2(\delta R^i_j + h^i_k R^k_j), \quad (16)$$

in which a semicolon denotes covariant differentiation. This equation is similar to that derived by Lifshitz[4]. From equation (10)

$$R^i_j = -(8\pi G/c^2)(T^i_j - \tfrac{1}{2}\delta^i_j T) + \delta^i_j \Lambda, \quad (17)$$

$$\delta R^i_j = -(8\pi G/c^2)\delta(T^i_j - \tfrac{1}{2}\delta^i_j T), \quad (18)$$

and therefore all that remains is to determine δT^i_j. It can be readily shown that

$$\delta T = (4-3\nu)c^2\delta\rho,$$

$$\delta T^o_o = c^2\delta\rho,$$

$$\delta T^o_\alpha = \nu c^2\rho(h^o_\alpha + g_{\alpha\alpha}\delta u^\alpha),$$

$$\delta T^{\alpha}_{\alpha} = -\ (\nu-1)c^2\delta\rho,$$
$$\delta T^{\alpha}_{\beta} = 0, \tag{19}$$

where $\alpha,\beta = 1, 2, 3$, $\alpha \neq \beta$, and there is no summation. Equation (16) is a set of ten equations for determining the ten quantities: $\delta\rho$; six of the h^i_j (since four can be eliminated by coordinate transformations); and δu^{α} ($\delta u^0 = -\frac{1}{2}h^0_0$).

At this stage it is helpful to consider the analogous case of the Newtonian equations of hydrodynamics. The perturbed fluid motion is governed by the gradients of the pressure and gravitational potential ψ, and hence the velocity is conserved. But the vorticity is zero prior to the perturbation and is therefore permanently zero, and the motion is irrational. Thus in the Newtonian treatment the equations of motion and continuity and Poission's equation are a set of three equations for the determination of $\delta\rho$, ψ, and φ, where φ is the velocity potential.

If we adopt $h_{o\alpha} = 0$, and demand that the motion is irrotational:

$$g_{\alpha\alpha}\delta u^{\alpha} = \frac{1}{c^2}\ \frac{\partial\varphi}{\partial x^{\alpha}},$$

it is found that h^{α}_{β} and $h^{\alpha}_{\alpha} - h^{\beta}_{\beta}$ are propagated independently of the fluid perturbation. These quantities are zero in the unperturbed state and therefore it can be assumed that they are permanently zero without affecting the fluid disturbance. Hence, all diagonal components of h^i_j are zero, and we have

$$h^o_o = -h^1_1 = -h^2_2 = -h^3_3. \tag{20}$$

If h^o_o is written as $2\psi/c^2$, the perturbed line element has the simple form

$$ds^2 = (1 + 2\psi c^{-2})dt^2 - \frac{R^2}{c^2} \frac{(1-2\psi c^{-2})}{(1+\frac{1}{4}\varkappa r^2)^2} (dr^2+r^2 d\Omega^2). \tag{21}$$

Equation (16) now reduces to the three equations

$$4\pi G c^2 \delta\rho = -3\dot{R}R^{-1}\dot{\psi} + (c^2\nabla^2 - 2\Lambda R^2 - 3\dot{R}^2 + 3\varkappa c^2)R^{-2}\psi, \tag{22}$$

$$4\pi G \delta p = \ddot{\psi} + 4\dot{R}R^{-1}\dot{\psi} + (2\ddot{R}R + \dot{R}^2 - \varkappa c^2)R^{-2}\psi, \tag{23}$$

$$\frac{1}{R} \frac{d}{dt}(R\psi) = 4\pi G(\rho + p/c^2)\varphi, \tag{24}$$

for determining $\delta\rho$, ψ and φ. In the important case when the pressure is small compared with the energy density these equations become

$$\left.\begin{aligned} &\ddot{\psi} + 4\dot{R}R^{-1}\dot{\psi} - (c_s^2\nabla^2 - \Lambda R^2 + 2\varkappa c^2)R^{-2}\psi = 0 \\ &4\pi G R^2 \delta\rho = \nabla^2\psi, \\ &\frac{1}{R}\frac{d}{dt}(R\psi) = 4\pi G\rho\varphi. \end{aligned}\right\} \tag{25}$$

where $c_s^2 = dp/d\rho$, and c_s is the speed of sound. They are identical with those derived by the Newtonian treatment[5]. For arbitrary ν in equations (6), (22)-(23) give

$$\ddot{\psi} + (1+3\nu)\frac{\dot{R}}{R}\dot{\psi} - \frac{1}{R^2}[(\nu-1)c^2\nabla^2-(3\nu-2)\Lambda R^2+(6\nu-4)\varkappa c^2]\psi=0 \quad (26)$$

In the above equations ∇^2 is the Laplacian in space of curvature $\varkappa = 0, \pm 1$. By separating the variables and using

$$\nabla^2\psi + k^2\psi = 0, \quad (27)$$

it can be shown that the eigenvalues are:

$$\begin{array}{lll} \varkappa = 0: & k^2 = \gamma^2, & \gamma^2 \geq 0, \\ \varkappa = +1: & k^2 = \gamma(\gamma+2), & \gamma = 1,2,3\ldots \quad (28) \\ \varkappa = -1: & k^2 = \gamma^2+1, & \gamma^2 \geq 0, \end{array}$$

We first consider Einstein's static universe of $\ddot{R} = \dot{R} = 0$, $\varkappa = +1$, and therefore

$$\Lambda = c^2(3\nu-2)/\nu R^2, \quad (29)$$

from equation (12). Equation (26) is now

$$\ddot{\psi} + \frac{c^2}{R^2}[\gamma(\gamma+2)(\nu-1) + (3\nu-2)(\nu-2)\nu^{-1}]\psi = 0, \quad (30)$$

and therefore

$$\psi \propto \delta\rho \propto \exp \pm \frac{c}{R}[(2-\nu)(3\nu-2)\nu^{-1} - \gamma(\gamma+2)(\nu-1)]^{1/2}t. \quad (31)$$

It is well known that the Einstein model is unstable against perturbations in R. When R + δR is used in equation (12), where R is the equilibrium Einstein value, it is found that to first order

$$\delta\ddot{R} = (3\nu-2)cR^{-1}\delta R,$$

and therefore

$$\delta R \propto \exp \pm \frac{c}{R}(3\nu-2)^{1/2}t. \quad (32)$$

An advantage of the Lemaître model, so it is argued, is that it possesses an extended or infinite past in a quiescent Einstein state during which disturbances can grow exponentially according to equation (31). But for $1 < \nu \leq \frac{4}{3}$ it is seen from equation (32) that δR grows more rapidly than any of the modes $\gamma = 1,2,3\ldots$ The rates of growth are equal in a cold universe of $\nu = 1$, in which the velocity of sound is zero, and $\delta\rho \propto \delta R \propto \exp ct/R$. Thus we see that the departure from the global equilibrium state in such a universe is just as likely, or even more likely, than the formation of condensations. The growth of condensations must therefore be considered within the framework of a nonstatic universe.

In the following we suppose that Λ is zero. By using the transformations (13)-(15), equation (26) becomes, for $\varkappa = 0$:

$$\alpha_0'' + (3\nu-2)^{-2}[4(\nu-1)k^2 - \frac{6\nu}{\chi^2}]\alpha_0 = 0, \qquad (33)$$

$$\alpha_0 = \psi\chi^{3\nu/(3\nu-2)},$$

for $\varkappa = +1$:

$$\alpha_1'' + (3\nu-2)^{-2}[4(\nu-1)k^2 + (4-3\nu)^2 - \frac{6\nu}{\sin^2\chi}]\alpha_1 = 0, \qquad (34)$$

$$\alpha_1 = \psi(\sin\chi)^{3\nu/(3\nu-2)},$$

and for $\varkappa = -1$:

$$\alpha_{-1}'' + (3\nu-2)^{-2}[4(\nu-1)k^2 - (4-3\nu)^2 - \frac{6\nu}{\sin^2\chi}]\alpha_{-1} = 0, \qquad (35)$$

$$\alpha_{-1} = \psi(\sinh\chi)^{3\nu/(3\nu-2)},$$

where dashes denote derivatives with respect to χ. For any value of k^2 and χ, and $1 \le \nu \le \frac{4}{3}$, it is easy to see that $\alpha_\varkappa''/\alpha_\varkappa$ have maximum positive values at $\nu = 1$. In other words, the growth of the $\alpha_\varkappa$ is maximum in a cold universe of zero pressure. This is of course what one would expect and occurs when the fluid consists of dust particles or other bodies having no peculiar motion of their own.

The growth of disturbances in a cold universe is of great interest. Should it turn out that even in this extremely favorable case the growth is too slow to establish a differentiated medium, then the instability hypothesis is in serious difficulty. For $\nu = 1$, equations (33)-(35) become

$$\varkappa = 0: \qquad \psi'' + 6\chi^{-1}\psi' = 0, \tag{36}$$

$$\varkappa = +1: \qquad \psi'' + 6\cot\chi\,\psi' - 8\psi = 0, \tag{37}$$

$$\varkappa = -1: \qquad \psi'' + 6\coth\chi\,\psi' + 8\psi = 0, \tag{38}$$

and their solutions are

$$\varkappa = 0: \qquad \psi = A_0\chi^{-5} + B_0, \tag{39}$$

$$\varkappa = +1: \qquad \psi = (\sin\chi)^{-3}[A_1P_2^1(i\cot\chi)+B_1Q_2^1(i\cot\chi)], \tag{40}$$

$$\varkappa = -1: \qquad \psi = (\sinh\chi)^{-3}[A_{-1}P_2^1(\coth\chi)+B_{-1}Q_2^1(\coth\chi)], \tag{41}$$

where $A_\varkappa$ and $B_\varkappa$ are constants. These results give the maximum growth possible for ψ in expanding and contracting models of the universe, and the contrast density $\delta\rho/\rho \propto \psi R$ is shown in

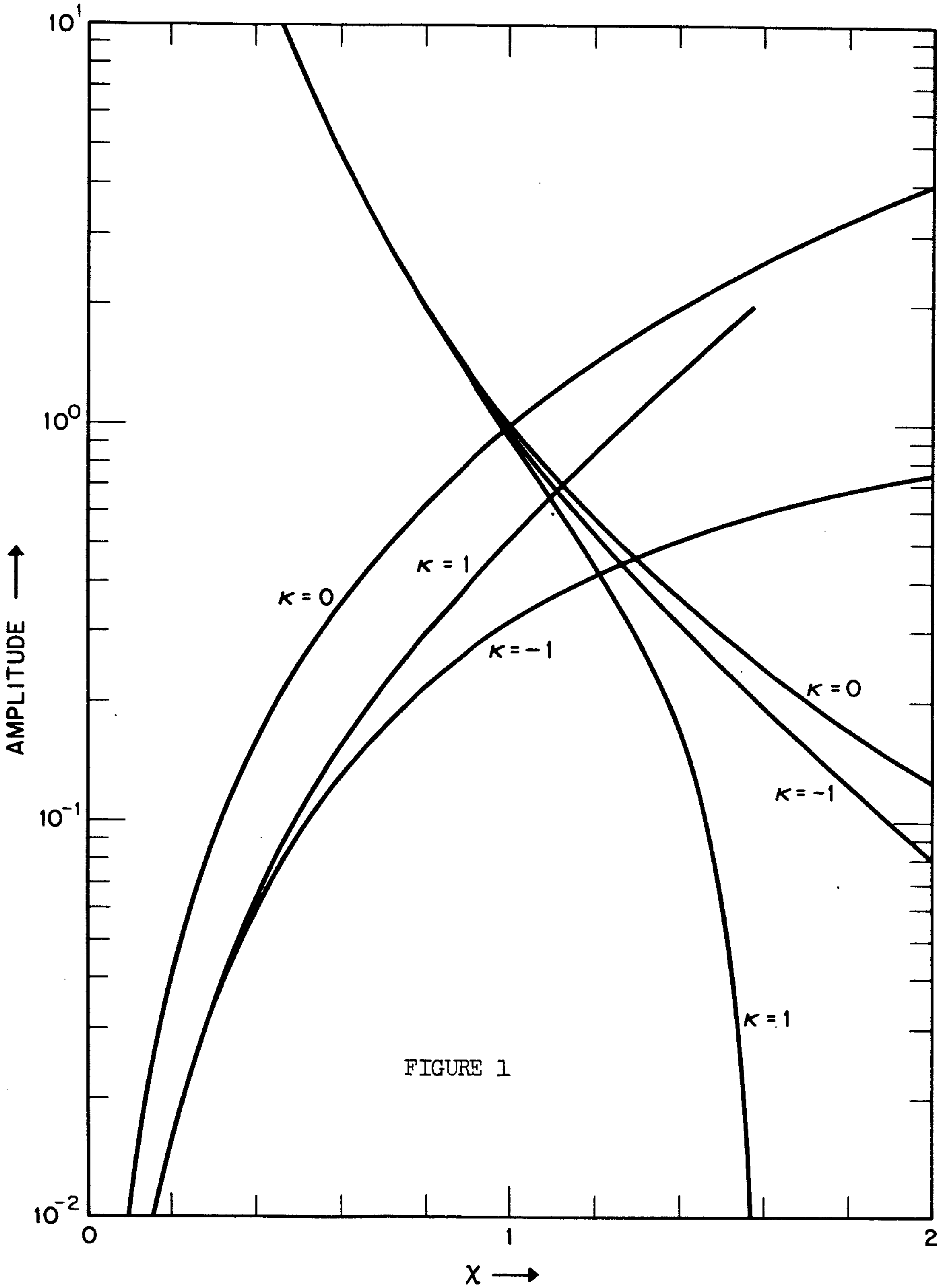

FIGURE 1

figure 1 as a function of χ.

The $\kappa = 0$ model is the simplest of all to study and possesses features representative of all three models. In this model

$$\delta\rho/\rho = AR^{-3/2} + BR, \qquad (42)$$

and therefore $\delta\rho/\rho^{2/3}$ is constant or diminishes in an expanding universe, and $\delta\rho/\rho^{3/2}$ is constant or diminishes in a contracting universe. Neglecting the decaying term, we have that in an expanding universe

$$\frac{\delta\rho}{\rho} \propto \frac{1}{\rho^{1/3}} \propto t^{2/3}. \qquad (43)$$

Strictly speaking, the condition $\nu = 1$ for the maximum rate of growth of the gravitational potential ψ is not necessarily also the condition for the maximum rate of growth of the contrast density. The maximum possible rate of growth of the contrast density for arbitrary ν is only slightly different, however, from equations (39)-(41).

3. THE INSTABILITY HYPOTHESIS

We now consider whether these results satisfy the requirements that were discussed earlier. The absence of exponential growth is typical of all nonstatic models. A comparison of equations (2) and (43) shows that $m = \frac{2}{3}$ falls a long way short of the large quantity desired. Our conclusion is that an expanding universe does not possess any pronounced instability. This conclusion is reinforced by a consideration of Jeans' theory of gravitational instability in which the unperturbed state is assumed to be static. According to this theory the maximum possible rate of growth gives

$$\frac{\delta\rho}{\rho} \propto \exp\frac{t}{\tau} \tag{44}$$

where $\tau = (4\pi G\rho)^{-1/2}$. But τ is the order of the age of the universe and therefore Jeans' theory cannot satisfy the inequality (1). In an expanding universe the rate of growth is even slower than that given by Jeans' theory.

It is required that during expansion the cosmic fluid fragments into islands. The gravitational potential of a disturbance must therefore increase with time and attain a value of $\psi \sim GM/\lambda$, where M and λ are the mass and radius of an island. The inadequacy of the gravitational theory to explain the origin of structure is shown clearly by equation (39) where it is seen that even in a cold expanding universe the gravitational potential of a disturbance cannot increase. Equations (40)-(41) for $\varkappa = \pm 1$ give essentially the same results.

For the sake of achieving maximum growth we have assumed a cold universe of zero pressure, and as a result all wavelengths have equal growth rates. The inclusion of pressure slows down or inhibits the growth of shorter wavelengths and there is no apparent mechanism whereby a limited range of wavelengths receive preferential treatment. If the growth were larger the results would be catastrophic and the universe would be violently unstable on the cosmic scale. We come therefore to the conclusion, previously arrived at in many different ways by various authors, that gravitational instability fails because perturbations grow too slowly and lack structural content.

At first glance our presentation of the instability hypothesis appears to contain several loopholes. The motion and properties of the cosmic fluid have been simplified and are obviously not very realistic. It seems plausible, however, that the inclusion of rotational effects will inhibit even further the formation of condensations. The neglect of rotational motions is a serious omission that must be corrected the moment a mechanism for adequate growth has been discovered. An initial state of turbulence[6] possesses many attractive features but forces us inevitably into the jaws of the primordial structure hypothesis. In addition we have considered a cosmic fluid of only rudimentary properties. Here again it seems unlikely that departures from a perfect fluid — in which the pressure is a scalar — will favor an increased growth. In fact, the growth of perturbations in a fluid which has a real velocity of sound (i.e., $dp/d\rho > 0$) must always be less than the expressions we have derived for a pressureless fluid.

Let us suppose that the instability is thermal in origin and not gravitational, and as a result ψ has the desired rate of growth of

$$\frac{\dot{\psi}}{\psi} \gg \frac{\dot{R}}{R} . \tag{45}$$

Equation (26) is then approximately

$$\ddot{\psi} + \frac{dp}{d\rho} \; \frac{k^2}{R^2} \, \psi = 0, \tag{46}$$

and the cosmological and curvature terms are omitted because they are unimportant and cannot effect the condition (45). Furthermore, we assume that the expansion index n, defined by

$$\frac{t}{R} \frac{dR}{dt} = n, \tag{47}$$

is constant in an interval of time t_1 to t. In general, $n \leq 1$, and in the early stages of the universe when the curvature term is unimportant, $n = 2/3\nu$. The solution of (46) is

$$\psi \propto t^{\frac{1}{2}} I_{\pm \frac{1}{2-2n}} \left((-dp/d\rho)^{\frac{1}{2}} \frac{k}{b} \frac{t}{1-n} \right),$$

in terms of the Bessel functions of imaginary argument and $R = bt^n$. Condition (45) is satisfied when the argument is large, and hence

$$\psi = \psi_o \ \exp \pm \left\{ \left(-\frac{dp}{d\rho} \right)^{\frac{1}{2}} \frac{t}{\lambda(1-n)} \left(1 - \frac{t_1^{1-n}}{t^{1-n}} \right) \right\},$$

where $\psi = \psi_o$ at $t = t_1$, and also $R/k = \lambda$ is the wavelength (divided by 2π). If $t = t_1 + \Delta t$, then to a first order

$$\psi = \psi_o \ \exp \pm \left\{ \left(-\frac{dp}{d\rho} \right)^{\frac{1}{2}} \frac{\Delta t}{\lambda} \right\}. \tag{48}$$

The age of the universe is $t \simeq (3/8\pi G\rho)^{\frac{1}{2}}$, and for a mass $M \simeq 4\pi\rho\lambda^3/3$ we have

$$\begin{aligned} \psi &= \psi_o \exp \pm x, \\ x &= \left(-\frac{dp}{d\rho} \frac{\lambda}{2GM} \right)^{\frac{1}{2}} \frac{\Delta t}{t} \ . \end{aligned} \tag{49}$$

Because $dp/d\rho$ is negative the speed of sound is imaginary; a region of space of density $\rho + \delta\rho$ has now a lower pressure than the region of density $\rho - \delta\rho$, and the pressure gradients now favor the formation of condensations. The question is: Can they succeed? Without specifying in detail the cause and

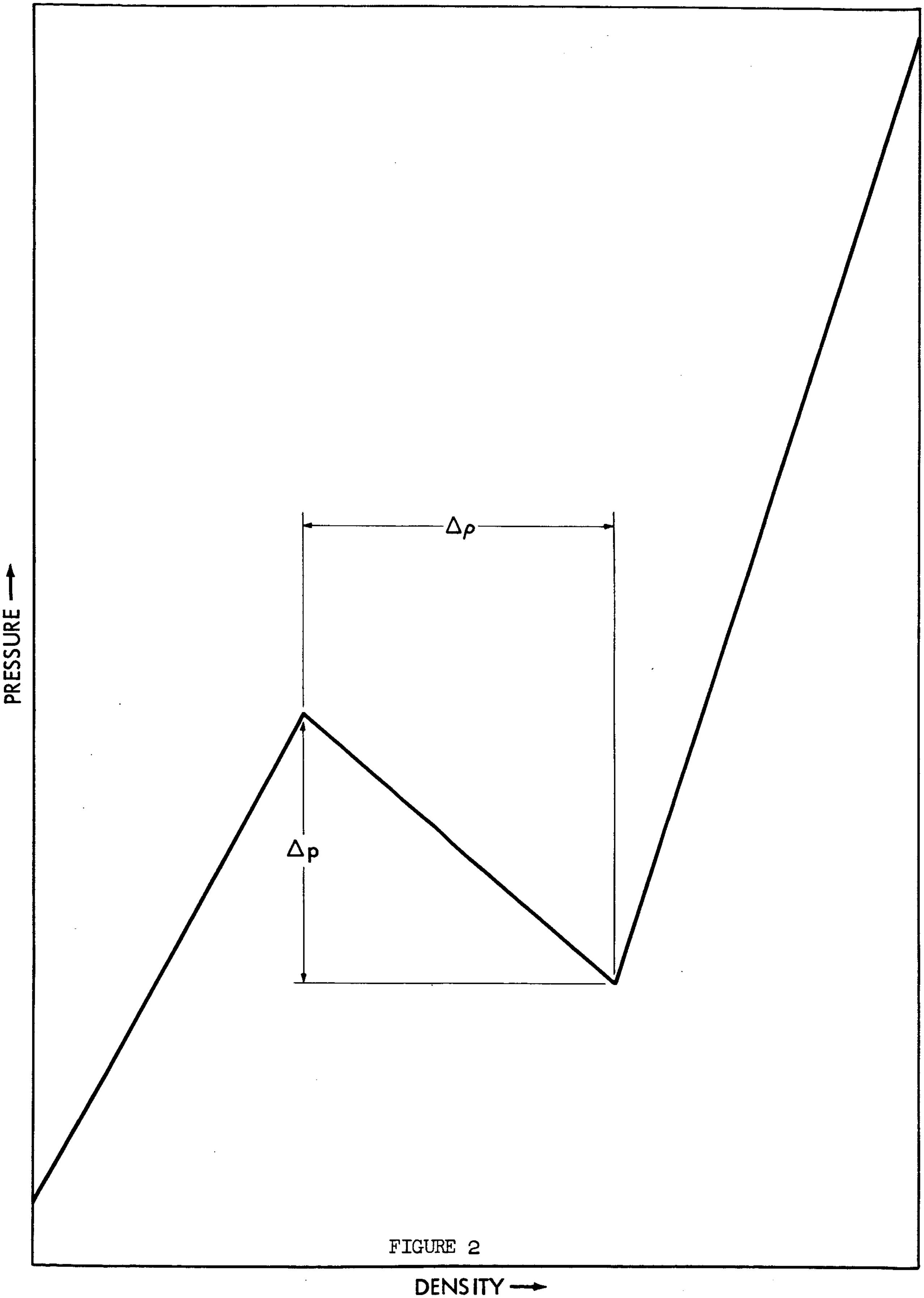

FIGURE 2

nature of the negative gradients it is possible to distinguish two classes of pressure instability.

In the first class we have a change of state such that $d\rho/dt < 0$ and $dp/dt > 0$ during expansion as shown in figure 2. From equations (11) and (42)

$$\frac{\Delta\rho}{\rho} \simeq -2\frac{\Delta t}{t},$$

and therefore

$$-\frac{\Delta p}{\Delta\rho} = \frac{t\Delta p}{2\rho\Delta t} = y\frac{c^2}{6}\left(\frac{t}{\Delta t}\right)^2, \tag{50}$$

where $$y = 3\frac{\Delta p}{\rho c^2}\frac{\Delta t}{t}.$$

We observe that the maximum possible value of y is unity when $\Delta t = t$, $\Delta p = p = \frac{1}{3}\rho c^2$. From equation (49) it follows that

$$12\frac{GM}{\lambda c^2} = \frac{y}{x^2}. \tag{51}$$

In terms of ρ and M this result becomes

$$(48)^2 G^3 M^2 \rho = y^3 x^{-6} c^6.$$

Now according to equation (4) x is at least of the order of 10^2 and therefore

$$\left(\frac{M}{M_\odot}\right)^2 \rho \lesssim 10^2 y^3 \text{ gcm}^{-3}, \tag{53}$$

where $M_\odot$ is the solar mass and $y < 1$. Thus stellar masses can in principle precipitate out when the density of the universe has dropped to less than $10^2 y^3$ gcm^{-3}, and galactic masses of $M = 10^{10} M_\odot$ when the density is less than $10^{-18} y^3$ gcm^{-3}. For a typical galactic density of 10^{-23} gcm^{-3}, y must have a value of at least 10^{-2}. Even if the galactic condensation time is $\Delta t \sim t$, we observe from equation (50) that as much as 1% of the mass at its final density must consist of relativistic particles. For a large cluster of galaxies the problem is even more severe.

In the second class of pressure instability we assume that there is radiation cooling either by photons or neutrinos, such that a region of $\rho - \delta\rho$ is heated at the expense of the region $\rho + \delta\rho$, such that where $d\rho/dx^i > 0$ we have $dp/dx^i < 0$. By writing

$$\begin{aligned} -\frac{dp}{d\rho} &= y\,\frac{c^2}{6}\left(\frac{t}{\Delta t}\right)^2, \\ y &= \frac{6\Delta p}{c^2 \Delta\rho}\left(\frac{\Delta t}{t}\right)^2, \end{aligned} \tag{54}$$

we obtain the same results as before.

On the face of it, without specifying any physical mechanism, thermal or pressure instability can provide an adequate rate of growth for stellar masses and a barely adequate rate of growth for galactic masses. The main difficulty is to discover an effective physical mechanism that can operate over a large range of density. The trouble is that at high density, where one might feel safe in claiming bizarre properties for the fluid, the universe expands rapidly and the time available is too short for pronounced growth. Over most of the density range radiation cooling at its best will merely make the fluctuations isothermal and therefore $dp/d\rho$ will be positive. The question of structure raises further difficulties. As the universe expands and the density diminishes we can imagine that different groups of wavelengths in succession are time-growing owing to various physical processes. Thus for each process y is small and therefore, provided the process permits, only relatively small masses have time to become differentiated. Given the right process equation (53) shows that planetesimal masses could form easily (and perhaps this accounts for their origin), but it is quite impossible to see how galactic masses can be carved out of the cosmic fluid by any reasonable process. Until a convincing physical basis can be found, and it is shown that an acceptable hierarchy of structures emerges out of an amorphous background, the thermal instability approach must be regarded as unfounded speculation.

It is suggested[7] that certain outstanding events, such as quasi-stellar radio sources and violent outbursts in galaxies, are the result of the expansion of objects from a radius close to their Schwarzschild singularity. It is visualized that in the early stages of the universe there occurs fragmentation into fluid cells which then remain partially encapsulated in the metric. To an internal observer the cell continues to expand and rapidly becomes an astronomical object; but to an external observer the cell lies dormant for a long period of time and is scarcely observable, and then bursts forth as a youthful and vigorous object. For a mass M of radius λ close to the Schwarzschild singularity, we have

$$2\frac{MG}{\lambda c^2} \simeq 1. \tag{55}$$

Now whatever the cause of the fragmentation it must involve propagation over a distance λ which cannot exceed ct, where t is at most the age of the universe. Therefore $\lambda \leq ct \leq (3c^2/8\pi G\rho)^{1/2}$, and for $M = 4\pi\rho\lambda^3/3$, it follows

$$2\frac{MG}{\lambda c^2} \leq 1, \tag{56}$$

and in principle the condition (55) is possible. Alternatively, we could argue that in the comoving coordinate system the maximum radial velocity is c, or

$$\lambda\dot{R} \leq Rc,$$

and from equation (11) we again derive the relation (56). A glance at equation (51) shows, however, that within the framework of the instability hypothesis encapsulation in the metric is impossible, for always $2MG/\lambda c^2 \ll 1$. If encapsulation does occur, then it must be studied on the basis of some other hypothesis.

4. PRIMORDIAL STRUCTURE HYPOTHESIS

We are confronted, so it seems, with a choice between an instability hypothesis which explains very little and a primordial structure hypothesis that leaves very little to be explained. We are driven by brute force to the conclusion that structure is implicit in the initial cosmological conditions and is not implicit in the known laws of physics. The origin of structure is thus apparently shrouded in the same inscrutable mystery as the fiat that created the universe.

This pessimistic view, however, is entirely unjustified and is a confession of our ignorance of the universe particularly in its earliest stages. It is quite possible that structural configurations with rotation are a necessary property of matter at exceedingly high densities. In this way the primordial structure hypothesis is not a policy of despair, but on the contrary it opens up a vista of exciting possibilities in which the initial conditions are determined naturally either by laws of physics which are as yet unknown or by the extrapolation of the known laws of physics to extreme conditions. Almost nothing is known about the universe during its earliest moments. We can expect that particle interactions are complicated by gravitational effects and quantum fluctuations of the metric. Possibly classical theories of the universe at very high density are inadequate or even invalid.[8] If indeed large scale structures such as the galaxies derive from primordial conditions then cosmology must grapple with this problem in order that eventually we shall understand the origin of structure in the universe.

Astronomy with its large telescopes has revealed the problem, and quite possibly cosmology must turn to high energy physics and its large accelerators for the solution.

Legends to Figures

Figure 1. Curves increasing from left to right show the growth in amplitude in an expanding cold universe; those increasing from right to left are for a contracting universe. $A_\varkappa = B_\varkappa = 1$ in equations (39)-(41).

Figure 2. Illustration of $dp/d\rho < 0$ in a given range of density.

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