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ANALYSIS OF THE MOTION OF A SATELLITE-REACTION WHEEL ASSEMBLY OPTIMIZED FOR WEIGHT AND POWER

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GODDARD SPACE FLIGHT CENTER

GREENBELT, MD.

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by
C. R. Hayleck, Jr.

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ABSTRACT

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This report contains an analysis of the slewing motion of a satellite and reaction wheel assembly during the execution of a slewing maneuver. The device analyzed obeys the law of conservation of momentum. Therefore any rotation of the reaction wheel is accompanied by a corresponding rotation of the satellite but in the opposite sense.

The motion is analyzed for two satellite slewing rates - 30° in 180 sec.; 2° in 30 sec. Three reaction wheel maximum speeds are considered - 500, 1000, and 1500 revolutions per minute. Both sets of values were taken from reports on work done on this problem in the past and were selected for the purpose of giving comparative data.

Based upon a typical (for this type of application) maximum permissible torque, T_o, the reaction wheel mass moments of inertia, I, were calculated.

From these values for I reaction wheel dimensions and weights were calculated and then optimized on the basis of minimum weight. Finally, on the basis of the optimized total weight, the power required was optimized for the reaction wheel speeds considered.

Although the data presented and the curves plotted are for specific conditions (i.e., reaction wheel maximum speed, vehicle slewing rate, steel reaction wheel, aluminum housing, maximum available torque, maximum available power) this analyses is intended to be general. It is intended that an investigator could, without difficulty, make a similar study for different boundary conditions by following the procedure presented here. This analysis differs from previous ones in that the motor was assumed to have constant torque over the working range.

Author

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SUBSCRIPTS

$_a$ - Acceleration

$_d$ - Deceleration

$_w$ - Reaction Wheel

$_v$ - Vehicle

$_{I.}$ - Phase I - [Acceleration]

$_{II.}$ - Phase II - [Constant Angular Velocity]

$_{III.}$ - Phase III - [Deceleration]

$_{max.}$ - Maximum Value For $\{\theta, \dot{\theta}$, etc.$\}$

$_c$ - Constant Velocity

NOMENCLATURE

I_v - Mass Moment of Inertia of Vehicle About Its Longitudinal Axis; Ft.-Lb.-Sec^2.

I_w - Mass Moment of Inertia of Reaction Wheel About Its Longitudinal Axis; Ft.-Lb.-Sec^2.

Note. Longitudinal Axes of Vehicle and Reaction Wheel coincide

To - Torque Supplied By The Motor To The Reaction Wheel; Ft. Lb. [To is constant]

t - Time; Seconds

t_{total} - Total Time For Maneuver; Seconds

θ - Angular Displacement; Radians

θ_{total} - Total Angular Displacement; Radians

$\dot{\theta}$ - Angular Velocity; Radians Per Second

$\ddot{\theta}$ - Angular Acceleration; Radians Per (Sec)^2

n - Per Cent Of Total Time, t_{total}

Per Cent Of Total Angular Displacement, θ_{total}

C_1, C_2, etc. - Constants {To Be Evaluated}

ANALYSIS OF VEHICLE MOTION AND REACTION WHEEL MOTION

PHASE I; ACCELERATION

<u>Vehicle</u> $I_v \ddot{\theta}_{v,a} = To$ [v-1]

<u>Integrating</u>

$I_v \dot{\theta}_{v,a} = To\ t_I + C_1$ [v-2]

when $t_I = 0$, $\dot{\theta}_{v,a} = 0$, $\therefore C_1 = 0$

t_{max} = The time required to rotate to a desired θ_{max}

$$t_{max} = n t_{total}$$

$$\theta_{max} = n\theta_{total}$$

$$\therefore \dot{\theta}_{v,a,max} = \frac{To}{I_v} t_{max} = \frac{To}{I_v} n t_{total} \qquad [v-3]$$

<u>Wheel</u> $I_w \ddot{\theta}_{w,a} = To$ [w-1]

<u>Integrating</u>

$I_w \dot{\theta}_{w,a} = To\, t_I + C_2$ [w-2]

when $t_I = 0$; $\dot{\theta}_{w,a} = 0$, $\therefore C_2 = 0$

Again

$$t_{max} = n\, t_{total}$$

$$\theta_{max} = n\theta_{total}$$

$$\therefore \dot{\theta}_{w,a,max} = \frac{To}{I_w} t_{max} = \frac{To}{I_w} n\, t_{total} \qquad [w-3]$$

Equating (To) $(n\,t_{total})$ from [w-3] and [v-3]

$$I_v \dot{\theta}_{v,a,max} = I_w \dot{\theta}_{w,a,max} \qquad [wv\text{-}1]$$

Vehicle Integrating Equation [v-2]

$$I_v \theta_{v,a} = To \frac{t_I^2}{2} + C_3 \qquad [v\text{-}4]$$

when $t_I = 0$, $\theta_{v,a} = 0$, $\therefore C_3 = 0$

$$\therefore I_v \theta_{v,a,max} = \frac{To}{2}(t_{max})^2 = \frac{To}{2}(n\,t_{total})^2 \qquad [v\text{-}5]$$

Wheel Integrating Equation [w-2]

$$I_w \theta_{w,a} = To \frac{t_I^2}{2} + C_4 \qquad [w\text{-}4]$$

when $t_I = 0$, $\theta_{w,a} = 0$, $\therefore C_4 = 0$

$$\therefore I_w \theta_{w,a,max} = \frac{To}{2}(t_{max})^2 = \frac{To}{2}(n\,t_{total})^2 \qquad [w\text{-}5]$$

PHASE II; CONSTANT VELOCITY

$$I_v \ddot{\theta}_{v,c} = 0 \quad \text{i.e., Torque} = 0 \qquad [v\text{-}6]$$

Integrating

$$I_v \dot{\theta}_{v,c} = C_5 \qquad [v\text{-}7]$$

At the beginning of the constant velocity phase

$$I_v \dot{\theta}_{v,c} = I_v \dot{\theta}_{v,a,max} = C_5$$

$$\therefore \dot{\theta}_{v,c} = \dot{\theta}_{v,a,max} \qquad [v\text{-}8]$$

Integrating Equation [v-8]

$$\theta_{v,c} = \dot{\theta}_{v,a,max}\ t_{II} + C_6 \qquad [v-9]$$

when $t_{II} = 0$ for $V =$ constant (Phase II)

then $\theta_{v,c} = 0 \quad \therefore C_6 = 0$

If the acceleration and deceleration phases occur in equal intervals of time i.e., $n\,t_{total}$

$$t_a + t_c + t_d = t_{total}$$

$$n\,t_{total} + t_c + n\,t_{total} = t_{total}$$

$$t_c = t_{total} - 2n\,t_{totol}$$

$$t_c = (1 - 2n)\ t_{total} \qquad [wv-2]$$

Wheel $\quad I_w \ddot{\theta}_{w,c} = 0 \qquad$ i.e., Torque $= 0 \qquad$ [w-6]

Integrating

$$I_w \dot{\theta}_{w,c} = C_7 \qquad [w-7]$$

when $t_{II} = 0,\ I_w \dot{\theta}_{w,c} = I_w \dot{\theta}_{w,a,max} = C_7 \qquad$ [w-8]

$$\therefore \dot{\theta}_{w,c} = \dot{\theta}_{w,a,max}$$

Integrating

$$\theta_{w,c} = \dot{\theta}_{w,a,max}\ t_{II} + C_8 \qquad [w-9]$$

when $t_{II} = 0,\ \theta_{wc} = 0 \ \therefore C_8 = 0$

From [w-9] and [v-9] using t_c from [wv-2]

<u>Vehicle</u> $\theta_{v,c,max} = \dot{\theta}_{v,a,max}\,(1 - 2n)\; t_{total}$ [v-10]

<u>Wheel</u> $\theta_{w,c,max} = \dot{\theta}_{w,a,max}\,(1 - 2n)\; t_{total}$ [w-10]

PHASE III; DECELERATION

<u>Vehicle</u> $I_v\,\ddot{\theta}_{v,d} = -\,To$ [v-11]

<u>Integrating</u>

$$I_v\,\dot{\theta}_{v,d} = -\,To\; t_{III} + C_9 \qquad [v\text{-}12]$$

when $t = 0$ {Beginning of Deceleration}

$$\dot{\theta}_{v,d} = \dot{\theta}_{v,a,max}$$

$$\therefore C_9 = I_v\,\dot{\theta}_{v,a,max}$$

$$I_v\dot{\theta}_{v,d} = -\,To\,t + I_v\,\dot{\theta}_{v,a,max} \qquad [v\text{-}13]$$

<u>Integrating</u>

$$I_v\,\theta_{v,d} = -\,To\,\frac{t_{III}^{\;2}}{2} + I_v\,\dot{\theta}_{v,a,max}\; t + C_{10} \qquad [v\text{-}14]$$

when $t_{III} = 0$ {for Phase III}

$$\theta_{v,d} = 0 \;\therefore\; C_{10} = 0$$

<u>Wheel</u> $I_w\,\ddot{\theta}_{w,d} = -\,To$ [w-11]

<u>Integrating</u>

$$I_w\,\dot{\theta}_{w,d} = -\,To\; t_{III} + C_{11} \qquad [w\text{-}12]$$

$$\text{when } t_{III} = 0, \; \dot{\theta}_{w,d} = \dot{\theta}_{w,a,max}$$

$$\therefore C_{11} = I_w \dot{\theta}_{w,a,max}$$

$$I_w \dot{\theta}_{w,d} = -\,To\, t_{III} + I_w \dot{\theta}_{w,a,max} \qquad \text{[w-13]}$$

<u>Integrating</u>

$$I_w \theta_{w,d} = -\,To\, \frac{t_{III}^{\;2}}{2} + I_w \dot{\theta}_{w,a,max}\, t_{III} + C_{12} \qquad \text{[w-14]}$$

$$\text{when } t = 0, \; \theta_{w,d} = 0 \;\; \therefore C_{12} = 0$$

If the angular displacement of the wheel during the deceleration phase is set equal to the angular displacement of the wheel during the acceleration phase, then:

$$\left.\begin{aligned} \theta_a &= n\,\theta_{total} \\ \theta_d &= n\,\theta_{total} \\ \theta_c &= (1 - 2n)\,\theta_T \end{aligned}\right\} \begin{array}{l} \text{General Statements} \\ \text{For Both} \\ \text{The Vehicle} \\ \text{And The} \\ \text{Reaction Wheel} \end{array}$$

Note $\theta_{w,a} \neq \theta_{v,a}$

$\theta_{v,d} \neq \theta_{w,d}$

<u>Analysis of Slewing Maneuver</u>

For a slewing maneuver that consists of a period of accelerating motion, a period of constant velocity motion, and a period of decelerating motion, in that sequence, the following displacement equation will apply;

$$\theta_{v,a} + \theta_{v,c} + \theta_{v,d} = \theta_{v,total} \qquad \text{[s-1]}$$

$$\text{since } \ddot{\theta}_{v,a} = -\,\ddot{\theta}_{v,d}$$

and, $\theta_{v,a} = \theta_{v,d}$

$$2\theta_{v,a} + \theta_{v,c} = \theta_{v,total} \qquad [s-2]$$

Since $\theta_{v,a}$ takes place during the period of time $n\,t_{total}$ the equation [v-5] can be rewritten:

$$\theta_{v,a} = \frac{1}{2}\frac{To}{I_v}\left(n\,t_{total}\right)^2 \qquad [s-3]$$

and equation [v-9] can be rewritten:

$$\theta_{v,c} = \dot{\theta}_{v,a,max}\ (1 - 2n)\ t_{total} \qquad [s-4]$$

and from equation [v-3]

$$\dot{\theta}_{v,a,max} = \frac{To}{I_v}\, n\ t_{total}$$

Equation [s-4] becomes

$$\theta_{v,c} = \frac{To}{I_v}\left(n\ t_{total}\right)\ (1 - 2n)\ t_{total} \qquad [s-5]$$

Therefore equation [s-2] becomes

$$2\left[\left(\frac{1}{2}\right)\left(\frac{To}{I_v}\right)\left(n\ t_{total}\right)^2\right] + \frac{To}{I_v}\left(n\ t_{total}\right)\ (1 - 2n)\ \left(t_{total}\right) = \theta_{v,total}$$

$$\frac{To}{I_v}\left[(n)^2 + n - 2(n)^2\right]\left(t_{total}\right)^2 = \theta_{v,total} \qquad [s-6]$$

$$[n - n^2]\ To = \frac{\theta_{v,total}\, I_v}{\left(t_{total}\right)^2} \qquad [s-7]$$

For a slewing rate of 2° in 30 seconds

$\theta_{v,total} = 2^o = 0.0349$ radians

$I_v = 1000$ Ft.-Lb.-Sec^2

$t_{total} = 30$ seconds

n varies from 0.0 to 0.5 [0% to 50%]

$$\therefore \frac{\theta_{v,total}\, I_v}{\left(t_{total}\right)^2} = \frac{0.0349 \times 1000}{(30)^2} = 0.0388 \text{ Ft.Lb.}$$

$$To = \frac{0.0388}{\left(n - n^2\right)} \text{ Ft.Lb.} \qquad [s-8]$$

For a slewing rate of 30° in 180 seconds

$\theta_{v,total} = 30^\circ = 0.524$ radians

$I_v = 1000$ Ft.-Lb.-Sec^2

$t_{total} = 180$ seconds

n varies from 0.0 to 0.5 [0% to 50%]

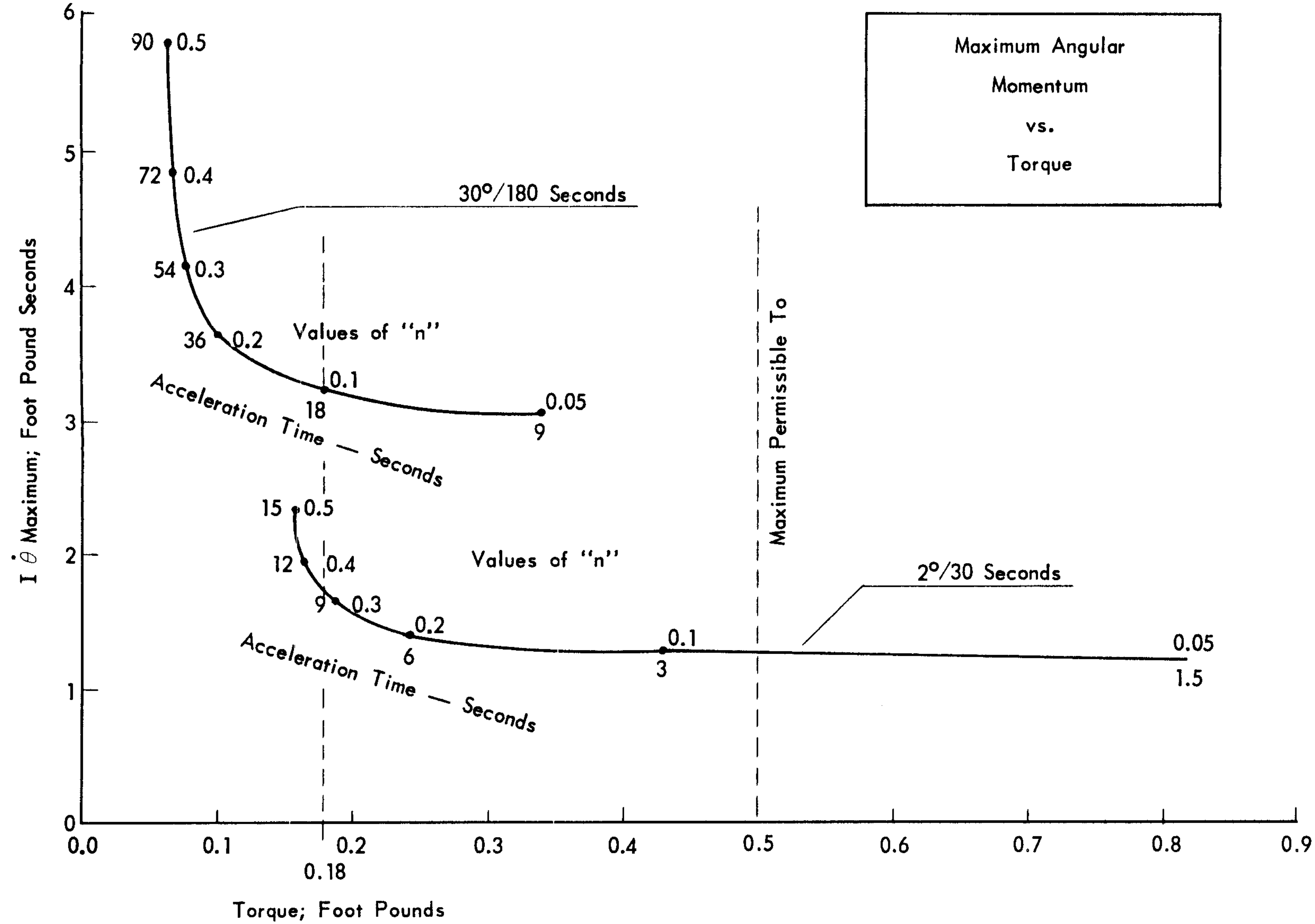
$$\therefore \frac{\theta_{v,total}\, I_v}{\left(t_{total}\right)^2} = \frac{0.524 \times 1000}{(180)^2} = 0.0161 \text{ Ft.Lb.}$$

$$To = \frac{0.0161}{\left(n - n^2\right)} \text{ Ft.Ib.} \qquad [s-9]$$

From the calculation of To for the corresponding values of n (Equation s-9) the maximum angular momentum can be calculated (see Equation v-3).

SLEWING RATE CALCULATIONS

①	②	③	④	⑤	⑥	⑦	⑧	⑨
n	n^2	$(n - n^2)$	2° in 30 Seconds			30° in 180 Seconds		
			To $\frac{.0388}{(n - n^2)}$ Ft.Lb.	$n\,t_t$ Sec.	$\frac{n\,t_t\,To}{I\dot{\theta}_v}$ Ft.Lb.Sec.	To $\frac{.0161}{(n - n^2)}$ Ft.Lb.	$n\,t_t$ Sec.	$\frac{n\,t_t\,To}{I\dot{\theta}_v}$ Ft.Lb.Sec.
0.5	0.25	0.25	0.155	15	2.32	0.0645	90	5.8
0.4	0.16	0.24	0.162	12	1.94	0.0671	72	4.84
0.3	0.09	0.21	0.185	9	1.67	0.0768	54	4.15
0.2	0.04	0.16	0.243	6	1.40	0.1005	36	3.62
0.1	0.01	0.09	0.431	3	1.29	0.179	18	3.22
0.05	0.0025	0.0475	0.818	1.5	1.23	0.340	9	3.06


Maximum Angular
Momentum
vs.
Torque
30°/180 Seconds
2°/30 Seconds
Values of "n"
Values of "n"
Acceleration Time — Seconds
Acceleration Time — Seconds
Maximum Permissible To
90
0.5
72
0.4
54
0.3
36
0.2
18
0.1
9
0.05
15
0.5
12
0.4
9
0.3
6
0.2
3
0.1
1.5
0.05
$I\dot{\theta}$ Maximum; Foot Pound Seconds
Torque; Foot Pounds
0.18
0
1
2
3
4
5
6
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

REACTION WHEEL

DETERMINATION OF WHEEL SIZE AND WEIGHT AND HOUSING SIZE AND WEIGHT

In this analysis the torque applied to the reaction wheel will be sufficient to maintain the angular acceleration of the wheel at a constant rate regardless of losses to the bearings, etc. This torque, "To", is also applied to the vehicle. Therefore from the conservation of angular momentum

$$I_v \dot{\theta}_v + I_w \dot{\theta}_w = 0$$

Assume the reaction wheel to be a hollow right circular cylinder.

Therefore

$$I_w = M R_M^2 = \frac{W}{g} R_M^2$$

where

W = weight or wheel - pounds
g = 386 inches/sec ^{2}
R_M = Mean radius of cylinder - inches

$W = 2\pi R_M t l w$

t = wall thickness - inches
l = length of cylinder - inches
w = specific weight of wheel material - pounds/(inch)^3

[The mass moment of inertia ($I = \int r^2 dm$) for a hollow circular cylinder is $M R_M^2$]

$$\text{Angular Momentum} = H = I\dot{\theta} = M R_M^2 \dot{\theta} = M \frac{D_M^2}{4} \dot{\theta} = \frac{W}{g} \frac{D_M^2}{4} \dot{\theta}$$

$$\therefore W = \frac{4 g H}{D_M^2 \dot{\theta}} = \text{weight of reaction wheel} \qquad [wT-1]$$

Reaction Wheel Angular Velocities

a. 500 rev/min = 52.4 rad/sec
b. 1000 rev/min = 104.8 rad/sec
c. 1500 rev/min = 157.2 rad/sec

$$g = 32.2 \text{ ft/sec}^2 = 386 \text{ inches/sec}^2$$

Set "To"(max) = 0.18 foot pounds

∴ For a slewing rate of;

a. 2°/30 sec; H=20.64 In.-Lb.-Sec. = 1.72 Ft.-Lb.-Sec.

b. 30°/180 sec; H=38.88 In.-Lb.-Sec. = 3.24 Ft.-Lb.-Sec.

REACTION WHEEL HOUSING

The reaction wheel housing is to consist of a cylindrical shell closed at the ends with circular cover plates.

$$W_H = \pi D_H L t w + 2\left[\frac{\pi D_H^2}{4} t w\right] \qquad \text{[wT-2]}$$

where D_H mean diameter of housing

L = length of housing

t = wall thickness of housing

w = specific weight of housing material

$$W_H = \pi D_H t w\left[\frac{D_H}{2} + L\right] \qquad \text{[wT-3]}$$

Total Weight = W_T = $W + W_H$

$$W_T = \frac{4 g H}{D_M^2 \dot{\theta}} + \pi D_H t w\left[\frac{D_H}{2} + L\right]$$

$$D_H = D_M + 2[t + c]$$

where C = clearance between wheel and housing
= 0.125 inches

$$W_T = \frac{4 g H}{D_M^2 \dot{\theta}} + \pi t w\left[D_M + 2(t + c)\right]\left[\frac{D_M + 2(t + c)}{2} + L\right]$$

for t = 0.250 inches [for both housing and reaction wheel]

$$D_H = D_M + 2[.250 + .125]$$

$$D_H = D_M + 0.750$$

$$W_T = \frac{4\,g\,H}{D_M^{\,2}\,\dot{\theta}} + \pi\,t\,w\,\left[D_M + 0.750\right]\left[\frac{D_M + 0.750}{2} + L\right]$$

$$W_T = \frac{4\,g\,H}{D_M^{\,2}\,\dot{\theta}} + \pi\,t\,w\,\left[D_M + 0.750\right]\left[\frac{D_M}{2} + 0.375 + L\right]$$

Reaction Wheel Weight

$$W = \frac{4\,g\,H}{D_M^{\,2}\,\dot{\theta}} \qquad [wT-4]$$

$$W = \pi\,D_M\,t\,L\,w \qquad [wT-5]$$

from [wT-4]

$$W = \frac{(4)\ (32.2)\ (3.24)}{D_M^{\,2}\,\dot{\theta}}$$

$$W = \frac{415}{D_M^{\,2}\,\dot{\theta}}$$ for worst condition

from [wT-5]

$$L = \frac{W}{\pi\,D_M\,t\,w}$$

$$L = \frac{W}{(\pi)\ (.250)\ (D_M\,w)}$$

$$L = 1.27\,\frac{W}{D_M\,w}$$

REACTION WHEEL WEIGHT FOR WHEEL VELOCITIES OF

500 rpm (52.4 rad/sec); 1000 rpm (104.8 rad/sec); 1500 rpm (157.2 rad/sec)

FOR H = 3.24 Ft. Lb. Sec.

$$W = \frac{415}{D_M^2 \dot{\theta}}$$

Mean Dia. (Inches)	1 D_M (F_t)	2 D_M^2 (F_t^2)	3 $\frac{415}{D_M^2}$ (Lb/Sec)	4 Weight $3/\dot{\theta}$ (Pounds) $\dot{\theta} =$ 52.4	$\dot{\theta} =$ 104.8	$\dot{\theta} =$ 157.2
6	0.5	0.25	1660	31.7	15.8	10.6
8	0.67	0.45	923	17.6	8.8	5.9
10	0.83	0.69	602	11.5	5.8	3.8
12	1.00	1.00	415	7.9	4.0	2.6
14	1.17	1.37	303	5.8	2.9	1.9
16	1.33	1.77	234	4.5	2.2	1.5
18	1.50	2.25	184	3.5	1.8	1.2

DETERMINATION OF LENGTH OF REACTION WHEEL "L"

FOR W = 0.283 lb/in (STEEL)

H = 3.24 Ft. Lb. Sec.

$L = 1.27 \frac{W}{D_M w}$

Mean Dia. Inches	$\frac{W}{D_M}\left[\frac{lb}{ins}\right]$			$1.27\frac{W}{D_M}\left[\frac{lb}{ins}\right]$			$\underset{(Inches)}{L} = \left(1.27\frac{W}{D_M}\right)\left(\frac{1}{w}\right)$		
	$\dot{\theta} = 52.4$	$\dot{\theta} = 104.8$	$\dot{\theta} = 157.2$	$\dot{\theta} = 52.4$	$\dot{\theta} = 104.8$	$\dot{\theta} = 157.2$	$\dot{\theta} = 52.4$	$\dot{\theta} = 104.8$	$\dot{\theta} = 157.2$
6	5.3	2.7	1.8	6.7	3.4	2.3	23.7	11.8	7.9
8	2.2	1.1	0.7	2.8	1.4	0.9	9.9	5.0	3.3
10	1.2	0.6	0.4	1.5	0.7	0.5	5.2	2.6	1.7
12	0.7	0.3	0.2	0.8	0.4	0.3	3.0	1.5	1.0
14	0.4	0.2	0.14	0.5	0.3	0.18	1.9	0.9	0.6
16	0.3	0.14	0.09	0.36	0.18	0.12	1.25	0.6	0.4
18	0.2	0.10	0.07	0.25	0.12	0.08	0.9	0.4	0.3

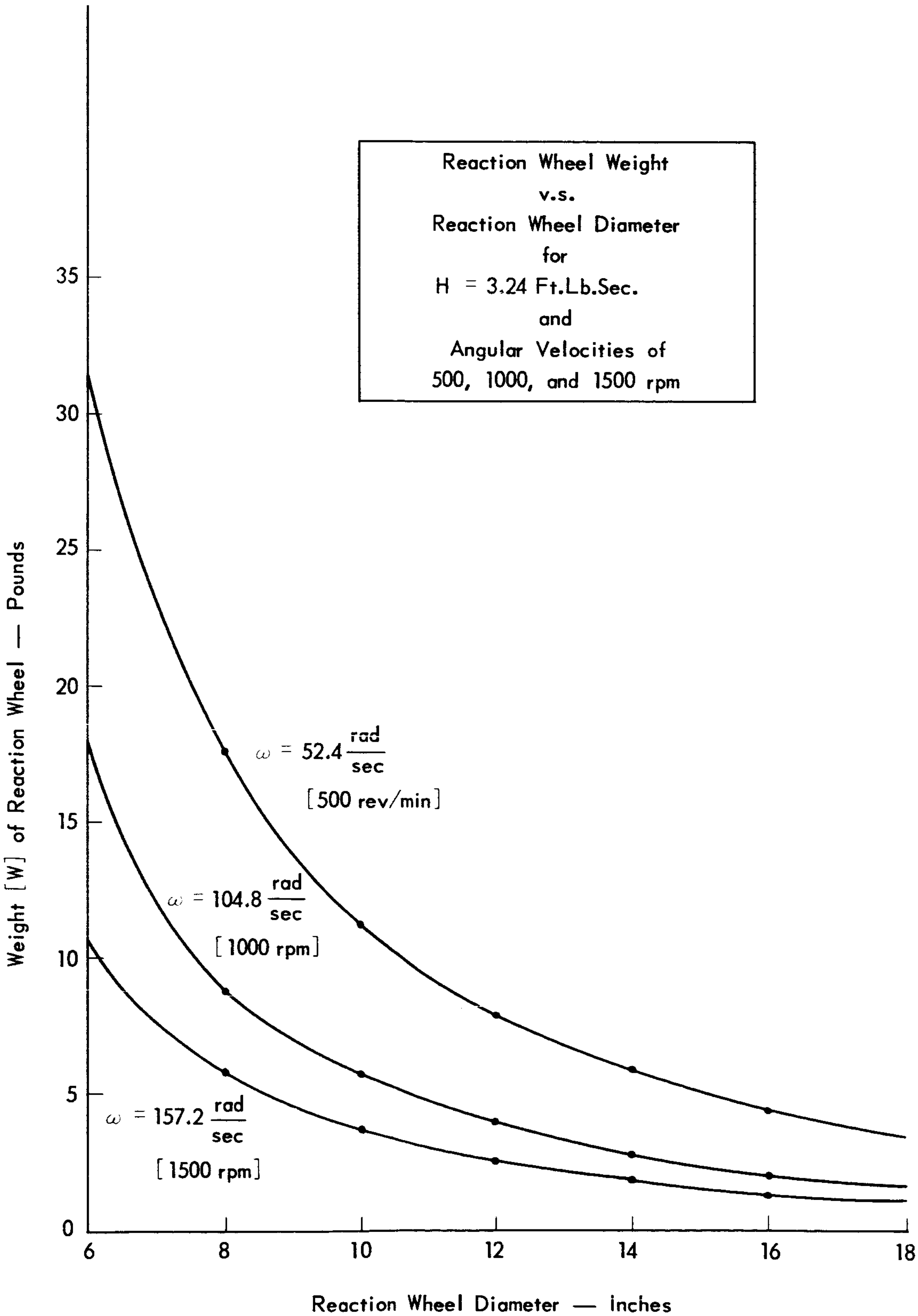
HOUSING WEIGHT FOR H = 3.24 Ft. Lb. Sec.

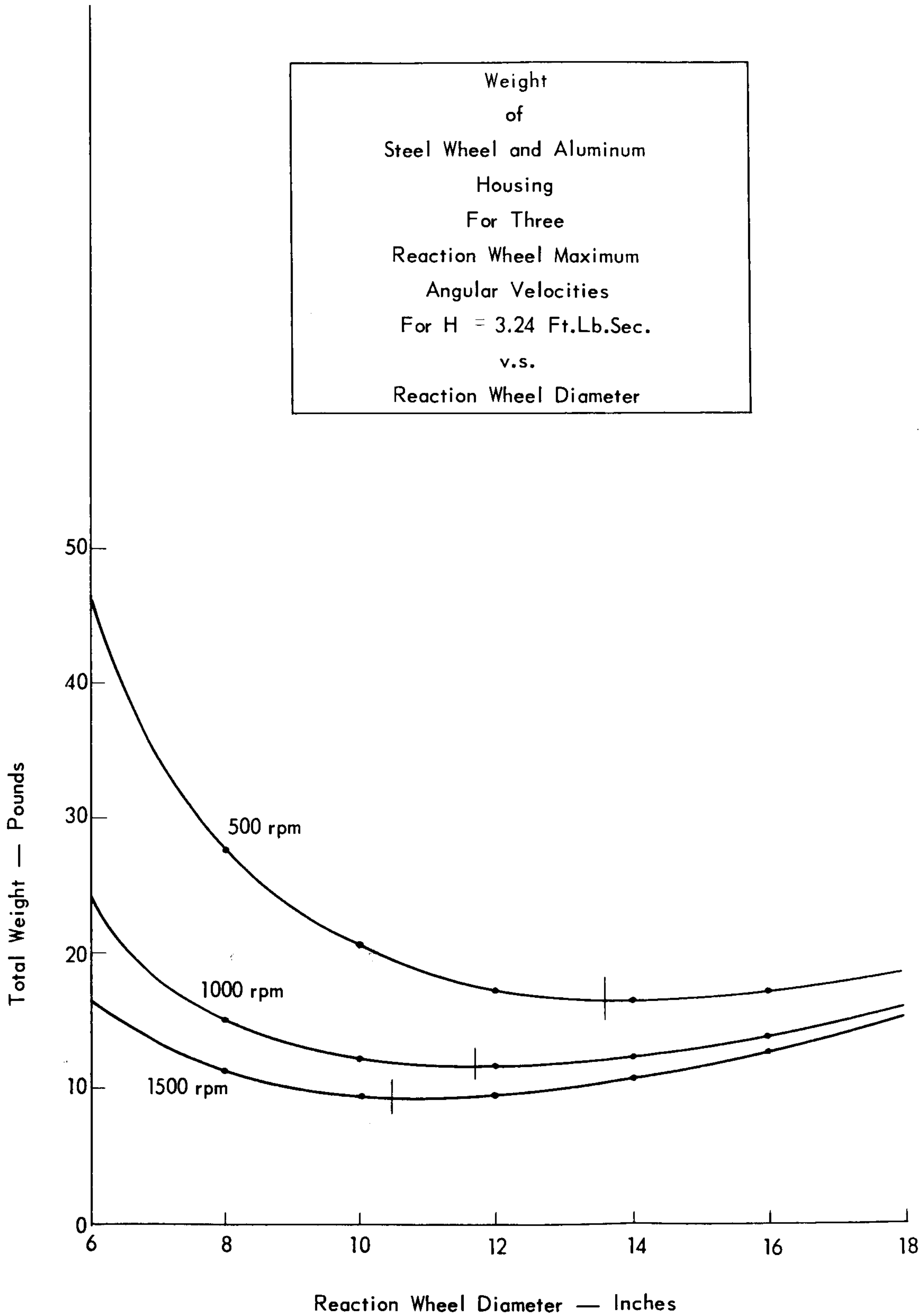
MATERIAL, ALUMINUM, w = 0.100 lb/in^3

(1) D_M (Inches)	(2) $(D_M + .750)$	(3) $\frac{D_M}{2}$	(4) $\frac{D_M}{2} + .375$	(5) $\left[\frac{D_M}{2}\right.$ $\dot{\theta}$ = 52.4	(6) + 0.375 $\dot{\theta}$ = 104.8	(7) $\left.+ L\right]$ $\dot{\theta}$ = 157.2	(8) (2) × (5) $\dot{\theta}$ = 52.4 $(IN)^2$	(9) (2) × (6) $\dot{\theta}$ = 104.8 $(IN)^2$	(10) (2) × (7) $\dot{\theta}$ = 157.2 $(IN)^2$
6	6.75	3	3.375	27.1	15.1	11.2	182.0	102.0	75.7
8	8.75	4	4.375	14.3	9.1	7.7	125.0	79.6	67.3
10	10.75	5	5.375	10.5	8.0	7.1	114.0	86.0	76.4
12	12.75	6	6.375	9.3	7.9	7.4	118.5	101.0	94.5
14	14.75	7	7.375	9.2	8.3	8.0	136.0	122.5	118.0
16	16.75	8	8.375	9.6	9.0	8.8	161.0	150.5	147.0
18	18.75	9	9.375	10.3	9.8	9.7	193.0	184.0	182.0

(1) D_M (Inches)	(11) $(\pi t w)$ lb/in^2	(12) Housing Weight Lbs. (11) × (8) $\dot{\theta}$ = 52.4	(13) (11) × (9) $\dot{\theta}$ = 104.8	(14) (11) × (10) $\dot{\theta}$ = 157.2	Total Weight (Pounds) (12) × (W) $\dot{\theta}$ = 52.4	(13) × (W) $\dot{\theta}$ = 104.8	(14) × (W) $\dot{\theta}$ = 157.2
6	0.0785	14.3	8.0	5.9	46.0	23.9	16.6
8	↓	9.8	6.3	5.3	27.4	15.1	11.2
10	↓	9.0	6.8	6.0	20.5	12.5	9.8
12	↓	9.3	7.9	7.4	17.2	11.9	10.1
14	↓	10.7	9.6	9.3	16.5	12.5	11.2
16	↓	12.6	11.8	11.5	17.1	14.0	13.0
18	0.0785	15.1	14.5	14.3	18.6	16.2	15.5

← W from P. 14


Reaction Wheel Weight
v.s.
Reaction Wheel Diameter
for
H = 3.24 Ft.Lb.Sec.
and
Angular Velocities of
500, 1000, and 1500 rpm
$\omega = 52.4 \frac{rad}{sec}$
[500 rev/min]
$\omega = 104.8 \frac{rad}{sec}$
[1000 rpm]
$\omega = 157.2 \frac{rad}{sec}$
[1500 rpm]
Weight [W] of Reaction Wheel — Pounds
Reaction Wheel Diameter — Inches
0
5
10
15
20
25
30
35
6
8
10
12
14
16
18


Weight
of
Steel Wheel and Aluminum
Housing
For Three
Reaction Wheel Maximum
Angular Velocities
For H = 3.24 Ft.Lb.Sec.
v.s.
Reaction Wheel Diameter
500 rpm
1000 rpm
1500 rpm
Total Weight — Pounds
0
10
20
30
40
50
6
8
10
12
14
16
18
Reaction Wheel Diameter — Inches

OPTIMIZATION OF REQUIRED POWER BASED ON MINIMUM WHEEL WEIGHT

The total weight involved includes the reaction wheel and housing plus 2.5 pounds of solar paddles and related equipment per watt of power required. Electrical and mechanical efficiency is taken as 60%. The slewing maneuver is assumed to occur an average of 4% of the time on a daily basis. The combined time for acceleration and deceleration is assumed to be 40% of the time required for the slewing maneuver. The total weight determination is based upon the minimum weight for each wheel speed from page 17.

a. Power = $T_o \times \omega_{max}$ [Ft. Lb./Sec.]

b. Watts = $\dfrac{\text{Power}}{0.738} \left[\dfrac{\text{Foot Pounds/Sec.}}{\text{Foot Pounds/Watt, Sec.}}\right]$

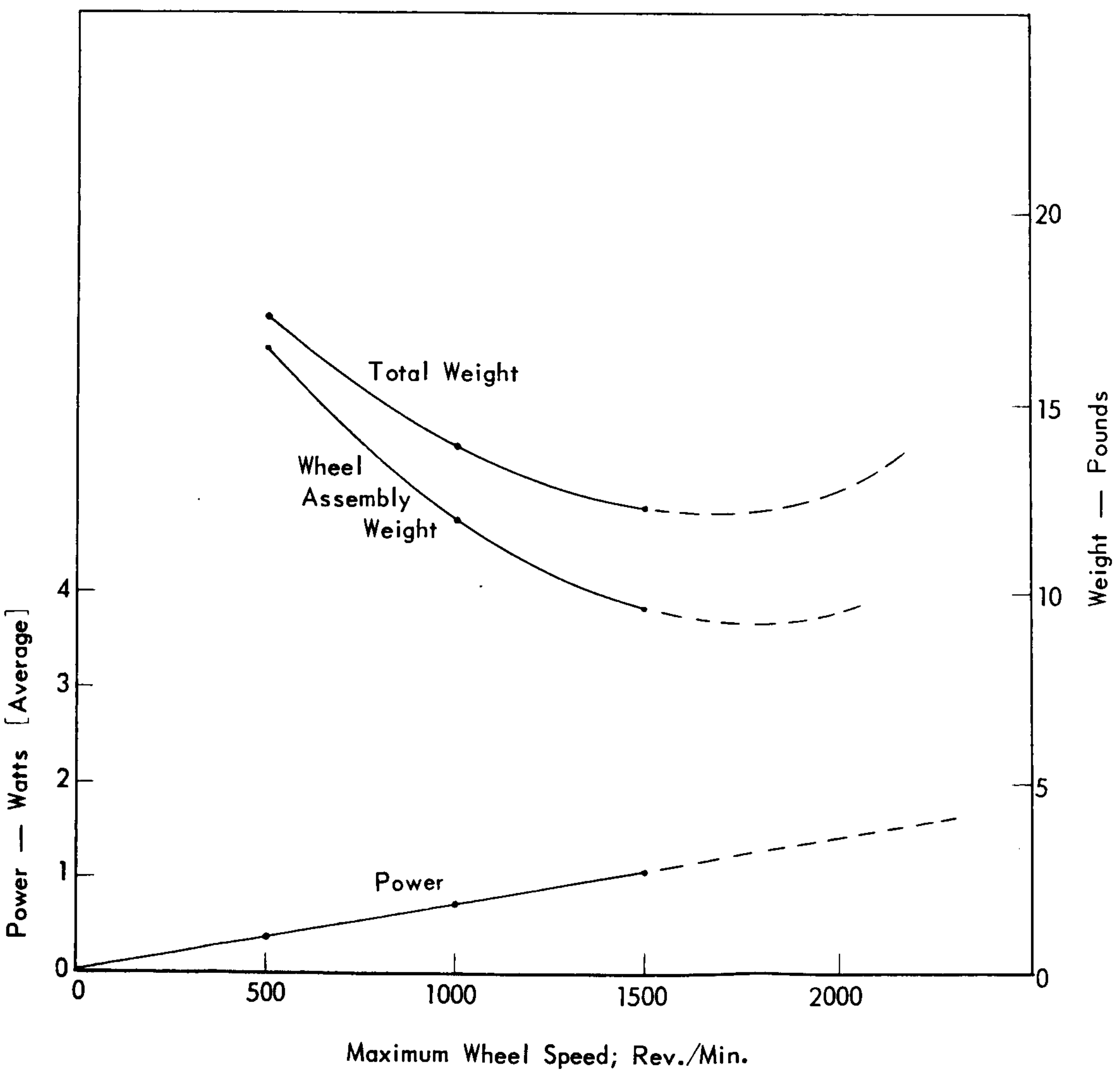
c. Watts required = $\dfrac{\text{Watts} \times 0.40 \times 0.04}{0.60}$ = (0.027) (Watts). Daily Average

d. Weight based on power = 2.5 × watts required

OPTIMIZED WEIGHT CALCULATIONS

① Revs./Min.	② ω rad/sec.	③ To Ft.Lb.	④ Power ② × ③ Ft.Lb./Sec.	⑤ Power Watts ④/.738	⑥ Power Watts Avg. .027×⑤	⑦ Weight Pounds ⑥ × 2.5	⑧ Wheel & Housing Weight	⑨ Total Weight Pounds ⑦ + ⑧
500	52.4	0.18	9.44	12.8	0.34	0.85	16.5	17.35
1000	104.8	0.18	18.9	25.6	0.68	1.70	12.0	13.70
1500	157.2	0.18	28.3	38.3	1.02	2.55	9.5	12.05

Weight and Power
Optimization
for
Steel Reaction Wheel
Aluminum Housing
and
Power Equipment



DISCUSSION AND CONCLUSIONS

The analysis presented here is one approach to the optimization of the design of a reaction wheel for a satellite. In this case the optimization is based on weight and power.

In this analysis the torque driving the reaction wheel was taken as constant over the working range. Restricting the analysis to a motor of constant driving torque simplifies the approach and final form for the result. A previous analysis by DeMarinis and Huttenlocher of the Grumman Aircraft Engineering Corporation was for the case where torque varied inversely with speed. For the constant output torque motor the various electrical and mechanical losses are overcome by supplying additional power sufficient to maintain the constant torque required of the reaction wheel.

For this analysis the conditions under which the satellite-reaction wheel assembly operate were assumed to satisfy the conservation of angular momentum criteria. The motion was defined as being divided between a period of constant acceleration, a period of constant velocity, and a period of constant deceleration. For the motions assumed, the design of the reaction wheel assembly was optimized for weight and power. The weight includes not only the motor, reaction wheel and its housing but related electrical equipment as well (solar array paddles, etc.) The weight of the related electrical equipment was taken as 2.5 pounds per watt of electrical power.

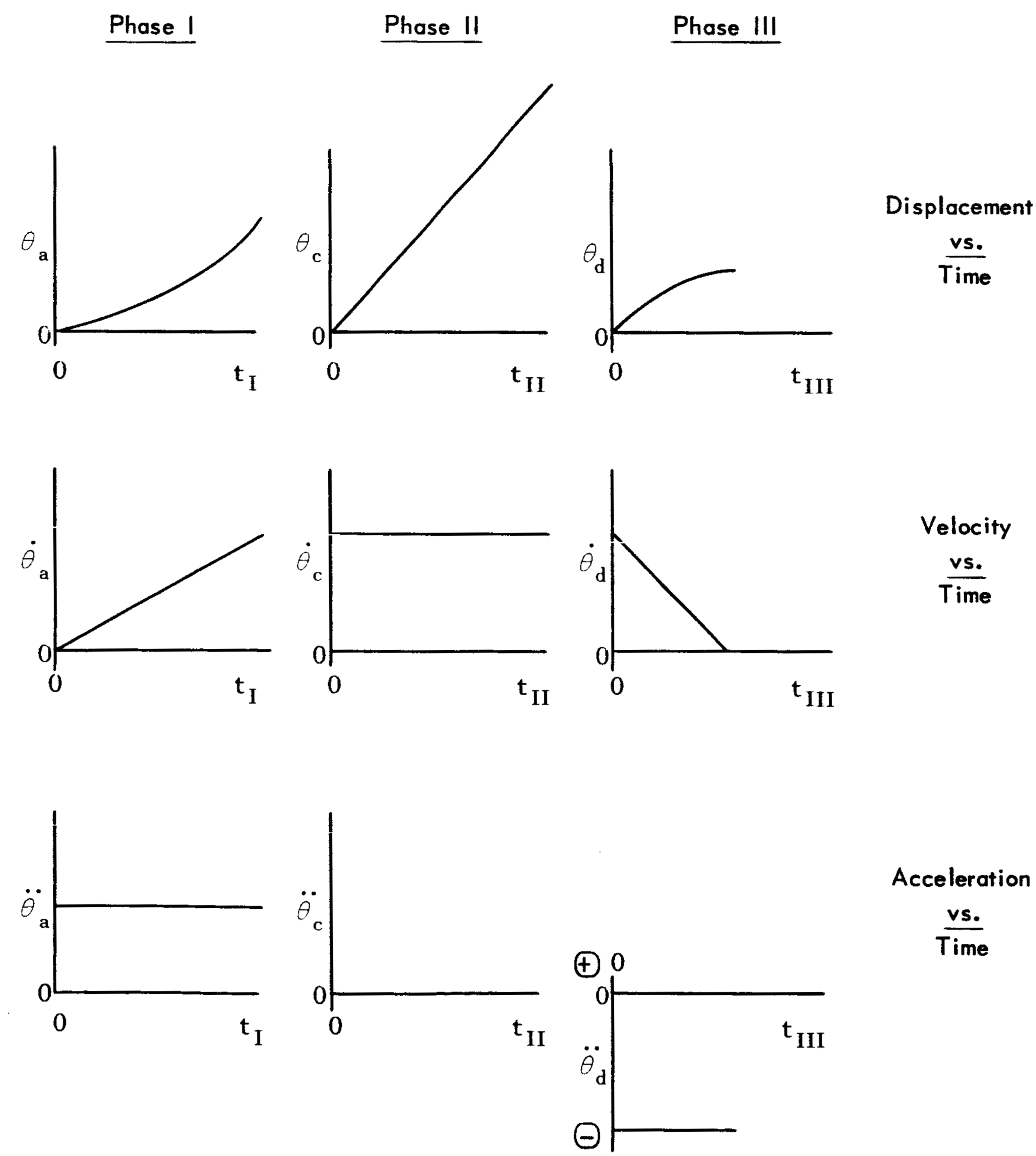
The maximum angular momentum for two rates of angular motion of the satellite (2° in 30 seconds, and 30° in 180 seconds) was determined as a function of the required torque (T_0). A torque (T_0) of 0.18 foot pounds was selected as a typical value and the weight of a steel reaction wheel plus that of a suitable aluminum housing was optimized (minimized) against the wheel diameter for three wheel speeds (500, 1000, and 1500 revolutions per minute). It was assumed in each case that the time for acceleration of the reaction wheel equalled the time for deceleration of the wheel.

The results presented in this report are for specific boundary conditions although the method established in this analysis is general. It is intended that this analysis can be used for any other set of values within the framework of the parameters established for this study.

APPENDIX

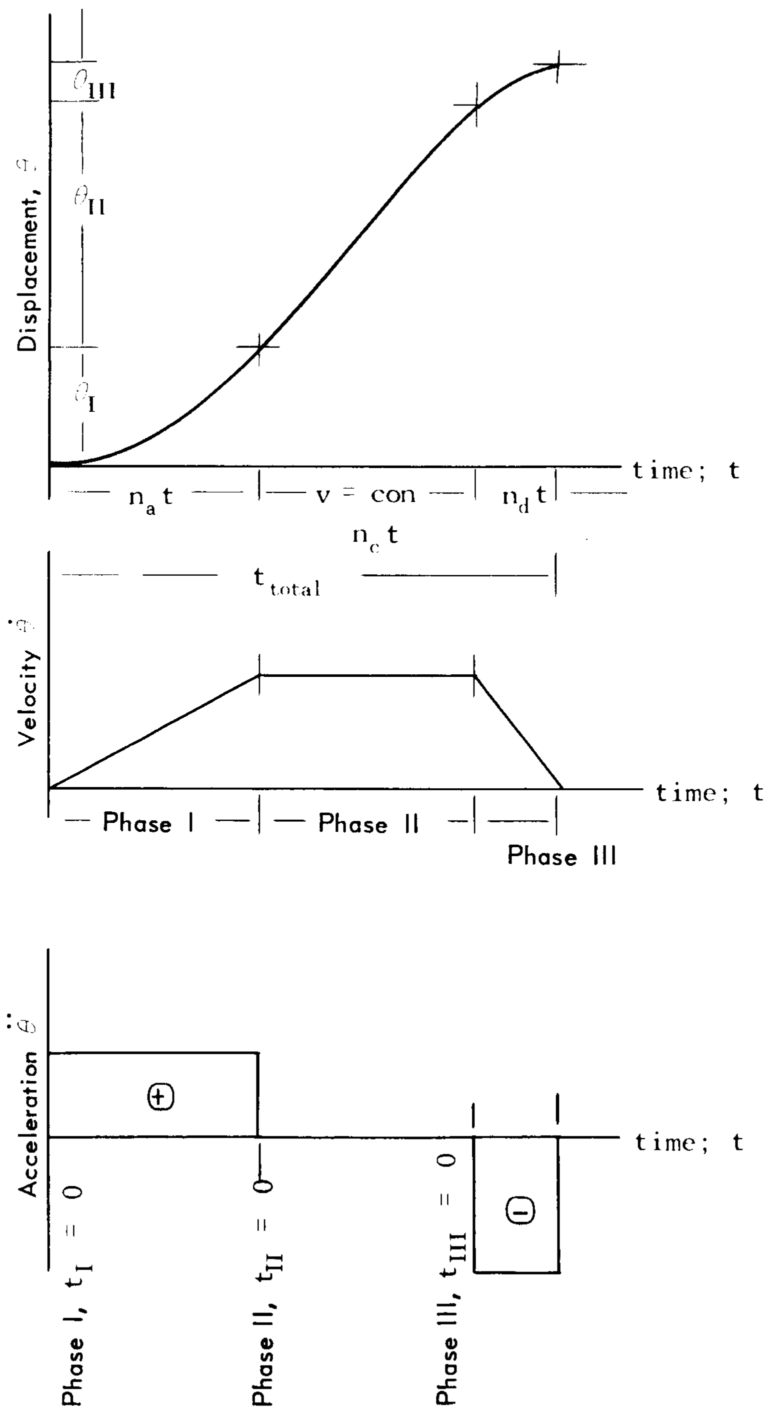
DISPLACEMENT, VELOCITY, AND ACCELERATION VS. TIME

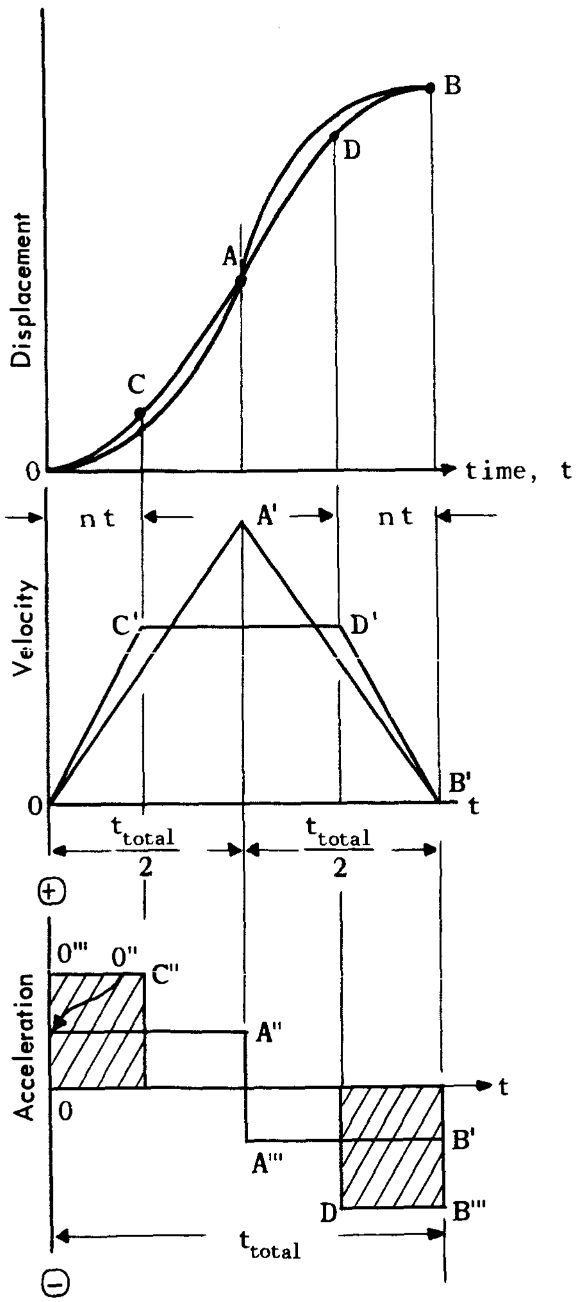
General Curves



DISPLACEMENT, VELOCITY, AND ACCELERATION VS. TIME

General Curves Combined





For a specified reaction wheel for which $t_a = t_d$ the displacement-time, velocity-time, and acceleration-time curves would be similar to the ones shown here.

When the torque, To, is at a minimum value the wheel would accelerate (0'' - A'') for $\frac{1}{2}\ t_{total}$ and decelerate (A''' - B') for $\frac{1}{2}\ t_{total}$. The velocity-time curve would be (0 - A') and (A' to B'). The displacement-time curve would be (0 - A) and (A - B).

When To is increased the acceleration will increase but the time required for the acceleration will decrease. The reaction wheel will arrive at a lower maximum velocity in less time than required for the case when To is a minimum. V_{max} for the increased To is represented by C'D' on the velocity curve plot whereas V_{max} is A' for minimum To. The product of the average velocity and total time is the same for both cases, i.e., the areas under the two velocity-time curves are equal. This is another way of stating that the impulses are equal.

In the preceeding report, three wheel speeds were selected based on previous work in the area and the moment of inertia of the reaction wheel was varied to meet the limitations imposed by selecting values for To.