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DISPLACEMENT THICKNESS OF THE BOUNDARY LAYER WITH BLOWING

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To calculate the inviscid flow field around a body we must know the displacement thickness of the boundary layer on the body as well as the body shape itself. Little consideration seems to have been given, however, to how the displacement thickness of the boundary layer should be defined in cases where fluid is injected at the wall. Mann[1] derived the correct formula for the two-dimensional compressible boundary layer along a flat plate with a large amount of blowing. [Here we consider the displacement thickness for the two-dimensional or axisymmetric compressible boundary layer with either large or small amounts of blowing for bodies of arbitrary shape. It is assumed that no chemical reaction occurs in the boundary layer.]

The coordinates (x, y), in which x is the distance along the wall from the forward stagnation point and y is the distance from the wall, have been used to define displacement thickness as

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \tag{1}$$

considering the effect of retardation due to the viscosity. Here u is the x-component of the velocity, ρ is the density, and the subscript e denotes the value at the outer edge of the boundary layer. This definition is valid

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only if no fluid is injected at the wall. When there is mass injection, we must use the density of the oncoming stream, ρ_1, in place of ρ. This consideration leads to the new definition of the displacement thickness

$$D^* = \int_0^\infty \left(1 - \frac{C\rho u}{\rho_e u_e}\right) dy > \delta^* \qquad (2)$$

where $C = \rho_1/\rho$ is the mass fraction or the concentration of the main-flow fluid in the mixture and varies through the boundary layer.

It should be noted that expressions (1) and (2) are exact for axisymmetric stagnation flows as well as for the general two-dimensional flows. Moreover, they are accurate for the general axisymmetric flows, provided the thickness of the boundary layer, δ, is far less than the distance between the axis and the surface, $r(x)$. In the following, such cases are considered.

To derive the relation between δ^* and D^*, Eq. (2) is rewritten as

$$D^* = \delta^* + (\rho_e u_e)^{-1} \int_0^\infty (1 - C)\rho u \, dy$$

The integral appearing in the right-hand side of this equation represents the mass flow of the injected fluid. For two-dimensional flows this integral is equal to $\int_0^x (1 - C_w)\rho_w v_w \, dx$ where v is the y component of the velocity, and the subscript w denotes the value at the wall. For axisymmetric flows the similar consideration gives

$$2\pi \int_0^\infty (1 - C)\rho u(r + y \cos\theta) dy = 2\pi \int_0^x r(1 - C_w)\rho_w v_w \, dx$$

where θ is the semivertex angle of the tangential cone at the point $(x, 0)$. Since the upper limit of integration appearing on the left side can be replaced by δ and $\delta \ll r$, the second term in the second parentheses is far less than

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the first term. Therefore the above relation reduces to

$$\int_0^\infty (1 - C)\rho u \, dy = r^{-1} \int_0^x r(1 - C_w)\rho_w v_w \, dx$$

Thus we obtain

$$D^* = \delta^* + (r^m \rho_e u_e)^{-1} \int_0^x r^m (1 - C_w)\rho_w v_w \, dx \qquad (3)$$

where $m = 0$ for two-dimensional flows and $m = 1$ for axisymmetric flows.

For the two-dimensional flow along a flat plate, Eq. (3) is reduced to Mann's formula[1] when the amount of injection is sufficiently large that the concentration of the oncoming fluid at the wall, C_w, can be neglected.

As an example, displacement thickness has been calculated for the incompressible stagnation flow, where $u_e = Kx$, K being a constant. In this calculation fluid properties were considered as constant, assuming that the temperature variation is small, and that the injected fluid is the same as the oncoming fluid. Dimensionless displacement thicknesses, $\delta_1 = (K/\nu)^{1/2}\delta^*$ and $D_1 = (K/\nu)^{1/2}D^*$, are plotted versus a dimensionless injection parameter

$$A = \frac{v_w}{(m + 1)(\nu K)^{1/2}}$$

in Figs. 1 and 2. Here ν is the kinematic viscosity of the fluid. We obtain, using Eq. (3),

$$D_1 = \delta_1 + A \qquad (4)$$

Pretsch[2] obtained the asymptotic velocity profiles

$$u/u_e = \sin (Ky/v_w), \quad (0 \leq y \leq \pi v_w/2K)$$

for two-dimensional flows, and

$$u/u_e = Ky/v_w, \quad (0 \leq y \leq v_w/K)$$

for axisymmetric flows, which have been proved to be accurate for $A \gg 1$.

Therefore we get

$$D_1 = \frac{1}{2} \pi A \tag{5}$$

for two-dimensional flows, and

$$D_1 = 2A \tag{6}$$

for axisymmetric flows. These relations are also plotted in the figures. It should be noted that $\delta^*/D^* = 1 - (2/\pi) = 0.3634$ for two-dimensional flows, and $\delta^*/D^* = 1/2$ for axisymmetric flows, provided the blowing rate is large. It may therefore be concluded that the conventional displacement thickness, δ^*, is grossly in error for the cases where a large amount of injection takes place at the wall. Such a large amount of injection is common for the ablation problem, where the order of the injection parameter, A, is between 10 and 100.

REFERENCES

1. Mann, W. M., Jr., "Effective displacement thickness for boundary layers with surface mass transfer," AIAA J. 1, 1181-1182 (1963).
2. Pretsch, J., "Die laminare Grenzschicht bei starkem Absaugen und Ausblasen," Z. W. B. Untersuch. u. Mitteil. Nr. 3091 (1944).

FIGURE TITLES

Fig. 1.- Displacement thicknesses for two-dimensional stagnation flows with blowing.

Fig. 2.- Displacement thicknesses for axisymmetric stagnation flows with blowing.

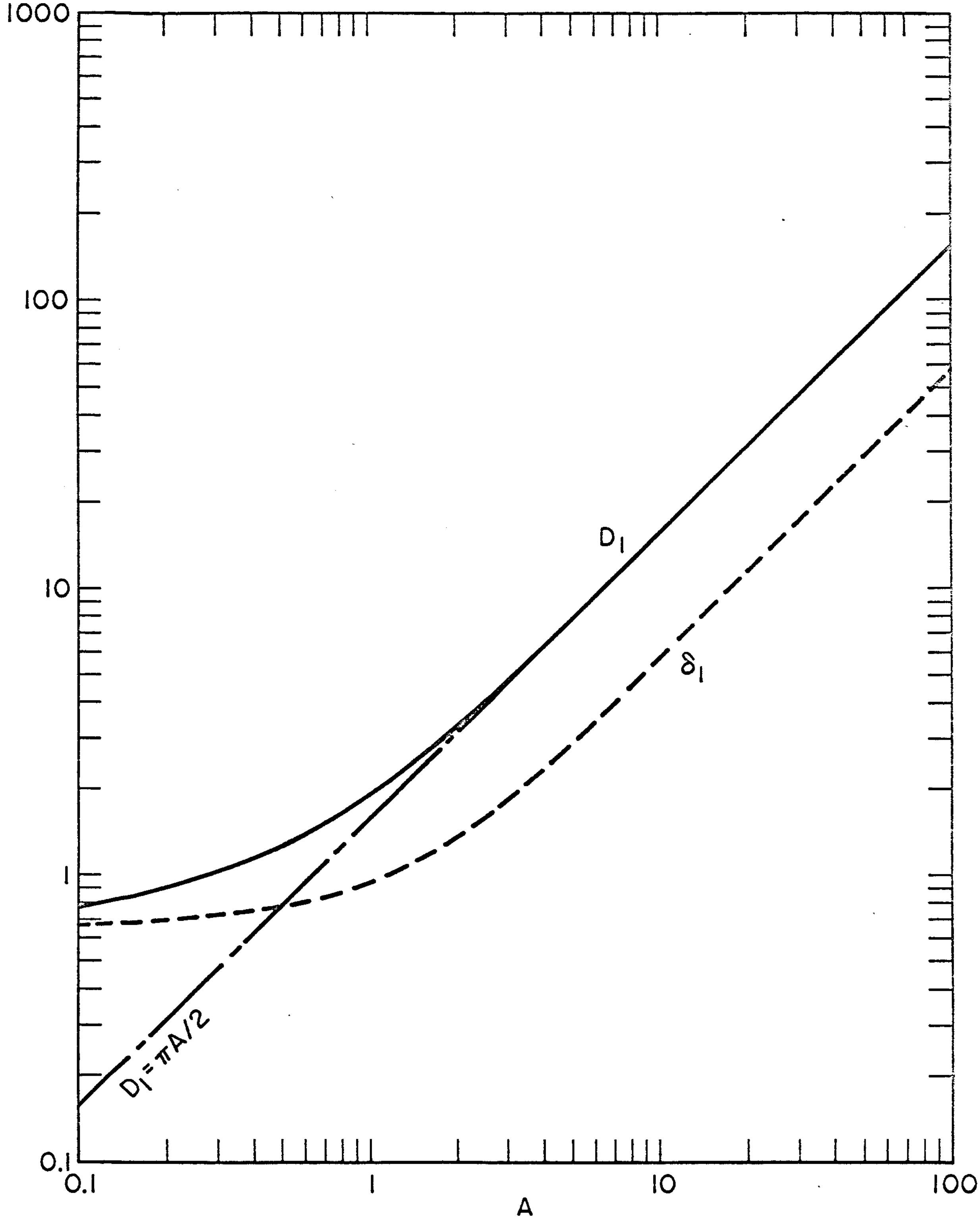

Fig. 1

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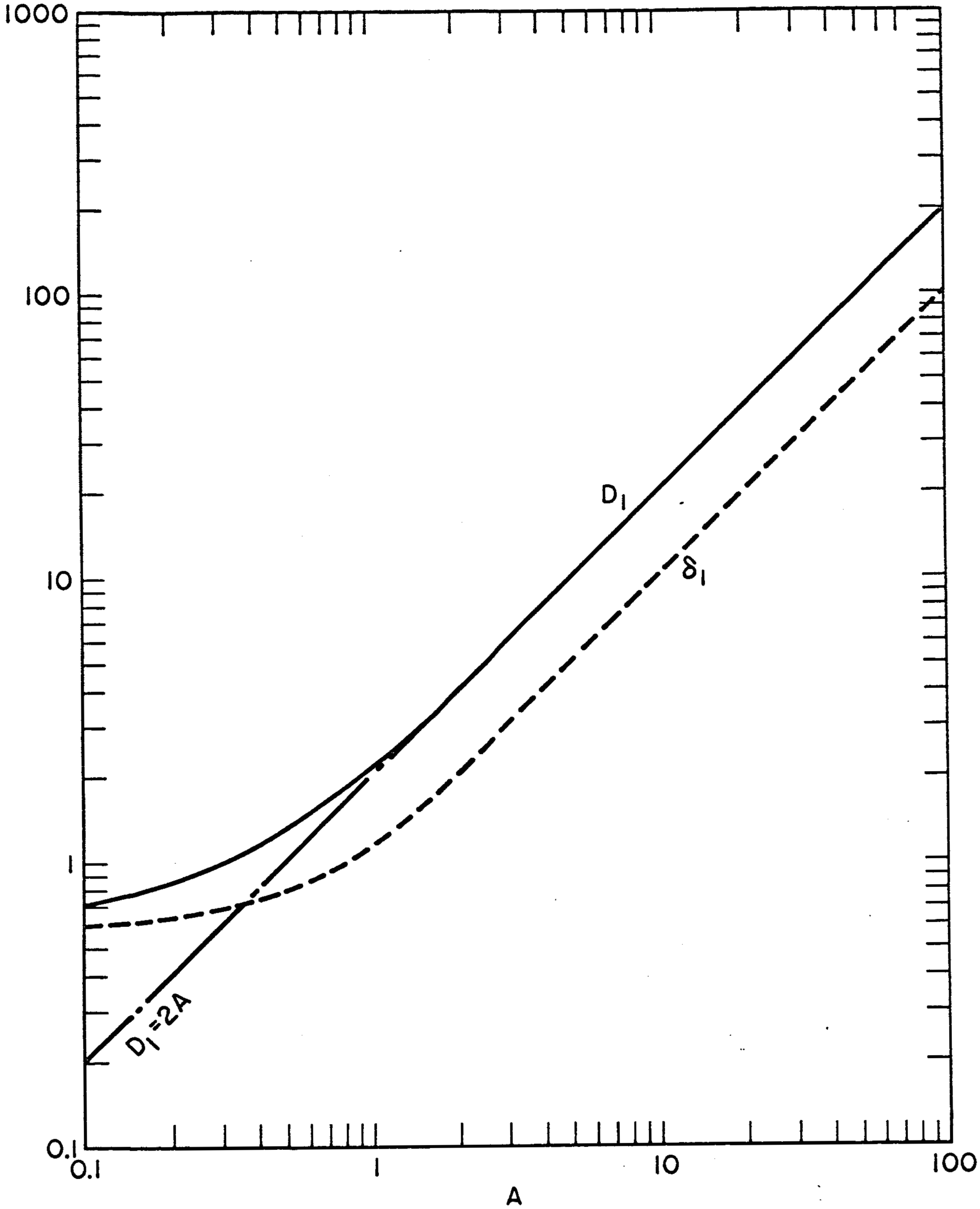

Fig. 2

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