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Hard copy (HC) 1.00

Microfiche (MF) .50

ff 653 July 65

CONDUCTIVE HEATING OF THE SOLAR WIND, III, BREAKDOWN OF THE NAVIER-STOKES EQUATIONS

Prepared for

National Aeronautics and Space Administration

Washington; D. C. 20546

Contract No. NASw-1226

4 JUNE to 4 DECEMBER 1965

Space Sciences Laboratory
Physical Research Division

TRW SYSTEMS
One Space Park
Redondo Beach, California 90278

CONDUCTIVE HEATING OF THE SOLAR WIND, III,
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F. L. Scarf

TRW SYSTEMS

In our most recent discussion of solar wind theory (Scarf and Noble, 1965), no detailed prediction of the flow pattern near the earth was given. For the best fit solution $nu \simeq 1.5 \times 10^{11}/cm^2$ sec at $r = 10R_{\odot}$ and the continuity relation, $nur^2 = c$, then yields an unambiguous flux value at the earth, $J_{av.} = 3.24 \times 10^8$ ions/cm^2 sec, but the proportions of n and u which make up this flux must be derived by solving the dynamical equations. Unfortunately, the validity of the Navier-Stokes equations with conventional transport coefficients becomes questionable long before the earth is reached.

The basic problem has to do with the assumption $\ell/L \ll 1$, used to derive the Navier-Stokes equation by the Chapman-Enskog technique and the correct L must first be identified. If L is simply defined by $L|dN/dr| \simeq N$, then the original solutions of the conductive heating equations indicate that $\ell \simeq L$ near 75 - 100 $R_{\odot}$. However, the problem is not so easily solved, and the complications become important much closer to the sun. It has already been noted that these conductive heating equations possess unphysical solutions with $T(r) = 0$, $r < \infty$ and $T(r) \to T_{\infty} > 0$, $r \to \infty$; in each case $u \to \infty$ at the singularity. With the hope that these annoying solutions were only present because the viscous forces (which presumably inhibit large velocity gradients) had been artificially omitted, Scarf and Noble (1964, 1965) introduced the appropriate viscous terms into the Navier-Stokes equations. For spherically symmetric flow the complete Navier-Stokes equations have the form

$$mnu \frac{du}{dr} + \frac{d}{dr}(nkT) + \frac{nGM_{\odot}m}{r^2} = \frac{4}{3} r \frac{d\mu}{dr} \frac{d}{dr}\left(\frac{u}{r}\right) + \frac{4}{3}\mu \frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr}(r^2u)\right] \quad (1)$$

and

$$\frac{mu^2}{2} - \frac{GM_{\odot}m}{r^2} + \frac{5}{2}kT - \frac{r^2k(T)}{c}\frac{dT}{dr} - \frac{4}{3}\frac{\mu(T)r^2}{c}\left(u\frac{du}{dr} - \frac{u^2}{r}\right) = E \quad (2)$$

where E and $c = nur^2$ are constants of motion. For fully ionized hydrogen, $\mu = \mu_0(T/T_0)^{5/2}$, with $\mu(T_0)$ given in Eq. (9) of Scarf and Noble (1965). However, unlike the conductive coefficient, k(T), the viscous coefficient does change significantly as the gas composition is varied. The addition of 10% helium reduces μ to about 70% of its value for fully ionized hydrogen. Thus for the solar wind the Prandtl number, $\kappa\mu/mk$, is approximately equal to 0.013. This explains why viscous contributions were previously neglected. When the thermal and kinetic energies and gradients are comparable, the viscous term in the energy equation is roughly (1 - 2)% of the conductive term and where $\kappa T > mu^2/2$, the ratio is even smaller.

This kind of argument is indeed appropriate when one discusses normal subsonic flow, but it should already be clear from Eq. (2) that a serious error is possible when the flow is supersonic. Even though $\mu(T)$ is numerically very small, if u is large and constant, the term $4\mu(T)u^2r/3$ can play a very important role in the energy balance equation as r increases.

Numerical integration of Eqs. (1), (2) confirms this expectation, but the way in which the viscous stresses modify the flow appears at first glance, to be unphysical. Figure 4 of Scarf and Noble (1965) shows some typical results derived by integrating in both directions from the crossover. The parameter B is essentially the Prandtl number times $A = 2k(T_0)GM_\odot m/\kappa^2 T_0 c$ where T_0 is the temperature at the coronal base. For the case shown there, A = 200, $T_0 = 1.5 \times 10^6$ K, and the curve labeled $B \equiv 0$ is the one derived earlier, with no viscous terms at all. The velocity profile labeled B = 2.46 is the numerical solution of the Navier-Stokes equation with the appropriate coefficient of viscosity for the coronal gas, and to the accuracy shown in this figure, the temperature distribution is unchanged by the addition of viscosity.

It was noted by Scarf and Noble that in the subsonic region the viscous terms produce a negligible change in u(r). This was expected because $\kappa\mu \ll mk$ and $(1/2)mu^2 \leq \kappa T$ here. It can also be seen that the viscous terms do induce a significant change in u(r) in the supersonic region. Again, this is not unexpected; although $\kappa\mu/mk$ remains small,

$(1/2)mu^2$ becomes large compared to kT, and the viscous contribution to the energy equation is comparable to the conductive term. However, it was found that $u(r,B)$ is greater than $u(r,B \equiv 0)$. Physically, one would expect viscous dissipation to yield a lower flow speed relative to the sun, and not a higher one.

The answer to this paradox was found by trying to vary the Prandtl number to approach $B = 0$. The curves for $B = 0.1$, 1.0, and 3.56 were calculated using artificial and unphysical values for $\mu(T_0)$, keeping $k(T_0)$ fixed. It was determined that as $\mu(T_0)$ (or B) is increased, the flow speed at any given radius does indeed decrease in a physically sensible manner. However, as $B \to 0$, we do not approach the curve $B \equiv 0$ which is the solution of the non-viscous conductive heating equations.

In fact, the "paradox" has an exact analogy when thermal conduction alone is considered. If μ and k are both neglected, then the flow is adiabatic and $T(r) \to r^{-4/3}$. However, if only the viscous terms are omitted, then $T(r) \to (k(T_0)r)^{-2/5}$ or $(r^{-2/7} + Ck(T_0)^{-1}r^{-4/7} + \ldots)$. The solutions with finite thermal conductivity do not yield adiabatic flow in the limit $k(T_0) \to 0$, and similarly, the solutions with small but finite $\mu(T_0)$ do not go over to the $\mu(T_0) \equiv 0$ solutions as $\mu(T_0) \to 0$. From a mathematical point of view, this is also clear. The differential equations are of the form $dT/dr = G/k(T)$, $du/dr = F/\mu(T)$ and $k \equiv 0$; $\mu \equiv 0$ represent singularities. The result is well known to aerodynamicists. In "Flow of Rarefied Gases," Schaaf and Chambre´ (1961) remark on page 33, "Solutions are thus singular in the viscosity (or equivalently, the mean free path), and the boundary layer solution (for $\mu \to 0$) cannot be obtained by perturbation schemes starting with the inviscid solution."

If the $B = 2.46$ curve in Figure 4 of Scarf and Noble (1965) is regarded as the "correct" one, then this solution can be used to investigate the limits of the continuum or fluid models of the corona. The first question has to do with the precise meaning of the restriction $\ell/L \ll 1$. In particular, it must be ascertained which scale length is involved. This can be determined by examining the higher order terms in the Chapman-Enskog treatment. The successive approximations are

$$
\begin{aligned}
p_{ij}^{(0)} &= 0 , \\
p_{ij}^{(1)} &= \mu\tau_{ij} , \\
p_{ij}^{(2)} &= \mu\tau_{ij} + \frac{\mu^2}{nkT}\,\tau_{ij}\,\vec{\nabla}\cdot\vec{u} + \ldots ,
\end{aligned} \tag{3}
$$

(Schaaf and Chambre', 1961, p. 30) and thus the Navier-Stokes equations should be valid if $\mu(du/dr)/nkT \ll 1$. Since $\mu \approx nm^{1/2}(kT)^{1/2}\ell$, this can be written as

$$
\frac{\ell}{L}\left(\frac{mu^2}{kT}\right)^{1/2} \ll 1, \qquad L\left|du/dr\right| \simeq u \tag{4}
$$

and the appropriate scale length turns out to be the one associated with the streaming velocity. (This is not a statement of general validity. The complete expression for $p_{ij}^{(2)}$ contains temperature gradients as well. The Navier-Stokes equations become questionable whenever one of these terms is significantly large. In our case, the velocity gradients are steep beyond the crossover while the thermal gradients remain moderate.) By inspection of the numerical solution, Scarf and Noble (1964) pointed out that the above inequality is not satisfied beyond $r = 20\ R_{\odot}$.

In the language of aerodynamics, we pass from the continuum region to the "slip flow" region when the appropriate Knudsen number approaches unity. For a neutral gas, the slip flow regime is bounded by the region of continuum flow ($\ell \ll L$) and free molecular flow ($\ell \gg L$). Actually, because the solar wind is a magnetized plasma, there is no region in which free flow occurs in the classical gasdynamics sense. Nevertheless, we can define three regions. For $(\mu\vec{\nabla}\cdot\vec{u}/nkT) \ll 1$, the full Navier-Stokes equations are valid. For $\mu(\vec{\nabla}\cdot\vec{u})/nkT \sim 1$, but $\ell\left|dT/dr\right| \ll T$, the Navier-Stokes equations are not valid, but there are enough collisions to maintain a statistical distribution function with a well-defined temperature. For $\ell\left|dT/dr\right| \geq T$, ordinary two-body collisions are unimportant and the characteristics of the particle distribution functions are determined by other phenomena.

In ordinary gasdynamics no rigorous formulation of the equations of motion is available for slip flow. Various complex systems of equations (the Burnett equations, the Thirteen Moment equations) have been proposed to replace the Navier-Stokes equations for moderate Knudsen numbers, but these replacements introduce severe difficulties, and considerable doubt exists about their physical validity. (Schaaf and Chambre', 1961, pp. 31-34.) It seems likely that no moment equations will really be of use here, so that the flow patterns have to be obtained by solving the Boltzmann equation itself. However, it appears that the Navier-Stokes equations, with some modifications, provide a better description of the gas in the slip flow regime than the higher order systems.

This discussion indicates that at present no rigorous treatment of the solar wind flow is possible beyond (15 - 20) $R_{\odot}$. In order to obtain some bound to the range of flow patterns, Scarf and Noble (1964) arbitrarily assumed that the Navier-Stokes equations are strictly valid up to 17 $R_{\odot}$, with adiabatic continuum flow beyond 17 $R_{\odot}$. The results for a quiet solar wind are shown in Table I. Although these numbers obviously are subject to considerable revision as more realistic treatments of the region beyond (15 - 20) $R_{\odot}$ are considered, the entries can be used to evaluate roughly the range of importance of two-body collisions. We find $\ell(T,n)$ $dT/dr \simeq T$ for $r \simeq 75\ R_{\odot}$, and two-body collisions cannot serve to maintain the Boltzmann distribution function beyond this radius. The radial flow itself tends to introduce a severe anisotropy in the absence of a randomizing mechanism (i.e., $T_{\parallel}$ is generally not equal to $T_{\perp}$ for collisionless spherical flow because the particles with $\vec{v}$ parallel to the mean flow direction tend to migrate farther than those with finite $v_{\perp}$; in the presence of a central force field this anisotropy is even more marked, because the particles travel on ballistic orbits). Thus, the close agreement between the distributions frequently observed at the earth and a Boltzmann distribution indicates that some mechanism other than two-particle scattering binds the solar wind into a fluid and maintains the statistical spectrum beyond (70 - 100) $R_{\odot}$.

One approach to this problem has been attempted by Sturrock and Hartle (1965). They point out that as the density and temperatures decline, the electrons and protons become thermally decoupled, and they suggest use of a two-fluid model in the outer corona. The predicted proton temperature near the earth is then about 3×10^{3} °K and $T_e \sim 10^6$ °K. Although T_e has not been well measured in interplanetary space, it is known from measurements on Mariner 2, IMP 1, 2, and VELA 2, 3 that T_p is generally much higher than this value ($\overline{T}_p \sim 2 \times 10^5$ °K). Thus, the particle-wave scattering must heat the protons.

Four kinds of instability which generate waves in a collisionless plasma have been discussed in the literature. For $nK(T_{\parallel}-T_{\perp}) > B^2/4\pi$, the "firehose" instability (Parker, 1958) allows Alfvén waves to grow. If $nK(T_{\perp})^2/T_{\parallel} > B^2/B\pi$, the system will be instable with respect to the growth of magnetosonic waves. Both of these instabilities would easily arise in the absence of collisions because only the wave scattering maintains $T_{\perp} \approx T_{\parallel}$. However, velocity space anisotropies also allow low frequency electrostatic plasma oscillations to grow to large amplitudes (Harris, 1961; Rosenbluth and Post, 1965). Finally, if fluctuations in the solar wind speed produce electric currents or longitudinal electric fields, ion wave drift instabilities will be triggered. The interplanetary electric field, $\vec{E} = -\vec{U} \times \vec{B}/C$, is normally about 10^7 greater than the runaway field. $E_R = KT_e/e\ell$, where ℓ is the mean free path, but $\vec{E}$ does not produce runaway since it is normal to $\vec{B}$. However, it would seem that fluctuations in $\vec{B}/|\vec{B}|$ yielding a fluctuating electric field $\delta\vec{E} = -\vec{U} \times \delta\vec{B}/C$ could easily lead to a runaway situation.

The available evidence suggests rather strongly that magnetosonic waves do not have large amplitudes in the solar wind. Holzer, McLeod, and Smith (1965) found a very rapid decrease in spectral intensity above (0.2 - 0.5) cps when OGO-1 was in interplanetary space. An indirect support for this contention comes from the IMP-1 magnetometer results (Ness, et al., 1964), since detection of large variances implies that the tilted fluxgate is contaminated by significant high frequency noise (Greenstadt, 1965; Fredricks and Sonnet, 1965). The general absence of large 5.46 minute variances in interplanetary space therefore suggests that little high frequency magnetic noise was present. Thus, the thermalizing waves are probably long period magnetic oscillations of the Alfvén variety, or electrostatic plasma oscillations.

The plasma oscillations are the ones which can account for proton heating most easily via a stochastic cyclotron acceleration process (Stix, 1964; Scarf, et al., 1965; Fredricks, et al., 1965; Sturrock, 1965), and electrons can also be accelerated by these waves. Indeed, the Mariner 4 interplanetary observations (Van Allen and Krimigis, 1965) of about 40 kev energy electrons strongly suggest that an interplanetary acceleration process is operative, as originally suggested by Parker (1965). However, further experimental study of the electromagnetic and electrostatic noise spectrum in the solar wind is clearly needed.

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