NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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An Adaptive Numerical Integration Routine for the IBM 1620 II

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ABSTRACT

This Report presents a Symbolic Programming System (SPS) coded numerical integration program designed to be used as a FORTRAN subroutine on the IBM 1620 II Computer. The program will solve differential equations by integration starting with known initial conditions. An adaptive option is available where the speed of integration is automatically varied in accordance with a predicted error.

I. GENERAL DESCRIPTION

This program is designed to solve, by integration, ordinary differential equations written in the form

$$\frac{dx}{dt} = f(t,x) \tag{1}$$

where x is an n vector and $f(t,x)$ is an n vector function of its arguments. The initial state

$$x(t_0) = x_0 \tag{2}$$

must be known.

Three options are available: (1) the Runge-Kutta-Gill integration method (Ref. 1), (2) the Adams method (Ref. 2), starting with the Runge-Kutta-Gill integration, and (3) the modified Adams or Moulton method with an adaptive step-size feature. In this last method, the solution is started by the Runge-Kutta-Gill method with automatic switching to the Adams method. The Adams method is used to predict the next point on the solution. This point is corrected by a closed-type formula (Ref. 2), and an error vector is formed from the absolute value of the difference between the predicted and corrected values. This error vector is compared to upper and lower tolerance vectors supplied by the user. If any component of the error vector is greater than its respective upper tolerance, the step size is automatically halved. On the other hand, if each component of the error is smaller than its lower tolerance for three successive steps, the step size is doubled. This halving and doubling can continue until h is truncated at h_{max} or h_{min} which are user supplied bounds.

With any of these methods the user can change the step size at any time. This is possible because the routine detects this change and automatically restarts the Adams or modified Adams methods.

All three methods have h^5 error terms. The Runge-Kutta-Gill integration, however, is generally somewhat more accurate than the Adams method when each is using the same step size. Offsetting this is the higher speed of the Adams method which often takes only one-fourth the running time required by the Runge-Kutta-Gill method. It is difficult to give a time comparison for the modified Adams method because of the adaptive feature. If one set $h_{max} = h_{min}$, thus removing the adaptive feature, the routine would have about the same accuracy

as the Runge-Kutta-Gill method but would require about one-half the running time.

The adaptive feature of the modified Adams method permits one to start with a small initial step size and allow the routine to double the step size until it is the maximum which is consistent with a given accuracy. Figure 1 shows how the adaptive feature operates. In this example, x is the solution to

$$\frac{d^2x}{dt^2} + 0.6\,\frac{dx}{dt} + 0.35\,x^3 = u\,(t - 25) \qquad (3)$$

where $u\,(\cdot)$ is the unit step function, and the initial conditions are

$$x\,(t = 0) = +\,0.2 \qquad (4)$$

$$\frac{dx}{dt}\,(t = 0) = -\,0.1 \qquad (5)$$

Notice that the initial step size of 0.5 sec was maintained until the time where dx/dt became large. At this point, the routine automatically reduced h to 0.125 to maintain accuracy. The step size h was later doubled and then doubled again as the truncation error decreased.

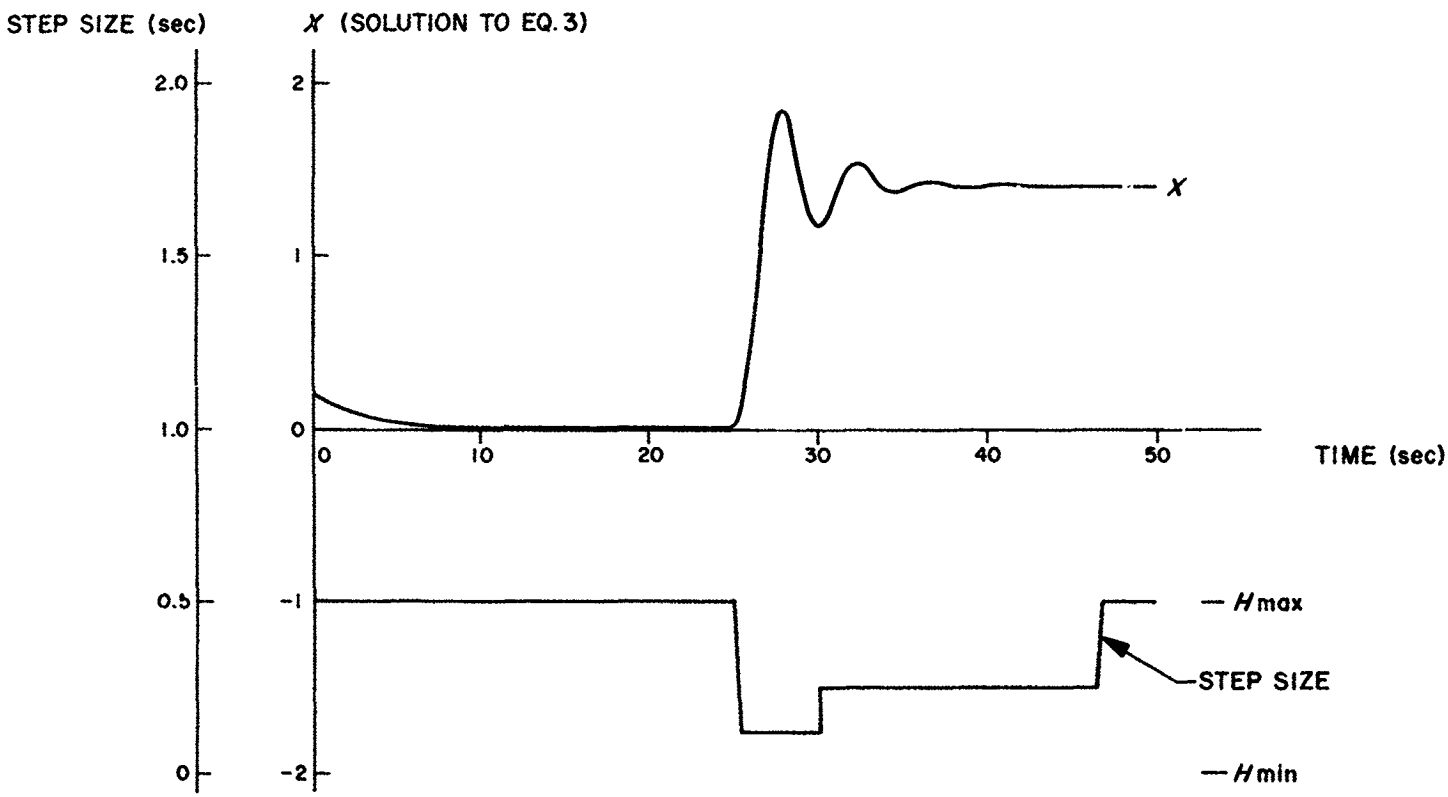


Fig. 1. Modified Adams integration with adaptive step size

II. PROGRAMMING CONSIDERATIONS

The user provides the information that the integration subroutine needs in the form of a derivative subroutine. This subroutine is written in the following manner. Let the differential equation to be integrated be

$$\dot{y} \triangleq \frac{dy}{dt} = f(t,y) \tag{6}$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad ; \dot{y} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \vdots \\ \dot{y}_n \end{bmatrix} \tag{7}$$

$$f(t,y) = \begin{bmatrix} f_1(t,y) \\ f_2(t,y) \\ \vdots \\ f_n(t,y) \end{bmatrix} \tag{8}$$

Next, the time t and the step size h must be adjoined to the differential equations as shown below. Let the augmented state be x; we then have

$$x \triangleq \begin{bmatrix} t \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad ; \dot{x} \triangleq \begin{bmatrix} h \\ \dot{y}_1 \\ \dot{y}_2 \\ \vdots \\ \dot{y}_n \end{bmatrix} \tag{9}$$

The purpose of the derivative subroutine is to calculate the derivative vector components $\dot{x}_2$ through $\dot{x}_{n+1}$ each time it is called. Since this subroutine is called by the integration routine, it can have *no* arguments in the call statement and all variables $x,\dot{x}$, etc. must be transferred through a *COMMON* statement.

The name of the derivative subroutine is *DER* since this is the name compiled into the integration routine. The user may change this to any valid FORTRAN name by changing the alphameric characters in the fourth from last card in the integration subroutine (LENGTH) to the desired name.

The integration subroutine is called *MARK* and is called by a two-statement sequence:

CALL *DER*

CALL *MARK* (*X,XD,N,IND,EU,EL,HMAX,HMIN,TS*)

where

X = $n + 1$ state vector as in *DER*

XD = $n + 1$ derivative vector as in *DER*

N = order of the equation

IND = indicator to show what integration system is to be used. *IND* = 0,1,2 for Runge-Kutta-Gill, Adams, modified Adams, respectively

EU = upper bound vector dimensioned n used only for modified Adams; must be positive

EL = lower bound vector dimensioned n used only for modified Adams; must be positive

HMAX = upper limit on step size, modified Adams

HMIN = lower limit on step size, modified Adams

TS = temporary storage dimensioned $(n + 1)$, $(5n + 1)$, $(5n + 2)$ for Runge-Kutta-Gill, Adams, or modified Adams, respectively

In the foregoing, the usual fixed/floating variable name rules apply. When using Runge-Kutta-Gill or the Adams method, *EU,EL,HMAX, HMIN* are not used and may be replaced by zeros in the call statement. For initialization purposes, *TS* (1) *must* be set to zero. When using Runge-Kutta-Gill or the Adams method, the x vector is initialized including the initial time in x (1). Next, the step size to be used, h, is placed into *XD* (1). Now each time that the two-statement call sequence is executed, the solution x will be moved from $x(t)$ to $x(t + h)$.

When the modified Adams option is selected, the initialization is as above; but after the solution is started

(3 steps), the predicted-corrected error is compared to *EU* and *EL*. The error for *X* (2) is compared to *EU* (1) and *EL* (1), and so on. The step size is then adjusted as described before. *XD* (1) always contains the step size which *will* be used, unless changed by the user, for the next step.

III. EXAMPLES

Equation (3) was written in state variable form

$$\dot{y}_1 = y_2 \tag{10}$$

$$\dot{y}_2 = -0.6\,y_2 - 0.35\,y_1^3 + u\,(t - 25) \tag{11}$$

where $y_1 = x$, $y_2 = \dot{x}$.

Time and step size were adjoined to obtain the proper form

$$\dot{Z}_2 = Z_3 \tag{12}$$

$$\dot{Z}_3 = -0.6\,Z_3 - 0.35\,Z_2^3 + u\,(Z_1 - 25) \tag{13}$$

and the derivative subroutine was written as:

```
1. SUBROUTINE  DER
2. DIMENSION   Z (3), ZD (3)
3. COMMON      Z, ZD
4. IF (Z (1) - 25.)  5,7,7
5. U = 0.
6. GO TO 8
7. U = 1.
8. ZD (2) = Z (3)
9. ZD (3) = -0.6 * Z (3) - 0.35 * Z (2) ** 3 + U
10. RETURN
```

A main program, which will integrate these equations from 0 to 50 sec for the initial conditions $x = 0.2$ and $\dot{x} = -0.1$, follows. The step size to be used is 0.1 sec, and the results are to be printed or plotted every 0.5 sec.

```
C
C  RUNGE-KUTTA-GILL
C
1. DIMENSION Z (3), ZD (3), TS (3)
2. COMMON Z ZD
3. TS (1) = 0
4. ZD (1) = 0.1
5. Z (1)  = 0
6. Z (2)  = 0.2
7. Z (3)  = -0.1
8. DO 11 I = 1,100
9. DO 11 J = 1,5
10. CALL DER
11. CALL MARK (Z, ZD, 3, 0, 0., 0., 0., 0., TS)
    .
    .
    . Output Routine
    .
    .
12. CONTINUE
13. CALL EXIT
```

The result of an integration of this type and one where h was increased to 0.5 sec, where a noticeable error occurred, are shown in Fig. 2.

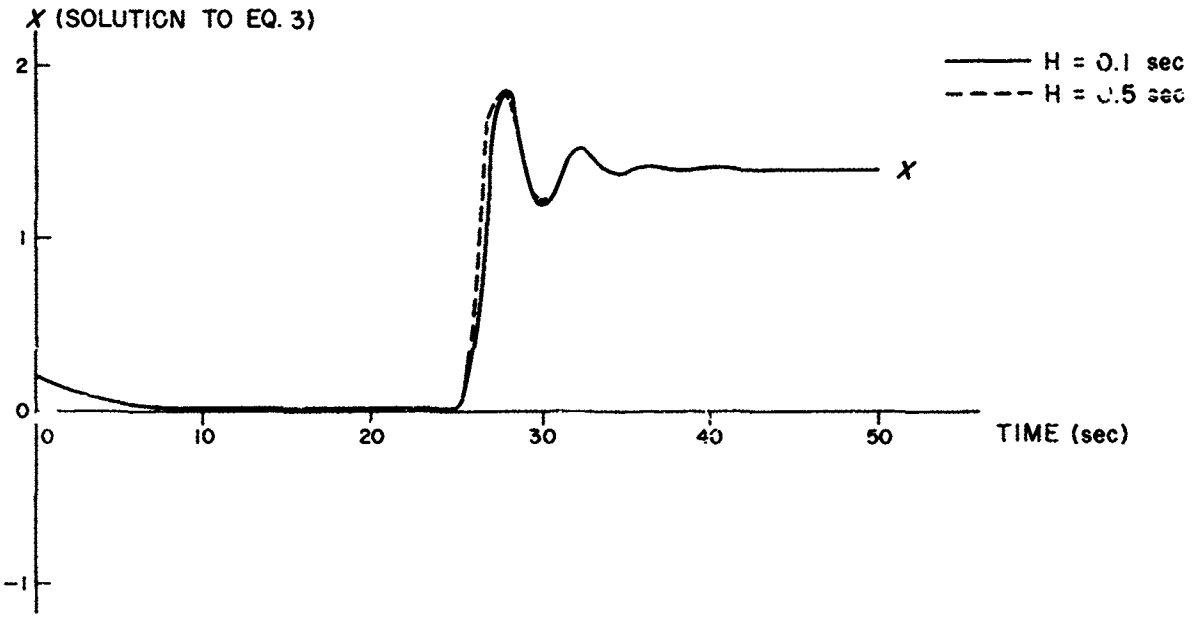


Fig. 2. Runge-Kutta-Gill integration

Returning back to Fig. 1, this was the result of a main program which forces the adaptive step-size feature to produce values at 0.5-sec intervals. This technique is illustrated below.

```
1. DIMENSION Z (3), ZD (3), TS (12), EU (2), EL (2)
2. COMMON Z, ZD
3. TS (1)    = 0.
4. Z (1)     = 0.
5. Z (2)     = 0.2
6. Z (3)     = −0.1
7. RESTOR = 0.5
8. DO 24 J = 1,100
9. T = Z (1)
10. DO 12 I = 1,2
11. EU (I) = 0.001 * ABSF (Z (I + 1)) + 0.001
12. EL (I) = 0.00001 * ABSF (Z (I + 1)) + 0.00001
13. ZD (1) = RESTOR
14. CALL DER
15. CALL MARK (Z, ZD, 2, 2, EU, EL, 0.5, 0.01, TS)
16. IF (ABSF (T + 0.5 − Z (1)) − 0.00001) 23, 23, 17
17. IF (T + 0.5 − Z (1) − ZD (1)) 18, 21, 21
18. RESTOR = XD (1)
19. XD (1) = T + 0.5 − X (1)
20. GO TO 14
21. RESTOR = XD (1)
22. GO TO 14
23. − − − Output Routine − − −
24. CONTINUE
```

As a comparison of running time, the integration using the Runge-Kutta-Gill method took 35 sec, while the modified Adams integration took 32 sec. Both yielded the same accuracy. Included in these times is a fixed time of 22 sec required to plot the output.

REFERENCES

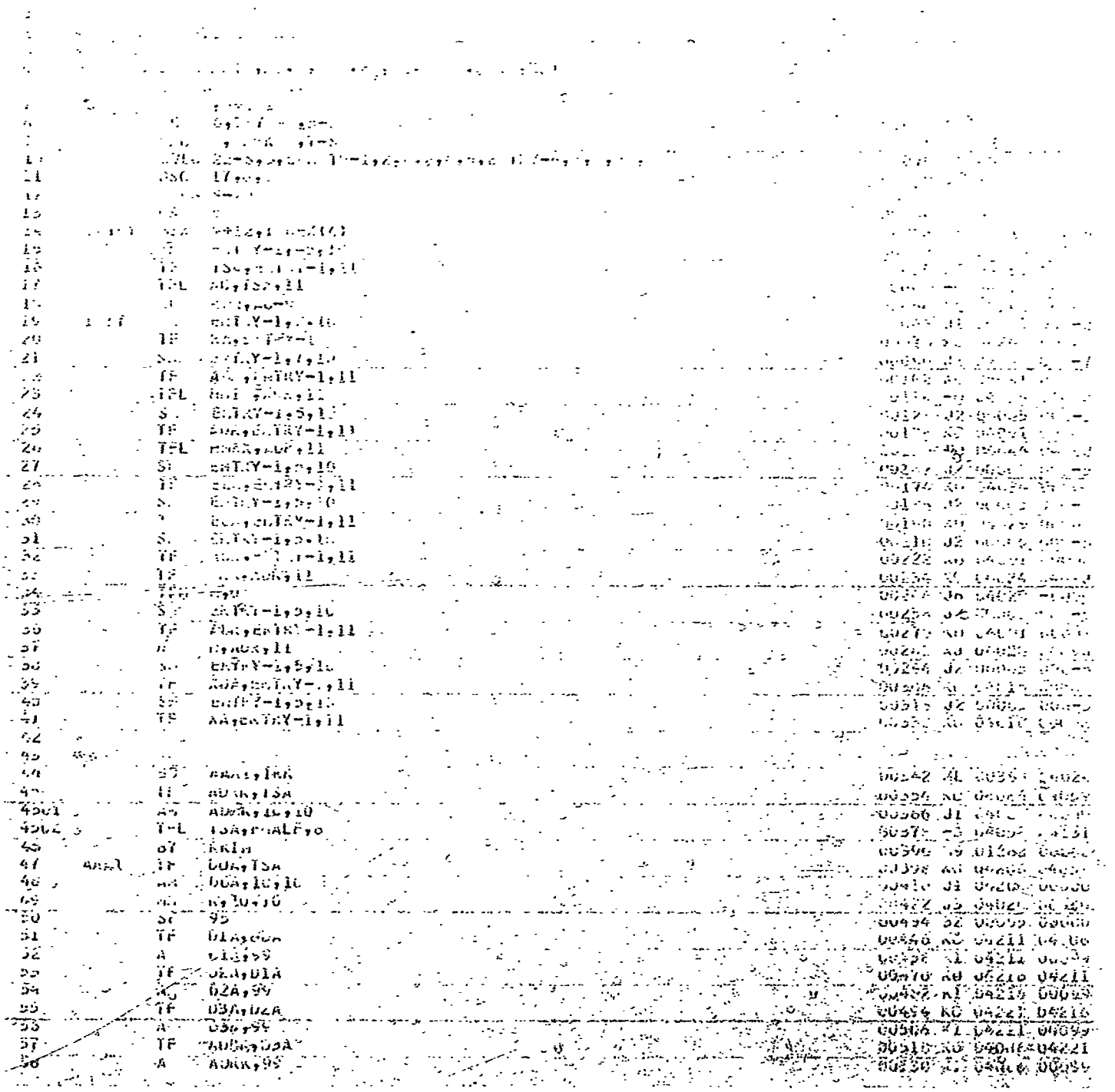
1. *Mathematical Methods for Digital Computers*, Ralston, A., and H. Wilf (Ed.), New York: John Wiley, 1962, p. 115, Eq. 26.

2. Hildebrand, F., *Introduction to Numerical Analysis*, New York: McGraw-Hill, 1956, p. 198.

APPENDIX

The Symbolic Programming System

The following pages contain sequential facsimiles of the Symbolic Programming System (SPS) as used in this Report.



```
                TF    AUK,XUA                                   01716 XC 0401  04121
                TF    AU1,XA                                    01750 XC 04056 04071
                TF    AUZ,AUKK                                  01742 XC 04121 04071
A3              AR    AU8,10,10                                 01754 J1 04071 00030
                AR    AU1,10,10                                 01756 J1 04056 00030
                TFL   AC,AUK,11                                 01778 -J 04071 04071
                FMUL  AC,XU1,11                                 01790 -L 04071 04121
                TFL   K,AC                                      01802 -K 04121 04071
                FSUB  AC,AU8,10                                 01814 -K 04071 04121
                FMUL  AC,FC2                                    01826 -L 04071 04121
```

```
52                FADD  AD1,AC,6                                  01638 -J 04060 04071
53                FMUL  K-FC5                                     01650 -L 04121 04151
54                TFL   AC,AD2,11                                 01662 -0 04071 04101
55                FMUL  AC,FC6                                    01674 -L 04071 04151
56                FADD  AC,K                                      01686 -J 04071 04121
57                TFL   AD2,AC,6                                  01698 -0 04101 04071
58                AM    AD2,10,10                                 01510 J1 04101 00030
59                BCXH  A3,1,011                                  01522 04 01754 0-004
60                TFL   AC,AD4,11                                 01534 -0 04071 04101
61                FMUL  AC,FHALF                                  01546 -L 04071 04151
62                FADD  XA,AC,6                                   01558 -J 04071 04071
63                STR   DER,*+12                                  0157C JP 04348 -1982
64                BLX   *+12,N(4)                                 01582 UN 01754 0-020
65                TF    ADR,XDA                                   01594 KO 04091 04015
66                TF    AD1,XA                                    01606 KO 04095 04010
67                TF    AD2,AER                                   02016 KO 04101 04086
68    A4          AM    ADR,10,10                                 02030 J1 04091 00030
69                AM    AD1,10,10                                 02042 J1 04095 00030
70                TFL   AC,ADR,11                                 02054 -0 04071 04091
71                FMUL  AC,XDA,11                                 02066 -L 04071 04101
72                FSUB  AC,AD2,11                                 02078 -K 04071 04101
73                FSUB  AC,AD2,11                                 02090 -K 04071 04101
74                FMUL  AC,FC7                                    02102 -L 04071 04201
75                FADD  AD1,AC,6                                  02114 -J 04090 04071
76                AM    AD2,10,10                                 02126 J1 04101 00030
77                BCXH  A4,1,011                                  02138 04 01736 0-004
78                ST    EXIT                                      02150 R9 04316 00000
1     *
2     *     ROUTINE OR ADARS OPTION
3     *
4     AA          TF    ADR,XDA                                   02158 KO 04091 04015
401               BLX   *+12,N(4)                                 02170 UN 02182 0002
5                 AS    ADR,10,10                                 02182 J1 04091 00030
6                 TF    AD1,DCA                                   02194 KO 04096 04206
7                 TF    AD2,DIA                                   02206 KO 04101 04211
8                 TF    AD3,FC3                                   02218 KO 04106 04216
9                 TF    AD4,D3                                    02230 KO 04111 04221
10                TFL   TS,XDA,611                                02242 -0 04095 04010
11                TFL   AC,ADR,11                                 02254 -0 04071 04091
12                FSUB  AC,AD1,11                                 02266 -K 04071 04091
13                TFL   STO,AC                                    02278 -0 04081 04071
14                TFL   AD1,ADR,611                               02290 -0 04050 04091
15                FSUB  AC,AD2,11                                 02302 -K 04071 04101
16                TFL   ADR,STO,6                                 02314 -0 04101 04081
17                TFL   STO,AC                                    02326 -0 04081 04071
18                FSUB  AC,AD3,11                                 02338 -K 04071 04106
19                TFL   AD4,AC,6                                  02350 -0 04111 04071
20                TFL   AD3,STO,6                                 02362 -0 04106 04081
21                AM    ADR,10,10                                 0237A JI 04091 00030
22                AM    AD1,10,10                                 0238E J1 04096 00030
23                AM    AD2,10,10                                 023AF J1 04101 00030
24                AM    AD3,10,10                                 023C2 J1 04106 00030
25                AS    AD4,10,10                                 023D4 J1 04111 00030
26                BCXH  A1,1,011                                  023E6 04 01754 0-004
27    *
28    *     ADARS ERROR OPTION
29                ZR    INCR,10                                   023F8 R6 02510 00000
30                TF    ADARS
```

```
6       STORX   TF    AC,R
7               ARS   AC,1,10
8               BLX   **12,AC(4)
9               TF    ADR,XA
10              TF    AD1,ADRX
11      *
12      *
13      *
14      *
15      *
16      C1      TFL   AD1,ADR,C12
17              AD    ADR,10,10
18              AD    AD1,10,10
19              BCAS  C1,1,C11
20      ADRXS   TF    AD1,D0A
21              TF    AD2,D1A
22              TF    AD3,D2A
23              TF    AD4,D3A
24              TF    ADR,XA
25              AM    AD5,10,10
26              FADD  XA,XDA,611
27              BLX   **12,1(4)
28      C2      TFL   AC,F36
29              FMUL  AC,AD4,11
30              TFL   STO,F512
31              FMUL  STO,AD3,1
32              FADD  AC,STO
33              TFL   STO,F16LF
34              FMUL  STO,AD2,11
35              FADD  AC,STO
36              FADD  AC,AD1,11
37              FMUL  AC,AD0,11
38              FADD  AD0,AC,S
39              AM    AD0,10,1
40              AM    AD1,10,10
41              AM    AD2,10,10
42              AM    AD3,10,10
43              AM    AD4,10,10
44              BCAS  C2,1,611
45              EM    1RR,1,10
46              EXIT
1       *
2       *       ROUTINE OPTION
3       *
4               STR   DER,**12
5               TFM   EQU,0,10
6               TFL   EL1,0,10
7               BLX   **12,R(4)
8               TF    ADR,XDA
9               AML   ADR,10,10
10              TF    AD1,D0A
11              TF    AD2,D1A
12              TF    AD3,D2A
13              TF    AD4,ADRX
14              TF    AD5,XA
15              TF    EQUS,EQR
16              TF    ELAS,ELA
17              AN    AD4,10,10
18              AM    AD5,10,10
19      E1      TFL   AC,ADR,11
```

02670 RU 04071 04020
02682 J1 04091 04000
02694 J1 0409[illegible] 04071
02706 RU 04091 04[illegible]
02718 RU 04096 04[illegible]
02730 -J 04[illegible] 04091
02742 J1 04091 04[illegible]
02754 J1 04096 04[illegible]
02766 -J 04[illegible] 04[illegible]
02778 RU 04[illegible] 04211
02790 RU 04116 04216
02802 RU 04121 04221
02814 RU 04091 04[illegible]
02826 J1 04091 04000
02838 -J 04[illegible] 04[illegible]
02850 -L 02[illegible] 02[illegible]
02862 -J 02[illegible] 04[illegible]
02874 -J 04[illegible] 04231
02886 -L 04071 04[illegible]
02898 -J 04061 04241
02910 -L 04081 04[illegible]
02922 -J 04071 04081
02934 -J 04081 04151
02946 -L 04081 04101
02958 -J 04071 04081
02970 -J 04071 04090
02982 -L 04071 04[illegible]
02994 -J 04093 04071
03006 J1 04091 000J0
03018 J1 04096 000J0
03030 J1 04101 000J0
03042 J1 04106 000J0
03054 J1 04111 000J0
02874 J4 02674 0-00J
02886 J4 [illegible]024 000-1
02898 J6 04316 01200
02902 JP 04346 -4914
02914 J6 04250 000-0
02926 J6 04252 000-0
02938 J6 04250 0R020
02950 RL 04091 04015
02962 J1 04091 000J0
02974 RU 04096 04206
02986 RU 04101 04211
02998 RU 04106 04216
03010 RU 04111 [illegible]4086
03022 RU 04317 04010
03034 RU 04307 04029
03046 RU 04312 04034
03058 J1 04111 000J0
03070 J1 04317 000J0
03082 -U 04071 0[illegible]9J

```
                FSUB  AC,AD1,11                                   03094 -K 04071 04090
                TFL   D3,AC                                       03104 -0 04282 04071
                FSUB  AC,AD2,11                                   03116 -K 04071 04100
                TFL   D2,AC                                       03130 -0 04272 04071
                FSUB  AC,  D3,11                                  03142 -K 04071 04100
                TFL   D3,C                                        03154 -0 04282 04071
*
*       CORRECTOR
*
                TFL   AC,FH12                                     03166 -0 04071 04302
                FMUL  AC,D3                                       03178 -L 04071 04282
                TFL   STO,F112                                    03190 -0 04081 04292
                FMUL  STO,D2                                      03202 -L 04081 04272
                FADD  AC,STO                                      03214 -J 04071 04081
                TFL   STO,FHALF                                   03226 -0 04081 04191
                FMUL  STO,D1                                      03238 -L 04081 04262
                FADD  AC,STO                                      03250 -J 04071 04081
                FSUB  AC,ADR,11                                   03262 -K 04071 04093
                TFL   STO,AD4,11                                  03274 -0 04081 04113
                FMUL  AC,ISA,11                                   03286 -L 04071 04038
                FSUB  STO,AC                                      03298 -K 04081 04071
                TFL   AC,STO                                      03310 -0 04071 04081
                FSUB  AC,AD5,11                                   03322 -K 04071 04312
                CF    AC-2                                        03334 13 04039 00000
*
*       ERROR CHECK
*
                TFL   AD5,STO,6                                   03346 -0 04312 04081
                TFL   STO,AC                                      03358 -0 04081 04071
                FSUB  AC,EDAS,11                                  03370 -K 04071 04300
                BL    E2                                          03382 M7 03414 01300
                TFR   EDI,1,10                                    03394 J6 04250 00051
                BF    E3                                          03406 K9 02450 00000
E2              FSUB  STO,ELAS,11                                 03414 -K 04081 04315
                BNL   E3                                          03426 M6 03450 01308
                AM    ELI,1,10                                    03438 J1 04252 000-1
E3              AR    ADR,10,10                                   03450 J1 04091 000J0
                AM    AD1,10,10                                   03462 J1 04093 000J0
                AM    AD2,10,10                                   03474 J1 04101 000J0
                AM    AD3,10,10                                   03486 J1 04106 000J0
                AM    AD4,10,10                                   03498 J1 04111 000J0
                AM    AD5,10,10                                   03510 J1 04317 000J0
                CM    ELAS,10,10                                  03522 J1 04312 000J0
                AM    EDAS,10,10                                  03534 J1 04307 000J0
                BCXR  E1,1,all0                                   03546 04 03082 0-00J
                CM    EDI,1,10                                    03558 J4 04250 000-1
                BNZ   E5                                          03570 M7 03694 01200
                TFH   DFLG,0,10                                   03582 J6 04248 000-0
                TFL   AC,XDA,11                                   03594 -0 04071 04031
                FMUL  AC,FHALF                                    03606 -L 04071 04191
                TFL   STO,AC                                      03618 -0 04081 04071
                FSUB  AC,EMIN                                     03630 -K 04071 04054
                BH    E4                                          03642 M6 03674 01300
                TFL   XDA,EXP,10                                  03654 -0 04031 00054
                BT    E4                                          03666 M6 03678 00000
                TFL   XDA,STO,6                                   03678 -0 04031 04081
                BT    E4                                          03690 M6 03678 00000
                C     N,E4                                        03702 M8 03678 00000
                BNZ   E5                                          03714 M7 03694 01200
```

```
79          AH    DFLG,1,10                                   03778 J1 04246 00001
80          CH    DFLG,3,10                                   03780 J4 04246 00003
81          BNZ   EXIT                                        03782 M7 04318 01260
82          TFH   DFLG,0,10                                   03784 M8 04246 00000
83          TFL   AC,ADA,11                                   03786 =0 04071 0401N
84          FADD  AC,ADA,11                                   03788 =J 04071 04011
85          TFL   STO,AC                                      03790 =0 04081 04071
86          FADD  AC,HMAX                                     03792 =K 04071 04044
87          BAL   E6                                          03814 M6 03846 01500
88          TFL   ADA,STO,6                                   03816 =0 04086 04081
89          B7    EXIT                                        03838 M9 04318 00000
90    E6    TFL   ADA,HMAX,6                                  03946 =0 04019 04044
91          B7    EXIT                                        03858 M9 04318 00000
92    XEX   TF    AC,A                                        03866 K0 04071 04020
93          AH    AC,1,10                                     03878 J1 04071 00001
94          BLX   *+12,AC(4)                                  03890 M4 03902 04071
95          TF    ADR,XA                                      03902 K0 04091 04010
96          TF    ADI,ADR(                                    03914 K0 04096 0[illegible]
97    E7    TFL   ADR,ADI,611                                 03926 =0 04093 04090
98          AH    ADR,10,10                                   03938 J1 04091 00010
99          AH    ADI,10,10                                   03950 J1 04096 00010
100         BCXH  E7,,311                                     03962 [illegible]
101         B7    EXIT                                        [illegible]
102   CLR   TFH   DFLG,0,10                                   03982 J6 04246 00000
103         B7    EXIT                                        [illegible]
1     *
2     *     STORAGE AREA
3     *
4     INDEX DS    5                                           04005 00005
5     XA    DS    5                                           04010 00005
6     XDA   DS    5                                           04016 00005
7     N     DS    5                                           04020 00005
8     LFA   DS    4                                           04024 00004
9     EUA   DS    6                                           04029 00006
10    BLA   DS    5                                           04034 00005
11    HMAX  DS    10                                          04039 00010
12    HMIN  DS    10                                          04049 00010
13    TSA   DS    6                                           04059 00006
14    FLG   DC    2,1                                         04065 00021
15    AC    DS    10                                          04071 00010
16    STO   DS    10                                          04081 00010
17    ADRX  DS    5                                           04091 00005
18    ADR   DS    5                                           04096 00005
19    ADI   DS    5                                           04101 00005
20    AD2   DS    5                                           04106 00005
21    AD3   DS    5                                           04111 00005
22    AD4   DS    5                                           04116 00005
23          DS    10                                          04121 00010
24          DC    8,20000000                                  04131 00008
25    FHALF DC    2,0                                         04139 00002
26          DC    8,292B9522                                  04141 00008
27    FL    DC    2,0                                         04149 00002
28          DC    8,36578644                                  04151 00008
29    FC2   DC    2,0                                         04159 00002
30          DC    8,12134014                                  04161 00008
31    FC3   DC    2,0                                         04169 00002
32          DC    8,17070056                                  04171 00008
33    FC4   DC    2,1                                         04179 00021
34          DC    8,34142136                                  04181 00008
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35    FC5     DC    2,1                                                  04141 00002
36            DC    6,-.1215203                                          04143 00004
37    FC6     DC    2,1                                                  04147 00002
38            DC    6,.16666667                                          04149 00004
39    FC7     DC    2,6                                                  04153 00002
40    DCA     DS    1                                                    04155 00001
41    DIA     DS    1                                                    04156 00001
42    DZA     DS    1                                                    04157 00001
43    DBA     DS    1                                                    04160 00001
44    *
45            DC    6,.57500000                                          04161 00004
46    FC8     DC    2,6                                                  04165 00002
47            DC    6,.41666667                                          04167 00004
48    FC9     DC    2,8                                                  04173 00002
49    I       DS    2                                                    04175 00002
50    DFLG    DS    2                                                    04177 00002
51    COI     DS    2                                                    04201 00002
52    ELI     DS    2                                                    04203 00002
53    D1      DS    10                                                   04205 00010
54    D2      DS    10                                                   04215 00010
55    D3      DS    10                                                   04225 00010
56            DC    8,.83333333                                          04235 00008
57    FIIZ    DC    2,-1                                                 04243 00002
58            DC    8,.41666667                                          04245 00008
59    FI2+    DC    2,+1                                                 04253 00002
60    EQAS    DS    5                                                    04255 00005
61    ELAS    DS    5                                                    04262 00005
62    ADS     DS    5                                                    04267 00005
63    *
1     *       EXIT
2     *
3     EXIT    BLX   *+1,INDEX(4)                                         04274 DI 04330 00005
4             B     RA,,0                                                04300 R9 04240 00000
5     *
6     *       CALL DER
7     *
8             DS    5                                                    04306 00005
9     DER     BTM   *,*+11                                               04312 JR 04334R -4329
10            B     DER+1,6                                              04320 R9 04334R 00000
11    *
12    *       TRAILING CARDS
13    *
14    LENGTH  DAC   6,DER                                                04333 00012
15            DVLC  ,5,DER+5                                             04340 00005
16            DC    2,0*                                                 04346 00002
17            DEND                                                       04350
```