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THE CENTER VENT SHAPE

FOR VENTING A TANK IN A LOW GRAVITY ENVIRONMENT

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SUMMARY

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Existing computations and liquid/liquid model tests have demonstrated the general zero-g equilibrium configuration of the Centaur upper-stage fuel-tank ullage. This report contains calculations and some discussion of the effect of the Center-Vent tube on this ullage-bubble configuration. The equilibrium shape about a two-inch tube is calculated. It appears that the liquid/gas interface will be displaced about 2.3 inches below the uppermost bubble surface. Because this effect is so small, it is concluded that, with certain restrictions discussed herein, the simple center tube will make a practical vent.

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THE CENTER VENT SHAPE

1.0 INTRODUCTION:

1.1 The practicality of a center vent to relieve the excess pressure produced by boil-off in the Centaur fuel tank has evoked considerable thought and deliberation. This device would certainly save weight and be simpler and more reliable than a centrifugal separator type of vent. But would it work?

1.2 Since Dr. Ta Li's "Zero-G" paper,* the existence of a central ullage in a tank of wall-wetting liquid in the absence of gravity has been generally accepted. However, it was felt that the all wetting liquid hydrogen would cover a vent pipe in the same manner that it covers the tank walls. The development of the two-liquid models stimulated renewed interest in the center vent. It was observed that wires or small tubes could pierce the liquid-liquid interface with what seemed to be a finite pucker. Also it was noticed that a wire loop pushed against the surface formed a depression which could be stretched into the ullage a short distance. Within a few diameters the distorted surface necked down, became unstable and pinched off. This report discusses the ideas and the mathematics generated by these observations.

* Li, Ta, Liquid Behavior in a Zero-G Field, Convair-Astronautics Report AE60-0682.

2.0 ASSUMED CONDITIONS:

2.1 Certain conditions are necessary on which to base the mathematics herein contained. First a perfect zero-g condition is assumed, undisturbed by sun-seeking rockets or other perturbations. Secondly, the heat received into the tank is assumed not to raise the wall temperature appreciably, i.e., the walls are everywhere wet. And, thirdly, the boiled off gas is assumed to enter the ullage quiescently, without causing bubbles or other disturbances.

The presence of small perturbations such as the sun-seeking rockets will probably not greatly influence the general result, but the effect of major rotations of the vehicle should be studied further. The mechanics of producing and transporting gas into the ullage can make or break the center vent system and should also be investigated further.

2.2 In the stylized Centaur considered here the center-vent tube extends along the tank center line from the forward end to about the tank center. See Figure 1.

3.0 MEMBRANE THEORY

3.1 The pressure, Δp , across the interfacial surface between two fluids can be related to the surface curvature by the following expression

$$\Delta p = \sigma \left(1/R_1 + 1/R_2 \right) \qquad (1)$$

Where σ is the surface tension and R_1 and R_2 are the principal radii of curvature. This general relation can be applied to the center vent but only with several pages of sticky algebra. However, before proceeding with the force balance approach in paragraph 4.0, several useful pieces of information should be noted. First, R_1 and R_2 are perpendicular to the surface at point of interest and lie in planes perpendicular to each other. In the case of the sphere these radii are equal while for a cylinder one is infinite. Equation (1) now may be rewritten for the sphere as

$$\Delta p = \frac{2\sigma}{R_s}$$

and for the cylinder as,

$$\Delta p = \frac{\sigma}{R_c}$$

where R_s and R_c are the related geometric radii.

4.0 FORCE BALANCE EQUATIONS: (See Table II)

4.1 Point Solutions:

Figure 1 shows a stylized Centaur upper-stage fuel tank. It is a cylinder of radius r_t containing two fluids. In the absence of the center vent the fluid interface would be a hemisphere with radius R_t ($= r_t$) producing:

$$\Delta p = \frac{2}{R_t} \qquad (2)$$

The introduction of a "small" center vent of radius r_v will distort this hemisphere. (This distortion and the vent size are exaggerated for clarity in the drawing.) For small distortions, i.e., a "small" vent the differential pressure remains approximately at its $\frac{2}{R_t}$ value. The two principal radii of curvature are measured as follows: R_1 in the plane of the paper and R_2 in a plane perpendicular to the paper.

4.1.1 Near the Vent:

As the surface approaches the vent R_2 decreases to r_v and R_1 decreases to a value just less than r_v. The interface approaches the vent tangentially with a radius (R_1) at the point of tangency very nearly equal to r_v.

4.1.2 At the Forward Point:

At the most forward point (or circle, to be exact) R_1 certainly is a finite radius but R_2 has become infinite. Note that the circle is in one plane (obviously). We shall designate R_1 at this point as R_f and its value is

$$R_f = \sigma / \Delta p$$

4.0 **FORCE BALANCE EQUATIONS: (CONTINUED)**

4.1.2 **At the Forward Point:** (Continued)

Hence: Referring to equation (2)

$$R_f = 1/2\, R_t \tag{3}$$

The radius, r_f, of this most forward circle is derived from the general force balance equation in paragraph 4.2.2

4.2 **The General Equation:**

4.2.1 The vertical components of the forces acting on a portion of the interfacial surface will be equated to obtain the general surface equation. Reference should be made to Figure 2. The surface considered extends from the vent tube to an arbitrary rim.

$$F_r \sin\alpha + \Delta p \cdot A = F_v$$

Where:

r is the radius of the rim

F_r is the surface tension force acting at the "rim";

α is the angle between this force and the horizontal;

Δp is the differential pressure across the surface;

A is the surface area within the rim projected onto a horizontal plane, and

F_v is the surface tension force acting along the vent tube.

4.0 FORCE BALANCE EQUATIONS: (CONTINUED)

4.2 The General Equation: (Continued)

4.2.1 (Continued)

This equation may be rewritten as

$$r \sin\alpha + \frac{r^2 - r_v^2}{r_t} = r_v \qquad (4)$$

4.2.2 Now let us digress from the general solution to find the r_f defined in paragraph 4.1.2. Equation (4) may be rewritten as

$$r_f \sin(o) + \frac{r_f^2 - r_v^2}{r_t} = r_v$$

Or:

$$r_f = \sqrt{r_v \left(r_t - r_v \right)} \qquad (5)$$

4.0 **FORCE BALANCE EQUATIONS:** (CONTINUED)

4.2 **The General Equation:** (Continued)

4.2.3 Returning to the main stream we now set r_v equal to 1.

Also,

$$\sin\alpha = \frac{1}{\sqrt{1 + (dr/ds)^2}}$$

Therefore equation (4) becomes

$$\frac{r}{\sqrt{1 + (dr/ds)^2}} + \frac{r^2 - 1}{r_t} = 1$$

Algebraic manipulation of this equation yields:

$$ds = \frac{1 - \dfrac{r^2}{r_t + 1}}{\sqrt{\left[1 - \left(\dfrac{r}{r_t + 1}\right)^2\right]\left[r^2 - 1\right]}}\, dr \qquad (6)$$

Equation (6) is real for

$$1 < r^2 < (r_t + 1)^2$$

The full significance of $r_t + 1$ will be left to the mathematicians for, as will presently be seen, it will not affect the final results of this report. However, the value 1 comes from r_e and is connected with the change in $\Delta\rho$ produced by the introduction of the vent; but, as we are interested only in values of r_t much greater than r_v, this lack of rigidity should be acceptable. See paragraph 4.1.

4.0 **FORCE BALANCE EQUATIONS:** (CONTINUED)

4.2 **The General Equation:** (Continued)

4.2.4 The integration of equation (6) requires the binomial expansion of

$$\left[1 - \left(\frac{r}{r_t + 1}\right)^2\right]^{-1/2}$$

and a term-by-term integration of the resulting series. An outline of this process is given here. From equation (6)

$$dz = \frac{dr}{\sqrt{1 - \frac{r}{B}}\ \sqrt{r^2 - 1}} - \frac{\left(\frac{r^2}{B}\right) dr}{\sqrt{1 - \frac{r}{B}^2}\sqrt{r^2 - 1}}$$

and,

$$z + z_o = I_1 - \frac{1}{B}\ I_2$$

Where:

B equals $r_t + 1$; z_o is a constant of integration; and I_1 and I_2 are the integrals expressed below:

$$I_1 = \int \frac{dr}{\sqrt{r^2 - 1}} + \tfrac{1}{2} B^{-2} \int \frac{r^2\, dr}{\sqrt{r^2 - 1}} + \frac{1.3}{2.4} B^{-4} \int \frac{r^4\, dr}{\sqrt{r^2 - 1}} + \ldots$$

$$= (\cosh^{-1} r) \left[1 + (\tfrac{1}{2})^2 B^{-2} + \left(\frac{1.3}{2.4}\right)^2 B^{-4} + \ldots\right.$$

$$\left. + \sqrt{r^2 - 1}\left[\tfrac{1}{2} B^{-2} \frac{r}{2} + \frac{1.3}{2.4} B^{-4}\left(\frac{r^3}{4} + \frac{1.3}{2.4} r\right) + \ldots\right]\right.$$

4.0 **FORCE BALANCE EQUATIONS:** (CONTINUED)

4.2 **The General Equation:** (Continued)

4.2.4 (Continued)

and,

$$I_2 = \int \frac{r^2\,dr}{\sqrt{r^2-1}} + \tfrac{1}{2} B^{-2} \int \frac{r^4\,dr}{\sqrt{r^2-1}} + \frac{1.3}{2.4} B^{-4} \int \frac{r^6\,dr}{\sqrt{r^2-1}} + \ldots$$

$$= (\cosh^{-1} r)\left[\tfrac{1}{2} + \tfrac{1}{2} B^{-2} \frac{1.3}{2.4} + \frac{1.3}{2.4} B^{-4} \frac{1.3.5}{2.4.6} + \ldots\right]$$

$$+ \sqrt{r^2-1}\left[\frac{r}{2} + \tfrac{1}{2} B^{-2}\left(\frac{r^3}{4} + \frac{1.3}{2.4} r\right) + \frac{1.3}{2.4} B^{-4}\left(\frac{r^5}{6} + \frac{5}{24} r^3 + \frac{5}{16} r\right) + \ldots\right]$$

Therefore,

$$z + z_o = (\cosh^{-1} r)\left[1 - \tfrac{1}{2} B^{-1} + (\tfrac{1}{2})^2 B^{-2} - \tfrac{1}{2}\left(\frac{1.3}{2.4}\right) B^{-3} + \left(\frac{1.3}{2.4}\right)^2 B^{-4} - \ldots\right]$$

$$- \sqrt{r^2-1}\left\{\left[\tfrac{1}{2} B^{-1} \; (\tfrac{1}{2})^2 B^{-2} + \tfrac{1}{2}\left(\frac{1.3}{2.4}\right) B^{-3} - \left(\frac{1.3}{2.4}\right)^2 B^{-4} + \ldots\right] r\right.$$

$$+ \left[\tfrac{1}{2} B^{-3} \tfrac{1}{4} - \frac{1.3}{2.4} B^{-4} \tfrac{1}{4} + \frac{1.3}{2.4} B^{-5} \frac{5}{24} - \ldots\right] r^3$$

$$\left. + \left[\frac{1.3}{2.4} B^{-5} \frac{1}{6} - \ldots\right] r^5 + \ldots \right\} \qquad (7)$$

Now let $z = 0$, $r = 1$ in equation (7):

$$0 - z_o = 0$$

4.0 FORCE BALANCE EQUATIONS: (Continued)

4.2 The General Equation: (Continued)

4.2.4 (Continued)

Now, having evolved a complex and cumbersome equation, what do we do with it? As usual, make approximations. Where $r \ll r_t$ we may use a short-tailed version of equation (7).

$$z \doteq \cosh^{-1}(r) - \frac{r\sqrt{r^2 - 1}}{2\,(r_t + 1)} \tag{8}$$

This accurate to about 1% for $r_t = 60$ and $r < 12$. Both (7) and (8) reduce to $z = \cosh^{-1} r$ for infinite r_t, i.e., Δp = zero.

4.2.5 The numerical configuration of equation (8) is presented in Table I and Figure 3 for $r_t = 60$. Both the one-unit radius circle and the inverse cosh curve are tangent to the vent-tube wall at $r = 1$. Note, too, that the "most forward point" occurs at $r = 7.8$ as the point solution given in paragraph 4.2.2 predicts.

4.2.6 It should be remembered that in the preceeding development the vent-tube radius is used as the unit of measure. For a greater r_t (this means a smaller vent for a fixed tank size) z_f will be numerically greater because of the reduced unit of measure. However, in terms of tank dimensions, decreasing the vent tube size will reduce z_f.

5.0 PHYSICAL REALITY:

5.1 The preceeding mathematics have been loosely connected with the Centaur upper stage. Although the exact size vent tube has not yet been chosen it will probably be about 1 inch in radius, perhaps smaller. The fuel tank is of course 60 inches in radius. The maximum quantity of fuel remaining in any zero-g period is expected to be about 30% and the resulting ullage in an unmodified tank will be a cylinder capped by two hemispheres. An allusion was made in paragraph 4.1 to an increased ΔP produced by the introduction of a vent tube and, while its effect on the surface shape about the vent is negligible, another effect will be produced. That effect is to push the ullage aft until the intermediate bulkhead distorts the aft interface to such an extent as to produce a balancing ΔP. Of course the use of an ullage support structure (USS), as has been suggested elsewhere, would completely override this effect.

5.2 The question may arise: If LH_2 wets everywhere, what happens aft of the tangent point on the vent? It appears that the tube will be wet with the liquid but that the film will be only a few molecules thick. It is conceivable that in the periods between ventings the tube could fill and thus a "tube-full" would be lost during each vent cycle. However, assuming the absurdly high values of 10^{-6} cm as the film thickness and 1 cm/sec as the liquid velocity, less than 0.1 cubic cm would be delivered to the tube each hour. The effect of evaporation/condensation on filling the tube is beyond the scope of this report but an estimate probably can be made easily.

6.0 CONCLUSION:

6.1 This report has, we hope, dispelled any question of the ability of a center vent to penetrate the liquid gas interface under the conditions listed in Section 2.0. The surface equation developed herein is for a a specific case and certain approximation were made; but the general result must be emphasized that a "small" tube passing through an interfacial surface causes little distortion of that surface. A large tube (or other shape) certainly will influence the tank contents and the Ullage Support Structure (USS) is based on this effect.

6.2 As mentioned previously two major problems have been ignored. They are the major vehicle rotations and the mechanics of gas production. While several General Dynamics/Astronautics groups are investigating related areas it is probable that only a full Centaur flight test will answer the latter question. If neither of these problems develop, the center vent should make a most reliable, light, and inexpensive vent system.

TABLE I

CENTER VENT SURFACE FOR 2" VENT TUBE IN 120" TANK

r	$\cosh^{-1} r$	$\frac{r\sqrt{r^2 - 1}}{122}$	z
1.0	0	0	0
1.1	0.44	0	0.44
1.2	0.62	0.01	0.61
1.3	0.76	0.01	0.75
1.5	0.96	0.01	0.95
2.0	1.31	0.03	1.28
3.0	1.76	0.07	1.69
4.0	2.06	0.13	1.93
6.0	2.48	0.29	2.19
7.81 (r_f)	2.75	0.50	2.25
10.0	3.00	0.82	2.18
12.0	3.17	1.19	1.98

TABLE II

NOMENCLATURE:

Variables:

R = Radius of curvature

r = Radial location from tank (and vent tube) centerline

z = Distance forward in the tank measured parallel to the centerline and from the point of surface/probe tangency.

F = Force produced by surface tension.

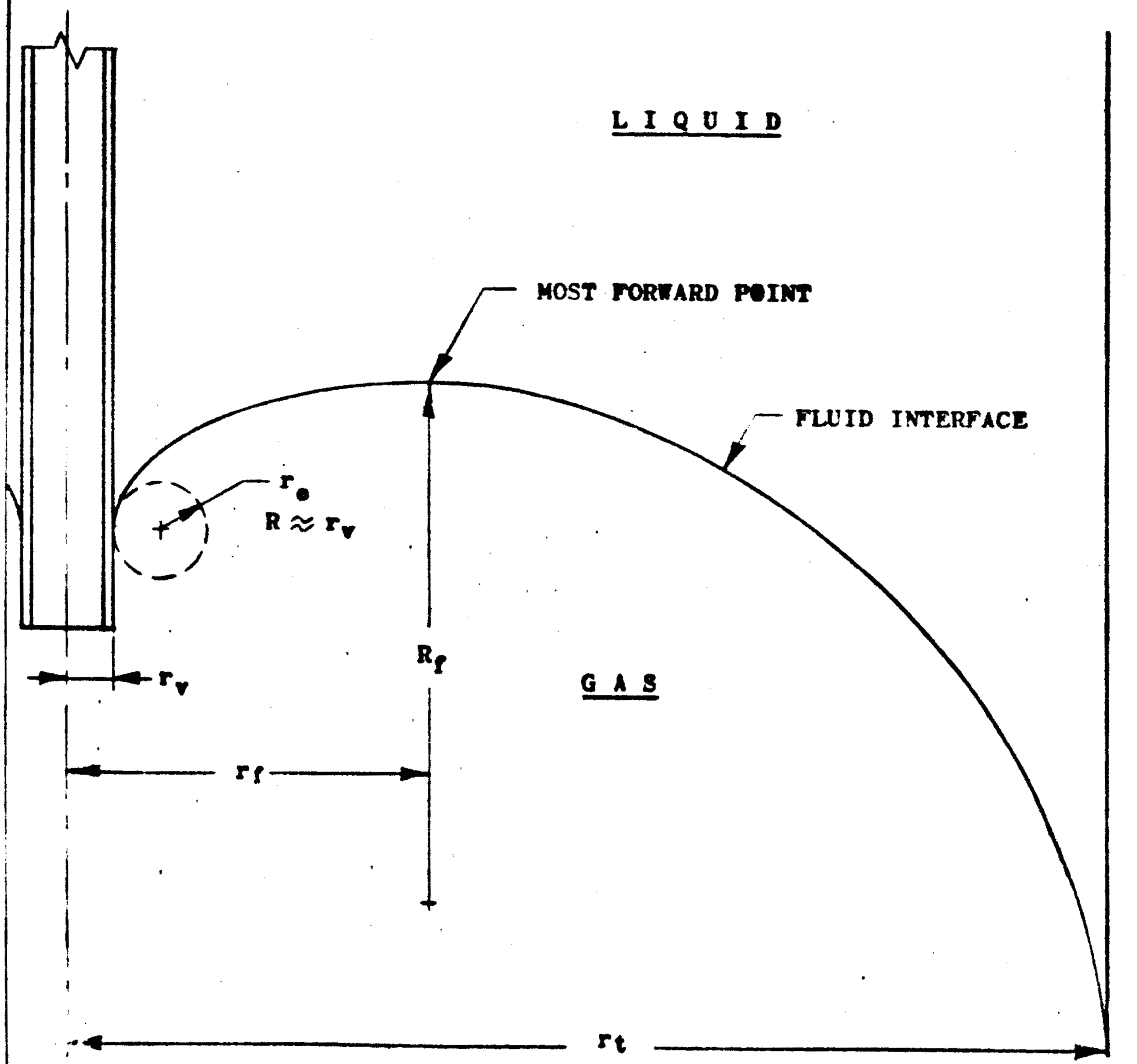
Subscripts:

v Pertains to the outside surface of the vent tube

f Pertains to the most forward point of the interfacial surface.

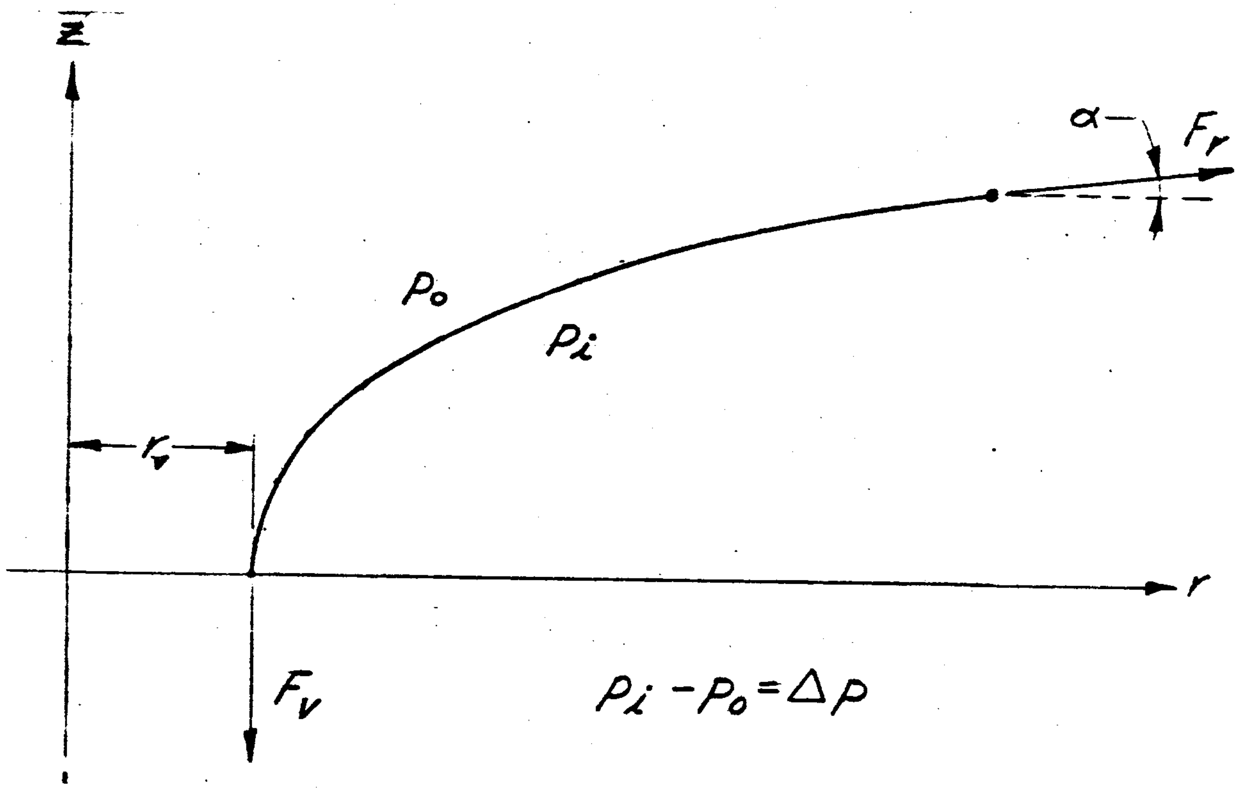
t Pertains to the inside surface of the tank

r Pertains to an arbitrary circular rim.



STYLIZED CENTAUR TANK
SHOWING FLUID INTERFACE

FIGURE 1



FORCE BALANCE DIAGRAM

FIG 2

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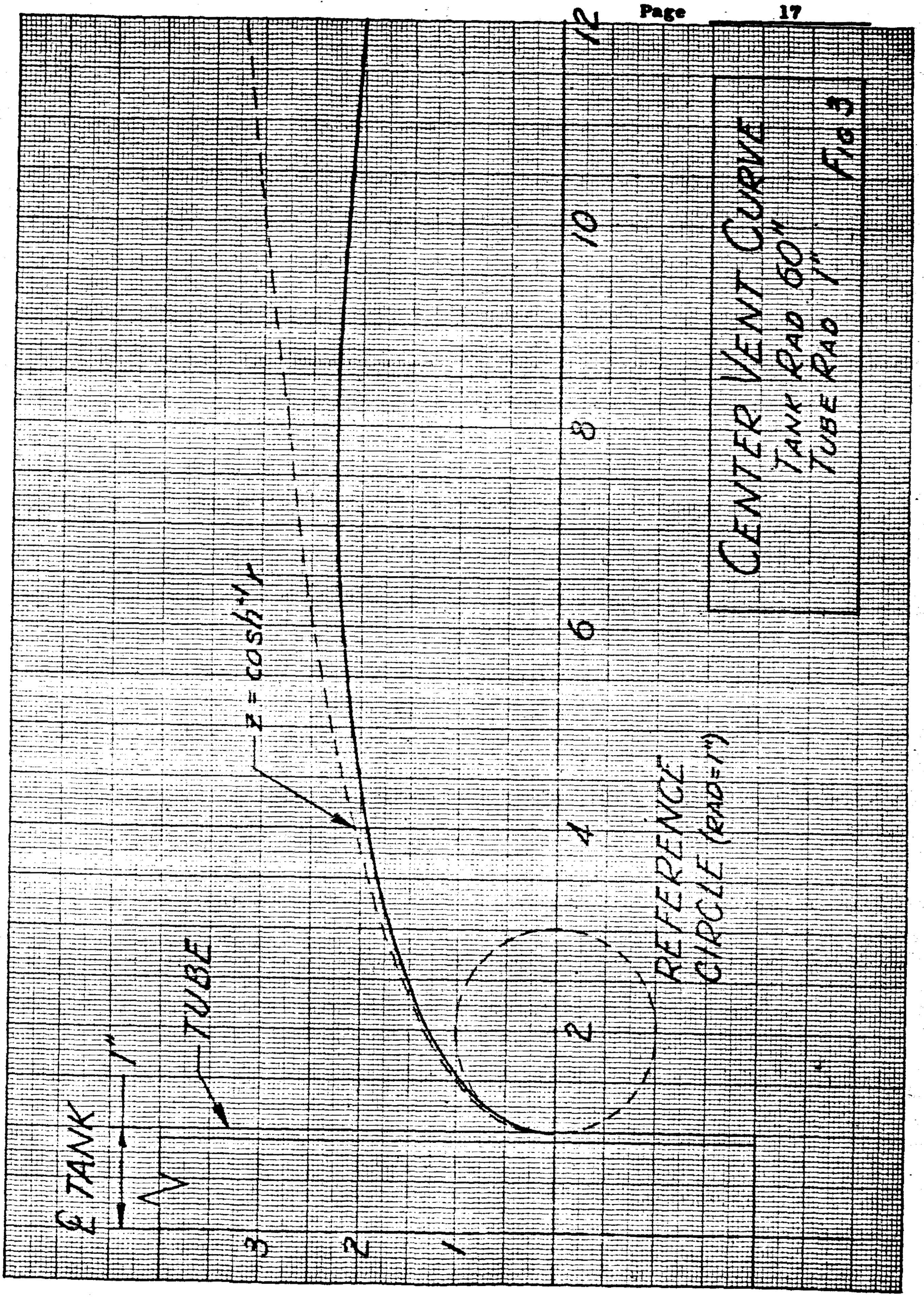
CENTER VENT CURVE
TANK RAD 60"
TUBE RAD 1"
FIG 3
REFERENCE CIRCLE (RAD=1")
Z = cosh⁻¹ r
TUBE
℄ TANK
1"
2
4
6
8
10
12
1
2
3

6 JUL 65 55081

1 G

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