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COLLECTIVE BREMSSTRAHLUNG EMISSION FROM PLASMAS CONTAINING ENERGETIC PARTICLE FLUXES

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ABSTRACT

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We have calculated the bremsstrahlung emitted from thermal plasmas which co-exist with a flux of energetic (suprathermal) electrons. We find that under some circumstances the radiation emitted can be greatly increased compared to the emission from a Maxwellian plasma with no energetic particles present. The enhanced emission occurs at the fundamental and second harmonic of the electron plasma frequency.

1. INTRODUCTION

There are many astrophysical and laboratory plasmas which consist of a "thermal" plasma co-existing with a flux of energetic particles. The energetic particles may be distributed isotropically in velocities, or possess a net streaming motion with respect to the "thermal" plasma. We have calculated the bremsstrahlung emission from such plasmas using several different non-relativistic velocity distribution functions which describe stable situations. We find that under some circumstances the bremsstrahlung emitted at ω_e and $2\omega_e$ (ω_e is the electron plasma frequency) can be enhanced by several orders of magnitude compared to the thermal emission from a Maxwellian plasma with no energetic particles present.

In the following calculations we make use of an expression for the emission of electromagnetic radiation by a field-free homogeneous plasma of ions and electrons previously derived by Dupree[1,2] (these references will be referred to as I and II throughout this paper). The formula involves integrals over products of spectral densities, $S_{ee}(\underline{k},\omega)$, $S_{ii}(\underline{k},\omega)$, etc., (see equation 2) for the fluctuating number densities of the electrons and ions. The complicated integrations involved in obtaining the emission from equation 2 have so far proved too difficult to carry out exactly, so that in order to convert (2) into something more transparent for some particular plasmas we have made a number of approximations based on the behavior of the spectral densities $S_{\mu\nu}(\underline{k},\omega)$ as functions of $\underline{k}$ and ω. In particular we note that for wave numbers $|\underline{k}| < k_D$, where k_D is the Debye wave number, the spectral density S_{ee} has resonances at $\omega \cong 0$ and ω_e, and S_{ii} has a resonance at $\omega \cong 0$. These resonances correspond to a spectrum of longitudinal electron plasma oscillations and ion waves excited in the medium in the sense that they represent Fourier components of the spectral densities for which ω and $\underline{k}$ are approximately related as they would be for propagating Vlasov ion and electron plasma waves. It turns out to be convenient to divide the contributions to the bremsstrahlung into two parts: a part from the range $|\underline{k}| > k_D$ (which we call the collisional contribution), and a part from $|\underline{k}| < k_D$ (wave contribution).

Now for a plasma in thermal equilibrium the collisional contribution to the radiation at frequency Ω is given by approximately (in the notation of II),

$$\left(\frac{dU}{dt}\right)^{coln}_{emiss} = \frac{\omega_e^6 m}{6\pi^2 n_o \Omega^2 V_e} \ln\left(\frac{k_o V_e}{\omega_L}\right) \tag{1}$$

where V_e is the electron thermal velocity, $k_o = KT/e^2$, and ω_L is the larger of Ω or ω_e. The wave-emission part in this case is represented by two small resonances at $\Omega \cong \omega_e$ and $2\omega_e$ with a negligible area under them.

If, on the other hand, we consider a plasma containing a flux of energetic particles, it turns out that the wave emission part of the spectrum - represented by two emission lines at ω_e and $2\omega_e$ - can become sufficiently enhanced that it becomes the dominant feature of the spectrum. One can visualize the suprathermal particles as driving the wave field part of the longitudinal fluctuation spectrum up to a high amplitude through a process of Cerenkov emission of electron plasma oscillations. These electrostatic waves, or components of the fluctuation spectrum, then "collide" with each other and with low frequency ion density fluctuations and emit electromagnetic radiation.

This enhanced radiation however depends sensitively on the velocity distribution of the energetic particles since they also re-absorb (through Landau damping) the Cerenkov electron plasma oscillations as well as emit them. We shall see that the most radiative plasmas are those for which the velocity distribution of the suprathermal particles in some given direction has a small derivative in velocity space. This minimizes the Landau damping of the electron plasma oscillations driven by the energetic particles, and consequently increases the height of the resonance in $S_{ee}(\underline{k},\omega)$ at $\omega \cong \omega_e$.

2. BASIC EQUATIONS

Consider the energy $U(\underline{K})$ in a transverse (radiation) mode of wave-number $\underline{K}$ in a plasma. Dupree's result (II 7.7) for the rate at which energy is emitted into this mode can be written as,

$$\left(\frac{dU}{dt}\right)_{emiss} = \sum_{\mu,\gamma,\alpha,\beta} n_\alpha q_\alpha n_\beta q_\beta \left(\frac{\omega_\mu^2 \omega_\gamma^2}{\Omega^2}\right) \int_{-\infty}^{\infty} \frac{d\underline{k}}{(2\pi)^3} \frac{1}{k^2} \int_{-\infty}^{\infty} \frac{d\omega}{(2\pi)} \cdot$$

$$\left\{ \frac{|\underline{k}\cdot\underline{\varepsilon}|^2}{k^2} S_{\mu\gamma}(\underline{K}-\underline{k},\omega)\, S_{\alpha\beta}(\underline{k},\Omega-\omega) + \frac{\underline{k}\cdot\underline{\varepsilon}^* \,[(\underline{K}-\underline{k})\cdot\underline{\varepsilon}]}{|\underline{K}-\underline{k}|^2} S_{\alpha\gamma}(\underline{k},\omega)\, S_{\mu\beta}(\underline{K}-\underline{k},\Omega-\omega) \right\} \quad (2)$$

where the summations are over charged species of number density n_α and charge q_α and plasma frequencies $\omega_\alpha = (4\pi n_\alpha q_\alpha^2/m_\alpha)^{1/2}$. The quantity $\underline{\varepsilon}$ is the electric field for an electromagnetic wave of unit energy density (i.e., $\underline{\varepsilon}$ is the polarization vector with normalization $\underline{\varepsilon}\cdot\underline{\varepsilon}^* = 2\pi$), and Ω and $\underline{K}$ are the frequency and wave-number of the emitted electromagnetic wave.

The functions $n_\alpha n_\beta S_{\alpha\beta}(\underline{k},\omega)$ are spectral densities for the fluctuating <u>number</u> densities, $n_\alpha \delta\rho_\alpha$, of the various components. They are defined as the Fourier transforms of autocorrelation functions for the normalized density fluctuations,

$$S_{\alpha\beta}(\underline{k},\omega) = \int_{-\infty}^{\infty} d\underline{x}\; e^{-i\underline{k}\cdot\underline{x}} \int_{-\infty}^{\infty} dt\; e^{-i\omega t} \left\langle \delta\rho_\alpha(\underline{X},T)\; \delta\rho_\beta(\underline{X}+\underline{x},T+t) \right\rangle . \quad (3)$$

It should be noted in equation (7.7) of reference 2 that $(dU/dt)_{emiss}$ is given in terms of a Laplace transform in time plus its complex conjugate. If one makes use of the relations (in the notation of II),

$$\left\langle \delta\rho_\alpha \; \delta\rho_\beta | \underline{k}, t \right\rangle^* = \left\langle \delta\rho_\alpha \; \delta\rho_\beta | -\underline{k}, t \right\rangle$$
$$= \left\langle \delta\rho_\beta \; \delta\rho_\alpha \; | \; \underline{k}, -t \right\rangle \quad , \tag{3b}$$

the two complex conjugate terms of II 7.7 can be combined to give the above equation (2) in terms of the Fourier transforms $S_{\alpha\beta}(\underline{k},\omega)$.

The spectral densities $S_{\alpha\beta}$ can be expressed solely in terms of the one-particle distribution functions, f_α , (see the Appendix). For a homogeneous plasma free of external magnetic or electric fields they become,

$$S_{\alpha\beta}(\underline{k},\omega) = \Gamma_{\alpha\beta}(\underline{k},i\omega) + \Gamma^*_{\beta\alpha}(\underline{k},i\omega)$$
$$+\left(\frac{n_\beta q_\beta}{n_\alpha q_\alpha} \frac{L_\alpha}{D} + \delta_{\alpha\beta}\right) U_\beta n_\beta^{-1} + \left(\frac{n_\alpha q_\alpha}{n_\beta q_\beta} \frac{L^*_\beta}{D^*} + \delta_{\beta\alpha}\right) U^*_\alpha n_\alpha^{-1} \quad , \tag{4}$$

where

$$\Gamma_{\alpha\beta} = \frac{1}{n_\alpha q_\alpha} \frac{L_\alpha}{D} \frac{1}{n_\beta q_\beta} \sum_\nu n_\nu q_\nu^2 \left[\frac{L^*_\beta}{D^*} + \delta_{\beta\nu}\right] U^* \quad , \tag{5}$$

$$U_\alpha(\underline{k},s) = \int_{-\infty}^{\infty} d\underline{v} \; \frac{f_\alpha(\underline{v})}{(s+i\underline{k}\cdot\underline{v})} \quad , \quad L_\alpha(\underline{k},s) = -\omega_\alpha^2 \int_{-\infty}^{\infty} d\underline{v} \; \frac{f_\alpha(\underline{v})}{(s+i\underline{k}\cdot\underline{v})^2} \quad ,$$

$$\mathrm{Re}(s) > 0 \; . \tag{6}$$

and D is the Landau denominator (or longitudinal dielectric constant)

$$D(\underline{k},s) = 1 - \sum_\mu L_\mu(\underline{k},s) \; . \tag{7}$$

The integrals in equation (6) are defined for $\mathrm{Re}(s) > 0$ and the functions U_α, L_α represent their continuations throughout the s-plane. It should also be noted that the arguments of all the functions on the right side of (4) and throughout (5) are $(\underline{k}, i\omega)$.

Equations (2)-(6) are our basic equations and will be used to compute $(dU/dt)_{emiss}$ for various distribution functions $f_\alpha(\underline{v})$. We shall include in the functions f_α both the thermal and the suprathermal particles.

The way in which these fluctuations "scatter" off each other and radiate can be represented diagramatically (Fig. 1). Thus consider the first term in (2) which involves a convolution of $S_{\mu\gamma}$ and $S_{\alpha\beta}$. The frequencies and wave-numbers add to give the frequency and wave number of the final electromagnetic wave.

For the particular case of an electron-ion plasma (subscripts e, i) for which $q_e = -q_i$, $n_e = n_i = n_o$, the spectral densities S_{ee}, S_{ii}, and S_{ei} readily reduce to

$$\frac{n_o}{2} S_{ee}(\underline{k}, \omega) = \frac{\mathrm{Re}(U_e)}{|D|^2} |1 - L_i|^2 + \frac{\mathrm{Re}(U_i)}{|D|^2} |L_e|^2 \quad , \qquad (8)$$

$$\frac{n_o}{2} S_{ii}(\underline{k}, \omega) = \frac{\mathrm{Re}(U_i)}{|D|^2} |1 - L_e|^2 + \frac{\mathrm{Re}(U_e)}{|D|^2} |L_i|^2 \quad , \qquad (9)$$

$$\frac{n_o}{2} S_{ei}(\underline{k},\omega) = \frac{n_o}{2} S^*_{ie}(\underline{k},\omega) = -\frac{\mathrm{Re}(U_e)}{|D|^2} L^*_i(1-L_i) - \frac{\mathrm{Re}(U_i)}{|D|^2} L_e(1-L^*_e)$$

where the arguments for <u>all</u> the functions on the right of the above equations are $(\underline{k}, i\omega)$. In the frequency ranges of interest to us, we shall only be concerned with S_{ee} and S_{ii}.

It is also useful to note that if we define,

$$F_\alpha(u) = \int_{-\infty}^{\infty} d\underline{v}\, \delta\left(u - \frac{\underline{k}\cdot\underline{v}}{k}\right) f_\alpha(\underline{v}) \tag{10}$$

then

$$L_\alpha(\underline{k},s) = \frac{\omega_\alpha^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_\alpha}{\partial u} \frac{du}{\left(u - \frac{is}{k}\right)}, \quad \mathrm{Re}(s) > 0$$

and

$$\mathrm{Re}[U_\alpha(\underline{k}, i\omega)] = \frac{\pi}{k} F_\alpha\left(\frac{\omega}{k}\right) .$$

Equation (8) for S_{ee} is thus in agreement with that given by Rostoker and Rosenbluth[3].

In order to obtain the emission intensity, I, from (2) we must multiply $(dU/dt)_{emiss}$ by the density of states, $dn/d\Omega$, for the electromagnetic modes. Thus we define

$$\frac{dI(\underline{K})}{d\Omega} = \left(\frac{dU}{dt}\right)_{emiss} \frac{dn}{d\Omega(\underline{K})} \tag{12}$$

where $dI/d\Omega$ is the energy emitted per second per unit frequency interval per unit volume of plasma. For plasmas in which the propagation properties of radiation are isotropic and for frequencies $\Omega > \omega_e$ we have approximately $\Omega^2 \cong \omega_e^2 + K^2 c^2$, and accounting for two polarizations,

$$\frac{dn}{d\Omega} \cong \frac{\Omega(\Omega^2 - \omega_e^2)^{1/2}}{\pi^2 c^3} \tag{13}$$

Now in the following calculations we shall be interested in the bremsstrahlung at frequencies $\Omega <$ several ω_e. In this range $K \cong O(k_D V_e/c) << k_D$ where V_e is the electron thermal velocity. For this reason it is useful to expand the integrand of (2) in powers of $|\underline{K}|$. The result of doing this gives for the first two terms,

$$\left(\frac{dU}{dt}\right)_{emiss} = \left(\frac{dU}{dt}\right)_1 + \left(\frac{dU}{dt}\right)_2 + O(K^4) \tag{14}$$

where for an electron-ion plasma

$$\left(\frac{dU}{dt}\right)_1 = \frac{n_o^2 e^2}{2\pi} \int_{-\infty}^{\infty} \frac{d\underline{k}}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \frac{|\underline{k}\cdot\underline{\varepsilon}|^2 \omega_e^4}{\Omega^2 k^4} \left\{ S_{ee}(\underline{k},\omega)\; S_{ii}(-\underline{k},\Omega-\omega) - S_{ie}(-\underline{k},\omega)\; S_{ei}(\underline{k},\Omega-\omega) \right\} . \tag{15}$$

$$\left(\frac{dU}{dt}\right)_2 = \frac{n_o^2 e^2 \omega_e^4}{\pi\Omega^2} \int_{-\infty}^{\infty} \frac{d\underline{k}}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \, \frac{|\underline{k}\circ\underline{\varepsilon}|^2 (\underline{k}\circ\underline{K})^2}{k^8} \, S_{ee}(\underline{k},\omega) \quad S_{ee}(-\underline{k},\Omega-\omega)$$

$$+ \, 0\left(K^2 \frac{\delta\rho_i}{\delta\rho_e}\right) \quad . \qquad (16)$$

In making the expansion, the spectral density $S_{\mu\gamma}(\underline{K}-\underline{k},\omega)$ in the integrand of (2) is also Taylor expanded and the resulting term in $\partial S_{\mu\nu}/\partial\underline{k}$ can be integrated by parts making use of the symmetry of the integrand.

In a non-relativistic plasma the first term, $(dU/dt)_1$, dominates $(dU/dt)_2$ in the region $\Omega <$ several ω_e, except in the neighbourhood $\Omega \cong 2\omega_e$. At this particular frequency $(dU/dt)_2$ dominates since it contains the effect of the wave-wave scattering of electron plasma oscillations. For $\Omega \cong 2\omega_e$, the ion number density fluctuations involved in (16) are such that $\delta\rho_i \cong 0\left(\frac{m}{M}\,\delta\rho_e\right)$. Thus we shall only retain the term involving the product $S_{ee}\ S_{ee}$ in $(dU/dt)_2$ in the following calculations.

3. ISOTROPIC DISTRIBUTIONS

In this section we shall consider several isotropic distributions $f_{e,i}(|\underline{v}|)$ which have high energy tails on them representing an isotropic flux of energetic particles in a "thermal" plasma. For such distributions the angular parts of the $d\underline{k}$ integrations can be carried out since the spectral densities become functions of $k = |\underline{k}|$. They will be written as $S_{\alpha\beta}(k,\omega)$. Further, noting that $|\underline{\varepsilon}|^2 = 2\pi$ and $\underline{K}\circ\underline{\varepsilon} = \underline{K}\circ\varepsilon^* = 0$, equations (15) and (16) reduce to

$$\left(\frac{dU}{dt}\right)_1 = \frac{8n_o^2e^2}{3(4\pi)^2}\int_o^\infty dk\,\frac{\omega_e^4}{\Omega^2}\int_{-\infty}^\infty d\omega\left\{S_{ee}(k,\omega)\quad S_{ii}(k,\Omega-\omega)\right.$$

$$\left.- S_{ie}(k,\omega)\quad S_{ei}(k,\Omega-\omega)\right\} \tag{17}$$

and

$$\left(\frac{dU}{dt}\right)_2 = \frac{4n_o^2e^2\omega_e^4K^2}{15\Omega^2(2\pi)^2}\int_o^\infty \frac{dk}{k^2}\int_{-\infty}^\infty d\omega\; S_{ee}(k,\omega)\quad S_{ee}(k,\Omega-\omega) \tag{18}$$

It should be noted that we sometimes obtain a divergence at zero in the k integrals in (17) and (18). This is due to a breakdown of the expansion in K/k of the original integrand in (2). In such cases we shall cut off the k integral for $|\underline{k}| < K \ll k_D$, and neglect the contribution to $(dU/dt)_{1,2}$ from the small range of wave numbers $0 \leq |\underline{k}| \leq K$.

The following three distributions will be considered:

$$\text{(i)*}\qquad f_e = \frac{4V_e^3}{\pi^2(v^2+V_e^2)^3}\ ,\quad f_i = \frac{4V_i^3}{\pi^2(v^2+V_i^2)^3}\ , \tag{19}$$

$$\text{(ii)}\qquad f_i = \frac{1}{(2\pi)^{3/2}V_i^3}\exp\left(-\frac{v^2}{2V_i^2}\right) f_{iMax}\ ,$$

$$f_e = \frac{\beta}{(2\pi)^{3/2}V_e^3}\exp\left(-\frac{v^2}{2V_e^2}\right) + \frac{(1-\beta)}{4\pi V_e^2}\,\delta(|\underline{v}| - V_E)\ , \tag{20}$$

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*These functions have a finite energy density, but the next higher moment, i.e., the flux of energy in one direction across a surface, diverges. This does not alter the general behaviour of the spectral densities however.

with $0 < \beta < 1$, $(1-\beta) \ll 1$.

(iii)
$$f_i = f_{i\,\mathrm{Max}} \quad ,$$

$$f_e = \beta\, f_{e\,\mathrm{Max}} + (1-\beta)\, I_s(|\underline{v}| - V_E)\, g(|\underline{v}|) \quad , \qquad (21)$$

where

$I_s = 0$ if $|\underline{v}| < V_E$ and $= 1$ otherwise, also $V_E >$ several V_e and

$$\int_{-\infty}^{\infty} d\underline{v}\, g\, I_s = 1 \quad .$$

Note that all the distribution functions are normalized to unity. Also in the above three cases we have taken

$$\frac{V_e}{V_i} = \left(\frac{M}{m}\right)^{1/2} \quad ,$$

i.e., the thermal components of the electrons and ions in cases (ii) and (iii) have equal temperatures, and the kinetic temperatures of the complete distributions in (i) are also equal.

Case (i)

Using the distributions (19) the integrations in (4)-(9) are readily done by contours and we find for the Landau denominator,

$$D(\underline{k}, i\omega) = 1 + \frac{\omega_e^2(3kV_e + i\omega)}{(kV_e + i\omega)^3} + \frac{\omega_i^2(3kV_i + i\omega)}{(kV_i + i\omega)^3} \qquad (22)$$

By similar methods the spectral densities become,

$$\frac{n_o}{2}\,|D(\underline{k},i\omega)|^2\, S_{ee}(\underline{k},\omega) = \frac{2k^3V_e^3}{(\omega^2+k^2V_e^2)^2}\left|1 + \frac{\omega_i^2(3kV_i+i\omega)}{(kV_i+i\omega)^3}\right|^2 + \frac{2k^3V_i^3}{(\omega^2+k^2V_i^2)^2}\,\omega_e^4\left|\frac{3kV_e+i\omega}{(kV_e+i\omega)^3}\right|^2 \tag{23}$$

and

$$\frac{n_o}{2}\,|D(\underline{k},i\omega)|^2\, S_{ii}(\underline{k},\omega) = \frac{2k^3V_i^3}{(\omega^2+k^2V_i^2)^2}\left|1 + \frac{\omega_e^2(3kV_e+i\omega)}{(kV_e+i\omega)^3}\right|^2 + \frac{2k^3V_e^3}{(\omega^2+k^2V_e^2)^2}\,\omega_i^4\left|\frac{3kV_i+i\omega}{(kV_i+i\omega)^3}\right|^2 \tag{24}$$

If we define dimensionless variables $x_e = \omega/kV_e$, $x_i = \omega/kV_i$, and the Debye length $L = k_D^{-1} = V_e/\omega_e = V_i/\omega_i$, then $S_{ee}(k,\omega)$ can be written

$$\frac{\omega_e n_o}{2}\,kL|D|^2\, S_{ee}(k,\omega) = \frac{2}{(1+x_e^2)^2}\left[1 + \frac{3-x_i^4-6x_i^2}{k^2L^2(1+x_i^2)^3}\right]^2 + \frac{128x_i^2}{k^4L^4(1+x_i^2)^8} + \frac{2}{k^4L^4}\left(\frac{V_e}{V_i}\right)\frac{(9+x_e^2)}{(1+x_e^2)^3(1+x_i^2)^2}, \tag{25}$$

with

$$|D|^2 = \left[1 + \sum_{\alpha=e,i} \frac{(3-x_\alpha^4-6x_\alpha^2)}{k^2L^2(1+x_\alpha^2)^3} \right]^2 + \left[\sum_{\alpha=e,i} \frac{8x_\alpha}{k^2L^2(1+x_\alpha^2)^3} \right]^2 \qquad (26)$$

We have plotted the dimensionless quantity $n_o\omega_e\, S_{ee}(k,\omega)$ as a function of (ω/ω_e) for several values of (k/k_D) in figure 2. It is clear that for $k < k_D$ there is a sharp resonance at $\omega \cong \omega_e$ and a low-frequency resonance near $\omega \cong 0$. These resonances become more pronounced as k becomes smaller. One can readily verify that the width of the resonance at $\omega \cong \omega_e$ is, for small k, given by the Landau damping decrement, γ_L, for longitudinal electron plasma oscillations. This can be obtained by calculating the zeros of (22) for small k which gives,

$$\gamma_L \text{ (Resonance distribution)} = 4\omega_e \left(\frac{k}{k_D}\right)^3 . \qquad (27)$$

The corresponding damping decrement for a Maxwellian plasma is for small k,

$$\gamma_L(\text{Maxwellian}) = \omega_e\left(\frac{\pi}{8}\right)^{1/2} e^{-3/2} \left(\frac{k_D}{k}\right)^3 \exp\left(-\frac{k_D^2}{2k^2}\right)$$

so that the resonance at $\omega \cong \omega_e$ and $k < k_D$ in the function S_{ee} is much sharper for the Maxwellian case. Its width would not be resolvable in figure 2 for $k = .1\, k_D$.

The function S_{ii} is similar to S_{ee} at low frequencies, but does not possess a resonance at $\omega = \omega_e$.

Case (ii)

The integrations using the distributions (20) in (4)-(12) are again tedious but straightforward. The Landau denominator for this case becomes,

$$D(\underline{k},i\omega) = 1 - \frac{\beta\omega_e^2}{k^2V_e^2}\left[-1 + \frac{\omega}{kV_e}\left(R\left(\frac{\omega}{kV_e}\right) + i\,I\left(\frac{\omega}{kV_e}\right)\right)\right]$$

$$+ \frac{(1-\beta)\omega_e^2}{(k^2V_E^2-\omega^2)} - \frac{\omega_i^2}{k^2V_i^2}\left[-1 + \frac{\omega}{kV_i}\left(R\left(\frac{\omega}{kV_i}\right) + i\,I\left(\frac{\omega}{kV_i}\right)\right)\right] , \qquad (28)$$

where

$$I(x) = \left(\frac{\pi}{2}\right)^{1/2} \exp\left(-\frac{x^2}{2}\right) ,$$

$$R(x) = \frac{1}{\sqrt{2\pi}}\, P\int_{-\infty}^{\infty} dy\, \frac{e^{-y^2/2}}{(y+x)} \equiv e^{-x^2/2}\int_0^x e^{y^2/2}\, dy . \qquad (29)$$

We note for future reference the asymptotic expansions,

$$x\,R(x) = 1 + \frac{1}{x^2} + \ldots \qquad \text{(large } x\text{)} ,$$

$$x\,R(x) = x^2 - \frac{x^4}{2} + \ldots \qquad \text{(small } x\text{)} . \qquad (29a)$$

Further, the spectral densities for the distributions (20) become

$$\frac{n_o}{2}\,|D(\underline{k},i\omega)|^2\,S_{ee}(\underline{k},\omega) =$$

$$\left\{\frac{\beta}{kV_e}\,I\left(\frac{\omega}{kV_e}\right)+\frac{\pi(1-\beta)}{2kV_E}\,I_S\left(1-\left|\frac{\omega}{kV_E}\right|\right)\right\}\left|1-\frac{\omega_i^2}{k^2V_i^2}\left[-1+\frac{\omega}{kV_i}\left(R\left(\frac{\omega}{kV_i}\right)+i\,I\left(\frac{\omega}{kV_i}\right)\right)\right]\right|^2$$

$$+\frac{1}{kV_i}\,I\left(\frac{\omega}{kV_i}\right)\left|\frac{\beta\omega_e^2}{k^2V_e^2}\left[-1+\frac{\omega}{kV_e}\left(R\left(\frac{\omega}{kV_e}\right)+i\,I\left(\frac{\omega}{kV_e}\right)\right)\right]-\frac{(1-\beta)\omega_e^2}{(k^2V_E^2-\omega^2)}\right|^2,$$

and

$$\frac{n_o}{2}\,|D(\underline{k},i\omega)|^2\,S_{ii}(\underline{k},\omega) =$$

$$\frac{1}{kV_i}\,I\left(\frac{\omega}{kV_i}\right)\left|1-\frac{\beta\omega_e^2}{k^2V_e^2}\left[-1+\frac{\omega}{kV_e}\left(R\left(\frac{\omega}{kV_e}\right)+i\,I\left(\frac{\omega}{kV_e}\right)\right)\right]+\frac{\omega_e^2(1-\beta)}{(k^2V_E^2-\omega^2)}\right|^2$$

$$+\frac{\omega_i^4}{k^4V_i^4}\left|-1+\frac{\omega}{kV_i}\left(R\left(\frac{\omega}{kV_i}\right)+i\,I\left(\frac{\omega}{kV_i}\right)\right)\right|^2\left[\frac{\beta}{kV_e}\,I\left(\frac{\omega}{kV_e}\right)+\frac{\pi(1-\beta)}{2kV_E}\,I_S\left(1-\left|\frac{\omega}{kV_E}\right|\right)\right], \tag{30}$$

where I_S is a step function, $I_S(x) = 0$ if $x < 0$, $= 1$ if $x > 0$.

If we set $\beta = 1$, this reduces to the usual spectral density for a hydrogen plasma in thermal equilibrium[3,4].

Next suppose $\beta \neq 1$. Then it is clear from (20) that there is a shell (in velocity space) of monoenergetic electrons in the thermal plasma, and their contribution to S_{ee} is represented by the terms multiplied by $(1-\beta)$ in (28)-(30). Now an extremely important feature of this distribution is that the energetic electrons do not contribute to the imaginary part of D, as is clear by inspection of (28). Equivalently, they do not contribute to the Landau damping of electrostatic oscillations.

In figure 3 we have drawn $f_e(|\underline{v}|)$ and $\bar{f}_e(v_x) = \int_{-\infty}^{\infty} dv_y \, dv_z \, f_e(|\underline{v}|)$ for the distribution (20). The function $\bar{f}_e$ is simply

$$\bar{f}_e(v_x) = \frac{\beta}{V_e\sqrt{2\pi}} \exp\left(-\frac{v_x^2}{2V_e^2}\right) + \frac{(1-\beta)}{2V_E} I_S(-|v_x| + V_E) \qquad (31)$$

where I_S is the usual step function. For the range of phase velocities $0 < |\omega/k| < V_E$ it is again evident that the contribution to the Landau damping of waves by the fast particles is zero since the damping decrement, γ_L, when small is proportional to $\partial\bar{f}_e(v_x)/\partial v_x \Big|_{v_x = \omega/k}$. This fact can lead to a greatly enhanced level of fluctuation in(30) for the wave number range $k < k_D$ and $V_E > \omega/k >$ several V_e, due to the fact that the <u>energetic particles are Cerenkov-emitting electrostatic (longitudinal) waves but not contributing to their reabsorption by Landau damping</u>.

<u>Case (iii)</u>

The distribution (21) has similar properties to that discussed above for case (ii). Instead of a δ-function at V_E however we consider a continuous flux of energetic particles confined to the region $|\underline{v}| > V_E$. The gap in velocity space from several $- V_e \leq |\underline{v}| \leq V_E$ is a region where the only particles present are the small number of thermal particles in the Maxwellian tail. The result is again that electrostatic waves in the region of phase velocities several $- V_e \leq \omega/k \leq V_E$ suffer very little Landau damping. However they are driven in this region by the fast electrons and so give rise to an enhancement of those Fourier components of S_{ee} in the above phase velocity range. In figure 4 we have sketched $f_e(|v|)$ and $\overline{f}_e(v_x)$ for case (iii).

The distributions (21) lead simply to

$$D(\underline{k},i\omega) = 1 - \frac{\omega_i^2}{k^2V_i^2}\left[-1 + \frac{\omega}{kV_i}\left(R\left(\frac{\omega}{kV_i}\right) + i\ I\left(\frac{\omega}{kV_i}\right)\right)\right]$$

$$- \frac{\beta\omega_e^2}{k^2V_e^2}\left[-1 + \frac{\omega}{kV_e}\left(R\left(\frac{\omega}{kV_e}\right) + i\ I\left(\frac{\omega}{kV_e}\right)\right)\right] - L_{Ge} \qquad (32)$$

with

$$\frac{n_o}{2}|D(\underline{k},i\omega)|^2\ S_{ee}(\underline{k},\omega) =$$

$$\left[\frac{\beta}{kV_e}\ I\left(\frac{\omega}{kV_e}\right) + \frac{\pi}{k}\ F_{Ge}\left(\frac{\omega}{k}\right)\right]\left|1 - \frac{\omega_i^2}{k^2V_i^2}\left[-1 + \frac{\omega}{kV_i}\left(R\left(\frac{\omega}{kV_i}\right) + i\ I\left(\frac{\omega}{kV_i}\right)\right)\right]\right|^2$$

$$+ \frac{1}{kV_i}\ I\left(\frac{\omega}{kV_i}\right)\left|\frac{\beta\omega_e^2}{k^2V_e^2}\left[-1 + \frac{\omega}{kV_e}\left(R\left(\frac{\omega}{kV_e}\right) + i\ I\left(\frac{\omega}{kV_e}\right)\right)\right] + L_{Ge}\right|^2 \qquad (33)$$

and

$$\frac{n_o}{2} |D(\underline{k}, i\omega)|^2 \quad S_{ii}(\underline{k}, \omega) =$$

$$\frac{1}{kV_i} \; I\left(\frac{\omega}{kV_i}\right) \left| \; 1 - \frac{\beta\omega_e^2}{k^2V_e^2} \left[-1 + \frac{\omega}{kV_e} \left(R\left(\frac{\omega}{kV_e}\right) + i \; I\left(\frac{\omega}{kV_e}\right) \right) \right] - L_{Ge} \right|^2$$

$$+ \; \frac{\omega_i^4}{k^4V_i^4} \left[\frac{\beta}{kV_e} \; I\left(\frac{\omega}{kV_e}\right) + \frac{\pi}{k} \; F_{Ge}\left(\frac{\omega}{k}\right) \right] \left| -1 + \frac{\omega}{kV_i} \left(R\left(\frac{\omega}{kV_i}\right) + i \; I\left(\frac{\omega}{kV_i}\right) \right) \right|^2 . \quad (34)$$

The functions R and I are those defined in (29). Also,

$$F_{Ge}(u) = 2\pi(1-\beta) \int_{(V_E, \; u \;)}^{\infty} v \; dv \; g(v)$$

where the larger of V_E or u is used at the lower limit, and

$$L_{Ge}\left(\frac{\omega}{k}\right) = \frac{(1-\beta)\omega_e^2}{2} \int_{-\infty}^{\infty} \frac{du \; F'_{Ge}(u)}{(u - \frac{\omega}{k} + i \; 0)} \quad .$$

4. APPROXIMATIONS FOR ISOTROPIC DISTRIBUTIONS

By inspection of figure 2 and equation (23) for $S_{ee}(k,\omega)$ we see that for $k < k_D$ this function is highly peaked at the two frequencies $\omega \approx \omega_e$ and 0 . Further, for $k > k_D$ the peak at $\omega \cong \omega_e$ vanishes but that at the origin remains. For this latter range of k the function $S_{ee}(k,\omega)$ has two "shelves" as a function of ω. Roughly speaking it is nearly constant for $0 \leq \omega \leq k \; V_i$, then for $\omega > k \; V_i$ it drops off rapidly until it reaches a second nearly constant value throughout the range several $k V_i \lesssim \omega \lesssim k V_e$. This second shelf then drops towards zero for $\omega > k V_e$. One can verify that these features are also characteristic of the other distributions (20) and (21) provided $(1 - \beta) << 1$.

In the following calculations we concern ourselves with electromagnetic radiation of frequency Ω in the range $\bar{\omega} < \omega <$ few ω_e , where $\omega_i << \bar{\omega} < \omega_e$. Now (17) involves S_{ee} , S_{ii} , S_{ie} and S_{ei} with arguments ω and $\Omega - \omega$. Thus with Ω in the above range of frequencies either ω or $\Omega - \omega$ must be $O(\Omega)$. However for a frequency argument of $O(\Omega)$, the spectral densities are of relative magnitudes, $S_{ee} \doteq O(1)$, $S_{ie} \cong S_{ei} \doteq O\left(\frac{m}{M}\right)$ and $S_{ii} \cong O\left(\frac{m}{M}\right)^2$, since $M >> m$ and the ions do not contribute much to high frequency number density fluctuations. Further, at low frequencies $S_{ee}(\omega \cong 0) \cong S_{ii}(\omega \cong 0)$. Thus from inspection of (17) we see that the dominant contribution to $(dU/dt)_1$, derives from the first term for emission frequencies in the range $\bar{\omega} < \Omega <$ few ω_e. It comes from the integration frequencies ω in the region $\omega \cong \Omega$ so that the product $S_{ee}(k,\omega)$ $S_{ii}(k,\Omega-\omega) \doteq O(1)$ in the integral (17).

Now we find it useful to split the k integrations in (17) and (18) into two parts as follows,

$$\int_0^\infty dk \rightarrow \underset{\begin{pmatrix}\text{wave}\\ \text{part}\end{pmatrix}}{\int_0^{k_D} dk} + \underset{\begin{pmatrix}\text{collisional}\\ \text{part}\end{pmatrix}}{\int_{k_D}^\infty dk} \tag{35}$$

and make approximations for S_{ee} and S_{ii} appropriate to these two ranges. The first integral derives mainly from the resonances in S_{ee} and S_{ii} for which ω and k are approximately related as if the spectral density described a spectrum of propagating Vlasov electron plasma oscillations and ion waves. The second integral derives from a wide range of ω for a given k. We shall refer to the above contributions to the integral over k as the wave and collisional contributions respectively. In general the separation of (35) into two parts is somewhat arbitrary. However, as we shall see the large enhancement of radiation emission over the thermal value comes from a resonance in S_{ee} for $k \ll k_D$, and in this case the separation is well defined indeed!

"Collisional" Contribution $(k > k_D)$

Consider the integral

$$\int_{k_D}^{\infty} dk \int_{-\infty}^{\infty} d\omega \quad S_{ee}(k,\omega) \quad S_{ii}(k,\Omega-\omega)$$

$S_{ee}(k,\omega)$ is largest in the region $|\omega/k\,V_e| < 1$, i.e., $-k\,V_e \leq \omega \leq k\,V_e$. Also $S_{ii}(k,\Omega-\omega)$ is appreciable only if $|(\Omega-\omega)/k\,V_i| < 1$. This latter condition allows us to replace,

$$\int_{-\infty}^{\infty} d\omega \rightarrow \int_{\Omega-kV_i}^{\Omega+kV_i} d\omega \qquad ,$$

and the condition for S_{ee} to be appreciable becomes $\Omega \pm kV_i < kV_e$, i.e., approximately $k > \Omega/V_e$. Thus, we now evaluate the above integrals approximately as,

$$\int_{(\text{larger of } k_D,\ \frac{\Omega}{V_e})}^{k_o} dk \int_{(\Omega - kV_i)}^{(\Omega + kV_i)} d\omega \ S_{ee}(k,\omega)\ S_{ii}(k,\Omega-\omega) \qquad (36)$$

The upper limit k_o is the usual inverse distance of closest approach ($k_o \gg k_D$). (The wave part of the k integral in (35) does not need to be "cut off" and therefore does not suffer from this "defect" of the theory.)

Now consider case (ii) with $(1 - \beta) = 0$. Using (29a), (30) reduces in the above range to,

$$\frac{\omega_e n_o}{2} \ S_{ee}(k,\omega) \cong \left(\frac{\pi}{2}\right)^{1/2} \frac{k_D k^3}{(k^2 + k_D^2)^2} \ ,$$

$$\frac{n_o}{2} \ S_{ii}(k,\Omega-\omega) \cong \left(\frac{\pi}{2}\right)^{1/2} \frac{1}{kV_i} \ \frac{k^4}{(k^2 + 2k_D^2)^2} \ . \qquad (37)$$

It then readily follows from the first term of (17) that

$$\left(\frac{dU}{dt}\right)_{emiss}^{coln} \cong \frac{\omega_e^6 \, m}{6\pi^2 n_o \Omega^2 V_e} \ \ln\left(\frac{k_o V_e}{\omega_L}\right) \qquad (38)$$

where ω_L is the larger of ω_e or Ω.

Equation (38) is the result usually given[2,5,6,7] for the bremsstrahlung emission from a plasma in thermal equilibrium. This "collisional contribution" varies only by multiplicative factors of $O(1)$ if we depart from the Maxwellian case and consider the resonance distributions (19).

To (38) we must also add the "wave-emission" contribution from the sharp resonances in S_{ee} or S_{ii} at $\omega \cong 0$, ω_e, $k < k_D$. For thermal equilibrium these resonances do not give an appreciable contribution to the emission since they have a neglibible area under them. However for some non-thermal situations they give rise to a resonance emission in $(dU/dt)^{wave}_{emiss}$ at $\Omega \cong \omega_e$, $2\omega_e$, and these resonances may become the dominant contribution to the total emission. We shall next calculate these.

Wave-contribution to the Emission $(k < k_D)$

Case (i)

Consider the resonance for $k < k_D$, $\omega \cong \omega_e$. Expanding (22) in kV_e/ω, we have

$$D(\underline{k}, i\omega) \cong \left(1 - \frac{\omega_e^2}{\omega^2} - \frac{3k^2V_e^2}{\omega_e^2}\right) - \frac{8ik^3V_e^3}{\omega_e^3} \tag{39}$$

and from (23),

$$\frac{n_o}{2} |D|^2 \; S_{ee}(\underline{k}, \omega) \cong \frac{2k^3V_e^3}{\omega_e^4} \tag{40}$$

Thus

$$\frac{n_o}{2} S_{ee}(\underline{k},\omega)\Bigg|_{k<k_D,\omega\cong\omega_e} \cong \frac{2k^3V_e^3}{(\omega^2-\omega_e^2-3k^2V_e^2)^2 + \frac{64k^6V_e^6}{\omega_e^2}} \qquad (41)$$

Next, we observe that at the low frequency end, S_{ee} is nearly flat in the region $0 < \omega < kV_i$ and for $\omega > kV_i$ S_{ee} decreases as ω^{-4}. In the low frequency region the second term of (23) dominates. We shall, for the purpose of being able to carry through the integrations, represent S_{ee} in this region by a resonance function with the same approximate height and width as (23) by simply evaluating (23) at $\omega \cong 0$ and multiplying the result by $k^4V_i^4/(\omega^4 + k^4V_i^4)$. Thus a function that reproduces S_{ee} in the resonance regions - but not accurately away from resonance - is,

$$\frac{n_o}{2} S_{ee}(\underline{k},\omega) \cong \frac{k^3V_i^3}{2(\omega^4+k^4V_i^4)} + \frac{2k^3V_e^3}{(\omega^2-\omega_e^2-3k^2V_e^2)^2 + \frac{64k^6V_e^6}{\omega_e^2}} \qquad (42)$$

Similarly,

$$\frac{n_o}{2} S_{ii}(\underline{k},\omega) \cong \frac{k^3V_i^3}{2(\omega^4+k^4V_i^4)} \qquad (43)$$

Note, for low frequency fluctuations, $S_{ee} \cong S_{ii}$.

Now consider the ω integrations in (17) and (18). If we neglect terms of order the line widths of the resonances we can for the purpose of integration further simplify things by writing δ-functions in (42) and (43) of equivalent weight to the areas under the resonances. Thus noting $k > 0$,

$$\frac{n_o}{2} S_{ee}(\underline{k},\omega) \cong \frac{\pi}{2}\delta(\omega) + \frac{\pi}{8}\left[\delta\left(\omega+\omega_e+\frac{3k^2V_e^2}{2\omega_e}\right)+\delta\left(\omega-\omega_e-\frac{3k^2V_e^2}{2\omega_e}\right)\right] , \quad (44)$$

$$\frac{n_o}{2} S_{ii}(\underline{k},\omega) \cong \frac{\pi}{2}\delta(\omega) , \quad (45)$$

and $\delta(\omega)$ is interpreted so that $\int_0^\infty \delta(\omega)\, d\omega = \frac{1}{2}$.

Equation (45) can also be viewed as an approximation based on taking the limit $M \to \infty$ for the ion mass. In this limit,

$$S_{ii} = \left\langle \delta\rho_i\ \delta\rho_i | \underline{k} \right\rangle 2\pi\ \delta(\omega) ,$$

where $\left\langle \delta\rho_i\ \delta\rho_i | \underline{k} \right\rangle$ is the <u>single</u> <u>time</u> density correlation function. A calculation of $\left\langle \delta\rho_i\ \delta\rho_i | \underline{k} \right\rangle$ will show that for $k < k_D$ is differs from $1/2n_o$ only by multiplicative factors of $O(1)$. Our final results for the emission will be similarly limited in accuracy.

It now readily follows from (17) and (18) that for the frequency range of interest,

$$\left(\frac{dU}{dt}\right)_1^{(wave)} \cong \frac{e^2\omega_e^2}{24} \int_0^{k_D} dk \left[\delta\left(\Omega-\omega_e - \frac{3k^2V_e^2}{2\omega_e}\right) + \delta\left(\Omega+\omega_e + \frac{3k^2V_e^2}{2\omega_e}\right)\right] , \quad (46)$$

$$\left(\frac{dU}{dt}\right)_2^{(wave)} \cong \frac{4n_o^2e^2\omega_e^4K^2}{15\Omega^2(2\pi)^2} \int_K^{k_D} \frac{dk}{k^2} \int_{-\infty}^{\infty} d\omega \; S_{ee}(\underline{k},\omega)\, S_{ee}(\underline{k},\Omega-\omega)$$

$$\cong \frac{e^2\omega_e^4K}{240\Omega^2} \left[\delta(\omega + 2\omega_e) + \delta(\Omega - 2\omega_e)\right] \quad (47)$$

The contribution to $\Omega \cong \pm\omega_e$ in $(dU/dt)_2^{(wave)}$ is neglected as small compared to $(dU/dt)_1^{(wave)}$.

If we next compute the total emission using (12) and (13) the contributions of the "wave-emission" to radiation at $\Omega \cong \omega_e$ and $\Omega \cong 2\omega_e$ become,

$$I_{\omega_e}^{(wave)} = \int d\Omega \left(\frac{dU}{dt}\right)_1^{(wave)} \frac{dn}{d\Omega}$$

$$= \frac{\sqrt{3}}{96\pi^3} \left(\frac{mV_e^2}{n_oL^3}\right) \left(\frac{V_e}{c}\right)^3 \frac{\omega_e}{L^3} \; \text{ergs/sec/cm}^3, \quad (48)$$

$$I_{2\omega_e}^{(wave)} = \frac{1}{320\pi^3} \left(\frac{mV_e^2}{n_oL^3}\right) \left(\frac{V_e}{c}\right) \frac{\omega_e}{L^3} , \quad (49)$$

where $L = k_D^{-1}$ is the Debye length.

It is interesting at this point to compare this with the corresponding emission at ω_e and $2\omega_e$ from a Maxwellian plasma. Similar approximations to those made above are carried out using (28)-(30) with $\beta=1$ and we find,

$$\frac{I_{2\omega_e}^{(wave)}\text{ (Resonance distribution)}}{I_{2\omega_e}^{(wave)}\text{ (Maxwellian)}} = \frac{\sqrt{3}}{16}\left(\frac{c}{V_e}\right) ,$$

$$\frac{I_{\omega_e}^{(wave)}\text{ (Resonance distribution)}}{I_{\omega_e}^{(wave)}\text{ (Maxwellian)}} = \frac{1}{4C} ,$$

where C is a constant of order unity and is defined following equation (55). Note, the emission contributions $I_{\omega_e,2\omega_e}^{(wave)}$ (Max) are in addition to the collisional or continuous spectrum part (38). We see that although there are differences between the resonance and Maxwellian distributions, they are not spectacular differences.

Cases (ii) and (iii)

The distributions (20) are a special case of (21), so we shall here consider case (iii) and later specialize the results for case (ii).

First, in the spectral density S_{ee} given by (33), we observe that the energetic particles contribute to the enhancement of fluctuations with $V_E \geq \frac{\omega}{k} >$ several V_e, but not to their Landau damping as represented in equation (32) for D. This range of phase velocities contributes most to the wave-emission part of the radiation and we shall neglect other contributions and make approximations to S_{ii} and S_{ee} appropriate to this range.

Now consider the resonance in $S_{ee}(\underline{k},\omega)$ in the neighbourhood $\omega \cong \omega_e$. Recalling that $I(x) = \sqrt{\pi/2}\ \exp(-x^2/2)$, we shall assume that throughout the range $V_E > \frac{\omega}{k} >$ several V_e, $\omega \cong O(\omega_e)$, $k < k_D$,

$$1 >> (1-\beta) >> I\left(\frac{\omega}{kV_e}\right) >> I\left(\frac{\omega}{kV_i}\right) \cong 0 . \qquad (50)$$

If we also make use of the asymptotic expansions (29a), equation (33) becomes,

$$\frac{n_o}{2}|D|^2 S_{ee}(\underline{k},\omega)\Big|_{\omega\cong\omega_e} \cong \frac{\pi}{k} F_{Ge}\left(\frac{\omega}{k}\right) ,$$

$$D(\underline{k},i\omega)\Big|_{\omega\cong\omega_e} \cong \left(1 - \frac{\omega_e^2}{\omega^2} - \frac{3k^2V_e^2}{\omega_e^2}\right) - i\,\frac{\omega_e^2}{k^2V_e^2}\ I\left(\frac{\omega}{kV_e}\right) . \qquad (51)$$

Next, define a dimensionless integral over the energetic particles,

$$G(V_E) = \int_{V_E}^{\infty} V_E\ v\ dv\ g(v) \qquad (52)$$

Then neglecting terms of order the line breadth of the resonance at $\omega \cong \omega_e$, the function $S_{ee}(\underline{k},\omega)$ can be replaced in the integrals by the following δ-functions,

$$S_{ee}(\underline{k},\omega)\Big|_{\omega\cong\omega_e} \cong \frac{2\pi^3 kV_e^2}{n_o\omega_e V_E}\ \frac{(1-\beta)G}{I\left(\frac{\omega_e}{kV_e}\right)} \left[\delta\left(\omega - \sqrt{\omega_e^2+3k^2V_e^2}\right) + \delta\left(\omega + \sqrt{\omega_e^2+3k^2V_e^2}\right)\right] \qquad (53)$$

We also require an approximate form for $S_{ee}(\underline{k},\omega) \cong S_{ii}(\underline{k},\omega)$ at $\omega \cong 0$. In this case these functions have a weak maximum near the origin at $\omega \cong kV_i$. Thus we proceed in a similar manner as before by evaluating (34) at $\omega = kV_i$ and multiplying the result by $k^4V_i^4[(\omega^2-k^2V_i^2)^2 + k^4V_i^4]$ so that the "equivalent resonance" has the same height and width as the true one at $\omega \cong kV_i$. In this case since $(1-\beta) << 1$ and using the asymptotic expansions (29a),

$$S_{ii}(\underline{k},\omega)\Big|_{\omega\cong kV_i} \cong \frac{2I(1)}{n_o[(2-R(1))^2+I^2(1)]} \; \frac{k^3V_i^3}{[(\omega^2-k^2V_i^2)^2 + k^4V_i^4]} \tag{54}$$

Note that the existence of a tenuous flux of energetic electrons in the medium does not appreciably alter the low frequency resonance in S_{ii}.

For the purposes of integration if we again neglect terms of order the line breadths we can write,

$$\frac{n_o}{2} S_{ii}(\underline{k},\omega) \cong \pi\ C\ \delta(\omega) \quad , \tag{55}$$

where the δ-function is defined as in (45) and $C = I(1)/[(2-R(1))^2 + I^2(1)]$.

As we pointed out in the paragraph following equation (45) we can also regard (55) as the result of taking $M \to \infty$ in which case $S_{ii} = \langle \delta\rho_i\ \delta\rho_i | k \rangle\ 2\pi\ \delta(\omega)$ which would lead to a value for the constant $C = n_o \langle \delta\rho_i\ \delta\rho_i | k \rangle$. In this case the well known equilibrium correlation function including self correlation is

$$\left\langle \delta\rho_i \; \delta\rho_i | k \right\rangle = \frac{1}{n_o} \frac{(k^2 + k_D^2)}{(k^2 + 2k_D^2)}$$

Thus for small $k < k_D$, $C \cong 1/2$ in this approximation. This is to be compared with $I(1)/[(2-R(1))^2 + I^2(1)] \cong .35$ which we obtained from our approximate representation (54) for S_{ii}.

It now follows that

$$\left(\frac{dU}{dt}\right)_1^{(wave)} \cong \frac{2G\pi^2 e^2 \omega_e C V_e^2 (1-\beta)}{3V_E} \int_{k_D\left(\frac{V_e}{V_E}\right)}^{k_D} \frac{k\,dk}{I\left(\frac{\omega_e}{kV_e}\right)} \left[\delta\left(\Omega - \sqrt{\omega_e^2 + 3k^2 V_e^2}\right) + \delta\left(\Omega + \sqrt{\omega_e^2 + 3k^2 V_e^2}\right)\right] , \tag{56}$$

$$\left(\frac{dU}{dt}\right)_2^{(wave)} \cong \frac{G^2 e^2 K^2 \pi^4 V_e^4 (1-\beta)^2}{15 V_E^2} \int_{k_D\left(\frac{V_e}{V_E}\right)}^{k_D} \frac{dk}{I^2\left(\frac{\omega_e}{kV_e}\right)} \left[\delta(\Omega - 2\omega_e) + \delta(\Omega + 2\omega_e)\right] \tag{57}$$

The k integrals have been cut off at a lower limit such that $\omega_e/k = V_E$ i.e., $k = k_D(V_e/V_E)$, since the spectral densities calculated do not apply in the range $\omega_e/k > V_E$ where Landau damping due to the fast electrons will reduce the level of fluctuation.

Multiplying (56) and (57) by the density of states (13), noting $|\underline{K}| = (\Omega^2 - \omega_e^2)^{1/2}/c$, and integrating over $d\Omega$, we obtain the emission intensities at the first and second harmonics,

$$I_{\omega_e}^{(wave)} \cong \frac{C(1-\beta)}{\pi\sqrt{3}} \; G \; J_1\left(\frac{mV_e^2}{n_o L^3}\right)\omega_e \left(\frac{V_e}{c}\right)^3 \frac{1}{L^3}\left(\frac{V_e}{V_E}\right) , \tag{58}$$

$$I_{2\omega_e}^{(wave)} \cong \frac{\pi\sqrt{3}(1-\beta)^2}{5} \; G^2 \; J_2\left(\frac{mV_e^2}{n_o L^3}\right)\omega_e \left(\frac{V_e}{c}\right)^3 \frac{1}{L^3}\left(\frac{V_e}{V_E}\right)^2 , \tag{59}$$

where

$$J_1 = \int_{k_D\left(\frac{V_e}{V_E}\right)}^{k_D} \frac{k^2 dk}{k_D^3} \left[I\left(\frac{k_D}{k}\right)\right]^{-1} \cong \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{V_e}{V_E}\right)^5 \exp\left(\frac{V_E^2}{2V_e^2}\right) ,$$

$$J_2 = \int_{k_D\left(\frac{V_e}{V_E}\right)}^{k_D} \frac{dk}{k_D} \left[I\left(\frac{k_D}{k}\right)\right]^{-2} \cong \frac{1}{\pi}\left(\frac{V_e}{V_E}\right)^3 \exp\left(\frac{V_E^2}{V_e^2}\right) , \tag{60}$$

and the approximations to J_1, J_2 follow from the inequality $\exp(V_E^2/2V_e^2) >> 1$.

It is clear that the emission of bremsstrahlung at ω_e and $2\omega_e$ represented by (58) and (59) can be many orders of magnitude larger than the thermal (Maxwellian) level for these two haromics. The exponential factors in (60) control this situation. The distribution (21) is also much more highly radiative than (19) for example although both distributions represent plasmas containing suprathermal electrons.

The basic reason for this is due to the gap in velocities $V_e < |\underline{v}| < V_E$ which only the few Maxwellian tail particles occupy. Longitudinal Cerenkov waves (included in S_{ee}) are emitted into the phase velocity range $V_e < \frac{\omega}{k} < V_E$ by the energetic electrons, but only reabsorbed (Landau damped) by the few thermal particles far out on the Maxwellian tail.

In applying (58), (59) to physical situations the following conditions should be noted. The fluctuation densities $S_{\alpha\beta}$ were calculated assuming an infinite homogeneous plasma. In such a plasma there is an equilibrium established between the emission and re-absorption of electrostatic (longitudinal) waves for the small wave-number part of the spectral density. This can only be the case for a bounded plasma if the propagation length $\ell = (\omega/k)\gamma_L^{-1}(k)$ for a Fourier component of wave-number k, is less than the smallest linear dimension L_S of the plasma, where γ_L is the Landau damping decrement. The first condition for the applicability of (58)-(60) to a bounded plasma is thus,

$$L_S \gg L \left[e^{3/2} \sqrt{\frac{8}{\pi}} \left(\frac{V_e}{V_E}\right)^2 \exp\left(\frac{V_E^2}{2V_e^2}\right) \right] \tag{61a}$$

where L is the Debye length.

Also (58) and (59) will not be valid if the fluctuation spectrum $S_{\alpha\beta}$ is so large as to invalidate the perturbation theory (in powers of the plasma parameter $g = (n_o L^3)^{-1} \ll 1$) which underlies (2), (4), and (5). From inspection of the expressions for $S_{ee}(\omega \cong \omega_e)$ we see that the expansion remains valid provided $\gamma_L \gg O(g)$. From physical considerations one would expect the resonance expressions for S_{ee} to be correct if $\gamma_L > \nu_c$ where ν_c is the electron collision frequency. In this case Landau damping rather than correlation (collisional) damping is primarily responsible for the re-absorption of electron plasma oscillations emitted by the energetic electrons in the plasma. However ν_c <u>is</u> an $O(g)$ quantity, so that the above requirement can be written $\gamma_L \gg \nu_c$, i.e., using the Spitzer collision frequency and neglecting terms $O(1)$ this condition becomes,

$$\left(\frac{V_E}{V_e}\right)^3 \exp\left(-\frac{V_E^2}{2V_e^2}\right) \gg \frac{1}{n_o L^3} \ln (n_o L^3) \quad . \tag{61b}$$

5. BREMSSTRAHLUNG FROM STABLE ELECTRON STREAMS IN A PLASMA.

In this section we are concerned with the case of an electron beam traversing a Maxwellian plasma of electrons and ions. The beam electrons are assumed to have a spread of velocities such that the distributions are <u>stable</u>, which of course is a necessary condition for applicability of the emission formula (2).

The following two cases are considered:

Case (i)

$$f_i = \delta(\underline{v} + \underline{U}_i) \text{ , } M \to \infty \text{ ,}$$

$$f_e = \beta\left(\frac{1}{2\pi V_e^2}\right)^{3/2} \exp\left(-\frac{v^2}{2V_e^2}\right) + \frac{(1-\beta)}{(2\pi v_1^2)^{3/2}} \exp - \frac{(\underline{v}+\underline{U}_e)^2}{2v_1^2} \text{ ,} \tag{62}$$

where

$$\underline{U}_i = \underline{U}_e(1-\beta) \text{ ,} \tag{63}$$

$$(1 - \beta) << 1 \text{ ,} \tag{64}$$

$$\left(\frac{v_1}{V_e}\right)^2\left(\frac{U_e}{V_e}\right) \exp\left(-\frac{U_e^2}{2V_e^2}\right) - (1 - \beta) = \delta > 0 \text{ .} \tag{65}$$

Equation (63) is the condition that there is no current in the plasma, and (65) the condition for the electron beam of density $(1 - \beta)$ to be stable. The above case is identical to that considered by Rostoker[8] in a related calculation of the coulomb energy density in wave-number space for a fluctuating plasma.

If we increase U_e to the point where $\delta \to 0$ in (65) then we reach a situation in which one wave number $\underline{k}$ parallel to $\underline{U}_e$ and of magnitude $\cong \omega_e/U_e$ first becomes unstable. For this unstable wave number the spectral density S_{ee} diverges. However the radiation emitted as given by (16) involves integrals over $\underline{k}$ and ω. We can approximate these if δ is small to estimate the contribution from the resonance at the nearly unstable wave-number. However it turns out that these integrals are finite as $\delta \to 0$, and represent an uninteresting change in the bremsstrahlung - i.e., there is no spectacular increase in the emission such as that represented by equations (58) and (59).

The reason appears to be that only a single wave number is first unstable. All the rest are Landau damped. For enhanced emission we require that a range of wave-numbers are bordering on instability - or at least that a range of wave-numbers have an extremely small Landau damping. We shall next treat an example of this latter case in more detail.

Case (ii)

Consider the distribution

$$f_e = \beta f_{eMax} + \frac{(1-\beta)}{V_E} I_0 \delta(v_y) \delta(v_z) \quad , \tag{66}$$

where we have set $M \to \infty$, i.e., the ions form a uniform positive background with a small amount of drift velocity to satisfy the zero-current condition. The function $I_0(v_x)$ will be chosen as a step-function $I_0 = 1$ for $0 \leq v_x \leq V_E$ and $I_0 = 0$ otherwise.

The spectral density for the electrons then readily follows as,

$$\frac{n_o}{2}\ S_{ee}(\underline{k},\omega) = \frac{\pi}{k|D|^2}\left\{\frac{\beta}{V_e\sqrt{2\pi}}\ \exp\left(-\frac{\omega^2}{2k^2V_e^2}\right) + \frac{k(1-\beta)}{k_x\ V_E}\ I_0\left(\frac{\omega}{k_x}\right)\right\}$$

$$|D|^2 = \left\{1 + \frac{\beta\omega_e^2}{k^2V_e^2}\left(1 - \frac{\omega}{kV_e}\ R\left(\frac{\omega}{kV_e}\right)\right) - \frac{\omega_e^2(1-\beta)}{\omega(\omega+k_xV_E)}\right\}^2 + \frac{\beta^2\omega_e^4\omega^2}{k^6V_e^6}\ I^2\left(\frac{\omega}{kV_e}\right) \quad . \tag{67}$$

For the wave emission contribution from $k < k_D$ and $\omega \cong \omega_e$ we assume that $\exp(-\omega^2/k^2V_e^2) << (1 - \beta) << 1$. Thus for $k_D > k_x > (V_e/V_E)k_D$,

$$\frac{n_o}{2}\ S_{ee}(\underline{k},\omega) \cong \frac{\pi(1-\beta)}{k_xV_E} \Big/ \left[\left(1 - \frac{\omega_e^2}{\omega^2}\right)^2 + \frac{\omega_e^6}{k^6V_e^6}\ I^2\left(\frac{\omega_e}{kV_e}\right)\right]$$

$$\cong \frac{\pi^2(1-\beta)\ k^3V_e^3}{2k_xV_E\omega_e^2\ I\left(\frac{\omega_e}{kV_e}\right)}\ [\delta(\omega-\omega_e) + \delta(\omega+\omega_e)] \quad . \tag{68}$$

Also for $-(V_e/V_E)k_D > k_x > -k_D$,

$$\frac{n_o}{2}\ S_{ee}(\underline{k},\omega) \cong \frac{\pi k^2V_e^2}{2\omega_e^2}\ [\delta(\omega-\omega_e) + \delta(\omega+\omega_e)] \quad . \tag{69}$$

It readily follows that the rate at which the second harmonic mode of the radiation field is excited is,

$$\left(\frac{dU}{dt}\right)_2^{(\text{wave})} \simeq \frac{e^2(1-\beta)}{4\pi\omega_e^2} \int_0^{k_D} dk_y \, dk_z \int_{\left(\frac{V_e}{V_E}\right)k_D}^{k_D} dk_x \, [\delta(\Omega+2\omega_e) + \delta(\Omega-2\omega_e)]$$

$$\cdot \left(\frac{2}{\pi}\right)^{1/2} \frac{|\underline{k}\cdot\underline{\varepsilon}|^2 \, (\underline{K}\cdot\underline{k})^2 V_\varepsilon^5}{k^3 k_x V_E} \exp\left(\frac{\omega_e^2}{2k^2 V_e^2}\right) . \qquad (70)$$

Since $(V_E/V_e) >> 1$, most of the contribution to the $\underline{k}$ integrals comes from $k_y \cong k_z \cong 0$ and $k_x \cong (V_e/V_E)k_D$. Thus the emission at the second harmonic reduces to,

$$I_{2\omega_e}^{(\text{wave})} \cong \frac{3\sqrt{6}(1-\beta)}{4\pi^4\sqrt{\pi}} \left(\frac{mV_e^2}{n_o L^3}\right)\frac{\omega_e}{L^3} \left(\frac{V_e}{c}\right)^5 \left(\frac{V_e}{V_E}\right)^6 |\underline{i}_x\cdot\underline{\varepsilon}|^2 \left(\frac{\underline{K}\cdot\underline{i}_x}{K}\right)^2 \exp\left(\frac{V_E^2}{2V_e^2}\right) , \qquad (71)$$

where $\underline{i}_x$ is a unit vector along the x axis.

It is clear that the emission is again considerably increased above its thermal level by the presence of the exponential factor in (71). The applicability of this result is as before subject to the conditions (61a) and (61b).

6. CONCLUDING REMARKS

The large amounts of radiation represented by formulas (58), (59) and (71) are essentially the result of choosing a particular class of distribution functions which describe a Maxwellian plasma co-existing with a flux of suprathermal electrons. These distributions have the property that the energetic particles do not contribute to the Landau damping of longitudinal waves but only to their emission into a certain range of phase velocities. Such waves are only damped by the Maxwellian particles. This results in an enhanced fluctuation spectrum and corresponding increased emission of radiation.

On the other hand we found that a resonance distribution (19), for which the suprathermal electrons do contribute appreciably to Landau damping at all phase velocities, gave little increase in the emission.

The above distributions represent two extremes and many physical situations may also be described by distributions which lie somewhere between them. However the radiative case for the isotropic function (21) which has a gap in velocity between the thermal and energetic particles is a physically quite realizable situation - and is likely to occur in many astrophysical plasmas. One application of our results, which we shall explore in detail in a later paper, is to theories of Type II solar radio outbursts. These bursts are generally thought to originate in electron plasma oscillations which in turn are driven by energetic electrons in the solar corona.

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9. Equations (I5.4) and (I5.5) are in error. In (I5.4) replace t_1 with t_1-t_2 and in (I5.5) replace t_2 with t_2-t_1 in the P operators.

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APPENDIX

Derivation of the Spectral Density

In this appendix we derive formulas (4-6) for the spectral density function $S_{\alpha\beta}(\underline{k},\omega)$. A formula for $S_{ee}(\underline{k},\omega)$ has been given by Rosenbluth and Rostoker[3]. A general expression for $S_{\alpha\beta}(\underline{k},\omega)$, including transverse fields, is given in (I5.7). But the evaluation of this expression directly is needlessly complicated since the spectral density functions required in (2) describe the fluctuations of charges interacting through Coulomb fields only. However, the derivation leading to (I5.7) is purely algebraic, and therefore it could have been carried through using only Coulomb forces. We now outline this procedure and use it to determine an explicit formula for $S_{\alpha\beta}(\underline{k},\omega)$.

As in I, $\delta N_\alpha(\underline{r},\underline{v},t)$ is a six dimensional phase space density fluctuation, and $\langle N_\alpha(\underline{v})\rangle = f_\alpha(\underline{v})$ is a one particle distribution function. In the Coulomb approximation, the operator in the linearized Vlasov equation (after Fourier transforming the $\underline{r}$ dependence) can be written

$$T^{(c)}(\underline{k},\alpha,\gamma) = \delta_{\alpha\gamma} i\underline{k}\cdot\underline{v} - \frac{q_\alpha}{m_\alpha}\frac{\partial f_\alpha(\underline{v})}{\partial\underline{v}}\cdot\frac{i\underline{k}}{k^2}\, 4\pi n_\gamma q_\gamma \int d\underline{v} \quad . \tag{A1}$$

And with this notation, the linearized Vlasov equation becomes

$$\frac{\partial}{\partial t}\delta N_\alpha + \sum_\beta T^{(c)}(\alpha,\beta)\delta N_\beta = 0 \quad , \tag{A2}$$

or simply

$$\frac{\partial}{\partial t} \delta N + T^{(c)} \delta N = 0 \tag{A3}$$

in the notation of I .

The solution to (A3) is given by the operator $P^{(c)}(\underline{k}, t)$ where

$$\frac{\partial P^{(c)}(\underline{k}, t)}{\partial t} + T^{(c)}(\underline{k}) \, P^{(c)}(\underline{k}, t) = 0 \quad , \tag{A4}$$

$$P^{(c)}(\underline{k}, 0) = \text{Identity} . \tag{A5}$$

The Laplace transform of $P^{(c)}(\underline{k}, t)$, i.e.,

$$P^{(c)}(\underline{k}, s) = [s + T^{(c)}(\underline{k})]^{-1} \tag{A6}$$

is explicitly given by (II6.3).

As in I we attach subscripts $\underline{1}$ and $\underline{2}$ to the coordinates and write the correlation function

$$\left\langle \delta N(\underline{r}_1, \underline{v}_1) \; \delta N(\underline{r}_2, \underline{v}_2) \right\rangle = \left\langle \delta N(\underline{1}) \; \delta N(\underline{2}) \right\rangle . \tag{A7}$$

The integral and summation operators $P(\underline{1})$ and $P(\underline{2})$ operate only on coordinates with $\underline{1}$ and $\underline{2}$ subscripts respectively. The Fourier transform of the correlation function (for a spatially homogeneous plasma) is denoted by

$$\left\langle \delta N(\underline{1})\ \delta N(\underline{2}) | \underline{k} \right\rangle = \int d(\underline{r}_1 - \underline{r}_2)\ e^{i\underline{k}\cdot(\underline{r}_1 - \underline{r}_2)} \left\langle \delta N(\underline{1})\ \delta N(\underline{2}) \right\rangle \quad . \tag{A8}$$

Now, according to $(5.4)^9$, the two-time correlation function is given by

$$\left\langle \delta N(\underline{v}_1, T+t)\ \delta N(\underline{v}_2, T) | \underline{k} \right\rangle$$

$$= P^{(c)}(\underline{1}, \underline{k}, t)\ \left\langle \delta N(\underline{1})\ \delta N(\underline{2}) | \underline{k} \right\rangle \ , \ t > 0 \ . \tag{A9}$$

The operand on the right-hand side is the time-asymptotic two particle correlation function including self-correlation. According to (I8.2) this function is given by

$$\left\langle \delta N_\alpha(\underline{1})\ \delta N_\beta(\underline{2}) | \underline{k} \right\rangle = \lim_{\tau\to\infty} \sum_{\mu,\nu} P^{(c)}(\underline{1}, \underline{k}, \tau, \alpha, \mu)$$

$$P^{(c)}(\underline{2}, -\underline{k}, \tau, \beta, \nu)\ \delta(\underline{v}_1 - \underline{v}_2)\ \delta(\mu, \nu)\ \frac{1}{n_\mu} \left\langle N_\mu(\underline{1}) \right\rangle \quad , \tag{A10}$$

or in the notation of I

$$\left\langle \delta N(\underline{1})\ \delta N(\underline{2}) | \underline{k} \right\rangle$$

$$= \lim_{\tau\to\infty} \quad P^{(c)}(\underline{1}, \underline{k}, \tau)\ \ P^{(c)}(\underline{2}, -\underline{k}, \tau)\ \Delta'^{(c)}(\underline{1}, \underline{2})\ \left\langle N(\underline{1}) \right\rangle \quad , \tag{A11}$$

which defines the quantity $\Delta'^{(c)}(\underline{1}, \underline{2})$. Using (A4) and (A5), this can be written

$$\left\langle \delta N(\underline{1})\ \delta N(\underline{2})|\underline{k}\right\rangle$$

$$= - \int_0^\infty d\tau\ P^{(c)}(\underline{1},\underline{k},\tau)\quad P^{(c)}(\underline{2},-\underline{k},\tau)\ [T^{(c)}(\underline{1},\underline{k}) + T^{(c)}(\underline{2},-\underline{k})]$$

$$\Delta'^{(c)}(\underline{1},\underline{2}) \left\langle N(\underline{1})\right\rangle + \Delta'^{(c)}(\underline{1},\underline{2}) \left\langle N(\underline{1})\right\rangle \qquad \text{(A12)}$$

Now using this formula in (A9) along with the relation $P^{(c)}(t)\ P^{(c)}(\tau) = P^{(c)}(t+\tau)$, we obtain

$$\left\langle \delta N(\underline{1},T+t)\ \delta N(\underline{2},T)|\underline{k}\right\rangle = - \int_0^\infty d\tau P^{(c)}(\underline{1},\underline{k},\tau+t)\ P^{(c)}(\underline{2},-\underline{k},\tau)$$

$$[T^{(c)}(\underline{1},\underline{k}) + T^{(c)}(\underline{2},-k)]\ \Delta'^{(c)}(\underline{1},\underline{2}) \left\langle N(\underline{1})\right\rangle$$

$$+ P(\underline{1},\underline{k},t)\ \Delta'^{(c)}(\underline{1},\underline{2}) \left\langle N(\underline{1})\right\rangle \qquad \text{(A13)}$$

Taking the <u>Laplace</u> transform (with respect to time) of both sides gives

$$\int_0^\infty dte^{-i\omega t} \left\langle \delta N(\underline{1},T+t)\ \delta N(\underline{2},T)|\underline{k}\right\rangle \equiv \left\langle \delta N(\underline{1})\ \delta N(\underline{2})|\underline{k},i\omega\right\rangle$$

$$= - \int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i}\quad \frac{1}{i\omega-s+\varepsilon}\quad P^{(c)}(\underline{1},\underline{k},s)\quad P^{(c)}(\underline{2},-\underline{k},-s)$$

$$[T^{(c)}(\underline{1},\underline{k}) + T^{(c)}(\underline{2},-\underline{k})]\ \Delta'^{(c)}(\underline{1},\underline{2}) \left\langle N(\underline{1})\right\rangle$$

$$+ \quad P^{(c)}(\underline{1},\underline{k},i\omega)\ \Delta'^{(c)}(\underline{1},\underline{2}) \left\langle N(\underline{1})\right\rangle \qquad \text{(A14)}$$

To obtain density fluctuations, we integrate both sides over $\underline{v}_1$ and $\underline{v}_2$.

$$\langle \delta\rho\delta\rho | \underline{k}, i\omega \rangle = \int d\underline{v}_1 \int d\underline{v}_2 \langle \delta N(\underline{1})\ \delta N(\underline{2}) | \underline{k}, i\omega \rangle \tag{A15}$$

Consequently the $P^{(c)}$ operators occurring on the right-hand side of (A14) will be replaced with the operators $\int d\underline{v} P^{(c)}$. For the case of zero magnetic field (II6.3) gives the familiar result

$$\int d\underline{v} P^{(c)}(\underline{k}, s, \alpha, \mu) = \left[\frac{n_\mu q_\mu}{n_\alpha q_\alpha} \frac{L_\alpha(\underline{k}, s)}{D(\underline{k}, s)} + \delta_{\alpha\mu} \right] \int \frac{d\underline{v}}{s + i\underline{k}\cdot\underline{v}} , \tag{A16}$$

with $D(\underline{k}, s)$ given by (7).

Using (A1) the $(\mu, \nu)^{th}$ component of

$$[T^{(c)}(\underline{1}, \underline{k}) + T(\underline{2}, -\underline{k})]\ \Delta'^{(c)}(\underline{1}, \underline{2}) \quad \langle N(\underline{1}) \rangle \tag{A17}$$

becomes

$$\begin{aligned} &\sum_\gamma T^{(c)}(\underline{1}, \underline{k}, \mu, \nu)\ \delta(\underline{v}_1 - \underline{v}_2) \quad f_\gamma(\underline{v}_1)\ \delta_{\gamma\nu}\ n_\gamma^{-1} \\ + &\sum_\gamma T^{(c)}(\underline{2}, -\underline{k}, \nu, \gamma)\ \delta(\underline{v}_1 - \underline{v}_2) \quad f_\gamma(\underline{v}_1)\ \delta_{\mu\gamma}\ n_\gamma^{-1} \\ = &- 4\pi \frac{q_\mu}{m_\mu} \frac{i\underline{k}}{k^2} \cdot \frac{\partial f_\mu(\underline{1})}{\partial \underline{v}_1} \quad q_\nu\ f_\nu(\underline{2}) \\ + &\quad 4\pi \frac{q_\nu}{m_\nu} \frac{i\underline{k}}{k^2} \cdot \frac{\partial f_\nu(\underline{2})}{\partial \underline{v}_2} \quad q_\mu\ f_\mu(\underline{1}) \end{aligned} \tag{A18}$$

Substituting (A18) and (A16) into (A14) and using (A15) yields

$$\left\langle \delta\rho_\alpha \delta\rho_\beta \middle| \underline{k}, i\omega \right\rangle = \int \frac{ds}{2\pi i} \frac{1}{i\omega - s + \varepsilon} \sum_{\mu,\nu} \left[\frac{n_\mu q_\mu}{n_\alpha q_\alpha} \frac{L_\alpha(\underline{k}, s)}{D(\underline{k}, s)} + \delta_{\alpha\mu} \right]$$

$$\int \frac{d\underline{v}_1}{s + i\underline{k}\cdot\underline{v}_1 + \varepsilon} \left[\frac{n_\nu q_\nu}{n_\beta q_\beta} \frac{L_\beta(-\underline{k}, -s)}{D(-\underline{k}, -s)} + \delta_{\nu\beta} \right] \int \frac{d\underline{v}_2}{-s - i\underline{k}\cdot\underline{v}_2 + \varepsilon}$$

$$4\pi \, q_\mu q_\nu \frac{i\underline{k}}{k^2} \cdot \left[\frac{1}{m_\mu} \frac{\partial f_\mu(\underline{1})}{\partial \underline{v}_1} f_\nu(\underline{2}) - \frac{1}{m_\nu} \frac{\partial f_\nu(\underline{2})}{\partial \underline{v}_2} f_\mu(\underline{1}) \right]$$

$$+ \left[\frac{n_\beta q_\beta}{n_\alpha q_\alpha} \frac{L_\alpha(\underline{k}, i\omega)}{D(\underline{k}, i\omega)} + \delta_{\alpha\beta} \right] \int \frac{d\underline{v}_1 \, f_\beta(\underline{1})}{i\omega + i\underline{k}\cdot\underline{v}_1 + \varepsilon} \; n_\beta^{-1} \quad . \tag{A19}$$

And with a little algebra, this becomes

$$\left\langle \delta\rho_\alpha \delta\rho_\beta \middle| \underline{k}, i\omega \right\rangle = \int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} \frac{1}{i\omega - s + \varepsilon} \left[\Gamma_{\alpha\beta} + \Gamma^*_{\beta\alpha} \right]$$

$$+ \left[\frac{n_\beta q_\beta}{n_\alpha q_\alpha} \frac{L_\alpha(\underline{k}, i\omega)}{D(\underline{k}, i\omega)} + \delta_{\alpha\beta} \right] U_\beta(\underline{k}, i\omega) \, n_\beta^{-1} \quad , \tag{A20}$$

where $\Gamma_{\alpha\beta}$ and U_β are given by (5) and (6) with $i\omega = s$.

The spectral density $S_{\alpha\beta}(\underline{k}, \omega)$ is the Fourier transform with respect to time, and (A20) is the Laplace transform. However, the two are easily related.

$$S_{\alpha\beta}(\underline{k},\omega) = \int_{-\infty}^{+\infty} dt\ e^{-i\omega t} \left\langle \delta\rho_\alpha(T+t)\delta\rho_\beta(T) \middle| \underline{k} \right\rangle$$

$$= \int_0^\infty dt\ e^{-i\omega t} \left\langle \delta\rho_\alpha(T+t)\delta\rho_\beta(T) \middle| \underline{k} \right\rangle + \int_0^\infty dt\ e^{i\omega t} \left\langle \delta\rho_\alpha(T-t)\delta\rho_\beta(T) \middle| \underline{k} \right\rangle$$

and using (3b)

$$= \left\langle \delta\rho_\alpha \delta\rho_\beta \middle| \underline{k}, i\omega \right\rangle + \left\langle \delta\rho_\beta \delta\rho_\alpha \middle| \underline{k}, i\omega \right\rangle^* \quad . \tag{A21}$$

Substituting (A20) into (A21), and using

$$\int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} \left(\frac{1}{i\omega - s + \varepsilon} - \frac{1}{i\omega - s - \varepsilon} \right) = \int_{-i\infty}^{+i\infty} ds\ \delta(s - i\omega)$$

one immediately obtains (4).

Figure Captions

Figure 1

Schematic representation of the scattering of longitudinal fluctuations into a transverse wave of wave number $\underline{K}$ and frequency Ω .

Figure 2

A plot of the spectral density $S_{ee}(k,\omega)$ for an electron-ion plasma with velocity distributions $f_{e,i} = 4V^3_{e,i}/\pi^2(v^2+V^2_{e,i})^3$ and $V_e = (M/m)^{1/2}\ V_i = 43V_i$.

Figure 3

The electron distribution functions $f_e(|\underline{v}|)$ and $\bar{f}_e(v_x)$ for a Maxwellian plasma co-existing with a mono-energetic flux of electrons.

Figure 4

The electron distribution functions $f_e(|\underline{v}|)$ and $\bar{f}_e(v_x)$ for a Maxwellian plasma with an isotropic flux of energetic electrons in the range $|\underline{v}| > V_E$.

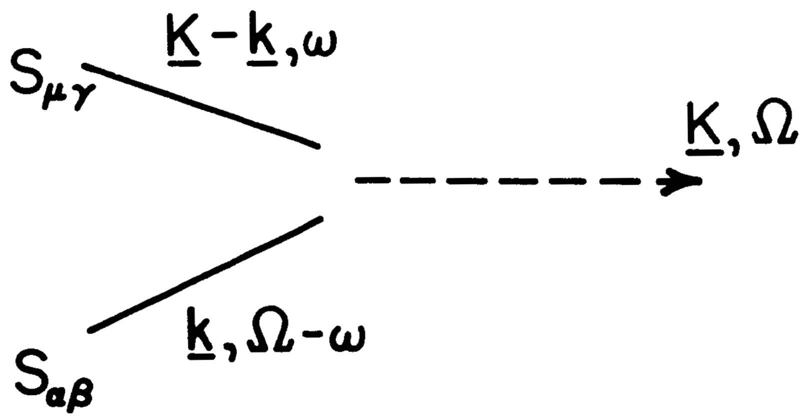
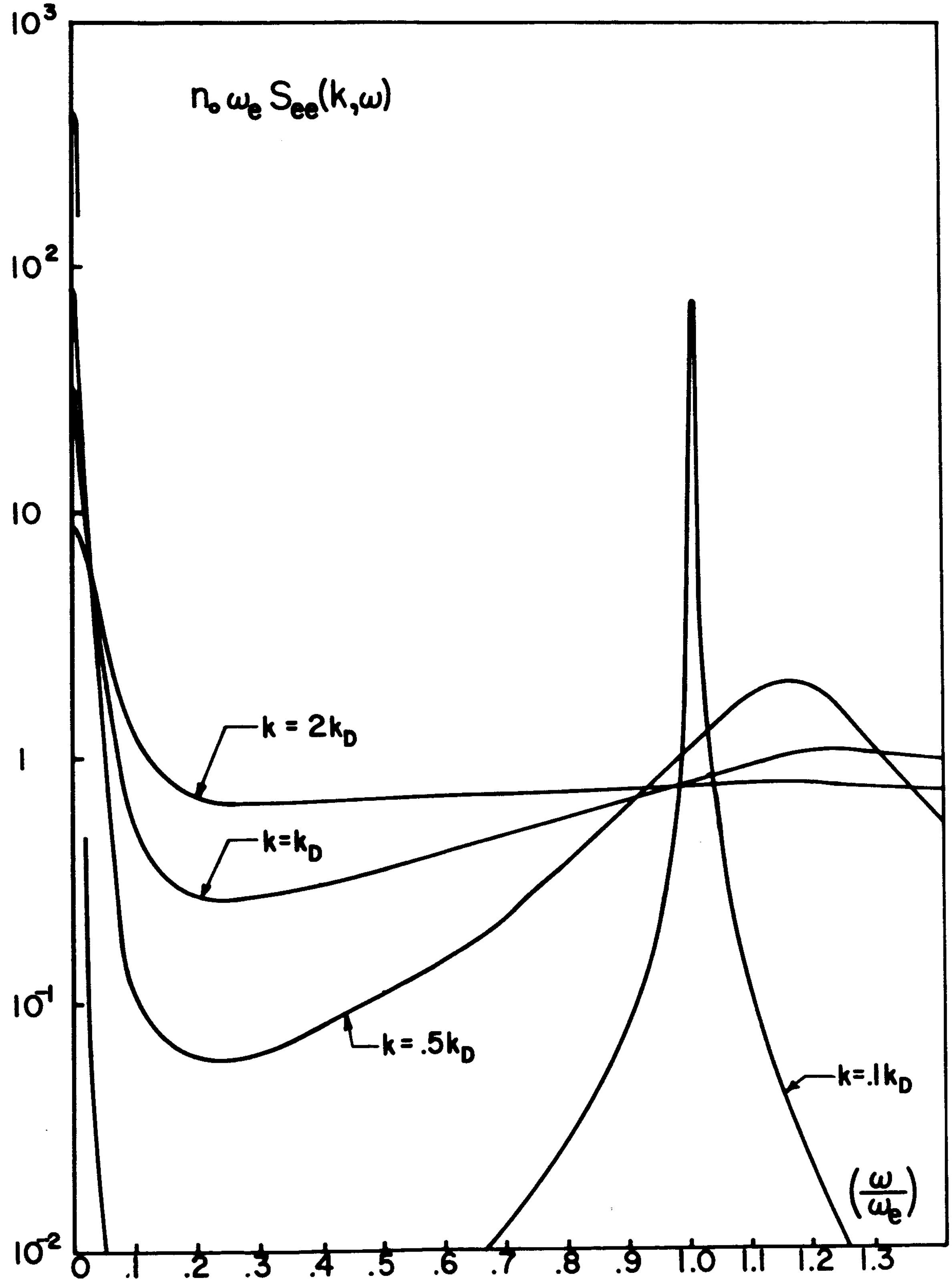
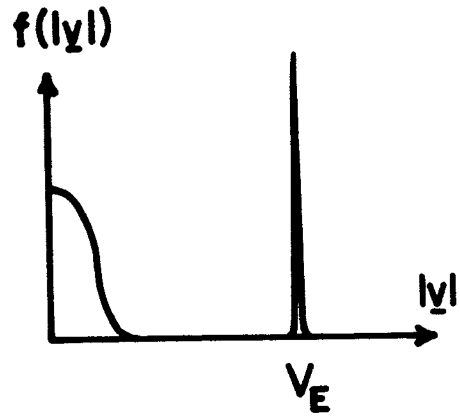

$\underline{K}-\underline{k},\omega$
$S_{\mu\gamma}$
$\underline{K},\Omega$
$\underline{k},\Omega-\omega$
$S_{\alpha\beta}$

Figure 1



$n_o \omega_e S_{ee}(k,\omega)$
10^3
10^2
10
1
10^{-1}
10^{-2}
k = 2k_D
k=k_D
k = .5k_D
k=.1k_D
$\left(\frac{\omega}{\omega_e}\right)$
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3

Figure 2



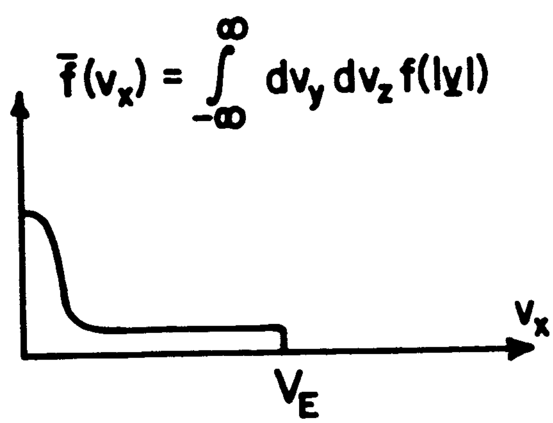


Figure 3

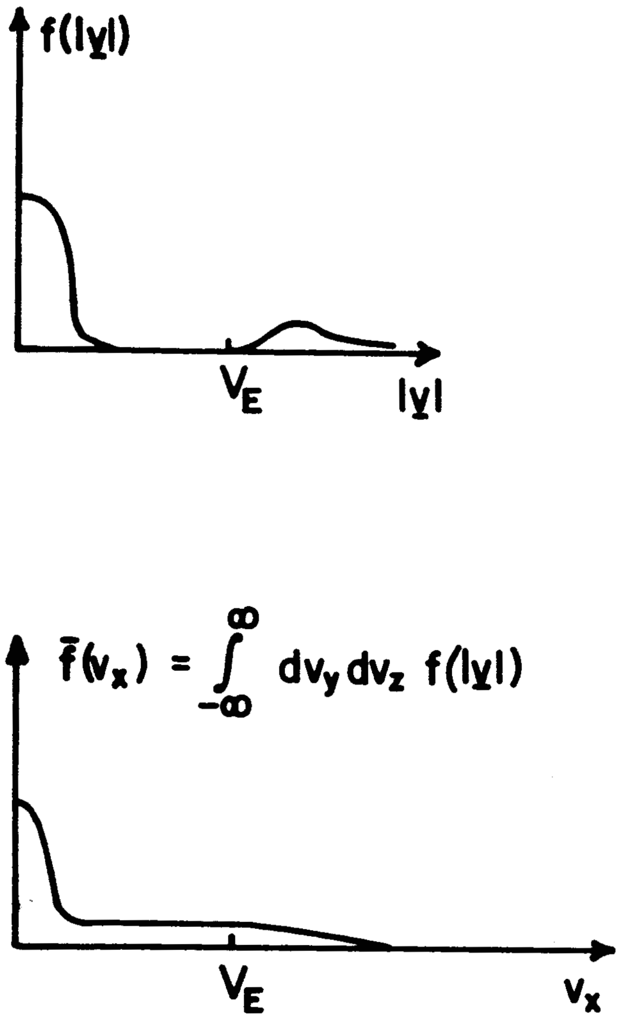


Figure 4