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THE EQUATORIAL GEOMAGNETIC ANOMALY AND ITS ASSOCIATED CURRENT SYSTEM

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ABSTRACT

From a detailed study of the momentum transport equation for charged gaseous fluids moving through a neutral gas, it is suggested that the equatorial F region electron density distribution (the geomagnetic anomaly) is simply the natural steady state distribution one would expect for charged fluids under the influence of gravitational, electric, and magnetic fields and production and loss, when interaction with the neutral medium is negligible. It is shown that this steady state distribution maintains itself by means of a longitudinal current system, and a study of the properties of this longitudinal current system leads to a technique for its complete mapping in the upper F region of the ionosphere. Quite importantly, it is shown that the latitudinal electron density distribution that characterizes the equatorial anomaly cannot exist when the longitudinal current system is absent.

The study of the longitudinal current system also demonstrates the possibility of large errors in temperature calculations which make use of vertical density slope measurements to determine scale height, and a technique is suggested to correct such calculations. Finally, using cited measurements for support, it is possible to suggest the mechanisms responsible for the diurnal and asymmetric behavior of the anomaly.

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INTRODUCTION

The expression "geomagnetic anomaly" refers to the observed geomagnetically controlled distribution of electrons in the equatorial F region of the ionosphere. Its properties have been studied by means of critical frequency, f_oF2 (Appleton, 1946; Rastogi, 1959; Rao, 1963; Lyon and Thomas, 1963; Rao and Malthotra, 1964); by means of the height of the F2 layer, h_mF2 (Thomas, 1962); by means of constant height profiles using ground-based sounders (Croom, Robbins, and Thomas, 1959 and 1960); and by means of Alouette Topside Sounder measurements (King et al., 1963; Lockwood and Nelms, 1964). The properties of this phenomenon are therefore, reasonably well known and will not be reviewed here.

Many recent papers have theoretically investigated different aspects of the geomagnetic anomaly making use of the steady state continuity equation for electrons in the ionosphere with limited degrees of success (Kendall, 1963; Rishbeth, Lyon, Peart, 1963; Baxter, 1964; Bramley and Peart, 1964; Moffet and Hanson, 1965). Some of this work is based on ideas concerning electrodynamic drift presented much earlier by Martyn (1947), and has the ultimate aim of completely describing the explicit causes of the geomagnetic anomaly. Unfortunately, it is clear that approaches which attempt to investigate the anomaly in this manner are extremely difficult because of the many unknown quantities which must be included, and hence the lack of complete success at this time.

On the other hand, Goldberg, Kendall and Schmerling (1964) and Chandra and Goldberg (1964) have studied the isothermal steady state transport equations for charged fluids moving in a neutral

medium (the advantage here being that explicit causes need not be known) and found that they generate, with the appropriate boundary conditions, accurate representations of the topside geomagnetic anomaly as measured by King et al., (1963). This semiphenomenological approach has also been successful in the upper bottomside F region when the isothermal restriction is lifted (Goldberg, 1965).

In this work we now investigate the momentum transport equations in further detail to demonstrate that the geomagnetic anomaly appears to be nothing more than the natural steady state distribution of electrons under the influence of production and loss in the earth's magnetic and gravitational fields in a region where their collisions with neutral particles are relatively small in number. This interpretation also leads to plausible suggestions for the causes of the solstice asymmetry and the diurnal behavior of the anomaly.

The analysis will demonstrate that steady state (no acceleration) requirements demand the existence of a longitudinal current system which not only produces the latitudinal distribution of the anomaly but also creates a topside vertical distribution different from a simple diffusive equilibrium shape. This makes derivations of electron temperature from the altitude profiles of electron density dubious in regions where latitudinal density gradients exist. A method will be suggested for correcting these measurements when information is available concerning the latitudinal gradient of electron density in the same region where temperature is being measured from vertical density slopes.

In executing the above analysis, we will first investigate the fundamental derivation of the general equations used in order to solidify the physical interpretations and implications of the theory involved. This is deemed necessary since we feel that some of the current work in this field has lost sight of the

correct physical interpretations of the equations.

THEORETICAL DEVELOPMENT

As first proposed by Johnson (1951) and Schlüter (1951), and later repeated by Chandra (1964), the general equation of motion of a multiple component gas under the action of external forces may be written as

$$\frac{\partial \vec{v}_s}{\partial t} + \vec{v}_s \cdot \frac{\partial \vec{v}_s}{\partial \vec{r}} = - \frac{1}{\rho_s} \frac{\partial}{\partial \vec{r}} \cdot \vec{p}_s + \vec{F}_s + \sum_k \frac{m_k \nu_{sk}}{m_s + m_k} (\vec{v}_k - \vec{v}_s) \qquad (1)$$

where the suffixes s, k stand for particle type and

$\vec{v}_s$ = macroscopic velocity

ρ_s = density

$\vec{p}_s$ = pressure tensor

$\vec{F}_s$ = external and internal (see below) forces per unit mass

m_s = mass

ν_{sk} = collision frequency between the sth and k^{th} particle

The last term on the right-hand side of (1) represents collisions, which are assumed to be elastic and two body, and may be thought of as a frictional drag term. The first term on the right-hand side of (1) is the concentration force term, and is the exclusive cause of diffusive motions.

The left-hand side of (1) is the total time derivative of velocity and thus represents the Newtonian acceleration of the gaseous fluids. Because the neglect of these terms is almost always assumed when studying transport problems in the ionosphere

with reasonable degrees of success, and because this neglect permits linearization of the transport equations, we will maintain this approach in the work which follows.

The force term $\vec{F}_s$ represents all possible forces including electrodynamic, electrostatic, gravitational, centrifugal, Coriolis, tidal, etc. We should note that $\vec{F}_s$ is not purely external since the electrostatic forces arise from electric fields of both external and internal origin. Since all conceivable forces are included in (1), the velocity $\vec{v}_s$ represents the entire macroscopic velocity of the sth type particle fluid. This point is emphasized since it seems to be ignored in many papers dealing with the F region diffusion problem.

The pressure tensor will be treated as an isotropic scalar by neglecting the off-diagonal viscosity elements in the tensor; c.f. Chandra (1964).

Finally, in the equatorial F region of the ionosphere, we neglect tital, Coriolis, and centrifugal forces as a first order approximation, leaving us with the following equations of motion for electron and ion fluids passing through a neutral gas:

$$\frac{n_e m_e m_i}{m_e + m_i} \nu_{ei} (\vec{v}_e - \vec{v}_i) + \frac{n_e m_e m_n}{n_e + m_n} \nu_{en} (\vec{v}_e - \vec{v}_n)$$

$$= - \nabla p_e + n_e m_e \vec{g} - e n_e (\vec{E} + \vec{v}_e \times \vec{B}) \qquad (2)$$

$$\frac{n_e m_i m_e}{m_e + m_i} \nu_{ei} (\vec{v}_i - \vec{v}_e) + \frac{n_i m_i m_n}{m_i + m_n} \nu_{in} (\vec{v}_i - \vec{v}_n)$$

$$= - \nabla p_i + n_i m_i \vec{g} + e n_i (\vec{E} + \vec{v}_i \times \vec{B}) \qquad (3)$$

since

$$\nu_{jk} \; n_j = \nu_{kj} \; n_k \tag{4}$$

In equations (2) and (3), the subscripts e, i, and n refer to electrons, ions, and neutrals, respectively; $\vec{E}$ is the electric field caused by both external and internal sources; $\vec{v}_s \; X \; \vec{B}$ is the electrodynamic force where $\vec{B}$ is magnetic field; and we have made use of

$$\rho_s = n_s \; m_s \tag{5}$$

where n_s is number density. It is also possible to write a similar equation of motion for the neutral particle flow but this is not needed here.

In (2) and (3), we now assume that

$$n_e = n_i = N \tag{6}$$

and since

$$m_e \; << \; m_i, \; m_n \tag{7}$$

summation of (2) and (3) gives

$$N \; m_e \; \nu_{en} \; (\vec{v}_e - \vec{v}_n) + \frac{N \; m_i m_n}{m_i + m_n} \; \nu_{in} \; (\vec{v}_i - \vec{v}_n)$$

$$= -\nabla \; (p_e + p_i) + N \; m_i \; \vec{g} + \vec{J} \; X \; \vec{B} \tag{8}$$

where $\vec{J}$ is the current density defined by

$$\vec{J} = Ne \; (\vec{v}_i - \vec{v}_e) \tag{9}$$

Equation (8) assumes only one type of ionizable constituent. However, this is not a serious limitation since the results which follow can easily be extended to ionospheric regions containing multiple ionizable constituents. Furthermore, the F region is usually strongly dominated by O^+ ions and the assumption is quite applicable in this region at most times.

We note again that the velocities $\vec{v}_e$ and $\vec{v}_i$ in (8) are the <u>total</u> macroscopic velocities of the ion and electron fluids. <u>Although the electric field $\vec{E}$ has been eliminated explicitly in</u> <u>(8), this has not removed its contribution to the general behavior</u> <u>of $\vec{v}_e$ and $\vec{v}_i$, e.g. electrodynamic drifts.</u> Note also that electron ion resistance terms have been eliminated explicitly.

We now neglect the remaining resistance terms in (8) by assuming that we are sufficiently high in the ionosphere for ν_{en} and ν_{in} type collisions to be negligible and thereby remove the effect of resistance between charged and neutral particle fluids. The validity of this assumption is justified at altitudes above the F2 maximum at the equator, and may even be justified at lower altitudes, c.f. Goldberg (1965).

We can then write

$$-\nabla(p_e + p_i) + N m_i \vec{g} + \vec{J} \times \vec{B} \approx 0 \tag{10}$$

Next, we assume the ideal gas law

$$p_s = n_s k T_s \tag{11}$$

and following Goldberg (1965), let

$$\tau = \frac{T_e + T_i}{2} \tag{12}$$

$$H_\tau = \frac{k\tau}{m_i g} \tag{13}$$

Then we can write (10) as

$$- \frac{\nabla N}{N} - \frac{\nabla T}{T} + \frac{\vec{j} \times \vec{B}}{2kT} - \frac{\vec{i}_r}{2H_T} = 0 \qquad (14)$$

where $\vec{i}_r$ is a unit vector in the radial direction and $\vec{j}$ is a velocity difference term defined by

$$\vec{j} = \frac{\vec{J}}{N} = e(\vec{v}_i - \vec{v}_e) \qquad (15)$$

Equation (14) is the general steady state description of the electron and ion densities under collision-with-neutral free conditions and under the influence of electric, magnetic, and gravitational fields (by steady state, we mean $d\vec{v}_s/dt = 0$). Previously, (15) has been integrated along a field line, using the vertical profile of density at the equator as a boundary condition, to provide a general description of the topside equatorial geomagnetic anomaly under isothermal (Goldberg, Kendall and Schmerling, 1964; Chandra and Goldberg, 1964; Baxter and Kendall, 1965) and non-isothermal conditions (Goldberg, 1965). The results obtained have agreed excellently with the topside and f_oF2 properties of the midday equinoctial anomaly (Appleton, 1946; Croom, Robbins, and Thomas, 1959; King et al., 1963; Lockwood and Nelms, 1964) and the inclusion of a realistic variable electron temperature has lead to an improved description of the observations including a partial explanation of the observed fine structure.

The convenience of integrating along a field line is rather obvious. It allows us to investigate the behavior of N without any knowledge of $\vec{j}$, i.e. if we take the component of (14) along line, we obtain

$$\left(\frac{\nabla N}{N} + \frac{\nabla T}{T} + \frac{\vec{i}_r}{2H_T} \right) \cdot \vec{h} = 0 \qquad (16)$$

where $\vec{h}$ is a unit vector along $\vec{B}$. Since (16) is independent of $\vec{j}$, it can be integrated yielding

$$N(r,\theta) = \frac{N(r_o)\tau(r_o)}{\tau(r,\theta)} e^{-\int_{r_o}^{r} \frac{dr}{2H_\tau}} \tag{17}$$

where θ is magnetic colatitude. Here the integration must be carried out along a field line that has an intersection with the equatorial normal at r_o; i.e., for a dipole,

$$r_o = r \csc^2\theta \tag{18}$$

As previously stated, evaluation of (17) requires a knowledge of $N(r_o)$, the vertical electron density distribution at the equator, thus making the theory semiphenomenological. Furthermore, it provides us with no knowledge concerning $\vec{j}$. In the next section, we shall investigate the behavior of $\vec{j}$ itself since a knowledge of $\vec{j}$ allows us a more useful tool in interpreting and calculating properties of the anomaly.

THE CAUSES AND EFFECTS OF AN ELECTRON-ION VELOCITY DIFFERENCE, $\vec{j}$

In the previous section, we reviewed the properties of N which can be obtained by considering a single component ($\vec{h}$) of the general steady state equation (14) for an electron-ion plasma flowing in a neutral gas. This leaves us with two more components from which to obtain additional information. We now write (14) in the three spherical polar coordinate components as

$$-\frac{1}{N}\frac{\partial N}{\partial r} - \frac{1}{\tau}\frac{\partial \tau}{\partial r} - \frac{1}{2H_\tau} + \frac{(\vec{j} \times \vec{B})_r}{2m_i g H_\tau} = 0 \tag{19}$$

$$- \frac{1}{Nr} \frac{\partial N}{\partial \theta} - \frac{1}{\tau r} \frac{\partial \tau}{\partial \theta} + \frac{(\vec{j} \times \vec{B})_\theta}{2m_i g H_\tau} = 0 \qquad (20)$$

$$- \frac{1}{Nr \sin\theta} \frac{\partial N}{\partial \phi} - \frac{1}{\tau r \sin\theta} \frac{\partial \tau}{\partial \phi} + \frac{(\vec{j} \times \vec{B})_\phi}{2m_i g H_\tau} = 0 \qquad (21)$$

In the analysis which follows, we treat $\vec{B}$ as a pure dipole field having its north pole in the geographic southern hemisphere. Furthermore, we assume that $\vec{j}$ is sufficiently small to allow neglect of any influences it might have on $\vec{B}$. Then

$$\vec{B} = (- \frac{\mu_o}{2\pi} \frac{M_p}{r^3} \cos\theta, - \frac{\mu_o}{4\pi} \frac{M_p \sin\theta}{r^3}, 0) \qquad (22)$$

where M_p is the permanent dipole moment of $\vec{B}$ and μ_o is the magnetic permeability of free space. Substitution of (22) into (19), (20), and (21) provides

$$\frac{1}{N} \frac{\partial N}{\partial r} + \frac{1}{\tau} \frac{\partial \tau}{\partial r} + \frac{1}{2H_\tau} - \frac{\mu_o M_p}{8\pi H_\tau m_i g} \frac{\sin\theta}{r^3} j_\phi = 0 \qquad (23)$$

$$\frac{1}{N} \frac{\partial N}{\partial \theta} + \frac{1}{\tau} \frac{\partial \tau}{\partial \theta} + \frac{\mu_o M_p}{4\pi H_\tau m_i g} \frac{\cos\theta}{r^2} j_\phi = 0 \qquad (24)$$

$$\frac{1}{N} \frac{\partial N}{\partial \phi} + \frac{1}{\tau} \frac{\partial \tau}{\partial \phi} + \frac{1}{H_\tau m_i g} \left[\frac{\mu_o M_p}{8\pi} \frac{\sin^2\theta}{r^2} j_r - \frac{\mu_o M_p}{4\pi} \frac{\cos\theta \sin\theta}{r^2} j_\theta \right] = 0 \qquad (25)$$

The advantage of writing (14) in the component form given by (23), (24) and (25) is immediately apparent. We find that the special two dimensional nature of the dipole field provides us with two independent equations, (23) and (24), for the longitudinal component of ion-electron velocity difference j_ϕ. This will enable us to obtain valuable information concerning j_ϕ and hence H_τ and N in the following discussions.

For the moment, let us concentrate on (25). We restrict ourselves to midday (or middle of the night) periods when $\partial \ln N\tau / \partial \phi$ can be considered negligible. We then obtain

$$j_r = j_\theta \tan I \qquad (26)$$

where I is the dip angle of the magnetic field and we have employed the dip angle condition

$$\tan I = 2 \cot \theta \qquad (27)$$

We observe that unless j_θ becomes infinite at the equator (a physically unrealizable situation), j_r must become zero there and electrons will not be able to separate from ions vertically in this region. This is the only information obtainable concerning j_r and j_θ without further assumptions, but fortunately, neither of these variables appear in (23) or (24), the expressions governing the distribution of N in the (r, θ) plane.

Having exhausted the information available in (25), let us return to (23) and (24), devoting the remainder of this section to j_ϕ. We first note that combination of these two equations with the elimination of j_ϕ simply produces the characteristic form obtained with (16), thereby providing no new information. We also note that j_ϕ is a coupling parameter between the

horizontal and vertical gradients of density (and temperature).
For the special case

$$j_\phi = 0 \qquad (28)$$

we find that the density distribution is horizontally stratified
and has the vertical hydrostatic distribution of exponential
decay with height, provided that we neglect gradients in temperature
compared to gradients in density (this is quite valid in the
topside F region). This is possibly what one might observe above
heights where the geomagnetic anomaly is known to occur.

In the topside anomaly region, we observe a non-zero lati-
tudinal distribution $(\frac{1}{N} \frac{\partial N}{\partial \theta})$ which requires the existence of a
j_ϕ from (24). From (23), we see that this simultaneously alters
the vertical distribution $(\frac{1}{N} \frac{\partial N}{\partial r})$ from the simple exponential decay
form we would obtain when (28) is satisfied. We can also see this
as follows: The solution of (23) is

$$N(r,\theta,\phi) = \frac{F_1(\theta,\phi)}{\tau(r,\theta,\phi)} e^{-\int_a^r \frac{1}{2H_\tau} (1 - \frac{\mu_o M_p}{4\pi m_i g} \frac{\sin\theta}{r^3} j_\phi) dr} \qquad (29)$$

where F_1 is an arbitrary function and the explicit dependence of
N on τ disappears if τ is a constant because the region concerned
is isothermal. If we evaluate (29) at the equator, neglecting ϕ
dependence for simplicity, we obtain

$$N(r,\pi/2) = \frac{N(a,\pi/2)\tau(a,\pi/2)}{\tau(r,\pi/2)} e^{-\int_a^r \frac{1}{2H_\tau} (1 - \frac{\mu_o M_p}{4\pi m_i g} \frac{j_\phi}{r^3}) dr} \qquad (30)$$

Thus, the vertical distribution at the equator is the simple
exponential decay law perturbed by a term containing j_ϕ. In the

region of the F2 peak, j_ϕ must have sufficient influence to distort the distribution into the required shape.

The physics involved can be thought of as follows: Since the steady state transport equations describe the geomagnetic anomaly, the anomaly is simply the natural steady state distribution of the electron density to be expected in a gravitational field under the influence of a dipole magnetic field and the effects of production and loss. Production and loss influence the distribution by contributing to the existence of the F2 peak. This in conjunction with the geomagnetic field requires a longitudinal current system, necessary to preserve the vertical distribution forced by production, loss, and transport combined. However, the existence of a longitudinal current system in the geomagnetic field simultaneously requires the latitudinal distribution of electron density known as the anomaly. The latitudinal peaks of the anomaly have special significance also, as will become evident from the analysis in the next section.

Experimental evidence indicating the stability of the anomaly has been given in studies of f_oF2 by (Rastogi, 1959; Lyon and Thomas, 1963; Rao and Malthotra, 1964). We find that the anomaly forms at about 8:00 to 10:00 am local time, depending on meridian, and then preserves its shape and amplitude throughout the day. After dark, the peaks remain relatively stable in latitudinal position but decay in amplitude. As morning progresses, the decay reaches completion and the sun begins to appear. This creates a non-steady state distribution which masks any effects associated with the anomaly. Finally, as transients disappear and the steady state reappears, the anomaly once again forms as the natural steady state distribution. Such empirical studies demonstrate that the anomaly is, as indicated here theoretically, the natural distribution of electrons and ions, one which disappears only when transients in the distribution caused by

sunrise temporarily remove steady state conditions and mask the visible anomaly.

We have shown at this point that the density in the (r, θ) plane cannot be affected by currents j_r and j_θ but only by j_ϕ (see (23), (24), and (25)). Hence, studies which neglect the perpendicular component of velocity in the continuity equation are incapable of generating the anomaly.

From (23), (24), and (29) we note that the vertical distribution of electron density is not influenced by scale height (and thus temperature) alone. We find that the simple scale height slope is distorted by a term containing j_ϕ. A simple order of magnitude calculation shows that a velocity separation of electrons and ions $(\vec{v}_i - \vec{v}_e)$ of only 3 or 4 cm/sec is sufficient to either cancel (j_ϕ positive) or be equal to (j_ϕ negative) the effect of gravity. In a region where j_ϕ is positive, i.e. between the latitudinal peaks of the anomaly and the equator, one will measure a vertical slope leading to a scale height larger than the true scale height. Thus, the measured temperatures will be larger than the true temperatures. At midlatitudes (higher than the position of the latitudinal peaks) or at heights above the anomaly where we observe a steady decay of density with latitude, j_ϕ is negative and one will measure effective scale heights leading to deduced temperatures smaller than the true temperatures.

The magnitude of j_ϕ can be obtained by measuring horizontal slopes of $\frac{1}{N} \frac{\partial N}{\partial \theta}$ and neglecting θ dependence of τ. Using (24) such calculations provide results in the cm/sec order of magnitude range further demonstrating that j_ϕ is not an insignificant quantity. In the next section, we will show how it is possible to separate out the contribution of j_ϕ to the vertical slope, provided information is available concerning $\frac{1}{N} \frac{\partial N}{\partial \theta}$ and $\frac{1}{T} \frac{\partial T}{\partial \theta}$,

thereby allowing us to correct vertical slope measurements for the effects of j_ϕ and thus obtain true scale heights and temperatures.

THE PROPERTIES OF j_ϕ

At this stage of development, no mention has been made of the properties of j_ϕ which must exist if (23) and (24) are to be satisfied consistently. We shall find that j_ϕ is not completely arbitrary, but instead possesses properties extremely useful in separating the true scale height from the measured vertical slope, thereby allowing us to obtain more accurate determinations of the electron temperature in the topside ionosphere.

The consistency requirement for equations (23) and (24) is easily obtained by rewriting them as

$$\frac{\partial \ln N_T}{\partial r} = \frac{\mu_o M_p}{8\pi \, kT} \frac{\sin\theta}{r^3} \, j_\phi - \frac{1}{2H_T} \tag{31}$$

$$\frac{\partial \ln N_T}{\partial \theta} = - \frac{\mu_o M_p}{4\pi \, kT} \frac{\cos\theta}{r^2} \, j_\phi \tag{32}$$

Then,

$$\frac{\partial}{\partial \theta} \left[\frac{\mu_o M_p}{8\pi \, kT} \frac{\sin\theta}{r^3} \, j_\phi - \frac{1}{2H_T} \right] = \frac{\partial^2 (\ln N_T)}{\partial r \partial \theta} = \frac{\partial}{\partial r} \left[- \frac{\mu_o M_p}{4\pi \, kT} \frac{\cos\theta}{r^2} \, j_\phi \right] \tag{33}$$

In the equatorial topside, the medium is to a first approximation isothermal, and (33) becomes

$$\tan\theta \, \frac{\partial j_\phi}{\partial \theta} + 2r \, \frac{\partial j_\phi}{\partial r} = 3 j_\phi \tag{34}$$

where we have also restricted ourselves to a region of constant m_i so that

$$\frac{\partial H_T}{\partial \theta} \approx 0 \tag{35}$$

Equation (34) represents the constraint equation on j_ϕ. It can be solved by separation of variables yielding

$$j_\phi = \int_{-\infty}^{\infty} G(\lambda) \, r^{(3-\lambda)/2} \sin^\lambda \theta \, d\lambda \tag{36}$$

where λ is the separation constant and $G(\lambda)$ is an arbitrary function. In order to guarantee j_ϕ finite as $r \to \infty$, (36) requires $\lambda \geq 3$. Then, with the following substitutions,

$$\beta = \lambda - 3 \tag{37}$$

$$\Psi(\beta) = G(\beta+3) \tag{38}$$

and

$$u = \ln(r^{\frac{1}{2}}/\sin\theta) \tag{39}$$

we have

$$j_\phi = \sin^3\theta \int_0^\infty \Psi(\beta) \, e^{-\beta u} d\beta \tag{40}$$

Thus, since the integral in (40) is a Laplace transform, j_ϕ is a rather arbitrary function of u depending on $\Psi(\beta)$.

Let us now consider lines of constant u, which are clearly the dipole field lines specified by (18). Since u is constant

along these paths, we can immediately conclude that

$$j_\phi = C \sin^3\theta = C(r/r_o)^{3/2} \tag{41}$$

along a field line, where C is a constant. By specifying j_ϕ at some given point on a field line, we can then obtain j_ϕ and $\frac{\partial \ln N_T}{\partial \theta}$ at all other points on the same field line using (41) and (32).

The above discussion then permits the following general procedure. If N and τ are known for a single constant height profile, we can calculate j_ϕ along this profile from the logarithmic slope of N_T. Then j_ϕ can be evaluated in the entire (r,θ) plane using (41). Finally, at any point that $\frac{\partial \ln N_T}{\partial r}$ is known from vertical measurements, we can evaluate the true scale height and temperature using (31). Naturally, great care must be taken in the evaluation of results using such a technique due to the large errors which can result from inaccurate slope determinations. Furthermore, because (41) depends on the assumption of isothermallity, this technique is restricted to regions where logarithmic gradients of N are much larger than those of τ.

Finally, (32) and (41) indicate the physical effect occurring at the field line along which the angular peaks lie, viz. that along this field line, j_ϕ is zero. Below this latitude j_ϕ is positive and hence west to east; above this latitude, j_ϕ is negative and hence east to west in direction.

SOME FURTHER PROPERTIES OF THE GEOMAGNETIC ANOMALY

Because the anomaly can be thought of as the natural steady state distribution of electrons in the vicinity of the geomagnetic equator, it is possible to explain another of its properties. Farley (1965), using incoherent backscatter data, has observed that the nighttime peak of the F2 layer often maintains itself

at the equator until local midnight at which time it begins to drop
into the higher loss regions. Studies of f_oF2 by Rastogi (1959),
Lyon and Thomas (1963), Rao and Malthotra (1964), and the author
(in process) indicate that the latitudinal distribution maintains
itself till approximately the same time period, at which time the
angular peaks begin to decay.

The time constant of loss appears relatively long due to
the length of time before observable decay begins to take place.
It therefore appears quite plausible to think of the density
distribution as a very slowly varying function in time always
extremely close to steady state. Under this assumption, the
mechanism causing the layer drop at the equator becomes evident.
The measurements of Thomas (1962) and the theory of Goldberg,
Kendall and Schmerling (1964) and Goldberg (1965) indicate that
the steady state vertical F2 peak be much higher at the equator
than at latitudes where the latitudinal f_oF2 peaks are observed.

Since the latitudinal peaks of f_oF2 are in a higher loss
rate region than the equatorial minimum they decay faster and
cause the steady state distribution to slowly readjust to
compensate for the fadeout of these latitudinal peaks. This
requires a lowering of the F2 layer at the equator, c.f. Goldberg,
Kendall, and Schmerling (1964) and Goldberg (1965). Thus, the
drop of the layer at the equator is nothing more than a slowly
varying readjustment of the entire F region distribution, never
deviating much from the steady state, to compensate for the
more rapid initial decay of the F2 layer at latitudes where the
latitudinal peaks in the anomaly occur than at the equator.
Naturally, as the equatorial layer finally lowers to more rapid
loss rate heights equivalent to the height of the layer at higher
latitudes, we would expect a more rapid decay at the equator,
and this is what Farley (1965) has observed.

The geomagnetic anomaly is also known to possess asymmetric

properties which are most exaggerated during months of solstice
(Croom, Robbins, and Thomas, 1960; Lyon and Thomas, 1963).
During such a period, the asymmetry exhibits itself as a higher
density peak in one hemisphere. This behavior is accounted for
in (17) by means of $\tau(r,\theta)$, which will certainly exhibit asymmetric
properties when heating of one hemisphere is larger than the other.
Although (17) does not explain or define the behavior of $\tau(r,\theta)$,
it allows us to calculate it as a function of θ. It can be shown
that $N(r,\theta)$ is far more sensitive to fluctuations of $\tau(r,\theta)$
exhibited in the coefficient of the exponential than in H_τ.
This allows us to think of $N(r,\theta)$ as being inversely proportional
to $\tau(r,\theta)$ to first order. It is then a simple matter to take a
constant height profile of the asymmetric anomaly and calculate
the temperature distribution necessary to make the distribution
deviate from the symmetric case. Naturally, this requires an
approximation of the distribution which would be present during
equinoctial symmetry, but it does not seem unreasonable to
select the average of the asymmetric values in the two hemispheres
as a first approximation.

SUMMARY AND CONCLUSIONS

We have suggested that the geomagnetic anomaly is nothing
more than the natural steady state distribution of electrons under
the influence of production-loss effects in the earth's magnetic
and gravitational fields and in a nearly collision free (charged-
neutral particle type) medium. Furthermore, properties of both
the vertical and latitudinal distribution satisfy this explanation.
Experimental evidence has been cited to justify these conclusions.
From this analysis, we have also been able to offer plausible
explanations for the asymmetric properties observed during solstice
and the nighttime behavior of the F2 layer observed at the equator.
We have also suggested a method for evaluating latitudinal temperature
variations.

The above analysis has been obtained by studying the momentum transport equations as applied to the equatorial ionosphere in greater detail than any time previously. This has lead to information concerning the current density system which must exist to maintain both the observed vertical and latitudinal electron density distribution. The analysis has displayed the character of magnetic influence on the density distribution, by proving that the (r, θ) plane density distribution is quite independent of currents in the (r, θ) plane but depends only on longitudinal currents.

The study of j_ϕ has also shown that any region possessing a latitudinal gradient in density must simultaneously have its vertical distribution distorted from the pure diffusive equilibrium type shape. This could lead to rather large errors in temperatures obtained using vertical slope techniques in the topside F region. However, it is possible to estimate the errors involved, and a method to evaluate such errors has been suggested.

The fact that the electron density distribution in the (r, θ) plane is influenced directly by variations in the longitudinal component of current lends itself nicely to the possibility of explaining more local type phenomena (such as Spread F). This could be achieved by study of various mechanisms capable of causing perturbations in the longitudinal current system thereby altering $N(r, \theta)$.

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