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PREDICTION OF THE DYNAMIC ENVIRONMENT OF SPACECRAFT COMPONENTS

Part I. General Concepts and Description of Computer Programs

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ABSTRACT

This report discusses the general types of motion involved in spacecraft response and the computer methods and programs for computing both ~~structural~~ and air path response. A discussion of various types of damping is included. A procedure for testing components is propo~~sed which depends in~~ part on the computations obtained from the computer ~~procedures.~~

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I. Introduction

Several years ago a program was undertaken by this author to obtain the dynamic response of structures under random excitation, with the goal being to apply the methods to determine the response of space vehicles to acoustic loading and other random loads that occur during flight. The main objective of the program is to help in formulating dyanamic specifications for future spacecraft. Thus far in this program methods have been developed for computing the response of cylindrical shell structures[1*] and for computing the mean square accelerations and internal loads of builtup structures with the attached spacecraft[2] under random loading. Recently a method has also been proposed[3] for computing the noise radiated to the inside through the air path due to random loading on the outside of the shroud.

It is obvious that theoretical computations alone are certainly not sufficient to form specifications for new spacecraft. Therefore, as part of the overall program of helping to form new dynamic specifications, a modelling study was made.[4] It is the contention of this author that a combination of modelling and theoretical calculations together with extrapolation from previously obtained full scale data will eventually be the answer to obtaining new dynamic specifications.

In the process of developing the theoretical methods and modelling laws it was found that joint or friction damping plays a major part in the problem. So during the past year time was taken to study the methods in which joint damping effects modelling and how it could be estimated for use in builtup structural analysis. Since it is not feasible to construct a complete scale model of the structure for dynamic testing one must be satisfied with testing segments of the structure. The segment boundary conditions and the damping in the model segment is a problem in this area which will be discussed later in this report.

It is the main purpose of this report to bring together the research conducted during the past several years on analytical methods, modelling and damping and show how these can be used jointly in helping to formulate new dynamic specifications. This report (which has been labelled "Part I) will be devoted only to a description of the concepts, computer programs and modelling ideas to be used. The actual mathematical description of the analysis is kept to a minimum in this report in order to concentrate on the physical concepts involved. Part II of the study which will follow shortly will give results of specific computations for a practical situation.

*Superscripts refer to references listed at the back of the report.

II. Dynamic Environment

A. General

There are many facets to space environment besides dynamics and all the factors should be considered in performing specification tests. The problem of dynamic environment is very complicated itself. To discuss the complete combined problem of all environments and how they effect each other would be folly at this point since we are not yet sure how the various dynamic environments interact.

In order to define the dynamic inputs to use for testing components of new space vehicles it is necessary to know the external loading and motion of the space vehicle during the entire flight path. For sinusoidal excitation we must know the amplitude and frequency of the forces. For random excitation we must know the cross spectral density of the exciting pressure and for transient excitation the load must be known as a function of time. There are inputs which are important for the design of the booster and shroud which may be only of secondary importance for the spacecraft inside. Of primary consideration to the spacecraft are

1. Rigid body dynamic motions of the booster due to lift off, manuevering and atmospheric phenomena
2. Large scale modal vibrations of the entire vehicle due to engine vibration and control forces
3. Vibrations from the engine which may contain resonance frequencies which coincide with a frequency in one component of the spacecraft
4. Jet noise, aerodynamic boundary layer noise and transonic buffeting which excite the booster and shroud. Their effect can be transmitted through the structure and through the air on the inside to the spacecraft.

B. Division of motions

The response to the types of excitations listed above can be divided into three separate types. These types of motion will be called primary motions, secondary motions and tertiary motions.* The primary motions include the rigid body movements and the oscillations of the entire booster as a beam as shown in Figure 1.

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* This division of motions has also been used in describing the motions of ships[19] and the stresses in ships[20].

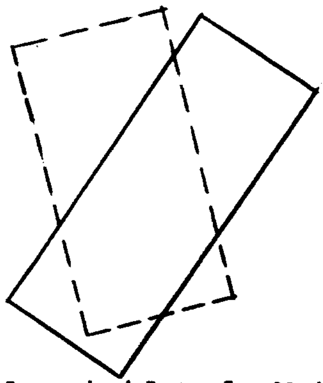

Fig. 1a Rigid Body Motion

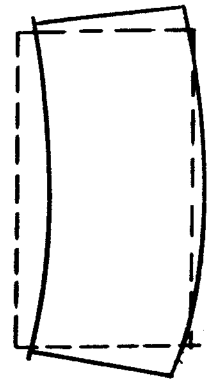

Fig. 1b Beam Type Motions

Fig. 1 Primary Motions

These motions will be excited by lift off, manuevering, atmospheric phenomena, and low frequency engine vibraion, as listed in 1, and 2 in section A above.

The secondary motions include vibration of the entire shell casing and shroud in the shell type modes which include the motions of ~~the~~ stiffeners such as shown in Fig. 2

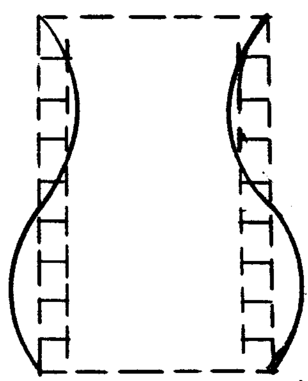

Fig. 2 Secondary Motions

These motions will be excited by engine vibration and by large scale external pressure such as jet noise and buffeting. Boundary layer noise will probably not excite these types of motions to any extent.

The tertiary motions are associated with vibrations between stiffeners in a stiffened shell such as shown in Fig. 3.

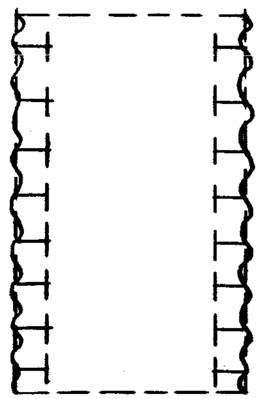

Fig. 3 Tertiary Motions

These motions will be excited primarily by engine vibration, jet noise, boundary layer noise and buffeting.

The spacecraft inside the shroud reacts either directly or indirectly to each of these types of motion. For rigid body and beam type motions (lateral or longitudinal) the axis of the vehicle oscillates so that the same motion is directly felt by the spacecraft. For secondary and tertiary motions the centerline or axis of the booster and shroud remains stationary and the outside skin and stiffeners vibrate. Thus for secondary and tertiary motions the spacecraft receives vibrations through the base attachments to the frame of the booster or capsule and it receives sound radiated to the inside through the skin.

III. Description of computational procedures

A. General formulas and computer programs

1. Statistical analysis of builtup structures* based upon matrix procedures

The total displacement η_j of a structure at point j can be written in terms of the displacements in each of its m modes of vibration as follows:

$$\eta_j(t) = \sum_m Z_{jm} q_m(t) \qquad [1]$$

where m is the mode number, Z_{jm} is the value of the m^{th} mode shape at point j, $q_m(t)$ is a function of time which satisfies the following equation

$$\ddot{q}_m(t) + 2\beta_m \omega_m \dot{q}_m(t) + \omega_m^2 q_m(t) = Q_m(t) \qquad [2]$$

in which β_m is the damping ratio (ratio of damping to critical damping) in the m^{th} mode, ω_m is the undamped natural frequency of the m^{th} mode and $Q_m(t)$ is the generalized force for the m^{th} mode. $Q_m(t)$ can be written in terms of the loading $P(t)$ as follows:

*This derivation is taken from Ref. 2 which is based on the theory developed in Ref. 4.

$$Q_m(t) = \sum_r Z_{rm} A_r P_r(t) \qquad [3]$$

where Z_{rm} is the value of the m^{th} mode shape at point r, A_r is the area over which $P_r(t)$ acts, and $P_r(t)$ is the pressure at point r on the structure. Z_{rm} is the component of the m^{th} mode shape in the direction of the load at point r.

Now take the Fourier Transform of eq. [2] and denote the Fourier Transform of $q_m(t)$ by $q_m(\Omega)$, then

$$[-\Omega^2 + i\,2\beta_m \omega_m \Omega + \omega_m^2]\, q_m(\Omega) = Q_m(\Omega) \qquad [4]$$

The cross spectral density between the displacements at point j and k is defined as

$$\eta_{jk}(\Omega) = \lim_{T\to\infty} \frac{\eta_k(\Omega)\eta_j^*(\Omega)}{T} \qquad [5]$$

(η_j * being the complex conjugate of η_j)

Now

$$\eta_j(\Omega) = \sum_m Z_{jm} q_m(\Omega) \qquad [6]$$

and

$$Q_m(\Omega) = \sum_r Z_{rm} A_r P_r(\Omega) \qquad [7]$$

Thus

$$\eta_j(\Omega) = \sum_m Z_{jm} Q_m(\Omega) Y_m(\Omega)$$

$$= \sum_m Z_{jm} Y_m \sum_r Z_{rm} A_r P_r(\Omega) \qquad [8]$$

and

$$\eta_{jk}(\Omega) = \sum_m Z_{jm} Y_m^*(\Omega) \sum_{r,s} Z_{rm} A_r P_{rs}(\Omega) A_s \sum_n Z_{sn} Y_n(\Omega) Z_{kn} \qquad [9]$$

$$P_{rs}(\Omega) = \lim_{T\to\infty} \frac{P_r^*(\Omega) P_s(\Omega)}{T}, \quad Y_m(\Omega) = 1/[\omega_m^2 - \Omega^2 + i\,2\beta_m\omega_m\Omega]$$

Y_m^* is the oomplex conjugate of Y_m, $P_{rs}(\Omega)$ is the cross spectral density of the loading at points r and s.

It is to be noted that Z_{rm} and Z_{sn} are mode shape components in the direction of the loading at r and s respectively since the generalized force depends upon the dot product between the force and modal displacement (e.g. see Ref. 5). Z_{jm} and Z_{kn} are mode shape components of the m^{th} and n^{th} modes in any desired direction at each of the points j and k for which the cross spectral density is desired. In matrix notation we write

$$\eta(\Omega) = \bar{Z}\, B(\Omega)\, \bar{Z}^T \qquad [10]$$

(superscript T denotes the transpose of the matrix)

where

$$B(\Omega) = Y^* Z^T A P(\Omega) A Z Y$$

in which $\eta(\Omega)$ is a square matrix of all cross spectral densities between displacements at all points in desired directions (which we choose in advance) at each point, $\bar{Z}$ is a real rectangular matrix consisting of the values of the mode shapes in the desired directions at all points, $P(\Omega)$ is the complex matrix of cross power spectral densities of pressure at all pairs of points, A is a real diagonal matrix of areas, Z is a real rectangular matrix containing the mode shapes of the modes in the direction of the loading at all points, and Y is a complex diagonal matrix of modal transfer functions.

The RMS value of the deflection at j over a frequency band $\Delta\Omega = \Omega_{max} - \Omega_{min}$ can then be written

$$\eta_j(RMS) = \left[\frac{1}{2\pi}\int_{\Omega_{min}}^{\Omega_{max}} \eta_{jj}(\Omega)\,d\Omega\right]^{1/2} \qquad [11]$$

where $\eta_{jj}(\Omega)$ is one of the diagonal (auto spectral density) terms of the matrix (the matrix being computed for each value of Ω).

If we are dealing with a system having relatively small damping we can neglect cross product terms in eq. [9], which reduces this equation to

$$\eta_{ij}(\Omega) = \sum_m Z_{jm} Z_{km} |Y_m|^2 \sum_{r,s} Z_{rm} A_r P_{rs}(\Omega) A_s Z_{sm} \qquad [12]$$

The mean square acceleration in a frequency band $\Delta\Omega$ can then be written

$$\bar{a}_{j\,\Delta\Omega}^{\,2} = \frac{1}{2\pi}\int_{\Omega_{min}}^{\Omega_{max}} a_{jj}(\Omega)\,d\Omega$$

$$= \frac{1}{2\pi}\int_{\Omega_{min}}^{\Omega_{max}} \left[\Omega^4 \sum_m Z_{jm}^2 |Y_m(\Omega)|^2 \overbrace{\sum_{r,s} Z_{rm} A_r P_{rs}(\Omega) A_s Z_{sm}}^{C_{mm}(\Omega)}\right. \qquad [13]$$

For cases of light damping this can be integrated directly (Ref. 6) to give

$$\frac{1}{2\pi}\int_{\Omega_1}^{\Omega_2} \Omega^4 C_{mm}(\Omega)\,d\Omega\,|Y_m(\Omega)|^2 \approx \frac{\omega_m^4 C_{mm}(\omega_m)}{2\pi}\int_{\Omega_1}^{\Omega_2} |Y_m(\Omega)|^2\,d\Omega \qquad [14]$$

$$\approx \frac{\omega_m^4 C_{mm}(\Omega)}{2\pi}\int_0^{\infty} |Y_m(\Omega)|^2 d\Omega = \frac{\omega_m C_{mm}(\omega_m)}{8\beta_m}$$

where $C_{mm}(\omega_m)$ is the joint acceptance evaluated at the natural frequency, ω_m (the ω_m's consist of these natural frequencies in the frequency band $\Delta\Omega$) and β_m is the damping ratio as given before. The mean square acceleration over frequency band $\Delta\Omega$ can then be written

$$\overline{a_{j\,\Delta\Omega}}^{\,2} = \sum_m \frac{\omega_m C_{mm}(\omega_m)}{8\beta_m} Z_{jm}^2 \qquad [15]$$

Note that Z_{jm} are the normalized modes (see eq. [2]). The actual modes are given in terms of the normalized modes by the equation (see Ref. 7)

$$Z_{jm} = \sqrt{M_m}\;\bar{Z}_{jm} \qquad [16]$$

where M_m is the generalized mass for the m^{th} mode and where $\bar{Z}_{jm}$ are the actual normal modes. In terms of actual modes eq. [15] becomes

$$\overline{a_j}\,\Delta\Omega^2 = \sum_m \frac{\omega_m \bar{C}_{mm}(\omega_m)\,\bar{Z}_{jm}^{\,2}}{8\beta_m M_m^{\,2}} \qquad [17]$$

where

$$\bar{C}_{mm} = \sum_{r,s} \bar{Z}_{rm} A_r P_{rs}(\Omega) A_s \bar{Z}_{sm} \qquad [18]$$

The time dependent internal load $\gamma_j(t)$ associated with the j^{th} structural element can be written in matrix notation as follows: (Ref. 8)

$$\gamma_j(t) = \lfloor \lambda_j \rfloor \left[A P(t) - K \ddot{\eta}(t) - G \dot{\eta}(t)\right] \qquad [19]$$

where K and G are respectively mass and dissipation matrices and $\lfloor \lambda_j \rfloor$ is a row matrix relating the forces on the structure to the internal load in the j^{th} element. The matrix of cross spectral densities of internal loads can then be written in matrix notation by employing the theory given in Ref. 2 and 8. The final equation is (Ref. 2)

$$\gamma(\Omega) = \lambda\, K\, Z\, \omega^2 B(\Omega)\, \omega^2 Z^T K \lambda^T \qquad [20]$$

where λ is the complete internal load influence matrix (see Ref. 8), K is the diagonal mass matrix, ω^2 is the diagonal matrix of natural frequencies squared. The equations for computing the cross spectral density of the displacements (spectral density of velocity and acceleration follow immediately by multiplying the C. P. D. of displacement by ω^2 and ω^4 respectively) and internal loads have been programmed for the electronic computer.

2. The computer program for statistical analysis of builtup structures

The computer program for obtaining the cross spectral densities of displacements and internal loads has been developed by the Martin Company under the author's direction using the theory given in Ref. 2,4. There are two programs. The first program is tied to the Martin SB038 program for computing the mode shapes and frequencies of builtup structures with attachments. This program develops the internal load influence matrix and computes the cross spectral density matrices of all displacements and internal loads as given by the relations described above.

The second program is a "stand alone" program which is not dependent on SB038 for computing modes and frequencies. This latter program must be given the modes and frequencies as input. It computes the cross spectral density of displacements alone as given by equation [10]. This "stand alone" program can prove very helpful in predicting the "in flight" statistical response of systems for which the mode shapes, frequencies and damping have been determined experimentally during ground testing.

The main objective of the above program is to estimate the response of complicated coupled systems such as the shroud and connected spacecraft. The main unknowns associated with this program are the statistical distribution of the loading (i.e. cross spectral density of loading) and the structural damping. This program can be used to predict the primary secondary and tertiary motions (see Sect. IIB) of structures, however due to the small scale nature of the tertiary motions of the outside structure it may be more efficient to employ a cylindrical shell program that will be described in Sec. III A3.

3. General program for cylindrical shell response

In the proveious section a general multi-purpose program was described for computing the response of booster-spacecraft systems. The main drawback of the complete program (which includes computing modes and frequencies) is that many degrees of freedom can be involved, especially since small scale motions require use of many points on the outside skin of the structure. In many cases the structure is close enough to a stiffened, unstiffened or sandwich cylindrical shell to analyze the structure as such. A general cylindrical shell program was originally developed for hull structures several years ago (Ref. 9) and with some slight changes and additions it can be applied to predict the secondary and tertiary motions (see section IIB) of cylindrical shell bodies of flight vehicles. A brief derivation and description of the appropriate relations follows:

Start with the general solution for the displacements of a cylindrical shell of length ℓ with freely supported ends

$$\text{Longitudinal} \quad u(x,\phi,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[u_{1mn}(t)\cos n\phi + u_{2mn}(t)\sin n\phi\right]\cos\frac{m\pi x}{\ell}$$

$$\text{Tangential} \quad v(x,\phi,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[v_{1mn}(t)\cos n\phi + v_{2mn}(t)\sin n\phi\right]\sin\frac{m\pi x}{\ell} \qquad [21]$$

$$\text{Radial} \quad w(x,\phi,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[w_{1mn}(t)\cos n\phi + w_{2mn}(t)\sin n\phi\right]\sin\frac{m\pi x}{\ell}$$

The loading on the structure is expanded into a Fourier Series. The three components of loading on the structure can be written

$$\text{Longitudinal} \quad U(x,\phi,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[U_{1mn}(t)\cos n\phi + U_{2mn}(t)\sin n\phi\right]\cos\frac{m\pi x}{\ell}$$

$$\text{Tangential} \quad V(x,\phi,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[V_{1mn}(t)\cos n\phi + V_{2mn}(t)\sin n\phi\right]\sin\frac{m\pi x}{\ell} \qquad [22]$$

$$\text{Radial} \quad W(x,\phi,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[W_{1mn}(t)\cos n\phi + W_{2mn}(t)\sin n\phi\right]\sin\frac{m\pi x}{\ell}$$

The cross spectral density of displacements (velocities, accelerations) stress and moment resultants as well as cross spectral densities of internal acoustic pressures inside the shell can be determined in terms of frequency response functions for each of these items. These frequency response functions have been programmed (see Ref. 3,9). The general form of the cross spectral density of response (whether it be displacements, velocities, accelerations, force or moment resultants, acoustic pressure, etc.) can be written as follows considering only lateral loading W on the cylindrical surface (which is the case of most practical interest)

$$\underbrace{\widetilde{R}(x_1,\phi_1,x_2,\phi_2,\omega)}^{\text{C.P.D. of Response}} = \frac{1}{2\pi}\sum_m\sum_n\sum_p\sum_q f_m(x_1)f_p(x_2)\times \int_{-\infty}^{+\infty}\Big[\langle\alpha_{mn}(t)\alpha_{pq}(t+\tau)\rangle\cos n\phi_1\cos q\phi_2 + \langle\beta_{mn}(t)\beta_{pq}(t+\tau)\rangle\sin n\phi_1\sin q\phi_2 + \langle\beta_{mn}(t)\alpha_{pq}(t+\tau)\rangle\sin n\phi_1\cos q\phi_2 + \langle\alpha_{mn}(t)\beta_{pq}(t+\tau)\rangle\cos n\phi_1\sin q\phi_2\Big]e^{-i\omega\tau}d\tau \qquad [23]$$

where

$$\int_{-\infty}^{+\infty}\langle\alpha_{mn}(t)\alpha_{pq}(t+\tau)\rangle e^{-i\omega\tau}d\tau = A_R(m,n,\omega)A_R^*(p,q,\omega)J_{\alpha_{mn}\alpha_{pq}}$$

$$J_{\alpha_{mn}\alpha_{pq}} = \frac{4}{\pi^2\ell^2}\int_0^\ell\int_0^\ell\int_0^{2\pi}\int_0^{2\pi}S_w(\xi_1,\eta_1,\xi_2,\eta_2,\omega)f_m(\xi_1)f_p(\xi_2)\cos n\eta_1\cos q\eta_2\,d\xi_1d\eta_1d\xi_2d\eta_2$$

$$\int_{-\infty}^{+\infty}\langle\beta_{mn}(t)\beta_{pq}(t+\tau)\rangle e^{-i\omega\tau}d\tau = B_R(m,n,\omega)B_R^*(p,q,\omega)J_{\beta_{mn}\beta_{pq}} \qquad [24]$$

$$J_{\beta_{mn}\beta_{pq}} = \frac{4}{\pi^2\ell^2}\int_0^\ell\int_0^\ell\int_0^{2\pi}\int_0^{2\pi}S_w(\xi_1,\eta_1,\xi_2,\eta_2,\omega)f_m(\xi_1)f_p(\xi_2)\sin n\eta_1\sin q\eta_2\,d\xi_1d\eta_1d\xi_2d\eta_2$$

$$\int_{-\infty}^{+\infty}\langle\beta_{mn}(t)\alpha_{pq}(t+\tau)\rangle e^{-i\omega\tau}d\tau = B_R(m,n,\omega)A_R^*(p,q,\omega)J_{\beta_{mn}\alpha_{pq}}$$

$$J_{\beta_{mn}\alpha_{pq}} = \frac{4}{\pi^2\ell^2}\int_0^\ell\int_0^\ell\int_0^{2\pi}\int_0^{2\pi}S_w(\xi_1,\eta_1,\xi_2,\eta_2,\omega)f_m(\xi_1)f_p(\xi_2)\sin n\eta_1\cos q\eta_2\,d\xi_1d\eta_1d\xi_2d\eta_2$$

$$\int_{-\infty}^{+\infty}\langle\alpha_{mn}(t)\beta_{pq}(t+\tau)\rangle e^{-i\omega\tau}d\tau = A_R(m,n,\omega)B_R^*(p,q,\omega)J_{\alpha_{mn}\beta_{pq}}$$

$$J_{\alpha_{mn}\beta_{pq}} = \frac{4}{\pi^2\ell^2}\int_0^\ell\int_0^\ell\int_0^{2\pi}\int_0^{2\pi}S_w(\xi_1,\eta_1,\xi_2,\eta_2,\omega)f_m(\xi_1)f_p(\xi_2)\sin q\eta_1\cos n\eta_2\,d\xi_1d\eta_1d\xi_2d\eta_2$$

In the above equations $S_w(\xi_1, \eta_1, \xi_2, \eta_2, \omega)$ is the cross spectral density of the lateral exciting pressure on the cylindrical surface. $\alpha_{mn}(t)$ denotes the factor of $\cos n\phi$ in the Fourier Expansion of displacement, stress or moment resultant, internal pressure, etc. and $\beta_{mn}(t)$ denotes the factor of $\sin n\phi$. $f_m(x_1)$ $f_p(x_2)$ denote either $\sin\frac{m\pi x_1}{l}\sin\frac{p\pi x_2}{l}$ or $\cos\frac{m\pi x_1}{l}\cos\frac{p\pi x_2}{l}$ depending upon which appears in the original expression for the deflection, moment resultant, etc.. The function A_p is a frequency response function for $\alpha_{mn}(t)$ which gives the response for a unit sinusoidal loading which has the spatial shape of the mode (see Ref. 9). Similarly for B_p.

The computer program obtains the A_p's and B_p's which constitute the major computing job of the response problem. These A_p's and B_p's will depend upon the geometrical and material properties of the structure, the damping, the speed of the vehicle, the densities of the air inside and outside (i.e. it includes air radiation damping).

4. Use of cylindrical shell program vs. use of general purpose builtup structure program

The builtup structure program described in Section IIIA1 includes the cylindrical shell as a special case. However the special program described in IIIA3 is a vastly more efficient one. When small scale tertiary motions such as skin vibrations under turbulent boundary layers are involved, it is no longer necessary to use the builtup structure program and we will get much more accurate and complete answers from the cylindrical shell program since it is basically a continuum approach vs. a pointwise approach that is used by the builtup structure program.

5. A discussion of the general form of the solution for structural response and air path transmission involving reverberation

The cross spectral density of the acceleration of any point on the structure can be written in integral form as (see eq. [9])

$$a(P_1, P_2, \Omega) = \Omega^4 \sum_m \sum_n Z_m(P_1) Z_n(P_2) Y_m(\Omega) Y_n^*(\Omega) C_{mn} \quad [25]$$

where

$$C_{mn} = \int_{\sigma_1}\int_{\sigma_2} Z_m(S_1) Z_n(S_2) P(S_1, S_2, \Omega) d\sigma_1 d\sigma_2$$

where $d\sigma_1$ and $d\sigma_2$ denote differential elements of loaded surface area and the integral is taken twice over the loaded surface area S_1, S_2. The ~~points~~ S_1 and S_2 are two points on the loaded surface and P_1, P_2 are two points within the enclosure. For systems with low damping and well separated modes the mean square acceleration over a frequency band is (see eq. [15])

$$\overline{a(P_1)^2}_{\Delta\Omega} = \sum_m \frac{\omega_m C_{mm}(\omega_m) Z_m^2(P_1)}{8\beta_m} \quad [26]$$

$$C_{mm} = \int_{\sigma_1}\int_{\sigma_2} Z_m(S_1) Z_m(S_2) P(S_1, S_2, \Omega) d\sigma_1 d\sigma_2$$

If we consider a shroud or surrounding structure which is exposed to the outside environment then the cross spectral density of the acoustic pressure induced inside assuming a reverberant field inside can be written in general form as (see Ref. 3)

$$G(P_1, P_2, \Omega) = \frac{1}{(4\pi)^2}\int_{\sigma_1}\int_{\sigma_2} \rho_0^{\,2} a_n(S_1, S_2, \Omega)\, \bar{g}(P_1, S_1, \Omega)\, \bar{g}^*(P_2, S_2, \Omega)\, d\sigma_1\, d\sigma_2 \qquad [27]$$

where $G(P_1, P_2, \Omega)$ is the cross spectral density of the pressure inside the structure at points P_1 and P_2 at frequency Ω

$a_n(S_1, S_2, \Omega)$ is the cross spectral density of the normal acceleration at the surface

$\bar{g}(P_1, S_1, \Omega)$ is the Green's Function for the medium inside of the structure whose normal derivative vanishes over the surface of the structure

$\bar{g}^*(P_2, S_2, \Omega)$ is the complex conjugate of $\bar{g}(P_1, S_1, \Omega)$ referred to coordinates in the x_2, y_2, z_2 space.

σ denotes the vibrating surface

The Green's Function for the inside chamber can be expanded in terms of eigenfunctions of the inside enclosure as follows: (Ref. 10, 11)

$$\bar{g}(P_1, S_1, \Omega) = \sum_p \frac{\psi_p(P_1)\psi_p(S_1)}{\Omega^2/C_i^{\,2} - \lambda_p^{\,2}} \qquad [28]$$

where C_i is the sound velocity on the inside, λ_p is the p^{th} eigenvalue of the enclosed region, and ψ_p is the corresponding eigenfunction. The λ_p's are in general complex since some energy is absorbed at the elastic boundary. Similarly

$$\bar{g}^*(P_2, S_2, \Omega) = \sum_q \frac{\psi_q(P_2)\psi_q(S_2)}{\Omega^2/C_i^{\,2} - \lambda_q^{*\,2}} \qquad [29]$$

Substituting into the expression for the cross spectral density of the inside pressure we have

$$G(P_1, P_2, \Omega) = \sum_m \sum_n \sum_p \sum_q \frac{\rho_0^{\,2}}{(4\pi)^2} \psi_p(P_1)\psi_q(P_2) C_{mn} Y_m(\Omega) Y_n^*(\Omega)\, \Omega^4 \int_{\sigma_1} \frac{Z_m(S_1)\psi_p(S_1)\,dS_1}{\Omega^2/C_i^{\,2} - \lambda_p^{\,2}} \int_{\sigma_2} \frac{Z_n(S_2)\psi_q(S_2)\,dS_2}{\Omega^2/C_i^{\,2} - \lambda_q^{*\,2}} \qquad [30]$$

$$C_{mn} = \int_{\sigma_1}\int_{\sigma_2} Z_m(S_1) Z_n(S_2) G(S_1, S_2, \Omega)$$

This equation is the general relation for the cross spectral density of the pressure inside an enclosure due to the motion of the boundary of the enclosure. This expression is indeed quite complicated and would not be practical to use in this form. The complications are quite apparent when one realizes that both structural and enclosure resonances are involved, both of which effect each other. It is for this reason that a simpler cylindrical model was chosen to compute acoustic pressures inside. As pointed out by Morse and Ingard (Ref. 10) for the higher frequencies (which are of most interest in air path transmission) many modes of the enclosure and many structural modes are involved so that the resonance peaks overlap each other. This is the reason why product terms cannot be neglected in either the structural response equation or the enclosure relations. This is why there is a quadruple sum in eq. [23] (there being one mode for each m, n or p, q). This is equivalent to using the complete expression as given by eq. [9]. The programs have therefore been set up to include all these product terms.

B. Damping

1. General

It is seen in the development of the relations in Section IIIA that one of the main parameters that must be estimated is the structural damping. Examination of eq. [15] clearly points to the critical way in which β_m (the ratio of damping to critical damping in the mth mode) goes into lightly damped systems where primarily resonant response is of greatest importance. In systems with higher damping where non-resonant response is of equal importance the damping need not be the focal point of interest. In present day structures vibrating in air or atmosphere which is close to vacuum one can be reasonably certain that resonant response in frequency bands is of great importance. Since we have derived the analysis in terms of viscous damping coefficients it is necessary that any quantitative results be determined in terms of such coefficients.

2. Material damping

In cases where all joints are tight or where only monocoque construction is used with very tight end enclosures the material damping is of considerable importance. A very informative experimental investigation on material and air damping has been carried out by Granick and Stern (Ref. 12). Their results show that pure material damping in aluminum follows the Zener relation (Ref. 13)

$$g = \frac{\alpha^2 E T}{c}\left[\frac{\omega \tau}{1+\omega^2\tau^2}\right] \qquad [31]$$

where g is the material damping coefficient
T is the absolute temperature
E is the modulus of elasticity
c is the specific heat per unit volume
α is the coefficient of linear expansion
τ is the relaxation time
ω is the radian frequency

For a great many practical cases $\omega^2\tau^2 >> 1$
Thus

$$g\omega \approx \frac{\alpha^2 E T}{c}\,\frac{1}{\tau} \qquad [32]$$

The damping ratio β_m is given in terms of g by

$$g_m = 2\beta_m \qquad [33]$$

Multiplying by ω_m we obtain

$$g_m \omega_m = 2\beta_m \omega_m \qquad [34]$$

Thus the coefficient of $\dot{g}_m(t)$ in eq. [2] is a constant for a given material. Granick and Stern have found that for aluminum

$$g_m \omega_m \approx 0.5 \text{ rad/sec.} \qquad [35]$$

Similar results for material damping in vibrations of aluminum shells have been determined by Sechler and Fung (Ref. 14).

3. Air damping

Granick and Stern (Ref. 12) have found by testing in air and vacuum that the air damping can be much larger than the material damping and that the proposed amplitude dependence of the material damping was no more than air damping entering the problem. This air damping could be due to viscosity or it could be radiation damping resulting from sound radiated from the structure. The latter type has been considered in the cylindrical shell analysis previously described. The viscosity in the air might be a factor when testing cantilevers alone but when considering an entire enclosed shell structure it is felt that especially at high frequencies the radiation damping could be as great if not greater than the viscous air damping. As the structure goes into thinner atmosphere both the radiation and viscous damping will be reduced and material damping should be the predominant of the three.

4. Joint or friction damping

The main proportional of total damping of a builtup structure is probably due to friction at joints and general solid friction at various rubbing surfaces in the structure. This is the type of damping for which it has been very difficult to get theoretical predictions because each situation is particular in itself. Slight changes in the pressure or coefficient of friction between the two surfaces can radically change the energy dissipated in this type of damping. However there are some general principles about friction damping that can be used as a guide in practical problems.

Since we have done all the theoretical analysis in terms of viscous damping coefficients it is necessary to also obtain the friction damping in these terms. Using the equivalent viscous damping concept of Jacobsen (Ref. 15, 16), the equivalent viscous damping coefficient of a single degree of freedom system with friction can be written

$$c = \frac{4F}{\pi A \omega} \qquad [36]$$

where $c = \zeta c_c$ in which c_c is the critical damping, ζ is the ratio of damping to critical damping. F is the friction force, A is the amplitude of motion and ω is the radian frequency of the motion (which is assumed to be harmonic). The above equation refers only to a single area. Let us employ the same concept for an entire structure. The displacement in harmonic motion of a structure can be written as follows:

$$\vec{q}(\vec{r},t) = \sum_m \vec{q}_m(\vec{r}) q_m(t)$$
$$q_m(t) = A_m \sin(\omega t - \alpha_m)$$ [37]

in which A_m is the modal amplitude of the motion and α_m is the phase angle associated with the mth mode. Let $\vec{F_i}$ be the friction force at point i of the structure (which we assume is independent of time) and let $\vec{d_i}$ be the relative maximum displacement between the joined or rubbing parts during motion. The work done per cycle against the friction force can be written:

$$W_F = 4 \sum_i \vec{F_i} \cdot \vec{d_i}$$ [38]

For the mth mode we have

$$W_{F_m} = 4 \sum_i \vec{F_i} \cdot \vec{d_{im}}$$ [39]

where $\vec{F_i}$ is the constant friction force at the ith station and $\vec{d_{im}}$ is the relative displacement at the ith station in the mth mode. This displacement must be determined by a separate analysis at the joint or at the rubbing point. The friction force can be written

$$\vec{F_i} = \mu_i \vec{N_i}$$ [40]

where μ_i is the coefficient of friction at the ith point and N_i is the normal force between the two structures. The normal force can be determined by a separate elastic analysis (e.g. the basic SB038 program for the static case that was described for the dynamic case earlier in the report) and the coefficient of friction can be estimated from recent work in lubrication and wear (e.g. see Ref. 17, 18).

Relation [39] says that the total work done by the friction force is the amplitude of the relative deflection at the points where friction occurs (ith point) multiplied by the friction force. The factor of 4 arises since we are dealing with a complete cycle.

The energy dissipated per cycle in terms of the viscous damping coefficients in the modes is given in the following analysis:

$K_m = C_m \dot{q}_m(\vec{r})$ = damping force per unit volume in the mth mode [41]

(C_m = damping force per unit velocity per unit volume)

Substituting [37] we obtain

$$K_m = C_m A_m \omega \cos(\omega t - \alpha_m) \vec{q}_m(\vec{r})$$ [42]

Thus the work done per cycle in the mth mode (integrating over the whole body) is

$$\begin{aligned} W_m &= \int_V \int_0^T [C_m A_m \omega \cos(\omega t - \alpha_m) \vec{q}_m(\vec{r})] \\ &\quad \cdot [\vec{q}_m(\vec{r}) \omega A_m \cos(\omega t - \alpha_m)] \, dV \\ &= C_m A_m^2 \omega \pi \int_V |\vec{q}_m(\vec{r})|^2 dV \end{aligned}$$ [43]

Now equating

$$W_m = W_{F_m}$$ [44]

We obtain (assuming C_m is independent of location)

$$C_m A_m^2 \omega \pi \int_V |q_m(\vec{r})|^2 dV = 4 \sum_i \vec{F}_i \cdot \vec{d}_{im} \quad [45]$$

The modal amplitude, A_m, will depend upon the frequency. However we can obtain its value at resonance which will depend upon the force distribution as follows:

$$q_m = A_m e^{i\omega t} \quad [46]$$

where q_m satisfies

$$\ddot{q}_m(t) + 2\beta_m \omega_m \dot{q}_m(t) + \omega_m^2 q_m(t) = \int_S \vec{P}(\vec{r}_s) e^{i\omega t} \cdot \vec{q}_m(\vec{r}_s) ds$$

$$\therefore A_m = \frac{\int_S \vec{P}(\vec{r}_s) \cdot \vec{q}_m(\vec{r}_s) ds}{\omega^2 - \omega_m^2 - 2i\beta_m \omega_m \omega} \quad [47]$$

$C_m A_m^2$ at resonance in the mth mode is*

$$(C_m A_m^2)_{res} = \frac{2M_m \omega_m \beta_m \left[\int_S \vec{P}(\vec{r}_s) \cdot \vec{q}_m(\vec{r}_s) ds\right]}{4\beta_m^2 \omega_m^4} \quad [48]$$

Thus

$$\frac{M_m \pi \left[\int_V |q_m(\vec{r})|^2 dV\right]\left[\int_S \vec{P}(\vec{r}_s) \cdot \vec{q}_m(\vec{r}_s) ds\right]^2}{2\beta_m \omega_m^2} = 4 \sum_i \vec{F}_i \cdot \vec{d}_{im} \quad [49]$$

Thus

$$\beta_m = \frac{M_m \pi \left[\int_V |q_m(\vec{r})|^2 dV\right]\left[\int_S \vec{P}(\vec{r}_s) \cdot \vec{q}_m(\vec{r}_s) ds\right]^2}{8\omega_m^2 \sum_i \vec{F}_i \cdot \vec{d}_{im}} \quad [50]$$

This gives the equivalent viscous damping coefficient for frictional forces at resonant frequency. It can be determined at other frequencies by using the complete equation ([47]).

IV. The way in which computations and modelling can be used in component testing and determining environmental specifications of components

The response of the components of a spacecraft depend upon the environment to which the component is exposed. Since it is impractical to construct models of the complete booster and spacecraft combination and since we cannot expose the entire vehicle to all inputs (dynamic, thermal, etc.) at one time it is necessary to devise schemes where one component can be tested individually. We will concentrate on dynamic environment and dynamic testing in this report. Even if the statistical characteristics (i.e. cross spectral density) of the in-flight loading could be described accurately, it is extremely difficult to duplicate the spatial characteristics of the cross spectral density in the laboratory or on the ground for model testing. Taking into accout all these factors it therefore

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*Using $C_m = (C_c)_m \beta_m$ where $(C_c)_m = 2M_m \omega_m$, M_m = mth mode generalized mass does not seem feasible to attempt modelling a complete spacecraft-booster combination and attempt to expose it to what we think are realistic inputs. How then can we determine the environment to use for testing an individual component and how should this component be supported? A suggested procedure for accomplishing this will be described in the next several paragraphs.

We can describe with some degree of assurance, the outside loading on the booster such as the spatial correlation of jet noise, boundary layer noise, transonic buffeting, gusts and peculiar inputs associated with the engine. The programs for calculating the response of the structure due to a known statistical loading such as these loadings are outlined in the previous sections. A mathematical model of the structure is then determined which gives the details of the outside of the structure only, but only the rudiments of the inside need by described. For example lumping the spacecraft and its components into a single mass will be sufficient. One item with which care must be taken is the attachments between spacecraft and booster; these attachments must be considered carefully in the analysis together with the proper damping. The response of the inside mass which models the spacecraft is then determined analytically. The structural response is determined from the builtup structure program by solving the problem of a booster and shroud with an attached mass on the inside connected by the appropriate supports to the shroud and booster. The air path acoustic loading can be determined by using the cylindrical shell program and computing the spectral density of the acoustic pressure inside the shroud. Specifically, the rigid body response of the mass which models the spacecraft is determined and the acoustic pressure around the vicinity of the spacecraft is computed.

From this point on it would be difficult to compute the response by analysis since the spacecraft usually consists of many components which can be attached in various ways to the shell or frame of the spacecraft itself. A rough model of the spacecraft is then constructed which does not contain any components but models the elastic and mass distribution and is the same material as the actual spacecraft to avoid any problems concerning material and joint damping. This model should then be exposed to a modelled acoustic field which was computed from the cylindrical shell acoustic pressure analysis and a modelled mechanical vibration which was determined from the rigid body motion calculations in the builtup structure program. The laws for modelling the proper acoustic pressure and vibration are contained in a previous reference (Ref. 4). The actual acoustic field and rigid body motion will depend upon the scale factor used in the model. One need be considered only with overall spectrum levels at this point and details of the spatial characteristics of the inputs can be neglected. Also a vibration input which gives the model spacecraft the computed accelerations of the CG and the accelerations around the CG will be adequate.

The resulting response (acoustic and vibratory) of the model spacecraft should then be measured at various locations where equipment will be placed. This is the dynamic environment of a particular component which can then be used to test this component by itself. In supporting the component during the proof test we should use the actual support conditions that will exist on the spacecraft so that no questions regarding damping will arise. In this way the damping of supports will be modelled exactly and one of the primary drawbacks in modelling components will be avoided.

To recapitulate, the computer programs are first used to compute the structural response and inside acoustic pressure around the spacecraft from known loadings on the outside. In this calculation the spacecraft is represented by a single rigid body which has the mass and moment of inertia distribution of the actual spacecraft. A model of the spacecraft frame is then constructed which has the mass and elastic characteristics of the actual spacecraft and it is exposed to the modelled input of the structural vibration and acoustic field which was computed above. The model response is then measured at critical points. It will consist of rigid body and elastic motion and an acoustic field. This response is then the combined dynamic environment to use as a specification for the individual component.

In the presentation above it was recommended to make the model of the same material as the full scale to avoid problems in material and joint damping. If different materials have to be used then it is imperative that the joint friction be modelled properly (see Ref. 4). The material damping is not expected to be critical if there are any joints in the model. If there are no joints then the same material as the full scale must be used since there is no way to model material damping properly. Note that in the scheme presented here computations are used to determine the input environment to the spacecraft as a whole. This enables us to obtain overall environment of the spacecraft without having to take out segments of the vehicle or to model individual parts until a later stage in the procedure; the problem of equivalent impedance at a break point (where part of the booster is taken out to test) is therefore circumvented. The only place where model damping enters is a later stage when the model spacecraft is constructed which has the appropriate mass and elastic properties of the full scale. Since the calculations would have been previously made on the rigid body and acoustic environment of the spacecraft as a whole we do not have to be concerned at this stage about the damping of attachment points of the model to the booster. The only problem remaining is damping in the joints of the model itself. If the same material is used in the model as the full scale this problem reduces to insuring the proper normal force between the attachment points and if different materials are used then also adjustments on the coefficient of friction must be made as explained in Ref. 4.

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