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WITH MAGNETIC FIELD AND ROTATION

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ABSTRACT

The problem of instability in contrastreaming streams of plasma or self-gravitating gas clouds is investigated for general propagation direction, using moment equations. A uniform rotation is also included in view of its astrophysical importance. Conditions for instability (monotonic or growing wave) are derived. It is found that the classical Jeans' wavelength for fragmentation of interstellar medium is considerably diminished due to interstreaming speeds. For a non-gravitating plasma it is concluded that perturbations propagating normal to the interstreaming direction lead to a montonic instability. This instability, though characterized by a small growth rate, should be possible to observe in laboratory plasmas if dimensions are suitably chosen to eliminate the conventional electrostatic two-stream instability.

I. INTRODUCTION

Contrastreaming plasmas are a common occurrence in nature e.g. in colliding galaxies, plasma streams from M-regions, solar flares in the background solar wind. This makes the investigation of the possibility of excitation of stable or unstable oscillations due to collective interactions in contrastreaming plasmas an important question. Various investigations of the two-stream instability in collisionless plasmas, both cold and warm, have been reported[1], but they have been restricted to <u>electrostatic perturbations in non-gravitating plasmas</u>. The electrostatic instability arises due to electrostatic interactions arising from a charge separation produced by wave propagation <u>along</u> the streaming direction so that the magnetic field remains unperturbed. In general, the system is subject to a perturbation propagating at any angle to the streaming motion. It is of interest, therefore, to explore whether electromagnetic interactions, due to a perturbed magnetic field, can lead to an instability in contrastreaming plasmas. Again in astrophysical situations (e.g. interpenetrating star streams) the streams are self-gravitating and endowed with a large-scale galactic rotation. The two problems--namely contrastreaming instability in collisionless plasmas and stellar streams--are essentially alike except for the important difference that gravitational interactions are always attractive (as against attractive and repulsive forces in charges constituting a plasma), and that the self-gravitational field is not neutralized as in an ionized gas which is electrically neutral. It may be mentioned that cooperative phenomena in collisionless stellar streams (in the absence of rotation and magnetic field) have recently been studied by Sweet[2].

The purpose of the present paper is to present a unified treatment of two-stream instability for general perturbations for ionized streams or self-gravitating streams of unionized gas including the effect of a uniform rotation and prevailing uniform interstellar magnetic field. We shall make use of the moment equations for a warm, collisionless plasma. These equations will naturally preclude phenomena like Landau damping. The results obtained, though exact for cold configurations, would, it is hoped, represent reasonably well the situations including thermal effects.

II. INITIAL STATE

Consider the two unbounded, homogeneous, plasma streams interpenetrating with equal and opposite speeds U_0,$-U_0$. The ions and electrons of either stream will be assumed to move together so that there is no initial electric current in either medium, and be characterized by equal temperatures. The two streams will be supposed to be self-gravitating and subject to the simultaneous effect of a homogeneous rotation and magnetic field. In homogeneous, isothermal streams we are required, as shown below, to take the prevailing magnetic field $\underline{B_0}$, the rotation vector $\underline{\Omega}$, and the streaming motion $\underline{U}_0$, all parallel to one another in order that the steady state equations are consistently satisfied for both streams.

The initial state is governed by the following equations with respect to a rotating frame of reference,

Beam 1

$$-\frac{1}{N_{01}}\nabla P_1^{(e)} - e\left[\underline{E}_0 + \frac{\underline{U}_0\times\underline{B}_0}{c}\right] + 2m_e(\underline{U}_0\times\underline{\Omega}) + m_e\nabla\Phi_0 - m_e\underline{\Omega}\times(\underline{\Omega}\times\underline{r}) = 0 \quad \text{(electrons)} \tag{1}$$

$$-\frac{1}{N_{01}}\nabla P_1^{(i)} + e\left[\underline{E}_0 + \frac{\underline{U}_0\times\underline{B}_0}{c}\right] + 2m_i(\underline{U}_0\times\underline{\Omega}) + m_i\nabla\Phi_0 - m_i\underline{\Omega}\times(\underline{\Omega}\times\underline{r}) = 0 \tag{2}$$

(ions)

Beam 2

$$-\frac{1}{N_{02}}\nabla P_2^{(e)} - e\left[\underline{E}_0 - \frac{\underline{U}_0 \times \underline{B}_0}{c}\right] - 2m_e(\underline{U}_0 \times \underline{\Omega}) + m_e \nabla \Phi_0 - m_e \underline{\Omega} \times (\Omega \times \underline{r}) = 0 \qquad \text{(electrons)} \tag{3}$$

and

$$-\frac{1}{N_{02}}\nabla P_2^{(i)} + e\left[\underline{E}_0 - \frac{U_0 \times \underline{B}_0}{c}\right] - 2m_i(\underline{U}_0 \times \underline{\Omega}) + m_i \nabla \Phi_0 - m_i \underline{\Omega} \times (\Omega \times \underline{r}) = 0 \qquad \text{(ions)} \tag{4}$$

$$\nabla^2 \Phi_0 = -4\pi G \sum m_j N_{oj} \tag{5}$$

$$\nabla \cdot \underline{B}_0 = 0 \quad , \quad \nabla \times \underline{B}_0 = 0 \tag{6}$$

$$\nabla \times \underline{E}_0 = 0 \quad , \quad \nabla \cdot \underline{E}_0 = 0 \tag{7}$$

where the symbols have their usual meaning. For gravitating streams of unionized gas the equations (6) and (7) have no meaning and we have equation (5) together with two equations out of equations (1) - (4) without the electromagnetic quantities.

Equating equations (1) and (3) we obtain,

$$-\frac{1}{N_{01}}\nabla P_1^{(e)} + 4 m_e \underline{U}_0 \times \left[\underline{\Omega} + \frac{\underline{\Omega}_e}{2}\right] = -\frac{1}{N_{02}}\nabla P_2^{(e)} \tag{8}$$

which must be satisfied for the initial state. Thus for beams characterized by homogeneous pressures we must have

$$\underline{U}_0 \times \left[\underline{\Omega} + \frac{\underline{\Omega}_e}{2}\right] = 0 \tag{9}$$

so that the three vectors $\underline{U}_0$, $\underline{\Omega}$, $\underline{\Omega}j$, [Larmor frequency $\frac{e_j B_0}{m_j c}$ for jth particle, (electron or ion)] are parallel.

Again for a gravitating gas stream we need to satisfy

$$\nabla \Phi_0 = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}) \tag{10}$$

for each beam. This leads to the equilibrium relation

$$\Omega^2 = 2\pi G \sum m_j N_{0j} \tag{11}$$

III. PERTURBATION EQUATIONS

The moment equations defining the time-dependent perturbed state of the streaming plasma are written as,

$$m_j\left[\frac{\partial \underline{v}_j}{\partial t} + (\underline{v}_j\cdot\nabla)\underline{v}_j\right] = -\frac{\nabla P_j}{N_j} + e_j\left[\underline{E} + \frac{\underline{v}_j\times\underline{B}}{c}\right] + 2m_j(\underline{v}_j\times\underline{\Omega}) + m_j\nabla\Phi - m_j\,\Omega\times(\Omega\times\underline{r}) \tag{12}$$

$$\frac{\partial N_j}{\partial t} + \nabla.(N_j\underline{v}_j) = 0 \tag{13}$$

$$\nabla^2\Phi = -4\pi G\sum m_j N_j \tag{14}$$

$$\frac{d}{dt}P_j = S_j^2\frac{d}{dt}\rho_j \tag{15}$$

$$\nabla\cdot\underline{B} = 0 \tag{16}$$

$$\nabla\times\underline{B} = \frac{4\pi}{c}\underline{J} + \frac{1}{c}\frac{\partial}{\partial t}\underline{E} = \frac{4\pi}{c}\sum N_j e_j\underline{v}_j + \frac{1}{c}\frac{\partial}{\partial t}\underline{E} \tag{17}$$

$$\nabla\times\underline{E} = -\frac{1}{c}\frac{\partial}{\partial t}\underline{B} \tag{18}$$

and

$$\nabla.\underline{E} = 4\pi\sum e_j N_j \tag{19}$$

Here

$$
\begin{aligned}
\underline{B} &= \underline{B}_0 + \underline{b} \\
\underline{E} &= \delta \underline{E} \\
\underline{V}_j &= \underline{U}_{0j} + \underline{u}_j \\
\underline{J} &= \underline{J} \\
\underline{\Phi} &= \underline{\Phi}_0 + \phi
\end{aligned}
\tag{20}
$$

The equilibrium quantities are shown with a suffix '0' and the corresponding perturbations are denoted by small letters. In equations (14), (17) and (19) summation includes ions and electrons for both the beams. The quantity Sj stands for the characteristic sound speed $\left(= \sqrt{kT_j/m_j} \right)$ for the jth particle.

From equations (17) and (18) we obtain

$$
\nabla \nabla \cdot \delta \underline{E} - \nabla^2 \delta \underline{E} + \frac{4\pi}{c^2} \frac{\partial}{\partial t} \underline{J} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \delta \underline{E} = 0 \tag{21}
$$

Assuming the perturbations to be of the form

$$
e^{i(\underline{k} \cdot \underline{r} - \omega t)} \tag{22}
$$

we write equation (21) as,

$$(\omega^2 - c^2k^2)\,\delta\underline{E} + c^2\underline{k}\,(\underline{k}\cdot\delta\underline{E}) + 4\pi i\omega\sum e_j\left(N_{oj}\,\underline{u}_j + n_j\,\underline{U}_{oj}\right) = 0 \tag{23}$$

The expression for nj is obtained from equation (13) as

$$n_j = \frac{N_{oj}\,\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{oj})} \tag{24}$$

The Poisson's equation (14) yields

$$\phi = \frac{4\pi G}{k^2}\sum\frac{m_j N_{oj}\,\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{oj})} \tag{25}$$

The equation of motion (12) can be rewritten, using equations (24) and (25) as

$$\begin{aligned}(-i\omega + i\underline{k}\cdot\underline{U}_{oj})\,\underline{u}_j &+ \frac{S_j^2\, i\underline{k}\;\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{oj})} + (\underline{\Omega}_j + 2\underline{\Omega})\times\underline{u}_j \\ &+ \frac{4\pi G}{ik^2}\,\underline{k}\sum\frac{m_j N_{oj}\,\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{oj})} = \frac{e_j}{m_j}\left[\delta\underline{E} + \frac{\underline{u}_j\times\underline{b}}{c}\right]\end{aligned} \tag{26}$$

Using equation (18) we rewrite equation (26) as

$$\begin{aligned}(-i\omega + i\underline{k}\cdot\underline{U}_{oj})\,\underline{u}_j + \frac{S_j^2\, i\underline{k}\;\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{oj})} + (\underline{\Omega}_j + 2\underline{\Omega})\times\underline{u}_j + \frac{4\pi G}{ik^2}\,\underline{k}\sum\frac{m_j N_{oj}\,\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{oj})} \\ = \frac{e_j}{m_j\omega}\left[\omega - \underline{k}\cdot\underline{U}_{oj} + \underline{k}\,\underline{U}_{oj}\cdot\right]\delta\underline{E}\end{aligned} \tag{27}$$

The equation (23) is rewritten as,

$$(\omega^2 - c^2k^2)\,\delta\underline{E} + c^2\underline{k}\,(\underline{k}\cdot\delta\underline{E}) + 4\pi i\omega \sum e_j \left(N_{0j}\,\underline{u}_j + N_{0j}\,\underline{U}_{0j}\,\frac{\underline{k}\cdot\underline{u}_j}{(\omega - \underline{k}\cdot\underline{U}_{0j})} \right) = 0 \qquad (28)$$

The equations (27) and (28) constitute the coupled set of equations for the problem under investigation. Clearly for gravitating inter-penetrating streams the perturbation equation is (27) with right hand side put equal to zero. It may also be noted that the rotation vector occurs in the perturbation equation (27) along with the Larmor frequency term, and we can therefore speak of an effective Larmor frequency vector $\underline{a} = (\underline{\Omega}_j + 2\underline{\Omega})$ in the presence of rotation.

Let us now fix the direction of the wave number vector $\underline{k}$ as the z-axis and assume the parallel vectors $\underline{U}_{0j}$ and $\underline{a}$ to have two components, for generality, in the x and z directions. We may now eliminate $\delta\underline{E}$ from equations (27) and (28) and write the final perturbation equations in component form as

$$(\omega^2 - c^2k^2)\left[u_{jx} - \frac{i a_z u_{jy}}{(\omega - \underline{k}\cdot\underline{U}_{0j})}\right] - \frac{4\pi e_j}{m_j}\sum N_{0j} e_j \left(u_{jx} + \frac{k U_{0jx}\, u_{jz}}{(\omega - \underline{k}\cdot\underline{U}_{0j})} \right) = 0 \qquad (29)$$

$$(\omega^2 - c^2k^2)\left[u_{jy} + \frac{i(a_z u_{jx} - a_x u_{jz})}{(\omega - \underline{k}\cdot\underline{U}_{0j})}\right] - \frac{4\pi e_j}{m_j}\sum e_j N_{0j} u_{jy} = 0 \qquad (30)$$

$$\left[kV_{ojx}\left(u_{jx} - \frac{ia_z u_{jy}}{\omega - \underline{k}.V_{oj}}\right) - (\omega - \underline{k}.V_{oj})u_{jz} - ia_x u_{jy} + \frac{S_j^2 k^2 u_{jz}}{\omega - \underline{k}.\underline{V}_{oj}} - 4\pi G \sum \frac{m_j N_{oj} u_{jz}}{\omega - \underline{k}.V_{oj}} \right] + 4\pi \frac{e_j}{m_j} \sum N_{oj} e_j \frac{u_{jz}}{\omega - \underline{k}.\underline{V}_{oj}} = 0 \tag{31}$$

The equations (29) - (31) constitute in all twelve equations (three for electrons and three for ions) for both beams taken together. The summation is over both electrons and ions in the two beams and would thus consist of four terms. For gravitating unionized streams there will be, in all, six equations (three for each beam) with each summation having only two terms. The self-gravitation term occurs only in the equation (31), which on comparison with the last summation term reveals that the contribution from self-gravitation in an ionized gas is negligible compared to the contribution from the charged particles as $\omega_{p_{i,e}}^2 / 4\pi G N_0 m_{i,e}$, independent of charged particle density, is always much larger than unity. Thus so far as we are dealing with ionized streams, the self-gravitation effect is entirely negligible. We may, therefore, discuss the case of unionized gravitating gas separately from the ionized (plasma) streams.

IV. ELECTRON OSCILLATIONS IN PLASMA STREAMS

Having seen that the self-gravitational effects are negligible so long as the plasma frequency is not zero, we may, for simplicity, consider the case of electron oscillations only in interpenetrating plasma streams. This approximation is reasonable in view of large mass of the ions which can therefore be regarded as unperturbed unless the frequency of oscillation is small.

A. Field-free non-rotating plasma streams

For a configuration of interpenetrating plasma streams in the absence of magnetic field and rotation, it is easy to obtain the dispersion relation for electron oscillations from equations (29) - (31). This is written as

$$\left[-(\omega-\underline{k}.\underline{U_0})^2+k^2S_1^2+\omega_{p_1}^2+\frac{k^2U_{0x}^2\,\omega_{p_1}^2}{\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)}\right]$$

$$+\,\omega_{p_2}^2\left[1-\frac{k^2U_{0x}^2}{\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)}\right]\left[\frac{-(\omega-\underline{k}.\underline{U_0})^2+k^2S_1^2+\dfrac{2k^2U_{0x}^2\,\omega_{p_1}^2}{\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)}}{-(\omega+\underline{k}.\underline{U_0})^2+k^2S_2^2+\dfrac{2k^2U_{0x}^2\,\omega_{p_2}^2}{\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)}}\right]=0 \qquad (32)$$

Here suffixes 1 and 2 refer to the two beams, and all quantities U_0, S, ω_p refer to electrons.

For the particular case of parallel propagation ($k \parallel U_0$), we put $U_{0x} = 0$ and obtain

$$\frac{\omega_{p_1}^2}{(\omega-\underline{k}.\underline{U_0})^2-k^2S_1^2}+\frac{\omega_{p_2}^2}{(\omega+\underline{k}.\underline{U_0})^2-k^2S_2^2}=1 \qquad (33)$$

This is the well-known dispersion relation for electrostatic instability in contrastreaming field-free plasmas. For identical plasma streams ($\omega_{p1} = \omega_{p2}$; $S_1 = S_2$) the configuration is stable for all wave lengths of perturbation so far as the streaming velocity is less than the thermal speed. In case $U_0 > S$, the configuration is monotonically unstable for $k < k_*$ where k_* is given by

$$k_*^2 = \frac{2\omega_p^2}{U_0^2 - S^2} \tag{34}$$

In the unstable range of wavelengths there exists a mode of maximum instability defined by,

$$\begin{aligned} n_m^2 &= \frac{\beta^2-1}{\beta^2+1}\,\omega_p^2\left[1 - \frac{(\beta^2-1)}{2}\left\{\frac{1}{\beta}\left(\frac{\beta+1}{\beta-1}\right)^{1/2} - 1\right\}\right] \\ &= \omega_p^2/4 \qquad \text{for } S = 0 \end{aligned} \tag{35}$$

and

$$\begin{aligned} k_m^2 &= \frac{\omega_p^2}{2S^2}\left[\frac{\beta^2+1}{\beta(\beta^2-1)^{1/2}} - 1\right] \\ &= 3\omega_p^2/4U_0^2 \qquad \text{for } S = 0 \end{aligned} \tag{36}$$

where $\beta = U_0/S$. n_m and k_m respectively denote the growth rate and the wave number of the mode of maximum instability.

To see whether instability may arise when wave propagation vector $\underline{k}$ is inclined to the streaming direction, let us consider the particular case of transverse propagation $k \perp U_0$. For this case we put $\underline{k} \cdot \underline{U}_0 = 0$ in equation (32) and obtain for identical interpenetrating plasma streams, the dispersion relation as,

$$\omega^4 - \omega^2 \left[(c^2 + s^2) k^2 + 2\omega_p^2 \right] + k^2 s^2 (c^2 k^2 + 2\omega_p^2) - 2k^2 U_{0x}^2 \omega_p^2 = 0 \tag{37}$$

The equation (37) shows that the configuration of field-free plasma streams are unstable monotonically for propagation normal to the streaming if the following inequality is satisfied,

$$k < \frac{2\omega_p^2 (U_0^2 - s^2)}{s^2 c^2} \tag{38}$$

Clearly the streams characterized by $U_0 \leq S$ are stable as was the case for $k \parallel U_0$. For a pressureless configuration (cold streams) it may be noted that there is instability for all wavelengths transverse to streaming motion. Thus even those wavelengths which were stable for wave vector along streaming are, strictly speaking, unstable when they propagate normal to the streaming direction. We conclude, therefore, that cold interpenetrating streams are, in general, unstable for all k. Again there exists a mode of maximum instability in this case, too, defined by

$$n_m^2 = \frac{2\omega_p^2 (U_0^2 - s^2) - 2k^2c^2s^2}{s^2 + c^2}$$

$$= 2\omega_p^2 U_0^2/c^2 \quad \text{for } s = 0 \tag{39}$$

and

$$k_m^2 = \frac{2\omega_p^2(c^2+2U_0^2)}{(c^2-s^2)^2}\left[\left\{1+\left(\frac{U_0^2}{s^2}-1\right)\frac{\left(1-\frac{s^2}{c^2}\right)^2\left(1+\frac{U_0^2}{c^2}\right)}{\left(1+\frac{2U_0^2}{c^2}\right)^2}\right\}^{1/2}-1\right]$$

$$\simeq \frac{2\omega_p^2(U_0-s)}{c^2 s} \quad \text{for } s \to 0 \tag{40}$$

The electromagnetic instability ($k \perp U_0$) is characterized by a very small growth rate and so should be masked by the electrostatic instability when both are simultaneously present. It should, however, be possible to observe $k \perp U_0$ instability by a proper choice of plasma dimensions thus getting rid of the electrostatic instability.

B. Plasma streams with field and rotation

When the interpenetration of plasma streams takes place along the direction of external uniform magnetic field, the results are well-known for the case of parallel propagation i.e. $k \parallel U_0$. We may recapitulate by noting that the longitudinal oscillations (involving the perturbation velocity component parallel to the magnetic field) are unaffected by the presence of the magnetic field, leading to the same dispersion equation as (33). In addition to a longitudinal mode we obtain a mixed transverse mode in the presence of a uniform magnetic field, the results for which are also well-known (Bernstein and Trehan[3]).

Let us now investigate in particular the case of propagation normal to the streaming motion ($k \perp U_0$) in the presence of a uniform magnetic field (and/or rotation). After some simplifications on equations (29), (30) and (31) we obtain the dispersion relation as

$$\left(-\omega^2+k^2S_1^2+\omega_{p_1}^2\right)\left(\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)\right)+\left[k^2U_{0x}^2\omega_{p_1}^2+a_x^2\left(\omega^2-c^2k^2-\omega_{p_2}^2\right)\right]$$
$$+\left[\omega_{p_2}^2\left(\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)\right)-\omega_{p_2}^2\left(a_x^2+k^2U_{0x}^2\right)\right]\frac{\left[\omega^2-k^2S_1^2-a_x^2-\dfrac{2k^2U_{0x}^2\,\omega_{p_1}^2}{\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)}\right]}{\left[\omega^2-k^2S_2^2-a_x^2-\dfrac{2k^2U_{0x}^2\,\omega_{p_2}^2}{\omega^2-c^2k^2-(\omega_{p_1}^2+\omega_{p_2}^2)}\right]}=0 \tag{41}$$

For identical plasma streams the equation (41) leads to

$$\omega^4-\omega^2\left[k^2(c^2+S^2)+a_x^2+2\omega_p^2\right]+(k^2S^2+a_x^2)(c^2k^2+2\omega_p^2)-2k^2U_{0x}^2\omega_p^2=0 \tag{42}$$

The equation (42) shows that the overstability (growing wave instability) is absent, but that the configuration of <u>cold plasmas</u> is monotonically unstable if the wave number of transverse perturbation exceeds a certain critical value given by

$$k_* = \frac{a_x\,\omega_p}{\sqrt{\omega_p^2U_{0x}^2-\frac{a_x^2}{2}c^2}} \tag{43}$$

Thus contrastreaming <u>cold</u> plasmas are completely stabilized by a strong enough prevailing magnetic field (or rotation) defined by

$$a_x^* = \frac{\sqrt{2}\,\omega_p U_{0x}}{c} \tag{44}$$

For warm plasma streams we conclude that there is no overstability possible and the configuration is stable for all k when $U_0 \leq S$, or the field is stronger than defined by

$$a_x^* = \frac{\sqrt{2}\,\omega_p (U_0 - S)}{c} \tag{45}$$

in case $U_0 > S$. The configuration shows monotonic instability only in case $a_x^2 < \frac{2\omega_p^2 (U_0 - S)^2}{c^2}$ for a range of wave-numbers defined by,

$$2c^2S^2k_{1,2}^2 = \left[a_x^2c^2 - 2\omega_p^2(U_0^2 - S^2)\right]\left[-1 \pm \left\{1 - \frac{8a_x^2\omega_p^2c^2S^2}{\left[a_x^2c^2 - 2\omega_p^2(U_0^2 - S^2)\right]^2}\right\}^{1/2}\right] \tag{46}$$

V. GRAVITATING STREAMS WITH ROTATION

For interpenetrating streams of unionized gas characterized by a self-gravitation the relevant equation is (27) with right hand side put zero. The summation is over the particles (or stars in stellar streams) constituting the two streams. The dispersion relation, as obtained by simplifying equation (27), is finally written as,

$$\left[(\omega-\underline{k}.\underline{U_0})^2\left\{1-\frac{a_x^2}{(\omega-\underline{k}.\underline{V_0})^2-a_z^2}\right\}-k^2S_1^2\right]$$
$$+4\pi G m_1 N_{01}\left[1+\frac{m_2}{m_1}\frac{N_{02}}{N_{01}}\left\{\frac{k^2S_1^2-(\omega-\underline{k}.\underline{U_0})^2+\dfrac{a_x^2(\omega-\underline{k}.\underline{U_0})^2}{(\omega-\underline{k}.\underline{V_0})^2-a_z^2}}{k^2S_2^2-(\omega+\underline{k}.\underline{V_0})^2+\dfrac{a_x^2(\omega+\underline{k}.\underline{V_0})^2}{(\omega+\underline{k}.\underline{U_0})^2-a_z^2}}\right\}\right] \tag{47}$$

To digress the general dispersion equation (47) we may consider the special cases of parallel propagation ($k \parallel U_0 \parallel \Omega$) and perpendicular propagation ($\underline{k} \perp U_0, \Omega$).

A. Parallel Propagation ($k \parallel U_0$).

In this case $a_x = 0$, and the dispersion relation (47) yields

$$(\omega^2-k^2U_0^2)^2-k^2S_1^2(\omega+kU_0)^2-k^2S_2^2(\omega-kU_0)^2+k^4S_1^2S_2^2$$
$$+4\pi G m_1 N_{01}\left[\omega^2\left(1+\frac{m_2}{m_1}\frac{N_{02}}{N_{01}}\right)+2\omega k V_0\left(1-\frac{m_2}{m_1}\frac{N_{02}}{N_{01}}\right)\right.$$
$$\left.+k^2V_0^2\left(1+\frac{m_2}{m_1}\frac{N_{02}}{N_{01}}\right)-k^2(S_1^2+S_2^2)\right]=0 \tag{48}$$

which is a fourth degree polynomial in ω, the parameter deciding the question of stability of the configuration. For streams of identical stellar masses and number densities and having same thermal velocities, the dispersion equation (48) reduces to

$$\omega^4 - 2\omega^2\left[k^2(U_0^2+S^2) - 4\pi G m N_0\right] + k^2(U_0^2-S^2)\left[k^2(U_0^2-S^2) + 8\pi G m N_0\right] = 0 \tag{49}$$

This expression, independent of rotation, reduces to the well-known Jeans' criterion for fragmentation of interstellar gas for the case $U_0 = 0$. The analysis of equation (49) reveals that two interpenetrating gravitating streams do not show any overstability if $U_0 \leq S$. In this case the configuration is monotonically unstable for

$$k^2 < \frac{8\pi G m N_0}{(S^2 - U_0^2)} \tag{50}$$

and stable for

$$k^2 > \frac{8\pi G m N_0}{(S^2 - U_0^2)}$$

The significance of this result (equation 50) is in the modification of the Jeans' criterion for fragmentation due to the interpenetrating interstellar clouds. The classical Jeans' theory ($U = 0$) led to a critical size so large that it is impossible for stars to be formed with masses less than about 500 times the solar mass. This prompted various workers[4] to think of some operative mechanisms which could result in star condensations of much smaller masses. In equation (50) we find that the critical wavelength $\lambda^*\left(= 2\pi/k^*\right)$ (above which there is montonic instability) is reduced by the presence of interpenetrating speeds and goes down to zero for $U_0 = S$. The interstellar medium is by no means quiescent and such interpenetrating speeds are quite likely to occur. We may, therefore, surmise that the essential condition for monotonic instability $\left(U_0 \leqslant S \right)$ is likely to be satisfied in various regions of interstellar gas and would thus play a part in fragmentations leading to star formation of much smaller masses than given by the classical Jeans' theory. Similar results were obtained by Sweet[2], although the question of whether instability would lead to fragmentation or only to a conversion of the initial streaming energy to disordered energy till the linearized theory breaks down, is still open.

Again the streaming motion may exceed the thermal velocities ($\sim$ 1 Km./sec.) for the interstellar conditions. In that case the equation (49) predicts a growing wave instability (overstability) for a range of wave numbers given by,

$$k_{1,2}^2 = \frac{2\pi G m N_0}{S^2}\left[1 \pm \left\{1 - \frac{S^2}{U_0^2}\right\}^{1/2}\right] \tag{51}$$

If we neglect the thermal effects altogether the corresponding conditions for instability to manifest are,

$$k^2 > \frac{\pi G m N_0}{U_0^2} \qquad \text{(overstability)}$$

and

$$k^2 < \frac{4\pi G m N_0}{U_0^2} \qquad \text{(monotonic instability)} \qquad (52)$$

Thus we may say that for cold gravitating stellar systems the situation is unstable for all relative streaming, through overstability for high values of streaming velocities (or wave number for a given U_0) and through monotonic increase of amplitude for slow streaming.

B. <u>Perpendicular Propagation ($k \perp U_0$)</u>

In this case $\underline{k}.\underline{U}_0 = 0$, and the dispersion relation (47) gives

$$\omega^4 - \omega^2\left[2a_x^2 + k^2(S_1^2+S_2^2) - 4\pi G(m_1 N_{01} + m_2 N_{02})\right] + a_x^2\left[a_x^2 + k^2(S_1^2+S_2^2) - 4\pi G(m_1 N_{01} + m_2 N_{02})\right] + k^4 S_1^2 S_2^2 - 4\pi G(m_1 N_{01} S_2^2 + m_2 N_{02} S_1^2) = 0 \qquad (53)$$

This equation for identical gravitating streams leads to

$$(\omega^2 - k^2S^2 - 4\Omega^2)\left[\omega^2 - (k^2S^2 + 4\Omega^2 - 8\pi G m N_0)\right] = 0 \qquad (54)$$

With equilibrium rotation as defined by equation (11) the equation (54) gives two stable modes.

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