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OCTOBER 1963 TO OCTOBER 1964

STUDY OF PROPAGATION OF COHERENT RADIATION AT VISIBLE OR NEAR VISIBLE WAVELENGTHS THROUGH THE ATMOSPHERE

by

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FOREWORD

The report was prepared by ITT Federal Laboratories, a Division of International Telephone & Telegraph Corporation, 15151 Bledsoe Street, San Fernando, California, on National Aeronautics and Space Administration contract No. NAS8-11103. The report covers the research program from 1 October 1963 to 1 October 1964. The work was administered under the direction of the George C. Marshall Space Flight Center, Huntsville, Alabama, with Dr. James B. Dozier as the Contracting Officer's Technical Representative.

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ABSTRACT

The degree of coherence for optical beams traversing a fluctuating medium is derived in terms of the correlation coefficient of the atmosphere. Under certain specific conditions, it is shown that the degree of coherence is determined solely by one atmospheric parameter, the mean square angular fluctuation of a ray. The degree of coherence of laser beams traversing artificially created fog atmospheres within the ITTFL optical tunnel has been measured by the two-pinhole interferometric technique. The results obtained are in good agreement with the theoretical predictions. Based on this work, the aperture limitation, imposed by the atmosphere, on an optical heterodyne receiver is predicted.

1.0 INTRODUCTION

The purpose of the present program is to investigate the propagation of coherent radiation traversing the atmosphere.

Initially, the program was organized to study the scattering of coherent light from aerosols. The intense, quasi-monochromatic laser source offered the possibility of obtaining improved high resolution scattering data. Moreover, it had been suggested that charged aerosols might "on the average" distribute themselves in some regular manner. If this effect were pronounced, it might lead to a characteristic lobe structure in the scattered radiation pattern. Several experimental and theoretical investigations were undertaken. These are described in more detail in Appendices A and B of this report.

In January 1964 the coordinating NASA personnel expressed a strong interest in determining the limitations imposed by the atmosphere on optical communication systems employing optical heterodyne receivers. The program was then revectored to emphasize the study of this problem. It was considered most expedient to conduct initial experiments in the atmosphere of the ITTFL fog tunnel. Here the semiquantitative control of atmosphere parameters would permit an easier interpretation of the experimental data. Only after the results of the tunnel work could be described in terms of an empirical or fundamental theory would consideration be given to performing experiments in the real atmosphere.

The bulk of this report is devoted to reviewing the work which has been performed in this context. In Section 2.0 the significance of the mutual coherence function relative to the spectral power density of photo-detector output current is demonstrated. The experimental determination of the mutual coherence function is described in Section 3.0. In Section 4.0 a simple theoretical expression for the mutual coherence function is derived. Finally, in Section 5.0 a comparison is made between the theoretical and experimental results.

2.0 OPTICAL HETERODYNING AND THE MUTUAL COHERENCE FUNCTION

Optical heterodyne receivers may be constructed with a broad range of sophistication. In addition to the front-end optical mixing and demodulation, any number of mixing stages may be used to demodulate further the subcarriers. In the following development only the simplest receiver is considered. The addition of further mixing simply requires a knowledge of the higher optical moments. When the signal is strong enough to be treated as a gaussian variate, the higher moments can be computed from the first moment (mutual coherence function). When gaussian statistics do not apply, each individual moment must be determined.

The current at the output of the photodetector originating in an elemental volume $d^3\vec{r}_1$ at the point $\vec{r}_1$ at time t_1 may be expressed as

$$di(\vec{r}_1,t_1) = kd^3 r_1 \int h(\vec{r}_1,t_1-\tau_1)\, V^2\, (\vec{r}_1,\tau_1)\, d\tau_1$$

where

- $V(\vec{r},t)$ is the optical field at $\vec{r}$ at t,
- $h(\vec{r},t)$ is the impulse response of the detector,
- k is a constant of proportunality including, among other things, the quantum efficiency of the detector.

The total current is given by the integral over the volume of the detector, i.e.,

$$i(t_1) = k\int_V d^3\vec{r}_1 \int h(\vec{r}_1,t_1-\tau_1)\, V^2(\vec{r}_1,\tau_1)\, d\tau_1 \,.$$

A quantity of extreme importance in evaluating communication systems is the spectral power density $G(\omega)$. This can be obtained from the Fourier transform of the autocorrelation function $A(S)$, where

$$A(S) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} i(t_1)\, i(t_1 + S)\, dt_1 .$$

When the processes are ergodic, the autocorrelation function can be obtained from the ensemble average, i.e.,

$$A(S) = < i(t_1)\ i(t_2) > \qquad t_2 - t_1 = S$$

where the symbols < > denote ensemble averaging. Introducing the previous expression for the current and interchanging the order of integrations and ensemble averaging, one obtains

$$A(S) = \int d^3 r_1 \int d^3 r_2 \int d\tau_2 \int d\tau_1\ h(\vec{r}_1, t_1 - \tau_1)\ h(\vec{r}_2, t_2 - \tau_2)$$

$$< V^2(\vec{r}_1, \tau_1)\ V^2(\vec{r}_2, \tau_2) > .$$

Because of the stationary hypothesis, the averaged product $< V^2(\vec{r}_1,\tau_1)\ V^2(\vec{r}_2,\tau_2) >$ is a function of the time difference $\tau_2 - \tau_1$. Introducing the notations

$$< V^2(\vec{r}_1,\tau_1)\ V^2(\vec{r}_2,\tau_2) > = R(\vec{r}_1,\vec{r}_2,\tau_2 - \tau_1)$$

$$h(\vec{r}_1, t_1 - \tau_1) = \frac{1}{2\pi} \int \hat{h}(\vec{r}_1, \omega_1)\ e^{+i\omega_1(t_1 - \tau_1)}\ d\omega_1$$

$$h(\vec{r}_2, t_2 - \tau_2) = \frac{1}{2\pi} \int \hat{h}(\vec{r}_2, \omega_2)\ e^{+i\omega_2(t_2 - \tau_2)}\ d\omega_2$$

$$R(\vec{r}_1,\vec{r}_2,\tau_2 - \tau_1) = \frac{1}{2\pi} \int \hat{R}(\vec{r}_1,\vec{r}_2,\omega_3)\ e^{+i\omega_3(\tau_2 - \tau_1)}\ d\omega_3$$

into the previous expression, and reversing the order of some of the integrations, one obtains

$$A(S) = \frac{1}{2\pi} \int_V d^3r_1 \int_V d^3r_2 \int d\omega_3 \, \hat{h}(\vec{r}_1, -\omega_3) \, \hat{h}(\vec{r}_2, \omega_3) \, \hat{R}(\vec{r}_1, \vec{r}_2, \omega_3) \, e^{+i\omega_3 S}$$

and

$$G(\omega) = \int_{-\infty}^{+\infty} A(S) e^{-i\omega S} \, d\omega = \int_V d^3r_1 \int_V d^3r_2 \, \hat{h}(\vec{r}_1, -\omega) \, \hat{h}(\vec{r}_2, \omega) \hat{R}(\vec{r}_1, \vec{r}_2, \omega) \ .$$

In many instances the response of the photodetector will be essentially the same for all portions of the detector. Then $\hat{h}(\vec{r}_1,\omega) = \hat{h}(\vec{r}_2,\omega) = \hat{h}(\omega)$. Moreover, since $h(\vec{r}_1,t)$ is real $\hat{h}(\vec{r}_1, -\omega) = \hat{h}^*(\omega)$ and one has, finally,

$$G(\omega) = \left|\hat{h}(\omega)\right|^2 \int_V d^3r_1 \int_V d^3r_2 \ \hat{R}(\vec{r}_1, \vec{r}_2, \omega) \ .$$

Thus, the spectral power density may be obtained if the quantity $\hat{R}(\vec{r}_1, \vec{r}_2, \omega)$ or its transform $< V^2(\vec{r}_1, t_1) \ V^2(\vec{r}_2, t_2) >$ is known. There has been no attempt to include the effects of shot and thermal noise in the previous expression. This may be done, however, in a straightforward, but tedious, manner following the approach of Davison[1].

If the optical field V intercepted by the photodetector is composed of a local oscillator field $V_1(\vec{r},t)$ and an incoming signal field $V_2(\vec{r},t)$,

1 W. F. Davison, "Shot Noise in Optical-to-Electric Converters, Part I," ITTFL Technical Memorandum No. 63-239, p. 2-11, December 1963.

the averaged product $< V^2(\vec{r}_1,t_1)\ V^2(\vec{r}_2,t_2) >$ may be expanded.

$$\begin{aligned} < V^2(\vec{r}_1,t_1)\ V^2(\vec{r}_2,t_2) > = < & V_1^2(\vec{r}_1,t_1)\ V_1^2(\vec{r}_2,t_2) + 2V_1(\vec{r}_1,t_1)\ V_2(\vec{r}_1,t_1)\ V_1^2(\vec{r}_2,t_2) \\ & + V_2^2(\vec{r}_1,t_1)\ V_1^2(\vec{r}_2,t_2) + 2V_1^2(\vec{r}_1,t_1)\ V_1(\vec{r}_2,t_2)\ V_2(\vec{r}_2,t_2) \\ & + 4V_1(\vec{r}_1,t_1)\ V_2(\vec{r}_1,t_1)\ V_1(\vec{r}_2,t_2)\ V_2(\vec{r}_2,t_2) \\ & + 2V_2^2(\vec{r}_1,t_1)\ V_1(\vec{r}_2,t_2)\ V_2(\vec{r}_2,t_2) + V_1^2(\vec{r}_1,t_1)\ V_2^2(\vec{r}_2,t_2) \\ & + 2V_1(\vec{r}_1,t_1)\ V_2(\vec{r}_1,t_1)\ V_2^2(\vec{r}_2,t_2) + V_2^2(\vec{r}_1,t_1)\ V_2^2(\vec{r}_2,t_2) > . \end{aligned}$$

Because the fields V_1 and V_2 in general are independent, and because $< V_1 > = < V_2 > = 0$, the above expression may be immediately reduced to

$$\begin{aligned} < V^2(\vec{r}_1,t_1)\ V^2(\vec{r}_2,t_2) > = & < V_1^2(\vec{r}_1,t_1)\ V_1^2(\vec{r}_2,t_2) > + < V_2^2(\vec{r}_1,t_1)\ V_2^2(\vec{r}_2,t_2) > \\ & + < V_2^2(\vec{r}_1,t_1) > < V_1^2(\vec{r}_2,t_2) > + < V_1^2(\vec{r}_1,t_1) > < V_2^2(\vec{r}_2,t_2) > \\ & + 4 < V_1(\vec{r}_1,t_1)\ V_1(\vec{r}_2,t_2) > < V_2(\vec{r}_1,t_1)\ V_2(\vec{r}_2,t_2) > . \end{aligned}$$

The first term is the first cross-moment of quantities proportional to the instantaneous power of the local oscillator. The second term has a similar meaning for the incoming signal. The third and fourth terms are just products of the average powers of the local oscillator and the incoming signal. The last term represents the heterodyne contribution to the spectral power density.

When the incoming signal is small compared to the local oscillator signal, the term $< V_2^2(\vec{r}_1,t_1)\ V_2^2(\vec{r}_2,t_2) >$ is of the second order in smallness compared with $< V_2(\vec{r}_1,t_1)\ V_2(\vec{r}_2,t_2) >$ and may be neglected. This is normal heterodyne operation. If the incoming signal is large and if the field process may be assumed gaussian, the higher moments can be expanded to give:

$$< V^2(\vec{r}_1,t_1)\ V^2(\vec{r}_2,t_2) > = \left\{ < V_1^2(\vec{r}_1,t_1) > + < V_2^2(\vec{r}_1,t_1) > \right\} \left\{ < V_1^2(\vec{r}_2,t_2) > + < V_2^2(\vec{r}_2,t_2) > \right\}$$

$$+\ 2 \left\{ < V_1(\vec{r}_1,t_1)\ V_1(\vec{r}_2,t_2) > + < V_2(\vec{r}_1,t_1)\ V_2(\vec{r}_2,t_2) > \right\}^2 .$$

In both cases the heterodyne contribution to the spectral power density can be determined if the first cross-moment $< V_2(\vec{r}_1,t_1)\ V_2(\vec{r}_2,t_2) >$ is known.

Evoking the stationary hypotheses, the moments become independent of the time origins and depend on the time difference $S = t_2 - t_1$. The first cross-moment may, then, be written as:

$$< V_2(\vec{r}_1,t_1)\ V_2(\vec{r}_2,t_2) > = < V_2(\vec{r}_1,0)\ V_2(\vec{r}_2,S) > .$$

This latter quantity is just one-half the real part of the mutual coherence function $\Gamma(\vec{r}_1,\vec{r}_2,S)$ which plays the central role in the theory of partially coherent light*. Since the first cross-moment, Γ, and the real part of Γ

* For a detailed description of the mutual coherence function, the reader is referred to one of the following texts: M. Born and E. Wolf, Principles of Optics, Pergamon Press, 1959; M. J. Beran and G. B. Parrent, Jr., Theory of Partial Coherence, Prentice Hall, 1964; E. L. O'Neill, Introduction to Statistical Optics, Addison-Wesley Pub. Co., Inc. 1963.

all determine one another through the expressions

$$\mathrm{Re}\left\{\Gamma(\vec{r}_1,\vec{r}_2,S)\right\} = 2 < V(\vec{r}_1,0)\; V(\vec{r}_2,S) >$$

$$\mathrm{Im}\left\{\Gamma(\vec{r}_1,\vec{r}_2,S)\right\} = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\mathrm{Re}\left\{\Gamma(\vec{r}_1,\vec{r}_2 S')\right\}}{S' - S}\, dS'$$

$$\Gamma = \mathrm{Re}\,\Gamma + i\; \mathrm{Im}\,\Gamma\ ,$$

we will, to shorten the expressions, use the appropriate Γ notation rather than write the moment expression explicitly. It is also convenient to introduce the complex degree of coherence $\gamma(\vec{r}_1,\vec{r}_2,S)$ defined by

$$\gamma(\vec{r}_1,\vec{r}_2,S) = \frac{\Gamma(\vec{r}_1,\vec{r}_2,S)}{\left\{\Gamma(\vec{r}_1,\vec{r}_1,0)\;\Gamma(\vec{r}_2,\vec{r}_2,0)\right\}^{1/2}}\ .$$

The quantities $\Gamma(\vec{r}_1,\vec{r}_1,0)$ and $\Gamma(\vec{r}_2,\vec{r}_2,0)$ just represent the average squared fields at the points $\vec{r}_1$ and $\vec{r}_2$.

Consider now an optical disturbance arriving at the points $\vec{r}_1$ and $\vec{r}_2$ after traversing some atmospheric path. The coherence between disturbances at the two points may, in principle, be investigated by introducing an opaque screen with small apertures at $\vec{r}_1$ and $\vec{r}_2$ and observing the intensity of transmitting light at some point Q in back of the screen (Figure 1.).

It is not difficult to show* that I(Q), the intensity at Q, is

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* M. Born and E. Wolf, loc cit.

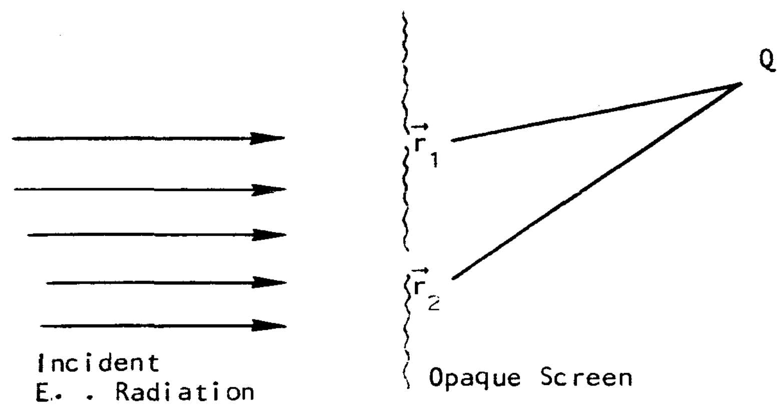

Figure 1. Measurement of γ with Two Pinhole Experiment

given by

$$I(Q) = I_1(Q) + I_2(Q) + 2\sqrt{I_1(Q)\ I_2(Q)}\ \ \mathrm{Re}\,\gamma(\vec{r}_1, \vec{r}_2, S)$$

where $I_1(Q)$ is the intensity observed at Q when the aperture at $\vec{r}_2$ is closed and $I_2(Q)$ is the intensity observed at Q when the aperture at $\vec{r}_1$ is closed. The different path lengths between the aperture at $\vec{r}_1$ and Q and the aperture at $\vec{r}_2$ and Q determine the time delay S. γ can then be computed from a knowledge of $I(Q)$, $I_1(Q)$, and $I_2(Q)$. Knowing γ one can compute the first cross-moment $< V(\vec{r}_1, 0)\ V(\vec{r}_2, S)>$.

While the simple arrangement just described works in principle, it is not very practical. When the two apertures are widely separated, one has difficulty finding a region of space for which both $I_1(Q)$ and $I_2(Q)$ are large enough to be measured. Moreover, one cannot vary the time delay between the two points arbitrarily; an upper bound to the path difference is always set by the aperture separation.

The next order of complexity involves folding the "path" of the light emerging from one or both of the apertures (Figure 2). This arrangement permits the more intense portions of the light from both apertures to be superimposed. A range of time delays could be obtained by varying

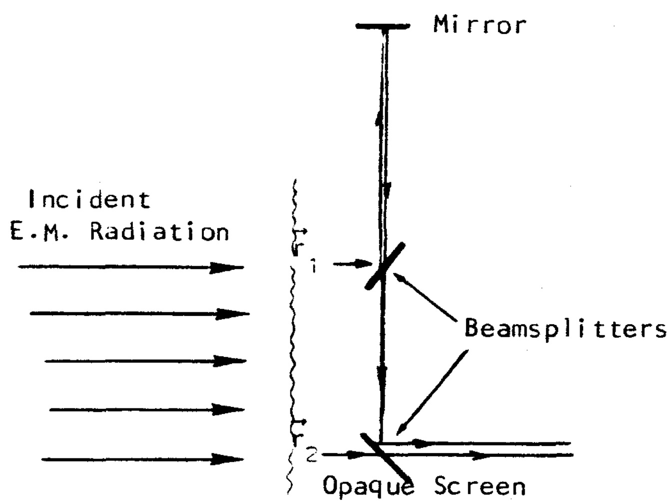

Figure 2. Modification of Two Pinhole Experiment

the length of one of the paths; however, for the cases of interest this is not necessary. Let us digress for a moment to see what properties the mutual conerence function of laser light traversing an atmosphere might possess.

Let the optical disturbances reaching the points at $\vec{r}_1$ and $\vec{r}_2$ in the absence of atmospheric modulation be $X_1(t)$ and $X_2(t)$. The optical disturbances in the presence of modulation may be written as $M_1(t)X_1(t)$ and $M_2(t)X_2(t)$. The first cross-moment may then be expressed as

$$< V(\vec{r}_1,0)\ V(\vec{r}_2,S) > = < M_1(0)M_2(S)X_1(0)X_1(S) > = < M_1(0)M_2(S) > < X_1(0)X_2(S) >$$

since the modulation may be assumed independent of the laser output. If the center band angular frequency of the laser source is ω_o, if the line width is $\Delta\omega_o$, and if the line shape is Lorentzian, the quantity $< X_1(0)X_2(S) >$ is proportional to $e^{-\Delta\omega_o|S|}\ e^{i\omega_o S}$. With similar assumptions about the modulation, one has

$$\Gamma(\vec{r}_1,\ \vec{r}_2,\ S) = \Gamma(\vec{r}_1,\vec{r}_2,0)e^{-\Delta\omega_A|S|}\ e^{i\omega_A S}\ e^{-\Delta\omega_o|S|}\ e^{i\omega_o S}$$

where ω_A and $\Delta\omega_A$ are the center band angular frequency and the linewidth of the modulation. Now it is certainly true that $\omega_o >>> \omega_A$. Moreover, laser linewidths are usually greater than 10 kc, while the atmosphere modulation is well below 1 kc. Then, generally, $\Delta\omega_o >>> \Delta\omega_A$ and the time portion of the mutual coherence function is dominated by the time coherence of the optical source. The assumption of a Lorentzian line shape for the modulation is ad hoc, but the conclusions are valid for an aribtrary line shape. Thus, because the correlation time associated with the atmospheric fluctuation is much longer than the correlation time of the optical source, the temporial portion of the mutual coherence function is determined principally by the source.

Consider again the problem of measuring the degree of coherence with the previous arrangement. It is now clear that any path difference for which $\Delta\omega_o S <<<< 1$ and $\omega_o S = 2n\pi$ (n an integer) will suffice. Then

$$\Gamma(\vec{r}_1,\vec{r}_2,\ \frac{2n\pi}{\omega_o}) \cong \Gamma(r_1,r_2,0)$$

and $\Gamma(\vec{r}_1,\vec{r}_2,S)$ can be inferred from the time coherence and/or spectral power density of the source.

To recapitulate then, the heterodyne contribution to the spectral power density can be related to the first cross-moment of the optical disturbance at the photodetector. This first cross-moment is simply related to the mutual coherence function and the complex degree of coherence. The spatial part of these latter functions can be determined from a simple interference experiment, and the temporal part can be inferred from the relative spectral power densities of the modulating atmosphere and the laser source.

3.0 MEASUREMENTS OF THE DEGREE OF COHERENCE

In Section 2 it was shown that the heterodyne signal can be computed if the complex degree of coherence is known. This quantity can be measured by many techniques, but the most straightforward approach is the interferometric arrangement outlined in the previous section.

After performing several crude interferometric experiments to acquire an appreciation of the environmental surroundings, the experimental apparatus shown schematically in Figure 3 was assembled. Light from the laser is split into two beams whose separation may be varied. The beams, after traversing the tunnel atmosphere, pass through two small sampling apertures, are combined, and directed onto the photodetector aperture. A lens and neutral density optical filters are used to adjust the size and intensity of the fringes. The two beams are chopped to give the intensity from first one sampling aperture, then the other, and finally, the intensity arising from the superposition of the fields from both apertures. The output from the photodetector is displayed on a chart recorder.

A typical data sample from this arrangement is shown in Figure 4. The two rows of small dots on the recording are proportional to the intensities from the individual sampling apertures, while the upper trace is proportional to the intensity with both apertures open.

Because the average power density over the beams is quite uniform, in later experiments it was found convenient to remove the chopper and simply monitor one of the beams. When this was done the receiving end was arranged as shown in Figure 5. Also when the sampling apertures were spaced within a single beam diameter, only one beam from the source was used.

The source is a Spectra Physics Model 115 He–Ne laser operating at the 6328 Å line. The maximum output of the source is 6 milliwatts; however in the current experiments the laser output is typically 0.5 milliwatts, to avoid complexities arising from multi-moding. Even at one milliwatt, two strong modes separated by 1 Gc are observed.

The laser is mounted on a 1/2 meter Ealing optical bench which is in turn mounted on a 5/8–inch aluminum plate. Also mounted on the 5/8–inch plate is a 1 meter Ealing bench, perpendicular to the laser axis, which

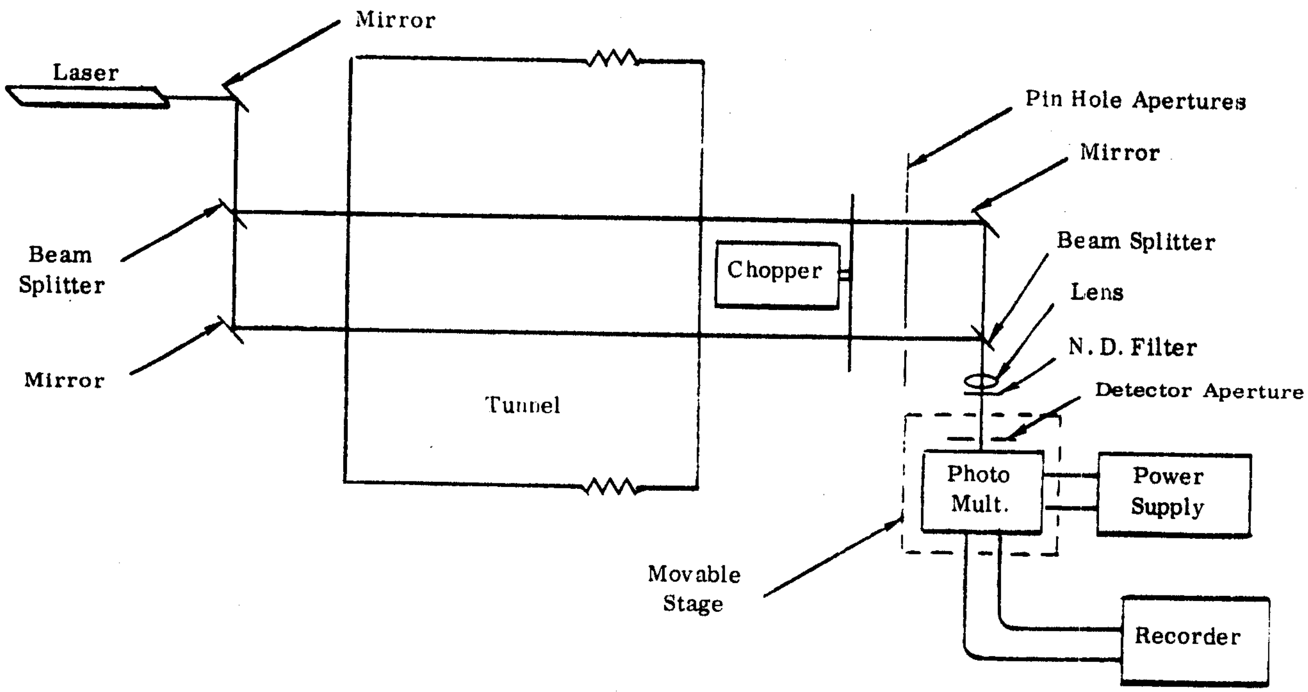

Figure 3. Schematic of Experimental Arrangement for Measurement of the Mutual Coherence.

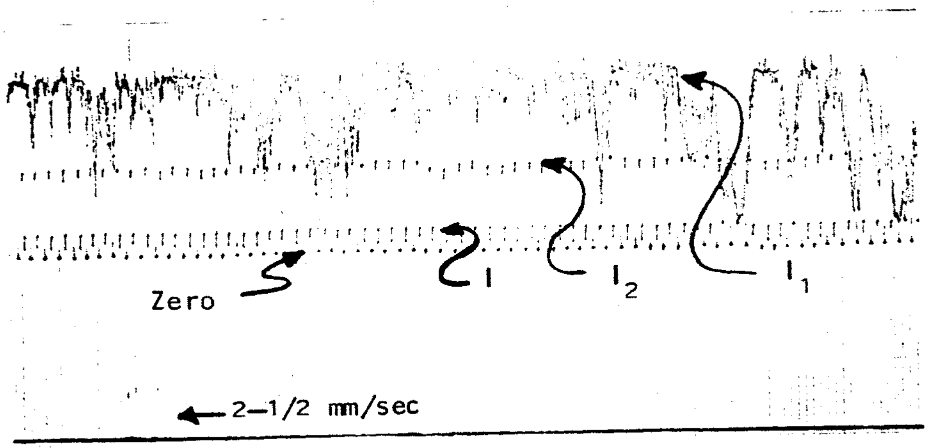

Figure 4. Typical Data Collected with Arrangement Shown in Figure 3.

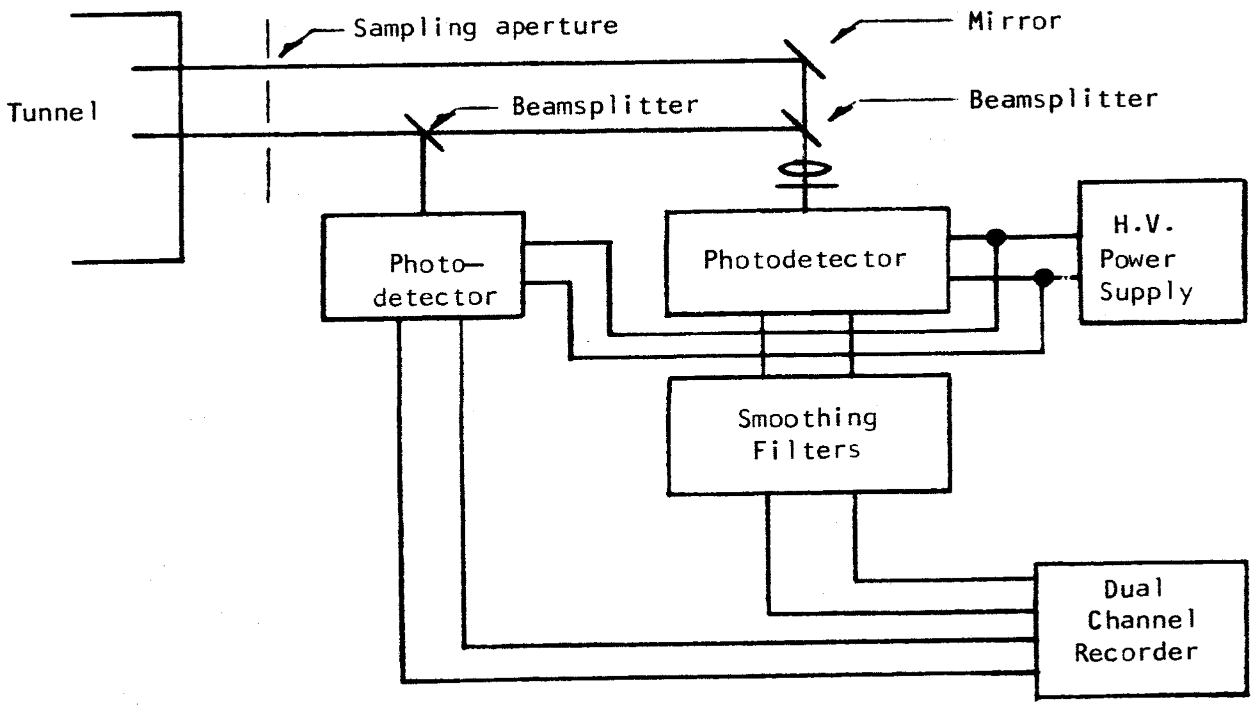

Figure 5. Schematic of Second Experimental Arrangement for the Measurement of the Mutual Coherence Function

supports the mirrors and beamsplitters. The plate itself rests on three 3/8-inch screws which are used to adjust the elevation of the plate. The entire construction rests on the slate top of a "stable table." A similar arrangement is used to mount the beamsplitters and mirrors at the other end of the tunnel.

All first surface mirrors and dielectric coated beamsplitters used have dimensions of the order of 2 inches square and are optically flat to within ten or twenty fringes. This is more than adequate, for only a small portion of the complete surface is used at any one time. Each element is mounted on a 3-inch square by 1/6-inch thick aluminum plate. This plate is attached to a base plate of the same dimensions by three spring loaded adjustment screws and the base is clamped in standard Ealing lens holders. The sampling apertures are generally one millimeter in diameter. When the sampling apertures are very close, their diameters are reduced to approximately one half of a millimeter. The aperture in front of the photodetector is one fourth of a millimeter in diameter.

The tunnel itself is constructed of 100-foot of 5-foot diameter concrete pipe buried in the ground beneath the laboratories. At each end of the tunnel there is a small 8 x 19 x 8 foot work area also beneath the ground level. Radiometric measurements of the tunnel and work area walls indicate that the temperature is uniform to within one degree Celcius. The air temperature as monitored by a recording thermometer is also constant to within one degree Celcius over a 24-hour period.

The tunnel is also equipped with apparatus for injecting and exhausting artificial fog atmospheres. Fog is injected into the tunnel by means of a Missimer Spraying system.

Two types of nozzles, giving 5 and 10 micron mean diameter fog particles are used. These figures are the manufacturer's specification. With the 10 micron nozzles fogs of optical densities greater than 2.5 can be obtained in less than 20 minutes. The exhaust system consists of a 1500 cubic feet per minute fan forcing air through a 12-inch duct. In addition to exhausting the fog in approximately 2 minutes, it can generate essentially laminar air flow in the tunnel with velocities up to 2 feet per second.

The "stable tables" consist of approximately 3/4 ton of concrete cinder block resting on MB Isomode Pads. The Isomode Pads are on tile-covered concrete floor of the work area which is in contact with the earth about ten feet below ground level. On top of the cinder blocks are alternate layers of plywood and polyurethane. Finally on the last sheet of plywood is a layer of tennis balls which support a 34-inch x 50-inch x 1-inch slate top.

The photodetector(s) is an RCA IP21 phototube operated in a very standard manner, i.e., with 470K ohm dynode resistances and a 10,000 ohm external load. The phototube which detects the fringes is mounted on the power driven, traveling-stage of an Intectron VS-12 microphotometer which is used to position the detector on the peak portions of the fringe patterns.

The smoothing filter, when used, is a simple variable RC circuit with time constants ranging between .01 and 2 sec. The recorder is a Sanborn Model 296 Dual Channel Recorder equipped with Model 350-1000 preamplifiers. Finally, the phototube power supply is a Fluke Model 408 A high voltage supply with 0.25 per cent regulation.

With this experimental arrangement, the real part of the degree of coherence at zero path difference was first measured for the nonturbulent air atmosphere of the tunnel. The quantities $I(Q)$, $I_1(Q)$, and $I_2(Q)$ were obtained from time averages of the intensities which extended over periods of the order of one minute. The values obtained for Re $\gamma(\vec{r}_1, \vec{r}_2, 0)$ were $0.95 \pm .05$ for spacing $|\vec{r}_2 - \vec{r}_1|$ extending from 1 to 15 inches. When the period of averaging is extended beyond a few minutes, the value of γ begins to drop rapidly toward zero. This reduction of the γ values is due to slow fringe drifts which could result from shifts of small thermal gradients within the tunnel, from small convective air currents, or from a relative shifting of the two tables at each end of the tunnel. The first two could represent true atmospheric phenomenon while the latter is clearly an instrument error.

To eliminate the possibility of relative table motion, two nonturbulent paths through the tunnel were constructed from a length of 2-inch diameter pipe and a second interferometer was assembled to sense the relative table motions. A schematic of this arrangement is shown in Figure 6. During the operation of the reference interferometer the current of the photodetector was found to vary quasi-periodically with the period monotonically increasing from about five to twenty minutes over a time span of four hours. A similar behavior was not observed in the other interferometer. The only plausible explanation for this result is that a small thermal gradient exists laterally across the tunnel.

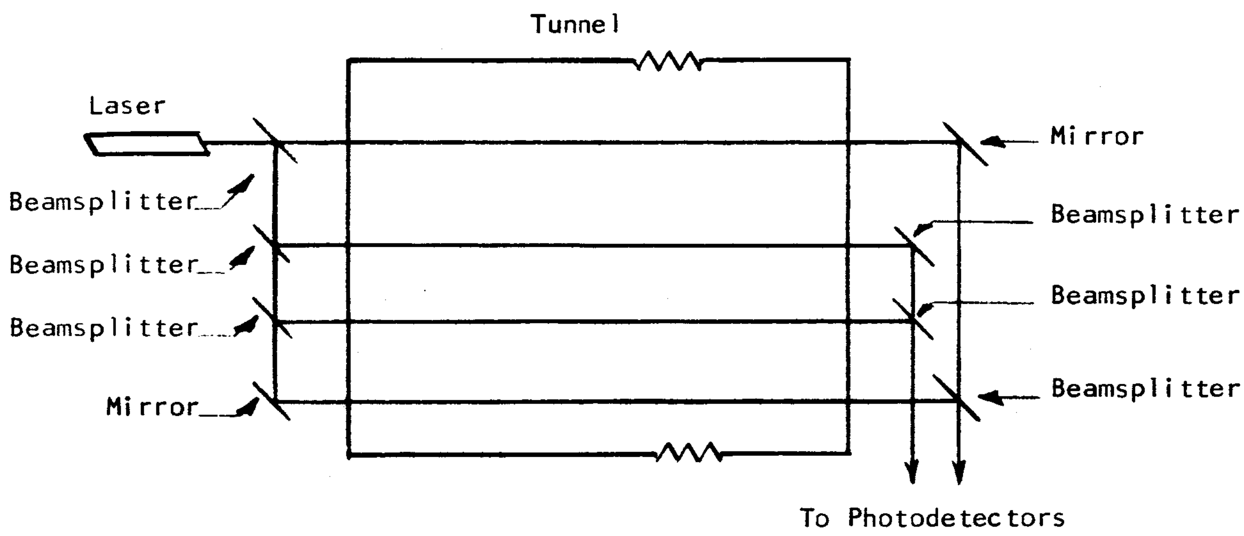

Figure 6. Double Interferometric Arrangement.

This is possible since the foundation of the outside wall of the building is close to one side of the tunnel. The interferometer operating in the tunnel atmosphere is not affected, since density changes caused by thermal heating of the air along the two beams become smoothed by convection. The pipes along the paths of the reference interferometer are individually sealed, and the corresponding density changes cannot be smoothed by convection.

A shift of one fringe in the reference interferometer would be caused by a mean temperature difference between the pipe of about 1/60 of a degree Celsius. To correct this problem, arrangements have been made to evacuate the pipes to pressures of a few millimeters of Hg. This would remove any thermal effects in the paths of the reference interferometer.

At the time of writing of this report, however, this experiment has not been performed.

In the interim, many measurements on the turbulent fog atmosphere have been made in which the degree of coherence is computed from the intensities averaged over periods of 5 to 60 seconds. In these experiments, the fogs have optical densities ranging between 0 and 2.

Several qualitative features are apparent from the experiments. When the turbulence is caused only by air issuing from the fog jets, the degree of coherence is only slightly affected. This indicates that for the path length used, the refractive index perturbations caused by convection are small. If a little water is introduced into the air streams, a substantial change in γ occurs. In this instance the atmosphere must contain turbulons of fog whose mean refractive index is much larger than the surrounding air. If the fog density is further increased, γ ceases to change rapidly. This would seem to indicate that the ratio of the mean refractive index of the turbulons to the mean refractive index over the total path is nearly constant.

Finally, if the fog jets are shut off and the fog is allowed to stabilize, the coherence is restored.

A determination of γ from quantities time averaged over periods of 5 to 60 seconds is, of course, not precisely correct since the periods should tend toward infinity. The values obtained, however, do place an upper bound on the degree of coherence. Figure 7 is a plot of the degree of coherence for various spatial separations, which is representative of the data collected to date.

To what extent the values obtained approximate the actual values of γ will depend on the period required to obtain a convergent time average. In parallel experiments, performed on Contract AF 30(602)-3284, the spectral distribution of the angular deflection of the optical beam traversing the same fogs has been measured for the frequency range 0.2 to 20,000 cycles per second. In these experiments, the spectral distribution peaks around

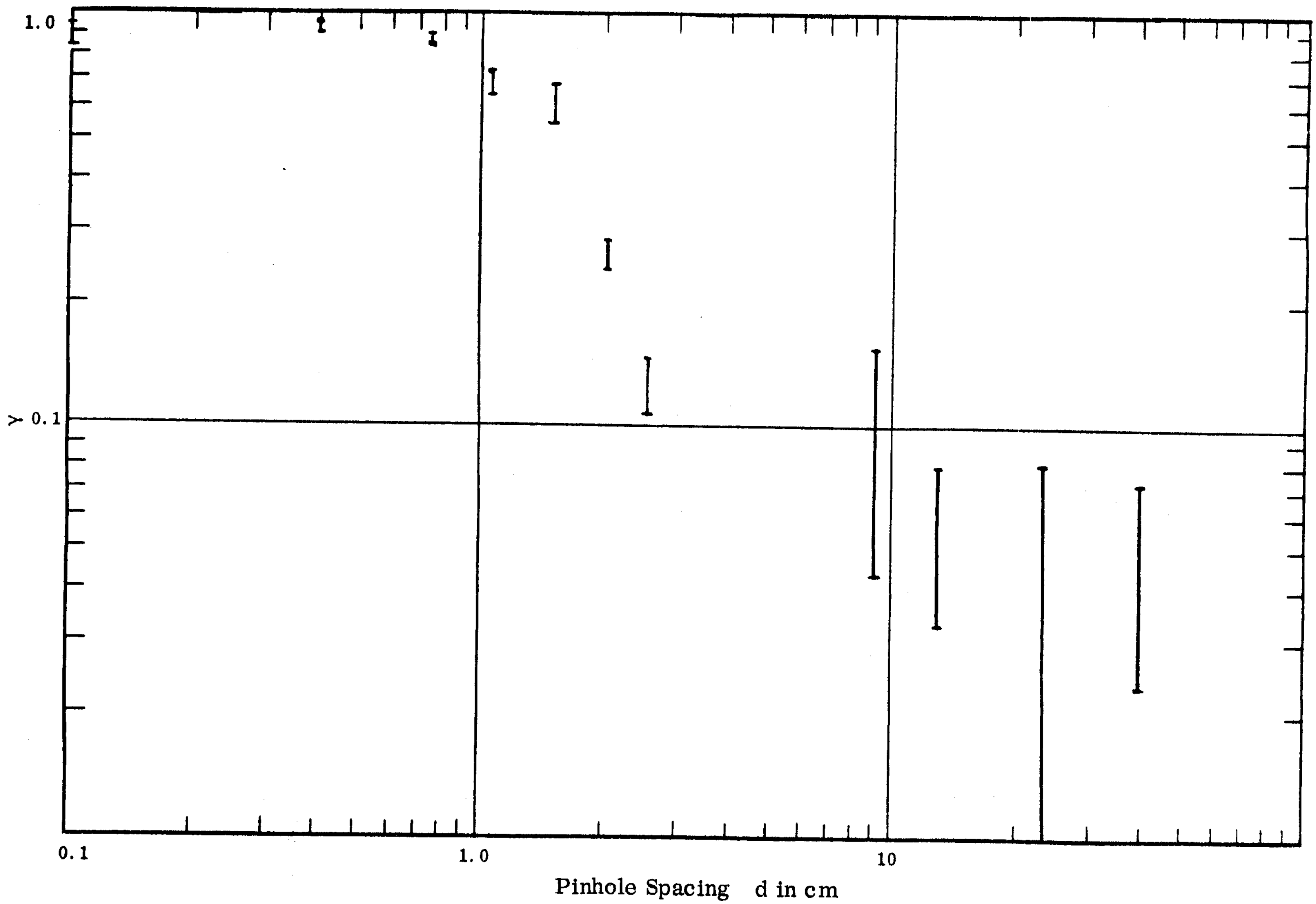

Figure 7. Measured Degree of Coherence γ Versus Pinhole Spacing in d

2 cps and has dropped sharply by .2 cps. If the distribution continues to drop rapidly below .2 cps, the very slow fringe fluctuations observed in the coherence experiment are probably due to relative table motions and should not be considered. Under these circumstances, the values displayed in Figure 7 would represent the true values of the degree of coherence rather than just upper bounds.

Also from these ray deflection experiments one can obtain the root mean square angular deflection for periods of at least 5 seconds. This quantity is a good parameter to represent the turbulence of the fog.

In the following section, it will be shown that the mean square angular deflection can be related to the degree of coherence, and in Section 5 a comparison will be made between the measured values of γ over short time periods, and those computed from the mean square angular deflection averaged over the same period.

4.0 THEORETICAL DESCRIPTION OF THE MUTUAL COHERENCE FUNCTION FOR QUASI–MONOCHROMATIC LIGHT TRAVERSING THE ATMOSPHERE

In the following paragraphs we relate the mutual coherence function of an optical disturbance to variations in refractive index which occur over the transmission path. The results are limited by the assumptions that: (1) The angular displacements of the optical rays are small; (2) The optical disturbance is quasi–monochromatic; (3) The statistics of the optical source and transmission medium are ergodic; (4) The statistics of the transmission medium are spatially homogeneous and isotropic.

Let $V(\vec{r}_1, t_1)$ be a component of the electromagnetic field at the point $\vec{r}_1$ at time t_1 due to a disturbance at 0 (Figure 8). Let $V(\vec{r}_2, t_2)$

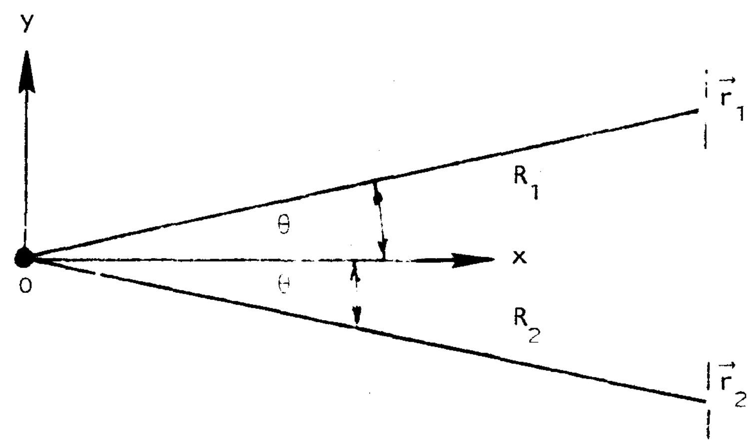

Figure 8. Geometry of Source and Observation Points

have a similar connotation for the point $\vec{r}_2$. Any scattered light arriving at $\vec{r}_1$ that has traversed a geometric path substantially greater than R_1 will contribute negligibly to the field at $\vec{r}_1$. Hence, we may write for the associated analytic signal

$$\mathcal{V}(\vec{r}_1, t_1) = \frac{A(t_1 - \xi_1)}{R_1}$$

where ξ_1 is the time required for the disturbance to travel the distance R_1, $|A(t)|$ is proportional to the strength of the source, and arg $A(t)$ is proportional to its phase. Similarly

$$\mathcal{V}(\vec{r}_2, t_2) = \frac{A(t_2 - \xi_2)}{R_2}$$

The mutual coherence function $\Gamma(\vec{r}_1, \vec{r}_2, t_1, t_2)$ is defined as the time average of the product $V(\vec{r}_1, t_1)\, V^*(\vec{r}_2, t_2)$. When the processes are time stationary $\Gamma(\vec{r}_1, \vec{r}_2, t_1, t_2) = \Gamma(\vec{r}_1, \vec{r}_2, \tau)$ where $\tau = t_2 - t_1$. Moreover, when the disturbance is quasi-monochromatic with mean frequency $\bar{\nu}$,

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau) = \Gamma(\vec{r}_1, \vec{r}_2, 0)e^{-2\pi i \bar{\nu}\tau}$$

provided τ is less than the coherence time of the disturbance.*

With the previous expressions for the fields V, Γ becomes

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau) = \frac{\overline{A(t_1 - \xi_1)\, A^*(t_2 - \xi_2)}}{R_1\, R_2} .$$

With the ergodic assumption the time average may be replaced by the

* M. Born and E. Wolf, loc cit.

ensemble average. Thus

$$\frac{\overline{A(t_1 - \xi_1)\ A^*(t_2 - \xi_2)}}{R_1\ R_2} = \frac{\langle A(t_1 - \xi_1)\ A^*(t_2 - \xi_2)\rangle}{R_1\ R_2}.$$

The ensemble average may be obtained by first averaging over the source process with fixed ξ_1 and ξ_2, and then averaging over the transmission process, i.e.,

$$\langle A(t_1 - \xi_1)\ A^*(t_2 - \xi_2)\rangle = \langle\langle A(t_1 - \xi_1)\ A^*(t_2 - \xi_2)\rangle_A\rangle_\xi .$$

Now $\langle A(t_1 - \xi_1)\ A^*(t_2 - \xi_2)\rangle_A = \langle A(0)\ A^*(\tau - \xi_2 + \xi_1)\rangle_A$ is just the time correlation $\rho(\tau - \xi_2 + \xi_1)$ of the source. Since the laser output is quasi-monochromatic, $\rho(\tau - \xi_2 + \xi_1)$ may be expressed as

$$\rho(\tau - \xi_2 + \xi_1) = \rho(0)e^{-2\pi i\bar{\nu}(\tau - \xi_2 + \xi_1)}$$

where $\bar{\nu}$ is the mean frequency of the source disturbance and $\tau - \xi_2 + \xi_1$ is small compared with the coherence time. The mutual coherence function may now be expressed as

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau) = \left\langle\frac{\rho(0)e^{-2\pi i\bar{\nu}(\tau - \xi_2 + \xi_1)}}{R_1\ R_2}\right\rangle_\xi$$

Or

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau) = \frac{\rho(0)}{R_1 R_2} \langle e^{+2\pi i \bar{\nu}(\xi_2 - \xi_1)} \rangle_{\xi} \, e^{-2\pi i \bar{\nu}\tau} .$$

Let the refractive index of the media be $n(X, Y, Z, t)$ and assume the variation of n at any point during the transit time of the disturbance is negligible. Then

$$\xi_i = \int_0^{R_i} n(X, Y, Z, t)\, d\sigma \qquad i = 1, 2$$

where $d\sigma$ is the element of path length. Writing $n = \bar{n} + \mu$ and $\xi_i = \bar{\xi}_i + \rho_i$ we have $\bar{\xi}_i = \frac{\bar{n} R_i}{c}$,

$$\rho_i = \frac{1}{c} \int_0^{R_i} \mu(X, Y, Z, t)\, d\sigma$$

and

$$\Gamma(\vec{r}_1, \vec{r}_2, 0) = \frac{\rho(0)}{R_1 R_2} e^{2\pi i \frac{\bar{\nu} \bar{n}}{c} \{R_2 - R_1\}} \langle e^{+2\pi i (\rho_2 - \rho_1)} \rangle_{\xi} .$$

Expanding the exponent and keeping only terms of second order or less in ρ_i, we have

$$\langle e^{2\pi i \bar{n}(\rho_2 - \rho_1)} \rangle_{\xi} = 1 - \frac{(2\pi\bar{\nu})^2}{2} \left\{ \langle \rho_1^2 \rangle_{\xi} + \langle \rho_2^2 \rangle - 2\langle \rho_1 \rho_2 \rangle_{\xi} \right\} .$$

Now

$$\langle \rho_i \rho_j \rangle_\xi = \frac{1}{c^2} \langle \int_0^{R_i} \mu(X_i, Y_i, Z_i, t)\,d\sigma_i \int_0^{R_j} \mu(X_j, Y_j, Z_j, t)\,d\sigma_j \rangle_\xi$$

$$= \frac{1}{c^2} \int_0^{R_i} d\sigma_i \int_0^{R_j} d\sigma_j \langle \mu(X_i, Y_i, Z_i, t)\, \mu(X_j, Y_j, Z_j, t) \rangle_\xi .$$

Since the statistics of the transmission path are assumed homogeneous and isotropic, the averages are only a function of the coordinate differences; then

$$\langle \mu(X_i, Y_i, Z_i, t)\, \mu(X_j, Y_j, Z_j, t) \rangle_\xi = \overline{\mu^2}\, N(X_i - X_j, Y_i - Y_j, Z_i - Z_j)$$

where $\overline{\mu^2}$ is mean square fluctuation at a point and N is the correlation coefficient. A common and useful functional form for N is

$$N(X_i - X_j, Y_i - Y_j, Z_i - Z_j) = e^{\frac{-r^2}{a^2}}$$

where $r^2 = (X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2$ and a indicates the scale of the correlation.

If the coordinate axes are chosen so that the Z = 0 plane coincides with the plane defined by the source and the two points at $\vec{r}_1$ and $\vec{r}_2$, and the X axis bisects the angle formed by the point at $\vec{r}_1$, the source, and the point at $\vec{r}_2$, one may write

$$Y_i(X_i) = X_i \tan\theta \cong \theta X_i,$$

$$Y_j(X_j) = -X_j \tan\theta \cong -\theta X_j,$$

$$Z_i = Z_j = 0.$$

Then setting $R = R_1 \cong R_j \ggg a$ and $d\sigma_i \cong dX_i$ and $d\sigma_j = dX_j$, one obtains

$$<\rho_k{}^2>_\xi = \frac{\bar{\mu}^2}{c^2} \int_0^R \int_0^R dX_1 dX_2 \exp\left\{- \frac{(X_1 - X_2)^2 + \theta^2(X_1 - X_2)^2}{a^2}\right\} \qquad k = 1, 2$$

and

$$<\rho_1\rho_2>_\xi = \frac{\bar{\mu}^2}{c^2} \int_0^R \int_0^R dX_1 dX_2 \exp\left\{- \frac{(X_1 - X_2)^2 + \theta^2(X_1 + X_2)^2}{a^2}\right\}$$

With the substitutions

$$u = \frac{X_1 - X_2}{2}, \quad v = \frac{X_1 + X_2}{2}$$

the two previous integrals reduce to

$$<\rho_k{}^2>_\xi = \frac{8\bar{\mu}^2}{c^2} \int_0^{R/2} dv \int_0^{R/2} du \exp\left\{- \frac{4(1 + \theta^2)u^2}{a^2}\right\} \qquad k = 1, 2$$

and

$$<\rho_1\rho_2>_\xi = \frac{4\bar{\mu}^2}{c^2} \int_0^{R/2} dv \left\{\exp(- \frac{4\theta^2 v^2}{a^2}) + \exp(- \frac{4\theta^2(R - v)^2}{a^2})\right\} \int_0^v du \exp(- \frac{4u^2}{a^2}) .$$

The inner integrands of both expressions are large only over a region $0 \le u \le u_0 \approx a/2$. When $R \ggg a$ and $\theta \lll 1$ the upper limit in the inner integral may be set equal to infinity. Then

$$\langle \rho_k{}^2 \rangle_\xi = \frac{\sqrt{\pi}\ \overline{\mu}^2\ a\ R}{c^2 \sqrt{1 + \theta^2}} \qquad k = 1, 2$$

and

$$\langle \rho_k{}^2 \rangle_\xi = \frac{\sqrt{\pi}\ \overline{\mu}^2\ a\ R}{c^2 \sqrt{1 + \theta^2}} \int_0^R dv \exp\left(- \frac{4\theta^2 v^2}{a^2} \right) . \qquad k = 1, 2$$

Collecting the previous results, one has for the mutual coherence function at zero time delay

$$\Gamma(\vec{r}_1, \vec{r}_2, 0) = \Gamma_0(\vec{r}_1, \vec{r}_2, 0)$$

$$\left\{ 1 - \frac{4\pi^{5/2} k^2\ \overline{\mu}^2\ a\ R}{\sqrt{1 + \theta^2}} \left[1 - \frac{\sqrt{\pi}\ a}{4\theta R} \operatorname{erf}\left(\frac{2\theta R}{a}\right) \right] \right\}$$

where $k = \frac{\overline{\nu}}{c}$ and

$$\Gamma_0(\vec{r}_1, \vec{r}_2, 0) = \frac{\rho(0)}{R_1 R_2}\, e^{2\pi i \frac{\overline{\nu} n}{c} \{R_2 - R_1\}} .$$

If the refractive index variation μ can be considered Gaussian at each point in space, then the quantity

$$\Psi(t) = \int_0^{R_2} \mu(X_i, Y_i, Z_i, t) d\sigma_i - \int_0^{R_1} \mu(X_j, Y_j, Z_j, t) d\sigma_j$$

is also Gaussian. The average value of $\exp \frac{2\pi i \overline{\nu}}{c} \Psi(t)$ is just the characteristic function evaluated at $\frac{2\pi \overline{\nu}}{c}$; then

$$< \exp \frac{2\pi i \overline{\nu}}{c} \Psi(t)>_\xi = e^{-2\pi^2 k^2 <\Psi^2(t)>_\xi}.$$

With the assumed form, $\overline{\exp \frac{r^2}{a^2}}$, for the correlation coefficient, the quantity $<\Psi^2>_\xi$ is just $\overline{\mu^2}\{<\rho_1{}^2>_\xi + <\rho_2{}^2>_\xi - 2<\rho_1\rho_2>_\xi\}$. If again R >>> a and θ <<< 1, one may write

$$< \exp\frac{2\pi i \overline{\nu}}{c} \Psi(t)>_\xi$$

$$= \exp \left\{ \frac{-4\pi^{5/2} k^2 \overline{\mu^2}\ a\ R}{\sqrt{1+\theta^2}} \left[1 - \frac{\sqrt{\pi}\ a}{4\theta R} \operatorname{erf}\left(\frac{2\theta R}{a}\right)\right] \right\}$$

which agrees with the previous result obtained from the second order expansion in smallness of μ.

It will be convenient to express the exponent in terms of the mean square angular deviation $\overline{\alpha^2}$ of a ray traversing the same atmosphere. With the previous assumption it may be shown• that $\overline{\alpha^2}$ is given by

$$\overline{\alpha^2} = 4\sqrt{\pi}\ \frac{\overline{\mu^2}}{a} R.$$

Substituting this quantity into the exponent and neglecting θ^2 in comparison with unity, one has

$$< \exp\frac{2\pi i \overline{\nu}}{c} \Psi(t)>_\xi = \exp\left\{- \pi^2 k^2\ \overline{\alpha^2}\ a^2 \left[1 - \frac{\sqrt{\pi}\ a}{4\theta R} \operatorname{erf}\left(\frac{2\theta R}{a}\right)\right]\right\}$$

• Lev A. Chernov, Wave Propagation in a Random Medium, McGraw-Hill, N.Y. (1960).

Two limiting forms of this expression may be easily explored. When $2\theta R \gg a$, the error function approaches one and

$$<\exp\frac{2\pi i \overline{\nu}}{c} \Psi(t)>_{\xi} \cong \exp\left\{-\pi^2 k^2 \overline{\alpha^2} a^2\right\}.$$

When $2\theta R \ll a$, the error function may be expanded to give

$$\operatorname{erf}\frac{2\theta R}{a} = \frac{2}{\sqrt{\pi}} \left\{ \frac{1}{1} \frac{2\theta R}{a} - \frac{1}{3(1!)} \left(\frac{2\theta R}{a}\right)^3 + \frac{1}{5(2!)} \left(\frac{2\theta R}{a}\right)^5 \cdot \cdot \cdot \right\}$$

and

$$<\exp\frac{2\pi i \overline{\nu}}{c} \Psi(t)>_{\xi} \cong \exp\left\{- \frac{\pi^2 k^2 \overline{\alpha^2} d^2}{3}\right\}$$

where $2\theta R = d$ is the distance between the two points at $\vec{r}_1$ and $\vec{r}_2$.

Thus, initially $\Gamma(\vec{r}_1, \vec{r}_2, 0)$ decreases expotentially with the square of the spacing between the two points. By the time the spacing has exceeded the scale of the atmosphere correlation only a residual correlation between the fields remains and this is independent of spacing. A graph of $\Gamma(\vec{r}_1, \vec{r}_2, 0)/\Gamma_0(\vec{r}_1, \vec{r}_2, 0)$ is shown in Figure 9, and in Figure 10 the same quantity is plotted for the limiting case $d \ll a$.

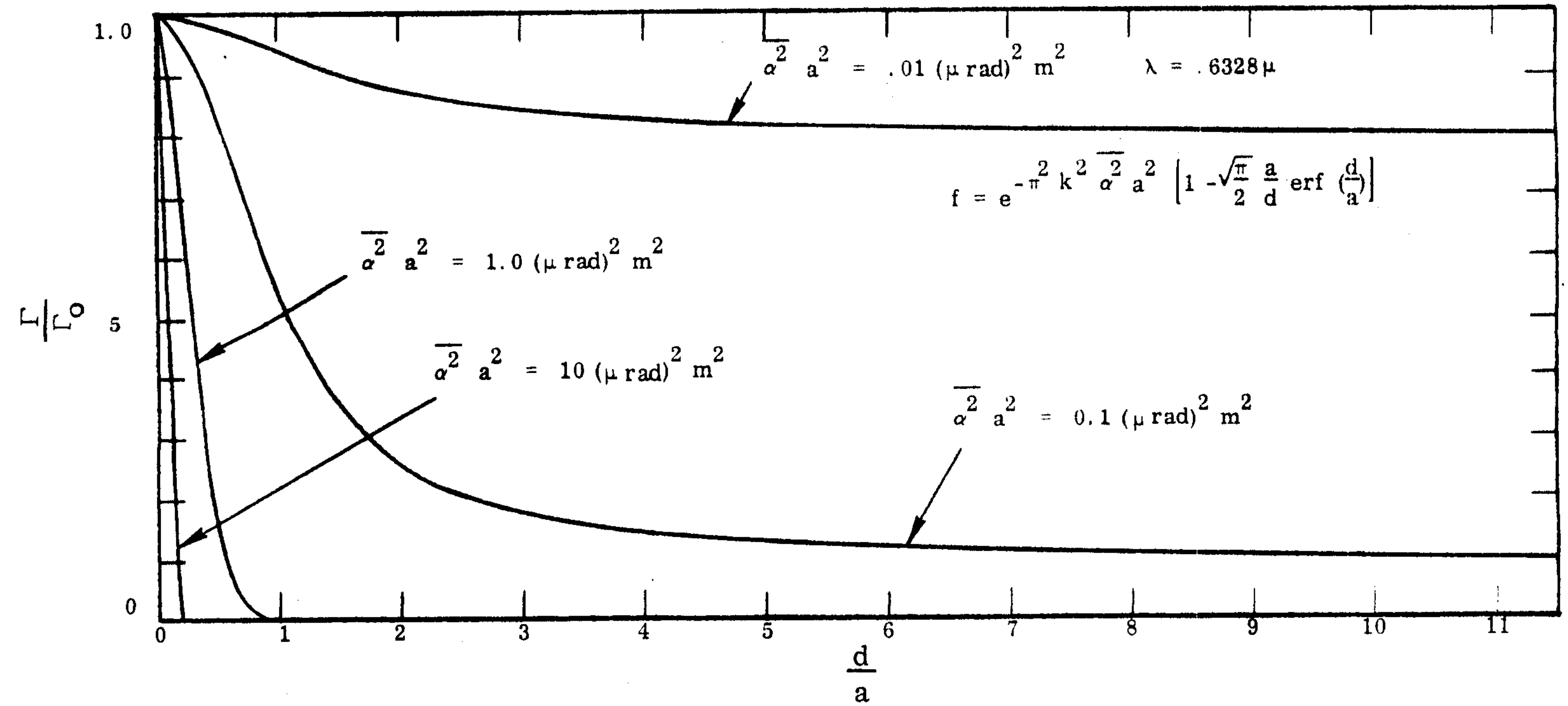

Figure 9. Spatial Variation of $\frac{\Gamma}{\Gamma_o}$ with Parameter $\frac{d}{a}$

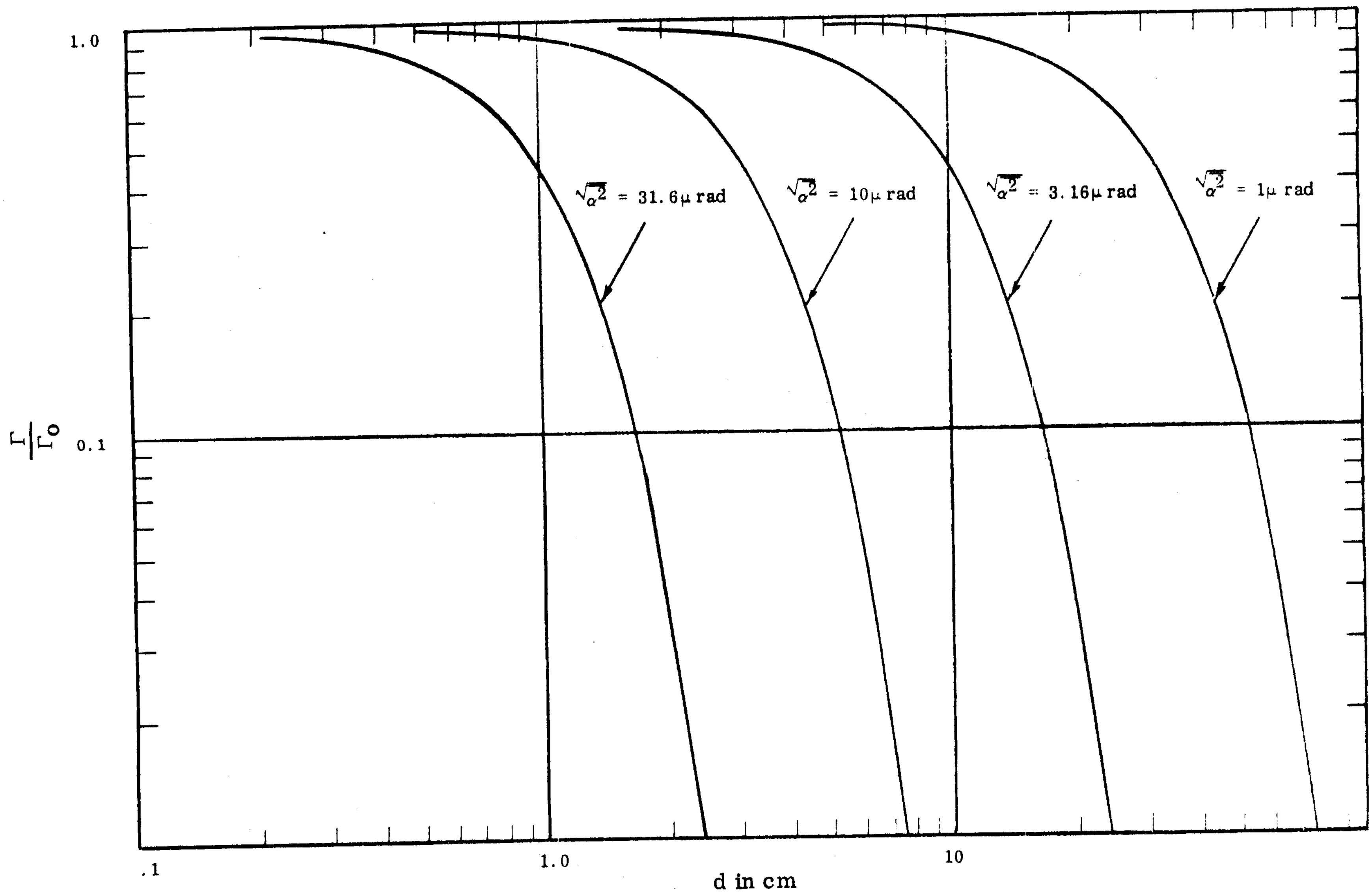

Figure 10. Spatial Variation of $\frac{\Gamma}{\Gamma_o}$ Versus d for d < a

5.0 DISCUSSION OF RESULTS

Because the fog atmosphere of the tunnel does not simulate closely naturally occurring atmospheres, the experimental data presented in Figure 7 of Section 3 does not in itself provide much direct information regarding the real atmosphere. It does, however, provide adequate information to check the simple theory presented in Section 4.0.

If the data for $\Gamma(\vec{r}_1, \vec{r}_2, 0)/\Gamma_0(\vec{r}_1, \vec{r}_2, 0)$ versus the separation $|\vec{r}_2 - \vec{r}_1|$ is fitted• in a mean square sense to a curve (Figure 11) of the form $e^{-b|\vec{r}_2 - \vec{r}_1|^2}$, the value of b obtained is 2940 meter^{-2}. Since $b = \frac{\pi^2}{3} k^2 d^2 \overline{\alpha^2}$ the corresponding value of the RMS angular deflection $\sqrt{\overline{\alpha^2}}$ is 19 μ radians. In the same fog atmospheres the RMS angular deflection measured in conjunction with Contract AF 30(602)-3284 is 18 ± 4 μ radians. The agreement is good.

It is certainly not surprising that the mean square angular deflection is related to the spatial coherence since both quantities are determined by the same effect, namely variations in atmospheric refractive index. For the same reason, one might also predict the spectrum of the angular deflections and the spectrum of the fringe intensity at a point are also very similar. In fact, a crude measurement of the spectrum of the fringe intensity was compared with the spectrum of angular deflections and semi-quantitative agreement was obtained. This result lends credence to the procedure of associating the degree of coherence computed from finite time averages with the squared angular deflections averaged over the same time intervals.

In any case, while the agreement between theory and experiment may be a little fortuitous, one is tempted to accept the theoretical relationship as valid, and proceed to the determination of the heterodyne signal.

From Section 2, it will be recalled that the spectral power density as given by

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• The points for $|\vec{r}_2 - \vec{r}_1| > 8$ cm were not considered in the curve fit because their absolute values are smaller than the experimental error. They may be significant, however.

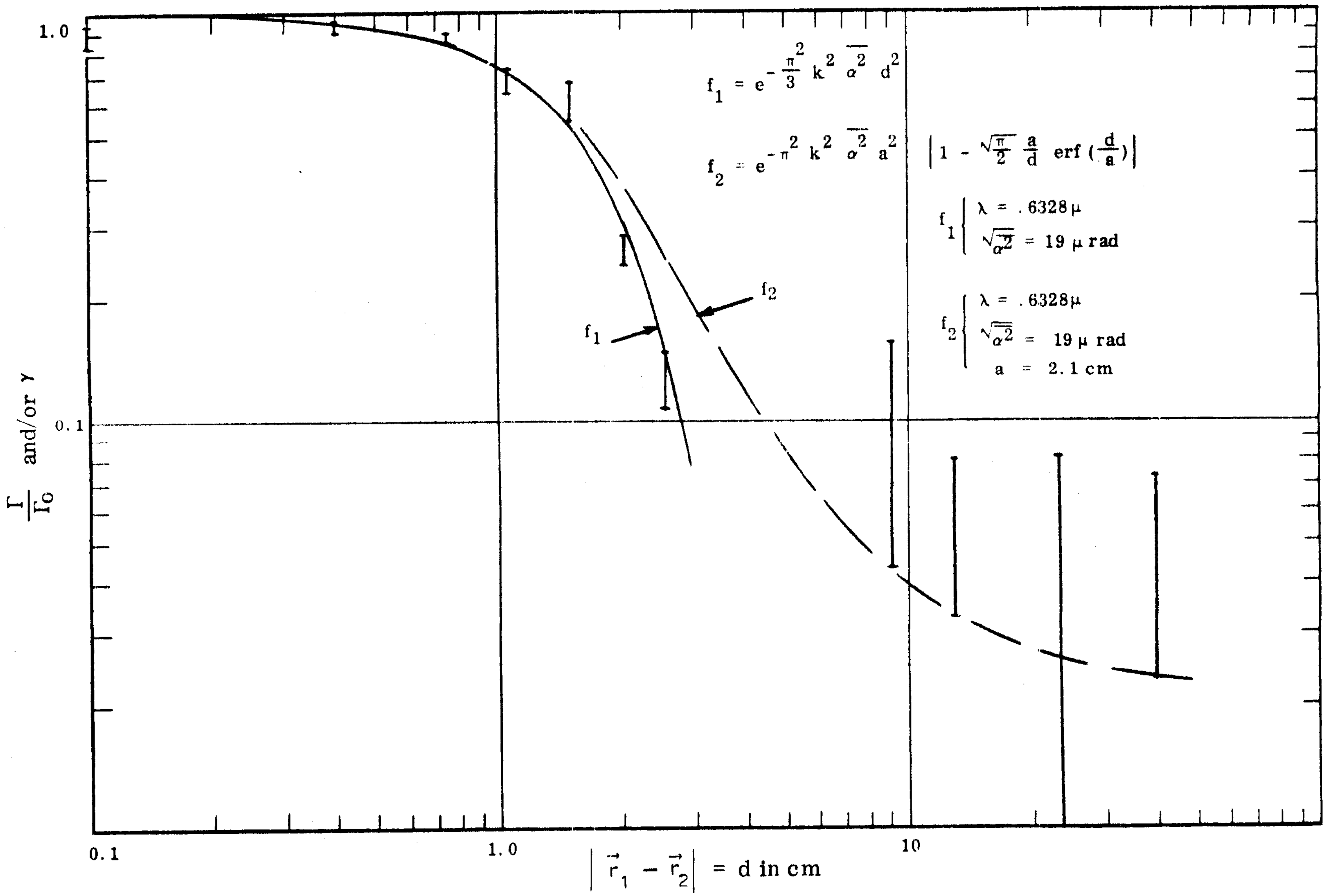

Figure 11. Spatial Dependence of the Mutual Coherence

$$G(\omega) = |\hat{h}(\omega)|^2 \int_V d^3r \int_V d^3r_2 \, R(\vec{r}_1, \vec{r}_2, \omega)$$

where R is the Fourier transform of $< V^2(\vec{r}_1, \tau_1) \, V^2(\vec{r}_2, \tau_2) >$. When this moment is expanded, the term which results in the heterodyne signal is

$$4 < V_1(\vec{r}_1, t_1) \, V_1(\vec{r}_2, t_2) > < V_2(\vec{r}_1, t_1) \, V_2(\vec{r}_2, t_2) >.$$

The heterodyne contribution to the spectral power density $G_H(\omega)$ becomes

$$G_H(\omega) = 4 |h(\omega)|^2$$

$$F \left\{ \int_V d^3r_1 \int_V d^3r_2 < V_1(\vec{r}_1, t_1) \, V_1(\vec{r}_2, t_2) > < V_2(\vec{r}_1, t_1) \, V_2(\vec{r}_2, t_2) > \right\}$$

where F denotes the Fourier transform. With the assumptions which lead to the results of Section 4.0, one may separate the spatial dependence of the expectations in the integrand, and complete the integrations.

Consider the case in which the local oscillator has a fixed output power which is distributed uniformly over the limiting aperture of the receiver. Let the average power density of the received optical signal also be constant over the limiting aperture. Then, the heterodyne contribution to the autocorrelation function and/or the spectral power density of the detector output due to an optical signal propagating through a turbulent atmosphere compared with the contribution from an optical signal propagating through vacuum is expressed by the ratio η, where

$$\eta = \frac{\int_0^{2\pi} d\varphi_1 \int_0^{2\pi} d\varphi_2 \int_0^r r_1 dr_1 \int_0^r r_2 dr_2 \, \exp\{-\frac{\pi^2}{3} k^2 \overline{\alpha^2} [r_1^2 + r_2^2 - 2r_1 r_2 \cos(\varphi_2 - \varphi_1)]\}}{\pi^2 r^4}$$

and r is the radius of the limiting aperture.

The integral may be immediately reduced to

$$\eta = \frac{4\int_0^{\xi} \xi_1 d\xi_1 \int_0^{\xi} \xi_2 d\xi_2 \, I_0(2\xi_1\xi_2) e^{-(\xi_1^2 + \xi_2^2)}}{\xi^4}$$

where $\xi = \{\frac{\pi^2}{3} k^2 \overline{\alpha^2}\}^{1/2} r$ and I_0 is the modified Bessel function of the first kind of zero order.

The previous expression for η was evaluated numerically on an IBM 1620 computer. The results are plotted in Figure 12. For good seeing conditions, which correspond to RMS angular deflections near 1 μ radian, the corresponding radius for a value of $\eta = 0.1$ is slightly larger than one meter for 6328 Å radiation.

In summary, then, the current work has shown a theoretical relationship between the spatial coherence and the mean square angular ray deflection, and this has been experimentally verified on artificial

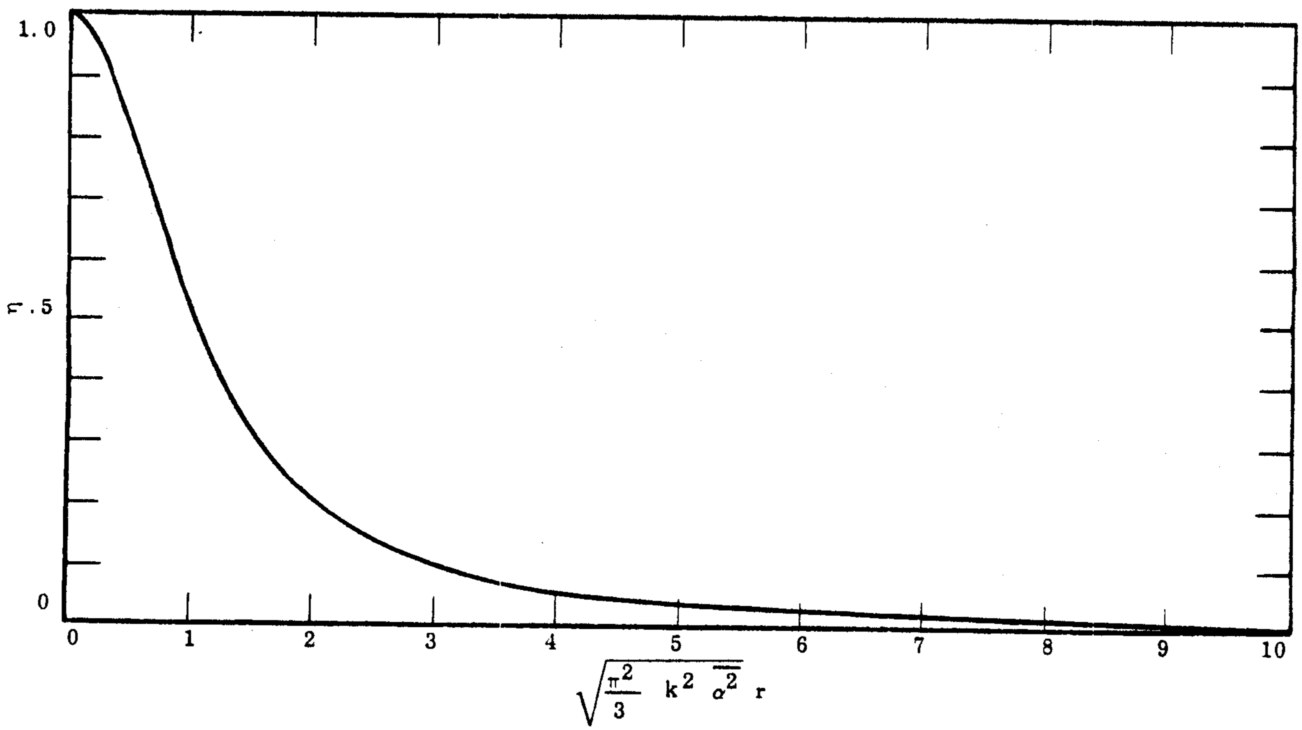

Figure 12. Ratio η of Heterodyne Signal Power in Presence of Turbulence to Heterodyne Signal Power Without Turbulence Versus $\sqrt{\frac{\pi^2}{3} k^2 \overline{\alpha^2}}\, r$

fog atmospheres over a limited range of the parameters. Increased confidence in the results could be obtained by extending the time over which averages are computed and performing the experiments over different optical paths. Ultimately, the results must be verified over long paths in the real atmosphere.

APPENDIX A

CONCERNING THE SCATTERING OF COHERENT LIGHT FROM RANDOMLY DISTRIBUTED SCATTERING CENTERS

A substantial portion of the initial phase of the current contract was devoted to a consideration of the scattering of coherent light from aerosols. Because the emphasis of the program was shifted to the heterodyne problem, this scattering work was not brought to its full conclusion. The initial effort is considered pertinent, however, and it is summarized in this and the following appendices. In this appendix, a partial treatment of the theoretical description of the scattering is presented, and in Appendix B the brief experimental efforts are summarized.

In the following paragraphs the problem of single elastic scattering of partially coherent light from a finite volume of random scattering centers will be outlined. In general, the differences in optical paths within a single scattering particle are small compared with the path difference between two scattering particles. The field scattered from a single particle may then be approximated by the field obtained from completely coherent radiation, and the total field becomes the sum of the single particle fields from a random assemblage of scattering particles.

Let a plane electromagnetic disturbance be normally incident on the plane $z = 0$, from the region $z < 0$. Let a component of the electric field of this primary disturbance in the region $z \geq 0$ be denoted by

$$E_p(t, z) = e^{-\alpha z} E_0(t - \frac{nz}{c}).$$

In the previous, α represents the absorption constant for removal of photons from the primary disturbance, n is the refractive index of the medium, and c is the vacuum velocity of light. Both α and n will be treated as frequency independent quantities.

It will be assumed that $E_0(t)$ represents a sample function of a stochastic process, that the process is ergodic, and that

$$\langle E_0(t) \rangle = 0$$

$$\langle E_0(t)\ E_0(t+\tau) \rangle = \mathcal{A}(\tau)$$

where the symbols $\langle \ldots \rangle$ indicate averages over the ensemble of sample functions, and $\mathcal{A}(\tau)$ is the autocorrelation function of the process.

Consider now the field at the point $\vec{R}$ due to the scattering of a single particle at the point $\vec{r}$. When $|\vec{R} - \vec{r}|$ is sufficiently large, the

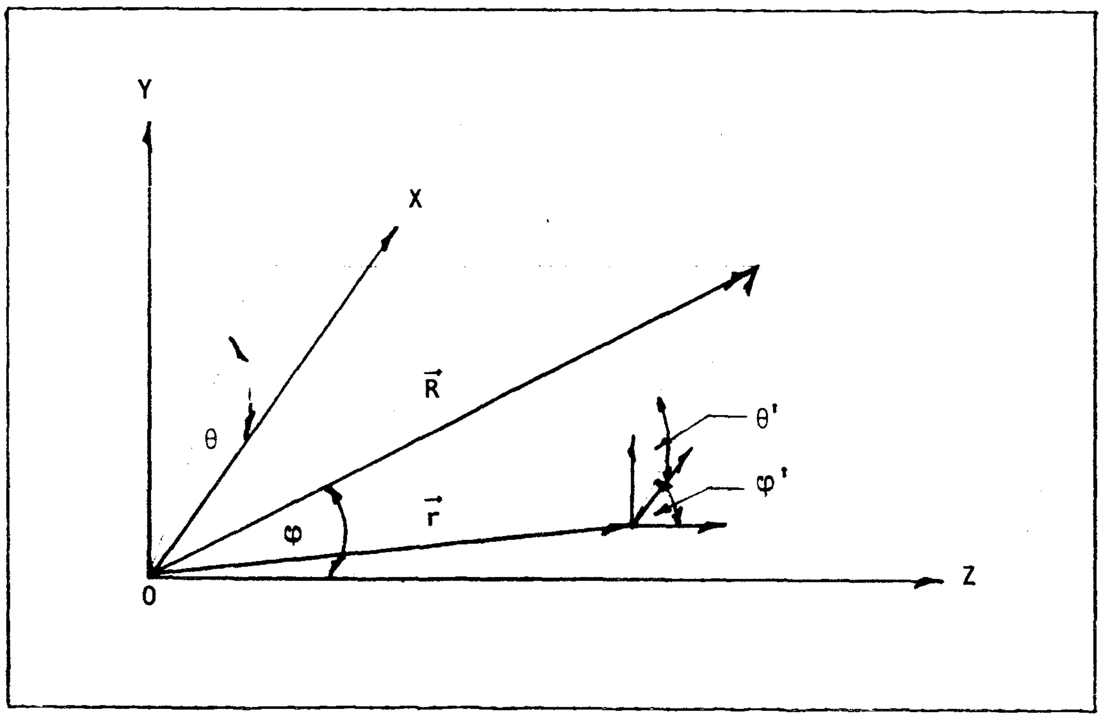

Figure A1. Geometry for Scattering Computation radiation field will appear to have originated at point with source strength S. Now this apparent source will be a function of the optical size of the scattering particle, the time, and the coordinates of the scatterer and observer, i.e.,

$$S = S(d, \vec{R}, \vec{r}, t)$$

where d is the optical diameter of the scattering particle. When the particles are fixed in space, S may be related to the incident field by

$$S(d, R, r, t) = \int_{-\infty}^{+\infty} h(d, \vec{R}, \vec{r}, \mu)\ E_p(t - \mu, z)d\mu.$$

To investigate the significance of h, assume for the moment that the primary disturbance is given by

$$E_p(t, z) = e^{i\omega(t - \frac{nz}{c})};$$

i.e., a completely coherent wave of frequency ω. Then

$$S(d, R, r, t) = \int_{-\infty}^{+\infty} h(d, \vec{R}, \vec{r}, \mu)e^{i\omega(t - \mu - \frac{nz}{c})}d\mu$$

or

$$S(d, \vec{R}, \vec{r}, t) = \hat{h}(d, \vec{R}, \vec{r}, \mu)e^{i\omega(t - \frac{nz}{c})},$$

where $\hat{h}$ is the Fourier transform of h. Thus $\hat{h}$ is just the field intensity factor derived from Mie scattering theory. More generally, the scatterers are moving in space, and one may write

$$S(d, \vec{R}, \vec{r}, t) = U(\vec{r}, t) \int_{-\infty}^{+\infty} h(d, \vec{R}, \vec{r}, \mu)\, E_p(t - \mu, z)\, d\mu$$

where $U(\vec{r}, t)$ is unity if a scatterer is at the point $\vec{r}$ at time t and zero otherwise. This representation is not precisely correct, for the on-off property of the scattering particles represented by the function $U(\vec{r}, t)$ should appear in the integrand. The present approximation is equivalent to neglecting the transient behavior of the light field.

The contribution to the field at $\vec{R}$ from the scattering center at $\vec{r}$ becomes proportional to

$$U\left(\vec{r}, t - \frac{n/\vec{R} - \vec{r}/}{c}\right) \frac{e^{-\alpha/\vec{R} - \vec{r}/}}{/\vec{R} - \vec{r}/} \cdot$$

$$\int_{-\infty}^{+\infty} h(d, R, r, \mu)\, E_0\left(t - \mu - \frac{nz}{c} - \frac{n/R - r/}{c}\right) d\mu .$$

The total field E_S at the point $\vec{R}$ at time t due to single scattering is just the sum of similar contributions from all scattering centers in the volume of interest. They may be written as

$$E_S(\vec{R}, t) = \int_V dV\, U\left(\vec{r}, t - \frac{n/\vec{R} - \vec{r}/}{c}\right) \frac{e^{-\alpha z - \alpha/\vec{R} - \vec{r}/}}{/\vec{R} - \vec{r}/}$$

$$\int_{-\infty}^{+\infty} d\mu\, h(d, \vec{R}, \vec{r}, \mu)\, E_0\left(t - \mu - \frac{nz}{c} - \frac{n/\vec{R} - \vec{r}/}{c}\right)$$

In the above integrations, $\mathcal{U}$ must be regarded as a sample function derived from the stochastic process governing the distribution of the scatterers.

With the usual far field approximations, for which $|\vec{R} - \vec{r}| \cong |\vec{R}| - \vec{R}\cdot\vec{r}/|\vec{R}|$ and θ', $\varphi' \cong \theta$, φ, one may rewrite E_S as

$$E_S \cong \frac{e^{-\alpha R}}{R} \int_V \mathcal{U}(\vec{r},\ t - \frac{nR}{c} - \frac{n\ \vec{R}\cdot\vec{r}}{cR}) e^{\alpha\left\{\frac{\vec{R}\cdot\vec{r}}{R} - z\right\}}$$

$$\int_{-\infty}^{+\infty} d\mu\ h(d,\ \theta,\ \varphi,\ \mu)\ E_0(t - \mu - \frac{nz}{c} - \frac{nR}{z} - \frac{n\ \vec{R}\cdot\vec{r}}{cR}) dV$$

where $R = |\vec{R}|$ and where h in the far field depends only on d, μ, and the angular direction of $\vec{R}$ as given by θ and φ.

To shorten the notation $h(d,\ \theta,\ \varphi,\ \mu)$ will be denoted by $h(\mu)$; however, it is to be understood that $h(\mu)$ will always be a function of the particle geometry, the refractive index, and the direction of $\vec{R}$.

The power per unit solid angle J may now be expressed as

$$J = Ce^{-2\alpha R} \int_V dV_1\ \mathcal{U}(\vec{r}_1,\ t - \frac{nR}{c} - \frac{n\ \vec{R}\cdot\vec{r}}{cR}) \exp\left\{\frac{\vec{R}\cdot\vec{r}_1}{R} - z_1\right\}$$

$$\int_{-\infty}^{+\infty} d\mu_1\ h(\mu_1)\ E_0(t - \mu_1 - \frac{nz_1}{c} - \frac{nR}{c} - \frac{n\ \vec{R}\cdot\vec{r}_1}{cR})$$

$$\int_V dV_2\ \mathcal{U}(r_2,\ t - \frac{nR}{c} - \frac{n\ \vec{R}\cdot\vec{r}}{cR}) \exp\left\{\frac{\vec{R}\cdot\vec{r}_2}{R} - z_2\right\}$$

$$\int_{-\infty}^{+\infty} d\mu_2\ h(\mu_2)\ E_0(t - \mu_2 - \frac{nz_2}{c} - \frac{nR}{c} - \frac{n\ \vec{R}\cdot\vec{r}_2}{cR})$$

where C is a constant. With the ergodic assumption the time average power per unit solid angle becomes

$$\bar{J} = \langle J \rangle = Ce^{-2\alpha R} \int dV_1 \int dV_2 \exp \alpha \left\{ \frac{\vec{R}\cdot(\vec{r}_2 + \vec{r}_1)}{R} - (z_2 + z_1) \right\}$$

$$\langle U(\vec{r}_1, t - \frac{nR}{c} - \frac{n\,\vec{R}\cdot\vec{r}_1}{cR}) \quad U(\vec{r}_2, t - \frac{nR}{c} - \frac{n\vec{R}\cdot\vec{r}_2}{cR}) \rangle$$

$$\int d\mu_1 \int d\mu_2 \; h(\mu_1)\; h(\mu_2)\; A(s)$$

where independence between the stochastic processes of the scatterers and the source is assumed and

$$s = \mu_1 - \mu_2 + \frac{n}{c} \; \frac{\vec{R}\cdot(\vec{r}_2 - \vec{r}_1)}{R} - (z_2 - z_1) \; .$$

Consider now the averaged product

$$\langle U(\vec{r}_1, t_0) \quad U(\vec{r}_2, t_0 + t) \rangle = \langle U(\vec{r}_1, 0) \quad U(\vec{r}_2, t) \rangle$$

for a random assemblage of particles obeying Maxwell–Boltzmann statistics. Assume that on the average a large volume V contains N particles, each occupying a volume ΔV. Let the $U(\vec{r}, t)$ be approximated by the function which has the value zero if no scattering particle is in the volume ΔV about $\vec{r}$ and the value unity if a particle is in the volume ΔV at $\vec{r}$. The product $U(\vec{r}, 0)\, U(\vec{r}_2, t)$ will also take on just two values, zero and one. The average $\langle U(\vec{r}_1, 0)\, U(\vec{r}_2, t)\rangle$ is then unity multiplied by the probability that the value unity is obtained, i.e.,

$$\begin{aligned}\langle U(\vec{r}_1, 0)\, U(\vec{r}_2, t)\rangle &= 0\cdot 0 \; P[U(\vec{r}_1, 0) = 0, \; U(\vec{r}_2, t) = 0] \\ &+ 1\cdot 0 \; P[U(\vec{r}_1, 0) = 1, \; U(\vec{r}_2, t) = 0] \\ &+ 0\cdot 1 \; P[U(\vec{r}_1, 0) = 0, \; U(\vec{r}_2, t) = 1] \\ &+ 1\cdot 1 \; P[U(\vec{r}_1, 0) = 1, \; U(\vec{r}_2, t) = 1]\end{aligned}$$

where $P[\mathcal{U}(\vec{r}_1, 0) = A, \mathcal{U}(\vec{r}_2, t) = B]$ denotes the probability that $\mathcal{U}(\vec{r}_1, 0) = A$ and $\mathcal{U}(\vec{r}_2, t) = B$.

The last probability in the previous expression may be expressed as

$$P[\mathcal{U}(\vec{r}_1, 0) = 1, \mathcal{U}(\vec{r}_2, t) = 1] = P[\mathcal{U}(\vec{r}_1, 0) = 1, \mathcal{U}(\vec{r}_2, 0) = 1]$$

$\cdot$ P[that $\mathcal{U}$ exhibits an even number of transitions between the values 0 and 1 in a time t.]

$+ P[\mathcal{U}(\vec{r}_1, 0) = 1, \mathcal{U}(\vec{r}_2, 0) = 0]$

$\cdot$ P[that $\mathcal{U}$ exhibits an odd number of transitions between the values 0 and 1 in a time t.]

It is clear that the probability that $\mathcal{U}(\vec{r}_1, t)$ is one or zero at time t is just equal to the probability that a particle is or is not in ΔV. With $N_0 = \frac{V}{\Delta V}$

$$P[\mathcal{U}(\vec{r}_1, 0) = 1, \mathcal{U}(\vec{r}_2, 0) = 1] = \frac{N}{N_0}\,\frac{N-1}{N_0-1} \cong \left(\frac{N}{N_0}\right)^2$$

and

$$P[\mathcal{U}(\vec{r}_1, 0) = 1, \mathcal{U}(\vec{r}_2, 0) = 0] = \frac{N}{N_0}\left(1 - \frac{N-1}{N_0-1}\right) \cong \frac{N}{N_0}\left(1 - \frac{N}{N_0}\right)$$

The time t that a particle remains in a volume ΔV is given by

$$\tau = \frac{a(\Delta V)^{1/3}}{\nu}$$

where "a" is a geometric factor near unity and ν is the magnitude of the velocity of the scattering particle. Since the scatterers are in equilibrium, the function $U(\vec{r}, t)$ over any long time interval must be different from zero over a fraction of the interval equal to the fraction of the volume which is filled with particles. Then, due to the particles with velocity magnitude ν, the function $U(r, t)$ makes

$$\eta_\nu = 2 \frac{dN_\nu}{N_0} \frac{1}{\tau}$$

transitions between the values zero and one.

The average number of transitions per unit time η obtained from the Maxwellian distribution in velocities is

$$\eta = \frac{4}{a(\Delta V)^{1/3}} \frac{N}{N_0} \sqrt{\frac{2kT}{\pi m}}$$

Assuming the transitions are Poisson distributed, one obtains the following expression for the probability of an even number of transitions in time t.

$$P[\text{even}] = \sum_{K=0}^{\infty} \frac{(\eta|t|)^{2K}}{(2K)!} e^{-\eta|t|} = 1/2[1 + e^{-2\eta|t|}].$$

Similarly,

$$P[\text{odd}] = \sum_{K=0}^{\infty} \frac{(\eta|t|)^{2K+1}}{(2K)!} e^{-\eta|t|} = 1 - P[\text{even}] = 1/2[1 - e^{-2\eta|t|}].$$

The cross-correlation of $\tilde{U}$ may now be expressed as

$$< U(\vec{r}_1, 0) \; U(\vec{r}_2, t) > = \left\{ f^2 \, e^{-b|t|} + 1/2 \, f(1 - e^{-b|t|}) \right\}$$

where $f = \frac{N}{N_0}$ is the fraction of the volume V occupied by the particles and

$$b = \frac{8f}{a(\Delta V)^{1/3}} \sqrt{\frac{2kT}{\pi m}}$$

Collecting the previous equations, one has

$$<J> = Ce^{-2\alpha R} \int dV_1 \int dV_2 \, \exp\{\frac{\vec{R} \bullet (\vec{r}_1 + \vec{r}_2)}{R}\} - (z_2 + z_1)$$

$$\left\{ f^2 \, \exp(\frac{-b|n\vec{R} \bullet (\vec{r}_2 - \vec{r}_1)|}{cR}) + 1/2 \, f(1 - \exp(\frac{-b|n\vec{R} \bullet (\vec{r}_2 - \vec{r}_1)|}{cR})) \right\}$$

$$\int d\mu \int d\mu_2 \, h(\mu_1) \, h(\mu_2) \quad A(s) \; .$$

The autocorrelation function A(s) may be expressed in terms of its Fourier transform $W(\omega)$ which is the spectral power density of the optical source. Then

$$\langle J\rangle = \frac{Ce^{-2\alpha R}}{2\pi} \int d\omega \left|\hat{h}(\omega)\right|^2 W(\omega) \int dV_1 \int dV_2 \exp\left[\alpha\left\{\frac{\vec{R}\bullet(\vec{r}_1 + \vec{r}_2)}{R} - (z_2 + z_1)\right\}\right]$$

$$\left\{f^2 \exp\left(- b\frac{\left|n\vec{R}\bullet(\vec{r}_2 - \vec{r}_1)\right|}{cR}\right) + 1/2\, f\left(1 - \exp\left(- b\frac{\left|n\vec{R}\bullet(\vec{r}_2 - \vec{r}_1)\right|}{cR}\right)\right)\right\}$$

$$\exp\left(i\,\frac{\omega n}{c}\left\{\frac{\vec{R}\bullet(\vec{r}_2 - \vec{r}_1)}{R} - (z_2 - z_1)\right\}\right) .$$

If one considers a scattering volume at 300° K containing 10 micron particles of specific gravity 1.0, the number b is of the order of 10^5 f $\sec^{-1}$. Then expotential terms containing b will essentially unity for all realizable volumes. The scattered power per unit solid angle is then identical to that obtained from a uniformly continuous volume, each point of which is a source of the characteristic Mie field. Since the effect of the random motion of the scatterers on the average power per unit solid angle is negligible, any departure from Mie scattering must be due to multiple scattering or departures from random behavior of the scatterers.

APPENDIX B

EXPERIMENTAL MEASUREMENTS OF THE SCATTERING OF COHERENT LIGHT FROM COLLOIDS

Before the program was revised to emphasize the heterodyne problem, several preliminary experiments comparing the scattering of coherent and incoherent light from colloidal solutions were performed. The experimental arrangement is sketched in Figure 1B.

The coherent source was a Model 7401 Admiral He—Ne gas laser operating at .6328 μ. The incoherent source was a 100 watt zirconium arc with a 100 Å filter centered at .63 μ. The two sources were adjusted to give the same radiance with the same beam size and degree of collimation.

The colloidal scattering medium was formed by precipitating sulfur from sodium thiosulfate and sulfuric acid in aqueous solution. Microscopic examination of the suspension revealed that the particles were spherical in shape with diameters between 3 to 5 μ. The density of scatterers ranges between 50 and 1000 particles per cubic centimeter.

The photodetector shown is an Amperex OAP12 germanium photodiode; however, this was later replaced with an RCA 7265 photomultiplier. The radiometer viewed from the scattering volume subtends a solid angle of .017 steradians, which is equivalent to a central angle of approximately 8 degrees.

The noise equivalent input of the detector is 10^{-12} watts/steradian at .63 μ.

The scattering data collected is summarized in Figure 2B. Since the total power from the incoherent source is equal to the total power in the polarized laser beam, the scattering data is essentially the same for the coherent and incoherent beams. There are, however, slight variations at 20°, 65°, and 105° through 130°, but the reproducibility of the data has not been sufficiently verified to attach any significance to these anomalies.

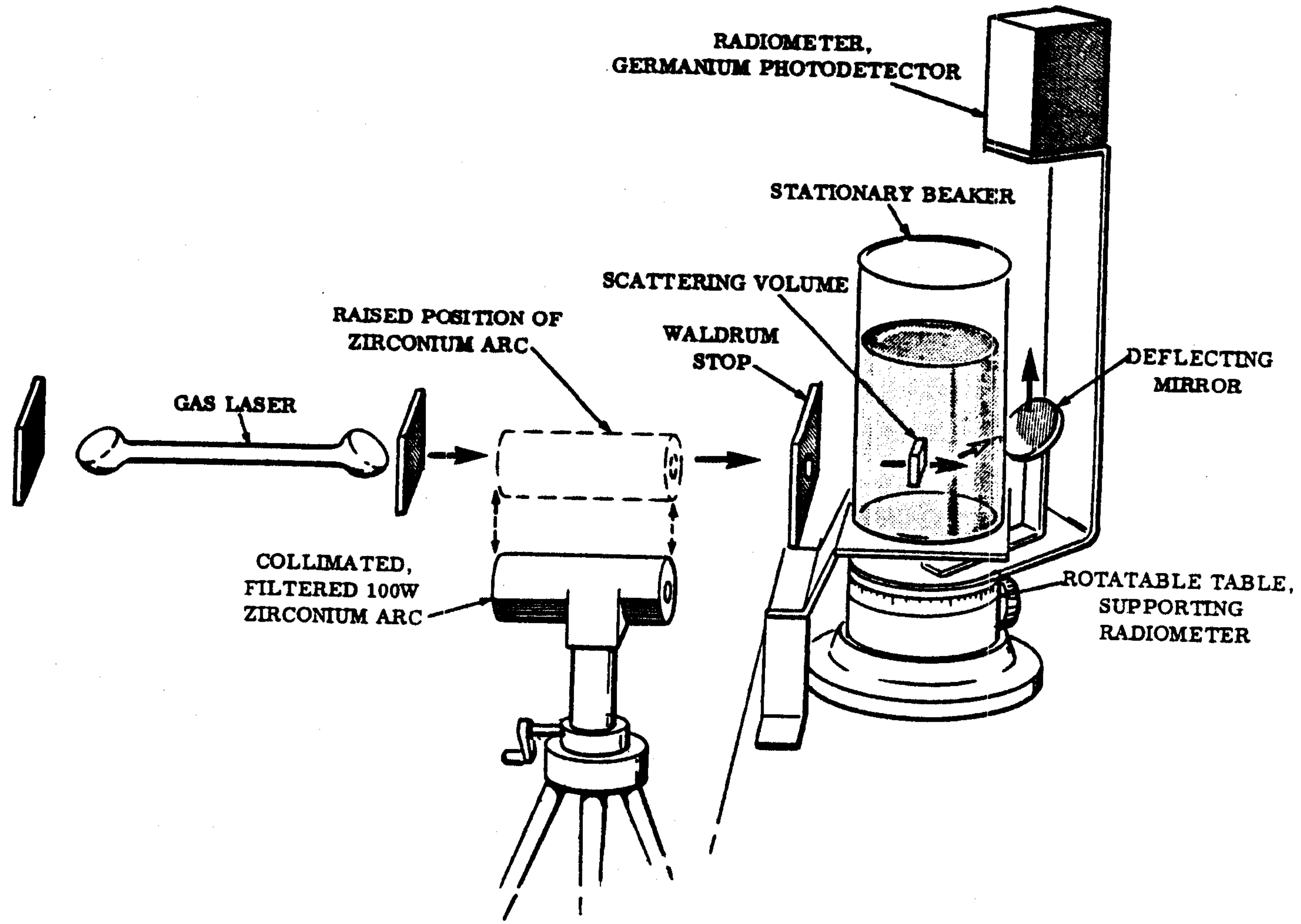

Figure B1. Scattering from Colloids Experimental Arrangement

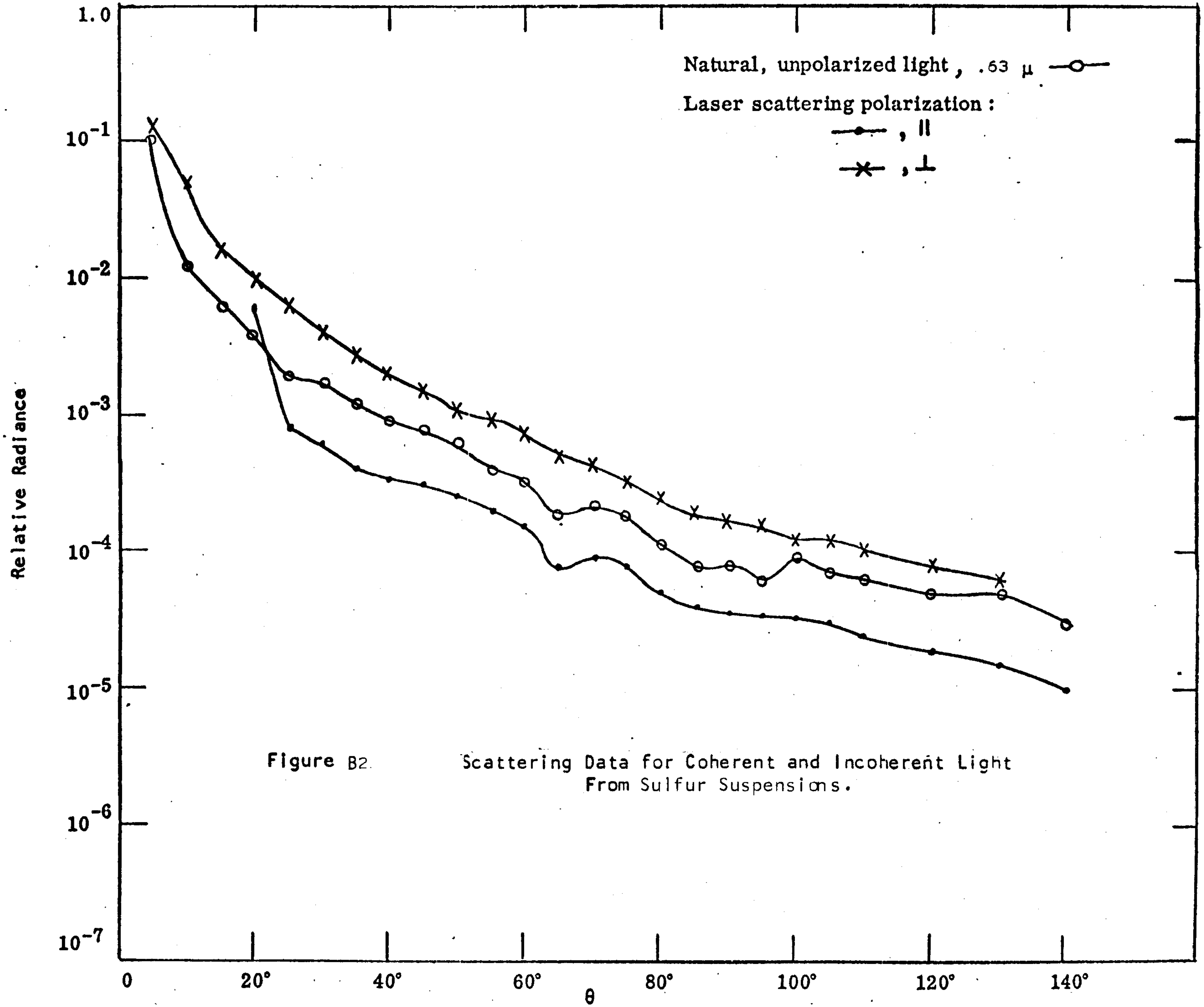

Figure B2 Scattering Data for Coherent and Incoherent Light From Sulfur Suspensions.