



COMPUTER PROGRAMS FOR THE GEOMAGNETIC FIELD

Francis J. Wall

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Greenbelt, Maryland

August 13, 1965

THE
Dikewood
CORPORATION

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1.0 INTRODUCTION

The geomagnetic potential function is usually represented by a spherical-harmonic expansion. The Dikewood Corporation has prepared two computer programs related to this spherical-harmonic expansion for the Goddard Space Flight Center. The first program, Jensen's Fit, is a least squares program for improving the precision of the parameters in the spherical-harmonic expansion. The second program, Wall's Error, is a statistical program for mapping the field from the spherical-harmonic expansion produced by Jensen's Fit and then computing the random error in the mapping.

These two programs are similar in much of the basic formulation. Hence, Section 2.0 of this report on basic formulation applies to both programs. Section 3.0 discusses in some detail Jensen's Fit program while Section 4.0 discusses in a similar manner Wall's Error program.

Most if not all of the information presented in this report can be found in the documents and books listed in the bibliography.

This document is a final report on work performed at The Dikewood Corporation and does not cover any program modifications later made at Goddard Space Flight Center for the accommodation of the Goddard computer system.

2.0 BASIC FORMULATION

We begin with the "terms of internal origin" in the Chapman and Bartels (1940, p. 639) formulation of the geomagnetic potential function to

write the following equation for the magnetic potential V:

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+1} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \quad (1)$$

where a = radius of the earth
 r = geocentric distance
 g, h = Gauss coefficients
 ϕ = longitude
 θ = colatitude

$P^{n,m}(\theta)$ = associated Legendre functions, Gauss normalized

The three orthogonal components are derived by taking the gradient $\vec{B} = +\nabla V$ to give

$$B_{\theta} = \frac{1}{r} \frac{\partial V}{\partial \theta} = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) \frac{\partial P^{n,m}(\theta)}{\partial \theta} \quad (2)$$

$$B_{\phi} = \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin \theta} (-g^{n,m} \sin m\phi + h^{n,m} \cos m\phi) P^{n,m}(\theta) \quad (3)$$

$$B_r = \frac{\partial V}{\partial r} = - \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} (n+1) (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \quad (4)$$

and, of course,

$$B = (B_{\theta}^2 + B_{\phi}^2 + B_r^2)^{1/2} \quad (5)$$

There have been several versions of Jensen's Fit. Basic formulation for the original versions ended with the above equations. However, the most recent version included provision for a field of external origin. The field specified was a simple magnetic field of intensity B at infinity (denoted by B_{∞}) and parallel to an arbitrary axis z' in the (x', y', z') coordinate system, and positive in the increasing z' direction.

To develop the formulation for this field, we begin with

$$\begin{aligned} \vec{B}_{\infty} &= +\nabla V = +\nabla (+B_{\infty} z') \\ V &= +B_{\infty} z' \\ &= +B_{\infty} r (\cos \gamma) P^{1,0}(\theta) \\ &= +B_{\infty} r P^{1,0}(\alpha) P^{1,0}(\theta) + B_{\infty} r P^{1,1}(\alpha) P^{1,1}(\theta) (\cos(\beta - \phi)) \end{aligned} \quad (6)$$

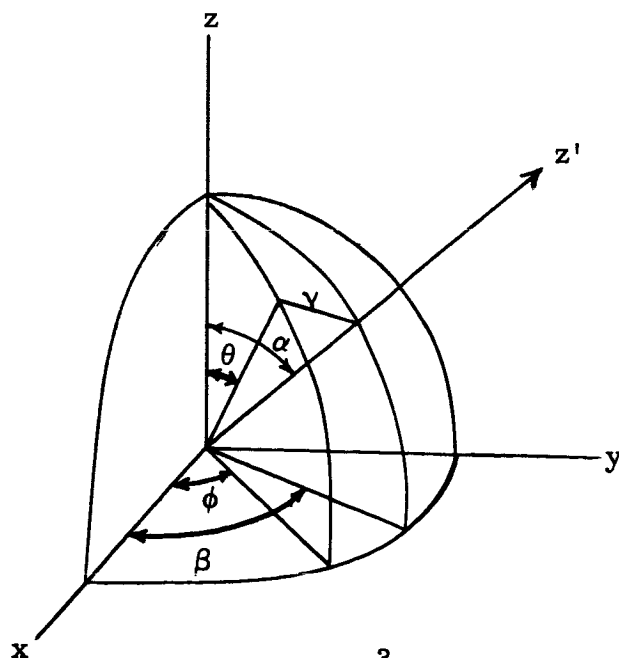


Fig. 1

Let $B_{\infty} \cos \alpha = E_1$

$$B_{\infty} \sin \alpha \cos \beta = E_2$$

$$B_{\infty} \sin \alpha \sin \beta = E_3$$

Then

$$V = rE_1 P^{1,0}(\theta) + rE_2 \cos \phi P^{1,1}(\theta) + rE_3 \sin \phi P^{1,1}(\theta) \quad (8)$$

The three orthogonal components are derived by taking the gradient $\vec{B} = +\nabla V$ to give

$$B_{\theta} = \frac{1}{r} \frac{\partial V}{\partial \theta} = E_1 \frac{\partial P^{1,0}(\theta)}{\partial \theta} + E_2 \cos \phi \frac{\partial P^{1,1}(\theta)}{\partial \theta} + E_3 \sin \phi \frac{\partial P^{1,1}(\theta)}{\partial \theta} \quad (9)$$

$$B_{\phi} = \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} = - \frac{E_2 \sin \phi}{\sin \theta} P^{1,1}(\theta) + \frac{E_3 \cos \phi}{\sin \theta} P^{1,1}(\theta) \quad (10)$$

$$B_r = \frac{\partial V}{\partial r} = E_1 P^{1,0}(\theta) + E_2 \cos \phi P^{1,1}(\theta) + E_3 \sin \phi P^{1,1}(\theta) \quad (11)$$

Substituting for the Legendre polynomials,

$$P^{1,0}(\theta) = \cos \theta$$

and

$$P^{1,1}(\theta) = \sin \theta$$

we have

$$B_{\theta} = -E_1 \sin \theta + E_2 \cos \phi \cos \theta + E_3 \sin \phi \cos \theta \quad (12)$$

$$B_{\phi} = -E_2 \sin \phi + E_3 \cos \phi \quad (13)$$

$$B_r = E_1 \cos \theta + E_2 \cos \phi \sin \theta + E_3 \sin \phi \sin \theta \quad (14)$$

Hence, the complete basic formulation including the simple external field is given by

$$\begin{aligned} V = a \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+1} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \\ + rE_1 P^{1,0}(\theta) + rE_2 \cos \phi P^{1,1}(\theta) + rE_3 \sin \phi P^{1,1}(\theta) \end{aligned} \quad (15)$$

The orthogonal components including the simple external field are

$$\begin{aligned} B_{\theta} = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) \frac{\partial P^{n,m}(\theta)}{\partial \theta} \\ - E_1 \sin \theta + E_2 \cos \phi \cos \theta + E_3 \sin \phi \cos \theta \end{aligned} \quad (16)$$

$$\begin{aligned} B_{\phi} = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin \theta} (-g^{n,m} \sin m\phi + h^{n,m} \cos m\phi) P^{n,m}(\theta) \\ - E_2 \sin \phi + E_3 \cos \phi \end{aligned} \quad (17)$$

$$\begin{aligned} B_r = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} (n+1) (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \\ + E_1 \cos \theta + E_2 \cos \phi \sin \theta + E_3 \sin \phi \sin \theta \end{aligned} \quad (18)$$

The external field is defined as positive if the direction of force is toward the positive z' axis. Hence from Fig. 1, E_1 is positive if the direction of force is out of the northern hemisphere. E_2 is positive if the direction of force is out of the hemisphere bisected by the Greenwich meridian (i. e., the half-circle passing through Greenwich) and E_3 is positive if the direction of force is out of the hemisphere to the right of the Greenwich meridian (i. e., east longitude).

In the above, $P^{n,m}(\theta)$ has been used to denote Gauss normalized associated Legendre polynomials. To simplify the computer program, the Legendre polynomials have been adjusted so that the coefficients of the highest order term in θ is one. Generating functions for these polynomials and their derivatives are given by

$$P^{0,0}(\theta) = 1.0$$

$$\frac{\partial P^{0,0}(\theta)}{\partial \theta} = 0.0$$

$$P^{n,n}(\theta) = \sin \theta P^{n-1,n-1}(\theta) \quad n \geq 2$$

$$\frac{\partial P^{n,n}(\theta)}{\partial \theta} = \sin \theta \frac{\partial P^{n-1,n-1}(\theta)}{\partial \theta} + \cos \theta P^{n-1,n-1}(\theta) \quad n \geq 2$$

$$P^{n,m}(\theta) = \cos \theta P^{n-1,m}(\theta) - K_{n,m} P^{n-2,m}(\theta) \quad n \geq 2, m \neq n$$

$$\frac{\partial P^{n,m}(\theta)}{\partial \theta} = \cos \theta \frac{\partial P^{n-1,m}(\theta)}{\partial \theta} - \sin \theta P^{n-1,m}(\theta) - K_{n,m} \frac{\partial P^{n-2,m}(\theta)}{\partial \theta} \quad (19)$$

where $K_{n,m} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)}$

These Gauss normalized polynomials are then converted to the Schmidt quasi-normalized functions $P_n^m(\theta)$ via the relationship:

$$P_n^m(\theta) = S^{n,m} P_n^{n,m}(\theta)$$

where

$$S^{0,0} = -1.0$$

$$S^{n,0} = S^{n-1,0} \left(\frac{2n-1}{n} \right)$$

$$S^{n,1} = S^{n,0} \left(\frac{2n}{n+1} \right)^{1/2}$$

$$S^{n,m} = S^{n,m-1} \left(\frac{n-m+1}{n+m} \right)^{1/2} \quad m \geq 3 \quad (20)$$

To introduce secular change coefficients into the equations for the potential function and its three orthogonal components, each coefficient in the terms of internal origin are expressed as follows:

$$g^{n,m} = g^{n,m,0} + g^{n,m,t}_t + g^{n,m,tt}_t{}^2$$

and

$$h^{n,m} = h^{n,m,0} + h^{n,m,t}_t + h^{n,m,tt}_t{}^2 \quad (21)$$

Hence, the complete basic formulation including the simple external field using the Schmidt quasi-normalized functions $P_n^m(\theta)$ is given by

$$\begin{aligned}
V = a \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+1} & \left((g^{n,m,0} + g^{n,m,t}_t + g^{n,m,tt}_t{}^2) \cos m\phi \right. \\
& \left. + (h^{n,m,0} + h^{n,m,t}_t + h^{n,m,tt}_t{}^2) \sin m\phi \right) P_n^m(\theta) \\
& - rE_1 P_1^0(\theta) - rE_2 \cos \phi P_1^1(\theta) - rE_3 \sin \phi P_1^1(\theta) \quad . \quad (22)
\end{aligned}$$

The orthogonal components are given by

$$\begin{aligned}
B_{\theta} = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} & \left((g^{n,m,0} + g^{n,m,t}_t + g^{n,m,tt}_t{}^2) \cos m\phi \right. \\
& \left. + (h^{n,m,0} + h^{n,m,t}_t + h^{n,m,tt}_t{}^2) \sin m\phi \right) \frac{\partial P_n^m(\theta)}{\partial \theta} \\
& + E_1 \sin \theta - E_2 \cos \phi \cos \theta - E_3 \sin \phi \cos \theta \quad (23)
\end{aligned}$$

$$\begin{aligned}
B_{\phi} = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin \theta} & \left((-g^{n,m,0} - g^{n,m,t}_t - g^{n,m,tt}_t{}^2) \sin m\phi \right. \\
& \left. + (h^{n,m,0} + h^{n,m,t}_t + h^{n,m,tt}_t{}^2) \cos m\phi \right) P_n^m(\theta) \\
& + E_2 \sin \phi - E_3 \cos \phi \quad (24)
\end{aligned}$$

$$B_r = \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+2} (n+1) \left((g^{n,m,0} + g^{n,m,t}_t + g^{n,m,tt}_t{}^2) \cos m\phi \right.$$

$$\begin{aligned}
& + (h^{n,m,0} + h^{n,m,t} + h^{n,m,tt^2}) \sin m\phi \Big) P_n^m(\theta) \\
& - E_1 \cos \theta - E_2 \cos \phi \sin \theta - E_3 \sin \phi \sin \theta
\end{aligned} \tag{25}$$

Secular change coefficients were not included for the external field.

The Dikewood Corporation has proposed to modify the formulation to include secular change coefficients for the external field in a future contractual effort.

To provide compatibility with the FORTRAN programming system, the spherical-harmonic expansion of the geomagnetic potential function (Eq. (22)) is redefined in terms of subscripts starting at one instead of zero, by letting $N = n+1$ and $M = m+1$. Thus,

$$\begin{aligned}
V = a \sum_{N=2}^{\infty} \sum_{M=1}^N \left(\frac{a}{r} \right)^N & \left((g_{N,M,0} + g_{N,M,t} + g_{N,M,tt^2}) \cos (M-1) \phi \right. \\
& + (h_{N,M,0} + h_{N,M,t} + h_{N,M,tt^2}) \sin (M-1) \phi \Big) P_{N,M}(\theta) \\
& - rE_1 P_{2,1}(\theta) - rE_2 \cos \phi P_{2,2}(\theta) - rE_3 \sin \phi P_{2,2}(\theta)
\end{aligned} \tag{26}$$

The orthogonal components of the magnetic field are given by

$$\begin{aligned}
B_{\theta} = \sum_{N=2}^{\infty} \sum_{M=1}^N \left(\frac{a}{r} \right)^{N+1} & \left((g_{N,M,0} + g_{N,M,t} + g_{N,M,tt^2}) \cos (M-1) \phi \right. \\
& + (h_{N,M,0} + h_{N,M,t} + h_{N,M,tt^2}) \sin (M-1) \phi \Big) \frac{\partial P_{N,M}(\theta)}{\partial \theta}
\end{aligned}$$

$$+ E_1 \sin \theta - E_2 \cos \phi \cos \theta - E_3 \sin \phi \cos \theta \quad , \quad (27)$$

$$B_\phi = \sum_{N=2}^{\infty} \sum_{M=1}^N \left(\frac{a}{r}\right)^{N+1} \frac{M-1}{\sin \theta} \left((-g_{N,M,0} - g_{N,M,t}^t - g_{N,M,tt}^{t^2}) \sin(M-1)\phi \right. \\ \left. + (h_{N,M,0} + h_{N,M,t}^t + h_{N,M,tt}^{t^2}) \cos(M-1)\phi \right) P_{N,M}(\theta) \\ + E_2 \sin \phi - E_3 \cos \phi \quad , \quad (28)$$

$$B_r = - \sum_{N=2}^{\infty} \sum_{M=1}^N \left(\frac{a}{r}\right)^{N+1} N \left((g_{N,M,0} + g_{N,M,t}^t + g_{N,M,tt}^{t^2}) \cos(M-1)\phi \right. \\ \left. + (h_{N,M,0} + h_{N,M,t}^t + h_{N,M,tt}^{t^2}) \sin(M-1)\phi \right) P_{N,M}(\theta) \\ - E_1 \cos \theta - E_2 \cos \phi \sin \theta - E_3 \sin \phi \sin \theta \quad . \quad (29)$$

The above formulation is rigorously correct only for a spherical earth. It is obvious that as the accuracy of evaluation of the geomagnetic field increases, it will eventually be necessary to take the earth's true shape into account. So long as the evaluation of the harmonic coefficients is done in spherical coordinates, r , θ , and ϕ , the resulting fields B_r , B_θ , and B_ϕ will be in strict geocentric directions. The only constant in the potential function pertaining to the earth is the radius a here chosen to be the mean radius or 6371.2 km. The only problem is that of converting positions in geodetic coordinates to geocentric coordinates. Both the Fit and

Error programs perform coordinate conversion for the oblateness of the earth only. Referring to Fig. 2, we can write:

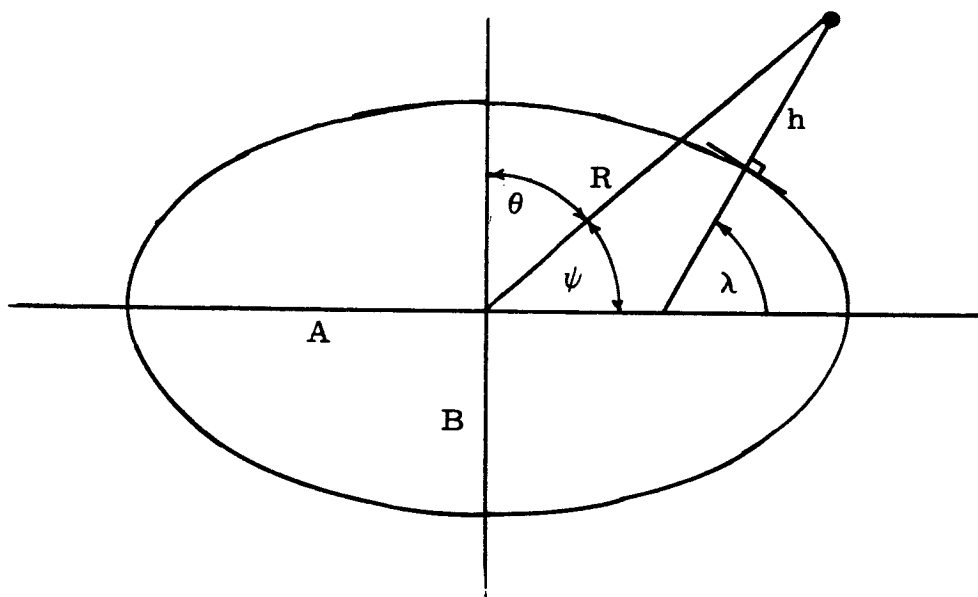


Fig. 2

$$\tan \psi = \frac{h \sqrt{A^2 \cos^2 \lambda + B^2 \sin^2 \lambda} + B^2}{h \sqrt{A^2 \cos^2 \lambda + A^2 \sin^2 \lambda} + A^2} \tan \lambda \quad (30)$$

and

$$R^2 = h^2 + 2h \sqrt{A^2 \cos^2 \lambda + B^2 \sin^2 \lambda} + \frac{A^4 \cos^2 \lambda + B^4 \sin^2 \lambda}{A^2 \cos^2 \lambda + B^2 \sin^2 \lambda} \quad (31)$$

where:

h = height above the geoid

ψ = geocentric latitude ($90^\circ - \theta$)

λ = geodetic latitude

R = geocentric distance

A = mean equatorial radius of 6378.165 km

B = polar radius of 6356.783 .

Using λ and h , the geocentric quantities $\theta = 90 - \psi$ and R can then be calculated. The conversion from B_r and B_θ to X and Z can then be done by the rotation:

$$X = -B_\theta \cos(\lambda - \psi) - B_r \sin(\lambda - \psi) \quad (32)$$

$$Z = B_\theta \sin(\lambda - \psi) - B_r \cos(\lambda - \psi) \quad (33)$$

To complete the coordinate systems, note that

$$Y = B_\phi .$$

Geomagnetic data have been assembled from many sources. Hence the reliability of the information varies. Since instrument accuracy and some other factors are known for the different data sources, quantitative estimates of reliability can be made. These reliability estimates are the basis for a system of geomagnetic data weights. Table 1 below lists the standard error associated with the different data sources.

The quantities measured in Gamma were weighted inversely as the standard error listed. Thus values of H measured at an observatory would have a weighting factor of 1/5 and values of H measured in Canada would have a weighting factor of 1/60, so that the observatory data would count 12 times as much as the Canadian data. For those quantities measured in

in degrees, the effect of the error is greater for points where the field is stronger. Thus, these data were weighted by the factor $\frac{1}{\delta D \cdot H}$ for D and $\frac{1}{\delta I \cdot F}$ for I.

Table 1
Estimated Standard Errors

	δD^O	δI^O	δH^Y	δZ^Y	δF^Y
Observatory	0.0033	0.006	5	15	15
Land Survey	0.1	0.1	30	50	-
Air Survey	0.3	0.1	-	-	30
Shipboard	0.083	0.083	25	-	-
Satellite	-	-	-	-	10
Towed magnetometer (proton)	-	-	-	-	10
Towed magnetometer (fluxgate)	-	-	-	-	40
Repeat observations	0.033	0.083	5	-	2
Canadian data	0.3	-	60	60	-

3.0 JENSEN'S FIT PROGRAM

Jensen's Fit Program is simply a computer program for determining small corrections to an already good set of parameters for the spherical-harmonic expansion of the geomagnetic function.

The general scheme for the program is a common least squares approach. The best available estimate of the coefficients of the potential function are used to estimate the magnetic field for observation locations.

Then the actual observations are compared with these estimates and corrections for the potential function coefficients are computed from all available data, both old and new, via the method of least squares. Since the corrections are not optimum, the procedure must be repeated until the corrections are no longer significant. Hence much computer time is consumed each time the coefficients are up-dated with new data.

The procedure is to express the geomagnetic measurements in terms of the field components X, Y, and Z developed in Section 2.0 of this report. As functions of the g's and h's, these expressions are expanded into Taylor series that include only linear terms. Then, via the method of least squares, corrections for the improvement of the g's and h's are estimated. These corrections are applied and the procedure repeated until the g and h parameters converge.

As an example of a Taylor's expansion, consider the measurement declination,

$$D = \tan^{-1} \frac{Y}{X} = \tan^{-1} \frac{B_{\phi}}{-B_{\theta} \cos(\lambda-\psi) - B_r \sin(\lambda-\psi)} \quad (34)$$

As a function of X and Y, we can write an algebraic expression for declination first in terms of B_r , B_{θ} , and B_{ϕ} as is done above and then in terms of the g's and h's of the potential function. The Taylor's expansion for D as a function of a single g and h is as follows:

$$D(g + \Delta g, h + \Delta h) \approx D(g, h) + \Delta g \frac{\partial D(g, h)}{\partial g} + \Delta h \frac{\partial D(g, h)}{\partial h} \quad (35)$$

Now the term on the left is the observation while $D(g, h)$ is a computed or expected measurement based on the best available set of g and h parameters. On rearranging this equation, we see that the error (or residue),

$$D(g + \Delta g, h + \Delta h) - D(g, h) = \Delta g \frac{\partial D(g, h)}{\partial g} + \Delta h \frac{\partial D(g, h)}{\partial h} \quad (36)$$

is linear in the correction terms Δg and Δh . Hence, a standard least-squares procedure can be used to find values for these corrections. These corrections are then applied to each of the coefficients, and the procedure repeated until the desired accuracy is obtained. It should be recognized at this point that g and h as here used can and do represent a large group of parameters. The number of parameters is determined by the limits on the summations in the potential function.

Since this linear expression resulted from a Taylor series that was truncated after the linear terms, it is accurate only as long as the summation term is small compared with the expected value. This requirement is not difficult to meet since there are several sets of coefficients available that represent the geomagnetic potential with very small error.

Similar expressions for dip,

$$\text{dip} = \tan^{-1} \frac{Z}{H}, \quad (37)$$

horizontal field,

$$H = (X^2 + Y^2)^{1/2}, \quad (38)$$

total field,

$$T = (X^2 + Y^2 + Z^2)^{1/2}, \quad (39)$$

and the X, Y, and Z field components can be used to estimate the corrections for the g's and h's. It is also possible to mix the use of geomagnetic measurements so long as the corresponding expression is used for the g and h corrections.

The coefficients of the set of simultaneous equations form a symmetric matrix. To conserve computer storage, and hence to permit the estimation of a larger number of g's and h's, only half of these coefficients are stored in the computer program. If one thinks of the complete array required for the normal equations as consisting of the square matrix of the coefficients of the unknowns plus two additional columns, one for the observation terms and the other for the computation check sums, then the program stores the upper half of the square matrix including the diagonal terms and the two additional columns. This array, which is illustrated in Fig. 3 for a system of five unknowns, is stored by rows in a one dimensional array named D in the computer program. In the illustration, the usual two dimensional Fortran subscript is equated with the corresponding position in the one dimensional D array. Given the subscript (I, J) for any element in the complete rectangular array, its position in the D array is determined by

$$K = I*(NOR + 2) - I*(I - 1) / 2 + J - NOR - 2 \quad (41)$$

where NOR is the number of rows in the complete matrix. To illustrate how this formula determines the position in the D array, it is rewritten as follows:

$$K = I * (NOR + 2) - I * (I - 1) / 2 + J - NOR - 2 \quad (41)$$

					Obser- vation	Check
					<u>Column</u>	<u>Sum</u>
(1,1)=D(1)	(1,2)=D(2)	(1,3)=D(3)	(1,4)=D(4)	(1,5)=D(5)	D(6)	D(7)
	(2,2)=D(8)	(2,3)=D(9)	(2,4)=D(10)	(2,5)=D(11)	D(12)	D(13)
		(3,3)=D(14)	(3,4)=D(15)	(3,5)=D(16)	D(17)	D(18)
			(4,4)=D(19)	(4,5)=D(20)	D(21)	D(22)
				(5,5)=D(23)	D(24)	D(25)

Fig. 3

Upper Triangular Matrix Storage
(illustrated with a 5 x 5 matrix)

or

$$K = (I - 1) * (NOR + 2) - I * (I - 1) / 2 + J \quad (42)$$

Now consider two positional notations. The first is the position in the complete array from which the array in Fig. 3 was taken. Number the elements in this complete array from left to right by rows. The second notation is the

one used for the subscripts of D in Fig. 3. Then the interpretation of each of the three terms in the K equation is as follows:

$(I-1) * (NOR+2)$ = position preceding the first position in row I of the complete rectangular array, i. e., each row has $NOR+2$ elements and this is multiplied by the row number less one. For row 1, this position is the position numbered zero or one less than position one.

$I * (I-1) / 2$ = the sum of the digits preceding the I^{th} digit. In the D array, one notes that in row 2, one position is not used; in row 3, two positions are not used, and so forth. Hence, the number of elements not used by row I is the sum of the positive digits less than I . Hence, the first two terms of K compute the position preceding the first position in a row of the D array.

J = column position which is added to the position preceding the first position in a row of the D array.

Each entry in the computation check column is the sum of the coefficients of the parameters in the respective row. For example, in Fig. 3, the third term in this column will be stored in $D(18)$ and is the sum

$$D(3) + D(9) + D(14) + D(15) + D(16) \quad .$$

The computation check column is a pseudo-observation column in which all of the unknowns are assumed to be 1.0. Hence on solution of the system with this pseudo-observation column, estimates of the unknowns should approximate 1.0. Their failure to do so indicates the precision of the true estimates made from the observation terms themselves.

The procedure employed for the solution of the least squares equations is a modification of the common Gauss elimination method. The modification consists of computing the "back solution" at the same time that the "forward solution" is computed, i. e. , the matrix of coefficients (here assumed to be the complete square array) is diagonalized at the same time as it is made into a triangle.

In the modified Gauss elimination procedure, the following sequence of operations is performed for each row:

- (1) Each element in the row (beginning with the diagonal element) is divided by the diagonal element. Let the diagonal row and diagonal column be defined as the matrix row and column respectively that contain this element.
- (2) Then beginning with the first column to the right of the diagonal column, each element in every row other than the diagonal row has subtracted from it the product of the corresponding element in the diagonal row and the corresponding element in the diagonal column.

From the procedure above, one can see that as each row is considered at step (1), the corresponding complete column is required for step (2). This column is contained in the triangle array D.

The solution of the set of simultaneous least squares equations yields adjustments or corrections for the parameters based on the average

observation time. The parameters for starting the program are based on the year 1960. Hence, the corrections must be computed for 1960 instead of the average observation time. To do this, we note that $g_{N,M,0}$ for time t is estimated by

$$g_{N,M,0} + g_{N,M,t} + g_{N,M,tt}t^2 \quad (43)$$

for time zero. Adding another subscript to indicate epoch, this relation may be put into an equation as follows:

$$g_{N,M,0,t} = g_{N,M,0,0} + g_{N,M,t,0}t + g_{N,M,tt,0}t^2 \quad (44)$$

Similarly,

$$g_{N,M,0,0} = g_{N,M,0,t} + g_{N,M,t,t}t + g_{N,M,tt,t}t^2 \quad (45)$$

Taking differences with respect to the g 's ,

$$\Delta g_{N,M,0,0} = \Delta g_{N,M,0,t} + \Delta g_{N,M,t,t}t + \Delta g_{N,M,tt,t}t^2 \quad (46)$$

we obtain the correction for $g_{N,M,0}$ at time zero from the corrections at time t . In a similar manner the correction for $g_{N,M,t}$ at time zero from the corrections at time t can be shown to be

$$\Delta g_{N,M,t,0} = \Delta g_{N,M,t,t} + 2 \Delta g_{N,M,tt,t}t \quad (47)$$

Similar expressions apply to the h 's .

In Appendix A, Jensen's complete Fit program is listed. Appendix B is a brief description and cross reference of the program. Appendix C is a flow chart of Jensen's Fit program.

4.0 WALL'S ERROR

The following derivation establishes the relationship between errors in the given data and errors in the coefficients determined by a least-squares procedure. Assume that the values y_i are measured at points having coordinates $X_i = (x_{1i}, x_{2i}, x_{3i}, x_{4i})$ and that it is desired to fit a curve of the form $y = \sum_n a_n f_n(X)$ to the data. In this equation, a_n corresponds to the g 's and h 's in the spherical-harmonic expansion of the geomagnetic potential function and $f_n(x)$ corresponds to the known coefficients which are, incidentally, all functions of the position X in time and space. The least squares procedure involves the calculation of a vector $\underline{v}$ whose elements are

$$v_n = \sum_i f_n(X_i) y_i$$

and a matrix $\underline{A}$ whose elements are

$$A_{mn} = \sum_i f_m(X_i) f_n(X_i) \quad .$$

The coefficients a_n of the fitted curve are then found by solving the matrix equations

$$\underline{A} \cdot \underline{a} = \underline{v}$$

for the coefficients a_n , the elements of vector $\underline{a}$. The solution is

$$\underline{a} = \underline{A}^{-1} \cdot \underline{v}$$

where $\underline{A}^{-1}$ is the inverse of $\underline{A}$, and the coefficients are

$$a_n = \sum_j A^{nj} v_j = \sum_j \sum_i A^{nj} f_j(X_i) y_i \quad (48)$$

Now, since the y_i are independent measurements, the standard error σ_{a_n} of coefficient a_n can be found from the expression

$$\sigma_{a_n}^2 = \sum_i \left(\frac{\partial a_n}{\partial y_i} \right)^2 \sigma_{y_i}^2$$

Assuming that all σ_{y_i} 's have the same value $\sigma_{y'}^*$ and using values for the partial derivatives obtained from Eq. (48),

$$\begin{aligned} \sigma_{a_n}^2 &= \sigma_{y'}^2 \sum_i \left(\frac{\partial a_n}{\partial y_i} \right)^2 = \sigma_{y'}^2 \sum_i \left[\sum_j A^{nj} f_j(X_i) \right]^2 \\ &= \sigma_{y'}^2 \sum_i \sum_j \sum_k A^{nj} A^{nk} f_j(X_i) f_k(X_i) \end{aligned}$$

* The assumption that all σ_{y_i} are the same can easily be satisfied if weighting factors are used when the a_n are determined. The value of $\sigma_{y'}$ should be thought of as $W_i \sigma_{y_i}$ where W_i is the normalized weight chosen to make the value a constant for all i .

Reordering this finite sum yields

$$\begin{aligned}\sigma_{a_n}^2 &= \sigma_{y'}^2 \sum_j \sum_k A^{nj} A^{nk} \sum_i f_j(X_i) f_k(X_i) \\ &= \sigma_{y'}^2 \sum_j \sum_k A^{nj} A^{nk} A_{jk} \quad .\end{aligned}$$

Note that

$$\sum_j A^{nj} A_{jk} = \delta_{nk}$$

and therefore

$$\sigma_{a_n}^2 = \sigma_{y'}^2 A^{nn} \quad . \quad (49)$$

Eq. (49) is the relation allowing the accuracy of the coefficients to be determined when the accuracy of the given data is known.

To calculate the accuracy of the field determined by the curve-fitting process, assume again that a function of the form $y = \sum_n a_n f_n(X)$ is to be fitted to the given data. The functional form itself gives

$$\frac{\partial y}{\partial y_i} = \sum_n \left(\frac{\partial a_n}{\partial y_i} \right) f_n(X) \quad , \quad (50)$$

and the fact that the values y_i are independently measured allows the expression

$$\sigma_y^2 = \sum_i \left(\frac{\partial y}{\partial y_i} \right)^2 \sigma_{y_i}^2$$

Assuming all $\sigma_{y_i} = \sigma_{y'}$, as before, and using Eq. (50),

$$\begin{aligned} \sigma_y^2 &= \sigma_{y'}^2 \sum_i \left(\frac{\partial y}{\partial y_i} \right)^2 = \sigma_{y'}^2 \sum_i \left[\sum_n \left(\frac{\partial a_n}{\partial y_i} \right) f_n(X) \right]^2 \\ &= \sigma_{y'}^2 \sum_i \sum_n \sum_m \frac{\partial a_n}{\partial y_i} \frac{\partial a_m}{\partial y_i} f_n(X) f_m(X) \end{aligned}$$

Evaluating the partial derivatives from Eq. (48),

$$\begin{aligned} \sigma_y^2 &= \sigma_{y'}^2 \sum_i \sum_n \sum_m \left\{ \left[\sum_j A^{nj} f_j(X_i) \right] \left[\sum_k A^{mk} f_k(X_i) \right] f_n(X) f_m(X) \right\} \\ &= \sigma_{y'}^2 \sum_i \sum_n \sum_m \sum_j \sum_k A^{nj} A^{mk} f_j(X_i) f_k(X_i) f_n(X) f_m(X) \end{aligned}$$

Reordering terms yields

$$\begin{aligned} \sigma_y^2 &= \sigma_{y'}^2 \sum_n \sum_m \sum_j \sum_k \left[A^{nj} A^{mk} f_n(X) f_m(X) \sum_i f_j(X_i) f_k(X_i) \right] \\ &= \sigma_{y'}^2 \sum_n \sum_m \sum_j \sum_k \left[A^{nj} A^{mk} f_n(X) f_m(X) A_{jk} \right] \end{aligned}$$

$$= \sigma_{y'}^2 \sum_n \sum_m \sum_k A^{mk} f_n(X) f_m(X) \delta_{nk}$$

since $\sum_j A^{nj} A_{jk} = \delta_{nk}$. Summing the remaining terms over k yields

$$\sigma_y^2 = \sigma_{y'}^2 \sum_n \sum_m a^{mn} f_n(X) f_m(X) \quad (51)$$

Wall's Error program is a computer implementation of Eq. (51). To provide data for the Error program, a minor change was made in Daniels' Matrix subroutine. Daniels' Matrix inverts the least squares matrix that is set up by Jensen's Fit program. However, Matrix records only the diagonal elements of this inverse. To estimate the error of the geomagnetic field computed from the parameters calculated by Jensen's Fit, the complete inverse is needed. (Reference Eq. (51) above. A^{mn} are the elements of this inverse.) Hence, the Matrix subroutine was modified so that the complete inverse plus the parameters of the potential function and certain other miscellaneous constants are recorded on magnetic tape 1. Then, whenever the Error program is used, the required data will be available from the last run of Daniels' Matrix subroutine.

On comparing the listing of Jensen's Fit in Appendix A and Wall's Error in Appendix D, one sees much similarity. The principal differences include: (1) the initial input data are different; (2) the Fit program receives

all data from the RDATA subroutine while the Error program generates its own data to form a grid over the earth's surface; and (3) instead of computing the D matrix as in the Fit program, the Error program computes the standard error of estimate according to Eq. (51) above.

The errors computed are all in gammas. For dip and declination, points and errors expressed in degrees are more meaningful. Hence, immediately before printout for a grid point, the generated declination and dip and the respective standard errors of estimate are converted to degrees. The formulae for converting these errors are developed below.

For declination, let

$$D_{\gamma} = \text{declination in gammas}$$

$$D_o = \text{declination in degrees}$$

Note that $H = \text{horizontal component of field intensity.}$

Then

$$D_{\gamma} = D_o H$$

$$D_o = \frac{D_{\gamma}}{H}$$

$$\sigma_{D_o}^2 = \frac{1}{H^2} \sigma_{D_{\gamma}}^2 + \frac{D_{\gamma}^2}{H^4} \sigma_H^2$$

$$\sigma_{D_o}^2 = \frac{1}{H^2} \sigma_{D_{\gamma}}^2 + \frac{D_o^2}{H^2} \sigma_H^2$$

$$\sigma_{D_o}^2 = \frac{\sigma_D^2 + D_o^2 \sigma_h^2}{H^2} \quad (52)$$

For dip, let

I_γ = dip in gammas

I_o = dip in degrees .

Note that F = total field intensity.

Then

$$I_\gamma = I_o F$$

$$I_o = \frac{I_\gamma}{F}$$

$$\sigma_{I_o}^2 = \frac{1}{F^2} \sigma_{I_\gamma}^2 + \frac{I_\gamma^2}{F^4} \sigma_F^2$$

$$\sigma_{I_o}^2 = \frac{1}{F^2} \sigma_{I_\gamma}^2 + \frac{I_o^2}{F^2} \sigma_F^2$$

$$\sigma_{I_o}^2 = \frac{\sigma_{I_\gamma}^2 + I_o^2 \sigma_F^2}{F^2}$$

In Appendix D, Wall's complete Error program is listed. Appendix E is a brief description and cross reference of the program. Appendix F is a flow chart of Wall's Error program.

5.0 CONCLUSIONS

While Jensen's Fit program including the external field provisions and Wall's Error program are in a sense complete, there are three minor tasks related to these projects that should be done. The first of these tasks concerns adding secular change coefficients for the external field. If an estimate of the external field based on a large data sample is as significant as Dikewood's initial estimate, then further calculations to determine if this field is time-dependent seem necessary.

The second task concerns the addition of the external field variables to Daniels' Matrix subroutine. This, of course, is absolutely essential if one wishes to include the external field parameters when using Wall's Error. The third task is the addition of the secular change coefficients for the external field to the Error program.

Future programs in geomagnetism will require new analytical techniques. The large volume of satellite data will necessitate a data reduction procedure preceding any data analysis. At two points per second, each orbit will produce about 10,800 data points. Some means of reducing these data to a more manageable quantity is imperative. One method recommended would fit a Fourier series to each orbit and then select points from this equation at equal distances along the orbit. Another recommendation is to try the same approach with elliptic functions.

Since this type of data reduction scheme preserves information on individual orbits, in a very short time the quantity of data will still become

massive. A scheme to preserve orbital data in terms of secular variables would provide for even more reduction of the data, and hence would have some merit. If one views the potential function as a changing surface in three dimensional space, then, at a specific time in a very small area, it will appear as a simple plane. (For this discussion, the size of such a small area is not delineated.)

Now assuming that a simple plane can be used to approximate the potential function at a specific time in a small area, one might use all data in that area without regard to orbit to determine this plane as a function of time. Then in a manner similar to that which would be used for orbital data, one or more points could be chosen on the plane to represent the small area. One would hope that the plane would be simple enough in form to permit direct as opposed to iterative estimation of the parameters required to describe it. If this is possible, the reduction of satellite data and the estimation of the potential function parameters from the reduced data will be a manageable problem. In fact, such reduced data can be used with existing programs to estimate the potential function parameters.

By generating pseudo-satellite data, any or all of these proposed data reduction procedures can be studied. The utility of the results of these studies will be limited only by our ability to generate the pseudo-data.

In an appendix to our December, 1964 report, a new technique for handling geomagnetic data was outlined. Several questions exist concerning

the general use of this method. For example, how are areas where no data exists handled? Most if not all of such questions do not apply where satellite data is concerned. Hence, the method as outlined can be developed into a system for processing satellite data only.

The problem of the westward drift of the geomagnetic field has been brought to the attention of Dikewood personnel. A cursory examination of the problem suggests that some form of correlation analysis may be useful in establishing the significance and hence validity of the drift.

The above problems were recognized by The Dikewood Corporation during previous work for the Goddard Space Flight Center. We look forward to future contractual effort in the field of geomagnetism.

APPENDIX A

PROGRAM LISTING FOR JENSEN'S FIT PROGRAM

```

C      * * * * *
C      JENSEN FIT PROGRAM      HENDRICKS VERSION      TRIANGULAR
C      * * * * *
C
COMMON /DD/D(7500)
COMMON /DATAR/ISKIP,FLATT,ELONG,ALT,TIME,DECL,DECLWT,DIP,DIPWT,HOR
$,HORWT,B,BWT,X,XWT,Y,YWT,Z,ZWT
COMMON /COEFS/G(9,9),H(9,9),GT(9,9),HT(9,9),GTT(9,9),HTT(9,9),MAXD
DIMENSION ERR(18,36),FNO2(18,36),JERR(18),F(127),SIDE(126)
DIMENSION SHMIDT(9,9)
DIMENSION DXDH(9,9),DYDH(9,9),DZDH(9,9)
DIMENSION DXDG(9,9),DYDG(9,9),DZDG(9,9)
DIMENSION CP(9),SP(9),P(9,9),DP(9,9),CONST(9,9)
DIMENSION IERR(200),TYPE(8),SIG1(8),FNO1(8),SWT1(8),WD(7)
INTEGER EXTFLD
DATA RAD,A,FLAT,(TYPE(I),I=1,8),PI,PI2,LINE/57.2957795,6378.165,29
18.3,1HD,1HI,1HH,1HB,1HZ,1HX,1HY,1H*,3.14159265,6.28318530,0/
C      * * * * *
C
C      FLAT=1.-1./FLAT
C      COMPUTATION WITH SPHERICAL EARTH
C
FLAT=1.
A=6371.2
DO 1 I=1,18
DO 1 J=1,36
FNO2(I,J)=0.
ERR(I,J)=0.
CONTINUE
MAXD=7500
A2=A**2
A4=A**4
B2=(A*FLAT)**2
A2B2=A2*(1.-FLAT**2)
A4B4=A4*(1.-FLAT**4)
READ (5,2) XID1,XID2
FORMAT (2A6,24X,12)
READ (5,3) NMAX,NMAXT,NMAXTT,NSKIP,ITER
FORMAT (5I5)
READ (5,4) ERRLLIM,AVETIM
FORMAT (2F10.0)
READ (5,5) EXTFLD
FORMAT (15)
WRITE (6,6) NMAX,NMAXT,NMAXTT,NSKIP,ITER,ERRLLIM
FORMAT (6H1NMAX=,15,3X,6HNMAXT=,15,3X,7HNMAXTT=,15,3X,6HNSKIP=,15,
$3X,5HITER=,15,3X,7HERRLLIM=,F10.0)
WRITE (6,7) XID1,XID2
FORMAT (1X,2A6)

```

C		COMPUTE CONSTANTS REQUIRED FOR GENERATING	51
C		LEGENDRE POLYNOMIALS	52
	DO 8 N=2,NMAX		53
	FN=N		54
	DO 8 M=1,N		55
	FM=M		56
	CONST(N,M)=((FN-2.0)**2-(FM-1.0)**2)/(FN+FN-3.0)/(FN+FN-5.0)		57
	CONTINUE		58
8		COMPUTE CONSTANTS TO CONVERT FROM GAUSS TO	59
C		SCHMIDT NORMALIZATION	60
	SHMIDT(1,1)=-1.0		61
	DO 9 N=2,NMAX		62
	FN=N		63
	SHMIDT(N,1)=SHMIDT(N-1,1)*(FN+FN-3.0)/(FN-1.0)		64
	FACT=2.0		65
	DO 9 M=2,N		66
	FM=M		67
	SHMIDT(N,M)=SHMIDT(N,M-1)*SQRT((FN-FM+1.0)*FACT/(FN+FM-2.0))		68
9	FACT=1.0		69
C		SET VALUE OF FIRST LEGENDRE POLYNOMIALS	70
	P(1,1)=1.0		71
	DP(1,1)=0.0		72
C		SET VALUE OF SIN(M-1)PHI AND COS(M-1)PHI WHEN M=1	73
	SP(1)=0.0		74
	CP(1)=1.0		75
C		READ BEST SET OF PARAMETERS AS FIRST APPROXIMATION	76
10	READ (5,11) N,M,GNM,HNM,GTNM,HTNM,GTTNM,HTTNM		77
11	FORMAT (2I3,6F11.4)		78
	IF (N) 12,13,12		79
12	G(N,M)=GNM		80
	H(N,M)=HNM		81
	GT(N,M)=GTNM		82
	HT(N,M)=HTNM		83
	GTT(N,M)=GTTNM		84
	HTT(N,M)=HTTNM		85
	GO TO 10		86
C		READ BEST SET FOR EXTERNAL FIELD	87
13	READ (5,14) E1,E2,E3		88
14	FORMAT (6X,3F11.4)		89
C		RECORD STARTING PARAMETERS	90
	WRITE (6,11) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)		91
	*,M=1,N),N=2,NMAX)		92
	WRITE (6,15) E1,E2,E3		93
15	FORMAT (4H0E1=,F13.4/4H E2=,F13.4/4H E3=,F13.4)		94
C		* * * * *	95
C		END INITIALIZATION, BEGIN DATA PROCESSING	96
C		* * * * *	97
	DO 120 ITNO=1,ITER		98
	REWIND 2		99
	DO 16 J=1,8		100

	SIG1(J)=0.	101
	FN01(J)=0.	102
	SWT1(J)=0.	103
16	CONTINUE	104
	DO 17 I=1,200	105
	IERR(I)=0	106
17	CONTINUE	107
	DO 18 I=1,MAXD	108
	D(I)=0.0	109
18	CONTINUE	110
	LINE=0	111
	SUMTM=0.0	112
	ISKIP=NSKIP	113
C	READ ONE DATA LINE	114
19	CALL RDATA	115
	WD(1)=DECL	116
	WD(2)=DIP	117
	WD(3)=HOR	118
	WD(4)=B	119
	WD(5)=Z	120
	WD(6)=X	121
	WD(7)=Y	122
	IF (ISKIP) 20,79,20	123
C	COMPUTE GEOCENTRIC THETA FROM	124
C	GEODETIC COORDINATES	125
20	FLATR=FLATT/RAD	126
	SINLA=SIN(FLATR)	127
	SINLA2=SINLA**2	128
	DEN2=A2-A2B2*SINLA2	129
	DEN=SQRT(DEN2)	130
	FAC=((ALT*DEN)+B2)/((ALT*DEN)+A2)	131
	THETA=ATAN(FAC*SINLA/(1.E-30+SQRT(1.-SINLA2)))	132
C	COMPUTE GEOCENTRIC R FROM GEODETIC COORDINATES	133
C	R=SQRT(ALT*(ALT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)	134
C	COMPUTE SINE AND COSINE OF DIFFERENCE BETWEEN	135
C	GEODETIC AND GEOCENTRIC LATITUDINAL COORDINATES	136
	SIND=SIN(FLATR-THETA)	137
	COSD=SQRT(1.0-SIND*SIND)	138
	AOR=6371.2/R	139
C	COS THETA MEASURED FROM POLAR AXIS	140
	CT=SIN(THETA)	141
C	SIN THETA MEASURED FROM POLAR AXIS	142
	ST=SQRT(1.0-CT*CT)	143
C	LONGITUDE INDEX	144
	LON=AMIN(AMAX(ELONG/10.0+18.0,1.0),36.0)	145
C	LATITUDE INDEX	146
	LAT=AMIN(AMAX(FLATT/10.0+9.0,1.0),18.0)	147
	SP(2)=SIN(ELONG/RAD)	148
	CP(2)=COS(ELONG/RAD)	149
	DO 21 M=3,NMAX	150

C	SIN(M-1)PHI, EQ.(5)	151
	SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)	152
C	COS(M-1)PHI, EQ.(6)	153
	CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)	154
21	CONTINUE	155
C	GENERATE ASSOCIATED LEGENDRE POLYNOMIALS	156
	DO 24 N=2,NMAX	157
	DO 24 M=1,N	158
	IF (N-M) 23,22,23	159
22	P(N,N)=ST*P(N-1,N-1)	160
	DP(N,N)=ST*DP(N-1,N-1)+CT*P(N-1,N-1)	161
	GO TO 24	162
23	P(N,M)=CT*P(N-1,M)-CONST(N,M)*P(N-2,M)	163
	DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-CONST(N,M)*DP(N-2,M)	164
24	CONTINUE	165
C	INITIALIZE TO COMPUTE X,Y,Z	166
	CX=0.0	167
	CY=0.0	168
	CZ=0.0	169
	AR=AOR*AOR	170
	TM=TIME-60.0	171
C	COMPUTE X,Y,Z USING BEST AVAILABLE	172
C	PARAMETERS	173
	DO 25 N=2,NMAX	174
	FN=N	175
	AR=AR*AOR	176
	DO 25 M=1,N	177
	FM=M-1	178
C	APPLY SCHMIDT NORMALIZATION CONSTANTS	179
C	AND MULTIPLY BY (A/R)**(N+1)	180
	P(N,M)=P(N,M)*AR*SHMIDT(N,M)	181
	DP(N,M)=DP(N,M)*AR*SHMIDT(N,M)	182
	TEMP=FN*P(N,M)*SIND-DP(N,M)*COSD	183
	DXDG(N,M)=TEMP*CP(M)	184
	DXDH(N,M)=TEMP*SP(M)	185
	TEMP=FM*P(N,M)/ST	186
	DYDG(N,M)=-TEMP*SP(M)	187
	DYDH(N,M)=TEMP*CP(M)	188
	TEMP=FN*P(N,M)*COSD+DP(N,M)*SIND	189
	DZDG(N,M)=TEMP*CP(M)	190
	DZDH(N,M)=TEMP*SP(M)	191
C	ADD TIME TERMS	192
	GNM=(TM*GTT(N,M)+GT(N,M))*TM+G(N,M)	193
	HNM=(TM*HTT(N,M)+HT(N,M))*TM+H(N,M)	194
	CX=CX+GNM*DXDG(N,M)+HNM*DXDH(N,M)	195
	CY=CY+GNM*DYDG(N,M)+HNM*DYDH(N,M)	196
	CZ=CZ+GNM*DZDG(N,M)+HNM*DZDH(N,M)	197
25	CONTINUE	198
	IF (EXTFLD) 26,27,26	199
26	T1=E2*CP(2)+E3*SP(2)	200

	T2=E1*ST-T1*CT	201
	T1=E1*CT+T1*ST	202
	CX=CX-T2*COSD+T1*SIND	203
	CY=CY+E2*SP(2)-E3*CP(2)	204
	CZ=CZ+T2*SIND+T1*COSD	205
C	COMPUTE HORIZONTAL, TOTAL FIELD, DIP, AND	206
C	DECLINATION	207
27	CH=SQRT(CX*CX+CY*CY)	208
	CF=SQRT(CH*CH+CZ*CZ)	209
	CI=2.0*ATAN(CZ/(CF+CH))	210
	CD=2.0*ATAN(CY/(CH+CX))	211
C	COMPUTE COEFFICIENTS OF G(N,M) AND	212
C	H(N,M) FOR THE NEXT APPROXIMATION	213
28	I=1	214
	IF (DECL) 35,29,35	215
29	IF (DIP) 42,30,42	216
30	IF (HOR) 45,31,45	217
31	IF (B) 48,32,48	218
32	IF (Z) 51,33,51	219
33	IF (X) 53,34,53	220
34	IF (Y) 55,19,55	221
C	COMPUTE COEFFICIENTS WHEN DECLINATION	222
C	(DECL) IS GIVEN	223
C	DECL=ARCTAN(Y/X)	224
C	T1=DDECL/DY	225
C	T2=DDECL/DX	226
35	T1=CX/CH	227
	T2=CY/CH	228
	!TYPE=1	229
	DO 37 N=2,NMAX	230
	DO 37 M=1,N	231
C	DDECL/DG(N,M)	232
	F(I)=(T1*DYDG(N,M)-T2*DXDG(N,M))	233
	IF (M-1) 36,37,36	234
36	I=I+1	235
C	DDECL/DH(N,M)	236
	F(I)=(T1*DYDH(N,M)-T2*DXDH(N,M))	237
37	I=I+1	238
	FI=DECL/RAD-CD	239
	IF (FI-PI) 39,39,38	240
38	FI=FI-PI 2	241
	GO TO 41	242
39	IF (FI+PI) 40,41,41	243
40	FI=FI+PI 2	244
41	FI=FI*CH	245
	WT=RAD/DECLWT/CH	246
	DECL=0.0	247
	GO TO 57	248
C	COMPUTE COEFFICIENTS WHEN DIP IS GIVEN	249
C	DIP=ARCTAN(Z/H)	250

C	T1=DDIP/DZ	251
C	T2=DDIP/DX	252
C	T3=DDIP/DY	253
42	T1=CH/CF	254
	T2=CZ*CX/CH/CF	255
	T3=CZ*CY/CH/CF	256
	ITYPE=2	257
	DO 44 N=2,NMAX	258
	DO 44 M=1,N	259
C	F(I)=DDIP/DG(N,M)	260
	F(I)=(T1*DZDG(N,M)-T2*DXDG(N,M)-T3*DYDG(N,M))	261
	IF (M-1) 43,44,43	262
43	I=I+1	263
C	F(I)=DDIP/DH(N,M)	264
	F(I)=(T1*DZDH(N,M)-T2*DXDH(N,M)-T3*DYDH(N,M))	265
44	I=I+1	266
	F1=(DIP/RAD-CI)*CF	267
	WT=RAD/DIPWT/CF	268
	DIP=0.0	269
	GO TO 57	270
C	COMPUTE COEFFICIENTS WHEN THE HORIZONTAL	271
C	COMPONENT (HOR) IS GIVEN	272
C	HOR=SQRT(X*X+Y*Y)	273
C	T1=DHOR/DX	274
C	T2=DHOR/DY	275
45	T1=CX/HOR	276
	T2=CY/HOR	277
	ITYPE=3	278
	DO 47 N=2,NMAX	279
	DO 47 M=1,N	280
C	F(I)=DHOR/DG(N,M)	281
	F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M))	282
	IF (M-1) 46,47,46	283
46	I=I+1	284
C	F(I)=DHOR/DH(N,M)	285
	F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M))	286
47	I=I+1	287
	F1=HOR-CH	288
	WT=1.0/HORWT	289
	HOR=0.0	290
	GO TO 57	291
C	COMPUTE COEFFICIENTS WHEN TOTAL	292
C	FIELD B IS GIVEN	293
C	B=SQRT(X*X+Y*Y+Z*Z)	294
C	T1=DB/DX	295
C	T2=DB/DY	296
C	T3=DB/DZ	297
48	T1=CX/B	298
	T2=CY/B	299
	T3=CZ/B	300

	ITYPE=4	301
	DO 50 N=2,NMAX	302
	DO 50 M=1,N	303
C	F(I)=DB/DG(N,M)	304
	F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M)+T3*DZDG(N,M))	305
49	IF (M-1) 49,50,49	306
	I=I+1	307
C	F(I)=DB/DH(N,M)	308
	F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M)+T3*DZDH(N,M))	309
50	I=I+1	310
	FI=B-CF	311
	WT=1.0/BWT	312
	B=0.0	313
	GO TO 57	314
C	COMPUTE COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN	315
51	DO 52 N=2,NMAX	316
	F(I)=DZDG(N,1)	317
	I=I+1	318
	DO 52 M=2,N	319
	F(I)=DZDG(N,M)	320
	F(I+1)=DZDH(N,M)	321
52	I=I+2	322
	ITYPE=5	323
	FI=Z-CZ	324
	WT=1.0/ZWT	325
	Z=0.0	326
	GO TO 57	327
C	COMPUTE COEFFICIENTS WHEN THE X COMPONENT IS GIVEN	328
53	DO 54 N=2,NMAX	329
	F(I)=DXDG(N,1)	330
	I=I+1	331
	DO 54 M=2,N	332
	F(I)=DXDG(N,M)	333
	F(I+1)=DXDH(N,M)	334
54	I=I+2	335
	ITYPE=6	336
	FI=X-CX	337
	WT=1.0/XWT	338
	X=0.0	339
	GO TO 57	340
C	COMPUTE COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN	341
55	DO 56 N=2,NMAX	342
	F(I)=DYDG(N,1)	343
	I=I+1	344
	DO 56 M=2,N	345
	F(I)=DYDG(N,M)	346
	F(I+1)=DYDH(N,M)	347
56	I=I+2	348
	ITYPE=7	349
	FI=Y-CY	350

	WT=1.0/YWT	351
	Y=0.0	352
C	ADD TIME**1 TERMS	353
57	NO=I-1	354
	TFACT=TIME-AVETIM	355
	IF (NMAXT) 58,60,58	356
58	J=1	357
	DO 59 N=2,NMAXT	358
	F(I)=F(J)*TFACT	359
	I=I+1	360
	J=J+1	361
	DO 59 M=2,N	362
	F(I)=F(J)*TFACT	363
	F(I+1)=F(J+1)*TFACT	364
	I=I+2	365
	J=J+2	366
59	CONTINUE	367
C	ADD TIME**2 TERMS	368
60	NONOT=I-1	369
	IF (NMAXTT) 61,63,61	370
61	J=1	371
	TFACT=TFACT*TFACT	372
	DO 62 N=2,NMAXTT	373
	F(I)=F(J)*TFACT	374
	I=I+1	375
	J=J+1	376
	DO 62 M=2,N	377
	F(I)=F(J)*TFACT	378
	F(I+1)=F(J+1)*TFACT	379
	I=I+2	380
	J=J+2	381
62	CONTINUE	382
63	IF (EXTFLD) 64,72,64	383
C	ADD EXTERNAL FIELD TERMS	384
64	DXDE1=CT*SIND-ST*COSD	385
	DZDE1=ST*SIND+CT*COSD	386
	DXDE2=CP(2)*DZDE1	387
	DXDE3=SP(2)*DZDE1	388
	DZDE2=-CP(2)*DXDE1	389
	DZDE3=-SP(2)*DXDE1	390
	GO TO (65,66,67,68,69,70,71), ITYPE	391
C	COEFFICIENTS WHEN DECLINATION (D) IS GIVEN	392
C	F(I)=DD/DE1	393
65	F(I)=-T2*DXDE1	394
	I=I+1	395
C	F(I)=DD/DE2	396
	F(I)=T1*SP(2)-T2*DXDE2	397
	I=I+1	398
C	F(I)=DD/DE3	399
	F(I)=-T1*CP(2)-T2*DXDE3	400

	I=I+1	401
	GO TO 72	402
	COEFFICIENTS WHEN DIP (I) IS GIVEN	403
	F(I)=D1/DE1	404
66	F(I)=T1*DZDE1-T2*DXDE1	405
	I=I+1	406
	F(I)=D1/DE2	407
	F(I)=T1*DZDE2-T2*DXDE2-T3*SP(2)	408
	I=I+1	409
	F(I)=D1/DE3	410
	F(I)=T1*DZDE3-T2*DXDE3+T3*CP(2)	411
	I=I+1	412
	GO TO 72	413
	COEFFICIENTS WHEN THE HORIZONTAL	414
	COMPONENT (HOR) IS GIVEN	415
	F(I)=DHOR/DE1	416
67	F(I)=T1*DXDE1	417
	I=I+1	418
	F(I)=DHOR/DE2	419
	F(I)=T1*DXDE2+T2*SP(2)	420
	I=I+1	421
	F(I)=DHOR/DE3	422
	F(I)=T1*DXDE3-T2*CP(2)	423
	I=I+1	424
	GO TO 72	425
	COEFFICIENTS WHEN TOTAL FIELD IS GIVEN	426
	F(I)=DF/DE1	427
68	F(I)=T1*DXDE1+T3*DZDE1	428
	I=I+1	429
	F(I)=DF/DE2	430
	F(I)=T1*DXDE2+T2*SP(2)+T3*DZDE2	431
	I=I+1	432
	F(I)=DF/DE3	433
	F(I)=T1*DXDE3-T2*CP(2)+T3*DZDE3	434
	I=I+1	435
	GO TO 72	436
	COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN	437
69	F(I)=DZDE1	438
	I=I+1	439
	F(I)=DZDE2	440
	I=I+1	441
	F(I)=DZDE3	442
	I=I+1	443
	GO TO 72	444
	COEFFICIENTS WHEN THE X COMPONENT IS GIVEN	445
70	F(I)=DXDE1	446
	I=I+1	447
	F(I)=DXDE2	448
	I=I+1	449
	F(I)=DXDE3	450

	I=I+1		451
	GO TO 72		452
C		COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN	453
71	F(I)=0.0		454
	I=I+1		455
	F(I)=SP(2)		456
	I=I+1		457
	F(I)=-CP(2)		458
	I=I+1		459
	GO TO 72		460
C		ADD OBSERVATION TERM	461
72	F(I)=FI		462
	NOR=I-1		463
	NOP=I		464
	NOPP=I+1		465
	CIDEG=C1*RAD		466
	CDDEG=CD*RAD		467
	IF (MOD(LINE,57)) 75,73,75		468
73	WRITE (6,74)		469
74	FORMAT (85H1 LAT LONG ALT TIME DECL DIP H B		470
	\$ Z X Y F(NOP) /1H0)		471
75	WRITE (6,76) FLATT,ELONG,ALT,TIME,(WD(KK),KK=1,7),TYPE(ITYPE),CDDE		472
	\$G,CIDEG,CH,CF,CZ,CX,CY,F(NOP)		473
76	FORMAT (1X2F6.1,F5.0,F5.1,2F7.2,5F7.0,5XA2/23X2F7.2,5F7.0,F7.0)		474
	LINE=LINE+1		475
C		SUM DEVIATIONS FROM COMPUTED VALUES, WEIGHTS, DATE	476
C		TYPE COUNTS, ETC. FOR STANDARD ERROR ESTIMATES	477
	K=AMAX(AMIN(F(I)/10.0+101.0,200.0),1.0)		478
	IERR(K)=IERR(K)+1		479
	SIG1(ITYPE)=SIG1(ITYPE)+F(NOP)*F(NOP)*WT		480
	FNO1(ITYPE)=FNO1(ITYPE)+1.		481
	SWT1(ITYPE)=SWT1(ITYPE)+WT		482
	SIG1(8)=SIG1(8)+F(NOP)**2*WT		483
	FNO1(8)=FNO1(8)+1.		484
	SWT1(8)=SWT1(8)+WT		485
	SUMTM=SUMTM+TM		486
	IF (ITNO-ITER) 78,77,78		487
77	FNO2(LAT,LON)=FNO2(LAT,LON)+WT		488
	ERR(LAT,LON)=ERR(LAT,LON)+F(NOP)*WT		489
C		COMPUTE SQUARES AND CROSS-PRODUCTS AND	490
C		SUM INTO TRIANGULAR MATRIX D	491
78	K=1		492
	CALL DLOOP (NOR,NOP,F,WT)		493
	GO TO 28		494
C		* * * * *	495
C		END DATA PROCESSING, BEGIN SOLUTION	496
C		OF THE LEAST SQUARES EQUATIONS	497
C		* * * * *	498
79	DO 80 IJ=1,8		499
	SIG1(IJ)=SQRT(SIG1(IJ)/SWT1(IJ))		500

80	CONTINUE	501
C	RECORD DATA FOR MATRIX SUBROUTINE	502
	WRITE (2) NMAX,NMAXT,NMAXTT,SWT1(8),FNO1(8),SIG1(8)	503
	WRITE (2) (D(I),I=1,MAXD)	504
C	COMPUTE SUMS FOR CHECK COLUMN	505
	DO 82 I=1,NOR	506
	SUMD=0.0	507
	DO 81 J=1,NOR	508
	NROW=MINO(I,J)	509
	NCOL=I+J-NROW	510
	K=(NROW*(NOR+NOR+5-NROW))/2+NCOL-NOR-2	511
	SUMD=SUMD+D(K)	512
81	CONTINUE	513
	K=(I*(NOR+NOR+5-I))/2	514
	D(K)=SUMD	515
82	CONTINUE	516
C	INVERT TRIANGULAR MATRIX	517
	DO 88 L=1,NOR	518
C	SET UP ONE COMPLETE COLUMN	519
	DO 83 I=1,NOR	520
	NROW=MINO(I,L)	521
	NCOL=I+L-NROW	522
	K=(NROW*(NOR+NOR+5-NROW))/2+NCOL-NOR-2	523
	SIDE(I)=D(K)	524
83	CONTINUE	525
	K=(L*(NOR+NOR+7-L))/2-NOR-2	526
	RDKK=1.0/D(K)	527
	DO 84 J=L,NOP	528
	D(K+1)=D(K+1)*RDKK	529
	K=K+1	530
84	CONTINUE	531
	DO 88 I=1,NOR	532
	IF (I-L) 85,88,85	533
85	DO 87 J=L,NOP	534
	IF (J+1-I) 87,86,86	535
86	K=(I*(NOR+NOR+5-I))/2+J-NOR-1	536
	KJ=(L*(NOR+NOR+5-L))/2+J-NOR-1	537
	D(K)=D(K)-SIDE(I)*D(KJ)	538
87	CONTINUE	539
88	CONTINUE	540
C	* * * * *	541
C	END SOLUTION OF THE LEAST SQUARES EQUATIONS,	542
C	BEGIN ESTIMATION OF THE PARAMETER CORRECTIONS	543
C	* * * * *	544
	WRITE (6,89) (TYPE(IJ),SIG1(IJ),FNO1(IJ),IJ=1,8)	545
89	FORMAT (6H1SIGMA,5X,6HPPOINTS/1X/(1X,A2,F5.0,5X,F6.0))	546
	WRITE (6,90)	547
90	FORMAT (5X,1HN,2X,1HM,15X,1HP,14X,2HDP,13X,3H1.0)	548
	TFACT=60.0-AVETIM	549
	I=1	550

	DO 100 N=2,NMAX	551
	DO 100 M=1,N	552
C	COMPUTE TIME ADJUSTMENT FOR THE G(N,M) CORRECTIONS	553
	K=(1*(NOR+NOR+5-1))/2-1	554
	KP=1+NO	555
	KP=(KP*(NOR+NOR+5-KP))/2-1	556
	KPP=1+NONOT	557
	KPP=(KPP*(NOR+NOR+5-KPP))/2-1	558
	IF (N-NMAXT) 91,91,93	559
91	D(K)=D(K)+D(KP)*TFACT	560
	IF (N-NMAXTT) 92,92,93	561
92	D(K)=D(K)+D(KPP)*TFACT*TFACT	562
	D(KP)=D(KP)+2.0*D(KPP)*TFACT	563
C	CORRECT G(N,M)	564
93	G(N,M)=G(N,M)+D(K)	565
C	RECORD NEW G(N,M), CORRECTION, AND THE	566
C	CORRESPONDING ITEM IN THE CHECK COLUMN	567
	WRITE (6,94) N,M,G(N,M),D(K),D(K+1)	568
94	FORMAT (3H G 213,4E20.8,F20.2)	569
	IF (M-1) 95,100,95	570
95	I=I+1	571
C	COMPUTE TIME ADJUSTMENT FOR THE H(N,M) CORRECTIONS	572
	K=(1*(NOR+NOR+5-1))/2-1	573
	KP=1+NO	574
	KP=(KP*(NOR+NOR+5-KP))/2-1	575
	KPP=1+NONOT	576
	KPP=(KPP*(NOR+NOR+5-KPP))/2-1	577
	IF (N-NMAXT) 96,96,98	578
96	D(K)=D(K)+D(KP)*TFACT	579
	IF (N-NMAXTT) 97,97,98	580
97	D(K)=D(K)+D(KPP)*TFACT*TFACT	581
	D(KP)=D(KP)+2.0*D(KPP)*TFACT	582
C	CORRECT H(N,M)	583
98	H(N,M)=H(N,M)+D(K)	584
C	RECORD NEW H(N,M), CORRECTION, AND THE	585
C	CORRESPONDING ITEM IN THE CHECK COLUMN	586
	WRITE (6,99) N,M,H(N,M),D(K),D(K+1)	587
99	FORMAT (3H H 213,4E20.8,F20.2)	588
100	I=I+1	589
	IF (NMAXT) 101,111,101	590
101	DO 105 N=2,NMAXT	591
	DO 105 M=1,N	592
C	CORRECT GT(N,M)	593
	K=(1*(NOR+NOR+5-1))/2-1	594
	GT(N,M)=GT(N,M)+D(K)	595
C	RECORD NEW GT(N,M), CORRECTION, AND THE	596
C	CORRESPONDING ITEM IN THE CHECK COLUMN	597
	WRITE (6,102) N,M,GT(N,M),D(K),D(K+1)	598
102	FORMAT (3H GT213,4E20.8,F20.2)	599
	IF (M-1) 103,105,103	600

103	I=I+1	601
C	CORRECT HT(N,M)	602
	K=(I*(NOR+NOR+5-1))/2-1	603
	HT(N,M)=HT(N,M)+D(K)	604
C	RECORD NEW HT(N,M), CORRECTION, AND THE	605
C	CORRESPONDING ITEM IN THE CHECK COLUMN	606
	WRITE (6,104) N,M,HT(N,M),D(K),D(K+1)	607
104	FORMAT (3H HT213,4E20.8,F20.2)	608
105	I=I+1	609
	IF (NMAXTT) 106,111,106	610
106	DO 110 N=2,NMAXTT	611
	DO 110 M=1,N	612
C	CORRECT GTT (N,M)	613
	K=(I*(NOR+NOR+5-1))/2-1	614
	GTT(N,M)=GTT(N,M)+D(K)	615
C	RECORD NEW GTT (N,M), CORRECTION, AND THE	616
C	CORRESPONDING ITEM IN THE CHECK COLUMN	617
	WRITE (6,107) N,M,GTT(N,M),D(K),D(K+1)	618
107	FORMAT (4H GTT,12,13,3E20.8)	619
	IF (M-1) 108,110,108	620
108	I=I+1	621
C	CORRECT HTT(N,M)	622
	K=(I*(NOR+NOR+5-1))/2-1	623
	HTT(N,M)=HTT(N,M)+D(K)	624
C	RECORD NEW HTT(N,M), CORRECTION, AND THE	625
C	CORRESPONDING ITEM IN THE CHECK COLUMN	626
	WRITE (6,109) N,M,HTT(N,M),D(K),D(K+1)	627
109	FORMAT (4H HTT,12,13,3E20.8)	628
110	I=I+1	629
C	RECORD NEW E1,E2,E3 AND THE	630
C	CORRESPONDING ITEM IN THE CHECK COLUMN	631
111	IF (EXTFLD) 112,116,112	632
112	K=(I*(NOR+NOR+5-1))/2-1	633
	E1=E1+D(K)	634
	WRITE (6,113) E1,D(K),D(K+1)	635
113	FORMAT (3H E1,6X,3E20.8)	636
	I=I+1	637
	K=(I*(NOR+NOR+5-1))/2-1	638
	E2=E2+D(K)	639
	WRITE (6,114) E2,D(K),D(K+1)	640
114	FORMAT (3H E2,6X,3E20.8)	641
	I=I+1	642
	K=(I*(NOR+NOR+5-1))/2-1	643
	E3=E3+D(K)	644
	WRITE (6,115) E3,D(K),D(K+1)	645
115	FORMAT (3H E3,6X,3E20.8)	646
	I=I+1	647
116	AVETIM=SUMTM/FNO1(8)+60.0	648
C	RECORD ENTIRE ARRAY OF G AND H PARAMETERS	649
	WRITE (6,117) ITNO	650

117	FORMAT (20H1OUTPUT COEFFICIENTS,39X,5HITNO=,14/1H0)	651
	WRITE (6,11) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)	652
	\$,M=1,N),N=2,NMAX)	653
	WRITE (6,15) E1,E2,E3	654
	WRITE (6,118) AVETIM	655
118	FORMAT (10H0AVETIM=,F10.2)	656
C	PUNCH CARDS FOR STARTING NEXT APPROXIMATION	657
	PUNCH 119,XID1,XID2,NMAX,NSKIP,SIG1(8)	658
	PUNCH 3,NMAX,NMAXT,NMAXTT,NSKIP,ITER	659
	PUNCH 4,ERRLIM,AVETIM	660
	PUNCH 5,EXTFLD	661
	PUNCH 11,((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M),M=1	662
	,\$,N),N=2,NMAX)	663
	PUNCH 14	664
	PUNCH 14,E1,E2,E3	665
119	FORMAT (2A6,6H NMAX ,11,7H NSKIP ,13,5HSIG ,F6.0)	666
120	CONTINUE	667
C	RECORD ERROR DISTRIBUTIONS	668
	WRITE (6,121) TYPE(1J)	669
121	FORMAT (23H1ERROR DISTRIBUTION FOR,3X,A2)	670
	DO 124 JK=1,200,10	671
	JL=JK+9	672
	IF (JK-101) 122,123,123	673
122	JM=JK-101	674
	GO TO 124	675
123	JM=JK-100	676
124	WRITE (6,125) JM,(1ERR(1K),1K=JK,JL)	677
125	FORMAT (15,3X1016)	678
C	RECORD MEAN DEVIATION FOR LAT-LONG BLOCKS	679
	WRITE (6,126) (L,L=10,90,10)	680
126	FORMAT (38H1MEAN DEVIATION FOR LAT-LONG BLOCK /1H0,58X,916)	681
	DO 128 K=1,36	682
	DO 127 J=1,18	683
127	JERR(J)=(ERR(J,K)/FNO2(J,K))	684
128	WRITE (6,129) K,(JERR(M),M=1,18)	685
129	FORMAT (1X12,3X,1816)	686
	CALL MATRIX	687
	RETURN	688
	END	689

	SUBROUTINE MATRIX	1
	COMMON /DD/D(3400)	2
	COMMON /COEFS/G(9,9),H(9,9),GT(9,9),HT(9,9),GTT(9,9),HTT(9,9),MAXD	3
	DIMENSION FONE(150),DIAG(150)	4
	DIMENSION ROW(150),SROW(150)	5
	REWIND 2	6
	REWIND 1	7
	READ (2) NMAX,NMAXT,NMAXTT,FWNP,FNP,SIGMA	8
	READ (2) (D(I),I=1,MAXD)	9
	NOR=NMAX*NMAX-1	10
	IF (NMAXT) 1,2,1	11
1	NOR=NOR+NMAXT*NMAXT-1	12
2	IF (NMAXTT) 3,4,3	13
3	NOR=NOR+NMAXTT*NMAXTT-1	14
4	NOP=NOR+1	15
	NOPP=NOR+2	16
	DO 6 I=1,NOR	17
	DO 5 J=1,NOR	18
	II=MINO(I,J)	19
	JJ=I+J-II	20
	K=((NOR+NOR+5-II)*II)/2+JJ-NOR-2	21
5	ROW(J)=D(K)	22
6	WRITE (1) (ROW(J),J=1,NOP)	23
	REWIND 2	24
	REWIND 1	25
	DO 27 K=1,NOR	26
	IF (MOD(K,2)) 7,8,7	27
7	READ (1) (SROW(L),L=1,NOP)	28
	GO TO 9	29
8	READ (2) (SROW(L),L=1,NOP)	30
9	IF (K-1) 12,10,12	31
10	SROW(NOP)=0.0	32
	DO 11 II=1,NOR	33
11	SROW(NOP)=SROW(NOP)+SROW(II)	34
12	RDKK=1.0/SROW(K)	35
	SROW(K)=1.0	36
	DO 13 J=1,NOP	37
13	SROW(J)=SROW(J)*RDKK	38
	DO 23 I=2,NOR	39
	IF (MOD(K,2)) 14,15,14	40
14	READ (1) (ROW(L),L=1,NOP)	41
	GO TO 16	42
15	READ (2) (ROW(L),L=1,NOP)	43
16	IF (K-1) 19,17,19	44
17	ROW(NOP)=0.0	45
	DO 18 II=1,NOR	46
18	ROW(NOP)=ROW(NOP)+ROW(II)	47
19	T=ROW(K)	48
	ROW(K)=0.0	49
	DO 20 J=1,NOP	50

20	ROW(J)=ROW(J)-T*SROW(J)	51
	IF (MOD(K,2)) 21,22,21	52
21	WRITE (2) (ROW(L),L=1,NOP)	53
	GO TO 23	54
22	WRITE (1) (ROW(L),L=1,NOP)	55
23	CONTINUE	56
	IF (MOD(K,2)) 24,25,24	57
24	WRITE (2) (SROW(L),L=1,NOP)	58
	GO TO 26	59
25	WRITE (1) (SROW(L),L=1,NOP)	60
26	REWIND 2	61
27	REWIND 1	62
	DO 31 I=1,NOR	63
	IF (MOD(NOR,2)) 28,29,28	64
28	READ (2) (ROW(L),L=1,NOP)	65
	GO TO 30	66
29	READ (1) (ROW(L),L=1,NOP)	67
	WRITE (2) (ROW(L),L=1,NOP)	68
30	DIAG(I)=ROW(I)	69
31	FONE(I)=ROW(NOP)	70
	WRITE (6,32) SIGMA,FWNP,FNP	71
32	FORMAT (19H1STATISTICS FOR FIT/1X,5HSIGMA,F5.0,3X15HWEIGHTED POINT	72
	\$S,F6.1,3X,6HPPOINTS,F6.0)	73
	WRITE (6,33)	74
33	FORMAT (5X1HN2X1HM8X1HP20X4HSIGP16X3H1.019X2HTC)	75
	I=0	76
	DO 37 N=2,NMAX	77
	DO 37 M=1,N	78
	I=I+1	79
	SIGP=SQRT(ABS(DIAG(I)))*SIGMA	80
	TC=ABS(G(N,M)/SIGP)	81
34	WRITE (6,34) N,M,G(N,M),SIGP,FONE(I),TC	82
	FORMAT (4H G 213,2E20.8,2F20.2)	83
	IF (M-1) 35,37,35	84
35	I=I+1	85
	SIGP=SQRT(ABS(DIAG(I)))*SIGMA	86
	TC=ABS(H(N,M)/SIGP)	87
36	WRITE (6,36) N,M,H(N,M),SIGP,FONE(I),TC	88
37	FORMAT (4H H 213,2E20.8,2F20.2)	89
	CONTINUE	90
	IF (NMAXT) 38,43,38	91
38	DO 42 N=2,NMAXT	92
	DO 42 M=1,N	93
	I=I+1	94
	SIGP=SQRT(ABS(DIAG(I)))*SIGMA	95
	TC=ABS(GT(N,M)/SIGP)	96
39	WRITE (6,39) N,M,GT(N,M),SIGP,FONE(I),TC	97
	FORMAT (4H GT 213,2E20.8,2F20.2)	98
	IF (M-1) 40,42,40	99
40	I=I+1	100

	SIGP=SQRT(ABS(DIAG(I)))*SIGMA	101
	TC=ABS(HT(N,M)/SIGP)	102
41	WRITE (6,41) N,M,HT(N,M),SIGP,FONE(I),TC	103
42	FORMAT (4H HT 2I3,2E20.8,2F20.2)	104
43	CONTINUE	105
44	IF (NMAXTT) 44,49,44	106
	DO 48 N=2,NMAXTT	107
	DO 48 M=1,N	108
	I=I+1	109
	SIGP=SQRT(ABS(DIAG(I)))*SIGMA	110
	TC=ABS(GTT(N,M)/SIGP)	111
45	WRITE (6,45) N,M,GTT(N,M),SIGP,FONE(I),TC	112
	FORMAT (4H GTT2I3,2E20.8,2F20.2)	113
46	IF (M-1) 46,48,46	114
	I=I+1	115
	SIGP=SQRT(ABS(DIAG(I)))*SIGMA	116
	TC=ABS(HTT(N,M)/SIGP)	117
47	WRITE (6,47) N,M,HTT(N,M),SIGP,FONE(I),TC	118
48	FORMAT (4H HTT2I3,2E20.8,2F20.2)	119
49	CONTINUE	120
	I=-1	121
	DO 53 N=2,NMAX	122
	DO 53 M=1,N	123
	I=I+2	124
	R=SQRT(G(N,M)**2+H(N,M)**2)	125
	SIGP=SQRT(G(N,M)**2*ABS(DIAG(I))+H(N,M)**2*ABS(DIAG(I+1)))/R*SIGMA	126
50	IF (M-1) 51,50,51	127
51	I=I-1	128
	TC=R/SIGP	129
52	WRITE (6,52) N,M,R,SIGP,TC	130
53	FORMAT (4H R 2I3,2E20.8,20X,F20.2)	131
	CONTINUE	132
54	IF (NMAXT) 54,59,54	133
	DO 58 N=2,NMAXT	134
	DO 58 M=1,N	135
	I=I+2	136
	R=SQRT(GT(N,M)**2+HT(N,M)**2)	137
	SIGP=SQRT(GT(N,M)**2*ABS(DIAG(I))+HT(N,M)**2*ABS(DIAG(I+1)))/R*SIG	138
	\$MA	139
55	IF (M-1) 56,55,56	140
56	I=I-1	141
	TC=R/SIGP	142
	WRITE (6,57) N,M,R,SIGP,TC	143
	PRINT 57,N,M,R,SIGP,TC	144
57	FORMAT (4H RT 2I3,2E20.8,20X,F20.2)	145
58	CONTINUE	146
59	IF (NMAXTT) 60,65,60	147
60	DO 64 N=2,NMAXTT	148
	DO 64 M=1,N	149
	I=I+2	150

	R=SQRT(GTT(N,M)**2+HTT(N,M)**2)	151
	SIGP=SQRT(GTT(N,M)**2*ABS(DIAG(1))+HTT(N,M)**2*ABS(DIAG(1+1)))/R*S	152
	SIGMA	153
	IF (M-1) 62,61,62	154
61	I=I-1	155
62	TC=R/SIGP	156
	WRITE (6,63) N,M,R,SIGP,TC	157
	PRINT 63,N,M,R,SIGP,TC	158
63	FORMAT (4H RTT213,2E20.8,20X,F20.2)	159
64	CONTINUE	160
65	IF (FNP-100.) 66,66,67	161
66	TCT95=0.0	162
	TCT50=0.0	163
	GO TO 68	164
67	TCT95=1.96	165
	TCT50=.674	166
68	WRITE (6,69) TCT95,TCT50	167
69	FORMAT (29H TC ABOVE SHOULD BE GREATER F10.3,26H FOR 95 PERCENT C	168
	ONFIDENCE/29X,F10.3,27H FOR 50 PERCENT CONFIDENCE)	169
	REWIND 1	170
	REWIND 2	171
	WRITE (1) NMAX,NMAXT,NMAXTT,FWNP,FNP,SIGMA,NOR,NOP,NOPP	172
	WRITE (1) ((G(N,M),H(N,M),M=1,N),N=2,NMAX)	173
	WRITE (1) ((GT(N,M),HT(N,M),M=1,N),N=2,NMAX)	174
	WRITE (1) ((GTT(N,M),HTT(N,M),M=1,N),N=2,NMAX)	175
	DO 70 I=1,NOR	176
	READ (2) (ROW(L),L=1,NOP)	177
	WRITE (1) (ROW(L),L=1,NOP)	178
70	CONTINUE	179
	END FILE 1	180
	REWIND 1	181
	REWIND 2	182
	RETURN	183
	END	184

APPENDIX B

JENSEN'S FIT

Introduction

The purpose of this appendix is to document the sequence of operations and to discuss various programming aspects of Jensen's Fit program. This program has been written to find time-dependent coefficients for a spherical-harmonic expansion of the geomagnetic potential function.

The mathematical formulas which form the basis of the computer program are not restated in this appendix. Each time that a formula is required to explain a Fortran variable, a reference is made to an equation in Sections 2.0 or 3.0 of this report or to one of the reports listed in the bibliography. When referencing this report, it should be noted that the Fortran variable N is equal to $n+1$. Similarly, $M = m+1$.

The computer program is relatively linear, i. e. , there are few alternate calculation sequences, as can be seen from the flow-charts in Appendix C. Hence, the calculation sequence will be described in a linear manner.

The program may be roughly divided into five phases as follows:
(1) initializing; (2) data processing for the coefficients in the least squares equations; (3) solution of the least squares equations; (4) estimation of the corrections for the coefficients of the spherical-harmonic expansion of the geomagnetic potential function; and (5) recording. This appendix will

likewise be divided into five principal sections to describe respectively these five phases. Within each section, the Fortran name for variables will be used whenever possible. A glossary identifying these variables is included at the end of this appendix.

Initialization

As with all computer programs, initialization consists of doing the things that must be done once at the beginning of the execution of the program. (Similarly, parts of a program, i. e., subprograms, may require initialization. While such initialization may subsequently be discussed, it is not the subject of this section of the appendix.) Initialization for this program includes setting or computing the value of certain constants that will be used throughout the other phases of the program. Among these are FLAT, A2, A4, B2, A2B2, A4B4, CONST(N,M), SHMIDT(N,M), P(1,1), DP(1,1), SP(1) and CP(1) all of which are identified in the glossary.

The equation for computing CONST(N,M) is found in Eq. (19) of Section 2.0. Note that $N = n+1$ and $M = m+1$. The equations for computing SHMIDT(N,M) are found in Eqs. (20) of Section 2.0. Again note that $N = n+1$ and $M = m+1$.

The value of the first associated Legendre polynomial, P(1,1), and its derivative, DP(1,1), are constants and may be found in Eqs. (19) of Section 2.0. Similarly the value of $\sin(M-1)\phi$ and $\cos(M-1)\phi$, i. e., SP(M) and CP(M) respectively, are constants for $M = 1$, i. e., $SP(1) = 0.0$ and $CP(1) = 1.0$.

Another common function of initialization is the clearing of tables. This program requires that the tables FNO2(I, J) and ERR(I, J) be cleared, i. e., all entries be made equal to zero. A third function of initialization is the input of variable control and starting data. Among the variables that must be input for this program are XID1, XID2, NMAX, NMAXT, NMAXTT, NSKIP, ITER, ERRLIM, AVETIM, EXTFLD, G(N, M), H(N, M), GT(N, M), HT(N, M), GTT(N, M), HTT(N, M), E1, E2, and E3. These variables are all identified in the glossary.

A final function of initialization is often the recording of initial values of pertinent variables. This program records NMAX, NMAXT, NMAXTT, NSKIP, ITER, ERRLIM, XID1, XID2, G(N, M), H(N, M), GT(N, M), HT(N, M), GTT(N, M), HTT(N, M), E1, E2, and E3 as a permanent record of the starting data used by the program. These variables are all identified in the glossary.

Except for setting or computing the values of certain other variables required by the initialization functions enumerated above, this completes the initialization phase of the program.

Data Processing

In Section 2.0 of this report, it was noted that the procedure must be repeated until the corrections estimated for the coefficients of the spherical-harmonic expansion of the potential function are no longer significant. The first Fortran statement in the data processing phase (card 98) is the DO

statement that controls the number of repeats, or iterations, that will be made for any particular run of the computer program.

Certain tables, SIG1(J), FNO1(J), SWT(J), IERR(I), and D(I) must be cleared, i. e. , all entries made equal to zero, at the beginning of each iteration. In addition to these tables, the value of LINE and SUMTM must be set equal to zero and ISKIP must be set equal to NSKIP which was input during the initialization phase. With the setting of ISKIP, the initialization of the data processing phase of the program is completed.

Beginning with the Fortran statement, CALL RDATA (card 115), the remainder of the data processing phase of the program is repeated for each observation that is to be used in the calculation. An observation input by the subroutine RDATA may consist of any combination of the following field measurements: DECL, DIP, HOR, B, Z, X, and Y. These are all identified in the glossary. In addition to these measurements, the location of the observation in time and space is recorded. This location is specified by the Fortran variables FLATT, ELONG, ALT, and TIME. All location data as well as measurement data are transmitted to the main program via the common field DATAR. RDATA signals the end of the data by setting the value of ISKIP to zero. To recognize this signal, the main program, following each CALL RDATA, examines ISKIP and terminates the data processing phase of the program upon sensing this signal.

Beginning with card 126, geocentric coordinates are computed from the geodetic measurements made for each observation. The equation for computing THETA is Eq. (30) of Section 2.0. The equation for computing geocentric R is Eq. (31) of Section 2.0.

The five variables SIND, COSD, AOR, CT, and ST which are all identified in the glossary are computed next. SIND and COSD are required for converting from geocentric to geodetic coordinates. CT and ST are required for the generation of the associated Legendre polynomials and AOR is a term that appears in the equations for estimating X, Y, Z, etc. from the best available set of parameters. One should note that the Fortran statement for computing CT and ST redefines θ to be measured from the polar axis instead of from the equatorial plane, i. e., colatitude.

The variables LON and LAT are computed next. These constants are required later for weight and error tabulations. Next, SP(2) and CP(2) are computed, followed by the computation of SP(M) and CP(M) for $M > 2$. The equations

$$\sin(M-1)\phi = \sin\phi \cdot \cos(M-2)\phi + \cos\phi \cdot \sin(M-2) \quad (B1)$$

and

$$\cos(M-1)\phi = \cos\phi \cdot \cos(M-2)\phi - \sin\phi \cdot \sin(M-2) \quad (B2)$$

used for computing SP(M) and CP(M) are available in standard texts on trigonometry under the subject, "functions of sums of angles."

Next the program evaluates the necessary associated Legendre polynomials, $P(N, M)$, and their derivatives, $DP(N, M)$, employing the

recurrence relationships which are found in Section 2.0, Eqs. (19). $K_{N,M}$ corresponds to the Fortran variable CONST(N,M) and was discussed above in the section on initialization.

Beginning with card 167, the Fortran variables CX, CY, and CZ are set equal to zero in preparation for the estimation of X, Y, and Z. AR and TM, two Fortran variables identified in the glossary, must also be initialized in preparation for the estimation of X, Y, and Z.

The Fortran statements through card 198 are required to estimate X, Y, and Z, i. e., the Fortran variables CX, CY, and CZ. The rotation formulas required for computing X and Z are given in Section 2.0, Eqs. (32) and (33). Equations for B_θ , B_r , and B_ϕ are given in several different forms in Section 2.0 of this report.

Near the beginning of the group of Fortran statements required to estimate X, Y, and Z (specifically cards 181 and 182), the Gauss normalized polynomials are Schmidt normalized and multiplied by the appropriate power of $\frac{6371.2}{r}$, i. e., AR.

Depending on when it is calculated, the Fortran variable TEMP is the common factor in the coefficients of the two parameters $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, or Z. DXDG, DXDH, DYDG, DYDH, DZDG and DZDH complete the calculation of the coefficients of $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, and Z. Next, the Fortran variables GNM and HNM are computed using Eqs. (21) of Section

2.0 and finally all terms are summed for the respective estimates of X, Y, and Z.

The X, Y, and Z components of the external field are estimated next and added to the respective estimates of X, Y, and Z (cards 200 through 205). This step is completely skipped if EXTFLD is zero.

From the estimates of X, Y, and Z, estimates of the horizontal field, total field, dip, and declination (Fortran variables CH, CF, CI, and CD respectively) are made using Eqs. (34) and (37-39) of Section 3.0.

The program now calculates the coefficients of the unknowns in the system of simultaneous least squares equations. Beginning with card 214, the first non-zero measurement is processed and then its value set equal to zero. The program returns to statement number 28 and since the value of the previous measurement was set equal to zero, processes the second non-zero measurement. This continues until all measurements have been processed and the value of the respective Fortran variables have all been set equal to zero.

Formulas for the coefficients of the unknowns in the system of simultaneous least squares equations may be derived easily from simple theorems in differential calculus and the various equations of Section 3.0. The calculus theorem results in the following:

$$\text{If} \quad x = f(u) \quad \text{and} \quad u = g(w)$$

$$\text{then} \quad \frac{dx}{dw} = \frac{df}{du} \cdot \frac{dg}{dw} \quad . \quad (B3)$$

Applying this theorem to Eq. (34) of Section 3.0, the following formulas can be derived for the declination of the total field strength:

$$\frac{d \text{DECL}}{d g_{N,M,0}} = \frac{d \text{DECL}}{d X} \cdot \frac{d X}{d g_{N,M,0}} + \frac{d \text{DECL}}{d Y} \cdot \frac{d Y}{d g_{N,M,0}} \quad (\text{B4})$$

$$\frac{d \text{DECL}}{d g_{N,M,0}} = - \frac{Y}{\text{HOR}} \frac{d X}{d g_{N,M,0}} + \frac{X}{\text{HOR}} \frac{d Y}{d g_{N,M,0}} \quad (\text{B5})$$

$$\frac{d \text{DECL}}{d h_{N,M,0}} = \frac{d \text{DECL}}{d X} \cdot \frac{d X}{d h_{N,M,0}} + \frac{d \text{DECL}}{d Y} \cdot \frac{d Y}{d h_{N,M,0}} \quad (\text{B6})$$

$$\frac{d \text{DECL}}{d h_{N,M,0}} = - \frac{Y}{\text{HOR}} \cdot \frac{d X}{d h_{N,M,0}} + \frac{X}{\text{HOR}} \frac{d Y}{d h_{N,M,0}} \quad (\text{B7})$$

Similarly, from Eqs. (37-39) of Section 3.0, the following can be derived for field dip:

$$\frac{d \text{DIP}}{d g_{N,M,0}} = - \frac{X \cdot Z}{H \cdot F} \frac{d X}{d g_{N,M,0}} - \frac{Y \cdot Z}{H \cdot F} \frac{d Y}{d g_{N,M,0}} + \frac{H}{F} \frac{d Z}{d g_{N,M,0}} \quad (\text{B8})$$

$$\frac{d \text{DIP}}{d h_{N,M,0}} = - \frac{X \cdot Z}{H \cdot F} \frac{d X}{d h_{N,M,0}} - \frac{Y \cdot Z}{H \cdot F} \frac{d Y}{d h_{N,M,0}} + \frac{H}{F} \frac{d Z}{d h_{N,M,0}} \quad (\text{B9})$$

for the horizontal component:

$$\frac{d \text{HOR}}{d g_{N,M,0}} = \frac{X}{\text{HOR}} \frac{d X}{d g_{N,M,0}} + \frac{Y}{\text{HOR}} \frac{d Y}{d g_{N,M,0}} \quad (\text{B10})$$

$$\frac{d \text{HOR}}{d h_{N,M,0}} = \frac{X}{\text{HOR}} \frac{d X}{d h_{N,M,0}} + \frac{Y}{\text{HOR}} \frac{d Y}{d h_{N,M,0}} \quad (\text{B11})$$

and for total field:

$$\frac{dB}{dg_{N,M,0}} = \frac{X}{B} \frac{dX}{dg_{N,M,0}} + \frac{Y}{B} \frac{dY}{dg_{N,M,0}} + \frac{Z}{B} \frac{dZ}{dg_{N,M,0}} \quad (B12)$$

$$\frac{dB}{dh_{N,M,0}} = \frac{X}{B} \frac{dX}{dh_{N,M,0}} + \frac{Y}{B} \frac{dY}{dh_{N,M,0}} + \frac{Z}{B} \frac{dZ}{dh_{N,M,0}} \quad (B13)$$

Similar expressions for derivatives with respect to $g_{N,M,t}$, $h_{N,M,t}$, $g_{N,M,tt}$, and $h_{N,M,tt}$ can be derived from the equations cited.

Beginning at card 227, the program employs Eqs. (B4-B7) to compute the coefficients of the $g_{N,M,0}$ and $h_{N,M,0}$ when declination is observed. The observation term (Fortran variable FI) is then computed followed by the weight assigned to the observation. Finally, the value of DECL is set equal to zero so that when program control is returned to statement number 28, the next data type will be processed.

In a similar manner the program processes field dip beginning with card 254, the horizontal field strength beginning with card 276, the total field strength beginning with card 298, the Z component beginning with card 316, the X component beginning with card 329, and the Y component beginning with card 342.

As each observation is processed, the program, beginning at card 354, adds the time terms. Then beginning at card 369, the squared time terms are added.

If external field terms are to be used and corrected, the program, beginning with card 385, computes the coefficients of the unknowns E_1 , E_2 ,

and E_3 in the system of simultaneous least squares equations. The equations employed are similar to equations (B4) through (B13) with $\frac{d}{dg_{N,M,0}}$ or $\frac{d}{dh_{N,M,0}}$ being replaced with $\frac{d}{dE_1}$, $\frac{d}{dE_2}$, or $\frac{d}{dE_3}$. Beginning with card 385, $\frac{dX}{dE_1}$, $\frac{dX}{dE_2}$, $\frac{dX}{dE_3}$, $\frac{dZ}{dE_1}$, $\frac{dZ}{dE_2}$, and $\frac{dZ}{dE_3}$ are computed. The quantities $\frac{dY}{dE_1} = 0$, $\frac{dY}{dE_2} = \sin \phi$, and $\frac{dY}{dE_3} = -\cos \phi$ are not set up explicitly. The program processes field declination beginning with card 394, field dip beginning with card 405, the horizontal field strength beginning with card 417, the total field strength beginning with card 428, the Z component beginning with card 438, the X component beginning with card 446, and finally the Y component beginning with card 454. Then at card 462, the observation term is added to the Fortran vector $F(I)$.

Beginning with card 463, the three Fortran constants NOR, NOP, and NOPP (identified in the glossary) required to specify the matrix size, etc. are computed.

Observations are recorded by cards 466 through 475. After the observation is recorded, various counts, weights, and errors (i. e., IERR, SIG1, FNO1, SWT1, and SUMTM which are all identified in the glossary) are computed and summed. On the last iteration, the Fortran variables FNO2 and ERR are summed.

Card 498 calls subroutine DLOOP. This subroutine computes the sums of squares and cross-products required for the coefficients of the unknown parameters in the system of simultaneous least squares equations.

A description of D as well as a description of the intricate manipulations required of D for the solution of the set of least squares equations is given in some detail in Section 3.0 of this report.

Solution of the Least Squares Equations

This phase of the program accomplishes four functions. First, calculations for standard errors are completed and a record made of the results. Second, the D array is recorded for the MATRIX subroutine calculations. Third, the computation check column is computed and finally, the least squares equations are solved.

Details of the D matrix storage and manipulation are presented in Section 3.0 of this report and will not be repeated here. However, note that cards 506 through 516 compute the computation check column.

From the procedure described in Section 3.0, one can see that as each row is considered at step (1), the corresponding complete column is required for step (2). This column is contained in the triangle matrix. As a column is needed, it is transferred to the vector SIDE by the following Fortran statements:

```
DO 83 I=1, NOR
  NROW=MIN(I, J)
  NCOL=I+L-NROW
  K=(NROW*(NOR+NOR+5-NROW))/2+NCOL-NOR-2
  SIDE(I)=D(K)
83  CONTINUE
```

where NOR = number of rows in the complete matrix
NROW = row number
NCOL = column number
MIN = function subprogram to choose minimum of the arguments
D = the triangle matrix stored.

(These variables are identified in the glossary.) The cards beginning with card 520 and ending with card 525 transfer the required column to the vector SIDE. Then beginning with card 526 and ending with card 531, step (1) above is accomplished. Step (2) follows ending with card 540. This completes the third phase of the program.

Corrections and Output

The last two phases of the program, viz., the estimation of the corrections for the parameters and the recording of all results, are intermingled so that while the functions are distinct, the Fortran statements are not. As is customary with Fortran programs, final results are not stored but are written as soon as available.

The output begins with a record of the Fortran variables TYPE (IJ), SIG1 (IJ), and FNO1 (IJ) which are all identified in the glossary.

The solution of the set of simultaneous least squares equations yields adjustments or corrections for the parameters based on the average observation time (AVETIM). The G, H, GT, HT, GTT, HTT input during the initialization phase are based on the year 1960. Hence, the corrections must be computed for 1960 instead of the average observation time. This time

adjustment in the corrections begins with card 549. Card 560 computes and adds the time adjustment while card 562 adds the squared time adjustment to the $g_{N,M,0}$ correction. Card 563 computes and adds the time adjustment to the $g_{N,M,t}$ correction. Card 565 adds the correction to the $g_{N,M,0}$ and the next two Fortran statements record the new $g_{N,M,0}$, the total correction, and the computer check column. In a similar manner, the cards from 573 through 589 compute, apply, and record the same information for the $h_{N,M,0}$ and $h_{N,M,t}$.

If time terms were used and are to be corrected, the cards beginning with 590 and going through 609 apply and record the corrections for $g_{N,M,t}$ and $h_{N,M,t}$. Finally, if squared time terms were used and are to be corrected, the cards beginning with 610 and going through 629 apply and record the corrections for $g_{N,M,tt}$ and $h_{N,M,tt}$.

If external field terms were used, and are to be corrected, the program beginning with card 632 and continuing through card 647 records the new values for E_1 , E_2 , and E_3 and the corrections applied.

Final Output

The final output consists of a permanent record of the results of the calculation plus the inputs required for the next updating of the parameters in the spherical-harmonic expansion of the geomagnetic potential function. First, a printed record is made of all the G, H, GT, HT, GTT, and HTT and the E_1 , E_2 , and E_3 . Next, a punched record is made of XID1, XID2,

NMAX, NSKIP, SIG1(8), TYPE(I), NMAX, NMAXT, NMAXTT, NSKIP, ITER, ERR LIM, AVETIM, all of the G, H, GT, HT, GTT and HTT, and the E1, E2, and E3. All of these Fortran variables are identified in the glossary.

With the punching of the new starting data, one iteration has been completed. The program now transfers control to the beginning of the data processing phase for the next iteration.

When all iterations have been completed, a printed record is made of the Fortran variables IERR(IK) and $ERR(J,K)/FNO2(J,K)$. $ERR(J,K)/FNO2(J,K)$ is the mean deviation for latitude-longitude blocks. This concludes the program.

GLOSSARY

Jensen's Fit

A	a, the mean equatorial radius of the earth in kilometers
A2	a^2
A4	a^4
A2B2	$(\text{mean equatorial radius of the earth})^2 - (\text{polar radius of the earth})^2$
A4B4	$(\text{mean equatorial radius of the earth})^4 - (\text{polar radius of the earth})^4$
ALT	altitude of the observation
AOR	radius of the sphere having volume equal to the earth's volume/R
AR	$(\text{radius of the sphere having volume equal to the earth's volume/R})^{N+1}$
AVETIM	average time for all observations
B	total observed field strength
B2	$(\text{polar radius of the earth})^2$
BWT	standard error of total field strength
CD	estimated declination in radians
CDDEG	estimated declination in degrees
CF	estimated total field strength
CH	estimated horizontal field strength
CI	estimated field dip in radians
CIDEG	estimated field dip in degrees
CONST	a set of constants required for the generation of the associated Legendre polynomials, P_n^m

$$\frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} = \frac{(N-2)^2 - (M-1)^2}{(2N-3)(2N-5)}$$

n and m are common formula notation while N and M are used in the computer program

COSD	cosine of difference between geodetic coordinate λ and geocentric coordinate θ
CP(M)	cosine of the product of (M-1) and the longitudinal coordinate ϕ
CT	cosine of $\pi/2$ minus the geocentric coordinate θ , i. e., coaltitude
CX	estimated X component of the field strength
CY	estimated Y component of the field strength
CZ	estimated Z component of the field strength
D	triangular matrix of sums of squares and cross-products of the coefficient of $g_{N,M,0}$, $h_{N,M,0}$, $g_{N,M,t}$, $h_{N,M,t}$, $g_{N,M,tt}$, and $h_{N,M,tt}$.
DECL	angle of declination D
DECLWT	standard error of the angle of declination
DEN	$(a^2 \cos^2 \lambda + b^2 \sin^2 \lambda)^{1/2}$ where a = mean equatorial radius of the earth b = polar radius of the earth λ = geodetic coordinate of latitude
DEN2	$(DEN)^2$
DIP	angle of dip, I
DIPWT	standard error of the angle of dip
DP	derivative of an associated Legendre polynomial
DXDE1	$\frac{dX}{dE_1}$ where X is the X component of the field strength

DXDE2	$\frac{dX}{dE_2}$	where X is the X component of the field strength
DXDE3	$\frac{dX}{dE_3}$	where X is the X component of the field strength
DXDG	$\frac{dX}{dg}$	where X is the X component of the field strength
DXDH	$\frac{dX}{dh}$	where X is the X component of the field strength
DYDG	$\frac{dY}{dg}$	where Y is the Y component of the field strength
DYDH	$\frac{dY}{dh}$	where Y is the Y component of the field strength
DZDE1	$\frac{dZ}{dE_1}$	where Z is the Z component of the field strength
DZDE2	$\frac{dZ}{dE_2}$	where Z is the Z component of the field strength
DZDE3	$\frac{dZ}{dE_3}$	where Z is the Z component of the field strength
DZDG	$\frac{dZ}{dg}$	where Z is the Z component of the field strength
DZDH	$\frac{dZ}{dh}$	where Z is the Z component of the field strength
E1	external field term along the polar axis	
E2	external field term in the equatorial and prime meridian planes	
E3	external field term in the equatorial plane but perpendicular to the plane of the prime meridian	
DLONG	longitudinal coordinate ϕ in degrees	
ERR	observations times weights summed for global grid-points	

ERRLIM	error limit for detecting measurement errors
EXTFLD	a code to identify when external field terms are to be used (EXTFLD $\neq$ 0) and when they are not to be used (EXTFLD = 0)
F	one of the numbers used to form the sums of squares and cross-products of the triangular matrix
FAC	$\tan \theta / \tan \lambda$ where θ and λ are the geocentric and geodetic latitudinal coordinates
FACT	a factor used in generating the factors for converting from Gauss normalization to Schmidt normalization. When $M=2$, FACT=2.0; when $M > 2$, FACT=1.0. Also used in forming squares and cross products. Hence a temporary storage.
FI	the "Y" or observation term used in forming the least squares matrix
FLAT	polar radius of the earth / mean equatorial radius of the earth
FLATR	λ in radians
FLATT	latitudinal coordinate λ
FM	the index M or $M-1$ in floating point notation
FN	the index N in floating point notation
FNO1	a set of eight storages to contain counts of the seven types of data and a count of the total number of data
FNO2	a set of (18, 36) storages to contain the sum of the data weights at points regularly spaced over the surface of the earth at 10° intervals
FWT	standard error of the field strength
G	the coefficient $g_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetic potential function
GNM	$G(N, M)$, i. e., a specific G

GT	the coefficient $g_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetic potential function
GTNM	GT(N, M), i. e. , a specific GT
GTT	the coefficient $g_{N,M,tt}$ in the spherical harmonic-expansion of the geomagnetic potential function
GTTNM	GTT(N, M), i. e. , a specific GTT
H	the coefficient $h_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetic potential function
HNM	H(N, M), i. e. , a specific H
HOR	observed horizontal component of field strength
HORWT	standard error of the observed horizontal component of field strength
HT	the coefficient $h_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetic potential function
HTNM	HT(N, M), i. e. , a specific HT
HTT	the coefficient $h_{N,M,tt}$ in the spherical-harmonic expansion of the geomagnetic potential function
HTTNM	HTT(N, M), i. e. , a specific HTT
I	an index
IERR	distribution of FI
IJ	an index
IK	an index
ISKIP	determines frequency of the data selected for a test calculation, e. g. , ISKIP = 3 selects every third observation, ISKIP = 10 selects every tenth, etc.

ITER	iteration limit, i. e. , maximum number of iterations to be performed	
ITNO	iteration counter	
ITYPE	identifies data type, e. g. ,	DECL = 1 DIP = 2 HOR = 3 B = 4 Z = 5 X = 6 Y = 7
J	an index	
JERR	average observation times weights for global grid-points	
JK	an index	
JL	an index	
JM	an index	
K	a computed subscript	
KJ	a subscript computed from J and L	
KK	an index	
KP	an index	
KPP	an index	
L	an index	
LINE	an index for counting the number of lines output for a page	
LON	a longitude code	
M	an index and subscript	
MAXD	maximum size of the triangular matrix C	
N	an index and subscript	

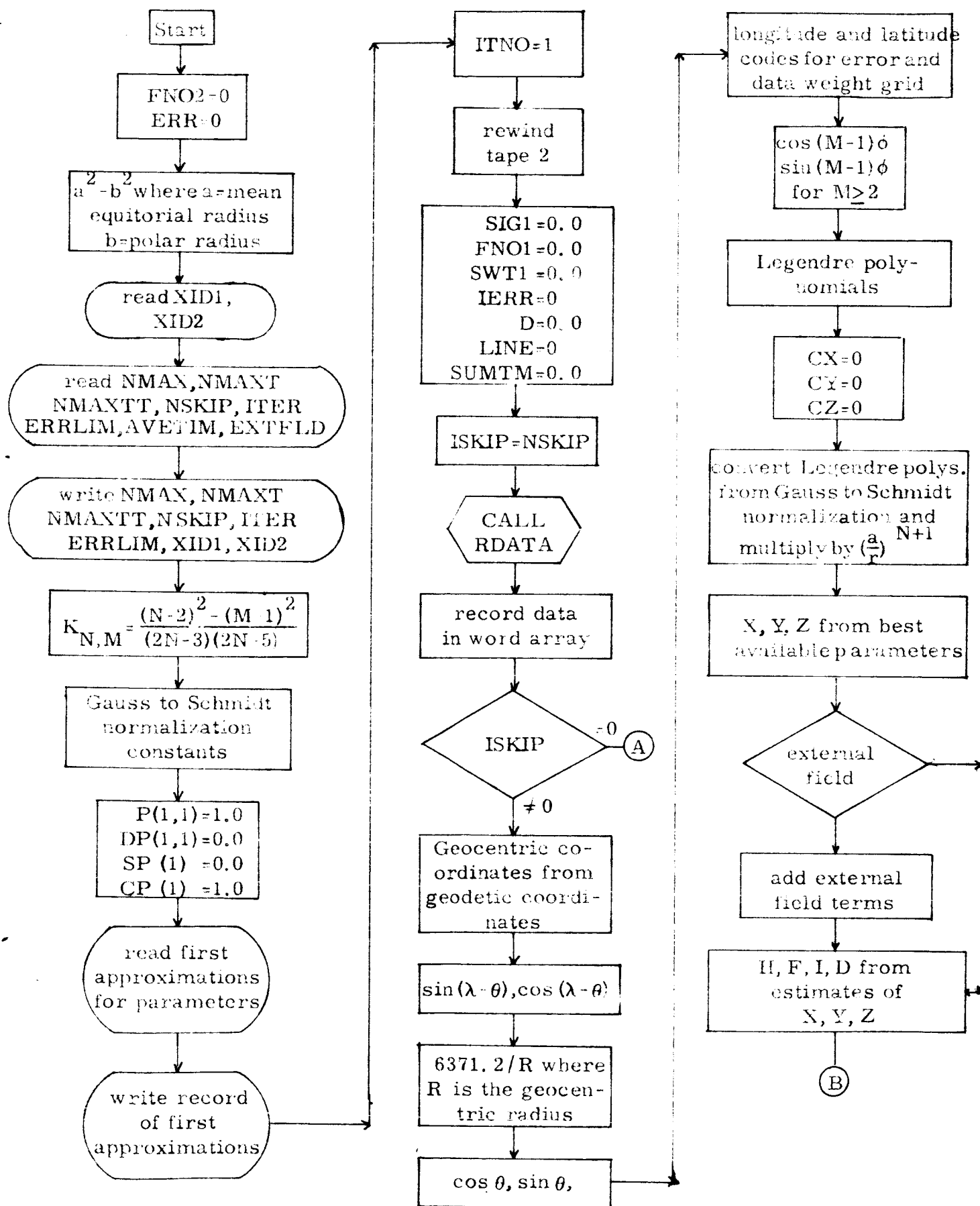
NCOL	column number in triangular matrix C
NMAX	maximum N in the terms of the form $g_{N,M,0} \cos(M-1)\phi$ or $h_{N,M,0} \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function
NMAXT	maximum N in the terms of the form $g_{N,M,t}^t \cos(M-1)\phi$ or $h_{N,M,t}^t \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function
NMAXTT	maximum N in the terms of the form $g_{N,M,tt}^{t^2} \cos(M-1)\phi$ or $h_{N,M,tt}^{t^2} \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function
NO	first row or column in the least squares equations for terms of the form $g_{N,M,t}^t \cos(M-1)\phi$ or $h_{N,M,t}^t \sin(M-1)\phi$
NONOT	first row or column in the least squares equations for terms of the form $g_{N,M,tt}^{t^2} \cos(M-1)\phi$ or $h_{N,M,tt}^{t^2} \sin(M-1)\phi$
NOP	number of parameters in the spherical-harmonic expansion of the geomagnetic potential function
NOPP	number of parameters plus one
NOR	number of rows in the triangular matrix D
NROW	row number in the triangular matrix D
NSKIP	constant used to set the value of ISKIP
P	an associated Legendre polynomial
PI	$\pi = 3.14159265$
PI2	$2\pi = 6.28318530$
R	geocentric coordinate of radius

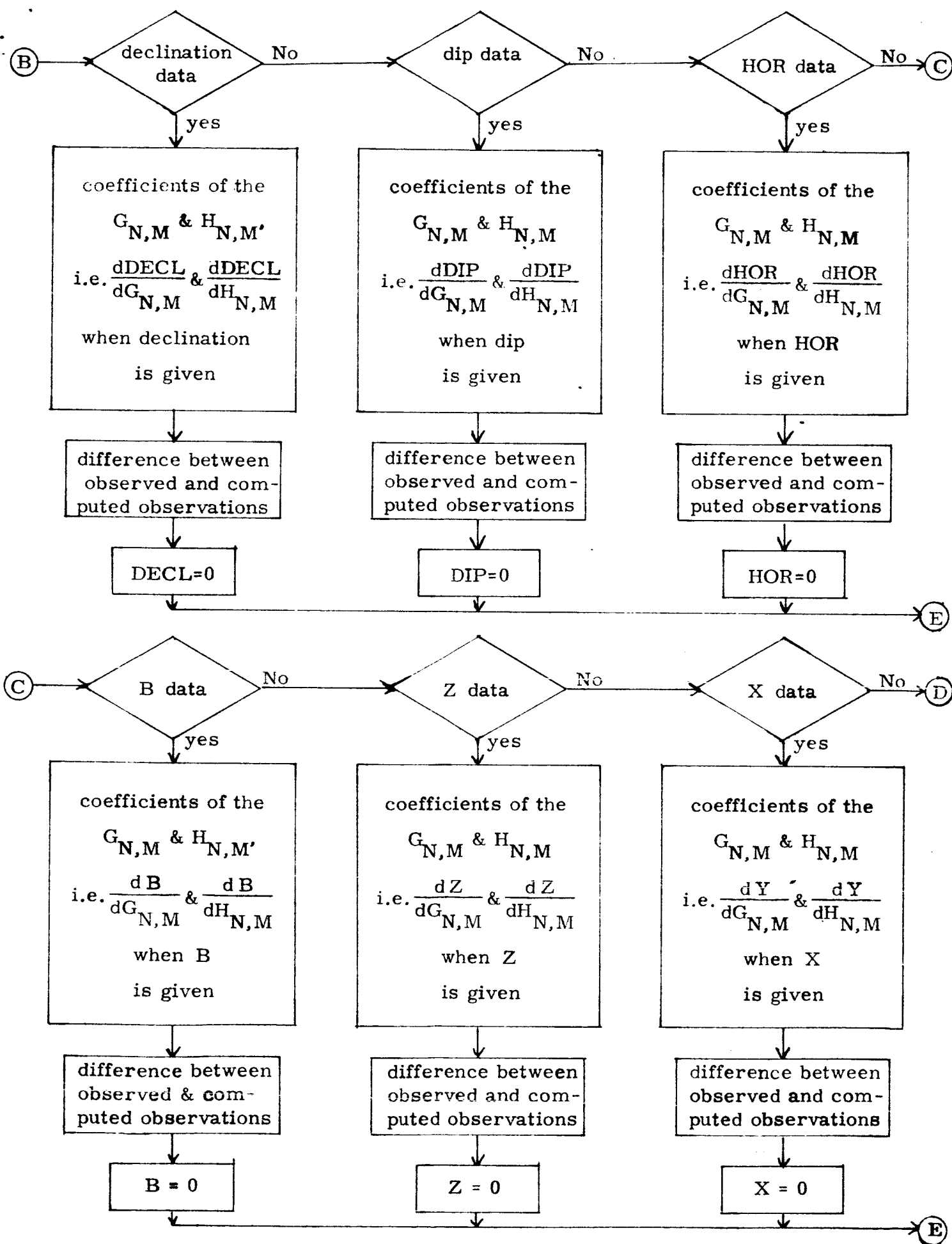
RAD	degrees in one radian = 57.2957795
RDATA	name of subroutine for reading data from magnetic tapes
RDKK	reciprocal of the (K, K) element in the triangular matrix D
SHMIDT	constants to convert from Gauss normalization to Schmidt normalization
SIDE	a "complete" column in the matrix D required for inversion
SIG1	a set of eight storages to contain the sums of the squared observations times the assigned weight types and the total for all data
SIND	sine of difference between geodetic coordinate λ and geocentric coordinate θ
SINLA	sine of the geodetic latitudinal coordinate λ
SINLA2	$(\text{SINLA})^2$
SP(M)	sine of the product of (M-1) and the longitudinal coordinate ϕ
ST	sine of $\pi/2$ minus the geocentric coordinate θ , i. e., coaltitude
SUMD	sum storage for forming check sum column of D matrix
SUMTM	sum of (time - 60.0)
SWT1	a set of eight storages to contain the sum of the weights of the seven types of data and a sum of the total weights of all data
T1	a temporary storage
T2	a temporary storage
T3	a temporary storage
TEMP	a temporary storage
TFACT	time factor
THETA	the geocentric coordinate of latitude

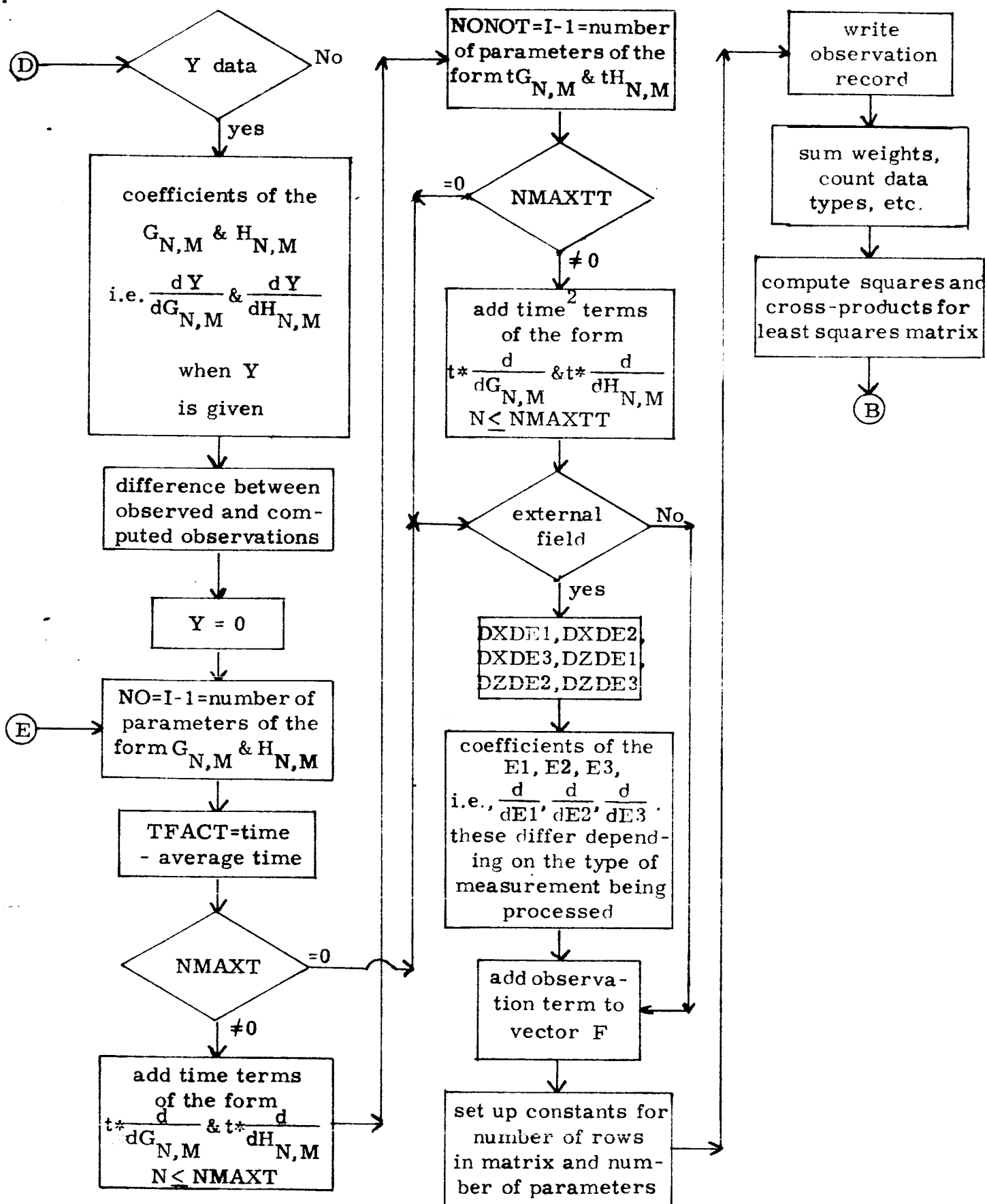
TIME	time of observation
TM	time - 60.0
TYPE	alphabetic code to identify ITYPE's on output
WD	seven storages for input data in the following order--DECL, DIP, HOR, B, Z, X, and Y
WT	an assigned data weight
X	observed X component of the field strength
XID1,XID2	storages to identify computer run
XWT	standard error of observed X component of the field strength
Y	observed Y component of the field strength
YWT	standard error of observed Y component of the field strength
Z	observed Z component of the field strength
ZWT	standard error of observed Z component of the field strength

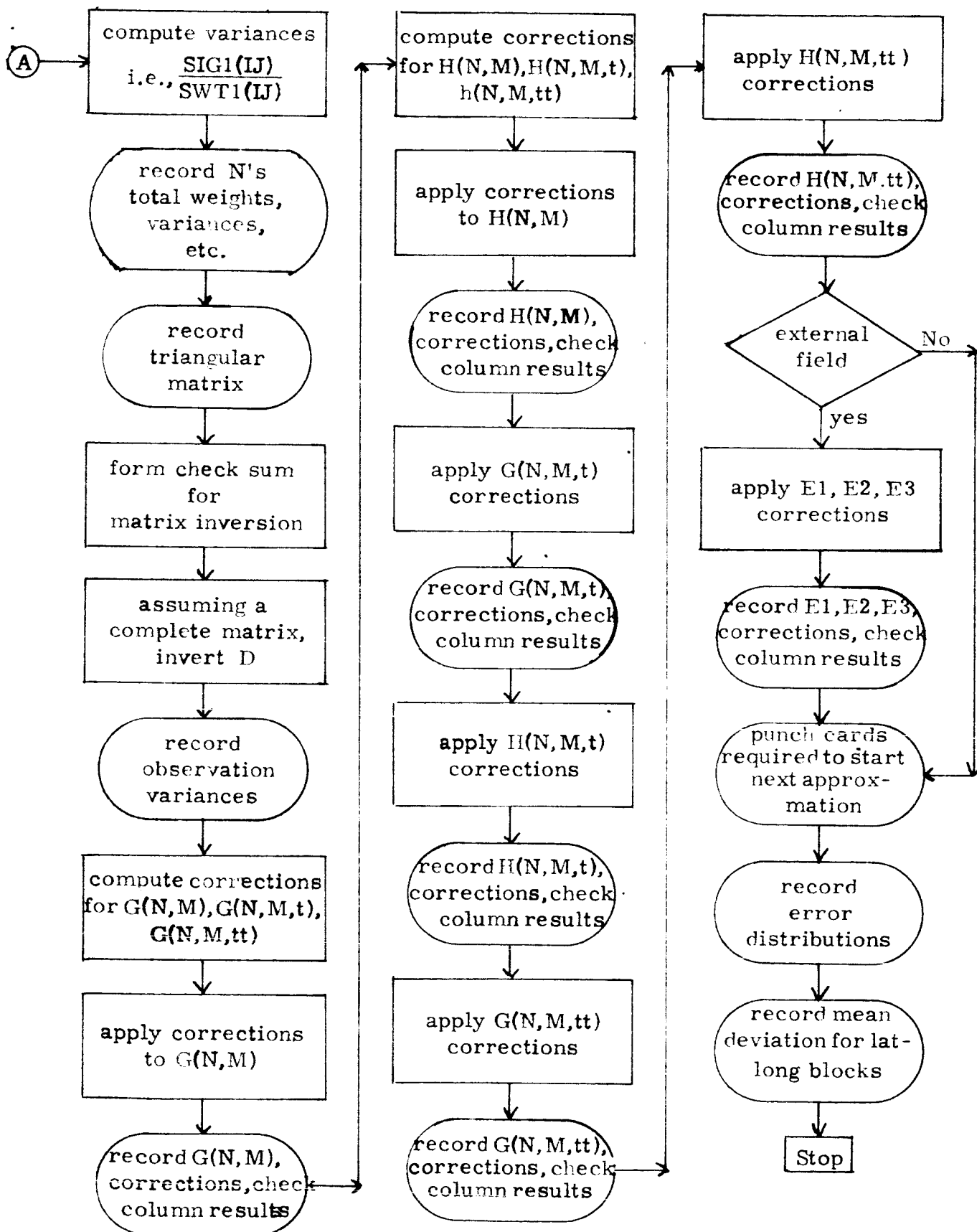
APPENDIX C

FLOW CHARTS FOR JENSEN'S FIT PROGRAM









APPENDIX D

PROGRAM LISTING FOR WALL'S ERROR PROGRAM

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* * * * *
WALL@S ERROR
* * * * *

D MUST EXCEED (NMAX**4+9*NMAX**2+14)/2
               +(NMAXT**4+9*NMAXT**2+14)/2
               +(NMAXTT**4+9*NMAXTT**2+14)/2
F AND SIDE MUST EXCEED NMAX**2+NMAXT**2+NMAXTT**2+6
DIMENSION G(9,9),H(9,9),GT(9,9),HT(9,9),GTT(9,9),HTT(9,9)
DIMENSION F(30),DINV(30,30),ERR(7)
DIMENSION SHMIDT(9,9)
DIMENSION DXDH(9,9),DYDH(9,9),DZDH(9,9)
DIMENSION DXDG(9,9),DYDG(9,9),DZDG(9,9)
DIMENSION CP(9),SP(9),P(9,9),DP(9,9),CONST(9,9)
INTEGER EXTFLD
DATA (RAD=57.2957795),(A=6378.165),(FLAT=298.3),(LINE=0)
DATA (PI=3.14159265),(PI2=6.28318530)
DATA (EXTFLD=0)
* * * * *

FLAT=1.-1./FLAT
      COMPUTATION WITH SPHERICAL EARTH
FLAT=1.
A=6371.2
MAXD=3400
A2=A**2
A4=A**4
B2=(A*FLAT)**2
A2B2=A2*(1.-FLAT**2)
A4B4=A4*(1.-FLAT**4)
REWIND 1
      READ DATA FROM MATRIX SUBROUTINE
READ (1) NMAX,NMAXT,NMAXTT,FWNP,FNP,SIGMA,NOR,NOP,NOPP
READ (1) ((G(N,M),H(N,M),M=1,N),N=2,NMAX)
READ (1) ((GT(N,M),HT(N,M),M=1,N),N=2,NMAX)
READ (1) ((GTT(N,M),HTT(N,M),M=1,N),N=2,NMAX)
DO 1 I=1,NOR
READ (1) (DINV(I,J),J=1,NOP)
CONTINUE
      READ DELTA LONGITUDE, DELTA LATITUDE, AND BASE TIME
READ 2,DLONG,DLATT,TIME
FORMAT (3F10.0)
VAR=SIGMA*SIGMA
TFACT=TIME-60.0
TFACT2=TFACT*TFACT
      FIND STARTING POINT FOR GRID
SLONG=0.0

```

3	SLATT=0.0	51
4	IF (SLONG-DLONG+180.0) 5,4,4	52
	SLONG=SLONG-DLONG	53
	GO TO 3	54
5	IF (SLATT-DLATT+90.0) 7,6,6	55
6	SLATT=SLATT-DLATT	56
	GO TO 5	57
7	SLATT=SLATT-DLATT	58
	ELONG=SLONG	59
	FLATT=SLATT	60
C		61
C	COMPUTE CONSTANTS REQUIRED FOR GENERATING LEGENDRE POLYNOMIALS	62
	DO 8 N=2,NMAX	63
	FN=N	64
	DO 8 M=1,N	65
	FM=M	66
	CONST(N,M)=((FN-2.0)**2-(FM-1.0)**2)/(FN+FN-3.0)/(FN+FN-5.0)	67
8	CONTINUE	68
C		69
C	COMPUTE CONSTANTS TO CONVERT FROM GAUSS TO SCHMIDT NORMALIZATION	70
	SHMIDT(1,1)=-1.0	71
	DO 9 N=2,NMAX	72
	FN=N	73
	SHMIDT(N,1)=SHMIDT(N-1,1)*(FN+FN-3.0)/(FN-1.0)	74
	FACT=2.0	75
	DO 9 M=2,N	76
	FM=M	77
	SHMIDT(N,M)=SHMIDT(N,M-1)*SQRT((FN-FM+1.0)*FACT/(FN+FM-2.0))	78
9	FACT=1.0	79
C		80
	SET VALUE OF FIRST LEGENDRE POLYNOMIALS	81
	P(1,1)=1.0	82
C	DP(1,1)=0.0	83
	SET VALUE OF SIN(M-1)PHI AND COS(M-1)PHI WHEN M=1	84
	SP(1)=0.0	85
	CP(1)=1.0	86
C		87
	RECORD PARAMETERS TO BE USED FOR THE GRID	88
10	WRITE (6,10) NMAX,NMAXT,NMAXTT,SIGMA,FNP,FWNP,NOR,NOP,NOPP	89
	FORMAT (6H1NMAX=,15,3X,6HNMAXT=,15,3X,7HNMAXTT=,15,3X,8HSIGMA *,F	90
	\$5.0,3X,7HPOINTS=,F6.0,3X,11HWEIGHT SUM=,E16.8/3X,4HNOR=,15,3X,4HNO	91
	\$P=,15,3X,5HNOPP=,15/1X)	92
11	WRITE (6,11) DLONG,DLATT,TIME	93
	FORMAT (12H DELTA LONG=,F8.2,3X,10HDELTA LAT=,F8.2,3X,10HBASE TIME	94
	\$=,F8.2/1X)	95
	WRITE (6,12) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)	96
	\$,M=1,N),N=2,NMAX)	97
12	FORMAT (213,6F11.4)	98
C		99
C	* * * * *	100
C	END INITIALIZATION, BEGIN FIELD GENERATION	
	* * * * *	
13	IF (FLATT+DLATT-90.0) 18,14,14	

14	IF (ELONG-180.0) 17,15,15	101
15	WRITE (6,16)	102
16	FORMAT (1H1/1H1/1H1)	103
	STOP END	104
17	ELONG=ELONG+DLONG	105
	FLATT=SLATT	106
	LINE=1	107
	WRITE (6,64)	108
18	FLATT=FLATT+DLATT	109
C	COMPUTE GEOCENTRIC THETA FROM	110
C	GEODETIC COORDINATES	111
	FLATR=FLATT/RAD	112
	SINLA=SIN(FLATR)	113
	SINLA2=SINLA**2	114
	DEN2=A2-A2B2*SINLA2	115
	DEN=SQRT(DEN2)	116
	FAC=((ALT*DEN)+B2)/((ALT*DEN)+A2)	117
	THETA=ATAN(FAC*SINLA/(1.E-30+SQRT(1.-SINLA2)))	118
C	COMPUTE GEOCENTRIC R FROM GEODETIC COORDINATES	119
	R=SQRT(ALT*(ALT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)	120
C	COMPUTE SINE AND COSINE OF DIFFERENCE BETWEEN	121
C	GEODETIC AND GEOCENTRIC LATITUDINAL COORDINATES	122
	SIND=SIN(FLATR-THETA)	123
	COSD=SQRT(1.0-SIND*SIND)	124
	AOR=6371.2/R	125
C	COS THETA MEASURED FROM POLAR AXIS	126
	CT=SIN(THETA)	127
C	SIN THETA MEASURED FROM POLAR AXIS	128
	ST=SQRT(1.0-CT*CT)	129
	SP(2)=SIN(ELONG/RAD)	130
	CP(2)=COS(ELONG/RAD)	131
	DO 19 M=3,NMAX	132
C	SIN(M-1)PHI, EQ.(5)	133
	SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)	134
C	COS(M-1)PHI, EQ.(6)	135
	CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)	136
19	CONTINUE	137
C	GENERATE ASSOCIATED LEGENDRE POLYNOMIALS	138
	DO 22 N=2,NMAX	139
	DO 22 M=1,N	140
	IF (N-M) 21,20,21	141
20	P(N,N)=ST*P(N-1,N-1)	142
	DP(N,N)=ST*DP(N-1,N-1)+CT*P(N-1,N-1)	143
	GO TO 22	144
21	P(N,M)=CT*P(N-1,M)-CONST(N,M)*P(N-2,M)	145
	DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-CONST(N,M)*DP(N-2,M)	146
22	CONTINUE	147
C	INITIALIZE TO COMPUTE X,Y,Z	148
	CX=0.0	149
	CY=0.0	150

	CZ=0.0	151
	AR=AOR*AOR	152
C		153
C	COMPUTE X,Y,Z USING BEST AVAILABLE PARAMETERS	154
	DO 23 N=2,NMAX	155
	FN=N	156
	AR=AR*AOR	157
	DO 23 M=1,N	158
	FM=M-1	159
C		160
C	APPLY SCHMIDT NORMALIZATION CONSTANTS AND MULTIPLY BY (A/R)**(N+1)	161
	P(N,M)=P(N,M)*AR*SHMIDT(N,M)	162
	DP(N,M)=DP(N,M)*AR*SHMIDT(N,M)	163
	TEMP=FN*P(N,M)*SIND-DP(N,M)*COSD	164
	DXDG(N,M)=TEMP*CP(M)	165
	DXDH(N,M)=TEMP*SP(M)	166
	TEMP=FM*P(N,M)/ST	167
	DYDG(N,M)=-TEMP*SP(M)	168
	DYDH(N,M)=TEMP*CP(M)	169
	TEMP=FN*P(N,M)*COSD+DP(N,M)*SIND	170
	DZDG(N,M)=TEMP*CP(M)	171
	DZDH(N,M)=TEMP*SP(M)	172
C		173
	ADD TIME TERMS	174
	GNM=(TM*GTT(N,M)+GT(N,M))*TM+G(N,M)	175
	HNM=(TM*HTT(N,M)+HT(N,M))*TM+H(N,M)	176
	CX=CX+GNM*DXDG(N,M)+HNM*DXDH(N,M)	177
	CY=CY+GNM*DYDG(N,M)+HNM*DYDH(N,M)	178
	CZ=CZ+GNM*DZDG(N,M)+HNM*DZDH(N,M)	179
23	CONTINUE	180
	IF (EXTFLD) 24,25,24	181
24	T1=E2*CP(2)+E3*SP(2)	182
	T2=E1*ST-T1*CT	183
	T1=E1*CT+T1*ST	184
	CX=CX-T2*COSD+T1*SIND	185
	CY=CY+E2*SP(2)-E3*CP(2)	186
	CZ=CZ+T2*SIND+T1*COSD	187
C		188
C	COMPUTE HORIZONTAL, TOTAL FIELD, DIP, AND DECLINATION	189
25	CH=SQRT(CX*CX+CY*CY)	190
	CF=SQRT(CH*CH+CZ*CZ)	191
	CI=2.0*ATAN(CZ/(CF+CH))	192
	CD=2.0*ATAN(CY/(CH+CX))	193
C		194
C	* * * * *	195
C	END FIELD GENERATION, BEGIN STANDARD ERROR ESTIMATION	196
C	* * * * *	197
	COMPUTE COEFFICIENTS OF G(N,M) AND H(N,M)	198
26	ITYPE=1	199
	I=1	200
C	GO TO (27,30,33,36,39,41,43,62),ITYPE	
	COMPUTE COEFFICIENTS FOR DECLINATION	

C	DECL=ARCTAN(Y/X)	201
C	T1=DDECL/DY	202
C	T2=DDECL/DX	203
27	T1=CX/CH	204
	T2=CY/CH	205
	DO 29 N=2,NMAX	206
	DO 29 M=1,N	207
C	DDECL/DG(N,M)	208
	F(I)=(T1*DYDG(N,M)-T2*DXDG(N,M))	209
28	IF (M-1) 28,29,28	210
	I=I+1	211
C	DDECL/DH(N,M)	212
	F(I)=(T1*DYDH(N,M)-T2*DXDH(N,M))	213
29	I=I+1	214
	GO TO 45	215
C	COMPUTE COEFFICIENTS FOR DIP	216
C	DIP=ARCTAN(Z/H)	217
C	T1=DDIP/DZ	218
C	T2=DDIP/DX	219
C	T3=DDIP/DY	220
30	T1=CH/CF	221
	T2=CZ*CX/CH/CF	222
	T3=CZ*CY/CH/CF	223
	DO 32 N=2,NMAX	224
	DO 32 M=1,N	225
C	F(I)=DDIP/DG(N,M)	226
	F(I)=(T1*DZDG(N,M)-T2*DXDG(N,M)-T3*DYDG(N,M))	227
	IF (M-1) 31,32,31	228
31	I=I+1	229
C	F(I)=DDIP/DH(N,M)	230
	F(I)=(T1*DZDH(N,M)-T2*DXDH(N,M)-T3*DYDH(N,M))	231
32	I=I+1	232
	GO TO 45	233
C	COMPUTE COEFFICIENTS FOR HORIZONTAL	234
C	FIELD	235
C	HOR=SQRT(X*X+Y*Y)	236
C	T1=DHOR/DX	237
C	T2=DHOR/DY	238
33	T1=CX/CH	239
	T2=CY/CH	240
	DO 35 N=2,NMAX	241
	DO 35 M=1,N	242
C	F(I)=DHOR/DG(N,M)	243
	F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M))	244
	IF (M-1) 34,35,34	245
34	I=I+1	246
C	F(I)=DHOR/DH(N,M)	247
	F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M))	248
35	I=I+1	249
	GO TO 45	250

C	COMPUTE COEFFICIENTS FOR TOTAL FIELD	251
C	B=SQRT(X*X+Y*Y+Z*Z)	252
C	T1=DB/DX	253
C	T2=DB/DY	254
C	T3=DB/DZ	255
36	T1=CX/CF	256
	T2=CY/CF	257
	T3=CZ/CF	258
	DO 38 N=2,NMAX	259
	DO 38 M=1,N	260
C	F(I)=DB/DG(N,M)	261
	F(I)=(T1*DXDG(N,M)+T2*DYDG(N,M)+T3*DZDG(N,M))	262
	IF (M-1) 37,38,37	263
37	I=I+1	264
C	F(I)=DB/DH(N,M)	265
	F(I)=(T1*DXDH(N,M)+T2*DYDH(N,M)+T3*DZDH(N,M))	266
38	I=I+1	267
	GO TO 45	268
C	COMPUTE COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN	269
39	DO 40 N=2,NMAX	270
	F(I)=DZDG(N,1)	271
	I=I+1	272
	DO 40 M=2,N	273
	F(I)=DZDG(N,M)	274
	F(I+1)=DZDH(N,M)	275
40	I=I+2	276
	GO TO 45	277
C	COMPUTE COEFFICIENTS WHEN THE X COMPONENT IS GIVEN	278
41	DO 42 N=2,NMAX	279
	F(I)=DXDG(N,1)	280
	I=I+1	281
	DO 42 M=2,N	282
	F(I)=DXDG(N,M)	283
	F(I+1)=DXDH(N,M)	284
42	I=I+2	285
	GO TO 45	286
C	COMPUTE COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN	287
43	DO 44 N=2,NMAX	288
	F(I)=DYDG(N,1)	289
	I=I+1	290
	DO 44 M=2,N	291
	F(I)=DYDG(N,M)	292
	F(I+1)=DYDH(N,M)	293
44	I=I+2	294
C	ADD TIME**1 TERMS	295
45	NO=I-1	296
	IF (NMAXT) 46,48,46	297
46	J=1	298
	DO 47 N=2,NMAXT	299
	F(I)=F(J)*TFACT	300

	I=I+1	301
	J=J+1	302
	DO 47 M=2,N	303
	F(I)=F(J)*TFACT	304
	F(I+1)=F(J+1)*TFACT	305
	I=I+2	306
	J=J+2	307
47	CONTINUE	308
C	ADD TIME**2 TERMS	309
48	NONOT=I-1	310
	IF (NMAXTT) 49,51,49	311
49	J=1	312
	DO 50 N=2,NMAXTT	313
	F(I)=F(J)*TFACT2	314
	I=I+1	315
	J=J+1	316
	DO 50 M=2,N	317
	F(I)=F(J)*TFACT2	318
	F(I+1)=F(J+1)*TFACT2	319
	I=I+2	320
	J=J+2	321
50	CONTINUE	322
51	IF (EXTFLD) 52,60,52	323
C	ADD EXTERNAL FIELD TERMS	324
52	DXDE1=CT*SIND-ST*COSD	325
	DZDE1=ST*SIND+CT*COSD	326
	DXDE2=CP(2)*DZDE1	327
	DXDE3=SP(2)*DZDE1	328
	DZDE2=-CP(2)*DXDE1	329
	DZDE3=-SP(2)*DXDE1	330
	GO TO (53,54,55,56,57,58,59), ITYPE	331
C	COEFFICIENTS WHEN DECLINATION (D) IS GIVEN	332
C	F(I)=DD/DE1	333
53	F(I)=-T2*DXDE1	334
	I=I+1	335
C	F(I)=DD/DE2	336
	F(I)=T1*SP(2)-T2*DXDE2	337
	I=I+1	338
C	F(I)=DD/DE3	339
	F(I)=-T1*CP(2)-T2*DXDE3	340
	I=I+1	341
	GO TO 60	342
C	COEFFICIENTS WHEN DIP (I) IS GIVEN	343
C	F(I)=D1/DE1	344
54	F(I)=T1*DZDE1-T2*DXDE1	345
	I=I+1	346
C	F(I)=D1/DE2	347
	F(I)=T1*DZDE2-T2*DXDE2-T3*SP(2)	348
	I=I+1	349
C	F(I)=D1/DE3	350

	$F(1)=T1*DZDE3-T2*DXDE3+T3*CP(2)$	351
	$I=I+1$	352
	GO TO 60	353
C		354
C	COEFFICIENTS WHEN THE HORIZONTAL	355
C	COMPONENT (HOR) IS GIVEN	356
	$F(1)=DHOR/DE1$	357
55	$F(1)=T1*DXDE1$	358
	$I=I+1$	359
C	$F(1)=DHOR/DE2$	360
	$F(1)=T1*DXDE2+T2*SP(2)$	361
	$I=I+1$	362
C	$F(1)=DHOR/DE3$	363
	$F(1)=T1*DXDE3-T2*CP(2)$	364
	$I=I+1$	365
	GO TO 60	366
C		367
C	COEFFICIENTS WHEN TOTAL FIELD IS GIVEN	368
	$F(1)=DF/DE1$	369
56	$F(1)=T1*DXDE1+T3*DZDE1$	370
	$I=I+1$	371
C	$F(1)=DF/DE2$	372
	$F(1)=T1*DXDE2+T2*SP(2)+T3*DZDE2$	373
	$I=I+1$	374
C	$F(1)=DF/DE3$	375
	$F(1)=T1*DXDE3-T2*CP(2)+T3*DZDE3$	376
	$I=I+1$	377
	GO TO 60	378
C		379
57	COEFFICIENTS WHEN THE Z COMPONENT IS GIVEN	380
	$F(1)=DZDE1$	381
	$I=I+1$	382
	$F(1)=DZDE2$	383
	$I=I+1$	384
	$F(1)=DZDE3$	385
	$I=I+1$	386
	GO TO 60	387
C		388
58	COEFFICIENTS WHEN THE X COMPONENT IS GIVEN	389
	$F(1)=DXDE1$	390
	$I=I+1$	391
	$F(1)=DXDE2$	392
	$I=I+1$	393
	$F(1)=DXDE3$	394
	$I=I+1$	395
	GO TO 60	396
C		397
59	COEFFICIENTS WHEN THE Y COMPONENT IS GIVEN	398
	$F(1)=0.0$	399
	$I=I+1$	400
	$F(1)=SP(2)$	
	$I=I+1$	
	$F(1)=-CP(2)$	
	$I=I+1$	
	GO TO 60	

C		COMPUTE STANDARD ERROR OF ESTIMATE	401
60	SUMD=0.0		402
	DO 61 I=1,NOR		403
	TEMP=F(I)		404
	DO 61 J=1,NOR		405
	SUMD=SUMD+TEMP*F(J)*DINV(I,J)		406
61	CONTINUE		407
	ERR(ITYPE)=SQRT(SUMD*VAR)		408
	ITYPE=ITYPE+1		409
	GO TO 26		410
C		* * * * *	411
C		END ERROR ESTIMATION, BEGIN ONE POINT OUTPUT	412
C		* * * * *	413
62	CD=CD*RAD		414
	CI=CI*RAD		415
	ERR(1)=SQRT((ERR(1)*ERR(1)+CD*CD*ERR(3)*ERR(3))/(CH*CH))*RAD		416
	ERR(2)=SQRT((ERR(2)*ERR(2)+CI*CI*ERR(4)*ERR(4))/(CF*CF))*RAD		417
	IF (MOD(LINE,51)) 65,63,65		418
63	WRITE (6,64)		419
64	FORMAT (118H1 LAT LONG DECLINATION FIELD DIP HORIZON		420
	\$TAL TOTAL FIELD Z COMPONENT X COMPONENT Y COMPONENT)		421
65	WRITE (6,66) FLATT,ELONG,CD,ERR(1),CI,ERR(2),CH,ERR(3),CF,ERR(4),C		422
	\$Z,ERR(5),CX,ERR(6),CY,ERR(7)		423
66	FORMAT (1X,F6.1,F7.1,2(F8.2,F7.3),5(F8.0,F7.2))		424
	LINE=LINE+1		425
	GO TO 13		426
	END		427

APPENDIX E

WALL'S ERROR

Introduction

The purpose of this appendix is to document the sequence of operations and to discuss various programming aspects of Wall's Error program. This program has been written to find standard errors of estimation for a grid of the geomagnetic potential over the earth's surface. The grid is based on the time-dependent coefficients estimated by Jensen's Fit program for the spherical-harmonic expansion of the geomagnetic potential function.

The mathematical formulas which form the basis of the computer program are not restated in this appendix. Each time that a formula is required to explain a Fortran variable, a reference is made to an equation in Sections 2.0, 3.0, or 4.0 of this report or to one of the reports listed in the bibliography. When referencing this report, it should be noted that the Fortran variable N is equal to $n+1$. Similarly, $M = m+1$.

The computer program is relatively linear, i. e., there are few alternate calculation sequences. Hence, the calculation sequence will be described in a linear manner.

The program can be roughly divided into four phases as follows:

- (1) initializing; (2) field generation; (3) standard error estimation; and
- (4) one point output or recording. This appendix will likewise be divided into

four principal sections to describe respectively these four phases. Within each section, the Fortran name for variables will be used whenever possible. A glossary identifying these variables is included at the end of this appendix.

Initialization

Initialization consists of doing the things that must be done once at the start of the execution of the program. For this program, these things include setting or computing the value of certain constants that will be used throughout the other phases of the program. Among these are FLAT, A2, A4, B2, A2B2, A4B4, VAR, TFACT, TFACT2, ELONG, FLATT, CONST(N,M), SHMIDT(N,M), P(1,1) DP(1,1), SP(1), and CP(1) all of which are identified in the glossary at the end of this appendix.

The equation for computing CONST(N,M) is found in Eqs. (19) of Section 2.0. Note that $N = n+1$ and $M = m+1$. The equations for computing SHMIDT(N,M) are found in Eqs. (20) of Section 2.0.

The value of the first associated Legendre polynomial, P(1,1), and its derivative, DP(1,1), are constants and may be found in Eqs. (19) of Section 2.0. Similarly, the value of SP(M) and CP(M) are constants for $M=1$, i. e., $SP(1) = 0.0$ and $CP(1) = 1.0$.

Another common function of initialization is the reading of variable control and starting data. Among the variables that must be read for this program are NMAX, NMAXT, NMAXTT, FWNP, FNP, SIGMA, NOR, NOP, NOPP, G(N,M), H(N,M), GT(N,M), HT(N,M), GTT(N,M),

HTT(N, M), and DINV(I, J). These variables, which are all identified in the glossary, are recorded by the modification of Daniels' Matrix subroutine specifically for this program. In addition to these, DLONG, DLATT, and TIME must also be supplied, these are identified in the glossary.

A final function of initialization is often the recording of initial values of pertinent variables. This program records all input data listed above except DINV.

Except for setting or computing the values of certain other minor variables required by the initialization functions enumerated above, this completes the initialization phase of the program.

Field Generation

Output page control and the incrementing of longitude and latitude is accomplished by cards 100 through 109.

Beginning with card 112, geocentric coordinates are computed from the geodetic grid point assignments made for each observation. The equation for computing THETA is Eq. (30) of Section 2.0. The equation for computing geocentric R is Eq. (31) of Section 2.0.

The five variables SIND, COSD, AOR, CT and ST which are all identified in the glossary are computed next. SIND and COSD are required for converting from geocentric to geodetic coordinates. CT and ST are required for the generation of the associated Legendre polynomials and AOR is a term that appears in the equations for estimating X, Y, Z, etc. from

the best available set of parameters. One should note that the Fortran statement for computing CT and ST redefines θ to be measured from the polar axis instead of from the equatorial plane, i. e., colatitude.

SP(2) and CP(2) are computed, followed by the computation of SP(M) and CP(M) for $M > 2$. The equations

$$\sin(M-1)\phi = \sin\phi \cdot \cos(M-2)\phi + \cos\phi \cdot \sin(M-2)\phi \quad (E1)$$

and

$$\cos(M-1)\phi = \cos\phi \cdot \cos(M-2)\phi - \sin\phi \cdot \sin(M-2)\phi \quad (E2)$$

used for computing SP(M) and CP(M) are available in standard texts on trigonometry under the subject, "functions of sums of angles."

Next the program evaluates the necessary associated Legendre polynomials, $P(N, M)$, and their derivatives, $DP(N, M)$, employing the recurrence relationships which are found in Section 2.0, Eqs. (19). $K_{N,M}$ corresponds to the Fortran variable $CONST(N, M)$ and was discussed above in the section on initialization.

Beginning with card 149, the Fortran variables CX, CY, and CZ are set equal to zero in preparation for the estimation of X, Y, and Z. AR and TM, two Fortran variables identified in the glossary, must also be initialized in preparation for the estimation of X, Y, and Z.

The Fortran statements through card 179 are required to estimate X, Y, and Z, i. e., the Fortran variables CX, CY, and CZ. The rotation formulas required for computing X and Z are found in Section 2.0, Eqs. (32)

and (33). Equations for B_θ , B_r , and B_ϕ are given in several different forms in Section 2.0 of this report.

Near the beginning of the group of Fortran statements required to estimate X, Y, and Z (specifically cards 162 and 163), the Gauss normalized polynomials are Schmidt normalized and multiplied by the appropriate power of $\frac{6371.2}{r}$, i. e., AR.

Depending on when it is calculated, the Fortran variable TEMP is the common factor in the coefficients of the two parameters $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, or Z. DXDG, DXDH, DYDG, DYDH, DZDG and DZDH complete the calculation of the coefficients of $g_{N,M,0}$ and $h_{N,M,0}$ in the formulas for computing X, Y, and Z. Next, the Fortran variables GNM and HNM are computed using Eqs. (21) of Section 2.0 and finally all terms are summed for the respective estimates of X, Y, and Z.

The X, Y, and Z components of the external field are next estimated and added to the respective estimates of X, Y, and Z (cards 180 through 186). This step is completely skipped if EXTFLD is zero.

From the estimates of X, Y, and Z, estimates of the horizontal field, total field, dip, and declination (Fortran variables CH, CF, CI, and CD respectively) are made using Eqs. (34) and (37-39) of Section 3.0.

This concludes the field generation phase of the Error program.

Standard Error Estimation

The program now computes the coefficients of the g's and h's in the spherical-harmonic expansion of the geomagnetic potential function. These coefficients are the f(x) in Eq. (51) of Section 4.0.

Formulas for the coefficients of the g's and h's in the system of simultaneous least squares equations may be derived easily from simple theorems in differential calculus and the various equations of Section 3.0. The calculus theorem results in the following:

$$\begin{aligned} \text{If} \quad & x = f(u) \quad \text{and} \quad u = g(w) \\ \text{then} \quad & \frac{dx}{dw} = \frac{df}{du} \cdot \frac{dg}{dw} \end{aligned} \quad (E3)$$

Applying this theorem to Eq. (34) of Section 3.0, the following formulas can be derived for the declination of the total field strength:

$$\frac{d \text{DECL}}{dg_{N,M,0}} = \frac{d \text{DECL}}{dX} \cdot \frac{dX}{dg_{N,M,0}} + \frac{d \text{DECL}}{dY} \cdot \frac{dY}{dg_{N,M,0}} \quad (E4)$$

$$\frac{d \text{DECL}}{dg_{N,M,0}} = - \frac{Y}{\text{HOR}} \frac{dX}{dg_{N,M,0}} + \frac{X}{\text{HOR}} \frac{dY}{dg_{N,M,0}} \quad (E5)$$

$$\frac{d \text{DECL}}{dh_{N,M,0}} = \frac{d \text{DECL}}{dX} \cdot \frac{dX}{dh_{N,M,0}} + \frac{d \text{DECL}}{dY} \cdot \frac{dY}{dh_{N,M,0}} \quad (E6)$$

$$\frac{d \text{DECL}}{dh_{N,M,0}} = - \frac{Y}{\text{HOR}} \cdot \frac{dX}{dh_{N,M,0}} + \frac{X}{\text{HOR}} \frac{dY}{dh_{N,M,0}} \quad (E7)$$

Similarly, from Eqs. (37-39) of Section 3.0, the following can be derived for field dip:

$$\frac{d \text{DIP}}{d g_{N,M,0}} = - \frac{X \cdot Z}{H \cdot F} \frac{dX}{d g_{N,M,0}} - \frac{Y \cdot Z}{H \cdot F} \frac{dY}{d g_{N,M,0}} + \frac{H}{F} \frac{dZ}{d g_{N,M,0}} \quad (\text{E8})$$

$$\frac{d \text{DIP}}{d h_{N,M,0}} = - \frac{X \cdot Z}{H \cdot F} \frac{dX}{d h_{N,M,0}} - \frac{Y \cdot Z}{H \cdot F} \frac{dY}{d h_{N,M,0}} + \frac{H}{F} \frac{dZ}{d h_{N,M,0}} \quad (\text{E9})$$

for the horizontal component:

$$\frac{d \text{HOR}}{d g_{N,M,0}} = \frac{X}{\text{HOR}} \frac{dX}{d g_{N,M,0}} + \frac{Y}{\text{HOR}} \frac{dY}{d g_{N,M,0}} \quad (\text{E10})$$

$$\frac{d \text{HOR}}{d h_{N,M,0}} = \frac{X}{\text{HOR}} \frac{dX}{d h_{N,M,0}} + \frac{Y}{\text{HOR}} \frac{dY}{d h_{N,M,0}} \quad (\text{E11})$$

and for total field:

$$\frac{d B}{d g_{N,M,0}} = \frac{X}{B} \frac{dX}{d g_{N,M,0}} + \frac{Y}{B} \frac{dY}{d g_{N,M,0}} + \frac{Z}{B} \frac{dZ}{d g_{N,M,0}} \quad (\text{E12})$$

$$\frac{d B}{d h_{N,M,0}} = \frac{X}{B} \frac{dX}{d h_{N,M,0}} + \frac{Y}{B} \frac{dY}{d h_{N,M,0}} + \frac{Z}{B} \frac{dZ}{d h_{N,M,0}} \quad (\text{E13})$$

Similar expressions for derivatives with respect to $g_{N,M,t}$, $h_{N,M,t}$, $g_{N,M,tt}$, and $h_{N,M,tt}$ can be derived from the equations cited.

Beginning with card 204, the Error program employs Eqs. (E4-E7) to compute the coefficients of the $g_{N,M,0}$ and $h_{N,M,0}$ in the equation for estimating declination. At card 221, the same coefficients in the equation

for dip are computed. Then at cards 239 and 256, these coefficients are computed for horizontal and total field respectively. Finally, at cards 270, 279, and 288, the coefficients for Z, X, and Y respectively are processed. Finally, coefficients for $g_{N,M,t}$ and $h_{N,M,t}$ are computed at 296 while coefficients for $g_{N,M,tt}$ and $h_{N,M,tt}$ are computed at card 310.

If external field terms are to be used and corrected, the program, beginning with card 325, computes the coefficients of the unknowns E_1 , E_2 , and E_3 in the system of simultaneous least squares equations. The equations employed are similar to Eqs. (E4) through (E13) with $\frac{d}{dg_{N,M,0}}$ or $\frac{d}{dh_{N,M,0}}$ being replaced with $\frac{d}{dE_1}$, $\frac{d}{dE_2}$, or $\frac{d}{dE_3}$. Beginning with card 325, $\frac{dX}{dE_1}$, $\frac{dX}{dE_2}$, $\frac{dX}{dE_3}$, $\frac{dZ}{dE_1}$, $\frac{dZ}{dE_2}$, and $\frac{dZ}{dE_3}$ are computed. The quantities $\frac{dY}{dE_1} = 0$, $\frac{dY}{dE_2} = \sin \phi$, and $\frac{dY}{dE_3} = -\cos \phi$ are not set up explicitly. Then beginning at cards 334, 345, 357, and 368 these derivatives are used to compute the coefficients of E_1 , E_2 , and E_3 for declination, dip, horizontal field and total field, respectively. At cards 378, 386, and 394, the E coefficients for Z, X, and Y respectively are processed.

Cards 402 through 408 compute the sum specified in Eq. (51) of Section 4.0. This concludes the Standard Error Estimation phase of the Error program.

One Point Output

This last phase of Wall's Error is very short, consisting only of cards 414 through 425. The first four of these cards convert declination and dip and their estimated errors from gammas to degrees. This conversion is discussed in Section 4.0 and specific equations for the conversion are (52) and (53). The remaining eight cards control paging and produce the output of one line.

This concludes Wall's Error program.

GLOSSARY

Wall's Error

A	a, the mean equatorial radius of the earth in kilometers
A2	a^2
A4	a^4
A2B2	$(\text{mean equatorial radius of the earth})^2 - (\text{polar radius of the earth})^2$
A4B4	$(\text{mean equatorial radius of the earth})^4 - (\text{polar radius of the earth})^4$
ALT	altitude of the observation
AOR	radius of the sphere having volume equal to the earth's volume/R
AR	$(\text{radius of the sphere having volume equal to the earth's volume/R})^{N+1}$
B2	$(\text{polar radius of the earth})^2$
CD	estimated declination in radians
CF	estimated total field strength
CH	estimated horizontal field strength
CI	estimated field dip in radians
CONST	a set of constants required for the generation of the associated Legendre polynomials, P_n^m $\frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} = \frac{(N-2)^2 - (M-1)^2}{(2N-3)(2N-5)}$ n and m are common formula notation while N and M are used in the computer program
COSD	cosine of difference between geodetic coordinate λ and geocentric coordinate θ

DZDE1	$\frac{dZ}{dE_1}$	where Z is the Z component of the field strength
DZDE2	$\frac{dZ}{dE_2}$	where Z is the Z component of the field strength
DZDE3	$\frac{dZ}{dE_3}$	where Z is the Z component of the field strength
DZDG	$\frac{dZ}{dg}$	where Z is the Z component of the field strength
DZDH	$\frac{dZ}{dh}$	where Z is the Z component of the field strength
E1		external field term along the polar axis
E2		external field term in the equatorial and prime meridian planes
E3		external field term in the equatorial plane but perpendicular to the plane of the prime meridian
ELONG		longitudinal coordinate ϕ in degrees
ERR		observations times weights summed for global grid-points
EXTFLD		a code to identify when external field terms are to be used (EXTFLD $\neq$ 0) and when they are not to be used (EXTFLD = 0)
F		one of the numbers used to form the sums of squares and cross-products of the triangular matrix
FAC		$\tan \theta / \tan \lambda$ where θ and λ are the geocentric and geodetic latitudinal coordinates
FACT		a factor used in generating the factors for converting from Gauss normalization to Schmidt normalization. When M=2, FACT =2.0; when M>2, FACT =1.0. Also used in forming squares and cross products. Hence a temporary storage.
FLAT		polar radius of the earth/mean equatorial radius of the earth
FLATR		λ in radians
FLATT		latitudinal coordinate λ
FM		the index M or M-1 in floating point notation

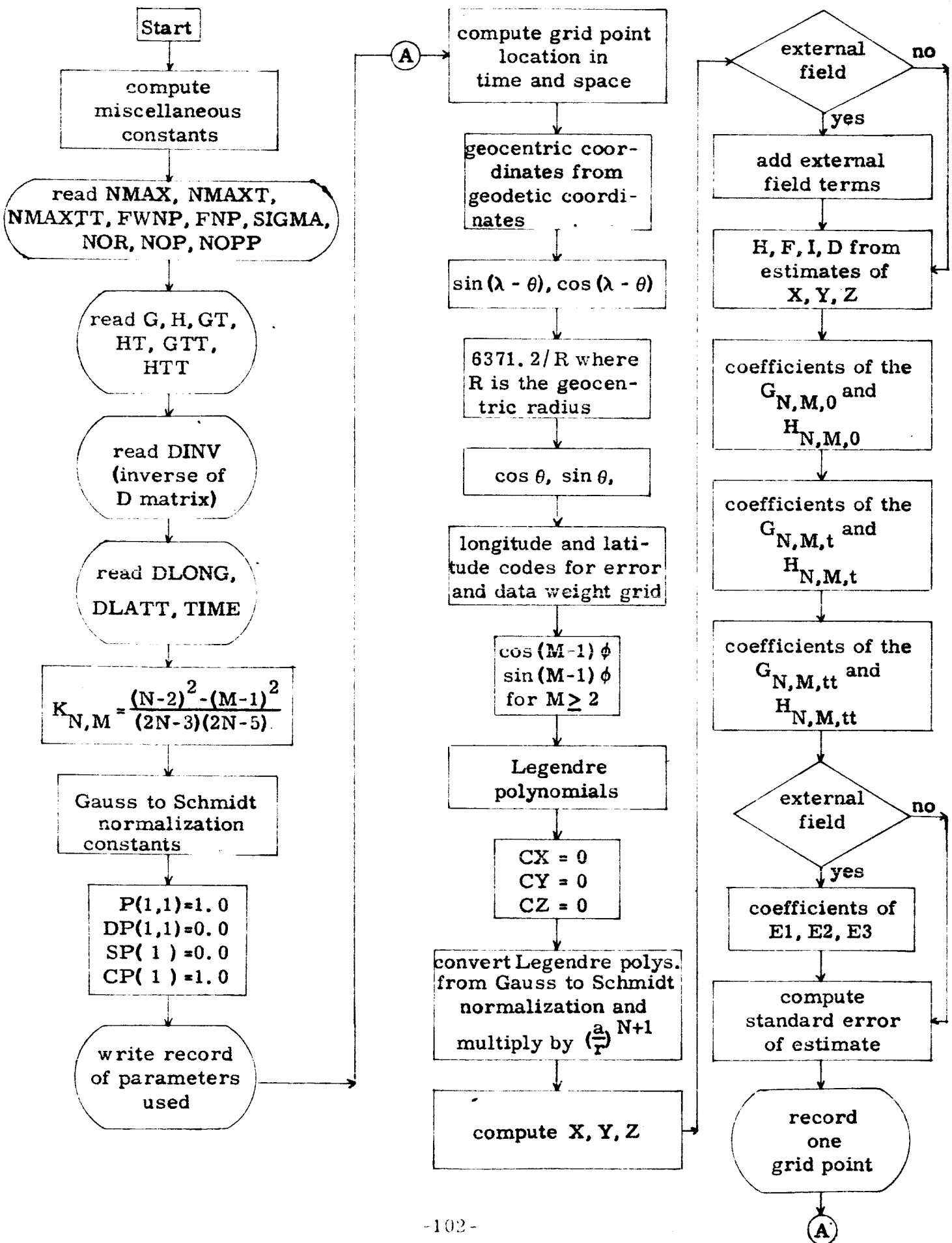
FN	the index N in floating point notation		
FNP	FNO1(8) in Jensen's Fit program		
FWNP	SWT1(8) in Jensen's Fit program		
G	the coefficient $g_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetic potential function		
GNM	G(N, M), i. e. , a specific G		
GT	the coefficient $g_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetic potential function		
GTT	the coefficient $g_{N,M,tt}$ in the spherical-harmonic expansion of the geomagnetic potential function		
H	the coefficient $h_{N,M,0}$ in the spherical-harmonic expansion of the geomagnetical potential function		
HNM	H(N, M), i. e. , a specific H		
HT	the coefficient $h_{N,M,t}$ in the spherical-harmonic expansion of the geomagnetical potential function		
HTT	the coefficient $h_{N,M,tt}$ in the spherical-harmonic expansion of the geomagnetical potential function		
I	an index		
ITYPE	identifies data type, e. g. ,	DECL	= 1
		DIP	= 2
		HOR	= 3
		B	= 4
		Z	= 5
		X	= 6
		Y	= 7
J	an index		
LINE	an index for counting the number of lines output for a page		

M	an index and subscript
MAXD	maximum size of the triangular matrix D
N	an index and subscript
NMAX	maximum N in the terms of the form $g_{N,M,0} \cos(M-1)\phi$ or $h_{N,M,0} \sin(M-1)\phi$ in the spherical harmonic expansion of the geomagnetic potential function
NMAXT	maximum N in the terms of the form $g_{N,M,t}^t \cos(M-1)\phi$ or $h_{N,M,t}^t \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function
NMAXTT	maximum N in the terms of the form $g_{N,M,tt}^{t^2} \cos(M-1)\phi$ or $h_{N,M,tt}^{t^2} \sin(M-1)\phi$ in the spherical-harmonic expansion of the geomagnetic potential function
NO	first row or column in the least squares equations for terms of the form $g_{N,M,t}^t \cos(M-1)\phi$ or $h_{N,M,t}^t \sin(M-1)\phi$
NONOT	first row or column in the least squares equations for terms of the form $g_{N,M,tt}^{t^2} \cos(M-1)\phi$ or $h_{N,M,tt}^{t^2} \sin(M-1)\phi$
NOP	number of parameters in the spherical-harmonic expansion of the geomagnetic potential function
NOPP	number of parameters plus one
NOR	number of rows in the triangular matrix D
P	an associated Legendre polynomial
PI	$\pi = 3.14159265$
PI2	$2\pi = 6.28318530$
R	geocentric coordinate of radius

RAD	degrees in one radian = 57.2957795
SHMIDT	constants to convert from Gauss normalization to Schmidt normalization
SIGMA	SIG1(8) of Jensen's Fit program
SIND	sine of difference between geodetic coordinate λ and geocentric coordinate θ
SINLA	sine of the geodetic latitudinal coordinate λ
SINLA2	$(\text{SINLA})^2$
SLATT	starting latitude for grid
SLONG	starting longitude for grid
SP(M)	sine of the product of (M-1) and the longitudinal coordinate ϕ
ST	sine of $\pi/2$ minus the geocentric coordinate θ , i.e., colatitude
SUMD	sum storage for forming check sum column of D matrix
T1	a temporary storage
T2	a temporary storage
T3	a temporary storage
TEMP	a temporary storage
TFACT	time factor
TFACT2	TFACT * TFACT
THETA	the geocentric coordinate of latitude
TIME	time of computed grid
TM	time - 60.0

APPENDIX F

FLOW CHART FOR WALL'S ERROR PROGRAM



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