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PROBABILISTIC DYNAMICS OF A GLOBAL HORIZONTAL SOUNDING SYSTEM

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PREFACE

The concept of using balloons as wind sensors and interrogating them via satellite raised the question of whether a large number of free-floating balloons can be maintained in a world-wide dispersion that would provide the required wind data. The study presented here is part of RAND's continuing research on satellite meteorology and is an attempt, specifically, to devise a simple model with which to check the feasibility of maintaining such a dispersion at no cost of balloons or launching paraphernalia.

ABSTRACT

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A method is developed for treating the position and motion of large numbers of constant-level balloons in a probabilistic fashion. The general-circulation model of Mintz is used to generate estimates of the motion of 1000 balloons at approximately the 400-mb level for a period of 45 days. The location of the balloons each day is expressed as the probability of being located in one of 34 possible regions of the earth. The motions are specified by the transition probabilities from one region to another in a 24-hour period. The degree to which the probabilities computed from the general circulation can be expected to match probabilities computed from real balloon trajectories is discussed. It is then shown how probabilities such as these, but more appropriate to the real atmosphere, could be used to show how balloons would be distributed and how reasonable variants of the expected distribution could be computed. It is suggested that the exercise of this type of probabilistic model could lead to a rational strategy for launching balloons to attain an optimum distribution of balloons over the globe.

Author

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I. INTRODUCTION

The need for an adequate global coverage of meteorological observations has been growing steadily. The advances in computer technology have provided meteorologists with the means to handle large numbers of data and to numerically integrate complex sets of differential equations. Meteorologists have been able to formulate increasingly realistic mathematical models of the atmosphere using the improved quantity and quality of weather data that have become available since World War II. The science of numerical weather prediction has now reached a state where the paucity of good observations over large segments of the globe can be shown to have a deleterious effect on the daily predictions. With the improved atmospheric models already operating as research tools and the increased computing capacity that is well within our grasp, we will have reached the point where our ability to handle data will greatly exceed the quantity and quality of the data available to us. One of the proposed solutions to the problem of an adequate world-wide weather observing system is the use of free-floating balloons in conjunction with an interrogation system contained in satellites. The purpose of this study is to examine such a system and to develop a technique for predicting and analyzing its operation.

The basic idea of the system was proposed in 1959 by Lally,[1] who realized before the first weather satellite was launched that cloud pictures, though useful, would not be adequate data for numerical forecast techniques. He proposed the GHOST (Global Horizontal Sounding Technique) system at that time. The GHOST proposal was to use superpressure balloons floating on constant density surfaces as wind sensors and as platforms for measuring temperature, and to interrogate these balloons from a satellite. Depending on the number of balloons and satellites and the distribution of the balloons it would then be possible to have frequent, world-wide reports of the dynamic elements needed by the forecast techniques.

The subject of this study is the distribution of the balloons. Because of semipermanent areas of convergence and divergence in the atmosphere it is entirely possible that the constant-level balloons might be swept out of some regions and concentrated in others. It would be particularly disastrous to the program if balloons were systematically removed from areas where they were needed and concentrated in areas where they were not useful. The evidence to date is not completely clear. Mesinger,[2] using data from the National Meteorological Center, computed divergent trajectories for a thirty-day period and compared the distribution of the computed positions to the distribution expected by a random placement of points. He concluded that the calculated distribution was not very different from that expected from a random distribution and that, therefore, the divergence problem was not important. Because of the possibly great economic implications of the divergence on the GHOST system, however, an independent set of calculations was made by Mesinger and Mintz[3] based on the general-circulation model of Mintz. These results were at variance with Mesinger's earlier results. Mesinger used as his measure of divergence the distribution of balloons from a series of randomly spaced, fixed points. Mesinger and Mintz used the same measure of divergence but applied it to the hypothetical trajectories computed from the Mintz model. We propose to use the trajectories computed from the Mintz model but will express the results as a concentration of balloons in specified regions.

We will divide the entire globe into a number of mutually exclusive regions. We will then use the trajectory data from the Mintz circulation model to compute a transition probability matrix for these regions. That is, we will compute the probability that a balloon in region i will move to a region j in a fixed time; these conditional probabilities of all regions into all other regions will form the transition-probability matrix.

We hope for two advantages by this treatment. First, it will be possible to test the transition probabilities so computed against some actual balloon flights made by the transosonde program.[4] This will provide at least a hint of how reasonable it is to assume that the trajectories of the general-circulation model represent the real atmosphere. Second, the probabilistic approach will make it possible to test the effect of various launching strategies on the balloon distribution. An efficient method for replacing lost balloons or supplying balloons to fill gaps caused by divergence may well spell the difference between an economically sound system and an expensive failure.

Probabilistic Dynamics

Consider a large number of balloons floating at a constant level in the atmosphere whose geographic position, $\underline{x}$, and horizontal velocity, $\underline{V}$, can be measured at a series of times, t_i. One way to summarize data of this type is to group them into nonoverlapping geographical regions that cover the earth. The velocity data can be grouped into nonoverlapping areas on the vector wind plot. If n geographical areas are defined and m areas are defined on the wind plot, there will be $s = nm$ states in which to group all of the balloon data.

The motion of the balloons can also be described in a statistical manner. If at time t the balloon is in state S_i and at a later time, say $t + \delta$, is found to be in state S_j, its change in position and velocity is represented by the change in state. The probability that a balloon will move from S_i to S_j in the time interval δ is denoted by $p_{ij}(\delta)$ and is called the transition probability. Since both i and j range from 1 to s, there will be s^2 possible transitions. The s x s matrix of these transition probabilities is denoted by $P(\delta)$ and is called the transition-probability matrix. (See Fig. 1.)

If the transition-probability matrix is constant over periods of time which are long compared with δ, the probable distribution among the states can be estimated as a function of time. Suppose that π_i represents the probability of finding a balloon in state i at time t. Then the vector probability at time t,

$$\Pi(t) = (\pi_1(t), \pi_2(t) \ldots \Pi_i(t) \ldots \pi_s(t)),$$

multiplied by the transition-probability matrix will yield the probability vector for the time $t + \delta$

$$\Pi(t+\delta) = \Pi(t)\, P(\delta)$$

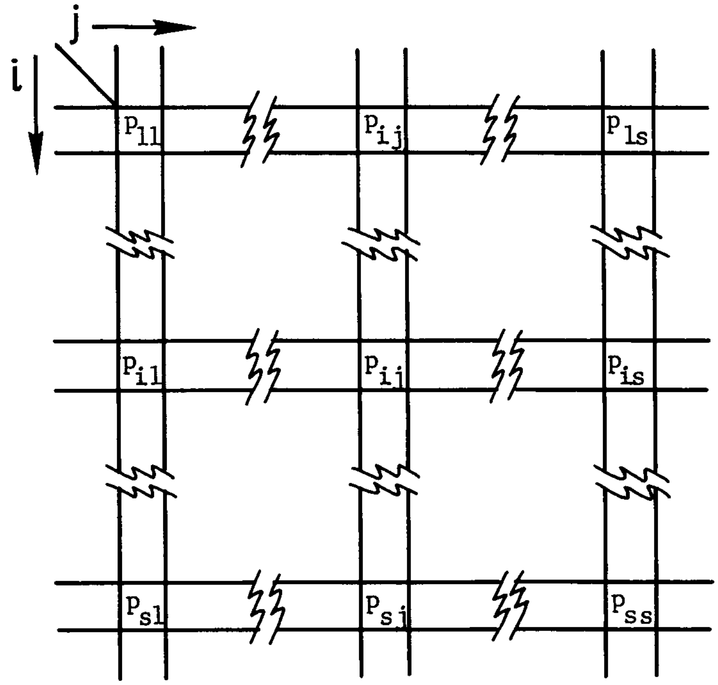

Fig. 1 -- Transition-probability matrix

If this process is repeated k times,

$$\Pi(t+k\delta) = \Pi(t)\ P^k(\delta)$$

If the actual distribution of a set of balloons is known at some time, t, then the expected distribution of these balloons at the time $t + k\delta$ can be found from

$$E[N(t+k\delta)] = N(t)\ P^k(\delta)$$

where $E[N(t+k\delta)]$ represents the expected number of balloons, $N(t)$ is a vector of random variables $(n_1, n_2 \ldots n_i \ldots n_s)$ giving the number of balloons in each state, and $P^k(\delta)$ is the transition matrix for the period $k\delta$. Thus if the position of a group of balloons is known at some time, the expected number in each region at some later time can be estimated from the transition matrix.

If, in addition to the expected number of balloons, a measure of the variation is desired, this can be developed from the binomial distribution. It is convenient for this purpose to drop the matrix notation and to simply deal with an individual cell of the matrix. If p_{ij} is the probability that a balloon will move from region i to region j, then the expected number of balloons that arrive in region j from region i is

$$E[n_{ij}(t+\delta)] = n_i(t)\ p_{ij}(\delta)$$

where n_i is the number of balloons that originate in region i. If it is assumed that the movement of balloons is independent from one region to another, the total number of balloons that arrive in region j is simply the sum over all of the regions i, that is,

$$E[n_j(t+\delta)] = \sum_i n_i(t)\ p_{ij}(\delta)$$

The variance of the number of balloons that arrive in region j from region i is

$$\text{var}\ [n_{ij}(t+\delta)] = n_i(t)\ p_{ij}(\delta)\ [1-p_{ij}(\delta)]$$

and, with the same assumption of independence, the variance of the number of balloons arriving in region j from all other regions is

$$\text{var}\ [n_j(t+\delta)] = \sum_i n_i(t)\ p_{ij}(\delta)\ [1-p_{ij}(\delta)]$$

If there are many balloons in each region initially, the probability distribution of balloons in a given region at a time k will be approximately normal with mean $E[n_j(t+k\delta)]$ and standard deviation $\{\text{var}\ [n_j(t+k\delta)]\}^{\frac{1}{2}}$.

The ratio of the standard deviation to the expected value is a crude measure of the degree to which the expected distribution is attained at any time. If the ratio is small compared to 1, an actual distribution would be expected to be close to the expected value. If the ratio is near 1, the actual distribution would not be close to the expected. If the expected number of balloons indicates a serious deficiency in some area, the effect of launching some balloons can be estimated by simply adding balloons to the appropriate areas and checking the expected distribution to determine whether or not the launch policy in fact improved the estimates of the future distribution. We hope that with a little experience with this type of model we will be able to define a technique for deciding upon a launch strategy that does not depend on a trial-and-error procedure.

II. THE DATA

In order to try this approach to the problem of insuring an adequate coverage of constant-level balloons, it is necessary to learn something about the transition-probability matrix. The data used by Mesinger and Mintz[3] were kindly lent to us by Dr. Mintz in the form of a magnetic tape which had hourly positions for ~1000 hypothetical balloons at 800 and 400 millibars. Because the information we needed to construct the transition-probability matrix could be easily obtained, we chose to use these data even though there was some question about their resemblance to real balloon trajectories. The problem of the adequacy of the Mintz trajectories for the representation of real trajectories will be considered in a later section.

The surface of the earth was divided into 34 approximately equal areas as shown in Fig. 2. Five different conditions of present balloon movement were established as shown in Fig. 3. If the balloon had moved more than 7.5 miles in the last hour, it was assigned a number corresponding to the quadrant into which it moved. If the balloon had moved less than 7.5 miles it was assigned the number 5, indicating an essentially stationary balloon. The size of the geographical areas had been selected as the approximate distance a balloon would travel in one day so that the probabilities that a balloon would be in more than two areas in one day would be very small. With 34 geographical areas and 5 possible wind conditions there are 170 possible states for the balloons. The transition matrix is therefore a 170 x 170 matrix.

Each fictitious balloon in the Mintz program had an identifying number so that individual balloons could be followed. The tape containing 45 days of travel of 1000 fictitious balloons at 400 mb was read into the machine. The motion of each balloon from hour 0 to hour 1 was read and each balloon was assigned a number from one to five according to the motion scheme shown in Fig. 2. The position of each balloon at hour 1 was then used to determine its geographical area. Thus each balloon was assigned to one of the 170 possible states. The motion of each balloon was then calculated for the period between hour 24 and

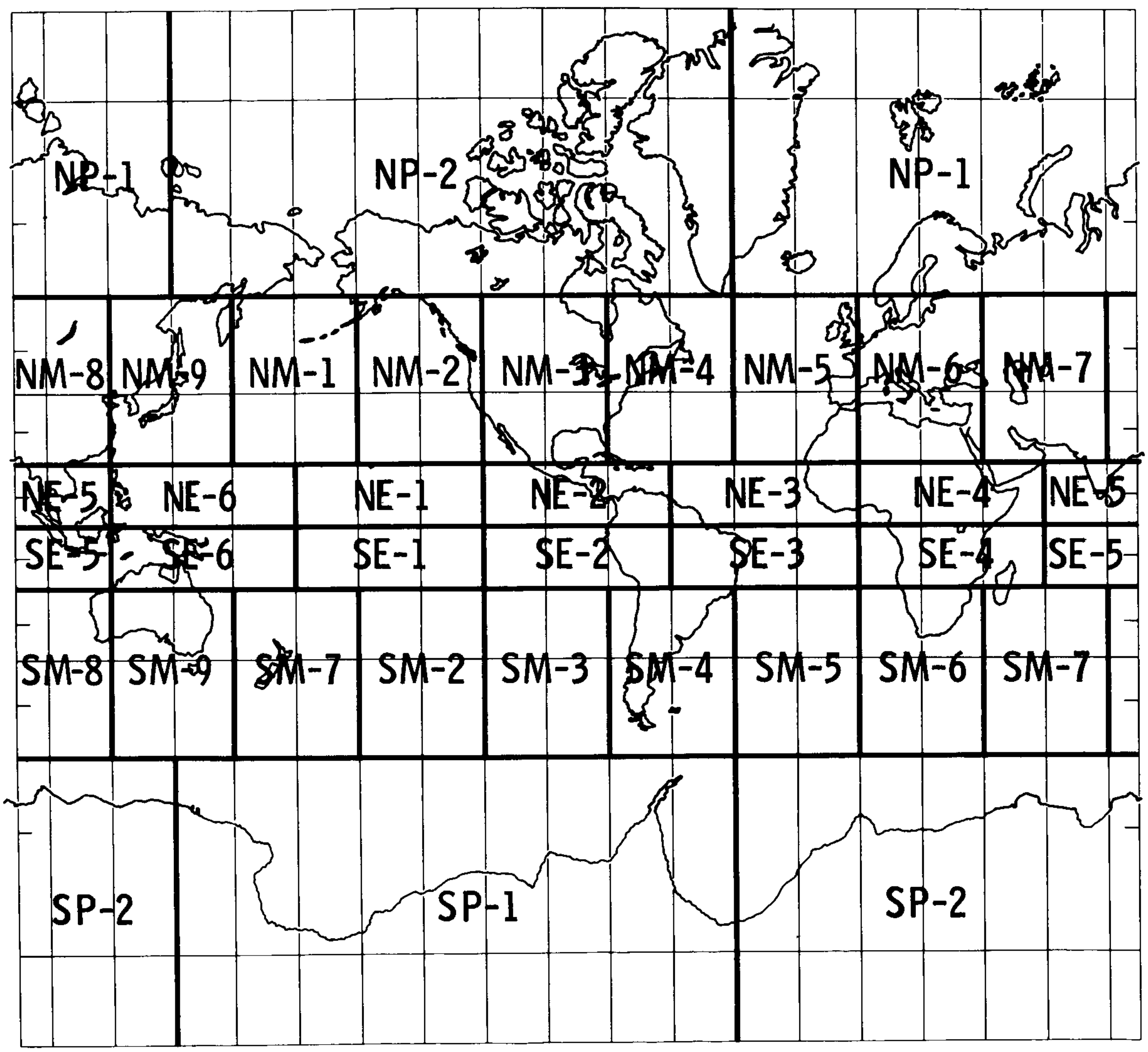

Fig. 2 -- Designations of surface areas used in computation of balloon distribution

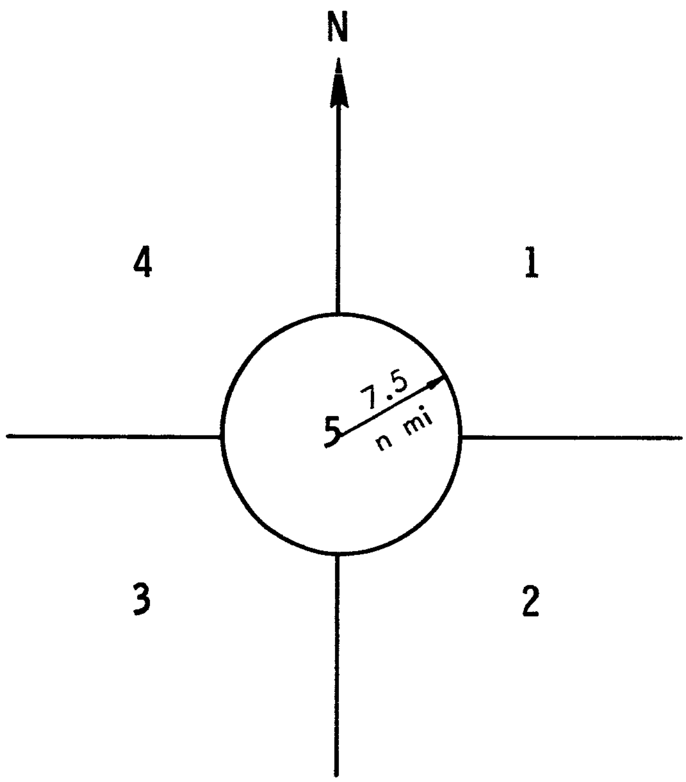

Fig. 3 -- States defined by the 1-hour movement of the balloon

hour 25, and the geographical area for hour 25 was noted. Each balloon was thus assigned to a state at hour 25. This provides an entry for each balloon in the 170 x 170 matrix. The process was repeated for each succeeding 24-hour period for the 45 days of data that were available. The result was approximately 2×10^9 transitions, from which the 28,900 elements of the transition matrix could be estimated. Table 1 shows a small segment of the matrix.

The reason for including some measure of the present motion of the balloon was the indication from some preliminary surveys and from Edinger and Rapp[(5)] that there was some sort of periodicity in the winds which would induce a day-to-day variation in the transition matrix. It is possible to note in the sample of the matrix shown in Table 1 that the initial motion of the balloon does indeed have some effect on the transition probabilities. Some of the probabilities are not reliable, however, because so few balloons were found in the wind regions 3 and 4 that the sample sizes were too small to provide a reasonable estimate of the probability. Moreover, the time for which the transition matrix was computed was sufficiently long to average out the effects of any periodicity. We therefore chose to ignore the effects of present motion on the balloon and to use the transition matrix for the 34 geographical areas. Table 2 shows the 24-hour transition matrix for the 34 areas which cover the globe.

Not unexpectedly, a balloon is most likely to stay in one area on any given day. There are two exceptions to this tendency: (1) balloons in NM-9 are most likely to move into NM-1, and (2) balloons in SM-4 are most likely to move into SM-5. Area NM-9 is that enclosing the east coast of Asia and Japan where the jet stream is strongest during the winter; therefore, there is a high probability that the balloons will move rapidly from west to east. SM-4 encloses the southern part of South America and, even though on the tape record it is summer in the Southern Hemisphere, it is not unreasonable to expect a jec stream in this area.

The matrix of Table 2 is $P(\delta)$ for a value of δ = one day. If we assume that one balloon starts in each region at time zero, the expected

Table 1

A PORTION OF THE LARGE TRANSITION MATRIX

From		To														
		NM-9					NM-1					NM-2				
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
NM-9	1	0.010	0.006	0.0	0.002	0.006	0.276	0.206	0.011	0.021	0.137	0.086	0.082	0.0	0.0	0.036
	2	0.018	0.013	0.0	0.001	0.011	0.278	0.208	0.010	0.020	0.146	0.075	0.074	0.0	0.0	0.035
	3	0.062	0.047	0.0	0.002	0.093	0.235	0.162	0.006	0.014	0.235	0.023	0.024	0.0	0.0	0.024
	4	0.024	0.020	0.0	0.012	0.058	0.073	0.051	0.002	0.004	0.078	0.006	0.007	0.0	0.0	0.007
	5	0.028	0.021	0.0	0.001	0.032	0.243	0.172	0.009	0.019	0.206	0.053	0.053	0.0	0.0	0.045
NM-1	1	0.0	0.0	0.0	0.0	0.0	0.215	0.154	0.009	0.018	0.121	0.116	0.101	0.0	0.0	0.055
	2	0.0	0.0	0.0	0.0	0.0	0.187	0.159	0.009	0.017	0.157	0.098	0.116	0.001	0.0	0.071
	3	0.0	0.0	0.0	0.0	0.0	0.225	0.186	0.011	0.020	0.183	0.086	0.097	0.0	0.0	0.057
	4	0.0	0.0	0.0	0.001	0.002	0.235	0.165	0.010	0.021	0.166	0.089	0.081	0.0	0.0	0.055
	5	0.0	0.0	0.0	0.0	0.0	0.180	0.120	0.009	0.020	0.241	0.080	0.080	0.0	0.0	0.110
NM-2	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.160	0.131	0.001	0.0	0.084
	2	0.0	0.0	0.0	0.0	0.0	0.002	0.0	0.0	0.0	0.0	0.156	0.178	0.002	0.0	0.131
	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.130	0.195	0.026	0.0	0.319
	4	0.0	0.0	0.0	0.0	0.0	0.0	0.001	0.0	0.0	0.001	0.081	0.081	0.001	0.0	0.236
	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.134	0.112	0.001	0.0	0.225

Table 2

ONE DAY 34 x 34 PROBABILITY MATRIX

From	NP-1	NP-2	NM-1	NM-2	NM-3	NM-4	NM-5	NM-6	NM-7	NM-8	NM-9	NE-1	NE-2	NE-3	NE-4	NE-5	NE-6	From
	To																	
NP-1	.695	.148	.030	.054	.026	.025	.009	.003	.001	.004	.002	.001	.000	.000	.000	.000	.001	NP-1
NP-2	.057	.737	.017	.003	.001	.001	.021	.045	.023	.066	.028	.000	.000	.000	.001	.000	.001	NP-2
NM-1	.057	.008	.567	.259	.064	.006	.000	.000	.000	.000	.001	.019	.002	.000	.000	.000	.016	NM-1
NM-2	.081	.005	.001	.473	.325	.093	.008	.000	.000	.000	.000	.009	.005	.000	.000	.000	.000	NM-2
NM-3	.019	.002	.000	.000	.465	.393	.093	.007	.000	.000	.000	.000	.017	.004	.000	...	.000	NM-3
NM-4	.159	.059	.002	.001	.001	.370	.308	.074	.006	.000	.000	.000	.001	.017	.001	.000	.000	NM-4
NM-5	.013	.200	.000	.000	.000	.000	.412	.283	.074	.010	.001	.000	.000	.003	.004	.000	.000	NM-5
NM-6	.012	.193	.000	.000	.000	.000	.001	.388	.300	.081	.010	...	.000	.000	.011	.003	.000	NM-6
NM-7	.013	.198	.025	.000	.000	.000	.001	.002	.379	.290	.080	.000	...	.000	.002	.009	.001	NM-7
NM-8	.013	.121	.246	.019	.000	.000	.000	.001	.001	.373	.216	.001	.000	.000	.000	.004	.006	NM-8
NM-9	.035	.140	.564	.126	.013	.000	.000	.001	.000	.006	.094	.006	.000	.000	.000	.000	.013	NM-9
NE-1	.010	.000	.105	.167	.071	.019	.001	.000	.000	.000	.000	.541	.074	.003	.000	.000	.007	NE-1
NE-2	.003	.001	.000	.000	.108	.173	.038	.003	.000	.000	.000	.006	.601	.066	.002	.000	.000	NE-2
NE-3	.002	.007	.000	.000	.000	.046	.246	.077	.021	.002	.000	.000	.013	.534	.049	.004	.000	NE-3
NE-4	.000	.004	.002	...	.000	.000	.000	.101	.165	.058	.011	.000	.000	.013	.556	.087	.003	NE-4
NE-5	.001	.003	.046	.002	.000	.000	.000	.000	.066	.147	.072	.004	.000	.000	.019	.586	.055	NE-5
NE-6	.006	.001	.128	.038	.006	.001	.000	.000	.000	.000	.016	.093	.004	.000	.000	.018	.683	NE-6
SE-1	.000	...	.000	.000	.000	.000	...	...	.000	...	.000	.019	.001	.000	.000	.000	.002	SE-1
SE-2	.000	.000	.000	.000	.000	.000	.000	.000	...	...	.000	.002	.027	.001	.000	.000	.000	SE-2
SE-3	.000	.000	.000	.000	.000	.000	.000	.000	.000	...	...	.000	.001	.031	.000	.000	.000	SE-3
SE-4	...	.000	...	...	...	.000	...	.000	.000	.000	.000	.000	.000	.001	.025	.001	.000	SE-4
SE-5	...	.000	.000	...	...	.000	...	.000	.001	.001	.000	.000	.000	.000	.006	.153	.007	SE-5
SE-6	.000	.000	.000	.000	.000	...	...	.000	.000	.000	.000	.002	.000	.000	.001	.042	.043	SE-6
SM-1	...	...	.000	.000	...	...	...	...	.000	...	...	.000	.000	...	.000	.000	.000	SM-1
SM-2	...	...	.000	.000	...	...	...	...	...	...	...	.000	.000	...	...	.000	.000	SM-2
SM-3	...	...	.000	.000	.000	.000	...	...	...	...	...	.000	.000	.000	...	...	.000	SM-3
SM-4	...	...	.000	.000	.000	.000	...	.000	...	...	...	.000	.000	.000	.000	...	.000	SM-4
SM-5	...	...	...	...	...	.000	...	.000	...	...	...	.000	.000	.000	.000	...	...	SM-5
SM-6	...	...	...	...	...	...	...	.000	.000	...	....	...	...	.000	.000	.000	...	SM-6
SM-7	...	...	...	...	...	...	...	.000	.000	...	...	...	...	.000	.000	.000	.000	SM-7
SM-8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	.000	.000	SM-8
SM-9	...	...	.000	...	...	...	...	...	.000	...	...	.000	...	...	.000	.000	.000	SM-9
SP-1	...	...	...	...	...	...	...	...	...	...	...	.000	.000	.000	...	.000	.000	SP-1
SP-2	...	...	...	...	...	...	...	...	...	...	...	...	...	.000	.000	.000	.000	SP-2
SUM	1.18	1.83	1.73	1.14	1.08	1.13	1.14	0.98	1.04	1.04	0.53	0.70	0.74	0.67	0.68	0.91	0.84	17.36

Table 2 -- continued

From	To																	From
	SE-1	SE-2	SE-3	SE-4	SE-5	SE-6	SM-1	SM-2	SM-3	SM-4	SM-5	SM-6	SM-7	SM-8	SM-9	SP-1	SP-2	
NP-1	.000	.000	...	...	...	.000	...	...	...	...	...	...	...	...	...	...	...	NP-1
NP-2	...	...	...	...	...	.000	...	...	...	...	...	...	...	...	...	...	...	NP-2
NM-1	.000	.000	...	...	.000	.000	.000	.000	...	...	...	...	...	.000	.000	...	...	NM-1
NM-2	.000	.000	...	...	...	.000	.000	.000	...	...	...	...	...	...	...	...	...	NM-2
NM-3	.000	.000	...	...	...	...	...	...	.000	.000	...	...	...	...	...	...	...	NM-3
NM-4	...	.000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	NM-4
NM-5	...	.000	...	...	.000	...	...	...	...	...	...	...	...	...	...	...	...	NM-5
NM-6	...	...	...	...	.000	...	...	...	...	...	...	...	...	...	...	...	...	NM-6
NM-7	...	...	...	...	.000	...	...	...	...	...	...	...	...	...	...	...	...	NM-7
NM-8	.000	...	...	...	.000	.000	...	...	...	...	...	...	...	...	...	...	...	NM-8
NM-9	.000	...	...	...	.000	.000	.000	...	...	...	...	...	...	.000	.000	...	...	NM-9
NE-1	.001	.000	...	...	.000	.000	.000	.000	.000	.000	...	...	...	.000	.000	.000	...	NE-1
NE-2	.000	.001	.000	...	...	.000	.000	.000	.000	.000	.000	...	...	...	...	.000	.000	NE-2
NE-3	.000	.000	...	...	.000	...	...	...	.000	.000	...	...	...	...	...	...	...	NE-3
NE-4	...	.000	...	.000	.000	.000	...	...	...	...	...	...	.000	.000	...	...	...	NE-4
NE-5	.000	...	.000	.000	.001	.000	.000	...	...	...	...	.000	.000	.000	.000	...	.000	NE-5
NE-6	.000	.000	...	.000	.000	.005	.000	.000	...	...	...	...	.000	.000	.000	.000	.000	NE-6
SE-1	.560	.041	.002	.000	.001	.025	.090	.179	.064	.012	.001	.000	.000	.000	.000	.003	.000	SE-1
SE-2	.035	.628	.022	.001	.000	.001	.000	.000	.119	.130	.028	.003	.000	.000	.000	.002	.000	SE-2
SE-3	.001	.038	.570	.028	.003	.000	.000	.000	.000	.037	.180	.089	.017	.001	.000	.000	.001	SE-3
SE-4	.000	.001	.028	.535	.059	.003	.000	...	.000	.000	.000	.138	.148	.051	.008	.000	.002	SE-4
SE-5	.000	.000	.000	.021	.504	.019	.008	.000	.000	.000	.000	.000	.051	.164	.063	.000	.001	SE-5
SE-6	.011	.001	.000	.002	.086	.786	.012	.005	.001	.000	.000	.000	.000	.003	.006	.000	.000	SE-6
SM-1	.004	.000	.000	.000	.000	.003	.557	.278	.057	.005	.000	.000	.000	.000	.000	.092	.003	SM-1
SM-2	.005	.000	.000	...	.000	.000	.001	.544	.312	.054	.004	.000	.000	.000	.000	.078	.003	SM-2
SM-3	.000	.002	.000	...	...	.000	.001	.003	.579	.272	.054	.003	.000	.000	.000	.082	.004	SM-3
SM-4	.000	.000	.004	.000	.000	.000	.001	.002	.002	.396	.399	.109	.011	.000	.000	.055	.021	SM-4
SM-5	.000	.000	.003	.000	.000	.000	.000	.000	.000	.000	.553	.307	.062	.005	.000	.002	.068	SM-5
SM-6	...	.000	.000	.003	.000	.000	.000	.000	.000	.000	.001	.547	.314	.058	.005	.002	.071	SM-6
SM-7	...	.000	.000	.001	.000	.000	.005	.000	.000	.000	.001	.002	.560	.295	.066	.002	.069	SM-7
SM-8	.000	...	.000	.000	.000	.001	.075	.004	.000	.000	.001	.001	.001	.527	.333	.010	.047	SM-8
SM-9	.000	.000	.000	.000	.000	.010	.329	.056	.005	.002	.000	.000	.000	.002	.513	.075	.007	SM-9
SP-1	.000	.000	.000	.000	.000	.000	.025	.064	.030	.056	.011	.002	.001	.001	.003	.743	.064	SP-1
SP-2	.000	.000	.000	.000	.000	.000	.008	.006	.002	.004	.019	.042	.024	.037	.018	.166	.675	SP-2
SUM	0.62	0.71	0.63	0.59	0.66	0.85	1.11	1.14	1.17	0.97	1.25	1.24	1.19	1.14	1.02	1.31	1.04	16.64
																TOTAL NUMBER OF BALLOONS		34.00

number of balloons is given by the sum of the columns in Table 2. The expected number in the equatorial regions after 1 day is only 8.6 balloons where there were 12 at the start. That is, after only 1 day there is evidence of the balloons being swept out of the equatorial regions. From the large blank area in the upper right of the matrix we note that the probability of going from the Northern Hemisphere is very small indeed. The maximum transfer southward across the equator is from NE-6 to SE-6, and the probability is 0.5 per cent. The maximum transport northward across the equator is about 15 per cent from SE-5 to NE-5. In other words, the net result would be a transfer of balloons northward across the equator.

By raising the transition matrix to the kth power, the expected distribution after k days can be readily calculated. Table 3 shows the three-day matrix. Although none of the transition probabilities is as large as it is for one day, the east--west flow of the air in midlatitudes is apparent in that the maximum of the probabilities has moved one or two regions to the east. The convergence area in NM-1 shows rather strongly with the expected number of balloons, assuming one balloon per region at day 0, reaching a value of 1.941. A secondary maximum is apparent at NM-5 with an expected number of 1.447. The tendency for the depletion of the equatorial regions is also apparent. The expected number of balloons in the equatorial region has been reduced to 6.5 after three days.

The trajectories computed from the Mintz model are all representative of Northern Hemisphere winter months, but the transition matrix must vary with the season. The matrix can be assumed constant for probably about one month. The matrix was cubed successively until a 27-day transition matrix was obtained. Assuming that one balloon started in each area, the expected number in each of the 34 regions was computed and is shown in Table 4. The trend toward an accumulation of balloons in NM-1 is quite apparent with a secondary maximum in NM-5. In the Southern Hemisphere there is a weak maximum in SM-3, and there are relatively high values through SM-1 and SM-2. A weak secondary maximum is indicated in SM-5. The zonal totals indicate that the equatorial

Table 3

THREE DAY 34 x 34 PROBABILITY MATRIX

	To																	
From	NP-1	NP-2	NM-1	NM-2	NM-3	NM-4	NM-5	NM-6	NM-7	NM-8	NM-9	NE-1	NE-2	NE-3	NE-4	NE-5	NE-6	From
NP-1	.408	.192	.043	.073	.067	.067	.050	.035	.019	.024	.011	.004	.003	.003	.001	.001	.003	NP-1
NP-2	.091	.403	.106	.042	.018	.009	.031	.072	.069	.097	.047	.003	.001	.001	.003	.003	.005	NP-2
NM-1	.090	.029	.175	.213	.201	.146	.061	.017	.004	.002	.002	.023	.011	.004	.000	.001	.022	NM-1
NM-2	.081	.028	.005	.099	.208	.268	.180	.069	.020	.005	.001	.007	.015	.010	.002	.000	.000	NM-2
NM-3	.037	.027	.003	.003	.078	.245	.298	.174	.073	.021	.004	.000	.014	.018	.005	.001	.000	NM-3
NM-4	.106	.091	.018	.013	.009	.077	.216	.214	.144	.065	.018	.001	.001	.013	.008	.003	.001	NM-4
NM-5	.027	.148	.057	.011	.003	.002	.102	.204	.224	.148	.052	.001	.000	.003	.009	.007	.002	NM-5
NM-6	.032	.138	.154	.037	.009	.002	.007	.085	.204	.212	.093	.002	.000	.000	.009	.011	.006	NM-6
NM-7	.047	.136	.277	.097	.032	.009	.008	.015	.085	.171	.094	.006	.001	.000	.002	.008	.013	NM-7
NM-8	.068	.090	.335	.193	.098	.038	.012	.010	.007	.061	.043	.015	.003	.001	.000	.003	.023	NM-8
NM-9	.088	.101	.217	.209	.159	.091	.034	.016	.009	.015	.008	.019	.007	.002	.000	.001	.023	NM-9
NE-1	.034	.009	.066	.135	.153	.142	.073	.023	.006	.001	.001	.227	.091	.018	.001	.001	.017	NE-1
NE-2	.014	.013	.002	.002	.052	.162	.175	.093	.039	.011	.002	.012	.299	.110	.010	.001	.000	NE-2
NE-3	.006	.031	.021	.003	.001	.013	.170	.175	.145	.073	.023	.001	.027	.253	.045	.011	.001	NE-3
NE-4	.009	.020	.128	.032	.008	.002	.004	.049	.154	.156	.071	.003	.002	.027	.252	.070	.014	NE-4
NE-5	.023	.013	.206	.086	.034	.011	.002	.002	.037	.094	.056	.020	.002	.002	.039	.273	.094	NE-5
NE-6	.025	.007	.101	.096	.069	.041	.014	.003	.002	.003	.005	.125	.020	.002	.003	.043	.427	NE-6
SE-1	.000	.000	.002	.004	.002	.001	.000	.000	.000	.000	.000	.046	.007	.001	.000	.002	.009	SE-1
SE-2	.000	.000	.000	.001	.002	.004	.002	.000	.000	.000	.000	.011	.066	.011	.000	.000	.001	SE-2
SE-3	.000	.000	.000	.000	.000	.001	.008	.003	.002	.000	.000	.001	.012	.075	.005	.000	.000	SE-3
SE-4	.000	.000	.001	.000	.000	.000	.000	.002	.004	.002	.001	.000	.001	.010	.056	.007	.001	SE-4
SE-5	.001	.001.	.015	.004	.001	.000	.000	.001	.007	.015	.008	.004	.000	.001	.024	.155	.030	SE-5
SE-6	.001	.000	.008	.004	.001	.001	.000	.000	.002	.005	.003	.013	.001	.000	.007	.072	.093	SE-6
SM-1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000	.000	.001	SM-1
SM-2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000	SM-2
SM-3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	SM-3
SM-4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000	SM-4
SM-5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000	SM-5
SM-6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.000	.000	SM-6
SM-7	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	SM-7
SM-8	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	SM-8
SM-9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.001	SM-9
SP-1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	SP-1
SP-2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	SP-2
SUM	1.19	1.48	1.94	1.36	1.21	1.33	1.45	1.26	1.25	1.18	0.54	0.54	0.59	0.57	0.48	0.68	0.79	17.04

Table 3 -- continued

	To																	
From	SE-1	SE-2	SE-3	SE-4	SE-5	SE-6	SM-1	SM-2	SM-3	SM-4	SM-5	SM-6	SM-7	SM-8	SM-9	SP-1	SP-2	From
NP-1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NP-1
NP-2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NP-2
NM-1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-1
NM-2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-2
NM-3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-3
NM-4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-4
NM-5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-5
NM-6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-6
NM-7	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-7
NM-8	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-8
NM-9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NM-9
NE-1	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NE-1
NE-2	.000	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NE-2
NE-3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NE-3
NE-4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NE-4
NE-5	.000	.000	.000	.000	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NE-5
NE-6	.000	.000	.000	.000	.002	.012	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	NE-6
SE-1	.338	.020	.003	.001	.009	.065	.040	.119	.143	.086	.039	.011	.003	.001	.001	.040	.005	SE-1
SE-2	.090	.345	.013	.001	.001	.008	.001	.003	.081	.125	.112	.057	.021	.005	.001	.022	.014	SE-2
SE-3	.011	.092	.334	.008	.002	.001	.004	.001	.004	.013	.087	.129	.105	.048	.018	.004	.030	SE-3
SE-4	.001	.008	.065	.263	.014	.007	.038	.009	.002	.001	.001	.085	.150	.141	.092	.008	.031	SE-4
SE-5	.001	.000	.006	.054	.240	.019	.079	.028	.008	.002	.001	.004	.028	.116	.117	.019	.012	SE-5
SE-6	.006	.000	.001	.017	.166	.516	.022	.012	.007	.003	.001	.001	.003	.016	.013	.004	.001	SE-6
SM-1	.012	.001	.000	.000	.001	.007	.181	.267	.212	.100	.037	.009	.002	.001	.002	.144	.021	SM-1
SM-2	.010	.002	.002	.000	.000	.001	.005	.179	.291	.205	.109	.034	.008	.002	.002	.124	.025	SM-2
SM-3	.001	.004	.005	.000	.000	.000	.005	.014	.191	.254	.224	.102	.034	.008	.003	.111	.043	SM-3
SM-4	.000	.002	.008	.001	.000	.000	.006	.008	.007	.104	.268	.243	.144	.053	.017	.052	.085	SM-4
SM-5	.000	.001	.005	.002	.000	.000	.012	.002	.001	.001	.169	.276	.236	.115	.045	.011	.123	SM-5
SM-6	.000	.000	.001	.005	.000	.001	.046	.010	.002	.001	.004	.161	.287	.221	.122	.018	.120	SM-6
SM-7	.000	.000	.000	.002	.001	.003	.131	.039	.010	.003	.004	.008	.166	.268	.228	.038	.099	SM-7
SM-8	.002	.000	.000	.000	.001	.009	.257	.117	.043	.012	.005	.005	.004	.155	.255	.083	.053	SM-8
SM-9	.005	.000	.000	.000	.003	.018	.289	.216	.113	.043	.013	.004	.001	.004	.117	.146	.025	SM-9
SP-1	.001	.000	.001	.000	.000	.001	.035	.095	.094	.108	.070	.034	.016	.012	.011	.395	.126	SP-1
SP-2	.000	.000	.000	.001	.000	.001	.038	.031	.021	.022	.038	.066	.070	.078	.060	.140	.433	SP-2
SUM	0.48	0.48	0.44	0.36	0.44	0.67	1.19	1.15	1.23	1.08	1.18	1.23	1.28	1.24	1.10	1.36	1.25	16.16

TOTAL NUMBER OF BALLOONS 33.20

Table 4

EXPECTED NUMBER OF BALLOONS, AS COMPUTED
FROM THE TWENTY-SEVEN-DAY TRANSITION MATRIX, ASSUMING ONE BALLOON
AT START OF PERIOD FOR EACH GEOGRAPHIC SECTOR

Zone	Expected number given by zone sector									Number in total zone	Number in hemisphere
	1	2	3	4	5	6	7	8	9		
NP	1.429	1.296	--	--	--	--	--	--	--	2.725	
NM	2.568	1.789	1.617	1.800	1.972	1.797	1.803	1.591	0.709	15.646	20.211
NE	0.347	0.247	0.287	0.187	0.234	0.488	--	--	--	1.840	
SE	0.196	0.089	0.108	0.064	0.097	0.207	--	--	--	0.761	
SM	1.159	1.157	1.168	1.033	1.147	1.105	1.119	1.095	1.003	9.986	13.789
SP	1.481	1.561	--	--	--	--	--	--	--	3.042	

regions are rapidly being depleted, and the hemispheric totals show that the balloons are being concentrated in the Northern Hemisphere.

These matrices provide a description of how balloons are expected to be distributed by a global flow pattern that is produced by the Mintz model. They contain essentially the same information as that of Mesinger and Mintz[(2)(3)] presented for the 400-mb data, but express the results as a concentration function rather than a mean distance between balloons. The conclusions that can be drawn about the possible clustering of the balloons are also essentially the same as those of Mesinger and Mintz.

The Applicability of the Mintz Data

Having discussed the probabilistic dynamics of the Mintz model trajectories, it is important to try to relate these data to the behavior of real balloons in the real atmosphere. Ideally, one would compare actual trajectories with trajectories computed from the same initial data. Unfortunately the only sizable body of balloon data that are sufficiently well documented is the collection of flights made by the Navy under the transosonde program.[(4)] To run the Mintz model for the dates when the transosondes were flown would be prohibitive. The next best comparison of the transition probabilities is between those computed from real balloon data and those computed from the Mintz model.

Data for transosonde flights were available for two winters -- December 1957 and 1958, and January--February 1958 and 1959. These flights originated at Iwakuni, Japan and were tracked across the Pacific and into the United States. They provided data for constructing a portion of the transition matrix which had balloons originating in regions NM-1, NM-2, NM-3, north equatorial (all regions), and north polar (both regions). The balloon count is shown in Table 5. These counts were converted to probabilities and are compared in Table 6 with the transition probabilities taken from Table 2. It is immediately apparent that the correspondence is not good. Except for those originating in NM-3, the real balloons show a much greater tendency to move from west to east than do the trajectories from the Mintz model. The discrepancies noted for NM-1 and NM-2 might well be due to a bias in the transosonde

Table 5

TRANSOSONDE BALLOON TRAJECTORIES

From	To						Total
	NM-1	NM-2	NM-3	NM-4	NE	NP	
NM-1	12	41	2	0	2	0	57
NM-2	2	9	22	2	5	1	41
NM-3	0	0	6	5	3	0	14

Table 6

COMPARISON BETWEEN FRACTION OF REAL BALLOONS AND FRACTION OF MINTZ TRAJECTORIES MOVING FROM SECTOR TO SECTOR

From	To						Total
	NM-1	NM-2	NM-3	NM-4	NE	NP	
NM-1 (Table 2)	.21 (.567)	.72 (.259)	.035 (.064)	.00 (.006)	.035 (.021)	.00 (.065)	1.0 (.982)
NM-2 (Table 2)	.05 (.001)	.22 (.473)	.54 (.325)	.05 (.093)	.12 (.014)	.02 (.086)	1.0 (.992)
NM-3	.00 (.000)	.00 (.000)	.43 (.465)	.36 (.393)	.21 (.017)	.00 (.021)	1.0 (.896)

data. The large sample of Mintz trajectories was made up of a thousand balloons, which were tracked for 45 days; the real balloons all originated in the same place and were tracked for 5 days. The balloons that were actually tracked in and out of NM-1 and NM-2, therefore, represent only a small subset of all possible tracks through these areas. In order to obtain a more comparable set of data from the Mintz trajectories a subset of these trajectories was traced for balloons that passed close to the origin of the real balloon trajectories. For convenience, an area of 5 degrees of latitude and 2½ degrees of longitude surrounding Iwakuni was chosen. The transitions are shown in Table 7. The transition probabilities computed from this subset of data are compared with the real balloon trajectories in Table 8. We note that the correspondence in this subset is somewhat better than the entire sample of Mintz trajectories, but that there are still some rather disturbing differences.

The question to be answered at this point is whether the differences between the transition probabilities as computed from the real balloons and the Mintz balloons are really significant or are caused by sampling variations. The hypothesis is therefore made that there is a single set of transition probabilities p_{ij} and that the results of both the real balloons and the Mintz balloons are simply two different samplings from this population. If distribution at day t+1 of the balloons which originate in a single region is considered as a multinomial distribution, then it is possible to use the technique developed by Goodman[(6)] to test the hypothesis that the real transition probabilities and the Mintz transition probabilities are different samples from the same population. If this hypothesis is rejected at the five-percent level (that is, there being only one chance in twenty that the samples are from one population), then we must conclude that the Mintz trajectories are not a good representation of the real motion of balloons.

Goodman proposes a statistic, y^2, which, if the two samples are drawn from one population, would have a χ^2 distribution with the number of degrees of freedom equal to 1-k, where k is the number of cells in the multinomial distribution.

Table 7

SELECTED MINTZ TRAJECTORIES

From	To						Total
	NM-1	NM-2	NM-3	NM-4	NE	NP	
NM-1	11	22	0	0	0	0	33
NM-2	0	13	19	0	0	0	32
NM-3	0	0	8	17	0	1	26
Total							91

Table 8

COMPARISON BETWEEN FRACTION OF REAL BALLOONS AND FRACTION OF SELECTED MINTZ TRAJECTORIES MOVING FROM SECTOR TO SECTOR

From	To						Total
	NM-1	NM-2	NM-3	NM-4	NE	NP	
NM-1*	.21	.72	.035	.00	.035	.00	1.0
(Table 7)**	(.33)	(.67)	(.00)	(.00)	(.00)	(.00)	1.0
NM-2	.05	.22	.54	.05	.12	.02	1.0
(Table 7)	(.00)	(.41)	(.59)	(.00)	(.00)	(.00)	1.0
NM-3							
(Table 7)	.00	.00	.43	.36	.21	.00	1.0
	(.00)	(.00)	(.31)	(.65)	(.00)	(.04)	1.0

*Real balloons.

**Fraction based on Table 7.

Table 9 shows the values of y^2 for the distribution of balloons starting in the regions NM-1, NM-2, and NM-3. Also in Table 9 are the values of χ^2 which would be exceeded 5 percent of the time if the hypothesis were true. The values of the y^2 statistic exceed the 5 percent values of χ^2 for one of the three cases which would indicate that the hypothesis was not true. We must therefore conclude that it is unlikely that the Mintz trajectories are a good representation of the real balloon trajectories.

Table 9

VALUES OF y^2 AND χ^2 FOR BALLOONS STARTING IN EACH OF THREE REGIONS (STATISTICAL TEST OF MINTZ TRAJECTORIES)

Sector	y^2	χ^2
NM-1	5.494	7.815
NM-2	14.411	12.592
NM-3	5.545	7.815

It can be seen in Table 7 that the Mintz trajectories do not show a finite probability of moving into the equatorial regions, whereas the real balloons indicate a quite respectable probability of being carried to these regions. It is possible, therefore, to propose a simple hypothesis that the probability of a transition to the equatorial regions is the same for both the Mintz and the real trajectories. For the real balloons, there are 10 transitions into the equatorial regions out of 112 transitions yielding a probability of 0.0894. For the Mintz trajectories there are no equatorial transitions out of 91 yielding a probability of zero. The difference in probabilities is therefore 0.0894. Assuming there is no real difference between these two, the pooled data yield a probability of 0.0493. The standard deviation of the difference of probability according to large-sample theory is 0.0305. The actual difference exceeds the standard deviation of the difference by a factor of more than two, so there is less than one chance in twenty that this sample of Mintz trajectories would have no equatorial transitions if it were drawn from the same population as the real balloons.

The results of these tests of the applicability of the Mintz model to the trajectories of balloons in the real atmosphere suggest that the depletion of balloons in the equatorial regions, which was discussed in connection with the transition-probability matrix, are an artifact of the model and not a real phenomenon. Mintz is aware of the shortcomings of the model in the tropical regions; he believes that the lack of an adequate humidity term in his equations produced a Hadley cell that was too strong and too broad.[7] A new model being programmed will carry water vapor as an explicit variable, which might produce results that are more in accord with the real atmosphere.

III. AN EXAMPLE OF THE USE OF THE TRANSITION MATRICES

Despite the fact that the Mintz trajectories are not truly representative of the real atmosphere, the transition matrices that have been constructed can be used to demonstrate how this probabilistic approach might be used to predict the areal distribution of balloons and to adjust the launchings to obtain an optimal distribution.

Let us suppose that at day zero an even distribution had been achieved with 30 balloons in each region. (The launching strategy to reach this state is difficult to determine, but it is hoped that further study will show how this can be done.) Nine days later the expected number of balloons and the standard deviation of these expected numbers are as shown in the first two columns of Table 10. The third column shows a hypothetical distribution of balloons that would be highly probable according to the mean and standard deviations given in the first two columns. Although the number of balloons in each region was chosen to be a reasonably probable result of the nine-day transition matrix, the numbers placed in the regions that would feed into NE-3 in three additional days were deliberately chosen to be low to show how an anomaly could develop in the pattern. If the numbers in the third column of Table 10 are now considered to be a reasonable observed distribution of the initial 1020 balloons and the three-day matrix of Table 3 is applied to this distribution, the expected number of balloons in NE-3 would be only 7.43.

If it were important to have good coverage in NE-3, additional balloons would have to be launched. The maximum transition probability from the three-day matrix is 0.253 that the balloons will stay in NE-3. The best chance of increasing the number of balloons in NE-3 after three days is to launch the balloons into NE-3. The land masses available for launching into this region are the western portion of Africa and the area of the Guianas in South America. Because NE-2 is the region with the next highest probability of transfer to NE-3, it might be logical to choose South American launch sites to achieve the desired coverage. In any event, it would require about 20 balloon launches to

Table 10

DISTRIBUTION OF BALLOONS BY "MINTZ CIRCULATION"
NINE DAYS AFTER A WORLD-WIDE UNIFORM DISTRIBUTION

Geographi-cal sector	Expected after 9 days, E_{30}	σ_{30}	Distribution based on Column 2	Geographi-cal sector	Expected after 9 days, E_{30}	σ_{30}	Distribution based on Column 2
NP-1	37.1	5.86	38	SP-1	45.50	6.34	46
NP-2	34.5	5.67	35	SP-2	45.10	6.37	45
NM-1	68.0	7.68	68	SM-1	37.3	5.81	38
NM-2	47.2	6.54	48	SM-2	36.8	5.79	37
NM-3	42.3	6.24	40	SM-3	36.9	5.80	37
NM-4	47.0	6.53	44	SM-4	32.1	5.42	34
NM-5	52.0	6.82	52	SM-5	35.2	5.65	35
NM-6	47.1	6.53	48	SM-6	34.0	5.57	34
NM-7	47.6	6.54	48	SM-7	34.5	5.62	35
NM-8	42.2	6.22	44	SM-8	34.6	5.63	35
NM-9	18.9	4.26	19	SM-9	31.8	5.36	33
NE-1	13.1	3.58	12	SE-1	11.37	3.22	12
NE-2	13.0	3.55	11	SE-2	7.80	2.68	8
NE-3	13.25	3.57	10	SE-3	7.80	2.70	7
NE-4	9.33	3.02	8	SE-4	5.37	2.28	6
NE-5	12.50	3.45	13	SE-5	7.27	2.59	8
NE-6	19.35	4.26	20	SE-6	11.91	3.31	12

gain an expected 5 additional balloons.

The above exercise represents only a crude demonstration of how the matrix could be utilized. It is hoped that more efficient methods for optimizing the balloon distribution can be developed from the transition matrix approach to the dynamics of a free-floating balloon system. We believe that further study of the system would provide a great deal of insight into the problem of achieving and maintaining a distribution of balloons that would yield the maximum amount of information for a given investment in balloons and launching equipment.

IV. CONCLUSION AND SUGGESTIONS

We have attempted to show how a probabilistic model of a system of free-floating balloons can be constructed and to suggest how such a model could be used to design efficient launching strategies. This report is only a feasibility study because the Mintz trajectories do not produce transition probability matrices which are adequate reflections of real atmospheric processes and because we have not had sufficient time to develop a theory of optimal launch strategies. We are convinced, however, that the approach we have attempted to demonstrate can be corrected and expanded into a useful tool for studying the problem of attaining an operationally useful free-floating balloon system.

An adequate set of transition matrices might well be produced from the actual weather data compiled by the National Meteorology Center. The same machine programs for constructing trajectories that were applied to the Mintz data could be adapted to the historical records. The program we developed to convert trajectories into transition matrices could then be applied. These transition matrices could subsequently be checked against the transosonde data and any other real balloon flights that may become available. We believe that such a program would avoid the bias in the Mintz trajectories and could provide for the necessary adjustment for seasonal change.

Given an adequate set of transition matrices, it should be possible to determine optimum launch strategies for any given set of launch sites and any desired distribution of balloons. Although such a program would require resources of men, money, and machines, its cost would be a small fraction of the cost of the failure of an operational system.

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