

USE OF THERMAL RERADIATIVE EFFECTS
IN SPACECRAFT ATTITUDE CONTROL

by

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ABSTRACT

A spacecraft to be launched into a solar orbit has been designed to use light pressure as its principle means of attitude control. However, the torques thus produced would be conservative (i.e. dependant only on angular position), and any initial oscillations of the spacecraft would be difficult to dissipate without the use of active damping. It is the purpose of this thesis to analyze the possibility of using the lags in thermal reradiation from the spacecraft's absorbing and emitting surfaces to produce non-conservative torques which will dissipate oscillations of a given axis. A physically reasonable design is analyzed which will align a spacecraft's figure axis with the direction of incident radiation whether or not the vehicle is spinning. It is observed that the hardware this technique calls for is completely passive --- it has no moving parts and requires no spacecraft power.

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CHAPTER 1

INTRODUCTION

A satellite can be designed to utilize electromagnetic momentum (i. e. light pressure) for attitude control by appropriate arrangement of its pressure surfaces with respect to its center of mass. Despite the inherent mechanical simplicity and reliability of such a system in providing simple conservative restoring torques, it is more complicated to adapt this technique to dissipate the energy contained in undesirable dynamical modes. However, thermal reradiative effects, which are non-conservative, can either add or subtract energy from these modes and thus may serve as a damping mechanism for vehicles using light pressure for stabilization.

A. The Rotating Cylinder

To illustrate one characteristic of thermal reradiative forces, let's use as an example, a black metal cylinder rotating freely about its figure axis at a rate, w radians/sec while being illuminated uniformly by light of intensity I_0 erg/cm²sec. If we measure ψ from entry "twilight", as shown in figure 1, then the cylinder is illuminated for $0 < \psi < \pi$, reaching maximum illumination at $\psi = \pi/2$. In a steady state situation we would expect that the surface temperature, $T(\psi)$, would be a function of ψ only and might be represented as

$$T(\psi) = T_0(1 + A \sin \psi + B \cos \psi) \quad (1.1)$$

where T_0 is the "average" value of $T(\psi)$ over the entire surface of the cylinder, while A and B are "small" fluctuations about T_0 . We might also expect that the maximum surface temperature of a cylinder element might occur after it had experienced maximum light intensity (i.e. for $\pi/2 < \psi < \pi$).

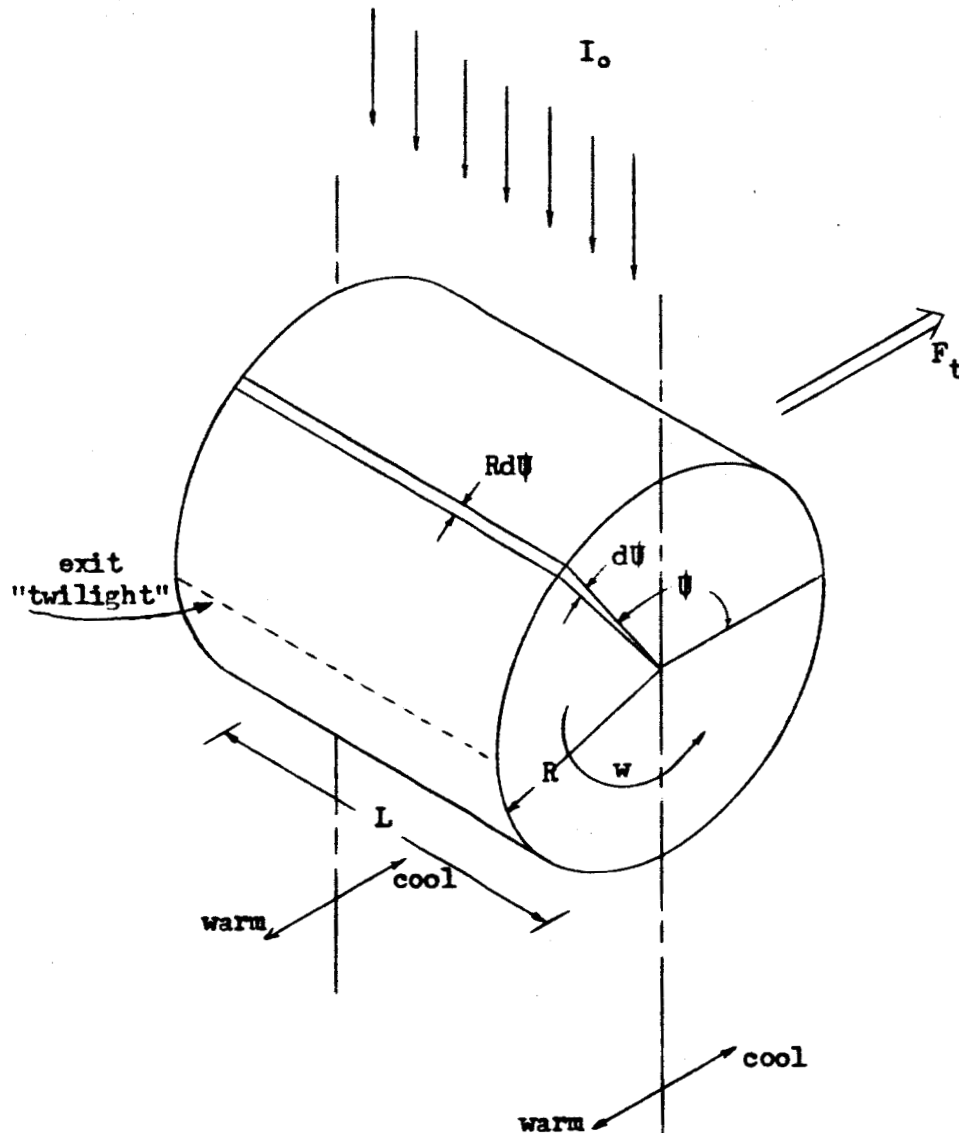


Figure 1

It is now helpful to divide the resulting radiation pressure on the cylinder into 2 parts. First is that component of pressure which is parallel to I_0 and proportional to the total energy intercepted by the cylinder. Second is the net pressure resulting from the asymmetric emission of thermal radiation from the cylinder surface since the surface temperature varies with ψ and the resulting pressure at each surface element is proportional to $S \sigma T^4(\psi)$, where $\sigma = 5.67 \times 10^{-5}$ erg/cm²sec°K⁴ and S is the area of the element.

Thus it can be seen that if the cylinder is divided into 2 halves as shown in figure 1, one half will always be (on the average) warmer than the other. Hence, more thermal radiation will be emitted from the "hot" side and therefore, there will be a net transverse component of radiation pressure which is in the direction of the cool half and perpendicular both to the rotation axis and to the direction of the incident radiation.

To find the magnitude of this pressure, consider a cylinder with the following properties:

- e = surface emissivity
- α = surface absorbtivity
- R = cylinder radius
- L = cylinder length
- w = angular velocity of cylinder
- $S = 2\pi RL$ = total area of cylinder
- $RL d\psi$ = area of a differential element

A differential element of the cylinder will emit radiation at the rate $dS e \sigma T^4(\psi) = RL d\psi e \sigma T^4(\psi)$. Now it can be shown (see appendix A) that the electromagnetic force on such a surface is normal to that surface and of a magnitude given by

$$dF = \frac{\text{power radiated}}{2c} = \frac{R d\psi L e \sigma T_o^4(\psi)}{2c} \quad (1.2)$$

where c is the speed of light in a vacuum. Substituting (1.1) into (1.2) we get

$$dF = \frac{R L d\psi e \sigma}{2c} T_o^4 (1 + 4A \sin \psi + 4B \cos \psi + \dots) \quad (1.3)$$

where the binomial expansion for $T^4(\psi)$ has been terminated at the first order terms.

In order to get the transverse force on the cylinder, F_t , we observe that the contribution to it from each differential surface element is

$$dF_t = dF \cos \psi \quad (1.4)$$

where ψ is the angle between the surface normal and the F_t direction, and also the same ψ that appears in (1.1). Therefore,

$$dF_t = \frac{R L d\psi e \sigma T_o^4}{2c} (1 + 4A \sin \psi + 4B \cos \psi) \cos \psi \quad (1.5)$$

and integrating this around the surface of the cylinder we get

$$F_t = \frac{R L e \sigma T_o^4}{2c} \int_0^{2\pi} (\cos \psi + \underbrace{4A \sin \psi \cos \psi}_{2A \sin 2\psi} + 4B \cos^2 \psi) d\psi. \quad (1.6)$$

However, this immediately reduces to

$$F_t = \frac{R L e \sigma T_o^4}{2c} 4B \int_0^{2\pi} \cos^2 \psi d\psi \quad (1.7)$$

since

$$\int_0^{2\pi} \sin(N\psi) d\psi = \int_0^{2\pi} \cos(N\psi) d\psi = 0 \quad \text{for } N=1, 2, 3, \dots \quad (1.8)$$

Evaluating (1.7) we get

$$F_t = \frac{2R L e \sigma T_o^4}{2c} B \pi = \frac{S e \sigma T_o^4}{c} B \quad (1.9)$$

where S is the total area of the cylinder.

Thus we see that the expression for the transverse force (1.9) takes the usual form of thermal emissive forces except that it is multiplied by the unknown factor "B". It is to the evaluation of this factor that chapter 2 is devoted.

However, it should be pointed out here that B also has a very special physical significance. The "direct" light pressure (i.e. that radiation pressure exclusive of reradiative forces) on the cylinder is simply equal to the total energy intercepted by it, divided by c ;

$$F_d = \frac{2RL\alpha I_0}{c}. \quad (1.10)$$

Now assume the cylinder receives energy from an area of $2RL$ and radiates it from an area of $2\pi RL$. Then we may write the energy balance for it as

$$\alpha I_0 2RL = \sigma e T_0^4 2\pi RL = \sigma e T_0^4 S \quad (1.11)$$

If we substitute (1.11) into (1.9) and then divide by (1.10), we get as our result

$$\frac{F_t}{F_d} = B \quad (1.12)$$

Thus we see that B is simply the ratio of the transverse reradiative pressure to the direct light pressure.

B. The Librating Slab

To illustrate another characteristic of thermal reradiative forces, consider a slab of material (perhaps part of a spacecraft) which is insulated on one side and librating about its equilibrium position (which is at $\theta = 0$) at angular frequency w as shown below in figure 2. If we require

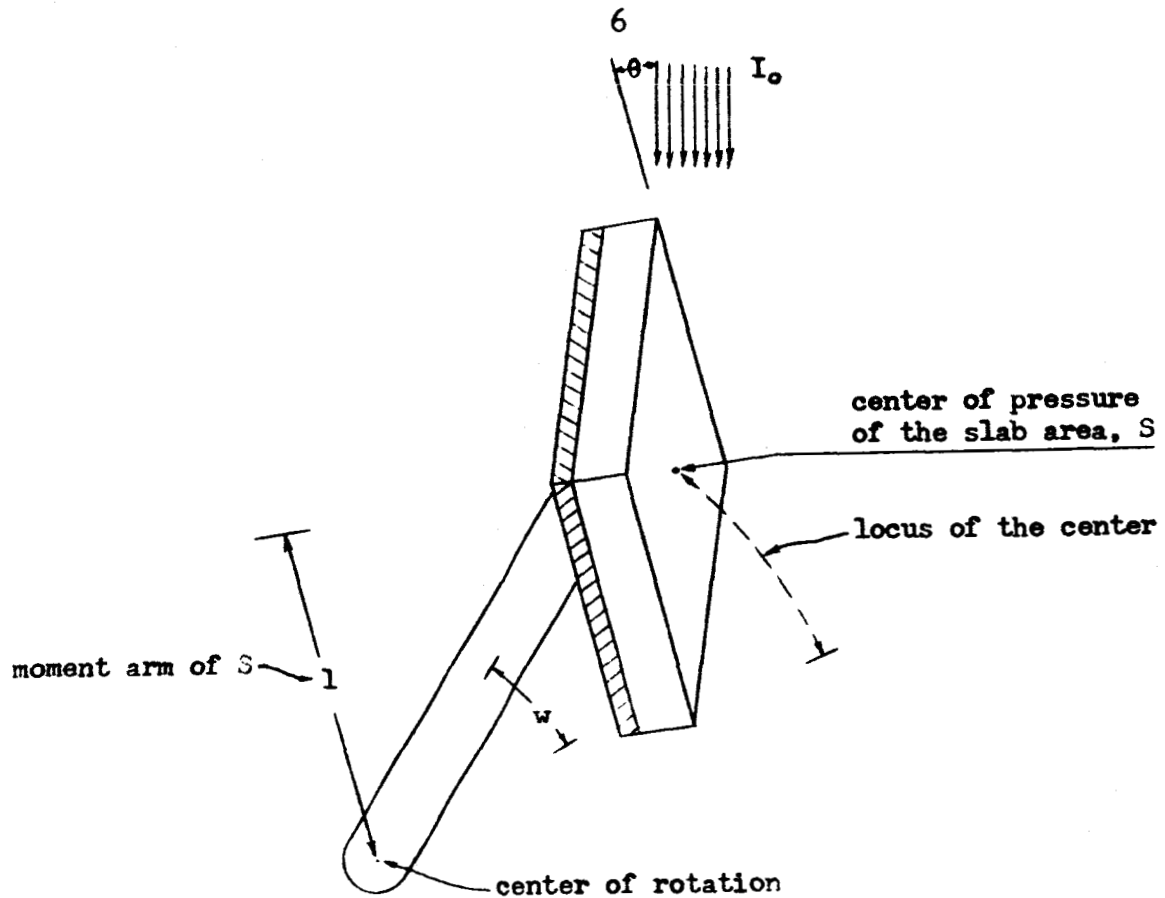


Figure 2

θ to be a periodic function of time, that is if

$$\theta = \theta_0 \sin \omega t \quad (1.13)$$

then the time dependent light intensity, $I(t)$, which the surface of the slab experiences will be periodic and identical in form with that experienced by a cylinder element in the previous section (see Appendix B).

Therefore, the surface temperature of the slab will also be a periodic function of time in the steady state case and might be represented in an analogous manner to equation (1.1)

$$T(t) = T_0(1 + A \sin \omega t + B \cos \omega t). \quad (1.14)$$

Now it should be expected that the "direct" light pressure on the slab, being a function of angular position only, would just give rise to conservative torques (i.e. torques that would do no net work on the system during one cycle). However, because the periodic surface temperature of the slab need not be exactly in phase with position due to the lag in thermal re-radiation, the resulting reradiative forces could be partially in phase with the angular velocity of the system. Thus, these latter torques would be non-conservative and actually do work over one cycle of oscillation.

To find the magnitude of this work, first note from (1.2) we have

$$F(t) = \frac{\sigma \epsilon S T^4(t)}{2c}.$$

Substituting (1.14) into (1.2) we get (as in (1.3))

$$F(t) = \frac{\sigma \epsilon T_o^4}{2c} (1 + 4A \sin \omega t + 4B \cos \omega t + \dots) \quad (1.15)$$

Now we want to find the work done, if any, by the slab's thermal re-radiative forces on the "system" during one cycle.

$$\text{Work} = W = \oint F(t) \text{ velocity}(t) dt = \int_0^{2\pi} F(t) l \dot{\theta}(t) dt \quad (1.16)$$

Differentiating (1.13) and substituting it into (1.16) there results

$$W = \frac{\theta_o l S \sigma \epsilon T_o^4}{2c} \int_0^{2\pi} (\cos \omega t + 4A \sin \omega t \cos \omega t + 4B \cos^2 \omega t) dt \quad (1.17)$$

$\underbrace{\hspace{10em}}_{2A \sin 2\omega t}$

which by (1.8) immediately reduces to

$$W = \frac{\theta_o l S \sigma \epsilon T_o^4}{2c} \int_0^{2\pi} 4B \cos^2 \omega t dt = \frac{2\pi \theta_o l S \sigma \epsilon T_o^4}{c} B \quad (1.18)$$

Now it should be pointed out that since the radiating surface of a librating slab and an element of a rotating cylinder experience identical

functions of light input intensity, the factor B which appears so prominently in both analyses is also identical. So although the case of the rotating cylinder will be used to calculate B in chapter 2, the results thus obtained also apply to this section.

CHAPTER 2

THE GENERAL EQUATION

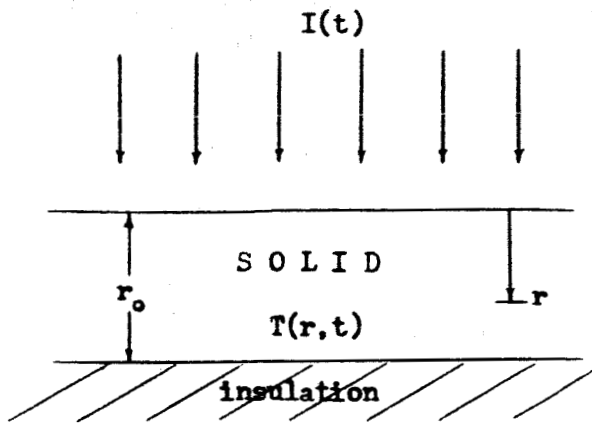
In chapter 1 it was assumed that the surface temperature was known and therefore, only the surface properties were needed to calculate the reradiative forces. However, the surface temperature of the cylinder is very much a function of the bulk thermal properties of the cylinder material, and these will now be put to use in the determination of the factor B. However, because the detailed derivation is extremely lengthy and tedious, it will only be sketched here and the remainder will be found in Appendix C.

A. The General Equation

Consider the case of one-dimensional heat flow in a solid (see figure 3) of thickness r_o , with insulation at one end and time dependant thermal radiation at the other end. Since no heat can flow through the surface, $r=r_o$, all thermal energy that is exchanged by the solid and its environment must enter the solid at $r=0$ at the rate $\alpha I(t)$ and leave the solid at the rate $e\sigma T^4(0,t)$. These two boundary conditions may be stated mathematically as;

$$K \frac{\partial T(r,t)}{\partial r} \bigg|_{r=r_o} = 0 \quad (2.1)$$

$$-K \frac{\partial T(r,t)}{\partial r} \bigg|_{r=0} = \alpha I(t) - e\sigma T^4(0,t) \quad (2.2)$$



Properties of Solid (constants)

K = thermal conductivity

C_p = specific heat

ρ = density

α = surface absorbtivity

e = surface emissivity

$T(r, t)$ = temperature as a function of depth and time

Figure 3

For our purposes $I(t)$ can be written as $I_0 f(t)$ (see Appendix B) where I_0 is the maximum light intensity and $f(t)$ is a unit half-rectified sinusoid as shown in figure 4. $f(t)$ can also be written as the Fourier series;

$$f(t) = \frac{1}{\pi} \left(1 + \frac{\pi}{2} \sin \omega t - \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t - \dots \right) \quad (2.3)$$

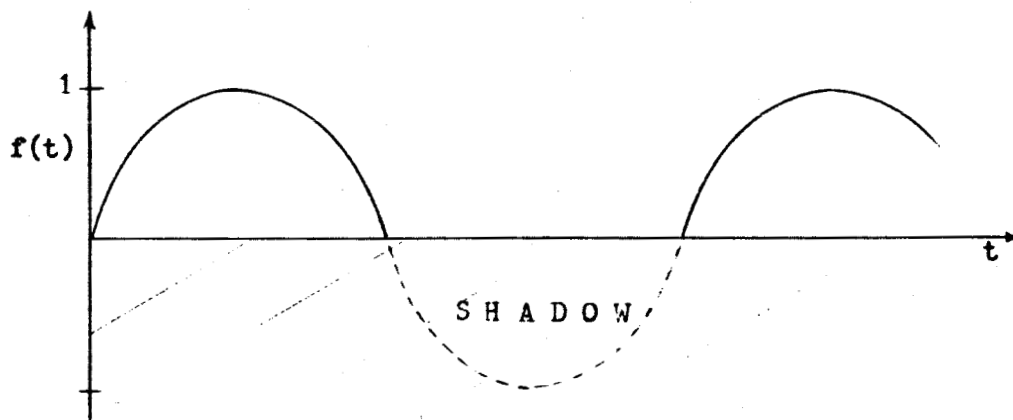


Figure 4

In order to find the surface temperature as a function of time (which is our goal), we must consider the flow of heat throughout the entire solid. In one dimension this heat flow obeys the well known equation

$$\frac{\partial T(r,t)}{\partial t} = k \frac{\partial^2 T(r,t)}{\partial r^2} \quad (2.4)$$

where $k = \frac{K}{C_p \rho}$.

Now since $I(t)$ is a periodic function of time with angular frequency w , it is reasonable to assume a solution to (2.4) of the form

$$T(r,t) = T_o(1 + A(r) \sin wt + B(r) \cos wt). \quad (2.5)$$

If we substitute (2.5) into (2.4), we will eventually get

$$\begin{aligned} T(r,t) = T_o(1 + [e^{-Zr}(-v \sin Zr + u \cos Zr) + e^{Zr}(-t \sin Zr + s \cos Zr)] \sin wt \\ + [e^{-Zr}(-u \sin Zr - v \cos Zr) + e^{Zr}(s \sin Zr + t \cos Zr)] \cos wt) \end{aligned} \quad (2.6)$$

where Z is a dimensionless parameter and u , v , s , and t are all arbitrary constants of integration. If we now require (2.6) to satisfy (2.1) and (2.2), we will get 4 algebraic equations in terms of the 4 arbitrary constants.

If we note that at the surface ($r=0$) equation (2.6) reduces to

$$T(0,t) = T_o[1 + (u+s) \sin wt + (t-v) \cos wt],$$

it follows from a previous argument that we want to solve for $t-v$, the cosine component of surface temperature (i.e. $t-v$ is the factor B that appeared in chapter 1). After many laborious algebraic manipulations, we finally obtain as our result:

$$B(r_o, w) = t-v = \frac{-\frac{1}{4}(\sinh M + \sin M)}{P(\cosh M - \cos M) + \frac{4}{\pi}(\sinh M - \sin M) + \frac{8}{\pi^2 P}(\cosh M + \cos M)} \quad (2.7)$$

where

$$P = \frac{T_o}{\alpha I_o} \sqrt{\frac{1}{2} C_p \rho K w}, \quad M = 2 Z r_o, \quad T_o^4 = \frac{\alpha I_o}{\pi e \sigma} \quad (2.8)$$

B. Special Cases of the General Equation

1. Infinite conductivity

It is of interest to examine (2.7) for the special case in which conductivity effects can be ignored. This may be done by letting $K \rightarrow \infty$ and using the small argument approximations for all the transcendental functions in (2.7). Thus we may write (since $M = 2r_0 \sqrt{\frac{\rho C_p w}{2K}} \rightarrow 0$ as $K \rightarrow \infty$).

$$\begin{aligned} \sinh 2Zr_0 + \sin 2Zr_0 &\approx 4Zr_0 \\ \sinh 2Zr_0 - \sin 2Zr_0 &\approx (8/3)Z^3 r_0^3 \\ \cosh 2Zr_0 + \cos 2Zr_0 &\approx 2 \\ \cosh 2Zr_0 - \cos 2Zr_0 &\approx 4Z^2 r_0^2 \end{aligned} \tag{2.9}$$

Substituting (2.9) into (2.7) we get

$$B = \frac{-\frac{1}{2} C_p \rho w r_0}{r_0^2 C_p^2 \rho^2 \frac{T_0 w^2}{\alpha I_0} + \frac{16}{\pi^2} \frac{\alpha I_0}{T_0} + \frac{8}{3\pi K} C_p^2 \rho^2 w^2}.$$

If we now let the remaining K in the expression go to infinity and define

$$\begin{aligned} S &= \text{total area of cylinder} \\ C_p^* &= C_p \rho S r_0 \\ Q &= \alpha S I_0 \end{aligned}$$

we finally get

$$B = \frac{-\frac{1}{2} C_p^* T_0 Q w}{(C_p^* T_0 w)^2 + \left(\frac{4}{\pi} Q\right)^2} \tag{2.10}$$

To find the maximum of (2.10), differentiate it with respect to w and set the result to zero. There results the following condition for maximum B :

$$w = \frac{4}{\pi} \frac{Q}{C_p^* T_0} \tag{2.11}$$

Substituting (2.11) into (2.10), one finds that the numerical value of this maximum is;

$$B_{\max} = \frac{-\pi}{16} = \underline{-0.1963}$$

Thus we find that in this case B can attain a value which is a significant fraction of T_0 . In fact, since B turns out to be the ratio of the transverse force to the total "direct" force, I think it must be surprising that a thermal reradiative force can be nearly 1/5 as large as direct light pressure itself.

In practice, this special case is readily applied to a thin metallic cylinder. Since the thermal conductivity of most metals is quite high, we may assume that there is no radial temperature gradient, and hence that the conductivity is "unimportant". However, since we must maintain a tangential thermal gradient ($T(\psi)$) to have a non-zero value for B, the conductivity of the metal will tend to smooth this gradient out thus degrading the B value. A quantitative analysis of this effect is given in the next section.

2. The Effect of Tangential Conductivity

The derivation which will only be sketched here may be found in detailed form in Appendix D. The result cannot be derived from equation (2.7) which considers only radial heat flow, an effect which will not be considered in this section. (It is assumed here that for a thin cylinder where tangential heat flow is important, radial effects may be neglected.)

Consider again the cylinder shown in figure 1. This cylinder will be assumed to be in exactly the same situation as before except that now heat may be exchanged between adjacent elements. Then, over a short angular

distance, we may assume linear one-dimensional heat flow and use (2.4) in the form

$$\frac{\partial T(\psi, t)}{\partial t} = k \frac{1}{R^2} \frac{\partial^2 T(\psi, t)}{\partial \psi^2} \quad (2.12)$$

The thermal energy balance for a cylinder element can be written as:

net heat influx rate = inflow rate - outflow rate

$$C_p^* \frac{\partial T(\psi, t)}{\partial t} = k \frac{1}{R^2} \frac{\partial^2 T(\psi, t)}{\partial \psi^2} + \alpha I_0 S f(t) - \sigma e S T^4(\psi, t) \quad (2.13)$$

Assuming a solution of the same form as (1.1) and substituting (1.1) into (2.13), we are finally led to

$$B = \frac{-\frac{1}{2} C_p^* Q w}{T_0 \left[C_p^{*2} w^2 + \left(\frac{4}{\pi T_0} Q + 2 \pi G \right)^2 \right]} \quad (2.14)$$

where

$$G = \frac{K L r_0}{R} \quad (2.15)$$

Now it can be readily verified that

$$w_{\max} = \frac{1}{C_p^*} \left(\frac{4}{\pi T_0} Q + 2 \pi G \right) \quad (2.16)$$

$$B_{\max} = \frac{-\frac{1}{2} Q}{\frac{8}{\pi} Q + 2 T_0 \pi G} \quad (2.17)$$

It can be seen that (2.10) and (2.14) are identical except for the factor G in the denominator whose presence serves both to reduce the maximum attainable value of B as well as to increase the value of w at which it is attained. It also seems reasonable that this factor G should be directly proportional to the product of the thermal conductivity and the tangential

cross section, and inversely proportional to the cylinder radius.

3. The Semi-infinite Solid

Another special case which is of interest is for "large" r_o , or for a cylinder in a situation where the temperature of its inner surface does not change with time. This special case occurs in the analysis of the radiation pressure on rotating bodies in orbit about the sun and was first mentioned in this context by Yarkovsky about 1900. Later quantitative work by Öpik led to an approximation for the B factor for a rotating sphere which was;

$$B \approx \frac{I_o}{T_o \sqrt{\rho C_p K w}} \quad (2.18)$$

and was derived in an entirely different manner than will be done here.

Note that (2.18) is not valid for small w .

I will now show that (2.7) reduces to (2.17) as $r_o \rightarrow \infty$ and as w becomes "large". First, we immediately note that the $\cosh(2Zr_o)$ and $\sinh(2Zr_o)$ are unbounded as $r_o \rightarrow \infty$ while the $\sin(2Zr_o)$ and $\cos(2Zr_o)$ remain between ± 1 . Therefore, (2.7) first reduces to

$$B = \frac{-\frac{1}{4} \sinh 2Zr_o}{P \cosh 2Zr_o + \frac{4}{\pi} \sinh 2Zr_o + \frac{8}{\pi^2 P} \cosh 2Zr_o} \quad (2.19)$$

If we divide both numerator and denominator of (2.18) by $\cosh(2Zr_o)$ and recognize that $\frac{\sinh(2Zr_o)}{\cosh(2Zr_o)} \rightarrow 1$ as $r_o \rightarrow \infty$, then (2.18) reduces to

$$B = \frac{-\frac{1}{4}}{\frac{T_o}{\alpha I_o} \sqrt{\frac{1}{2} C_p \rho K w} + \frac{4}{\pi} + \frac{8 \alpha I_o}{\pi^2 T_o} \frac{1}{\sqrt{\frac{1}{2} C_p \rho K w}}} \quad (2.20)$$

and as w gets "large" (2.20) approaches

$$B \approx \frac{-\alpha I_o}{2\sqrt{2} T_o \sqrt{C_p \rho K w}} \quad (2.21)$$

which agrees with (2.18). However, even though (2.20) is strictly valid only for a cylinder, it would probably be a better approximation if normalized to a sphere than (2.18).

C. A Numerical Example

Because of the complex character of equation (2.7), it seemed advisable to graph the function. From this graph, which is shown in Appendix C, it can be demonstrated that (2.10) is valid over a very large range, which encompasses the entire region corresponding to a rotating thin metallic cylinder. Therefore, the refinement of (2.10) which takes tangential conductivity into account, (2.14), could also be used with some confidence in this special case.

Let's take a specific numerical example, appropriate to the Sunblazer program, and use (2.10) to see ideally what type of hardware is called for. Since weight is a consideration of all spacecraft and beryllium has a large specific heat, we will use this metal in our design. The parameters we will use are;

$\rho = 1.85 \text{ gm/cc}$	$R = 25 \text{ cm}$
$C_p = 1.12 \times 10^7 \text{ erg/}^\circ\text{K gm}$	$L = 20 \text{ cm}$
$K = 2.3 \times 10^7 \text{ erg/cm}^2 \text{ sec } ^\circ\text{K}$	$w = .03 \text{ rad/sec } (.3 \text{ rpm})$
$\alpha = 1$	$r_o = ?$
$e = 1$	

If we choose to study the behavior of this system at 1 astronomical unit, then $I_o = 1.5 \times 10^6 \text{ erg/cm sec}$. Our problem is to find the value for r_o which maximizes (2.10) (i.e. the value for r_o which satisfies (2.11)).

Solving (2.11) for r_o we get

$$r_o = \frac{4Q}{\pi T_o \rho S C_p w} \quad (2.21)$$

Evaluating (2.21), r_0 turns out to be about 0.1 mm and the corresponding mass of the cylinder is about 60 grams. Also, using equation (1.12), we find that the maximum transverse force on the cylinder is approximately 0.01 dynes. This force is admittedly quite small, but since it is not necessarily conservative and therefore free to do work on the system, this value could become significant over an extended period of time. The effect of such forces on the various dynamical modes of a spacecraft will be considered in chapter 3.

It should be pointed out here that tangential conductivity is quite important in the previous example. In fact, if the resulting value for r_0 as well as the conductivity are plugged into (2.14), we find that B is decreased to less than half the value it would have in the absence of tangential conductivity. However, a continuous metal cylinder is not the only possible design, and a cylinder constructed of properly segmented strips of metal separated by insulating materials could probably approach ideal performance.

CHAPTER 3

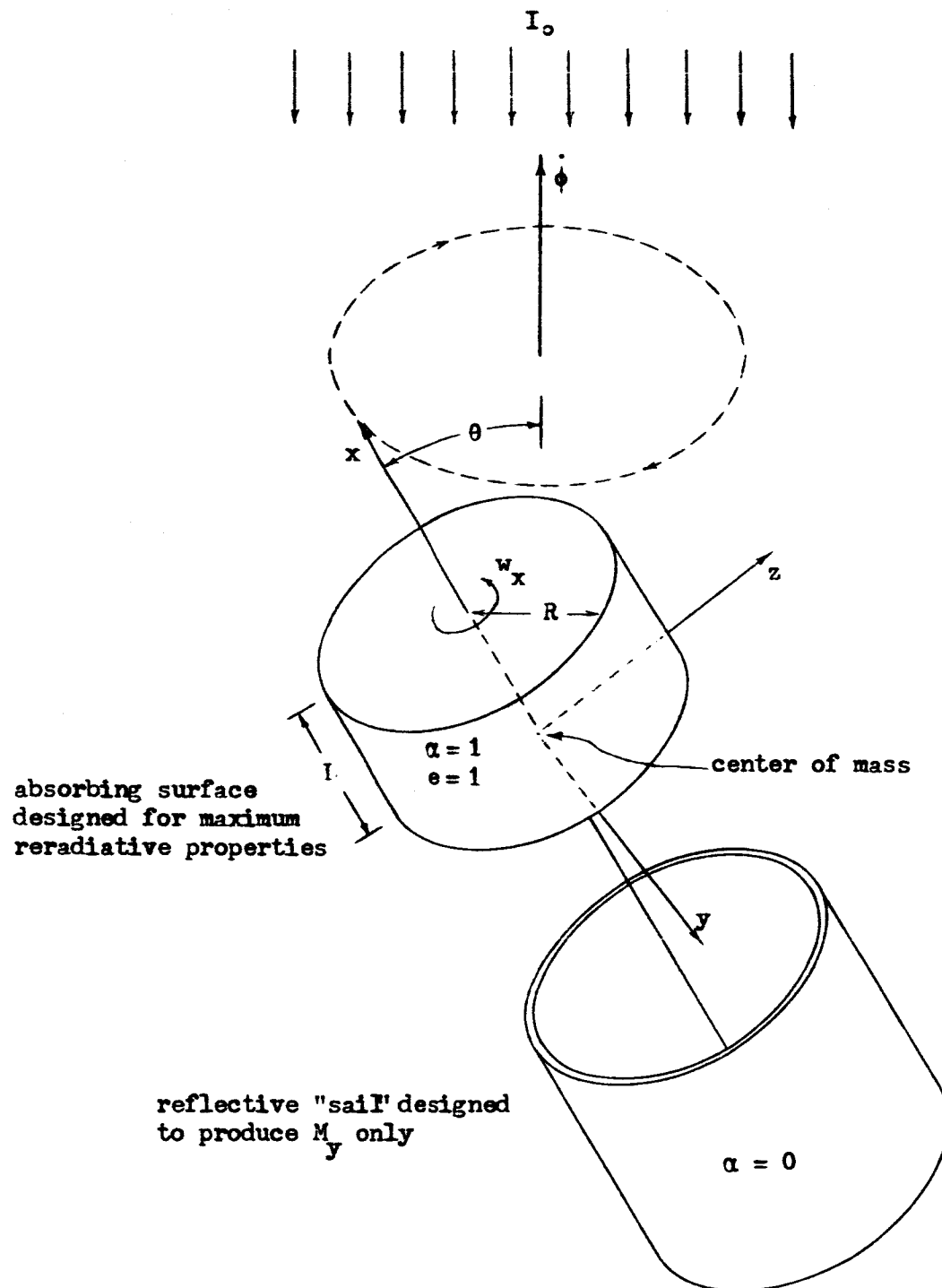
SPACECRAFT DYNAMICS

We have seen that an illuminated cylinder rotating about its axis of symmetry may experience a transverse reradiative force which is perpendicular both to the cylinder axis and to the direction of incident radiation. If this force were to have a moment arm about a spacecraft's center of mass, then the resulting torque could be capable of dissipating oscillations of the spacecraft's spin axis (i.e. precession).

A. A Spinning Axially Symetric Spacecraft

Consider an axially symetric spacecraft, as shown in figure 5, which has its center of pressure placed so as to always produce a torque tending to align its axis with the direction of incident radiation. If the vehicle is not spinning, it will simply librate about one of its transverse axes. If, however, it is rotating about its figure axis and we ignore reradiative forces, the spacecraft will, in general, both precess and nutate at an amplitude which will be undiminished with time and determined by the initial conditions.

To show this mathematically, consider a spacecraft whose axial and transverse moments of inertia are I_x and I_t respectively. Let its angular velocity be w about the x axis, which makes an angle θ with the direction of illumination, and let its y axis be chosen perpendicular to the direction



Schematic of Spacecraft

Figure 5

of illumination. Then because the spacecraft is symmetric, we may use the "flywheel" equations in the form

$$\begin{aligned}
 M_x &= I_x \frac{dw_x}{dt} \\
 M_y &= I_t \frac{d\Omega_y}{dt} - \Omega_x I_t \Omega_z + \Omega_z I_x w_x \\
 M_z &= I_t \frac{d\Omega_z}{dt} - \Omega_y I_x w_x + \Omega_x I_t \Omega_y
 \end{aligned} \tag{3.1}$$

where the M 's are applied moments about their respective axes. Now if the azimuth of the x axis about the direction of illumination is ϕ , then we have

$$\begin{aligned}
 \Omega_x &= \dot{\phi} \cos \theta \\
 \Omega_y &= \dot{\theta} \\
 \Omega_z &= \dot{\phi} \sin \theta
 \end{aligned} \tag{3.2}$$

Substituting (3.2) into (3.1) we get

$$\begin{aligned}
 M_x &= I_x \frac{dw_x}{dt} \\
 M_y &= I_x w_x \dot{\phi} \sin \theta + I_t (\ddot{\theta} - \dot{\phi}^2 \sin \theta \cos \theta) \\
 M_z &= -I_x w_x \dot{\theta} + I_t (\ddot{\phi} \sin \theta + 2\dot{\phi} \dot{\theta} \cos \theta)
 \end{aligned} \tag{3.3}$$

Now in general, $M_x = 0$ and if we exclude thermal reradiative forces, $M_z = 0$ also. However, M_y will usually be present and depend on θ only. If we are just interested in the steady state solution of (3.3), then we may further require $\dot{\theta} = \dot{\phi} = 0$ and there results

$$\dot{\phi} = - \frac{I_x w_x}{2I_t \sin \theta} \pm \sqrt{\left(\frac{I_x w_x}{2I_t \sin \theta} \right)^2 - \frac{M_y(\theta)}{I_t \sin \theta \cos \theta}} \tag{3.4}$$

$$\dot{\theta} = 0 \quad (3.5)$$

which corresponds to a steady precession of the spacecraft at two possible rates about the I_0 direction. It is important to note here that θ , the cone angle, is constant in this case where we have assumed $M_z = 0$.

B. Effects of Reradiative Forces on Steady Precession

We have seen that in the absence of thermal reradiative forces, the spacecraft spin axis will precess at a constant angle to the direction of illumination. Now since its direction is perpendicular both to the spin axis (x axis) and to the I_0 direction (see figure 5), this force must be parallel to the y direction. Hence, if it is generated primarily by the thermal characteristics of that portion of material forward of the spacecraft's center of mass, the resulting torque will be entirely about the z axis and (from the positive sense of w_x) in the positive sense.

Let's see what the effect of this positive z moment will be on the steady precession of the previous section. If we assume that the precession rate is very nearly constant, (i.e. $\ddot{\phi} \approx 0$) then we may write

$$M_z = -I_{xx} w_x \dot{\theta} + 2I_t \dot{\phi} \dot{\theta} \cos \theta \quad (3.6)$$

Solving for $\dot{\theta}$ we get

$$\dot{\theta} = - \frac{M_z}{I_{xx} w_x - 2I_t \dot{\phi} \cos \theta} \quad (3.7)$$

But since $w_x \gg \dot{\phi}$ (usually), we may further reduce (3.7) to

$$\dot{\theta} = - \frac{M_z}{I_{xx} w_x} \quad (3.8)$$

Thus, we see that an important effect of thermal reradiative forces on

a precessing spacecraft is to produce a negative $\dot{\theta}$, that is, an "erecting" torque is generated which tends to align the spacecraft's spin axis with the direction of incident radiation. Once this alignment has been accomplished, however, both M_y and M_z vanish and the vehicle's figure axis will thereafter remain fixed in space (unless the direction of the incident radiation changes).

We might briefly consider the effect of these same reradiative forces on a non-rotating spacecraft which is librating about one of its transverse axes. First recall from section 1.B that work may be done on such a system by reradiative forces. If we observe from figure 5 that in this case (with the reradiating surface in front of the center of mass) these forces will produce torques which oppose the angular velocity of libration, it becomes apparent that energy is removed from this type of oscillation during each cycle. Therefore, the amplitude of the libration will decrease and the spacecraft axis will eventually become stationary and aligned with the direction of incident radiation. It is a fortunate coincidence that the same reradiating surface which reduces the precession cone angle when the spacecraft is spinning, also damps out oscillations about the transverse axis when w_x is zero.

C. A Numerical Example

Suppose we examine the dynamical response of a spinning Sunblazer-type spacecraft, similar to that shown in figure 5, with parameters for the upper cylinder which are near those used in section 2.C.

$\alpha = 1$	$w_x = .01 \text{ rad/sec}$
$e = 1$	$I_x = 10^6 \text{ gm cm}^2$
$R = 25 \text{ cm}$	$C_p^* = 1.74 \times 10^9 \text{ erg/}^\circ\text{K}$
$L = 20 \text{ cm}$	$I_o = 1.5 \times 10^6 \text{ erg/cm}^2\text{sec } (\odot 1 \text{ A.U.})$
$r_o = .017 \text{ cm}$	

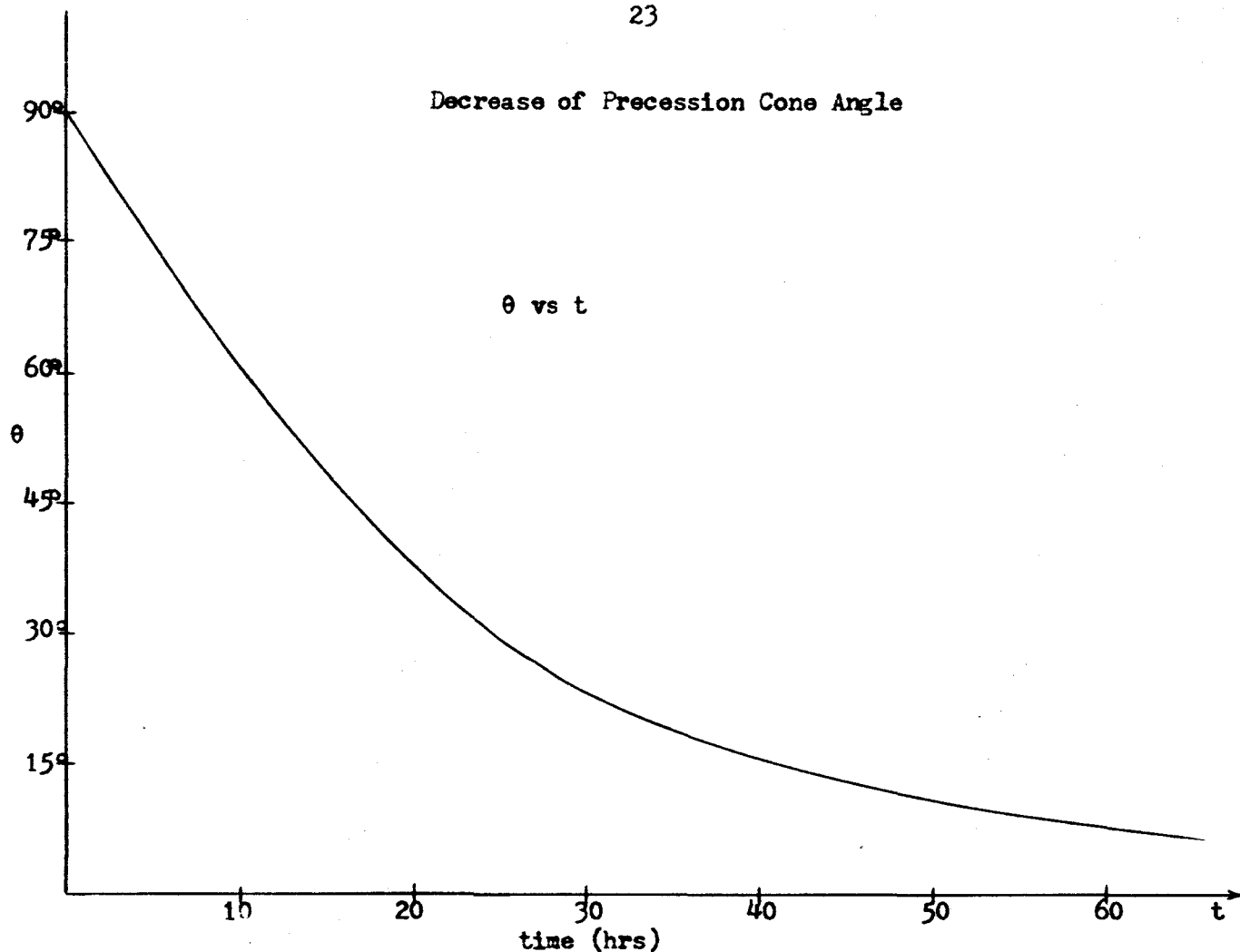


Figure 6

If we choose the initial value for θ to be 90° and assume that the cylinder surface has a moment arm of $l = 15$ cm about the center of mass, then we can calculate M_z from (2.10) as a function of θ . With this value for M_z , (3.8) can be used to calculate $\dot{\theta}$ as a function of θ . When this relationship is numerically integrated, there results a response for the spacecraft (θ vs t) as shown in figure 6.

Although the "erecting" torque is small in this example ($.15$ dyne-cm), it can be seen from the graph above that it is still capable of rotating the spacecraft's spin axis nearly 60° in 24 hours. However, this rate quickly decays as θ becomes less than 30° , but it is still sufficient to

reduce the cone angle to less than 6° after 3 days have elapsed. One way faster responses may be produced is to increase either the area of the radiating surface or its moment arm about the spacecraft's center of mass. Another way to do this would be to increase the erecting torque by amplifying the reradiative forces themselves, and this latter technique is considered in Chapter 4.

CHAPTER 4

AMPLIFICATION OF RERADIATIVE FORCES

In the previous chapters, only the momentum contained in electromagnetic radiation was considered in the production of thermal reradiative forces. However, since the ratio of energy to momentum in this type of radiation is c , the velocity of light, it might be advantageous to use some of this abundant energy to eject a small amount of mass from the reradiative surface.

Suppose a certain amount of energy, E , is radiated from a surface. Then the impulse, δp , transferred to that surface will be

$$\delta p = \frac{E}{2c} \quad (4.1)$$

on the average. If the same amount of energy is carried away from the surface by a particle of mass m , then the impulse transferred to the surface will be

$$\delta p = \frac{E}{v} = \sqrt{\frac{Em}{2}} \quad (4.2)$$

on the average where v is the velocity of the particle. Since $v \ll c$ in general, the particle will transfer much more momentum to the surface than radiation can for the same amount of energy. Therefore, thermal reradiative forces could be greatly amplified if even a small amount of the incident energy were used to expel mass from a surface.

To illustrate one way this might be done, consider a porous reradiating surface which is continuously supplied with a gas under low pressure. Now as this gas effuses through the surface pores and escapes, it will acquire an rms velocity corresponding to the temperature of the material with which it was in contact. Consequently, the molecules effusing from the warmer half of the cylinder will leave with a higher velocity than those escaping from the cooler side and this will add to the reradiative forces already present. A workable system of this type, which could be sustained by the example in section 3.C., might release up to 100 grams of N_2 gas per day and generate a continuous transverse force of about 2 dynes. This would be 200 times greater than the corresponding reradiative forces.

Unfortunately, the above system would have no weight advantage over a gas jet thruster system since gas bottles, valves, and piping would be required. However, suppose that instead of storing the gas in a bottle, we store it in a compound which is applied over the reradiative surface. If we also require that this compound remain non-volatile until after it is exposed to the extreme ultra-violet of the sun's spectrum after which it dissociates and evaporates thermally from the surface, then the reradiative forces will be amplified in much the same way as before. Also, when the spacecraft's axis is properly aligned, no sunlight reaches the compound and it would not be further expended unless the spacecraft axis were somehow disturbed. Therefore, it would not have to expel gas continuously as in the previous example.

We might attempt further amplification by another order of magnitude by coating the surface with a material which photoionizes easily. If the

spacecraft has acquired a positive charge from photoelectric emission, then the positive ions would be repelled from the spacecraft and leave the surface at a rather large velocity.

CHAPTER 5

CONCLUSIONS AND RECOMMENDED FURTHER STUDY

The main results which have been established here are as follows:

1. Any absorbing surface which is illuminated by radiation of time varying intensity will re-emit this energy with a certain time lag with respect to the incident radiation. If this surface is part of a rotating cylinder, then this lag will also result in the reradiated energy being emitted in a different direction, on the average, than the incident radiation. It is the momentum transferred to the surface by this reradiation which gives rise to a transverse force, which is a component perpendicular to the incident direction. This transverse force can be as large as $1/5$ of the total "direct" light pressure on a cylinder.
2. If this absorbing surface is part of a system which oscillates harmonically and which receives radiation in phase with angular position, then this lag in reradiation can produce a torque which is partially in phase with angular velocity. This allows the reradiative forces to do work on the system.
3. When the reradiating material is a thin flat sheet of metal, equation (2.10) may be used. For a thin metallic cylinder, equation (2.14) is preferred over (2.10). However, whenever radial

conductivity is "important" (see Appendix C), equation (2.7) must be used.

4. If a cylinder of reradiating material is attached in front of the center of mass of a spinning spacecraft, then the resulting reradiative torques can align the spacecraft's spin axis with the direction of incident radiation. (This feature is independent of the spacecraft's spin direction.) This same reradiating material can damp out librations of the spacecraft when it is not rotating. Now it should be noticed here that the hardware called for is completely passive; there are no moving or even bending members involved and no spacecraft power is required. Furthermore, the mass of this hardware need be no more than 1% of the total mass for a 10 kilogram spacecraft.

So far nothing has been said of the stability of nutational motions of the spacecraft's axis with the above mentioned reradiative technique. However, because only steady state situations have been considered, an analysis of nutational stability is not possible. A qualitative guess as to the effect of a "slowly" increasing I_0 on the rotating cylinder is that the reradiative forces will increase beyond the corresponding instantaneous steady state value for I_0 . The reverse should occur if I_0 slowly decreases. However, this speculation is not sufficient to lead to a clear understanding of nutational stability.

Therefore, I would recommend for further study an analysis of the behavior of thermal reradiative forces as the incident intensity is varied. This analysis could then be used to study the nutational stability problem.

However, even if the proper damping of this oscillation is not provided by reradiative forces, passive mechanical dampers should be able to operate satisfactorily at the relatively high angular accelerations of this mode.

I would also recommend the further investigation of techniques for amplifying reradiative forces. Although compounds are known which evolve N_2 gas when exposed to extreme ultra-violet radiation, I have not verified that any specific choice would be suitable for my purpose as stated in Chapter 4.

APPENDIX A

RERADIATIVE PRESSURE ON A FLAT SURFACE

Consider a flat surface of temperature T and area S which is isotropically radiating over a hemisphere. The rate at which it loses energy (the power radiated) is given by

$$\frac{dE}{dt} = e\sigma ST^4 \quad (A.1)$$

Then the energy flux per unit area through a unit hemisphere surrounding S will be

$$\frac{dE}{dt} \frac{1}{2\pi} = \frac{e\sigma ST^4}{2\pi} \quad (A.2)$$

and the corresponding momentum flux will be

$$dp = \frac{e\sigma ST^4}{2\pi c} \quad (A.3)$$

Our problem is to find that component of the total momentum flux through the hemisphere which is normal to the surface, S . We can do this by first dividing up the hemisphere into differential areas, dA , corresponding to the usual conventions of polar coordinates. (i.e. θ is the elevation angle from the horizontal and ϕ is the azimuth angle.) Therefore,

$$dA = d\phi \cos \theta d\theta$$

and the corresponding flux through the differential surface which is

normal to the surface, S , is

$$dp_n = \frac{e \sigma S T^4}{2\pi c} d\phi \cos \theta d\theta \sin \theta \quad (A.4)$$

Integrating (A.4) over the entire surface of the hemisphere, we get

$$F_n = \int dp_n = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \frac{e \sigma S T^4}{2\pi c} \sin \theta \cos \theta = \frac{e \sigma S T^4}{2c} \quad (A.5)$$

The force on the surface must be equal and opposite to the momentum flux through the hemisphere and therefore, the validity of equation (1.2) is demonstrated.

APPENDIX B

TIME DEPENDANT RADIATION INPUT

First consider the light intensity which an element of a rotating cylinder experiences. If we describe an axially symmetric body by a coordinate system like that shown in figure 5 with $w_x = \dot{\psi}$ and the direction of incident radiation defined as the x' axis, then we may relate the "prime" coordinate system to the body coordinate system by an orthogonal matrix transformation of the form

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (B.1)$$

where R is a rotation matrix. Now R can be found by multiplying together the individual matrices from successive rotations about body axes, and is given by

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \quad (B.2)$$

Multiplying (B.2) out and substituting it into (B.1) there results

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \sin \psi \\ \sin \phi \sin \theta & \cos \phi \cos \psi - \sin \phi \cos \theta \sin \psi \\ -\cos \phi \sin \theta & \sin \phi \cos \psi + \cos \phi \cos \theta \sin \psi \end{bmatrix}$$

$$\begin{bmatrix} \sin \theta \cos \psi \\ -\cos \phi \sin \psi - \sin \psi \cos \theta \cos \phi \\ -\sin \psi \sin \phi + \cos \phi \cos \theta \cos \psi \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (\text{B.3})$$

Now if the incident radiation is from the x' direction, then the light intensity which a surface element of the cylinder experiences will be proportional to the component of its surface normal in the x' direction. Since the surface normal of any cylinder element can be written as a linear combination of y and z , we want the projections of these axes on the x' axis which from (B.3) are $\sin \theta \sin \psi$ and $\sin \theta \cos \psi$ respectively. Therefore, for an element

$$I(t) = I_0 \sin \theta (\sin \psi + \cos \psi) \quad (\text{B.4})$$

where θ is constant. However, since we cannot have a negative value for the input intensity, $I(t)$ must be zero for negative values of (B.4), which corresponds to an element being in the shadow of the cylinder. Since $\psi = \omega_x t$, a cylinder element experiences a light input as shown in figure 4.

Now the angular position of a librating slab as shown in figure 2, which is oscillating harmonically, can be described by

$$\theta(t) = \theta_0 \sin \omega t \quad (\text{B.5})$$

Therefore, the light input intensity which the surface of the slab receives is

$$I(t) = I_0 \sin(\theta_0 \sin \omega t) \approx I_0 \sin \theta_0 \sin \omega t \quad (\text{B.6})$$

which is also of the same form as figure 4.

APPENDIX C

THE GENERAL EQUATION

Our problem is to find a solution to the equation

$$\frac{\partial T(r,t)}{\partial t} = k \frac{\partial^2 T(r,t)}{\partial r^2} \quad (C.1)$$

subject to the boundary conditions

$$k \frac{\partial T(r,t)}{\partial r} \bigg|_{r=r_0} = 0 \quad (C.2)$$

$$-k \frac{\partial T(r,t)}{\partial r} \bigg|_{r=0} = \alpha I_0 f(t) - e \sigma T^4(0,t) \quad (C.3)$$

We assume a solution to (C.1) of the form

$$T(r,t) = T_0(1 + A(r)\sin \omega t + B(r)\cos \omega t) \quad (C.4)$$

and upon substituting (C.4) into (C.1) we obtain 2 total differential equations for $A(r)$ and $B(r)$

$$\frac{d^4 A(r)}{dr^4} = -k^2 \omega^2 A(r) \quad (C.5)$$

$$\frac{d^2 A(r)}{dr^2} = -k \omega B(r) \quad (C.6)$$

Substituting $A = e^{\lambda r}$ in (C.5) there results

$$\lambda^4 = -k^2 w^2 = |k^2 w^2| \oplus 180^\circ \quad (C.7)$$

and applying DeMoivre's theorem to (C.7) we get

$$\lambda = \sqrt{k w} \left(\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} i \right), \quad -\sqrt{k w} \left(\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} i \right) \quad (C.8)$$

as the 4 roots. If we define

$$Z = \sqrt{\frac{k w}{2}}, \quad (C.9)$$

use (C.8) with 4 arbitrary constants of integration to get $A(r)$, and then use (C.6) to get $B(r)$, there results

$$A = e^{Zr} (-t \sin Zr + s \cos Zr) + e^{-Zr} (-v \sin Zr + u \cos Zr) \quad (C.10)$$

$$B = e^{Zr} (s \sin Zr - t \cos Zr) + e^{-Zr} (-v \cos Zr - u \sin Zr) \quad (C.11)$$

where s , t , u , and v are the arbitrary constants. If (C.10) and (C.11) are substituted into (C.4), it can be easily seen that (2.6) results.

In order to evaluate the 4 constants of integration in (2.6), we will use (C.2) and (C.3) to each give 2 algebraic equations. Using (C.2) with (2.6) and requiring the coefficients of $\sin wt$ and $\cos wt$ to separately vanish we get

$$e^{-Zr_0} [v(\sin Zr_0 \cos Zr_0) - u(\sin Zr_0 + \cos Zr_0)] + e^{Zr_0} [-s(\sin Zr_0 - \cos Zr_0) - t(\sin Zr_0 + \cos Zr_0)] = 0 \quad (C.12)$$

$$e^{-Zr_0} [v(\sin Zr_0 + \cos Zr_0) + u(\sin Zr_0 - \cos Zr_0)] + e^{Zr_0} [s(\sin Zr_0 + \cos Zr_0) - t(\sin Zr_0 - \cos Zr_0)] = 0 \quad (C.13)$$

In order to simplify this, define

$$\beta = \sin Zr_0 + \cos Zr_0 \quad (C.14)$$

$$\gamma = \sin Zr_0 - \cos Zr_0 \quad (C.15)$$

and (C.12) and (C.13) can be written as

$$e^{-Zr_0} (\gamma v - \beta u) + e^{Zr_0} (-\gamma s - \beta t) = 0 \quad (C.16)$$

$$e^{-Zr_0} (\beta v + \gamma u) + e^{Zr_0} (\beta s - \gamma t) = 0 \quad (C.17)$$

which are the 2 desired algebraic equations.

To get 2 more equations, we use (C.3) together with (2.6) and the first 2 terms of (2.3). If we define

$$C_1 = \frac{\alpha I_a}{K}, \quad C_2 = \frac{e \sigma T_a^4}{K} \quad (C.18)$$

we will get

$$\begin{aligned} ZT_0 [(-u - v + s - t) \sin wt + (v - u + t + s) \cos wt] = C \left(\frac{1}{\pi} + \frac{1}{2} \sin wt \right) \\ + C_2 (1 + 4(u + s) \sin wt + 4(t - v) \cos wt + \dots) \end{aligned} \quad (C.19)$$

Since (C.19) must be an identity, we equate the constant, sine, and cosine terms to get (after some simplification)

$$C_1 / \pi = C_2 \quad (C.20)$$

$$(ZT_0 + 4C_2)v - ZT_0 u + ZT_0 s + (ZT_0 - 4C_2)t = 0 \quad (C.21)$$

$$ZT_0 v + (4C_2 + ZT_0)u - (ZT_0 - 4C_2)s + ZT_0 t = \frac{1}{2}C_1 \quad (C.22)$$

Equations (C.18) and (C.20) can be used to find

$$T_0 = \sqrt[4]{\frac{\alpha I_a}{\pi e \sigma}} \quad (C.23)$$

However, (C.21) and (C.22) together with (C.16) and (C.17) provide enough

algebraic equations to solve explicitly for the constants s , t , u , and v .

Now at this point, the analysis becomes extremely messy and only intermediate results will be given. It is clear from the argument on page 11 that we want to solve for t and v . Therefore, we will first use (C.16) and (C.17) to get the following expressions for u and s ;

$$u = \left(\frac{2\beta\gamma}{\beta^2 - \gamma^2} \right) v - \left(\frac{\beta^2 + \gamma^2}{\beta^2 - \gamma^2} \right) e^{2Zr} t \quad (C.24)$$

$$s = - \left(\frac{\beta^2 + \gamma^2}{\beta^2 - \gamma^2} \right) e^{-2Zr} v + \left(\frac{2\beta\gamma}{\beta^2 - \gamma^2} \right) t \quad (C.25)$$

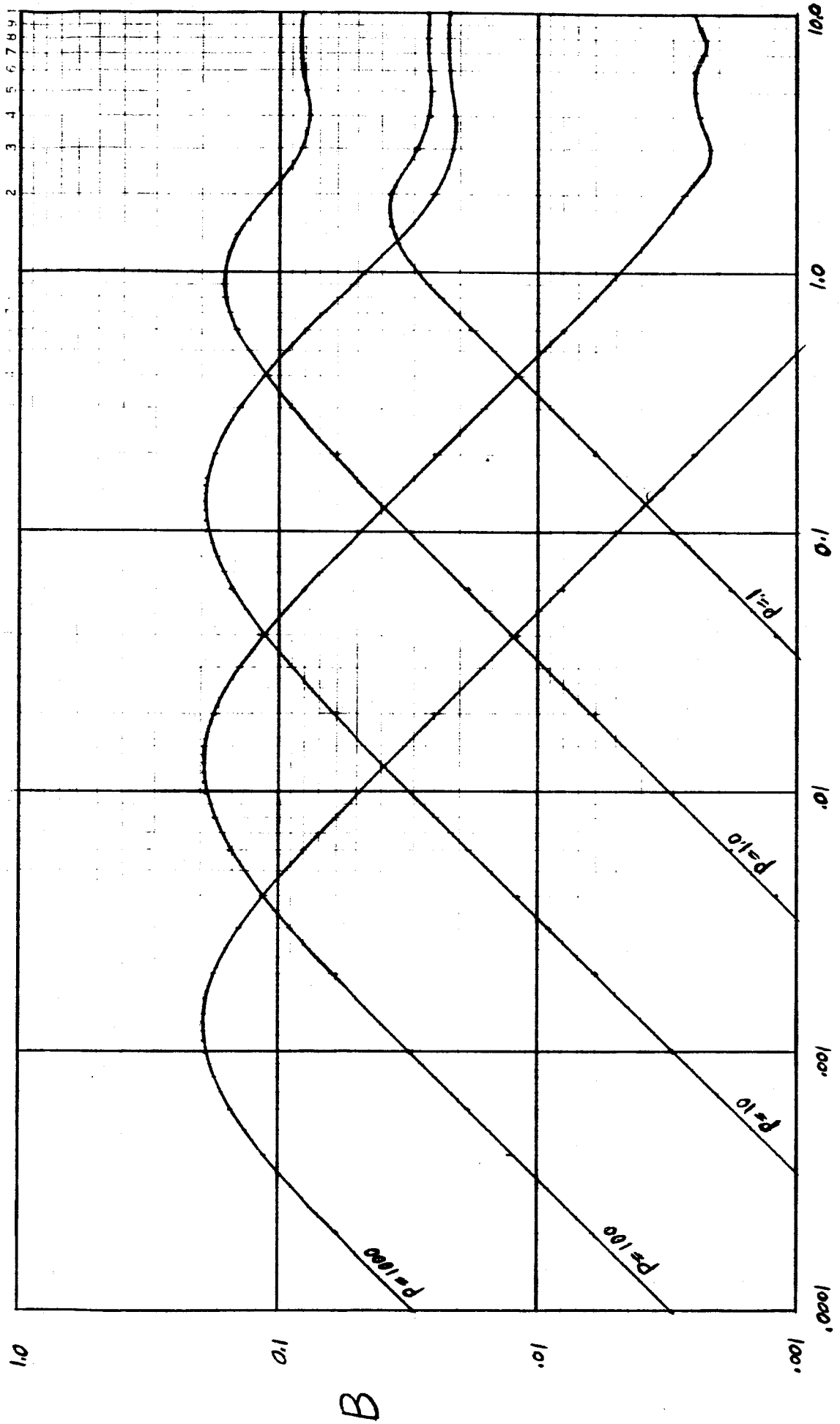
When (C.24) and (C.25) are substituted into (C.21) and (C.22), we get 2 equations for the 2 unknowns, t and v , which are quite lengthy. In proceeding to solve these equations for t - v , we find that all exponential terms coalesce into sinh's and cosh's, and we finally obtain

$$B(P, M) = t - v = \frac{-\frac{1}{4}(\sinh M + \sin M)}{P(\cosh M - \cos M) + \frac{4}{\pi}(\sinh M - \sin M) + \frac{8}{\pi^2 P}(\cosh M + \cos M)} \quad (C.26)$$

where

$$P = \frac{T_0}{\alpha I_0} \sqrt{\frac{1}{2} C_p \rho K W} \quad , \quad M = 2 Z r_0 \quad (C.27)$$

Equation (C.26) is graphed in figure 7 and it is clear that for $M < 1.0$, B takes the same characteristic form as P is varied. In fact, $M < 1.0$ is the condition necessary for (2.10) to be valid.



$$M = 2Z/k_0$$

Figure 7

APPENDIX D

TANGENTIAL HEAT FLOW

Our problem is to find a solution to (2.13). However, since $U = wt$, the temperature will just be a function of time and total derivatives of T may be used. Equation (2.13) may be converted to the form

$$C_1 \frac{dT}{dt} + T^4 = C_2 f(t) + C_3 \frac{d^2 T}{d(wt)^2} \quad (D.1)$$

where

$$\begin{aligned} C_1 &= \frac{r \rho C_p}{e \sigma} \\ C_2 &= \frac{\alpha I_0}{e \sigma} \\ C_3 &= \frac{K r}{e \sigma R^2} \end{aligned} \quad (D.2)$$

Now assume a solution to (D.1) of the form

$$T = T_0(1 + A \sin wt + B \cos wt) \quad (D.3)$$

If we substitute (D.3) into (D.1) we will get

$$\begin{aligned} C_1 T_0 (wA \cos wt - wB \sin wt) + T_0^4 (1 + 4A \sin wt + B \cos wt + \dots) = \\ C_2 \left(\frac{1}{\pi} + \frac{1}{2} \sin wt \right) + C_3 T_0 (-A \sin wt - B \cos wt) \end{aligned} \quad (D.4)$$

Since (D.4) must be an identity, we can equate the constant, sine, and cosine coefficients to get

$$T_o^4 = \frac{C_2}{\pi} = \frac{\alpha I_o}{\pi \epsilon \sigma} \quad (D.5)$$

$$(4T_o^4 + C_3 T_o)A - C T_o w B = \frac{C_2}{2} \quad (D.6)$$

$$C T_o w A + (4T_o^4 + C_3 T_o)B = 0 \quad (D.7)$$

Solving (D.6) and (D.7) for B we get

$$B = -\frac{w C_1 C_2}{2 T_o} \frac{1}{[C_1^2 w^2 + (4T_o^3 + C_3)^2]} \quad (D.8)$$

and using equations (D.2) and (D.5), we finally get

$$B = \frac{-\frac{1}{2} C_p^* Q w}{T_o [C_p^{*2} w^2 + (\frac{4}{\pi T_o} Q + 2\pi G)^2]} \quad (D.9)$$

where

$$G = \frac{K L r_o}{R} \quad (D.10)$$

REFERENCES

Opik, E. J., Proc. Roy. Irish Acad. 54 (1951), p 165