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ABSTRACT

A study has been made of the problem of elastic collisions and energy transfer between gases which have separate Maxwellian velocity distributions. It is shown that the expression for the energy transfer rate obtained by Desloge (1962) for gases of arbitrary temperature and particle mass can be adapted into a convenient form which involves a ratio of the particle masses, the difference in the gas thermal energies, and a collision frequency for energy transfer. An analysis is then made of the collision frequency in terms of an average momentum transfer cross section which is defined for conditions of thermal nonequilibrium. The general equations are then specialized to consider the problem of elastic electron collisions in heavy particle gases. To obtain useful numerical expressions for electron-neutral particle collision frequencies and energy transfer rates, an analysis has been made of the momentum transfer cross sections for N_2, O_2, O, H, and He. Calculations have also been made of the Coulomb momentum transfer cross section, collision frequency, and energy transfer rate. These results are then compared with previous work.

I. - INTRODUCTION

In order to understand the thermal behavior of ionized atoms or molecules which are subjected to selective heating processes it is necessary to know the different rates of collisional energy transfer between charged and neutral particle species. Once these have been determined it is possible to use energy balance equations to derive appropriate temperatures for each of the species present. Hence, in all generality it is necessary to have expressions for electron-ion, electron-neutral, and ion-ion energy transfer rates. The use of the full set of such energy transfer relations is currently required in the theoretical description of electron and ion temperatures in the upper atmosphere.

In this paper an investigation is made of elastic collisional energy transfer between mixed gases of arbitrary particle mass having separate Maxwellian velocity distributions. It is shown in Section II that the exact equation for energy transfer, derived by Desloge (1962), can be separated into three fundamental factors, each of which depends upon a different aspect of the collision process and gas composition. The concept of a nonequilibrium collision frequency for energy transfer is introduced for particle interactions of somewhat arbitrary cross section. In a similar manner the equations leading to the development of an average nonequilibrium momentum transfer cross section are derived.

Following the presentation of the general relations, which are valid for particles of arbitrary mass and temperature, the results are specialized to consider elastic collisions between electrons and heavy particles. In Section III we consider the difficulties involved in deriving average momentum transfer cross sections for electron - heavy particle interactions.

For electron collisions with other charged particles it is found that the classical Rutherford differential scattering cross section can be used to arrive at results which are in accord with previous calculations. The problem of elastic electron collisions with neutral particles is more difficult and it is necessary to analyze both laboratory data and theoretical derivations in order to arrive at useful sections for N_2, O_2, O, He, and H.

The application of the cross section data is made in Section IV to obtain expressions for momentum transfer collision frequencies and rates of electron energy transfer. A comparison is then made between the present results and those which have been used in earlier studies of electron energy transfer rates as applied to the problems of the ionospheric energy balance.

Section V is devoted to a general summary of the results of this study.

II. - BASIC EQUATIONS

1. General Derivation

The derivation of the equation which describes the rate of energy exchange between two gases with Maxwellian velocity distributions having different temperatures and particle masses has been made by Desloge (1962). By applying velocity distribution techniques to the mechanics of elastic collisions he was able to evaluate the average rate of change of the total kinetic energy of one gas as

$$\frac{dU_1}{dt} = -4\pi n_1 n_2 \frac{(m_1 m_2)^{1/2} (T_1 - T_2)}{(m_1 + m_2)^2 (2\pi k)^{3/2} (m_1 T_1 + m_2 T_2)^{5/2}} \int_0^\infty g^5 q_D(g) \exp(-Kg^2) \qquad (1)$$

where

$$U_1 = \int \frac{1}{2} m_1 v_1^2 f_1 d^3\vec{v}_1 \tag{2a}$$

$$K = \left(\frac{2kT_1}{m_1} + \frac{2kT_2}{m_2} \right)^{-1} \tag{2b}$$

$$q_D(g) = 2 \pi \int \sigma(g,\theta) \, (1-\cos \theta) \, \sin \theta \, d\theta \tag{2c}$$

and

U - gas total kinetic energy

n - particle number density

m - particle mass

T - Maxwellian temperature

k - Boltzmann's constant

g - relative velocity between particles

$q_D(g)$ - velocity dependent momentum transfer cross section

v - particle velocity in laboratory system

$d^3\vec{v}$ - velocity space volume element

θ - center of mass scattering angle

$\sigma(g,\theta)$ - differential scattering cross section

f - velocity distribution function.

Equation (1) is valid for conditions where separate Maxwellian velocity distributions can be maintained and where a suitable momentum transfer cross section can be found. In particular, Desloge (1962) has shown that this equation accurately describes the energy transfer rates for both elastic spheres and Coulomb particles.

In the interests of further clarity, it will now be shown that it is possible to rearrange equation (1) in such a way that a deeper physical insight can be obtained into the problem of elastic collisional energy transfer between Maxwellian distributions of particles.

As a brief guide, the following discussion is based upon a simple model of energy transfer for a single particle moving in a gas. By defining an appropriate collision frequency for energy transfer and a momentum transfer cross section it is possible to derive a general functional form for the energy exchange rate between two gases. Such a form can be compared with equation (1) to obtain specific equations for the collision frequency and momentum transfer cross section which are applicable to the problem of energy transfer.

We consider first the average energy loss per collision of a single particle of mass m_1 and kinetic ε_1 traveling through a gas composed of particles of mass m_2 and average energy $\bar{\varepsilon}_2$. The average loss of kinetic energy per collision, $\Delta\varepsilon_1$, for this single particle is, as shown by Crompton and Huxley (1962),

$$\Delta\varepsilon_1 = -\frac{2m_1 m_2}{(m_1+m_2)^2}\,(\varepsilon_1 - \bar{\varepsilon}_2) \tag{3}$$

To describe the rate at which the single particle losses energy per unit time we may introduce the concept of the single particle collision frequency given by

$$\nu_{12} = n_2 g\, q_D. \tag{4}$$

with n_2 the ambient gas number density. Since this quantity represents the collision rate of a single particle in a gas we may now combine equations (3) and (4) to obtain the average rate at which the single particle losses energy as

$$\frac{\overline{\Delta\varepsilon}}{\Delta t} = -\frac{2m_1 m_2}{(m_1+m_2)^2}\,(\varepsilon_1 - \bar{\varepsilon}_2)\,\nu_{12}. \tag{5}$$

If, instead of a single particle, we have a large number of particles combining to form a Maxwellian gas mixed with the original gas, we may approximate the total average energy exchange rate by the expression

$$\frac{dU_1}{dt} = - \frac{2m_1 m_2}{(m_1+m_2)^2} n_1 (\bar{\varepsilon}_1 - \bar{\varepsilon}_2) \bar{\nu}_{12} \tag{6}$$

$$U_1 = \frac{n_1}{2} m_1 \overline{v_1^2} = n_1 \overline{\varepsilon_1} \tag{7}$$

where $\overline{\varepsilon_1}$ is now an average energy which corresponds to the Maxwellian distribution of single particles which we permitted to become the mixed gas. Likewise, $\bar{\nu}_{12}$ is now an average collision frequency which, unlike equation (4), must now be suitably defined to take account of the many different relative velocities between the various gas particles.

The derivation leading to equation (6) is not rigorous since the proper averaging techniques of kinetic theory needed to arrive at an exact expression have not been used. It gives, however, a functional form for the energy exchange rate between two gases which can be used to decompose the original exact result of Desloge (1962), given in equation (1), into three factors ; a ratio of masses, a difference in average particle energies, and an energy transfer collision frequency. The first two quantities are independent of the mode of interaction between the two gas species, depending only upon the appropriate masses and average gas thermal energies. It is thus the collision frequency which must contain the factors which relate to the interparticle forces.

In order that the correct form for an average collision frequency may be synthesized from the comparison of equations (1) and (6) we require that the functional form of the average collision frequency be

$$\bar{\nu}_{12} \propto n_2 \, \bar{g} \, \bar{Q}_D \tag{8}$$

which allows for the presence of an arbitrary numerical factor in the final result for $\bar{\nu}_{12}$. The quantity $\bar{g}$ is the Maxwellian average relative velocity between the particles of the two gases while $\bar{Q}_D$ is the defined average momentum transfer cross section appropriate for conditions of thermal nonequilibrium. The quantity $\bar{g}$ can be derived directly for the motions of two gases having distribution functions f_1 and f_2 by the equation

$$\bar{g} = \iint f_1 f_2 \, | \, \bar{v}_1 - \bar{v}_2 \, | \, d^{3\rightarrow}v_1 \, d^{3\rightarrow}v_2 \tag{9}$$

where $d^{3\rightarrow}v_{1,2}$ are the respective velocity space volume elements for the two velocity distributions. For f_1 and f_2 representing separate Maxwellian velocity distribution functions it is possible to integrate equation (9) to obtain,

$$g = \left(\frac{8k}{\pi}\right)^{1/2} \left[\frac{T_1}{m_1} + \frac{T_2}{m_2}\right]^{1/2} , \tag{10}$$

the subscripts applying to the parameters of each respective gas.

With this result it is now possible to synthesize the necessary expressions for the average energy transfer collision frequency and the average momentum transfer cross section. Through manipulation of equations (1), (6), (8), and (10) the latter quantity becomes

$$\bar{Q}_D = K^3 \int_o^\infty g^5 \, q_D(g) \, \exp(-Kg^2) \tag{11}$$

where, from (2b),

$$K = \left[\frac{2kT_1}{m_1} + \frac{2kT_2}{m_2}\right]^{-1} \tag{12}$$

and it is assumed that $\bar{\varepsilon}_1 = \frac{3}{2} kT_1$ and $\bar{\varepsilon}_2 = \frac{3}{2} kT_2$.

Equation (11) is the generalization of the momentum transfer cross section to situations where thermal equilibrium does not prevail between gases composed of particles with different masses and different Maxwellian distributions. Under a condition of equilibrium we may take $T_1 = T_2$ which reduces $\bar{Q}_D$ immediately to the standard form for the average momentum transfer cross section given by Dalgarno, et al. (1958).

In a similar manner the comparison of equations (1), (6), and (8) and (11) permits the recognition of the average momentum transfer collision frequency as

$$\bar{\nu}_{12} = \frac{4}{3} n_2 \bar{g} \bar{Q}_D \tag{13}$$

or, using equation (10),

$$\bar{\nu}_{12} = \frac{4}{3} n_2 \left(\frac{8k}{\pi}\right)^{1/2} \left[\frac{T_1}{m_1} + \frac{T_2}{m_2}\right]^{1/2} \bar{Q}_D \tag{14}$$

which represents the generalization to conditions of different Maxwellian velocity distributions of two gases. For an equilibrium state such as $T_1 = T_2 = T$ and with μ representing the two particle reduced mass,

$$\bar{\nu}_{12} = \frac{4}{3} n_2 \left(\frac{8kT}{\pi\mu}\right)^{1/2} \bar{Q}_D , \tag{15}$$

which is a factor of 4/3 larger than the total scattering collision frequency derived by Chapman and Cowling (1952). This same factor has, however, been noted by Nicolet (1953) in an analysis of electron collision frequencies based upon an analysis of collision intervals and diffusion coefficients derived by the velocity distribution method.

As a final result it is now possible to express equation (1) in terms of the collision frequency, difference in energy, and mass factor as

$$\frac{dU_1}{dt} = -3n_1 \frac{m_1 m_2}{(m_1+m_2)^2} k(T_1 - T_2) \bar{\nu}_{12}. \qquad (16)$$

This equation represents the final goal of the derivation since we now have decomposed the general equation into a form which relates to different aspects of the collision process for energy transfer.

2. Application to Electron Energy Transfer

We now extend the preceding equations to consider the problem of an electron gas mixed with another gas composed of heavy particles such that $m_e \ll m_2$. The equations derived for this situation will be applicable to elastic electron-neutral and electron-ion collisions.

From equation (11) the average momentum transfer cross section becomes

$$\bar{Q}_D = \left(\frac{m_e}{2kT_e}\right)^3 \int_o^\infty v^5 q_D(v_e) e^{-\frac{m_e v^2}{2kT_e}} dv \quad , \qquad (17)$$

where v is the electron velocity since, for $m_e \ll m_2$, the relative velocity g is determined almost entirely by the motions of the electrons alone.

The average electron collision frequency can likewise be obtained from equation (14) under the assumption that $T_e/m_e \gg T_2/m_2$ as

$$\bar{\nu}_e = \frac{4}{3} n_2 \left(\frac{8kT_e}{\pi m_e}\right)^{1/2} \bar{Q}_D \quad , \qquad (18)$$

dependent upon the electron temperature alone.

Finally, the rate of exchange of kinetic energy between the electron gas and the second gas is obtained from equation (16) as

$$\frac{dU_e}{dt} = - 3n_e \frac{m_e}{m_2} k (T_e - T_2) \bar{\nu}_e , \tag{19}$$

or, in terms of $\bar{Q}_D$,

$$\frac{dU_e}{dt} = - 4 n_e n_2 \frac{m_e}{m_2} k \left(\frac{8kT_e}{\pi m_e} \right)^{1/2} \bar{Q}_D (T_e - T_2) \tag{20}$$

It is interesting to note that the energy transfer collision frequency, $\bar{\nu}_e$, can be directly related to the theory of the electrical conductively of a plasma. From Shkarofsky, et al. (1961) the equivalent collision frequency of electrons which limits the conduction of current in a plasma subjected to a weak electric field can be derived as

$$\bar{\nu}_{equiv} = \frac{\int_o^\infty \left[\frac{\nu_e}{\nu_e^2 + \omega^2} \right] e^{- \frac{m_e v^2}{2kT_e}} dv}{\int_o^\infty v^4 \left[\frac{1}{\nu_e^2 + \omega^2} \right] e^{- \frac{m_e v^2}{2kT_e}} dv} \tag{21}$$

where ν_e is the velocity dependent electron collision frequency for momentum transfer defined in equation (4) and ω is the angular frequency of the applied electric field. It the radio frequency ω is much larger than the collision frequency such that $\omega^2 \gg \nu_e^2$ this equation reduces to

$$\bar{\nu}_{equiv} = \frac{m_e}{3kT_e} \int_o^\infty v^4 \nu_e e^{- \frac{m_e v^2}{2kT_e}} dv \tag{22}$$

which is exactly the same as the electron collision frequency given in equation (18). Thus, by means of high frequency radio experiments in dilute plasmas it should be possible to obtain experimental data which can be used directly to calculate elastic electron energy transfer rates.

The application of the preceding equations to the problem of determining electron collision frequencies and energy exchange rates in heavy gases is made in the following sections. First, however, it is necessary to adopt adequate expressions for the momentum transfer cross sections. For electron-neutral collisions there exist no convenient analytical results and it is necessary to analyze existing laboratory and theoretical results. The problem of Coulomb collisions, however, is amenable to a direct theoretical approach.

III.- ELECTRON CROSS SECTIONS FOR MOMENTUM TRANSFER

1. Cross Sections for Neutral Particles

Since theoretical methods usually do not yield accurate values of q_D for low energy electron-neutral collisions it is necessary to rely upon the available experimental measurements. Descriptions of the current methods used to obtain momentum transfer cross sections for electron-neutral collisions can be found in Massey and Burhop (1952), McDaniel (1964), and Hasted (1964). As has been emphasized, it is the collision cross section for momentum transfer which is of dominating importance in determining the form of the energy equations. Unfortunately, most early experiments were designed to give values of the total scattering cross section and it is only within the past 15 years, with the advent of the microwave conductivity and drift velocity methods, that accurate values of the momentum transfer cross section have been determined.

In the following sections each atmospheric gas is considered separately with respect to electron collisions and appropriate expressions for the momentum transfer section are adopted.

1.1 Molecular Nitrogen

The results of theoretical studies and experimental measurements for q_D by various workers over the past 28 years are given by Shkarofsky, et al. (1961). Corrections have been made by these authors for the velocity distributions of the colliding electrons in all the previous experiments. Nevertheless, the values of the measured cross sections vary by as much as 65%. More recent studies have been based upon microwave and electron mobility experiments. These methods have tended to produce much more consistent data and are capable of covering a wide range of electron energies. The results of recent experiments are presented in Figure 1.

At low electron energies in the range 0.003 to 0.05 ev, Pack and Phelps (1961) have measured the drift velocity of electrons under the influence of a constant electric field. Their data on the momentum cross section agree well with the earlier measurements of Pack, et al. (1951) who used a microwave conductivity device over the energy range 0.02 to 0.03 ev. Anderson and Goldstein (1956a), employing a slightly different microwave technique, obtained results which diverge from other work, showing a substantial increase in q_D at low electron energies. When the work of Crompton and Huxley, as reported by Shkarofsky, et al. (1961), and Crompton and Sutton (1952) is considered, it appears that experimental errors probably exist in Anderson and Goldstein's work. Further, Huxley (1956) obtained results consistent with the earlier measurements of Crompton and Sutton and the later data of Pack and Phelps.

Frost and Phelps (1962) and Englehardt, et al. (1964) have used a method of integrating the Boltzmann equation to choose proper values for q_D. Their method consisted of adopting appropriate sets of elastic and inelastic cross sections and then solving the Boltzmann

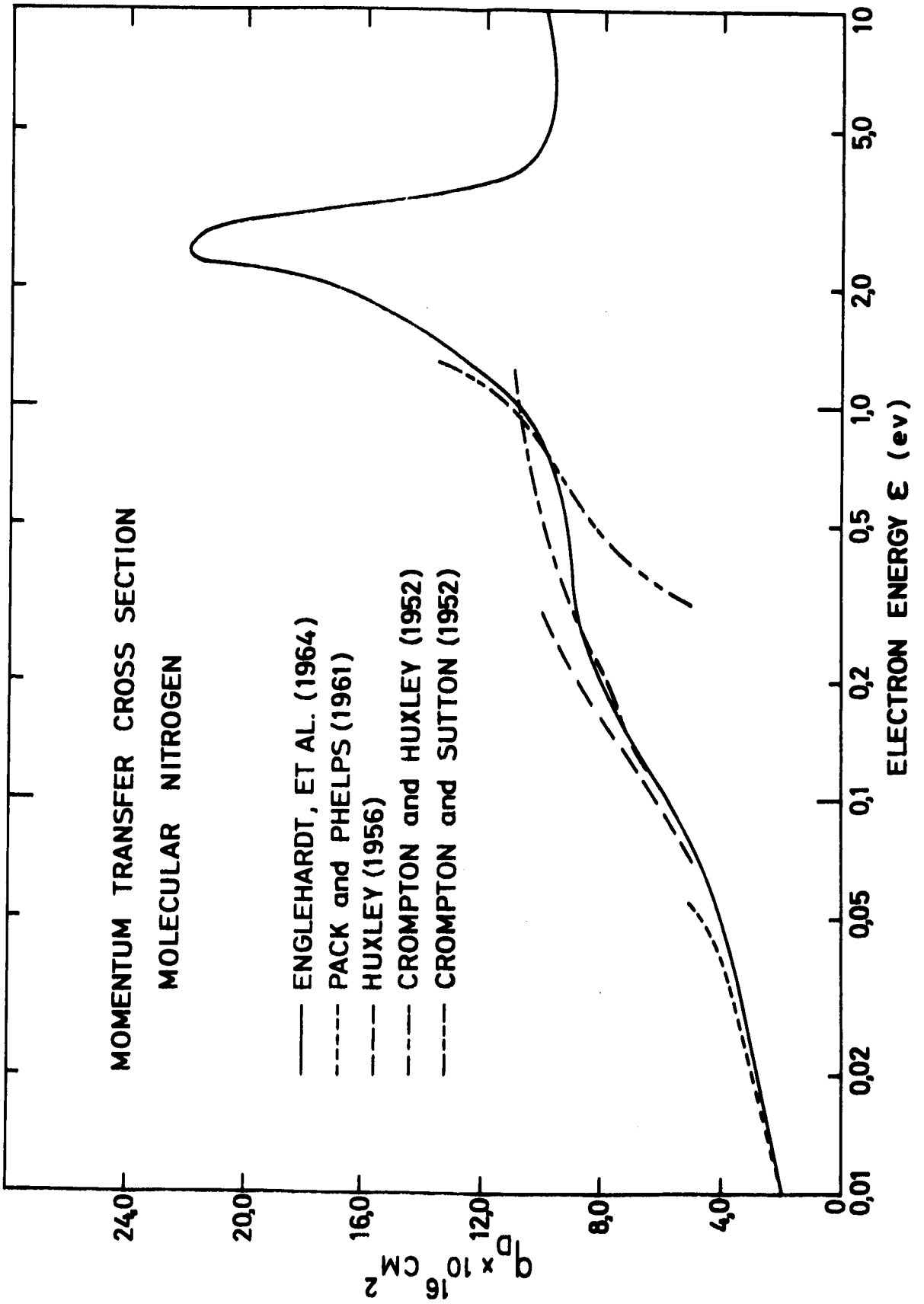
MOMENTUM TRANSFER CROSS SECTION
MOLECULAR NITROGEN
ENGLEHARDT, ET AL. (1964)
PACK and PHELPS (1961)
HUXLEY (1956)
CROMPTON and HUXLEY (1952)
CROMPTON and SUTTON (1952)
$q_D \times 10^{16}$ CM^2
ELECTRON ENERGY ε (ev)
24,0
20,0
16,0
12,0
8,0
4,0
0
0,01
0,02
0,05
0,1
0,2
0,5
1,0
2,0
5,0
10

FIGURE 1

equation for the equilibrium electron velocity distribution function in the presence of an electric field energy source. Next, they computed the electron mobilities and diffusion coefficients. Since these are experimentally known quantities, the theoretical results could be compared with the measurements. When differences existed, suitable changes were made in the cross sections until consistent results followed.

For electron energies between 0.02 ev and 0.1 ev the data of Pack and Phelps (1961) can be represented by the equation

$$q_D = 18.8 \times 10^{-16} \varepsilon^{1/2} \text{ cm}^2 \quad , \tag{23}$$

where ε is the electron energy measured in electron volts (ev). For energies above 0.1 ev this expression leads to an overestimate of the true cross section. From the data of Englehardt, et al. (1964) a suitable generalization to include the region 0.1 - 1.0 ev is

$$q_D = (18.3 - 7.3\ \varepsilon^{1/2})\ \varepsilon^{1/2} \times 10^{-16} \text{ cm}^2. \tag{24}$$

Applying equation (17) to equation (24) yields

$$\bar{Q}_D(N_2) = (2.82 - 3.41 \times 10^{-4} T_e)\ T_e^{1/2} \times 10^{-17} \text{ cm}^2 \tag{25}$$

In essence, this result represents the first correction to the work of Pack and Phelps (1961) such that the collision frequency and energy loss rate can now be evaluated over the range $100° \leq T_e \leq 4500°K$. For low temperatures the correction term, $3.41 \times 10^{-4} T_e$, is small. At temperatures above 2000°K, however, there is a significant reduction of the cross section below that which follows from the original cross section of Pack and Phelps.

1.2 Molecular Oxygen

The results of all experimental and thecretical studies of the electron-molecular oxygen momentum transfer cross section conducted

prior to 1958 have been compiled by Shkarofsky, et al. (1961). In general, the various results do not yield consistent values. However, the early work of Crompton and Huxley, as reported by Cook and Lorents (1960), over the energy range 0.2 to 2.0 ev has been found to agree well with later data. In particular, Phelps (1960) analyzed the 9.3 Mc microwave conductivity data given by Van Lint (1959) and matched it to an assumed first power dependence upon the electron velocity to determine a single particle collision cross section. Recently, Mentzoni (1965) has made a direct measurement of the electron collision frequency. By assuming the cross section to be proportional to the electron energy, he found a collision cross section which was a factor of 1.6 smaller than that given by Phelps.

Phelps (1963) conducted an analysis using the Boltzmann equation to evaluate the drift velocity and ratio of the diffusion coefficient to the electron mobility. By adjusting the various cross sections, he was able to find agreement between predicted and measured values. Phelps and Hake (1965) repeated the analysis using more refined measurements of electron mobility and diffusion coefficients. Their results, shown in Figure 2, should be accurate to within 20% in the electron energy range $0.2 \leq \varepsilon \leq 2.0$ ev. A difficulty arises in adopting a simple expression to represent the energy dependence of the cross section. As a first approximation a good fit to the experimental data is

$$q_D = (2.2 + 5.1\ \varepsilon^{1/2}) \times 10^{-16}\ \text{cm}^2 \qquad (26)$$

over the energy range $0.02 \leq \varepsilon \leq 1.0$ ev. From equation (17) $\bar{Q}_D$ is given by

$$\bar{Q}_D\ (O_2) = 2.2 \times 10^{-16}\ (1 + 3.6 \times 10^{-2}\ T_e^{1/2})\ \text{cm}^2\ , \qquad (27)$$

and applies for $150° \leq T_e \leq 5000°$.

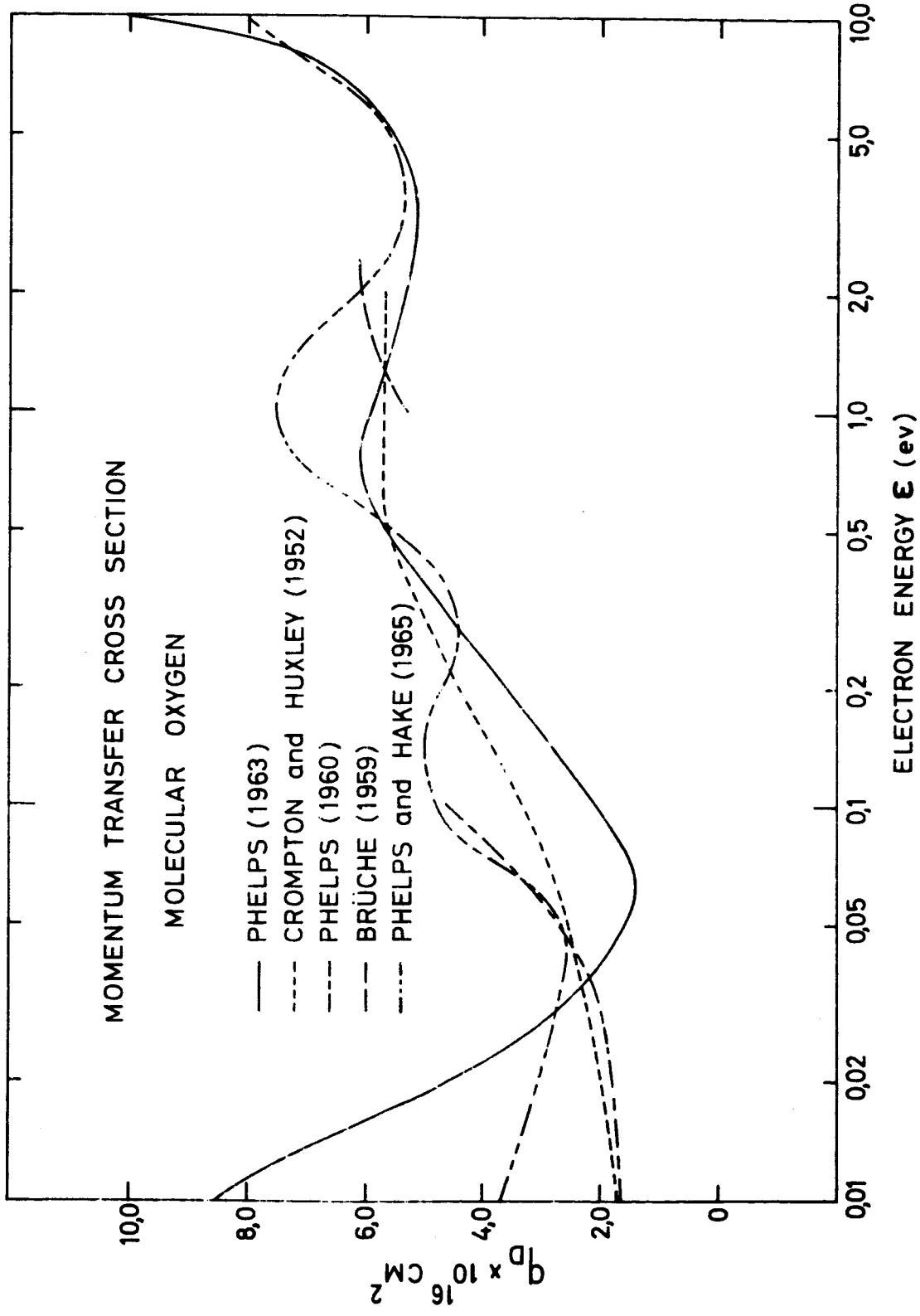
MOMENTUM TRANSFER CROSS SECTION
MOLECULAR OXYGEN
PHELPS (1963)
CROMPTON and HUXLEY (1952)
PHELPS (1960)
BRÜCHE (1959)
PHELPS and HAKE (1965)
$q_D \times 10^{16}$ CM^2
10,0
8,0
6,0
4,0
2,0
0
0,01
0,02
0,05
0,1
0,2
0,5
1,0
2,0
5,0
10,0
ELECTRON ENERGY $\mathcal{E}$ (ev)

FIGURE 2

1.3. Atomic Oxygen

There are several experimental results for the values of the total scattering cross section, q_T, in atomic oxygen, but none for the momentum transfer cross section since the chemical activity of oxygen makes measurements difficult in any closed container. Neynaber, et al. (1961) conducted a total scattering experiment but no data were taken below electron energies of 2 ev, far above the thermal energies of the upper atmosphere. At this energy it was found that $q_T = 5.5 \times 10^{-16}$ cm^2. Another experiment by Lin and Kivel (1959) was made at a lower energy. They found a momentum transfer cross section of 1.5×10^{-16} cm^2 at a mean electron energy of 0.5 ev.

From quantum theory, Klein and Bruckner (1958) derived a method of relating scattering phase shifts to measurements of photodetachment cross sections. It was later pointed out by Cooper and Martin (1962) that the calculated photodetachment cross sections did not match recent results and, further, that the effective range theory used by Klein and Bruckner was not valid at low electron energies. Cooper and Martin then recalculated the entire problem, obtaining new values of the phase shifts. In the absence of direct experimental results these values can be used to obtain an expression for the momentum transfer cross section according to the relation (McDaniel, 1964)

$$q_D = \frac{4\pi}{k_1^2} \sum_L (L+1) \sin^2 (\delta_L - \delta_{L+1}) \quad , \tag{28}$$

Where $k_1 = \frac{2\pi m_e v_e}{h}$ is the quantum mechanical wave number of relative motion, h is Planck's constant, L is the angular momentum quantum number, and δ_L is the L-th wave partial wave phase shifts of the radial solution to Schrödinger's equation.

Temkin (1958) also approached the problem in a theoretical manner and made a calculation of the S-wave contribution to the total scattering cross section. He compared his results with those of Bates and Massey (1947) and concluded that the true value of the total scattering cross section was bracketed by the two calculations. The results of these calculations are shown in Figure 3. To give an indication of the true value of q_D, the values of P-wave phase shifts given by Cooper and Martin have been added to the S-wave values of Temkin, and Bates and Massey. The values for the scattering phase shifts given by Cooper and Martin are accepted here as providing a basis for determining the momentum transfer cross section for atomic oxygen. Extrapolating from the good agreement found by these authors for the problem of negative ion photodetachment, it appears that the error involved in using the theoretical phase shifts for determining q_D should be less than 30% for electron energies below 0.5 ev. Thus, it does not appear unreasonable to accept an average value of $q_D = (3.4 \pm 1.0) \times 10^{-16}$ cm^2 for electrons in atomic oxygen. Using this expression the average momentum transfer cross section becomes

$$\bar{Q}_D(0) = (3.4 \pm 1.0) \times 10^{-16} \text{ cm}^2, \tag{29}$$

independent of the electron temperature for $T_e < 4000°K$.

1.4. Atomic Hydrogen

No experimental measurements have been made for the electron-hydrogen momentum transfer cross section. However, recent theoretical treatments of electron scattering in hydrogen have produced predicted total cross sections which are in good agreement with the measured total cross section as determined by Neyaber, et al. (1961) and Brackman, et al. (1958). Thus, the error involved in using the same partial wave phase shifts to determine the momentum transfer cross section by means of equation (28) should not be large. Two similar

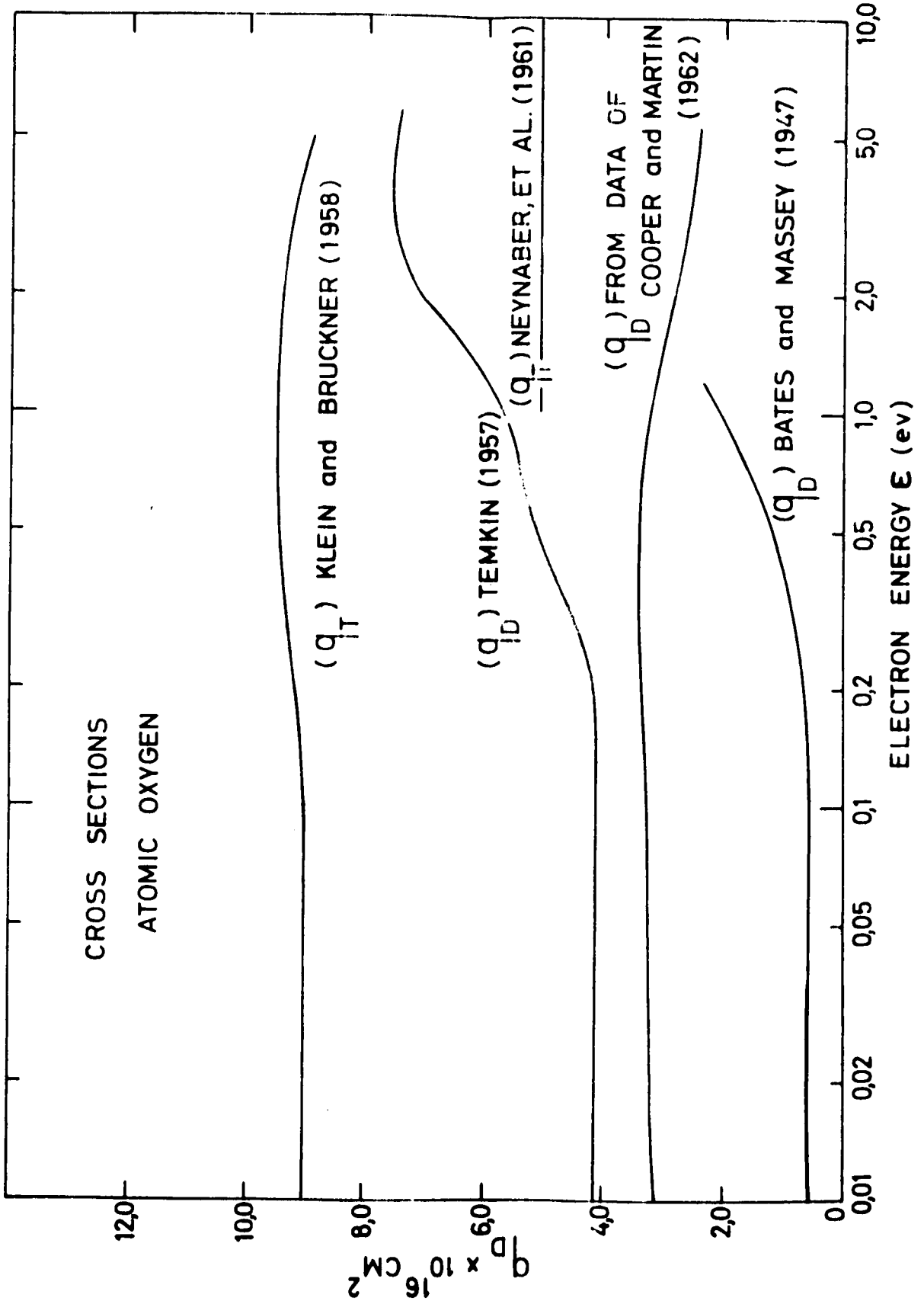
CROSS SECTIONS
ATOMIC OXYGEN
(q_T) KLEIN and BRUCKNER (1958)
(q_D) TEMKIN (1957)
$(q_{\overline{11}})$ NEYNABER, ET AL. (1961)
(q_D) FROM DATA OF COOPER and MARTIN (1962)
(q_D) BATES and MASSEY (1947)
$q_D \times 10^{16}$ CM^2
ELECTRON ENERGY ε (ev)
12,0
10,0
8,0
6,0
4,0
2,0
0
0,01
0,02
0,05
0,1
0,2
0,5
1,0
2,0
5,0
10,0

FIGURE 3

theoretical calculations of the scattering phase shifts for electrons in atomic hydrogen have been published by Smith, et al. (1962) and Burke and Schey (1962). Both derivations employ a close coupling approcimation where the scattering wave function is expanded in terms of hydrogen atom stationary eigenstates. For the present treatment, the results of Smith et al. are used.

In calculating q_D from the partial wave phase shifts for atomic hydrogen it is necessary to include both the singlet and triplet contributions to the scattering. Using equation (28) with the proper weighting factors yields a form for q_D shown in Figure 4. A suitable analytic expression for the energy dependence is

$$q_D = (54.7 - 28.7\ \varepsilon) \times 10^{-16}\ cm^2, \qquad (30)$$

which gives a cross section considerably larger than that found for the other atmospheric constituents. Using this expression the average momentum transfer cross section is

$$\bar{Q}_D(H) = (54.7 - 7.45 \times 10^{-3}\ T_e) \times 10^{-16}\ cm^2\ , \qquad (31)$$

over the temperature range $150° \leq T_e \leq 5000°$. It is difficult to assess the error involved in deriving $\bar{Q}_D$ but an arbitrary estimate of $\pm$ 25%, based on the correspondance between theoretical and experimental results for the total cross section, should give a reasonable indication. A difficulty is noted, however, in that there exist no reported measurements of the scattering cross section below 1 ev, and it is possible that there may be errors in the application of theoretical values to this region.

1.5. Helium

The experimentally determined values for q_D are in good agreement. Pack and Phelps (1961) conducted an electron drift experiment over the energy range $0.003 < \varepsilon < 0.05$ ev obtaining a constant cross section of

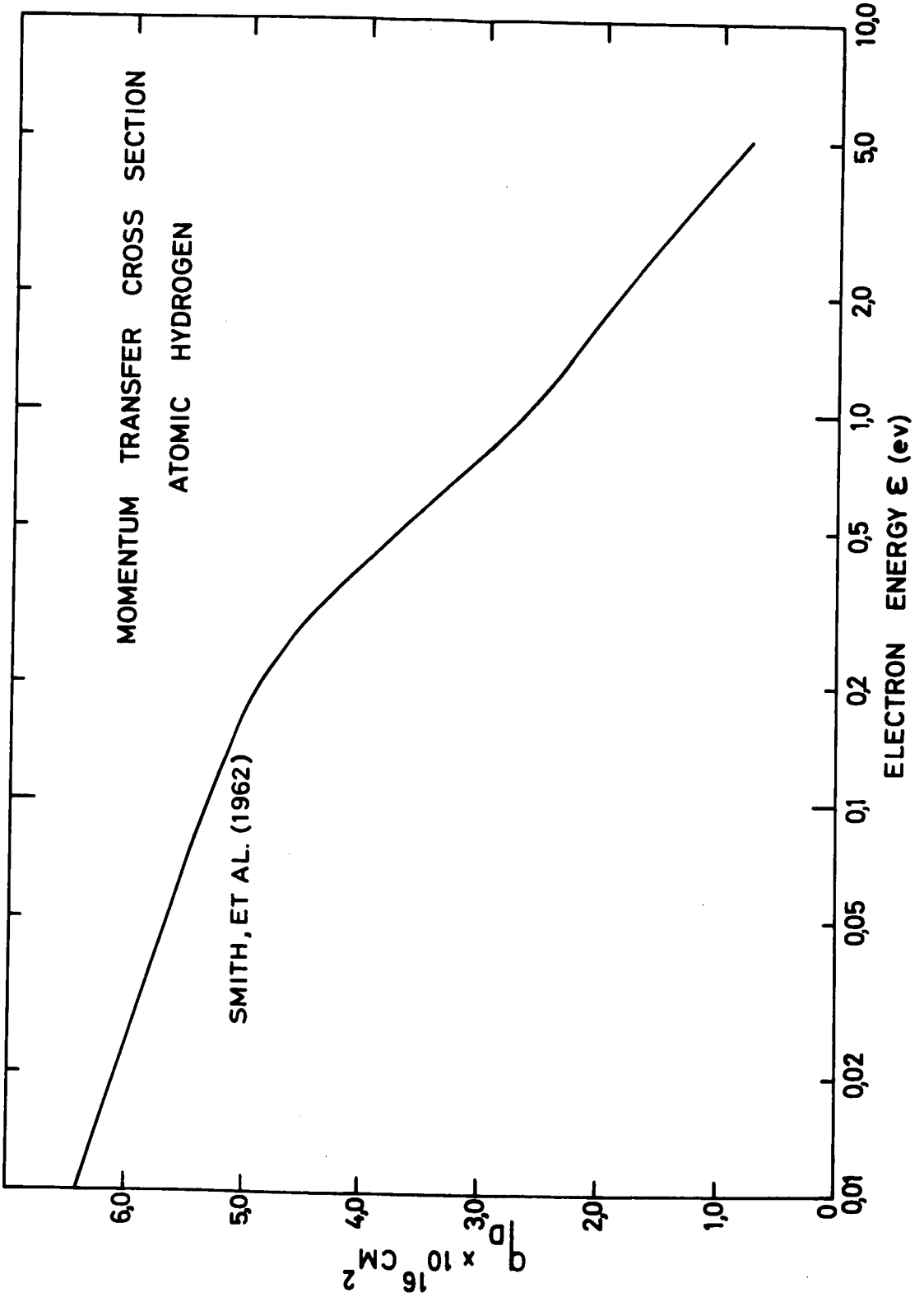
MOMENTUM TRANSFER CROSS SECTION
ATOMIC HYDROGEN
SMITH, ET AL. (1962)
$q_D \times 10^{16}$ CM^2
ELECTRON ENERGY ε (ev)
0
1,0
2,0
3,0
4,0
5,0
6,0
0,01
0,02
0,05
0,1
0,2
0,5
1,0
2,0
5,0
10,0

FIGURE 4

5.6×10^{-16} cm^2. Phelps, et al. (1951) performed a microwave conductivity experiment over the range $0.02 < \varepsilon < 0.04$ ev, finding virtually the same value. Gould and Brown (1954) made a separate determination by a different microwave technique which gave the value 5.2×10^{-16} cm^2 for all energies between 0.0 and 4.0 ev. Anderson and Goldstein (1956b) made microwave conductivity measurements down to electron energies of 0.05 ev and found a constant cross section of 6.8×10^{-16} cm^2.

Thus, for the case of electron-helium scattering, it appears reasonable to accept the value $q_D = (5.6 \pm 0.6) \times 10^{-16}$ cm^2, corresponding to an uncertainty of 10%, over the energy range 0.0 to 5.0 ev. The average momentum transfer cross section is

$$\bar{Q}_D(He) = (5.6 \pm 0.6) \times 10^{-16} \text{ cm}^2 \quad , \tag{32}$$

independent of the electron temperature.

The values of $\bar{Q}_D$ for the different gases considered here are shown in Figure 5 as a function of electron temperature. The largest cross sections are associated with H and N_2, these reaching values of 60×10^{-16} cm^2 and 12×10^{-16} cm^2, respectively. The cross sections of He an O are essentially constant over the range of temperatures indicated here. However, there must exist some uncertainty in the velocity dependence of q_D for several gases. For N_2 and O_2 the uncertainty in $\bar{Q}_D$ should be less than 20% (Englehardt, et al. 1964 ; Phelps and Hake, 1965) while for He a value of 10% is adequate. It is difficult to assess the possible error in the quantum calculations of q_D for H and O but the previous arbitrary estimates of $\pm$ 25% and $\pm$ 30%, respectively, should be reasonable. In fact, further experimental studies of H and O are needed to check the theoretical cross sections presented here.

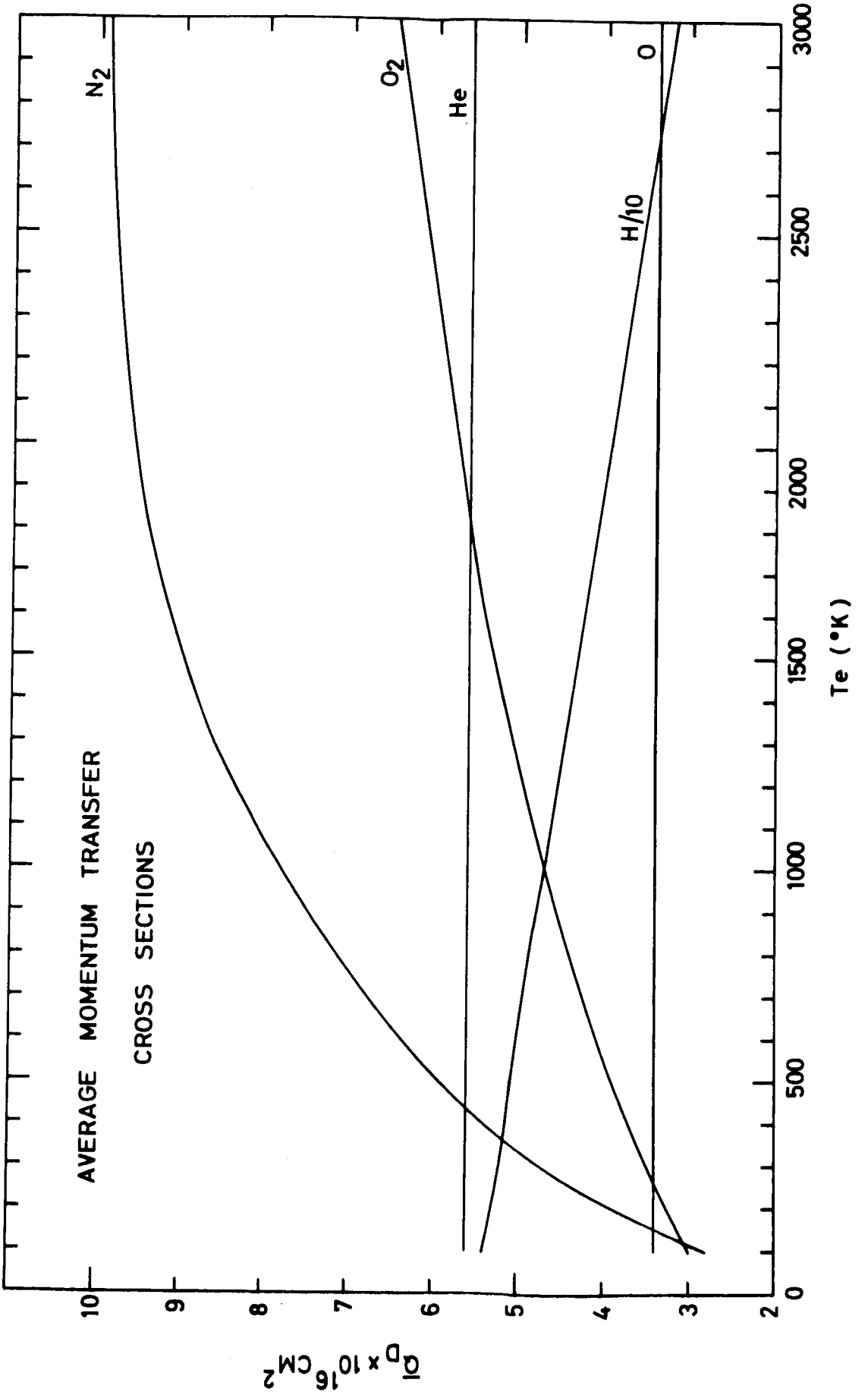
AVERAGE MOMENTUM TRANSFER
CROSS SECTIONS
$\bar{Q}_D \times 10^{16} CM^2$
Te (°K)
N_2
O_2
He
O
H/10
10
9
8
7
6
5
4
3
2
0
500
1000
1500
2000
2500
3000

FIGURE 5

2. Cross Section for Charged Particles

The momentum transfer cross section for charged particles of arbitrary mass can be derived through a knowledge of the differential scattering cross section and application of equations (2c) and (11). For electrical forces the interaction varies as r^{-2} and the Rutherford differential scattering cross section applies in the form (McDaniel, 1964)

$$\sigma(\theta,g) = \left(\frac{Z_1 Z_2 e^2}{2\mu g^2}\right)^2 \sin^{-4}\left(\frac{\theta}{2}\right) \quad , \tag{33}$$

where μ is the two particle reduced mass, $Z_{1,2}$ are the respective atomic charges, e is the electron charge, and θ is the center of mass scattering angle. Applying this to equation (2) yields

$$q_D = 8\pi\left(\frac{Z_1 Z_2 e^2}{2\mu g^2}\right)^2 \ln\left[\frac{1}{1 - \cos\theta_m}\right] \quad . \tag{34}$$

The normal limits of integration for equation (2) should cover all scattering angles between 0 and π radians. However it is found that the use of zero for the lower limit causes the Coulomb integral to diverge. To prevent this, the integration is arbitrarily truncated at a minimum angle, θ_m, whose value must be determined from the parameters of the charged particle gas. From Bachynski (1965) the relation between the impact parameter, b, the scattering angle, θ, and the relative velocity, g, is given by

$$(1 - \cos\theta) = \frac{2}{1 + (b/b_o)^2} \tag{35}$$

where $b_o = \frac{Z_1 Z_2 e^2}{\mu g^2}$ using previously defined quantities. From this equation it is seen that the minimum scattering angle, θ_m, is determined by the maximum value of the impact parameter b. As discussed bu Montgomery and Tidman (1964), collisions leading to large scattering

angles are rare in a plasma since the long range Coulomb force tends to deflect the slow thermal particles only through small angles. A good approximation is that $b \gg b_o$, with the result that equation (34) becomes

$$(1 - \cos\theta_m) = 2\left(\frac{b_o}{b_1}\right)^2 \tag{36}$$

where b_1 is the maximum impact parameter corresponding to θ_m.

Several different approaches have been taken to relate the characteristic parameters of the plasma to the maximum impact parameter b_1. Chapman and Cowling (1952) assumed that the maximum interaction distance was limited to the average interparticle spacing. This ignored, however, the influence of the longer range collisions which are responsible for the small angular deviations of the electrons. A more accurate treatment was introduced by Cohen, et al. (1950) who took into consideration the shielding of charge due to electrostatic polarization effects. By means of Poisson's equation in conjunction with the Boltzmann equation for nearly equilibrium conditions, it can be shown (Salpeter, 1963) that the potential, $\varphi(r)$ of a particle of charge $Z_1 e$ at an origin of coordinates within a plasma is given by

$$\varphi(r) = \frac{Z_i e}{r} \exp(-r/\lambda_D) \quad , \tag{37}$$

where r is the radial separation distance and λ_D is the Debye shielding distance, defined as

$$\frac{1}{\lambda_D^2} = 4\pi e^2 \left\{ \frac{Z_1 n_1}{kT_1} + \frac{Z_2 n_2}{kT_2} \right\} \quad , \tag{38}$$

with $T_{1,2}$ the respective Maxwellian temperatures and $n_{1,2}$ the number densities. For a plasma, the Debye length represents the maximum distance over which microscopic density fluctuations are correlated

by random electric fields. The Debye length can also be interpreted as a measure of the effective range of the Coulomb interaction between two charged particles. Cohen, et al. (1950) essentially prove this and conclude that the Debye length should be used in equation (34) as the maximum impact parameter. Therefore,

$$q_D = 16\pi\left(\frac{Z_1 Z_2 e^2}{2\mu g^2}\right)^2 \ln\left[\frac{\mu g^2}{Z_i e^2}\ \lambda_D\right] \tag{39}$$

for the velocity dependent momentum transfer cross section. This equation is a general relation for particles of arbitrary mass and can be used for electron-ion, electron-electron, and ion-ion interactions.

The argument of the logarithm in equation (39) can be rewriten in terms of the energy ε of two colliding particles, as viewed in the center of mass system, in the form

$$\Lambda = \frac{\mu g^2}{Z_1 Z_2 e^2}\ \lambda_D = \frac{2\varepsilon}{Z_1 Z_2 e^2}\ \lambda_D. \tag{40}$$

This term is common to all calculations of ionized gases and, according to Chapman (1956), can introduce a possible error of 10% into the derivation of the cross section. Table 1 lists the values of $\ln \Lambda$ for various particle energies and Debye lengths.

For the ionospheric conditions of particle energies and Debye lengths, it is found that most normal variations lie within the indicated uncertainty of 10% at 15.0 ± 1.5. However, for some problems involving very energetic photoelectrons, a higher value may be required.

Table 1 Values of ln Λ

ε (ev) \ λ_D (cm)	0.1	0.5	1.	2.	5.	10.	20.	50.	100.
1×10^{-2}	9.5	11.1	11.8	12.5	13.5	14.1	14.8	15.8	16.4
5×10^{-2}	11.1	12.8	13.5	14.1	15.1	15.8	16.4	17.4	18.1
1×10^{-1}	11.8	13.5	14.1	14.8	15.8	16.4	17.1	18.1	18.8
5×10^{-1}	13.5	15.1	15.8	16.4	17.4	18.1	18.8	19.7	20.4
1×10^{0}	14.1	15.8	16.4	17.1	18.1	18.8	19.4	20.4	21.1
5×10^{0}	15.8	17.4	18.1	18.8	19.7	20.4	21.1	22.0	22.7
1×10^{1}	16.4	18.1	18.8	19.4	20.4	21.1	21.7	22.7	23.4
5×10^{1}	18.1	19.7	20.4	21.1	22.0	22.7	23.4	24.3	25.0
1×10^{2}	18.8	20.4	21.0	21.7	22.6	23.4	24.0	24.9	25.7

With equations (20) and (39) it is possible to derive $\bar{Q}_D$ for two gases in the different Maxwellian temperatures. The result is

$$\bar{Q}_D = \frac{\pi}{2}\left(\frac{Z_1 Z_2 e^2}{\mu}\right)^2 \frac{\ln \Lambda}{\left(\frac{kT_1}{m_1} + \frac{kT_2}{m_2}\right)^2} \qquad (41)$$

and is valid for particles of arbitrary mass and charge. An interesting feature of this cross section is its rapid decrease with increasing gas temperatures.

For electron-ion scattering this equation may be reduced by taking $T_e/m_e \gg T_i/m_i$, $Z_1 = 1$, $Z_2 = Z_i$, giving

$$\bar{Q}_D\ (e\text{-}i) = \frac{\pi}{2}\,\frac{(Z_i e^2)^2\ \ln \Lambda}{(kT_e)^2} \qquad (42)$$

dependent only upon the electron temperature. Numerically this becomes, with $\ln \Lambda = 15$,

$$\bar{Q}_D(e\text{-}i) = (6.6 \pm 0.6)\ 10^{-5}/T_e^2\ cm^2 \qquad (43)$$

A brief comparison can be made here regarding the relative importance of electron-neutral and electron-ion collisions. For numerical purposes a general electron-neutral cross section of $5x10^{-16}$ cm^2 is adequate. Thus, the ratio of the electron-neutral cross sections can be written as

$$R = \bar{Q}_{ei}/Q_{en} = 1.3 \times 10^{11}/\ T_e^2\ . \qquad (44)$$

Since the electron temperature, T_e, generally assumes values between 250° and 3600°K in the upper atmosphere, we see that the ratio of the cross sections varies from $2x10^6$ to $1x10^4$. This implies that the effects of electron-ion collisions will become important when the ratios of the ion to neutral densities reach $5x10^{-7}$ and $1x10^{-4}$, respectively.

IV. ELECTRON COLLISION FREQUENCIES AND ENERGY TRANSFER RATES

1. Neutral Gases

The momentum transfer cross sections for electron-neutral collisions which were adopted in the previous section can be used to arrive at expressions for the electron energy transfer collision frequencies and energy transfer rates. Tables 2 and 3, respectively, give the final results.

A comparison of these values can be made with those previously reported. Care must be taken, however, to consider only elastic energy loss processes since, as shown by Gerjuoy and Stein (1955) and Frost and Phelps (1962), the impact exciatation of rotational and vibrational states in diatomic molecules can be an efficient energy loss process for an electron gas.

Table 2 Electron Collision Frequencies (sec^{-1})

N_2 : $\bar{\nu}_e = 2.33\times10^{-11}\, n(N_2)\, [1 - 1.21\times10^{-4}\, T_e]\, T_e$

O_2 : $\bar{\nu}_e = 1.82\times10^{-10}\, n(O_2)\, [1 + 3.6\times10^{-2}\, T_e^{1/2}]\, T_e^{1/2}$

O : $\bar{\nu}_e = 2.8\times10^{-10}\, n(O)\, T_e^{1/2}$

H : $\bar{\nu}_e = 4.5\times10^{-9}\, n(H)\, [1 - 1.35\times10^{-4}\, T_e]\, T_e^{1/2}$

He : $\bar{\nu}_e = 4.6\times10^{-10}\, n(He)\, T_e^{1/2}$

Table 3 Elastic Electron Energy Transfer Rates (ev cm^{-3} sec^{-1})

N_2 : $dU_e/dt = - 1.77\times10^{-19}\, n_e n(N_2)\, [1 - 1.21\times10^{-4}\, T_e]\, T_e\, (T_e - T)$

O_2 : $dU_e/dt = - 1.21\times10^{-18}\, n_e n(O_2)\, [1 + 3.6\times10^{-2}\, T_e^{1/2}]\, T_e^{1/2}\, (T_e - T)$

O : $dU_e/dt = - 3.74\times10^{-18}\, n_e n(O)\, T_e^{1/2}\, (T_e - T)$

H : $dU_e/dt = - 9.63\times10^{-16}\, n_e n(H)\, [1 - 1.35\times10^{-4}\, T_e]\, T_e^{1/2}\, (T_e - T)$

He : $dU_e/dt = - 2.46\times10^{-17}\, n_e n(He)\, T_e^{1/2}\, (T_e - T)$

For N_2, Dalgarno, et al. (1963) used an energy loss equation which was based upon the cross section measurements of Pack and Phelps (1961). This gave

$$\frac{dU_e(N_2)}{dt} = - 9.85\times10^{-20}\, T_e (T_e - T)\, n(N_2)\, n_e \text{ ev } cm^{-3} \text{ sec}^{-1} \quad (45)$$

A discrepancy is noted, however, if one uses the value of $\bar{Q}_D$, given in equation (25), which is also valid for the low temperatures where the data of Pack and Phelps (1961) apply. It appears that equation (45) is a factor of 1/2 smaller than would be found through application of the energy transfer rate given by equation (18). Hence, it is found

that equation (45) underestimates the elastic energy transfer rate for $T_e < 3600°K$.

In considering O_2, Dalgarno, et al. (1963) adopted the value

$$\frac{dU_e(O_2)}{dt} = -6.1\times10^{-20}\, T_e(T_e - T)\, n(O_2)n_e \text{ ev cm}^{-3} \text{ sec}^{-1}, \quad (46)$$

based upon the microwave studies of A.V. Phelps (Dalgarno, 1961). This equation yields energy loss rates which are 20 to 30% larger than those given in Table 3. This difference arises from the use here of newer cross section data and the application of the exact energy transfer equation.

The problem of electron energy loss in atomic oxygen has been considered by Hanson and Johnson (1961) and Hanson (1963). In the latter paper a cross section of 2×10^{-16} cm^2 was adopted, leading to an energy transfer rate of

$$\frac{dU_e(O)}{dt} = -1.42\times10^{-18}\, T_e^{1/2}\, (T_e - T)\, n(O)n_e \text{ ev cm}^{-3} \text{ sec}^{-1}. \quad (47)$$

If, however, equation (18) had been applied with the stated cross section, this rate would be a factor of 1.55 greater. In the same way, Dalgarno, et al. (1963) took a cross section of 6×10^{-17} cm^2 and arrived at the expression

$$\frac{dU_e(O)}{dt} = -1.3\times10^{-18}\, T_e^{1/2}(T_e - T)\, n(O)n_e \text{ ev cm}^{-3} \text{ sec}^{-1}. \quad (48)$$

Again, the direct use of their cross section in equation (18) leads to the value

$$\frac{dU_e(O)}{dt} = -6.6\times10^{-19}\, T_e^{1/2}\, (T_e - T)\, n(O)n_e \text{ ev cm}^{-3} \text{ sec}^{-1}. \quad (49)$$

which is a factor of 5.7 lower than the present rate. However, if the rate given by equation (48) is their final result, then the actual momentum transfer cross section used was 1.8×10^{-16} cm^2 which is close to the value adopted here.

For atomic hydrogen and helium there appear to be no determinations of energy transfer rates which could be used for comparison with this work.

2. Charged Particles Gases

Using the previous value for $\bar{Q}_D$, the charged particle collision frequency of a particle of mass m_1 in a gas of particles of mass m_2 is

$$\bar{\nu}_{12} = \frac{4}{3} \sqrt{2\pi}\, n_2 \left(\frac{Z_1 Z_2 e^2}{\mu}\right)^2 \frac{\ln \Lambda}{\left(\frac{kT_1}{m_1} + \frac{kT_2}{m_2}\right)^{3/2}} \qquad (50)$$

To reduce this to electron-ion scattering we take $T_e/m_e \gg T_i/m_i$ and $Z_1 = 1$ giving

$$\bar{\nu}_e = \frac{4}{3} \sqrt{2\pi}\, n_i \frac{(Z_i e^2)^2 \ln \Lambda}{(kT_e)^{3/2}} \qquad (51)$$

or, numerically with $\ln \Lambda = 15.$,

$$\bar{\nu}_e = (54. \pm 5.) n_i / T_e^{3/2} \ sec^{-1}. \qquad (52)$$

This result agrees with that derived by Nicolet (1953) from the work of Chapman and Cowling (1952).

There have been three experimental studies which have substantiated the expression adopted here. Anderson and Goldstein (1956a) conducted microwave experiments in a decaying nitrogen plasma and found a collision frequency which agrees with equation (52) to within 10%.

Chen (1964) has reported a more refined experimental technique which permitted him to measure the electron-ion collision frequency in a neon plasma. He found that equation (52) correctly represents the collision frequency over a wide range of temperatures and ion densities. The numerical results of his experiments agree to within 6%. Mentzoni (1965) has also been able to confirm the validity of equation (52) to within the indicated accuracy of 10%.

The energy transfer rate between two charged gases having Maxwellian velocity distributions but no restriction on the temperatures or masses is

$$\frac{dU_1}{dt} = - 4\sqrt{2\pi}\, n_1 n_2 \frac{(Z_1 Z_2 e^2)^2}{m_1 m_2} k(T_1 - T_2) \frac{\ln \Lambda}{\left(\frac{kT_1}{m_1} + \frac{kT_2}{m_2}\right)^{3/2}} , \qquad (53)$$

which, for electron-ion energy transfer, reduces to the well known form,

$$\frac{dU_e}{dt} = - 4\sqrt{2\pi}\, n_e n_i \frac{m_e^{1/2}}{m_i} k(T_e - T_i) \frac{(Z_i e^2)^2 \ln \Lambda}{(kT_e)^{3/2}} . \qquad (54)$$

Numerically, this becomes for singly charged ions,

$$\frac{dU_e}{dt} = - (7.7 \pm 0.8) \times 10^{-6} n_e n_i \frac{(T_e - T_i)}{A_i T_e^{3/2}} \text{ ev cm}^{-3} \text{ sec}^{-1} \qquad (55)$$

where A_i is the ion atomic mass in amu.

It is interesting to note that for a fixed temperature T_2 in equation (53) there occurs a maximum energy transfer rate which, for the general case, is found at a temperature $T_1 = (3 + 2m_1/m_2)T_2$. For electron-ion energy transfer this reduces to the usual result that $T_e = 3T_i$. This analysis ignores, however, the contribution of the

temperature dependence of the term ln Λ. For electron-ion energy transfer inclusion of this effect leads, for ln $\Lambda = 15$, to the relation $T_e = 3.5\ T_i$, an increase in the critical temperature by 16%.

The importance of equation (53) lies in the possibility of describing all charged particle energy transfer in terms of one general result ; the reduction to electron-ion, electron-electron, or ion-ion cases being made simply through a proper choice of subscripts, charges and mass ratios.

V.- SUMMARY AND CONCLUSIONS

It has been shown that a suitable synthesis can be made of the elastic energy transfer equation such that a generalized energy transfer collision frequency can be defined for conditions of thermal nonequilibrium. In considering the specific problem of electron-neutral collisions it was necessary to analyze both laboratory data and theoretical studies of scattering phase shifts in order to arrive at satisfactory expressions for the average momentum transfer cross sections. Thus, while $\bar{Q}_D$ for N_2, O_2, and He are founded upon experimental results, the values for O and H have been newly derived from the recent theoretical calculations of scattering phase shifts.

The derivation of the charged particle momentum transfer cross section was shown to follow from the standard expression for the Rutherford differential scattering cross section and the final expressions are valid for charged gases of arbitrary temperatures and particle masses, relativistic effects being ignored. A comparison of the electron-ion and electron-neutral cross sections was made to indicate the much larger value which is associated with charged particle collisions.

Using the different momentum transfer cross sections electron-neutral collision frequencies and energy transfer rates were derived and compared with expressions previously used. In general, the differences between various authors can be as large as a factor of two.

The problem of ion collision frequencies and energy transfer will be discussed in a subsequent paper.

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