Lubarsky

THEORETICAL INVESTIGATION OF ULTRASONIC ATTENUATION FOR

FREE ELECTRONS IN THE PRESENCE OF A MAGNETIC FIELD

A Thesis Submitted to
the Graduate School of
John Carroll University
in Partial Fulfillment of the Requirements
for the Degree of
Master of Science

by

Lawrence Flax

1966

NASA Lewis Research Center

THEORETICAL INVESTIGATION OF ULTRASONIC ATTENUATION FOR FREE ELECTRONS IN THE PRESENCE OF A MAGNETIC FIELD

A Thesis Submitted to
the Graduate School of
John Carroll University
in Partial Fulfillment of the Requirements
for the Degree of
Master of Science

by

Lawrence Flax

1966

The thesis of Lawrence Flax is hereby accepted:

Adviser Date

ACKNOWLEDGEMENTS

I thank Professor Joseph Trivisonno who suggested the problem. His advice and criticism during the entire project were invaluable. I also express my appreciation to Professor Said for many helpful discussions during the course of the work.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

SYMBOLS

B	magnetic induction
c	speed of light
F	force per particle
f	distribution function
g_n, g_n, r_n, s_n	integrals of Bessel functions
H	magnetic field intensity
l	electrons mean free path
l	electrons mean free path
m	mass of the particle
n	number of particles per unit volume
P	momentum of an electron
Q	power per unit volume
q	phonon wavelength
R	radius of an electron orbit
R_{ij}	reciprocal tensor
S_{11}	relative attenuation coefficient for a longitudinal wave moving perpendicular to a magnetic field
S_{22}	relative attenuation coefficient for a transverse wave moving perpendicular to a magnetic field

S_{33}	relative attenuation coefficient for a transverse waves whose direction of polarization is parallel to the magnetic field
V	velocity of the particle in K space
V_f	Fermi velocity
V_s	velocity of sound
X	product of phonon wave number and classical orbit radius
α	attenuation coefficient
$\alpha(H)$	attenuation in a magnetic field
β	ratio of the classical to the phonon wave length
λ	wavelength of sound
ξ	electric field
ρ	density
σ_{ij}	conductivity tensor
σ'_{ij}	effective conductivity tensor
σ_0	D.C. conductivity
τ	relaxation time
ω	frequency of the ultrasonic wave
ω_c	cyclotron frequency

I. INTRODUCTION

Theoretical investigations of ultrasonic attenuation consist in calculating the power loss from the ultrasonic wave to the conduction electron. The theory of ultrasonic attenuation in the absence of a magnetic field using the free electron model have been considered by Pippard (ref. 1). The free electron model in the presence of a magnetic field has been given by Kjeldaas and Holstein (ref. 2) and independently by Cohen, Harrison and Harrison (ref. 3). Theoretical treatments of real metals were investigated by Pippard (ref. 4), Kanner (ref. 5), Akhiezer (ref. 6) and Blount (ref. 7) in the case of zero magnetic field and by Pippard (ref. 4), Gurevich (ref. 8) when a nonzero magnetic field is present.

These theories have demonstrated that the attenuation of ultrasonic waves propagating through a metal depend strongly on the product of the wavelength q and the electrons mean free path l. In the short mean free path region where $ql \ll 1$ the attenuation varies as the frequency squared ω^2. When the mean free path is long $ql \gg 1$ the observed electronic attenuation is found to be dependent on the first power of the frequency ω.

When a magnetic field is applied longitudinally or transversely to the direction of propagation the electrons begin to move in spiralling orbits. If the orbit of the electron is of the same magnitude as the ultrasonic wave various resonance situations occur. This case represents the phenomena of magnetoacoustic oscillations in which the attenuation shows as oscillatory dependence on magnetic field and is periodic in H^{-1}.

The Cohen, Harrison and Harrison theory as well as Kjeldaas and Holstein theory on magnetoacoustic show good quantitative results for a variety of experimental measurements for metals which can be represented by a free electron model. However, if we were to use the former theory for the case of a longitudinal wave moving perpendicular to the magnetic field in order to plot attenuation coefficient versus qR where R is the orbital radius of the electron, one would find no shifts in the extrema for various ql values greater than 1. In contrast, Kjeldaas and Holstein graphs on attenuation shows shifts in the minima to be present.

In the limiting case where the magnetic field is negligible we would expect the expressions for the attenuation coefficient to approach the equations obtained by Pippard (ref. 1) in his theory on ultrasonic attenuation for zero magnetic field. However in the Cohen, Harrison and Harrison theory in its present form these expressions are not readily obtainable, whereas in the Kjeldaas and Holstein theory their equations do approach the limiting situation. Another distinct difference between the two theories is the requirements on the ql values. In the former, the ql range is restricted to values much greater than 1. In the latter there is no stringent conditions for the range of ql.

Recent experiments by Trivisonno and Said (ref. 9) on Potassium at John Carroll University have shown that the shift in the extrema for the longitudinal case do occur in agreement with Kjeldaas and Holstein's theory.

This paper will extend Cohen, Harrison and Harrison's theory and show that this theory also predicts the shift in extrema as well as being adequate for all ql values. It will also be shown that this theory goes directly over to the zero magnetic field case.

II. GENERAL DISCUSSION

Propagation of a sound wave in a metal causes the positive ions to oscillate around their position of stable equilibrium. Since the metal contains a free electron gas in addition to the ions the electrons will be forces to follow the ions in their motion in order to screen out any local charge imbalance and keep the metal electrically neutral. However, if a phase difference develops between the ions and electrons an electric current is generated. These electric currents induce electromagnetic fields which are able to transfer energy to the conduction electron. As a result of collisions energy is transferred back to the lattice or thermal phonons. Thus an irreversible flow of energy from the acoustic phonons to the thermal phonons.

The attenuation can be regarded as the reduction in amplitude of the wave per unit distance or rather, the decrease in the number of acoustic phonons per unit distance as it progresses through the metal. The attenuation coefficient α is defined as

$$\alpha = \frac{2Q}{\rho U^2 V_s} \tag{1}$$

where ρ is the density of the metal, $1/2(\rho U^2)$ is the energy of the acoustic wave, V_s is the velocity of sound and Q the power per unit volume absorbed by the electrons.

The attenuation of the sound wave by a metal depends greatly on the size of the electrons mean free path. At room temperature the attenuation is negligible because the mean free path of the electrons is so short that collisions are very frequent. Hence the energy transferred from the sound wave to the electrons is passed back nearly in phase. However, at low temperatures the mean free path of the electrons is so long that energy transferred between the sound wave and electrons is passed back with considerable lag. Thus, ultrasonic attenuation in a metal is a low temperature phenomena and can only be measured if the electrons mean free path is comparable to the size of the wavelength.

It should be pointed out that although ultrasonic attenuation is a recent phenomena its roots lie back to the old problem of electron scattering by elastic waves. Many transport problems such as electrical and thermal conductivity can be readily explained by the interaction of electron and phonons. Hence, even though the range of frequencies are completely different for ultrasonic and thermal waves they are otherwise identical in nature and have a common theoretical description. The major difference between ultrasonic attenuation and its older counterpart is that for the latter the mean free path of the conduction electrons is usually ignored.

To date, the most direct method for generating elastic waves is the use of piezoelectric transducers. In brief, a piezoelectric crystal will develop a net electrical polarization if it is placed under elastic strain along certain crystal directions. Thus, if we apply an electric field which varied with time, between the faces of piezoelectric crystal a strain field is set up with the same time variation produced at the free surface of the crystal and propagates into the interior. Longitudinal or transverse waves may be produced depending upon the crystal. The waves are introduced into the solid through a bond and electrical energy is converted into ultrasonic energy. The waves are attenuated during passage through the metal.

The difference between transverse and longitudinal waves propagating in a metal is that in the former no density changes occur and hence no electric fields resulting from space charges. However, the ionic current may not necessarily compensate the electric current in which case a magnetic field is generated and from these fields an electric field is developed.

The first theoretical investigation of ultrasonic attenuation in metals is the absence of a magnetic field was performed by Akhiezer (ref. 6), He predicted a at at low temperatures the conduction electrons would act as absorbers of ultrasonic waves. Many years later Bommel (ref. 10) and MacKinnon (ref. 11) experimentally investigating attenuation of waves in superconductors discovered that upon crossing the superconducting transition region the electrons contributed significantly to attenuation thus verifying Akhiezer's assumption.

The first complete theory of ultrasonic attenuation for a free electron model of a metal was developed by Pippard (ref. 1). The underlying assumption was that in the absence of collisions the ultrasonic waves adiabatically distorts the Fermi surface. For example, a spherical Fermi surface under a small distortion transforms into an ellipsoid. When the collisions between electrons and ions are taken into account this transformation is never completed, for the electron-phonon interaction attempts to restore the surface back to its original shape. Using this concept in conjunction with kinetic methods of following a single electron through the lattice, Pippard computed the coefficient of attenuation for normal metals.

From the above methods it has been also shown that the attenuation varies as the square of the frequency for $ql \ll 1$ and for $ql \gg 1$ where the sound wave length becomes comparable or less than the electrons mean free path, the attenuation varies proportionally with the first power of the frequency. Pippard's free electron theory has successfully accounted for most experimental features of ultrasonic attenuation.

Most of the recent theories of ultrasonic attenuation in metals is based on the Boltzmann equation for an electron distribution function. Its major advantage over the kinetic method is that if we were to incorporate the effect of an applied field, the calculations appear to be less formidable. This point may be debatable. Steinberg (ref. 12) and Blount (ref. 7), have used this method in calculating the coefficient of attenuation. These results are in agreement with Pippard for arbitrary ql values in zero applied magnetic field.

The main objective in the use of the Boltzmann transport equation is to study the distribution function $f(V,r,t)$, which represents the local concentration of particles in a state K in phase space in the neighborhood of the point r in real space. In order to extract the information about f, it is necessary to consider the causes which would tend to produce a change of f with time. The basic assumption in this technique is the use of Liouvilles' theorem on the invariance of volume occupied in phase space.

The Boltzmann transport equation in the presence of a sound wave is determined by

$$\left(\frac{\partial f}{\partial t}\right)_c = \frac{\partial f}{\partial t} + (V \cdot \nabla)f + \frac{\partial f}{\partial t} + \left(\frac{\vec{F}}{m} \cdot \nabla_K\right)F \qquad (2)$$

where F is the force per particle obtained from the Lorenz equation V is the velocity of the particle in K space and $(\partial f/\partial t)_c$ represents collisions between electrons and phonons.

Using the free electron model Pippard (ref. 1) calculated the attenuation for longitudinal and transverse waves with no restriction on the ql values. The results for the longitudinal wave in the absence of a magnetic field is that

$$\alpha = \frac{nm}{\rho V_s \tau}\left[\frac{(ql)^2 \tan^{-1} ql}{3(ql - \tan^{-1} ql)} - 1\right] \qquad (3)$$

where τ is the relaxation time and n the number of particles per unit volume. For $ql \ll 1$, equation (3) reduces to

$$\alpha = \frac{4}{15}\,\frac{nmV_F \omega q l}{\rho V_s^2} \qquad (4)$$

where w is the frequency and V_F the Fermi velocity. In the other extreme; $ql >> 1$ the coefficient of attenuation becomes

$$\alpha = \frac{\pi}{6} \frac{nmV_F\omega}{\rho V_s} \tag{5}$$

independent of l.

The attenuation for transverse waves is given by

$$\alpha_T = \frac{nm}{\rho V_s \tau} \left\{ \frac{2(ql)^2}{3} \left[\frac{(ql)^2 + 1}{ql} \tan^{-1} ql - 1 \right]^{-1} - 1 \right\} \tag{6}$$

At low frequencies for which the mean free path is smaller than the wavelength, that is $ql << 1$, the attenuation coefficient may be expressed as

$$\alpha_T = \frac{1}{5} \frac{nmV_F\omega}{\rho V_s{}^2} \tag{7}$$

When the product of ql attains values greater than unity ($ql >> 1$), the attenuation coefficient is found to be

$$\alpha_T = \frac{4}{3\pi} \frac{nmV_F\omega}{\rho V_s^2} \tag{8}$$

At extreme frequencies where $\omega\tau >> 1$ the attenuation coefficient is given by

$$\alpha_T = \frac{nm}{\rho V_s \tau} \tag{9}$$

beind independent of frequency. If we further let $\tau \to \infty$ then $\alpha_T \to 0$. This agrees with the usual conclusion for ideal metals that there is no electron-phonon interaction for shear waves.

We turn now to the effect of a magnetic field on the attenuation, considering a free electron gas. When a magnetic field is applied the electrons move in spiralling orbit. This causes the electron's mean free path to be reduced, thus having more collisions with the lattice. Therefore, one would expect that as we increase the magnetic field the attenuation decreases monotonically. However, this phenomena depends primarily on the ql values used.

For $ql \ll 1$ the attenuation decreases for all values of magnetic field since the effective mean free path of the electrons is decreased. Steinberg (ref. 13) showed that for this case and where the magnetic field is perpendicular to the direction of propagation and polarization (shear waves) the ratio of the attenuation coefficient in the presence of a magnetic field $\alpha(H)$ to the attenuation coefficient in zero magnetic field is given by

$$\frac{\alpha(H)}{\alpha_T(0)} = \frac{1}{1 + (2\omega_c\tau)^2} \approx \frac{1}{H^2} \tag{10}$$

where ω_c is the cyclotron frequency. If the magnetic field H goes to infinity the attenuation coefficient approaches zero.

For $ql \gg 1$ the attenuation varies in an oscillatory manner for certain geometries. This phenomena was first explained by Pippard (ref. 14) and independently by Morse, Bohm and Gavenda (ref. 15). Their interpretation which appears to agree with experiment relates the variation of the attenuation coefficient with the relative sizes of the wavelengths and orbit diameter of the electron.

This phenomena can be explained by noticing that the electrons Fermi velocity is several hundred times the velocity of the ultrasonic wave, the electrons can complete many orbits before interacting with the wave. Thus, due to the electrons velocity the variation of the local electric fields appear to be effectively stationary in space. Consequently, the effect of the magnet is to create a coherence between the electrons velocity and the ions velocity. It should also be remembered that its only the electrons at the Fermi surface which can absorb energy from the sound wave and lose it by relaxation processes.

By adjusting the diameter of the electrons orbit to equal one half the wavelength of the sound wave a resonance condition is obtained. Thus, in ultrasonic attenuation maximum attenuation is obtained by the orbit dimensions rather than the extremal areas as in the Hass Van Alphen effects.

Using the resonance and cyclotron relations

$$2R = \left(n + \frac{1}{2}\right)\lambda$$

$$R = \frac{mV_F}{eB} = \frac{Pc}{eH}$$

$$\omega_c = \frac{eB}{m} = \frac{eH}{mc}$$

which yields

$$P = \frac{eHR}{c}$$

$$cP = \left(n + \frac{1}{2}\right)\lambda$$

$$\Delta \frac{1}{H} = \frac{e\lambda}{cP} \tag{12}$$

Thus, by studying ultrasonic attenuation for single crystals a great deal of information is obtained about the shape of the Fermi surface.

To calculate the attenuation coefficient in the presence of a magnetic field we make use of the Boltzmann equation. This is essentially the same method as used previously but with the exception that F in equation (2) is modified to include the external magnetic field. F is the Lorenz force,

$$F = -e\left(\vec{\xi} + \frac{\vec{V}}{c} X\vec{H}\right)$$

$$H = H_0 + H_1 \tag{13}$$

where H includes both the applied field H_0 and the magnetic field H_1 associated with the sound wave.

The solution of the above equation is obtained by an ingenious method due to Chamber (ref. 16). The assumptions used were, that

(1) the relaxation time is a constant and

(2) $\delta f = f - f_0$

which represents the change in the electron distribution function from the perturbed f to the unperturbed state f_0. This change is set equal to zero immediately after collisions.

An electron contributes to the distribution function $f(r_0,V_0,t_0)$ only if it is the point r_0,V_0 with energy E in phase space at time t_0. This electron will have followed a certain trajectory since its last collision. Thus, at time t_1 it was scattered onto trajectory T at r_1,V_1. The number of electrons scattered onto T is $f(r_1,V_1,t_1)dt_1/\tau$ and the probability that an electron will not scatter before reaching r,V is $\exp[-(t_0 - t_1)]/\tau$. Thus, the distribution function $f(r_0,V_0,t_0)$ is found by integrating the number scattered onto the trajectory at previous points before reaching r_0, weighted by their probability of reaching r_0:

$$f(r_0,V_0,t_0) = \int_{-\infty}^{t_0} \frac{dt_1}{\tau} f(r_1,V_1,t_1)e^{-(t_0-t)/\tau} \qquad (14)$$

This technique is denoted as Chamber's trajectory method.

III. CALCULATIONS

The relative attenuation coefficient is determined from the nonvanishing components of the conductivity tensor σ_{ij}. Using the equations developed by Cohen, Harrison and Harrison the attenuation coefficient under the application of an applied magnetic field is given by

$$S_{11} = \mathrm{Re}\left[\frac{\sigma_{22}' + i\beta}{\sigma_{11}'\sigma_{22} + (\sigma_{12}')^2 + i\beta\sigma_{11}'}\right] - 1 \tag{15}$$

$$S_{22} = \mathrm{Re}\left[\frac{(1 + i\beta)^2}{\sigma_{22}' + i\beta + \dfrac{(\sigma_{12}')^2}{\sigma_{11}'}}\right] - 1 \tag{16}$$

$$S_{33} = \mathrm{Re}\left[\frac{(1 + i\beta)^2}{\sigma_{33}' + i\beta}\right] - 1 \tag{17}$$

where S_{11} represents the relative attenuation coefficient for a longitudinal wave moving perpendicular to a magnetic field, S_{22} corresponds to a transverse wave moving perpendicular to an applied field and S_{33} corresponds to a transverse wave moving parallel to the field. The attenuation is obtained by multiplying S_{ij} by $nm/\rho V_s\tau$.

The effective conductivity σ_{ij}' is derived from the conductivity tensor σ_{ij} by means of a reciprocal tensor R_{ij} where

$$R_{ij} = - \frac{i\omega\tau V_F^2}{3\sigma_0(1 - i\omega\tau)V_s^2} \sigma_{ij} \tag{18}$$

and

$$\bar{\sigma}' = [1 - \bar{R}]^{-1} \cdot \frac{\bar{\sigma}}{\sigma_0} \tag{19}$$

The conductivity tensor is given by

$$\sigma_{11} = \frac{3\sigma_0}{q^2 l^2} (1 - i\omega\tau) \left[1 - \sum_{n=-\infty}^{\infty} \frac{(1 - i\omega\tau) g_n}{1 + i(n\omega_c - \omega)\tau} \right] \tag{20}$$

$$\sigma_{22} = 3\sigma_0 \sum_{n=-\infty}^{n=\infty} \frac{s_n}{1 + i(n\omega_c - \omega)\tau} \tag{21}$$

$$\sigma_{33} = 3\sigma_0 \sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega_c - \omega)\tau} \tag{22}$$

$$\sigma_{12} = -\sigma_{21} = \frac{3\sigma_0}{2ql} \sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau) g_n'}{1 + i(n\omega_c - \omega)\tau} \tag{23}$$

Here

$$g_n(X) = \frac{1}{X} \int_0^X J_{2n}(2t)dt \tag{24}$$

$$g_n'(X) = \frac{d}{dx} g_n(X) \tag{25}$$

$$r_n(X) = \int_0^X t^2 g_n(t)dt \tag{26}$$

$$s_n = 3r_n - \left(1 - \frac{n^2}{X^2}\right)g_n(X) \tag{27}$$

where $X = qV_F/\omega_c$. This term can be written as the product of the ultrasonic wavelength and the orbital radius of an electron moving perpendicular to a magnetic field. The radius can be represented as

$$R = \frac{V_F}{\frac{eH}{mc}} = \frac{V_F}{\omega_c}$$

therefore,

$$X = qR$$

The above equations are due to Cohen, Harrison and Harrison. The calculations of these equations in their present form prove to be a formidable task. This is due to the nature of the series which involves integral Bessel functions. Thus, it was necessary in the Cohen, Harrison and Harrison paper to limit the equations for the case where $n = 0$, hence, eliminating the sums. It was also feasible throughout products involving ql terms where this product was not much greater than one. Due to this limitation no shift in the relative attenuation for S_{11} was observed and no single analytic expression which can approach both extremes where H equals zero and H equals infinity were obtained.

For the new extension the above difficulties are removed. The only assumptions used throughout this paper will be that terms involving $\omega\tau$ as well as terms containing the square of the ratio of the classical skin depth to the phonon wavelength is small and set equal to zero. Most metals fulfill this requirement.

In order to extend the former theory it is necessary to take all the terms into account. Thus, the relative attenuation coefficient must contain the complete series involving integral Bessel functions. By using equation (18) in conjunction with equation (20) the effective conductivity tensor can be written as

$$\sigma'_{11} = -\frac{3i\omega t}{q^2 l^2}(1 - i\omega\tau)\frac{1 - \sum_{n=-\infty}^{n=\infty}\frac{(1 - i\omega\tau)\, g_n}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=-\infty}^{n=\infty}\frac{(1 - i\omega\tau) g_n}{1 + i(n\omega_c - \omega)\tau}} \tag{28}$$

$$\sigma'_{12} = -\frac{3i\omega\tau}{ql}\,\frac{\sum_{n=-\infty}^{n=\infty}\frac{\frac{g'_n}{2}(1 - i\omega\tau)}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=\infty}^{n=\infty}\frac{g_n(1 - i\omega\tau)}{1 + i(n\omega_c - \omega)\tau}} \tag{29}$$

$$\sigma_{22} = \frac{3}{1 - i\omega\tau}\sum_{n=-\infty}^{n=\infty}\frac{s_n}{1 + i(n\omega_c - \omega)\tau}$$

$$+ \frac{\sum_{n=-\infty}^{n=\infty}\frac{\frac{g'_n}{2}(1 - i\omega\tau)^2}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=-\infty}^{n=\infty}\frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}} \tag{30}$$

The relative attenuation S_{ij} for ultrasonic waves propagating in an ideal metal under the application of an applied magnetic field is derived in appendix A. However, even though all the terms are included in a compact form we are still left with the problem of summing over integrals which contain Bessel functions. To remove this complexity we use a direct approach to the problem of summing infinte series in closed form. This method is outlined in detail in appendix B.

Combining the results of appendixes A and B the relative attenuation coefficient can be written down as

$$S_{11} = \frac{(ql)^2}{3}\left[\frac{1}{b + \frac{\left(\frac{u}{2}\right)^2}{\bar{w}}} - 1\right] - 1 \tag{31}$$

$$S_{22} = \frac{1}{3\left[\bar{w} + \frac{\left(\frac{u}{2}\right)^2}{b}\right]} - 1 \tag{32}$$

$$S_{33} = \left(\frac{1}{3V} - 1\right) \tag{33}$$

Here as derived in Appendix B

$$b = -\sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} \tag{34}$$

$$u = \sum_{n=1}^{n=\infty} \frac{(-1)^n 2n X^{2n-1}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} \tag{35}$$

$$\bar{w} = -\sum_{n=1}^{n=\infty} \frac{(-1)^n 2n X^{2n}}{(2n+1)(2n+3)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]}$$

$$+ \frac{1}{(ql)^2} \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} \tag{36}$$

$$V = \frac{1}{3} + \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+3)(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} \tag{37}$$

These expressions are extremely easy to work with. No difficulty in any of the limiting cases or in any oscillatory situation. The ease in handling the equations will further be explored in section IV.

IV. RESULTS AND DISCUSSION

Confining ourselves to propagation along directions of high symmetry we can avoid some of the complexities which can arise. There are three cases we shall analyze in detail. For each case the relative attenuation coefficient, S_{11}, S_{22}, and S_{33} under the application of a magnetic field will be discussed. For the phenomena of magnetoacoustic oscillations graphs, as well as tables are presented. Whenever it is possible comparison between experiment and theory will be made.

A. High Field Limit

When the magnetic field is extremely large the attenuation coefficient tends to a limit, different for each of the three attenuation coefficients. This limit is found simply from equations (34) to (37) by allowing X to go to zero as H approaches infinity. As X goes to zero the series approaches zero rapidly. Hence, only the first term of each series is necessary since any further terms will be zero automatically. Thus, the following is obtained

$$b = - \sum_{\substack{n=1\\ H\to\infty\\ X\to 0}}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2\ell^2}\right) \cdot \cdot \cdot \left(n^2 + \frac{X^2}{q^2\ell^2}\right)\right]} \to \frac{X^2}{3} \qquad (38)$$

$$u = \sum_{\substack{n=1\\ H\to\infty\\ X\to 0}}^{n=\infty} \frac{(-1^n 2nX^{2n-1}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2\ell^2}\right) \cdot \cdot \cdot \left(n^2 + \frac{X^2}{q^2\ell^2}\right)\right]} \to \frac{2}{3} X \qquad (39)$$

$$\bar{w} = - \sum_{\substack{n=1\\ H\to\infty\\ X\to 0}}^{n=\infty} \frac{(-1)^n 2nX^{2n}}{(2n+1)(2n+3)\left[\left(1^2 + \frac{X^2}{q^2\ell^2}\right) \cdot \cdot \cdot \left(n^2 + \frac{X^2}{q^2\ell^2}\right)\right]}$$

$$+ \frac{1}{q^2\ell^2} \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2\ell^2}\right) \cdot \cdot \cdot \left(n^2 + \frac{X^2}{q^2\ell^2}\right)\right]} \qquad (40)$$

$$\to \frac{2X^2}{15} + \frac{X^2}{3q^2\ell^2} \qquad (40)$$

$$V = \sum_{\substack{n=1 \\ H\to\infty \\ X\to 0}}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+3)(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} + \frac{1}{3}$$

$$\to \frac{1}{3} - \frac{X^2}{15} \qquad (41)$$

Inserting the above expressions into equations (31) to (33) we obtain the following values for the relative attenuation coefficient:

$$S_{11} = \frac{q^2 l^2}{3}\left[\frac{1}{\frac{X^2}{3} + \frac{\frac{X^2}{9}}{\frac{2X^2}{15} + \frac{X^2}{3q^2 l^2}}} - 1\right] - 1 = \frac{q^2 l^2}{15} \qquad (42)$$

$$S_{22} = \left[\frac{1}{3\left(\frac{2}{15}X^2 + \frac{X^2}{3q^2 l^3} + \frac{\frac{X^2}{9}}{\frac{X^2}{3}}\right)} - 1\right] = 0 \qquad (43)$$

$$S_{33} = \left[\frac{1}{3\left(\frac{1}{3} - \frac{X^2}{15}\right)} - 1\right] = 0 \qquad (44)$$

The results that S_{11} saturates in a high magnetic field can readily be explained by the fact that the electron's gyro radius becomes smaller and smaller and thus tends to approach the zero field value. For the case of shear waves the attenuation coefficient tends to zero as H^{-2}. This is due to the fact that the attenuation decreases since the effective mean free path of the electrons is decreased. The predictions from the free electron model in this respect have great validity.

B. Low Field Limit

In the low field limit we expect that the attenuation coefficient to approach Pippard's result for zero magnetic field. Thus, allowing X to approach infinity while H goes to zero we obtain the following:

$$b = - \sum_{\substack{n=1 \\ H\to 0 \\ X\to\infty}}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]}$$

$$= - \sum_{n=0}^{n=\infty} \frac{(-1)^n (ql)^{2n}}{(2n+1)} = 1 - \frac{1}{ql} \tan^{-1} ql \tag{45}$$

$$u = \sum_{\substack{n=1 \\ H\to 0 \\ X\to\infty}}^{n=\infty} \frac{(-1)^n 2n X^{2n-1}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} = 0 \tag{46}$$

$$\bar{w} = -\sum_{\substack{n=1\\H\to 0\\X\to\infty}}^{n=\infty} \frac{(-1)^n 2n X^{2n}}{(2n+1)(2n+3)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \ldots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]}$$

$$+ \frac{1}{(ql)^2} \sum_{\substack{n=1\\H\to 0}}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \ldots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]}$$

$$= \frac{\tan^{-1} ql}{2ql}\left[1 + \frac{1}{(ql)^2}\right] - \frac{1}{2q^2 l^2} \qquad (47)$$

$$V = \frac{1}{3} + \sum_{\substack{n=1\\H\to 0\\X\to\infty}}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+3)(2n+1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \ldots \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]}$$

$$= \frac{1}{2ql} \tan^{-1} ql\left[1 + \frac{1}{(ql)^2}\right] - \frac{1}{2(ql)^2} \qquad (48)$$

Substituting the above expressions into equations (31) to (33) we obtain the following:

$$S_{11} = \frac{(ql)^2}{3}\left(\frac{1}{1 - \tan^{-1} ql} - 1\right) - 1 = \frac{1}{3}\left[\frac{(ql)^2 \tan^{-1} ql}{ql - \tan^{-1} ql} - 1\right] \qquad (49)$$

$$S_{22} = \left(\frac{1}{3\left\{\frac{\tan^{-1} ql}{2ql}\left[1 + \frac{1}{(ql)^2}\right] - \frac{1}{2(ql)^2}\right\}} - 1 \right)$$

$$= \frac{2(ql)^2}{3}\left\{\left[\frac{(ql)^2 + 1}{ql}\tan^{-1} ql - 1\right]^{-1} - 1\right\} \qquad (50)$$

$$S_{33} = \frac{2(ql)^2}{3}\left\{\left[\frac{(ql)^2 + 1}{ql}\tan^{-1} ql - 1\right]^{-1} - 1\right\} = S_{22} \qquad (51)$$

These results are in exact agreement with Pippard's theory for the case of ultrasonic wave propagating in a free electron model of a metal in the absence of a magnetic field.

C. Magnetoacoustic Oscillations

When the field is of such magnitude that the electrons orbit dimensions are comparable with the wavelength, the attenuation is oscillatory. The effect of attenuation on the electrons mean free path for longitudinal and transverse waves is shown by figures 1 to 6. These plots may be regarded as relative attenuation as a function of the product of phonon wave number and the radius of an electron. R is inversely proportional to the field.

An important anomaly arises for the case of a longitudinal wave moving perpendicular to the magnetic field. This anomaly is strikingly revealed in figures 1 and 2, which shows that for different ql values a shift in the minimum occurs. The maximum positions are not affected by varying ql. In table I, points of maximum and minimum positions for S_{11} are given.

The above anomaly has not been mentioned explicitly by either Cohen, Harrison and Harrison or Kjeldaas and Holstein. Recent experimental investigations at John Carroll University by Trivisonno and Said on Potassium have verified that these shifts do exist. Calculations from equations (31), (34), (35), and (36) for various ql values are in good agreement with the magnitudes of the shifts in the minimum reported by Trivisonno and Said. The magnitudes for the relative attenuation also coincide with their experimental results.

The case for a transverse wave moving perpendicular to the field shows an anomaly in the maximum positions. Here, however, the shifts in the maximum are extremely small for various ql values compared to the shifts in the minimum positions of S_{11}. The minimum points of S_{22} show no appreciable change. Due to the extremely small shift in the maximum values no experimental verification can be obtained. In table II the points of maximum and minimum positions are given. In figures 3 and 4 a plot of the attenuation of a transverse wave moving perpendicular to the magnetic field as a function of the electron mean free path is presented.

Figures 5 and 6 represent the case where the transverse wave is in a transverse magnetic field polarized parallel to the field.

Experimental investigations by Trivisonno and Said[9] as well as Foster, Meijer and Mielczarek[17] on potassium have shown that the free electron model is valid, that is, the Fermi surface is spherical. The oscillations and magnetic field dependence of attenuation are in accord with the free electron theory.

APPENDIX A

The effective conductivity tensor σ_n' can be written as

$$\sigma_{11}' = -3i\omega\tau(1 - i\omega\tau) \times \left[\frac{1 - \sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}} \right] \tag{A1}$$

Let

$$a = (n\omega_c - \omega)\tau$$

Now

$$\sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)}{1 + ia} \frac{(1 - ia)}{(1 - ia)} = \sum_{n=-\infty}^{n=\infty} \frac{g_n(1 - a\omega\tau)}{1 + a^2} = y$$

Let

$$1 - y = b$$

then

$$\sigma_{11}' = \frac{3\omega\tau(1 + \omega^2\tau^2)b}{q^2l^2(\omega\tau + ib)(1 + i\omega\tau)}$$

For σ'_{12} we can write it as follows

$$\sigma'_{12} = -\frac{3i\omega\tau}{ql}\left[\frac{\displaystyle\sum_{n=-\infty}^{n=\infty}\frac{\frac{g'_n}{2}(1 - i\omega\tau)}{1 + ia}}{1 - i\omega\tau - \displaystyle\sum_{n=-\infty}^{n=\infty}\frac{g_n(1 - i\omega\tau)}{1 + ia}}\right] \tag{A2}$$

Let

$$\bar{u} = \sum_{n=-\infty}^{n=\infty}\frac{(1 - a\omega\tau)\frac{g'_n}{2}}{1 + a^2}$$

$$\sigma'_{12} = -\frac{3i\omega\tau\bar{u}}{ql(b - i\omega\tau)} = \frac{3\omega\tau\bar{u}}{ql(\omega\tau + ib)}$$

$$(\sigma_{12})^2 = \frac{9\omega\ \tau^2(\bar{u})^2}{q^2l^2(\omega\tau + ib)^2}$$

Let

$$c = (\omega\tau + ib)$$

$$(\sigma_{12})^2 = \frac{9\omega^2\tau^2}{q^2l^2}\frac{\bar{u}^2}{c^2}$$

Now

$$\sigma_{22}' = \frac{3}{1 - i\omega\tau}\left[\sum_{n=-\infty}^{n=\infty} \frac{s_n(1 - i\omega\tau)}{1 + ia} + \frac{\displaystyle\sum_{n=-\infty}^{n=\infty} \frac{g_n'}{2}\,\frac{(1 - i\omega\tau)^2}{1 + ia}}{1 - i\omega\tau - \displaystyle\sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + ia}}\right] \quad \text{(A3)}$$

Let

$$\bar{w} = \sum_{n=-\infty}^{n=\infty} \frac{s_n}{1 + a^2}$$

Hence,

$$(\sigma_{22}') = \frac{3(1 + i\omega\tau}{1 + \omega^2\tau^2}\left(\frac{\bar{w}_c + u^2 i}{b - c}\right)$$

$$\left[\frac{\sigma_{12}'}{\sigma_{22}}\right]^2 = \frac{3\omega^2\tau^2(1 + \omega^2\tau^2)u^2}{(ql)^2(1 + i\omega\tau)c[(\omega c) + u^2 i]}$$

$$S_{11} = \mathrm{Re}\left[\sigma_{11}' + \frac{(\sigma_{12}')^2}{\sigma_{22}}\right] - 1$$

Let

$$d = \omega\tau(\bar{u}^2 + \omega b)$$

$$e = b\bar{u}^2 + b^2\bar{w}$$

Thus,

$$S_{11} = \frac{q^2 l^2}{3(1 + \omega^2\tau^2)} \left\{ \frac{-1 + e\bar{w}[b + (\omega\tau)^2] - d\bar{w}(b - (\omega\tau)}{d^2 + e^2} + 1 \right\}$$

Making the approximation

$$\omega\tau \to 0$$

$$S_{11} = \frac{q^2 l^2}{3}\left(\frac{\bar{w}b}{e} - 1\right) - 1$$

$$= \frac{(ql)^2}{3}\left[\frac{1}{\left(\frac{\bar{u}^2}{\bar{w}}\right) + b} - 1\right] - 1$$

In the same manner S_{22} and S_{33} can be obtained.

APPENDIX B

To remove the difficulty in the integral Bessel summation we begin by writing

$$\sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau} = \frac{g_0}{\omega_0} + 2\omega_0 \sum_{n=1}^{n=\infty} \frac{g_n}{\omega_0^2 + n^2\omega_c^2\tau^2} \qquad \text{(B1)}$$

where

$$\omega_0 = 1 - i\omega\tau$$

Let

$$\frac{2}{\omega_0} \sum_{n=1}^{n=\infty} \frac{g_n}{\omega_0^2 + n^2\omega_c^2\tau^2} = \frac{2}{\omega_0} \sum_{n=1}^{n=\infty} \frac{g_n\omega_0^2\pi^2}{\omega_0^2\pi^2 + n^2\omega_c^2\tau^2\pi^2}$$

$$= \frac{\frac{\omega_0^2\pi^2}{\omega_c^2\tau^2}}{\omega_0\pi^2} \sum_{n=1}^{n=\infty} \frac{g_n}{\frac{\omega_0^2\pi^2}{\omega_c^2\tau^2\pi^2} + n^2}$$

$$= -\frac{2a^2}{\omega_0} \sum_{n=1}^{n=\infty} \frac{g_n}{n^2 - a^2} \qquad \text{(A1)}$$

where

$$a^2 = -\left(\frac{\omega_0}{\omega_c \tau}\right)^2$$

Now

$$g_n(X) = \frac{1}{X}\int_0^X J_{2n}(2t)dt$$

$$J_{2n}(2t) = \frac{2}{\pi}\int_0^{\pi/2} (-1)^n \cos 2n\,\theta \cos(2t\cos\theta)d\theta$$

$$-\frac{4a^2}{\omega_0 \pi X}\int_0^X \int_0^{\pi/2} \sum_{n=1}^{n=\infty} \frac{(-1)^n \cos 2n\theta \cos(2t\cos\theta)d\theta\, dt}{n^2 - a^2}$$

but

$$\sum_{n=1}^{n=\infty} \frac{(-1)^n \cos 2n\theta}{n^2 - a^2} = \frac{1}{2a^2}\left[1 - \frac{\pi a \cos(2\theta a)}{\sin(a\pi)}\right]$$

Hence

$$-\frac{2}{\omega_0 \pi X}\int_0^X \int_0^{\pi/2} \left[\frac{1 - \pi a \cos(2\theta a)}{\sin(a\pi)}\right] \cos(2t\cos\theta)d\theta\, dt$$

$$= -\frac{1}{\omega_0 X}\int_0^X J_0(2t)dt$$

$$+\frac{2a}{\omega_0 X}\int_0^X \int_0^{\pi/2} \frac{\cos(2\theta a)\cos(2t\cos\theta)dt\, d\theta}{\sin(a\pi)} \qquad \text{(A1)}$$

Thus,

$$-\frac{g_0}{\omega_0} + \frac{2a}{\omega_0 X \sin(a\pi)} \int_0^X \int_0^{\pi/2} \cos(2\theta u)\cos(2t \cos \theta)d\theta\, dt \qquad \text{(A1)}$$

However,

$$\int_0^{\pi/2} \cos(2t \cos \theta)\cos 2\theta a\, d\theta = \frac{1}{2a} \sin(a\pi)$$

$$\times\, 1 + \sum_{n=1}^{n=\infty} \frac{(-1)^n (t)^{2n}}{(1^2 - a^2)(2^2 - a^2)\; .\; .\; .\; (n^2 - a^2)}$$

Hence,

$$-\frac{g_0}{\omega_0} + \frac{1}{\omega_0 X} \int_0^X \left[1 + \sum_{n=1}^{n=\infty} \frac{(-1)^n (t)^{2n}}{(1^2 - a^2)\; .\; .\; .\; (n^2 - a^2)}\, dt\right] = -\frac{g_0}{\omega_0} + \frac{1}{\omega_0}$$

$$+\, \frac{1}{\omega_0} \sum_{n=1}^{n=\infty} \frac{(-1)^n (X)^{2n}}{(2n + 1)\left[\left(1 + \frac{X^2}{q^2 l^2}\right)\; .\; .\; .\; \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} \qquad \text{(A1)}$$

Therefore,

$$\sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau}$$

$$= \frac{1}{\omega_0}\left[1 + \sum_{n=1}^{n=\infty} \frac{(-1)X^{2n}}{(2n+1)\left[\left(1+\frac{X^2}{q^2l^2}\right)\cdots\left(\frac{X^2+n^2}{q^2l^2}\right)\right]}\right] \tag{B1}$$

$$\omega_0 = 1 - i\omega\tau \approx 1$$

In the report the above equation is always subtracted by 1 thus we denoted, the summation portion as b.

Using the same procedure we find that

$$\sum_{n=-\infty}^{n=\infty} \frac{g_n'}{1 + i(n\omega_c - \omega)\tau} = \frac{d}{dx}\sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau}$$

$$= \sum_{n=-\infty}^{n=\infty} \frac{(-1)^n 2nX^{2n-1}}{(2n+1)\left[\left(1^2+\frac{X^2}{q^2l^2}\right)\cdots\left(\frac{n^2+X^2}{q^2l^2}\right)\right]} = u \tag{B2}$$

For r_n we use the following relationships:

$$\sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega\ \ - \omega)\tau} \tag{B3}$$

$$r_n = \frac{g_n}{2} - \frac{1}{2X^3}\int_0^X t^2 J_{2n}(2t)\,dt$$

Using the same method as was done for B - 1 we obtain

$$\sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega_c - \omega)\tau} = \frac{1}{3}$$

$$+ \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n + 3)(2n + 1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdot \cdot \cdot \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} = V \qquad \text{(B3)}$$

For s_n the same procedure is carried out,

$$\sum_{n=-\infty}^{n=\infty} \frac{s_n}{1 + i(n\omega_c - \omega)\tau} = 3 \sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega_c - \omega)\tau} - \sum_{n=-\infty}^{n=\infty} \frac{g_n}{(1 + i(n\omega_c - \omega)\tau}$$

$$- \frac{\omega_0}{q^2 l^2} \sum_{n=-\infty}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n + 1)\left[\left(1^2 + \frac{X^2}{q^2 l^2}\right) \cdot \cdot \cdot \left(n^2 + \frac{X^2}{q^2 l^2}\right)\right]} = \bar{W} \qquad \text{(B4)}$$

BIBLIOGRAPHY

1. Pippard, A. B.: "Ultraonic Attenuation in Metals." Phil. Mag., 46, 1104 (1955).

2. Kjeldaas, T. Jr., and Holstein T.: "Oscillatory Magnetoacoustic Effect in Metals." Phys. Rev. Lett. 2, 340 (1959). Erratum:

3. Cohen, M. H., Harrison, J. J., and Harrison, W. A.: "Magnetic Field Dependence of Ultrasonic Attenuation in Metals." Phys. Rev. 117, 937 (1960).

4. Pippard, A. B.: "Theory of Ultrasonic Attenuation in Metals and Magnetoacoustic Oscillations." Proc. Roy. Soc. A257 165 (1960).

5. Kanner, E. A.: "On the Theory of Ultrasonic Absorption by Metals in a Strong Magnetic Field." Soviert Phys. JETP 11, 154 (1960).

6. Akhiezer, A. I.: "Absorption of Sound in Solid Bodies." Soviet Phys. JETP 8, 1330 (1938).

7. Blount, E. I.: "Ultrasonic Attenuation by Electrons in Metals." Phys. Rev. 114, 418, (1959).

8. Gurevitch, V. L.: "Ultrasonic Absorption in Metals in a Magnetic Field I." Soviet Phys. JETP 10, 51 (1960).

9. Trivisonno, J. and Said, S.: To be Published, (in print Phys. Rev.).

10. Bommel, H. E.: "Ultrasonic Attenuation in Superconducting and Normal-Conducting Tin at Low Temperatures." Phys. Rev. 100, 758, (1955).

11. Mackinnon, L.: "Relative Absorption of 10 Mc/sec Longitudinal Sound Waves in Superconducting Polycrystaline Tin Rod." Phys. Rev. 100, 1965

12. Steinberg, M. S.: "Ultrasonic Attenuation and Dispersion in Metals at Low Temperatures." Phys. Rev. 111, 425, (1958).

13. Steinberg, M. S.: "Viscosity of the Electron Gas in Metals." Phys. Rev. 109, 1486, (1958).

14. Pippard, A. B.: "A Proposal for Determining the Fermi Surface by Magnetoacoustic Resonance." Phil. Mag. 2, 1147 (1957).

15. Morse, R. W., Bohm, H. V., and Gavenda, J. D.: "Electron Resonances with Ultrasonic Waves in Cu." Phys. Rev. 109, 1394 (1958).

16. Chambers, R. G.: "The Kinetic Formulation of Conduction Problems." Proc. Phys. Soc. A65, 458 (1952).

17. Foster, H. J., Meijer, P. H. and Mielczarek, E.: Magnetoacoustic Absorption and the Fermi Surface in Potassium.

TABLE I. - S_{11} EXTREMA.

ql	Maximum, X	Relative attenuation	Minimum, X	Relative attenuation
25	0	41.7	2.85	28.62
	4.05	38.39	5.95	17.01
	7.25	21.27	9.05	13.95
	10.45	16.35	12.15	12.92
18	0	21.6	2.80	15.23
	4.05	20.4	5.90	9.99
	7.25	12.21	9.00	8.96
	10.45	10.12	12.15	8.78
15	0	15	2.75	10.81
	4.0	14.54	5.85	7.63
	7.25	9.17	8.95	7.20
	10.45	7.94	12.10	7.20
13	0	11.27	2.70	8.29
	4.0	11.18	5.80	6.27
	7.25	7.41	8.95	6.12
	10.45	6.62	12.10	6.18
11	0	8.07	2.6	6.12
	4.0	8.30	5.8	5.05
	7.25	5.85	8.95	7.20
	10.45	7.94	12.1	7.20
9	0	5.40	2.45	4.27
	4.0	5.87	5.75	3.97
	7.25	4.47	8.90	4.11
	10.50	4.27	12.05	4.18

TABLE II. - S_{22} EXTREMA.

ql	Maximum, X	Relative attenuation	Minimum, X	Relative attenuation
25	4.20 7.40 10.55	8.80 10.20 10.66	0 5.6 8.85	0 3.73 6.17
18	4.20 7.40 10.55	7.56 7.86 7.72	0 5.6 8.85	0 3.50 5.34
15	4.20 7.40 10.55	6.70 6.56 6.30	0 5.60 8.85	0 3.32 4.79
13	4.15 7.35 10.55	5.98 5.61 5.34	0 5.6 8.85	0 3.15 4.33
11	4.15 7.35 10.55	5.11 4.61 4.37	0 5.6 8.85	0 2.91 3.79
9	4.15 7.35 10.55	4.11 3.60 3.44	0 5.60 8.85	0 2.59 3.17

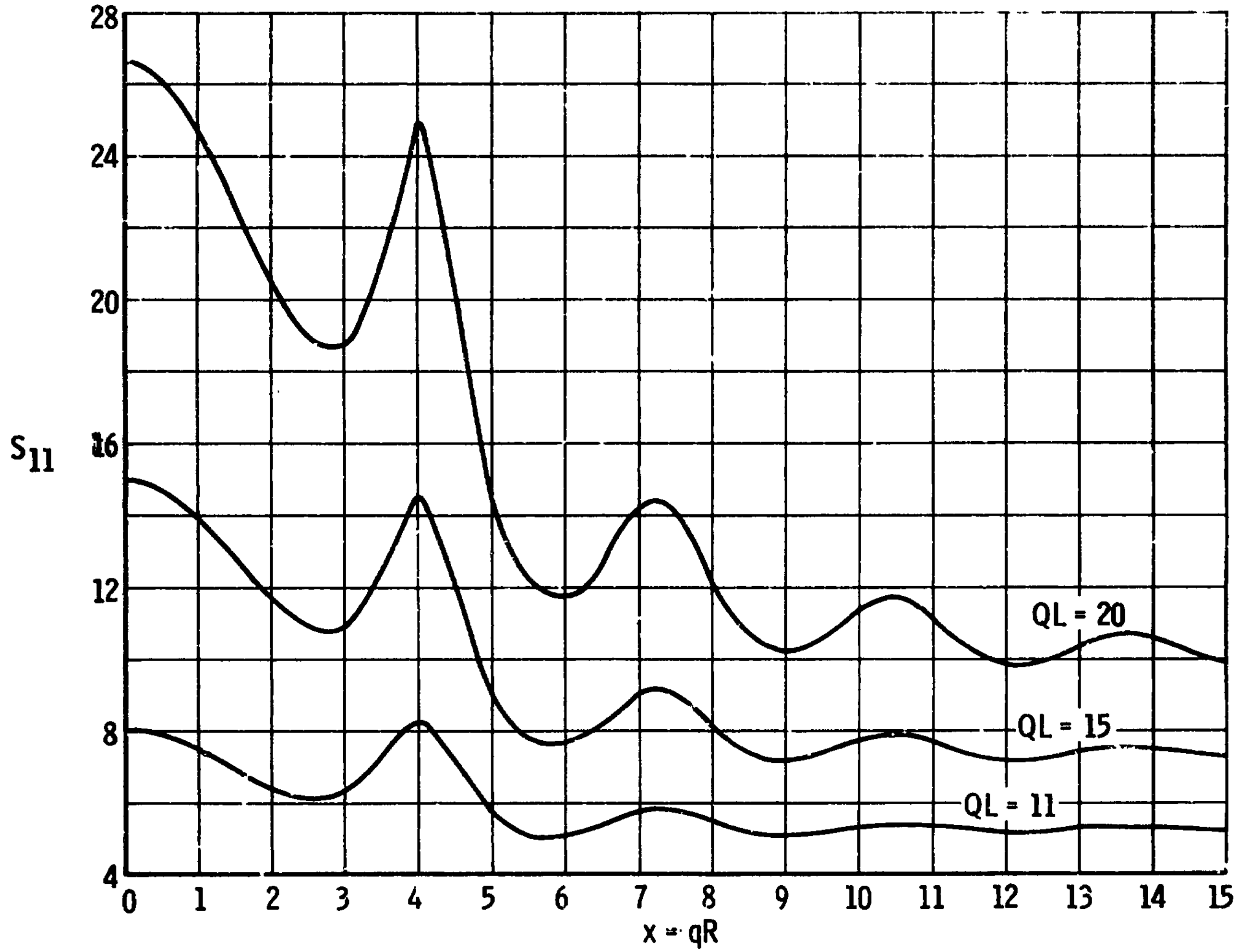

Figure 1. - The relative attenuation of a longitudinal wave in a transverse magnetic field as a function of the electron mean free path.

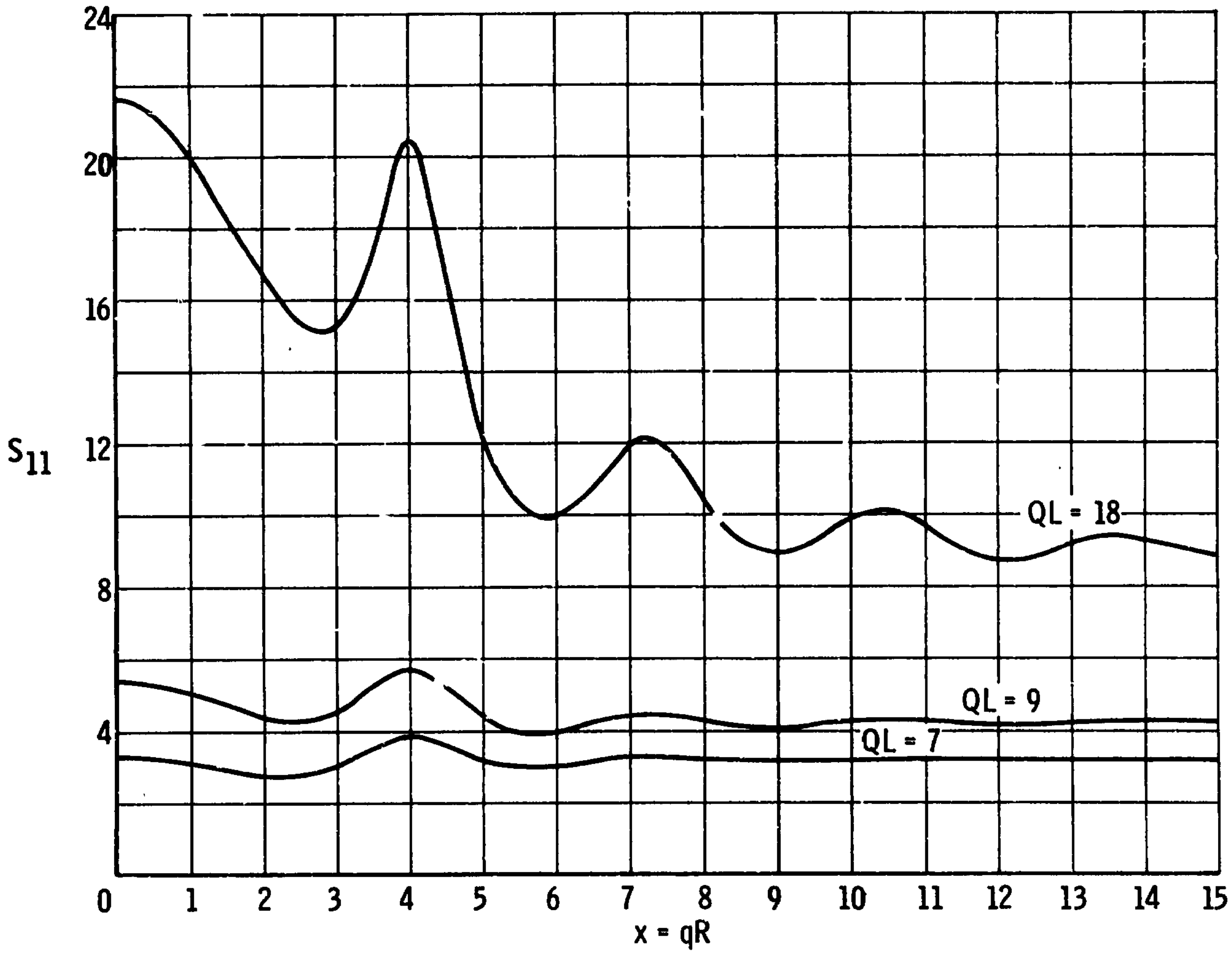

Figure 2. - The relative attenuation of a longitudinal wave in a transverse magnetic field as a function of the electron mean free path.

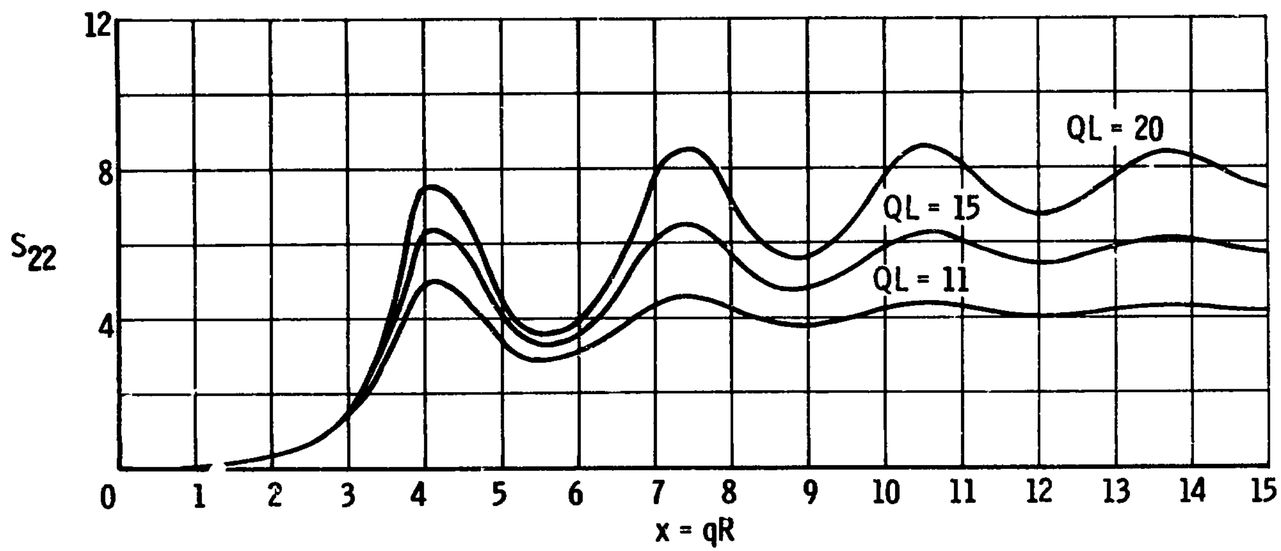

Figure 3. - The relative attenuation of a transverse wave in a transverse magnetic field as a function of the electron mean free path.

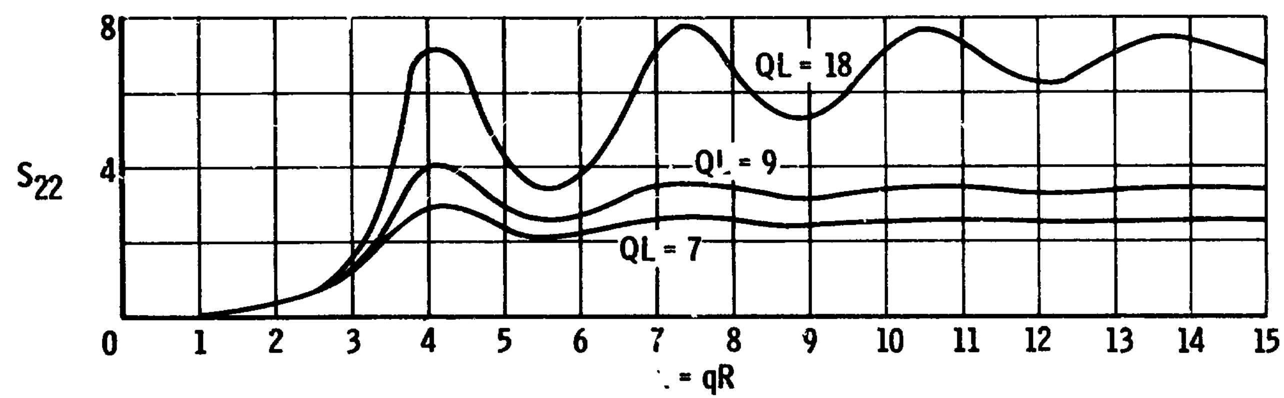

Figure 4. - The relative attenuation of a transverse wave in a transverse magnetic field as a function of the electron mean free path.

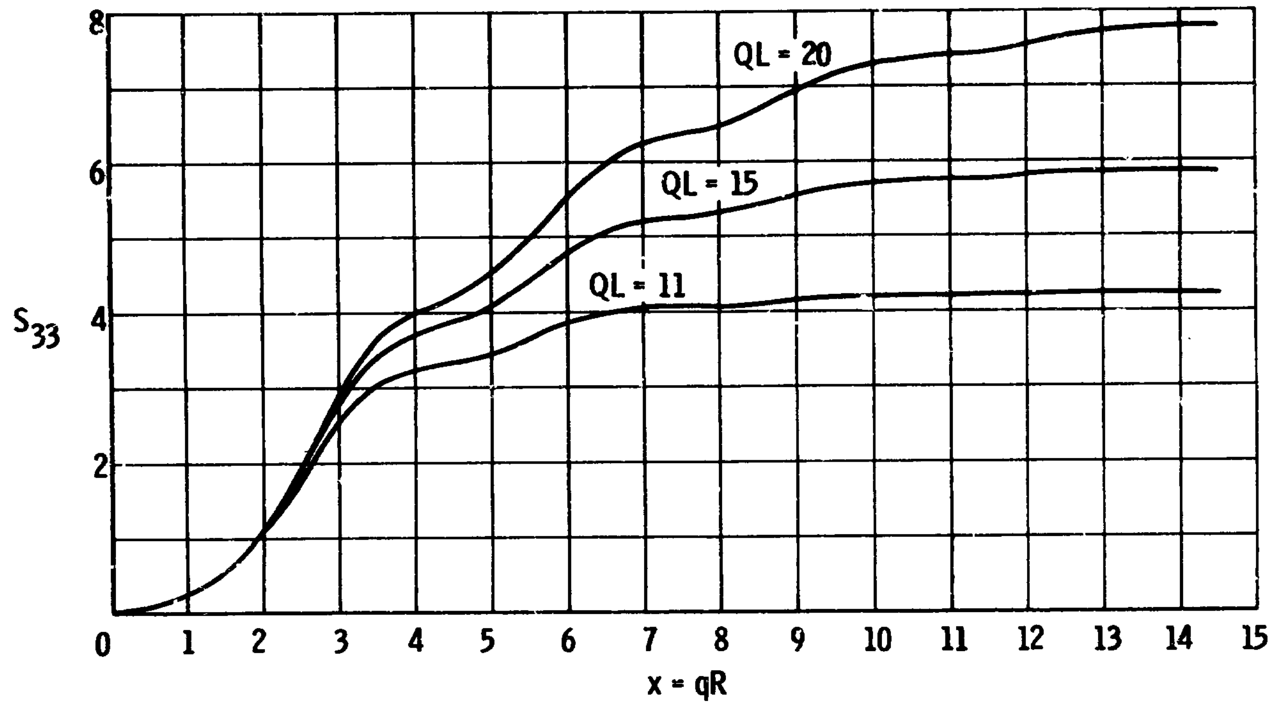

Figure 5. - The relative attenuation of a transverse wave in a magnetic field parallel to the polarization direction as a function of the electron mean free path.

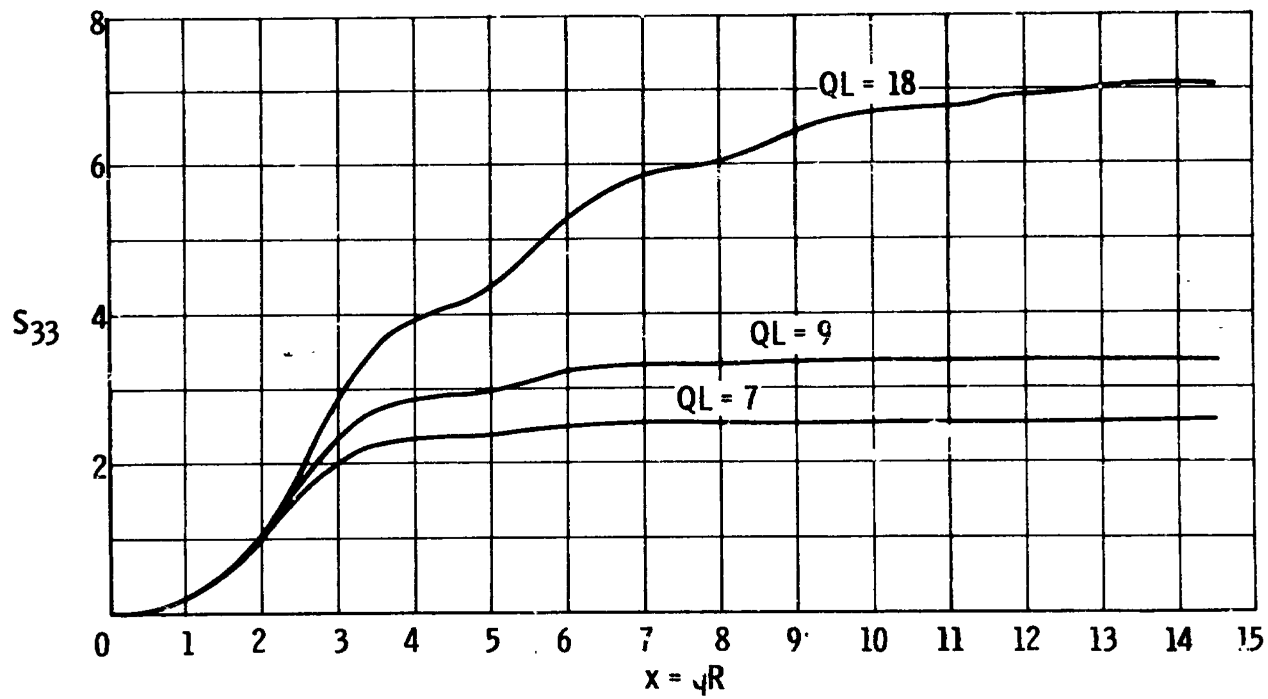

Figure 6. - The relative attenuation of a transverse wave in a magnetic field parallel to the polarization direction as a function of the electron mean free path.