ON THE EFFECTS OF THE NON-CONSERVATION OF BARYONS IN COSMOLOGY*†

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ABSTRACT

The idea underlying this paper is that the massive objects, now known to be associated with radio sources and with quasi-stellar objects, are of cosmological significance; and that the energy output of these objects is associated with the problem of creation in cosmology. We have used the same equations describing creation of matter, as before; but have taken account of the fact that they contain far wider cosmological implications than have been explored hitherto. The present paper only deals with the more conventional aspects of the steady-state theory. Full exploration of the creation process leads to a radical departure from the old steady-state concept. For convenience of presentation, these less conventional ideas are described in a separate paper.

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INTRODUCTION

No classical Friedmann cosmology ($\lambda = 0$) is known that is consistent with the empirical law of baryon conservation. Cosmologies that demand a singular origin for the whole universe are obviously inconsistent with this law, while so far no satisfactory mathematical description of an oscillating model has been given. The latter, if it could be found, would be consistent with baryon conservation. It is possible in this situation to take one or other of two points of view. The conservative view is to regard particle physics as a far more definitive branch of knowledge than cosmology and therefore to adhere to baryon conservation, implying that the mathematical difficulties of the oscillating model will eventually be overcome. The less conservative attitude contemplates non-conservation of baryons in strong gravitational fields, on the grounds that the empirical facts, on which particle physics is based, all relate to conditions in weak gravitational fields. In cosmology, on the other hand, we have to deal with strong fields.

While curiosity alone is sufficient to require that both these possibilities be explored, the point of view that is regarded as the more promising must evidently turn on an assessment of the mathematical difficulties of the oscillating model. It is well known that no switch from contraction to expansion is possible if the universe is homogeneous and isotropic; and according to an investigation by Narlikar (1963) this situation is not likely to be changed by a departure from isotropy. The issue therefore turns on the mathematically very difficult case of an inhomogeneous universe. It is easy to see how the motion of a finite cloud can change from contraction to expansion, through pressure gradients operating either directly throughout the whole cloud or within a number of fragments into which the cloud divides. Inward-moving fragments can 'miss' each other as they approach the center

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provided they are small enough, as the stars of a cluster miss each other when they move in and out of the central region of the cluster. By dividing the universe into finite clouds it is possible to reverse the contraction of each separate cloud; but this is not the same thing as a switch from contraction to expansion of the whole universe. Our doubts are increased further by the circumstance that a static configuration always exists in association with oscillating systems, and no static configuration of the required type has ever been found for the universe. If the universe is oscillatory, a dissipative damping of the oscillations would be expected, gradually leading to a static state. Why has such a static state not already been attained?

Owing to these difficulties of the oscillating model, it seems to us worthwhile to explore the cosmological implications of the other, more radical, approach — that involving the non-conservation of baryons. To permit non-conservation of baryons it is necessary to introduce a new field. This was done by Hoyle (1948) in an attempt to describe the so-called steady-state theory within the framework of general relativity. Later it was shown by M.H.L. Pryce (private communication) that the problem could be treated in terms of an action principle, using a zero rest-mass scalar field C. This formulation and its analogue in terms of the direct particle action have been discussed in earlier papers (Hoyle and Narlikar 1963, 1964a,b,c). For the sake of completeness we have summarized the formal aspects of the theory in the Appendix.

Classical considerations already incorporate the conservation of energy and momentum, and the relation of the C-field to gravitation. In the present paper, we shall have occasion to refer to explicit particles and for this something more than classical theory is needed. We take spin to be conserved, so that a baryon created by the C-field must also be accompanied by at least one other fermion. In particular, we shall be concerned with the creation of a baryon-lepton pair. The possibility of creation of mesons along with baryons and leptons cannot be excluded, however. A more complete theory, incorporating quantum ideas, is needed to make more definitive statements.

C-FIELD PHYSICS — A BRIEF RESUME

In the present paper we are concerned with exploring the practical applications to astronomy of C-field physics. Particularly, we need to know three important properties of the field:

(i) the nature of its sources,

(ii) the form of the interaction with matter,

(iii) the effect on gravitation.

These properties will now be described.

(i) The sources of the C-field — The C-field arises whenever a baryon (and its accompanying lepton) is created or destroyed — i.e., it has a source in the beginning or end of a world line.* Thus if the world line $\underline{a}$ has a beginning or end at $\underline{A}$, the C-field originating from it is given at a point X by

$$C^{(a)}(X) = -\frac{\epsilon}{f}\bar{G}(A,X) \quad , \tag{1}$$

where $\bar{G}(A,X)$ is the 2-point scalar Green's function described in the Appendix. $\epsilon = +1$ if there is creation at A, $\epsilon = -1$ if there is destruction at A. f is a coupling constant. The total C-field at X is defined by the sum of all

* The concepts of creation/destruction or beginning/end presuppose the existence of a direction of time. This can be introduced ad-hoc in a mathematical discussion; but it has a physical significance only when related to some observed time — asymmetric phenomenon such as the expansion of the universe.

such contributions:

$$C(X) = \sum_a C^{(a)}(X) , \tag{2}$$

and it identically satisfies the wave equation

$$C^k{}_{;k} = n/f \qquad , \tag{3}$$

where $C_k = \partial C/\partial x^k$ and n = number of baryons (and leptons) created per unit proper volume per unit proper time.

(ii) <u>The interaction of the C-field with matter</u> — <u>The C-field does not affect the world line of a particle during its existence</u>. It only affects the ends of world lines — it determines the condition of creation or destruction at any point in space-time. Thus, provided at X

$$C^k C_k = E^2 \qquad , \tag{4}$$

a baryon-lepton pair of total energy E may be created at X with the momentum

$$p_i = C_i \qquad . \tag{5}$$

In the same way the condition for creation of more than one pair is a generalization of (5):

$$p_i^{(1)} + p_i^{(2)} + \cdots = C_i \quad , \tag{6}$$

where $p_i^{(1)}$, $p_i^{(2)}$, ... refer to the momenta of different pairs. This result follows from the variation of world lines of particules, as shown in the Appendix. In equation (5) or (6) the C-field does not contain the contributions from the newly created (or destroyed) particles, i.e., there is no self-action. The equation (5) or (6) is the conservation law that must be satisfied at creation.

(iii) Gravitational influence of the C-field. The C-field affects the geometry of space-time through the contribution of an energy-momentum tensor to Einstein's equations. The energy-momentum tensor of the C-fields arising from baryons a, b is given by

$$H_{ik}^{(a,b)} = -f\left[C_i^{(a)}\,C_k^{(b)} + C_i^{(b)}\,C_k^{(a)} - g_{ik}\,C_l^{(a)}\,C^{(b)l}\right] . \tag{7}$$

The method of deriving this formula is briefly described in the Appendix. The total contribution from all pairs (a,b) is

$$H_{ik} = \sum_{a<b}\sum H_{ik}^{(a,b)} . \tag{8}$$

If the number of particles is large enough to justify a "smooth-fluid" approximation, equation (8) becomes

$$H_{ik} = -f\left[C_i\,C_k - \tfrac{1}{2}\,g_{ik}\,C_l\,C^l\right] . \tag{9}$$

The Einstein equations with matter and the C-field are given in this case by

$$R_{ik} - \tfrac{1}{2}\,g_{ik}\,R = -8\pi G\left[T_{ik} - f\,(C_i\,C_k - \tfrac{1}{2}\,g_{ik}\,C_l\,C^l)\right] , \tag{10}$$

where we have taken the velocity of light as unity.

Divergence of equation (10) gives

$$T^{ik}{}_{;k} = f\,C^i\,C^k{}_{;k} \tag{11}$$

Thus matter alone is not conserved; to get a conservation law one must include the contribution from the C-field. In most problems $H_{44} < 0$; the C-field therefore has negative energy density.

To conclude this section we note that in order to preserve CP-invariance in K-meson decay, Lee (1964) has postulated the existence of a zero rest mass, zero spin field φ and that the Lagrangian for φ has a formal similarity to that used here for C.

COSMOLOGY WITH COMPLETE HOMOGENEITY AND ISOTROPY

The C-field alters the mathematical structure of the cosmological equations in a very important way, <u>and this is true even if there is conservation of baryons</u>. Provided $\dot{C} \neq 0$, there can be no singularity of the kind that is so worrying in the usual Friedmann cosmologies. Thus it is possible to obtain an oscillating model <u>without departing from homogeneity and isotropy</u>. Writing

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\, d\Omega^2\right] , \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2 , \tag{12}$$

where $k = 0, \pm 1$ for the general Robertson-Walker line element, the Einstein equations, including C-field, are:

$$3\,\frac{\dot{S}^2 + k}{S^2} = 8\pi G\left(\rho - \tfrac{1}{2} f \dot{C}^2\right) , \tag{13}$$

$$2\,\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + k}{S^2} = 4\pi G f \dot{C}^2 . \tag{14}$$

(In a completely homogeneous and isotropic situation, only the time derivative of C can be non-zero.) For baryon conservation,

$$C^k{}_{;k} = \ddot{C} + 3\,\frac{\dot{S}}{S}\,\dot{C} = 0, \qquad \text{i.e.,} \quad \dot{C} = \frac{A}{S^3} \tag{15}$$

where A = constant. The constant A is non-zero; for, otherwise $\dot{C} = 0$ and the C-field makes no contribution at all and we should have the classical

Friedmann cosmologies. With $A \neq 0$ we have,

$$\dot{S}^2 = -k + \frac{B}{S} - \frac{1}{2} f \frac{A^2}{S^4} \quad . \tag{16}$$

In the case $k = 1$, $f > 0$, there are two roots for $\dot{S} = 0$, provided the maximum value of the right-hand side of equation (16) is positive. So long as the coupling constant f is positive, we have a situation in which the $-k$ term produces the switch from expansion to contraction at maximum S and in which the C-field term produces the switch from contraction to expansion at minimum S.

This avoidance of the Friedmann-type cosmological singularity suggests that singularities of the Oppenheimer-Snyder type for a local imploding body might also be prevented by the C-field terms in the gravitational equations. We have investigated this problem in a previous paper (Hoyle and Narlikar 1964d) and have shown that singularities are indeed prevented. The C-field terms thus resolve the otherwise very worrying question of singularities in the theory of relativity.

However, while the above scheme is satisfactory when applied to a local imploding body, it is not so when applied to the universe as a whole. The C-field was introduced into equations (13), (14), and (16) as a solution of the homogeneous wave equation. In our view the C-field must arise from sources, in accordance with equation (3). Writing N for the proper density in the spaces t = constant, we have

$$n = \dot{N} + 3 \frac{\dot{S}}{S} N \quad . \tag{17}$$

Equation (3) then gives

$$\ddot{C} + 3 \frac{\dot{S}}{S} \dot{C} = \frac{1}{f} (\dot{N} + 3 \frac{\dot{S}}{S} N) \; , \tag{18}$$

which integrates to

$$\dot{C} = \frac{N}{f} + \frac{A}{S^3} \quad , \tag{19}$$

in which A is a constant of integration. Equation (19) contains the classical Friedmann cosmologies ($\dot{C} = 0$) and the case discussed above ($\dot{C} \propto S^{-3}$) as special cases. There is a further possibility contained in equation (19), which emerges in the following way. As an illustration we consider the case $k = 0$. Define m_o by

$$m_o = \rho/N \quad , \tag{20}$$

and define H, T, ξ, u by

$$3H^2 = 4\pi G f m_o^2 \quad , \qquad T = 3\,Ht \quad ,$$

$$\xi = S^3 \quad , \qquad u = \left(\frac{d\xi}{dT}\right)^2 + \frac{2A\xi}{m_o} \quad . \tag{21}$$

Eliminate $\dot{C}, \rho$ between the equations (13), (14) and (19). The result can be expressed in the form

$$\frac{1}{4}\left(\frac{du}{d\xi}\right)^2 = \xi \frac{du}{d\xi} - u \quad . \tag{22}$$

The zero of T can be chosen so that $\xi\,(T = 0) = 1$. Assigning a numerical value for $u(T = 0)$ allows equation (22) to be solved for $du/d\xi$ at $T = 0$. Either solution of the quadratic can be followed by a numerical integration to give $u(T)$, so that we have a one-parameter family of solutions corresponding to the choice of u at $T = 0$. The analytical solution is

$$u = 3\lambda\xi - \lambda^2 \quad , \tag{23}$$

where λ is an assignable parameter. Insertion of equation (23) in the definition of u gives a differential equation for $\xi(T)$. The solution of this equation is

$$\xi = \frac{\lambda^2}{\lambda - A} + (\lambda - A)(T - T_o)^2 \quad . \tag{24}$$

The case $\lambda = 0$ gives the classical Friedmann cosmology corresponding to $k = 0$ (the so-called Einstein - de Sitter cosmology). For $\lambda \neq 0$, we get the case described above where $\dot{C} \neq 0$, $C^k{}_{;k} = 0$.

Apart from these solutions, a new solution is obtained by considering the singular solution of equation (22):

$$u = \xi^2 \quad . \tag{25}$$

This leads to

$$\xi = A\left[1 + \cosh\,(T - T_o)\right] \quad . \tag{26}$$

Without loss of generality we can take $T_o = 0$ so that

$$\xi = S^3 = A\,(1 + \cosh T) \quad . \tag{27}$$

For this solution $\dot{C} = m_o$ and $C_k\, C^k = m_o{}^2 = (\rho/N)^2$, which is condition (4) in a different notation. We also have

$$\rho = fm_o{}^2 \cosh T/(1 + \cosh T). \tag{28}$$

The universe tends to a steady state as $T \to \pm\,\infty$, and $\rho \to 3H^2/4\pi G$. The point $T = 0$, at which $\rho = 3H^2/8\pi G$ is minimum, is a point of symmetry as shown in Fig. 1.

In a previous paper (Hoyle and Narlikar 1963) the possibility of electrodynamic and thermodynamic arrows of time being directed in the manner shown in Fig. 1 was considered. An observer at a random point on the T-axis will most likely be at $T \pm \infty$, i.e., in one or other of the two steady states.

In this way it was suggested that a completely time-symmetric universe could show an apparently asymmetric time direction to a particular observer. If by some 'magic', two observers, one at each end of the time axis, could compare notes, they would find that while one had a world of 'matter', the other had a world of 'anti-matter', and that the Co^{60} experiment showed opposite handedness in the two cases.

The situation concerning the non-conservation of baryons is shown in Fig. 2. The 'ends' of world lines are also symmetrically distributed with respect to $T = 0$. If the particles to the left of $T = 0$ are taken with those on the right, there is an obvious sense in which conservation applies. As many 'ends' lie to the left as to the right. The C-field supplies the connection between the ends according to the definition (1). The relation $p_i = C_i$, which requires conservation of energy-momentum at each end, includes the total C-field, however. The situation therefore is analogous to the phonon problem in a crystal rather than to the emission or absorption of a single quantum.

In the asymptotic case, we have

$$\frac{\dot{S}}{S} = H, \quad \rho = m_o N = f m_o^2 = \frac{3H^2}{4\pi G} \quad . \tag{29}$$

The source-equation for the C-field has the form

$$C^k{}_{;k} = 3H\, m_o \quad . \tag{30}$$

It is easy to verify that $C = m_o t$ is a solution of equation (30). Also, any other time dependent solution of (30) tends asymptotically to the same solution.

The point we wish to make is that the solution $C = m_o t$ arises from the expansion of the universe. In ordinary flat-space physics, a constant source term in the inhomogeneous wave equation gives a solution proportional to t^2, not to t.

The above analysis applies to the case $k = 0$. The asymptotic solutions for the cases $k = \pm 1$ are unchanged, since the terms involving k tend to zero as $S \to 0$.

Finally, we resolve an apparent paradox. Equation (13) is the (4,4) component of the gravitational equations, and the right-hand side is the (4,4) component of the energy-momentum tensor in equation (10). According to equation (4), energy is conserved at the 'end' of a world line. The (4,4) component is therefore unchanged by the non-conservation of baryons. How then can the expansion rate $\dot{S}$ of the universe be in any way affected? The answer lies in the differing subsequent behavior of the two initially equal terms that appear on the right-hand side of equation (13). Suppose non-conservation causes terms Δ to be added to ρ and to $\frac{1}{2} f \dot{C}^2$. Initially the two terms cancel, but that added to ρ decreases as S^{-3}, while that added to $\frac{1}{2} f \dot{C}^2$ decreases as S^{-6}, as S increases. Subsequently the term added to ρ is greater in magnitude than that added to $\frac{1}{2} f \dot{C}^2$, so that the effect of $\Delta > 0$ is to increase $\dot{S}^2$. In the asymptotic case the average effect of the terms added to ρ is exactly twice the effect of the terms added to $\frac{1}{2} f \dot{C}^2$ – i.e., $\rho = f \dot{C}^2$. At the point of symmetry $T = 0$; on the other hand, the terms added to ρ do cancel those added to $\frac{1}{2} f \dot{C}^2$, and $\rho = \frac{1}{2} f \dot{C}^2$ at this point.

NON-HOMOGENEITY – QUALITATIVE CONSIDERATIONS

In the above homogeneous case we had $C_i = (0,0,0,\dot{C})$ and $p_i = (0,0,0,m_o)$, $C_k C^k = m_o^2$. The energy-momentum condition necessary for creation is satisfied at all points of space-time. At what stage of the argument have we introduced this condition into the theory? The answer is by our choice of the singular solution of equation (22). If instead of choosing the singular solution, we had imposed $C_k C^k = m_o^2$, we should have arrived at the singular solution. (In the homogeneous case $C_k C^k = m_o^2$ leads to $\dot{C} = m_o$ and $C^k_{;k} = 3m_o \dot{S}/S \neq 0$, when $\dot{S} \neq 0$. The singular solution of equation (22) is the only solution for which $C^k_{;k} \neq 0$). Thus so long as we are considering the homogeneous-isotropic case the two conditions — choice of a singular solution or the requirement $C_k C^k = m_o^2$ — are equivalent.

The situation is changed, however, as soon as inhomogeneities are admitted. There is no longer a unique solution with $C^k_{;k} \neq 0$. The C-particles would gain or lose energies as they travel through the gravitational fields of inhomogeneities, in a manner analogous to the behavior of light quanta in gravitational fields. Consider a spherically symmetrical concentration of mass M. A light quantum falling from large distance to distance R has its frequency increased by the factor $(1 - 2GM/R)^{-\frac{1}{2}}$. Similarly, if $C_k C^k = m^2$ at large distance, then at distance R from M, we expect

$$C_k C^k = m^2\left(1 - \frac{2GM}{R}\right)^{-1} . \tag{31}$$

This conjecture will be proved in the next section. Equation (31) applies to the field incident at large distance, and does not include the field arising from any baryon non-conservation in the locality of M.

Let m_o be the least energy required for the creation of a baryon-lepton pair. If $m < m_o$, there can be no creation in the smooth-fluid cosmological sense, but there can be creation near M, provided the radius of the mass concentration permits $2GM/R$ to satisfy the inequality

$$\frac{2GM}{R} > 1 - \left(\frac{m}{m_o}\right)^2 . \tag{32}$$

Plainly, we no longer have the yes/no situation in which we either have creation or not, and in which, if we have creation, the rate is precisely determined everywhere by the coupling constant f. To obtain anything different from the classical Friedmann cosmologies, we must still make the postulate that creation/destruction can occur if $C_k C^k$ exceeds the threshold m_o^2; but granted this postulate there is now an infinity of possibilities depending on the value of m and the distribution of inhomogeneities. Galaxies, and highly collapsed objects such as are believed to be present in radio sources, would be expected to play an important role in the creation process. We shall call such mass concentrations 'pockets' of creation.

Given a value of $m < m_o$, together with any distribution of pockets, with equation (32) satisfied sufficiently close to them, it is clear that there will be a time interval Δt such that in that time interval as much mass will be created as was originally present in the pockets. A new kind of 'steady-state' expansion of the universe is then possible, with

$$\frac{\dot{S}}{S} = H = (\Delta t)^{-1} \quad . \tag{33}$$

The expansion rate depends on m and on the number and nature of the pockets — not on the coupling constant f.

The term 'steady-state' is something of a misnomer here since it does not follow that the universe necessarily takes up a steady-state situation for a given m, and a given distribution of pockets. If at some stage $\Delta t \ll (\dot{S}/S)^{-1}$, we have a distribution of pockets giving a rapid creation rate that in turn would set up an increase in $\dot{S}/S$, until the system was brought into an approximately steady state. But if $\Delta t \gg (\dot{S}/S)^{-1}$, the pockets would be expanded apart before they had sufficient time to reproduce themselves. The universe, in such a case, would tend asymptotically to a classical Friedmann cosmology with no creation of matter. We shall return to this question at a later stage.

The present concepts relate the expansion of the universe intimately to the inhomogeneities in the mass distribution. This appears to us to be an important advance on the previous situation in cosmology. It is now more than forty years since Friedmann obtained his cosmological models, and in this considerable span of time the progress made in understanding the relation between cosmology and astrophysics has been essentially nil. The best that can be done is to pass off the galaxies as chance fluctuations, the problem being treated in the following way. For any observer at rest relative to the cosmological stratum of gas, choose coordinates so that space-time is locally flat, i.e.,

$$ds^2 = dt^2 - (dr^2 + r^2\, d\Omega^2) + O_4 \tag{34}$$

$\underline{r}$ being a Euclidean radial coordinate measured from the observer. O_4 represents terms of fourth order, arising from the curvature of space-time.

It can be shown that the local behavior of the gas relative to the observer is determined by the equation

$$\ddot{r} = -\frac{4\pi}{3} G\rho r \quad , \tag{35}$$

where ρ is the gas density. This is exactly the Newtonian equation for the expansion of a gas cloud, and with $\rho \propto r^{-3}$, an integral is immediately obtained in the form

$$\dot{r}^2 = \frac{2\mu}{r} + k \tag{36}$$

where μ is a constant depending on the constant of proportionality in $\rho \propto r^{-3}$ and k is a constant of integration. If $k < 0$ the cloud eventually falls back on itself, if $k \geq 0$ it expands to infinity. When k is small and negative, slight changes in k can give large changes in the value of r at which the cloud reverses its radial motion. Slight local changes in k could yield local condensations in an expanding cosmological stratum of gas. But there is no suggestion here of the tightly controlled situation apparently demanded by observation. Galaxies occur typically in small groups of about twenty members, each group usually possessing one member that is brighter than the rest, by one or two magnitudes. The brightest member is frequently an elliptical of the early type, E0 to E2, the others are often spirals. Sandage has pointed out the remarkable fact that the brightest members in small groups attain closely the same luminosity as the brightest members of major clusters. There appears to be a well-defined upper limit of luminosity that is attained regardless of whether rich or poor clusters be considered, which is not at all the situation that would be expected if inhomogeneities arose from random fluctuations of the constant of integration $\underline{k}$.

The homogeneous steady-state theory has prompted certain fruitful lines of investigation in astrophysics. Considerable effort has been made to discuss the problem of galaxy formation within the framework of the homogeneous theory described in the previous section. However, this has proved to be the least successful part of the theory. Apparently hopeful new concepts, e.g., the "hot universe" — have presented themselves from time to time, but the resulting investigations have led to difficulties just as formidable as those it was hoped to resolve. By abandoning the homogeneous theory along the lines indicated above, the situation is changed in a drastic way, since condensations enter the theory from the outset. The expansion of the universe is related immediately to the inhomogeneities themselves, so that cosmology and astrophysics are linked from the beginning. Ideas of a similar general nature have recently been put forward by McCrea (1964), who has emphasized exactly these latter points.

The mathematical structure of the theory is immediately more difficult, since we are now involved in partial differential equations from the outset. This causes the simple characteristics of the steady-state theory to be lost. This may seem a disadvantage to many who feel these simple characteristics to be an attractive feature of the theory. But we see no reason why the mathematical structure of the large scale properties of the universe should be any simpler than that of such a minor phenomenon as the Falls of Niagara.

NON-HOMOGENEITY — QUANTITATIVE CONSIDERATIONS

Suppose we have baryon-lepton pairs, $P_o, P_1, P_2, \ldots$, say, with energy thresholds, $m_o, m_1, m_2, \ldots$, in increasing order. Suppose further that the average cosmological value of $C_i C^i$ is $m^2 < m_o^2$. None of the pairs P_o, $P_1, P_2, \ldots$ can then be created in the average situation. However, near a massive object $C_i C^i$ is increased and creation of $P_o, P_1, P_2, \ldots$ can take place, provided the relativistic parameter, $2GM/R$, is sufficiently large. This will now be shown.

Consider a spherical object of mass M and radius R_b. The line-element outside the object is described in terms of Schwarzschild coordinates R, Θ, φ, T by

$$ds^2 = e^{\nu}\, dT^2 - e^{\lambda}\, dR^2 - R^2\, d\Omega^2 \quad . \tag{37}$$

At large distance from the object the line element is essentially of the homogeneous cosmological form, which for the steady-state cosmology is

$$ds^2 = (1 - H^2R^2)\, dT^2 - \frac{dR^2}{1 - H^2R^2} - R^2\, d\Omega^2 \quad , \tag{38}$$

with the C-field given by

$$C = mT + \frac{m}{2H} \ln (1 - H^2R^2) \quad . \tag{39}$$

The time-scale H^{-1} need not be related to the coupling constant f by equation (21) – we are now dealing with a 'steady-state' produced entirely by pocket-creation. The terms in H do not enter the following argument.

The C-field incident on the object satisfies the homogeneous wave equation, $C^k{}_{;k} = 0$, i.e.,

$$\frac{\partial}{\partial T}\left[e^{-\frac{\nu-\lambda}{2}} R^2\dot{C}\right] = \frac{\partial}{\partial R}\left[e^{\frac{\nu-\lambda}{2}} R^2 C'\right]; \qquad \dot{C} = \frac{\partial C}{\partial T},\ C' = \frac{\partial C}{\partial R}. \tag{40}$$

To calculate the C-field near the object we shall assume that the metric outside the object is not significantly affected by the C-field terms. (The validity of this assumption will be examined at a later stage.) The e^ν, e^λ are then given by

$$e^\nu = e^{-\lambda} = 1 - \frac{2GM}{R}, \tag{41}$$

and equation (40) becomes

$$\ddot{C} = \frac{1}{R^2}\left(1 - \frac{2GM}{R}\right)\frac{\partial}{\partial R}\left[\left(1 - \frac{2GM}{R}\right)R^2 C'\right]. \tag{42}$$

This partial differential equation may be solved by a separation of variables. Writing

$$C = X(T)\,Y(R), \tag{43}$$

X, Y satisfy the ordinary differential equations

$$\frac{d^2X}{dT^2} = \alpha X, \quad \frac{d}{dR}\left[\left(1 - \frac{2GM}{R}\right)R^2\frac{dY}{dR}\right] = \alpha R^2\left(1 - \frac{2GM}{R}\right)^{-1} Y, \tag{44}$$

in which α is an arbitrary constant. The general solution of equation (40) is obtained by a superposition of solutions (43) for different α. We require, however, a solution which can be matched to equation (39) as R tends to the finite value, small compared to H^{-1}, at which we consider the effect of the object to be negligible, i.e., $2GM/R$ very small. Clearly only $\alpha = 0$ is admissible. For $\alpha \neq 0$, X varies exponentially or sinusoidally with T and such a solution cannot be matched to equation (39), since the matching must apply at all values of T.

With $\alpha = 0$ we get the required solution as

$$\dot{C} = m \quad , \quad C' = -\frac{Am}{R^2}\left(1 - \frac{2GM}{R}\right)^{-1} , \tag{45}$$

in which $\dot{C} = m$ is chosen to match the cosmological solution (37) which also has $\dot{C} = m$. Equation (45) applies only in the locality of M. The quantity A is a constant of integration. Near the object, therefore, we have

$$C_k C^k = m^2 \frac{1 - A^2/R^4}{1 - 2GM/R} = m^2 K(R) \text{ , say.} \tag{46}$$

If we set $A = 0$, we have the result conjectured in equation (31). The next step is to understand the physical significance of A and to see why $A = 0$ if there is no creation/destruction.

Consider equation (45) in the case of weak gravitational fields. Then we get

$$C = mT + \frac{Am}{R} \quad . \tag{47}$$

This satisfies the wave equation $C^k{}_{;k} = 0$ in flat-space. There is a mathematical equivalence between the present case and that of the potential φ in electrostatics. The solution $\varphi = Am/R$ requires an electric charge Am interior to R. In the present problem the term Am/R represents a rate of creation of $4\pi Amf$ baryons in the interior to R, as seen by using the inhomogeneous wave equation (3).

The same interpretation of A holds when the gravitational field is not weak. Suppose N particles are created per unit proper time per unit area of a thin shell of radius R_1. The C-field arising from this creation satisfies the inhomogeneous wave equation

$$f\, C^k{}_{;k} = N\, \delta(R - R_1)\ . \tag{48}$$

In order that the C-field shall match a solution of the form (45) for $R > R_1$, it is necessary that $\dot{C} = m$. This gives

$$C' = -\frac{\text{constant} + \theta(R - R_1)\, N\, R_1{}^2\, f^{-1}}{R^2\,(1 - 2GM/R)}\ , \tag{49}$$

in which

$$\theta(x) = \begin{cases} +1 & \text{if} \quad x > 0\ , \\ 0 & \text{if} \quad x < 0\ . \end{cases} \tag{50}$$

The effect of creation in the shell is therefore to change the parameter A. As R increases through R_1, A is increased by $NR_1^2 f^{-1} m^{-1}$. Since the number created in the shell is $4\pi NR_1^2$, we have

$$\begin{array}{l}\text{Number of particles created} \\ \text{per unit proper time}\end{array} = 4\pi m f\ (\text{change in } A). \tag{51}$$

The parameter A represents the cumulative effect of creation in all such shells. In the absence of creation, $A = 0$, and equation (46) reduces to the conjecture (31).

Before using the above results to estimate possible creation rates there are two issues we wish to discuss, even though the second issue — the meaning to be attached to the term 'Schwarzschild radius' — requires a considerable interruption of our main argument.

The first issue is that of creation/destruction. In terms of our electrostatic analogy, we can regard the C-field generated from creation as corresponding to the field of positive charge and that arising from destruction as corresponding to the field of negative charge. Why, if both creation and destruction are possible, do we speak only of creation? For a baryon-lepton pair of threshold energy m_o, if the gradient of the C-field C_i satisfies $C_iC^i > m_o^2$, it is possible for such a pair to appear with the appropriate momentum $p_i = C_i$. For destruction, on the other hand, a baryon and lepton must arrive together at a place with total momentum $p_i = C_i$. In thermodynamic equilibrium the two processes would, of course, balance each other. However, just as in the electromagnetic case a thermodynamic equilibrium is not reached between matter and radiation in the universe, in the present case we similarly think that creation predominates over destruction in the cosmological situation. Indeed, so long as there is no strict detailed balancing, we can always choose the sense of our time axis so that there is a preponderance of 'creation'. In making these remarks we do not exclude the possibility that in special localized regions there could be an approximately thermodynamic situation for the C-field, just as there is in stars for the electromagnetic field.

The Schwarzschild radius is usually defined by $R_s = 2GM$ where M is given by

$$M = 4\pi \int_0^{R_b} \underset{m}{T}{}_4^4 \, R^2 \, dR \quad , \tag{52}$$

$\underset{m}{T}{}_4^4$ being the mixed (4,4) component of the energy-momentum tensor of matter. When the matter tensor is the only tensor on the right-hand side of Einstein's equations, this is a sensible procedure. When there are other energy-momentum tensors present, their contribution must also be included in the definition of

R_s. When the C-field is present we have

$$R_s = 8\pi G \int_0^{R_b} \left(\underset{m}{T_4^4} + H_4^4 \right) R^2 \, dR; \qquad H_4^4 < 0. \tag{53}$$

The C-field reduces the effective mass and hence decreases the effective Schwarzschild radius. Now when equation (45) is used to construct H_4^4, we obtain

$$H_4^4 = - fm \frac{1 + A^2/R^4}{1 - 2GM/R} \quad . \tag{54}$$

Since $\int H_4^4 R^2 \, dR$ diverges logarithmically at $R = 2GM$, it is clear that we have the following situation. As R_b decreases towards $2GM$, the H_4^4 term increases in importance, and for R_b sufficiently close to $2GM$, $R_s \to 0$. We are here regarding M as still being given by equation (52), but of course the present argument does not possess strict quantitative validity when H_4^4 becomes comparable with $\underset{m}{T_4^4}$, because equation (45) for the C-field does not include the effect of the C-field on the line-element. What the argument does show, however, is that as R_b decreases toward the usual Schwarzschild radius, the C-field contribution to the energy-momentum tensor has an important effect on the line element and that the Schwarzschild radius, defined by equation (53), is reduced below the usual value.

This clarifies a situation which arose in an earlier paper (Hoyle and Narlikar, 1964d), which was used by Hoyle, Fowler, Burbidge and Burbidge (1964). It was proved quite generally that the C-field prevents an infalling object from imploding into a singularity. The object 'bounces' at a minimum radius which can lie <u>inside</u> the usual Schwarzschild radius. This causes no violation of causality, since the usual Schwarzschild radius, defined in terms of matter alone, is irrelevant here. The oscillating object would always lie outside its <u>effective</u> Schwarzschild radius.

We now return to our main problem of calculating the creation rate near an object of mass M and radius R_b. The function $K(R)$, defined by equation (46), is sketched in Fig. 3 for different values of the parameter A. All curves tend to unity as $R \to \infty$, but their behavior is different as $R \to 2GM$, according as $A \gtreqless (2GM)^2$. For $A < (2GM)^2$, $K(R) \to +\infty$, while for $A > (2GM)^2$, $K(R) \to -\infty$, as $R \to 2GM$. For $A = (2GM)^2$, $K(R) \to 4$ as $R \to 2GM$. In the case $A > (2GM)^2$ the curves rise from $-\infty$ to a positive maximum and then fall back to unity, as R increases from $2GM$ to infinity. A horizontal line drawn at $(m_o/m)^2$ above the R-axis separates pairs of values A, R for which creation can take place (the portions of the curve that lie above the line) from pairs A, R for which creation cannot take place (curves or portions of curves that lie below the line).

We proceed by calculating the maximum possible rate of creation. Our postulate for this is that once $C_k C^k \geq m_o^2$, creation proceeds as fast as it can. Since we are considering the case $m < m_o$, there will be a radius R_c such that $C_k C^k < m_o$ for all $R > R_c$, but $C_k C^k \geq m_o^2$ for R immediately interior to R_c. Evidently $C_k C^k = m_o^2$ at $R = R_c$. From equation (46) we get A as a function of R_c:

$$\left[\frac{A}{(2GM)^2}\right]^2 = \left(\frac{R_c}{2GM}\right)^4 \left[1 - \frac{m_o^2}{m^2}\left(1 - \frac{2GM}{R_c}\right)\right] . \qquad (55)$$

The total creation rate in the interior of R_c is $4\pi fmA$ baryons per unit time. The maximum creation rate is obtained by choosing the maximum value of A consistent with equation (55). Let $A = A_c$ denote this maximum. Then A_c and the corresponding R_c are given by

$$A_c = \frac{3(3)^{\frac{1}{2}}}{4} (GM)^2 \frac{m_o}{m}\left(1 - \frac{m^2}{m_o^2}\right)^{-3/2} , \qquad (56)$$

$$R_c = \frac{3GM}{2} \left(1 - \frac{m^2}{m_o^2}\right)^{-1} . \qquad (57)$$

The corresponding creation rate in terms of mass is given by

$$Q = 4\pi f m m_o A_c = 3(3)^{\frac{1}{2}} \pi f m^2 (GM)^2 \left(1 - \frac{m^2}{m_o^2}\right)^{-3/2} . \qquad (58)$$

This is the maximum creation rate. The theory does not guarantee that every massive object would necessarily create matter at this rate. Indeed, the full range of cosmological possibilities is bracketed between zero creation rate and the maximum creation rate (58). The interesting fact emerges that the theory does not permit arbitrarily large creation rates.

We anticipate the conclusion that in order for creation to be cosmologically significant m must be close to m_o, the average cosmological value of $C_k C^k$ should not be much less than the creation threshold. Writing

$$m_o = m(1 + \eta) , \quad 0 < \eta \ll 1 , \qquad (59)$$

and using equation (21) to relate f and m (a good approximation in the steady-state situation), equation (58) becomes

$$Q = \frac{9(3)^{\frac{1}{2}}}{8(2)^{\frac{1}{2}}} GM^2 H^2 \eta^{-3/2} , \qquad (60)$$

a result very sensitive to η.

It is of interest to compare this result with the creation rate for the homogeneous steady-state theory, in the following way. The average cosmological mass density in the latter is $3H^2/4\pi G = f m_o^2$ from equation (21). The volume occupied by a mass M of this homogeneous cosmological matter is $M/f m_o^2$. The creation rate per unit volume is $3Hf m_o^2$. The creation rate in the volume occupied by M is therefore just $3HM$. The ratio of the creation rate given by equation (60) to this is

$$\frac{Q}{3HM} = \frac{8(2)^{\frac{1}{2}}}{3(3)^{\frac{1}{2}}} \left(\frac{GM}{H^{-1}}\right) \eta^{-3/2} \quad . \tag{61}$$

Now H^{-1} is the cosmological distance scale, and the amount of matter contained in a sphere of radius H^{-1} and density $fm_o{}^2$ is

$$\mathcal{M} = \frac{4\pi}{3} H^{-3} fm_o{}^2 = \frac{H^{-1}}{G} \quad . \tag{62}$$

Therefore the ratio in equation (61) is of the order $M/\mathcal{M} \cdot \eta^{-3/2}$. We see that the creation rate associated with a localized mass M is less than that which would be associated with the volume occupied by mass M in the homogeneous theory by the ratio $M/\mathcal{M}$, but is greater by the factor $\eta^{-3/2}$. For η sufficiently small the creation rate is more in the inhomogeneous case.

In the homogeneous case the mass M reproduces itself in time $1/3\ H^{-1}$. In the inhomogeneous case therefore, for M to double itself in the cosmological time scale, we must have

$$\eta^{-3/2} > \sim \mathcal{M}/M \ . \tag{63}$$

Numerically $\mathcal{M} \approx 10^{23}\ M_\odot$. Since a typical inhomogeneity consists of a small cluster of galaxies, probably with total mass $\sim 10^{12}\ M_\odot$, the factor $M/\mathcal{M} \approx 10^{-11}$. We require $\eta \approx 10^{-8}$ in order that inhomogeneities shall play a critical role. Creation occurs out to the radius R_c given by equation (57), which, for $\eta \ll 1$, takes the form

$$R_c \approx \frac{3}{4} \frac{GM}{\eta} \quad . \tag{64}$$

Putting $\eta = 10^{-8}$, $M = 10^{12}\ M_\odot$ gives $R_c \approx 10^{25}$ cm, about 3 megaparsecs, consistent with the radius of a cluster of galaxies.

The above considerations are based on the assumption that newly created matter escapes from the pocket. In general it seems more likely that the gravitational field of the pocket would hold most of the matter created. Thus leptons may escape but baryons would remain with the pocket. This has the effect of increasing M. The C-field generated by creation is, however, radiated away and contributes towards the cosmological energy density of the C-field. This will maintain the value of $C_k C^k$ at m^2. Thus in this theory we cannot impose a fixed average value of m^2. Both m and η must be calculable in terms of the C-field radiation from all inhomogeneities taken together. A more complete theory would require

(i) that the average spatial arrangement of inhomogeneities be maintained,

(ii) that there be self-consistency in the value chosen for η, in the sense that η affects the creation rate for all inhomogeneities and creation rates for the inhomogeneities, taken together, determine η,

(iii) that the self-consistent solution be stable.

We think that (ii) can be satisfied for any assigned distribution of inhomogeneities. The creation rate is very sensitive to η but the level of $C_k C^k$ depends on $(1 + \eta)^{-2}$ and is not sensitive to variations in η when $\eta \ll 1$.

The situation is less clear about (i) and (iii). If created matter is retained in the pocket the mass grows at the rate

$$\frac{dM}{dT} = \left(\frac{9(3)^{\frac{1}{2}}}{8(2)^{\frac{1}{2}}} \frac{G H^2}{\eta^{3/2}}\right) M^2 = \alpha M^2, \text{ say.} \qquad (65)$$

The solution of equation (65) is

$$M(T) = M_i (1 - \alpha M_i T)^{-1} , \tag{66}$$

where M_i is the initial value of M. Thus the mass grows to infinity in time $(\alpha M_i)^{-1}$ — only twice the time required for the mass to double its original value. The growth of the pocket is therefore an unstable process. It seems likely that the 'runaway' solution (66) will be prevented by a break-up of the growing mass into smaller inhomogeneities, which in turn would serve as individual pockets of creation. In this way it might be possible to satisfy both (i) and (iii).

This picture is essentially the one advanced by McCrea (1964). Our quantitative considerations show the picture to be possible, but the situation relating to the growth and fragmentation of inhomogeneities remains qualitative. An important feature of the picture is that the expansion rate of the universe turns out to be determined by the coupling constant f, the constant H being essentially determined by equation (21). We do not have the possibility discussed briefly in the previous section where the creation process may stop because of a sudden drop in the pocket creation. This is because η — the difference from the threshold value — is very small and $C_k C^k$ differs from m_o^2 by only a small amount. The cosmological situation is therefore much the same as it is in the homogeneous theory.

In a later paper we explore the applications of the concept of pocket creation to the production of high energy particles and to the energy output of radio galaxies. It turns out that these and other interesting phenomena associated with galaxy formation could be accounted for by means of creation induced by inhomogeneities, provided the picture of gentle creation suggested here is abandoned. This leads to a radical departure from the steady-state cosmology.

APPENDIX

The properties of the C-field can be derived from the principle of direct interparticle action. The action function is formally written as

$$J = \frac{1}{16\pi G}\int R(-g)^{\frac{1}{2}}\, d^4x - \sum_a \int m\, da + f^{-1}\sum_{a<b}\sum \tfrac{1}{2}\iint \bar{G}_{;i_A i_B}\, da^{i_A}\, db^{i_B} \quad . \tag{A1}$$

In this formula, the particle world lines are labelled by letters <u>a</u>, <u>b</u>, ... A typical point on the world line <u>a</u> is denoted by <u>A</u>. The proper time at A along the world line <u>a</u> is given by the line element

$$da^2 = g_{i_A k_A}\, da^{i_A}\, da^{k_A} \quad , \tag{A2}$$

where g_{ik} describe the Riemannian geometry of space-time.

The first term in the action is gravitational and the second inertial. The third term, which has the form of interactions between particle pairs, is the C-field term with a coupling constant f. Actually the first two terms in the action can be combined into a single term which has the form of direct particle action. The action then takes the form

$$J = \sum_{a<b}\sum \iint \tilde{G}(A,B)\, da\, db + f^{-1}\sum_{a<b}\sum \tfrac{1}{2}\iint \bar{G}(A,B)_{;i_A i_B}\, da^{i_A}\, db^{i_B} \quad . \tag{A3}$$

This leads to a new theory of gravitation (cf. Hoyle and Narlikar 1964e). For the purposes of this paper, however, the results are identical to those given by equation (A1). We shall therefore use (A1), although it is less satisfactory than (A3) from a formal point of view.

The interaction between A,B is described by the Green's function $\bar{G}(A,B)$ which is the elementary symmetric solution of the wave equation

$$g^{i_A k_A}\, \bar{G}(A,B)_{;i_A k_A} = -\frac{\delta^{(4)}(A,B)}{[-\bar{g}(A,B)]^{\frac{1}{2}}} \quad . \tag{A4}$$

In equation (A4), $\delta^{(4)}(A,B)$ is the 4-dimensional delta function corresponding to points A,B. $\bar{g}(A,B)$ is the determinant of the parallel propagators $\bar{g}_{i_A i_B}$ (Synge 1960). The explicit form of $\bar{G}(A,B)$ is given by DeWitt and Brehme (1960).

The C-field at X due to the world line a is defined by

$$C^{(a)}(X) = f^{-1} \int \bar{G}(X,A)_{;i_A} \, da^{i_A} \quad . \tag{A5}$$

If the world line has ends at A_1, A_2 we get

$$C^{(a)}(X) = f^{-1} \left[\bar{G}(X,A_2) - \bar{G}(X,A_1) \right] . \tag{A6}$$

The C-field therefore arises only from the ends of world lines. If the end $A_2 \to \infty$ (i.e., the world line is created at A_1) we get $C^{(a)}(X) = - f^{-1} \bar{G}(X,A_1)$. If $A_1 \to -\infty$ (i.e., the world line is destroyed at A_2), $C^{(a)}(X) = f^{-1} \bar{G}(X,A_2)$. Both these results can be summarized into the statement

$$C^{(a)}(X) = - \frac{\epsilon}{f} \bar{G}(X,A) \quad , \tag{A7}$$

where $\epsilon = +1$ if the world line is <u>created</u> at A and $\epsilon = -1$ if it is <u>destroyed</u> at A. The terms creation and destruction have a meaning only in the sense of integration along the particle world lines.

A small variation of the world line of a typical particle a shows that the action is unchanged provided

$$\frac{d^2 a^i}{da^2} + \Gamma^i_{k\ell} \frac{da^k}{da} \frac{da^\ell}{da} \quad ; \tag{A8}$$

along the world line, and also provided

$$\frac{m \, da^i}{da} = \sum_{b \neq a} C^{(b);i} \tag{A9}$$

at the ends of a world line. The C-field therefore does not affect the motion of the particle — it only affects the conditions at the ends. The condition (A9) shows that the energy and momentum of a created particle is balanced by the C-field in a Mössbauer fashion.

The C-field identically satisfies the source equation

$$fC^{(a)k}{}_{;k} = \frac{\delta^{(4)}(X,A_1)}{[-\bar{g}(X,A_1)]^{\frac{1}{2}}} - \frac{\delta^{(4)}(X,A_2)}{[-\bar{g}(X,A_2)]^{\frac{1}{2}}} \quad . \tag{A10}$$

This follows immediately from equations (A4) and (A6). If we add up the contributions from all particles, the total C-field at X is

$$C(X) = \sum_a C^{(a)}(X) \quad , \tag{A11}$$

which satisfies the equation

$$fC^k{}_{;k} = n \quad , \tag{A12}$$

where n = the net number of particles created per unit proper volume per unit proper time. (Destruction is counted as negative creation.)

Finally, a variation of g_{ik} produces a change in the Green's functions and hence a change in the C-field part of the action. Taken together with the changes in the first two terms we get the Einstein equations

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -8\pi G\left(\underset{m}{T}_{ik} + H_{ik}\right) \quad , \tag{A13}$$

where

$$H_{ik} = \sum_{a<b}\sum H^{(a,b)}_{ik} \quad , \tag{A14}$$

with

$$H^{(a,b)}_{ik} = -f\left[C^{(a)}_i C^{(b)}_k + C^{(a)}_k C^{(b)}_i - g_{ik}\, C^{(a)}_\ell C^{(b)\ell}\right] \quad . \tag{A15}$$

Divergence of $H^{ik(a,b)}$ gives

$$H^{ik(a,b)}{}_{;k} = -\tfrac{1}{2} f \left[C^{(a)i} C^{(b)k}{}_{;k} + C^{(b)i} C^{(a)k}{}_{;k} \right] ,$$

i.e.,

$$H^{ik}{}_{;k} = - f \sum_{a} C^{(a)k}{}_{;k} \left(\sum_{b \neq a} C^{(b)i} \right) . \qquad \text{(A16)}$$

This vanishes except at the ends of world lines. The same applies to $T^{ik}{}_{;k}$. At the ends of world lines we have, using equation (A10),

$$H^{ik}{}_{;k} = - \sum_{A} \frac{\delta^{(4)}(X,A)}{(-\bar{g})^{\frac{1}{2}}} \left(\sum_{b \neq a} C^{(b)}(A)^{;i} \right) , \qquad \text{(A17)}$$

where the sum on A includes both ends A_1, A_2 if the world line is finite. We similarly have, at the ends of world lines

$$T^{ik}{}_{;k} = \sum_{A} \frac{\delta^{(4)}(X,A)}{(-\bar{g})^{\frac{1}{2}}} \left(m_a \frac{da^i}{da} \right) . \qquad \text{(A18)}$$

Since $R^{ik} - \frac{1}{2} g^{ik} R$ has zero divergence, we must equate (A17) to (A18) at all ends. This gives

$$m_a \frac{da^i}{da} = \sum_{b \neq a} C^{(b)}(A)^{;i} \qquad \text{(A19)}$$

to be satisfied at the ends. This is precisely the condition (A9) obtained by a variation of the world lines. This is in fact just a verification of the general result that a variation of world lines of particles produces the same equations of motion as are obtained by the divergence of the gravitational equations. It is given here in detail because it has not been discussed before in connection with broken world lines.

REFERENCES

DeWitt, B. A. and Brehme, R. W., 1960, Ann. Phys., 9, 220.

Hoyle, F., 1948, Mon. Not. Roy. Ast. Soc., 108, 372.

Hoyle, F., Fowler, W. A., Burbidge, G. R., and Burbidge, E. M., 1964, Ap. J. 139, 909.

Hoyle, F. and Narlikar, J. V., 1963, Proc. Roy. Soc., A277, 1.

__________ 1964a, Proc. Roy. Soc., A282, 178.

__________ 1964b, Proc. Roy. Soc., A282, 184.

__________ 1964c, Proc. Roy. Soc., A282, 191.

__________ 1964d, Proc. Roy. Soc., A278, 465.

Lee, T. D., 1964 (preprint).

McCrea, W. H., 1964, Mon. Not. Roy. Ast. Soc., 128, 335.

Narlikar, J. V., 1963, Mon. Not. Roy. Ast. Soc., 126, 204.

Synge, J. L., 1960, Relativity: The General Theory (North Holland Publishing Company).

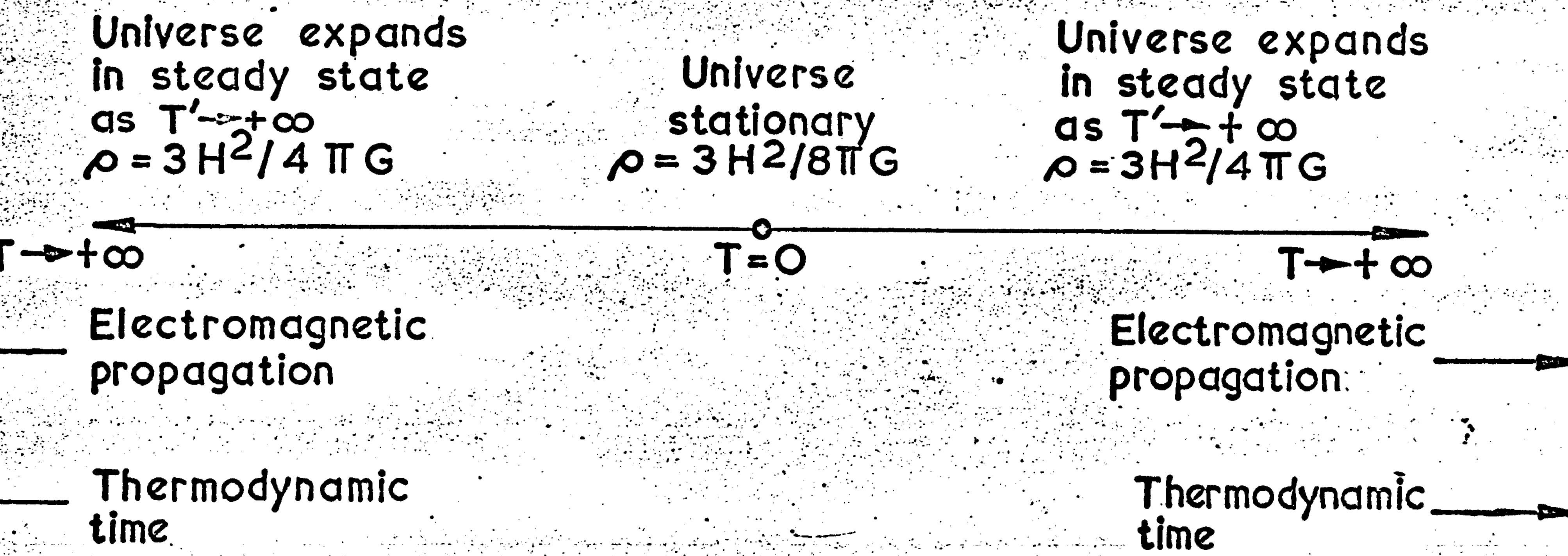
Universe expands
in steady state
as T′→+∞
ρ = 3H²/4πG
Universe
stationary
ρ = 3H²/8πG
Universe expands
in steady state
as T′→+∞
ρ = 3H²/4πG
T→+∞
T=O
T→+∞
Electromagnetic
propagation
Electromagnetic
propagation
Thermodynamic
time
Thermodynamic
time

FIG. 1

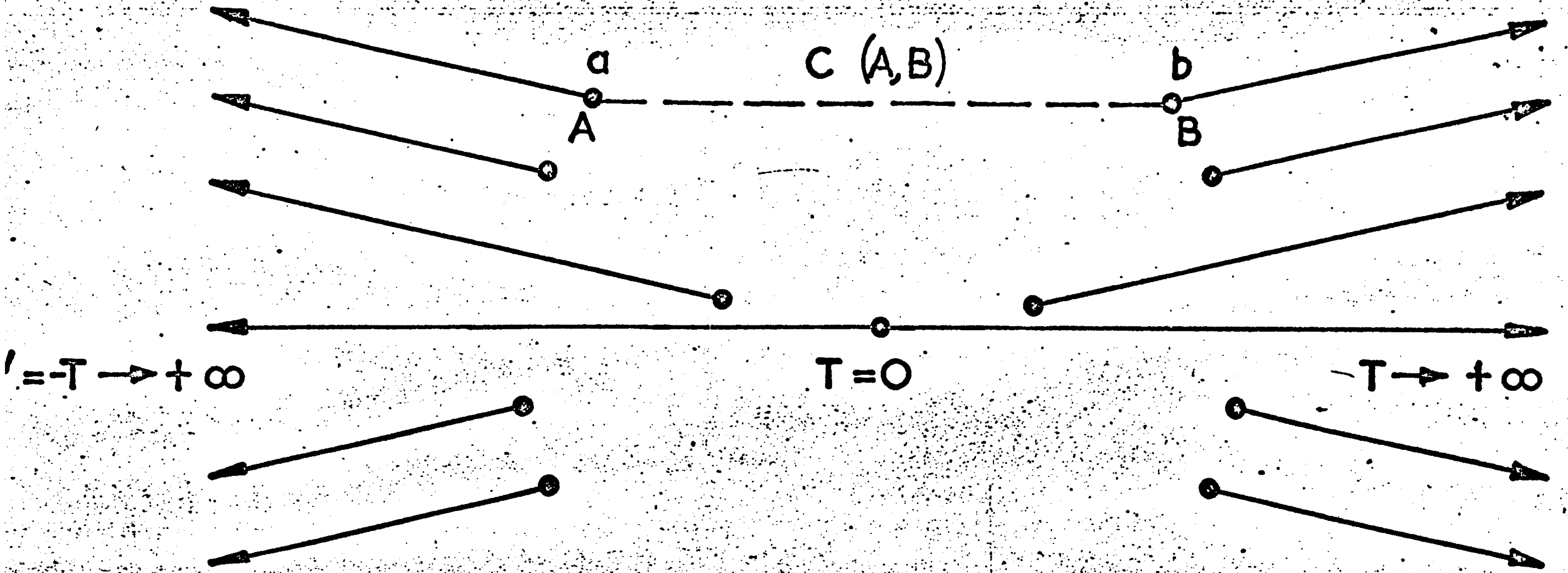

FIG. 2

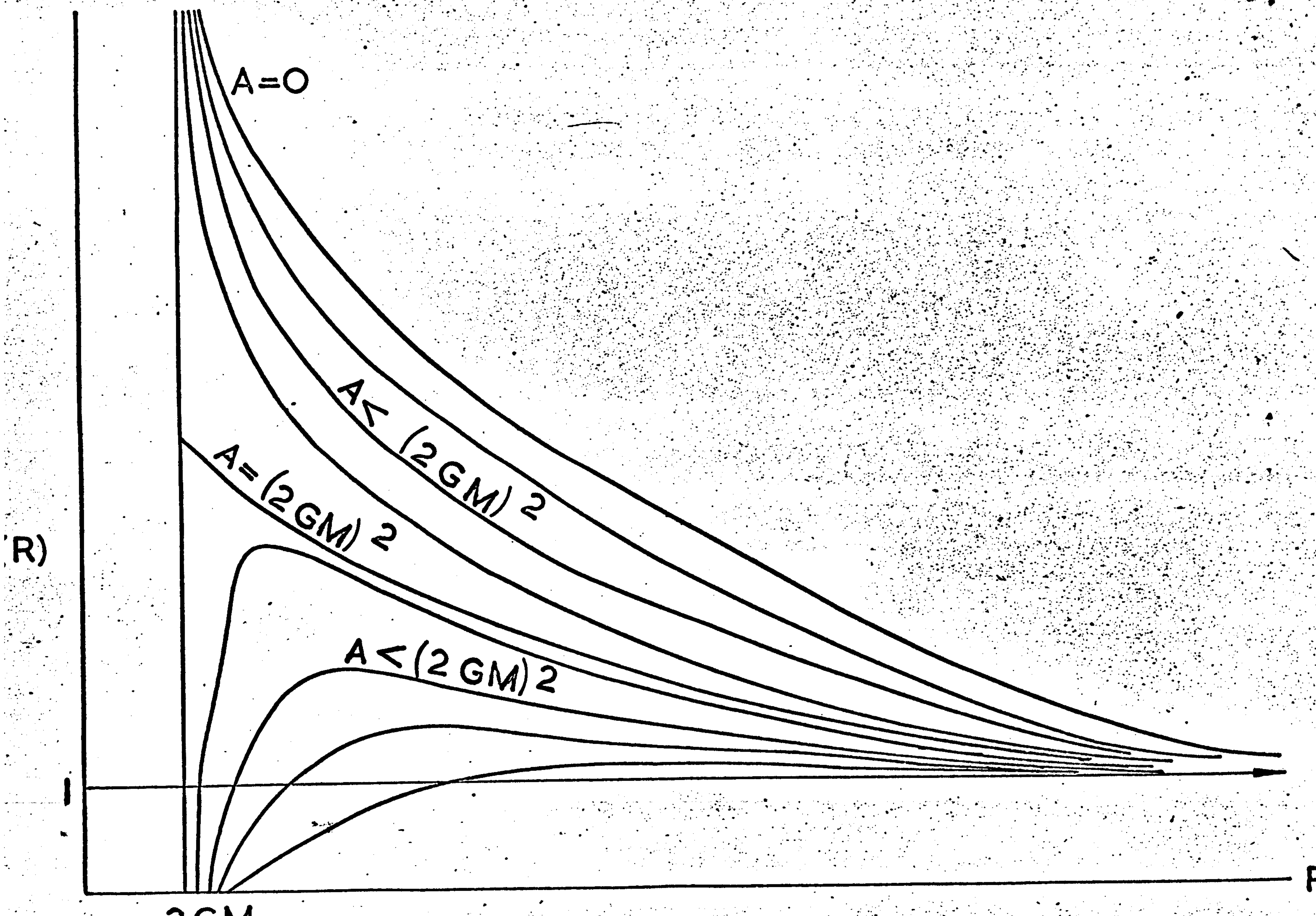

FIG. 3