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LONG PERIOD LUNAR AND SOLAR EFFECTS ON THE MOTION OF RELAY 2

BY

THEODORE L. FELSENTREGER

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SUMMARY

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Long period perturbations in the orbital elements of Relay 2, arising from luni-solar gravitation and solar radiation pressure, are analyzed. An analysis is presented of the effects on the eccentricity and inclination caused by the existence of a near-resonant condition in two of the trigonometric terms in the luni-solar disturbing function.

Author

CONTENTS

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INTRODUCTION

In a previous paper (see Reference 1), a study was made concerning the influence of long period luni-solar gravitational effects and solar radiation pressure on the motion of close earth satellites. The formulas presented therein were applied to the Relay 1 and Telstar 2 satellites — as a consequence, the observed long period variations in the orbital elements of the satellites were found to be attributable to lunar and solar forces.

Similar variations in the orbital elements of Relay 2 have been noted. However, the problem is complicated by the existence of a near-resonant condition caused by a longitude of perigee which is practically constant for the time interval under study (period is about 550 years). It will be shown that this condition causes almost secular increases in the eccentricity and, to a much lesser extent, in the inclination.

This paper, then, will show that the principal observed long period variations in the orbital elements of Relay 2 are caused by lunar and solar effects. In addition, the secular motion of the longitude of perigee and the remaining variations in eccentricity are explained through the use of values for the zonal harmonic coefficients J_3 and J_4 in line with recent determinations. Data for Relay 2 spanning a period of approximately 654 days was used — this data consists of mean values of orbital elements computed at the Goddard Space Flight Center, using formulas given by Brouwer (Reference 2) and tracking data obtained from the minitrack network. A list of symbols is presented in Appendix A.

LUNAR AND SOLAR PERTURBATION FORMULAS

The formulas for the long period luni-solar perturbations in the orbital elements as presented in Reference 1 were used here. However, the precaution was taken to exclude the near-resonant terms $2(\Omega_{\odot} - g - h)$ and $2(\Omega_{☾} - g - h)$ from the initial computations. The effects caused by these terms were left to a separate analysis. Elements which have been corrected for luni-solar effects, excluding the near-resonant terms, are designated by the subscript "c". Values for e_c, i_c, g_c, and h_c, in addition to mean values, may be found in Tables 1, 2, 3, and 4 (Appendix B), respectively.

The portion of the luni-solar gravitational disturbing function which includes the two near-resonant terms is

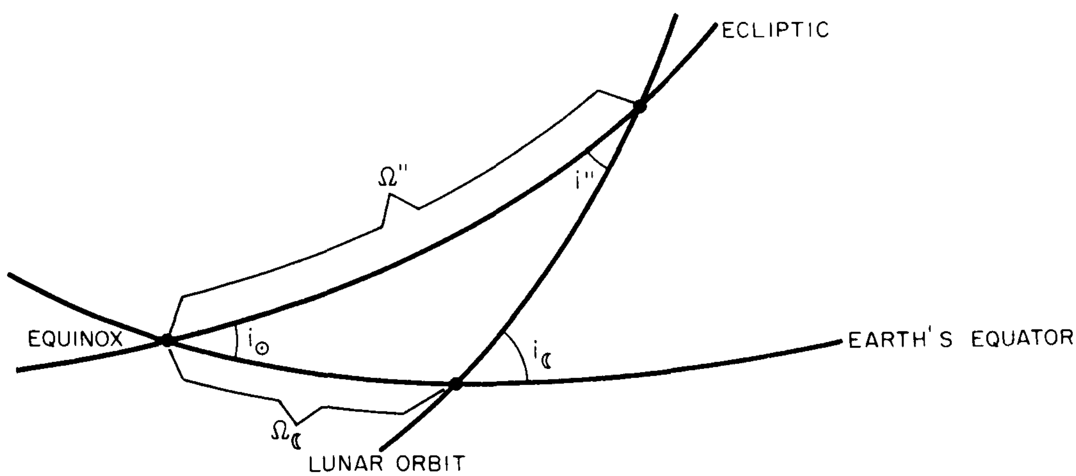
$$F_R = \frac{15}{64} a^2 e^2 (1 + \cos i)^2 \left[n_\odot^2 m_\odot \sin^2 i_\odot \cos 2(\Omega_\odot - g - h) + n_☾^2 m_☾ \sin^2 i_☾ \cos 2(\Omega_☾ - g - h)\right]. \tag{1}$$

The method of integration employed in Reference 1 cannot be directly applied to the "moon" term in its present form inasmuch as the motions of $i_☾$, $\Omega_☾$, and $g + h$ are all commensurate; in addition, since $i_☾$ and $\Omega_☾$ have been referred to the earth's equatorial plane, their motions are not sufficiently constant over the time interval under study to allow accurate computation of the perturbations.

However, using the ecliptic as a reference plane the "moon" term can be rewritten in terms of angular variables having motions which are essentially constant. First,

$$\sin^2 i_☾ \cos 2(\Omega_☾ - g - h) = \sin^2 i_☾ \left[\cos 2\Omega_☾ \cos 2(g + h) + \sin 2\Omega_☾ \sin 2(g + h)\right]. \tag{2}$$

The necessary transformations can be derived making use of the following diagram:



Hence,

$$\cos i_☾ = \cos i'' \cos i_\odot - \sin i'' \sin i_\odot \cos \Omega''$$

$$\sin \Omega_{\mathbb{C}} = \frac{\sin i'' \sin \Omega''}{\sin i_{\mathbb{C}}}$$

$$\cos \Omega_{\mathbb{C}} = \frac{\cos i'' - \cos i_{\odot} \cos i_{\mathbb{C}}}{\sin i_{\odot} \sin i_{\mathbb{C}}} .$$

Consequently,

$$\sin^2 i_{\mathbb{C}} \cos 2\Omega_{\mathbb{C}} = \sin^2 i_{\odot} \left(\cos^2 i'' - \frac{1}{2} \sin^2 i''\right)$$
$$+ 2 \sin i'' \cos i'' \sin i_{\odot} \cos i_{\odot} \cos \Omega''$$
$$+ \sin^2 i''\left(\cos^2 i_{\odot} + \frac{1}{2} \sin^2 i_{\odot}\right) \cos 2 \Omega''$$

$$\sin^2 i_{\mathbb{C}} \sin 2\Omega_{\mathbb{C}} = 2 \sin i'' \cos i'' \sin i_{\odot} \sin \Omega'' + \sin^2 i'' \cos i_{\odot} \sin 2 \Omega''.$$

Upon substitution of these expressions into Equation 2, Equation 1 becomes

$$F_R = \frac{15}{64} a^2 e^2 (1 + \cos i)^2 \left\{ n_{\odot}^2 m_{\odot} \sin^2 i_{\odot} \cos 2(\Omega_{\odot} - g - h) \right.$$
$$+ n_{\mathbb{C}}^2 m_{\mathbb{C}} \left[\sin^2 i_{\odot} \left(\cos^2 i'' - \frac{1}{2} \sin^2 i''\right) \cos 2(g+h) - \sin i'' \cos i'' \sin i_{\odot} \cdot \right.$$
$$\cdot (1 - \cos i_{\odot}) \cos(\Omega'' + 2g + 2h) + \sin i'' \cos i'' \sin i_{\odot} (1 + \cos i_{\odot}) \cos(\Omega'' - 2g - 2h)$$
$$\left.\left. + \frac{1}{4} \sin^2 i'' (1 - \cos i_{\odot})^2 \cos 2(\Omega'' + g + h) + \frac{1}{4} \sin^2 i'' (1 + \cos i_{\odot})^2 \cos 2(\Omega'' - g - h) \right] \right\} . \tag{3}$$

Here, $i'' = 5^\circ\!.1453964$, and Ω'' has an essentially constant motion (period is approximately 18.6 years).

The variation equations for e and i are

$$\frac{d(\delta e)_R}{dt} = -\frac{\sqrt{1-e^2}}{e\sqrt{a}} \frac{\partial F_R}{\partial g}$$

$$\frac{d(\delta i)_R}{dt} = \frac{\cos i}{\sqrt{a}\sqrt{1-e^2}\sin i} \frac{\partial F_R}{\partial g} - \frac{1}{\sqrt{a}\sqrt{1-e^2}\sin i}\frac{\partial F_R}{\partial h}. \tag{4}$$

Substituting Equation 3 into Equations 4 and integrating, the result is

$$(\delta e)_R = \frac{15}{32} a\sqrt{a}\; e\sqrt{1-e^2}(1+\cos i)^2 \Bigg\{ -\frac{n_\odot^2 m_\odot \sin^2 i_\odot}{2(\dot{g}+\dot{h})} \cos 2(\Omega_\odot - g - h)$$

$$+ n_{\mathbb{C}}^2 m_{\mathbb{C}} \Bigg[-\frac{\sin^2 i_\odot \cos^2 i'' - \frac{1}{2}\sin^2 i''}{2(\dot{g}+\dot{h})} \cos 2(g+h)$$

$$+ \frac{\sin i'' \cos i'' \sin i_\odot (1-\cos i_\odot)}{\dot{\Omega}'' + 2(\dot{g}+\dot{h})} \cos(\Omega'' + 2g + 2h)$$

$$+ \frac{\sin i'' \cos i'' \sin i_\odot (1+\cos i_\odot)}{\dot{\Omega}'' - 2(\dot{g}+\dot{h})} \cos(\Omega'' - 2g - 2h)$$

$$- \frac{\sin^2 i''(1-\cos i_\odot)^2}{8(\dot{\Omega}''+\dot{g}+\dot{h})} \cos 2(\Omega'' + g + h)$$

$$+ \frac{\sin^2 i''(1+\cos i_\odot)^2}{8(\dot{\Omega}''-\dot{g}-\dot{h})} \cos 2(\Omega'' - g - h) \Bigg] \Bigg\}$$

$$(\delta i)_R = \frac{e(1-\cos i)}{(1-e^2)\sin i} (\delta e)_R. \tag{5}$$

Similar expressions for $(\delta g)_R$ and $(\delta h)_R$ involve integration of $(\delta e)_R$ and $(\delta i)_R$, and produce perturbations which are far too large. Hence, the perturbations for g and h cannot be determined in this fashion.

However, there still remains to be found a suitable value for the motion of the longitude of perigee $g + h$ before $(\delta e)_R$ and $(\delta i)_R$ can be computed.

LONGITUDE OF PERIGEE

It was noted that the longitude of perigee has a period of about 550 years. In order to obtain a sufficiently accurate value for the motion of $g + h$, the corrected values $g_c + h_c$ (Table 4) were fit (by least squares) to an expression of the form

$$A_0 + A_1 (t - t_0)$$

with the result

$A_0 = 48^\circ\!.32003$

$A_1 = 1^\circ\!.7428435 \times 10^{-3}$ /day.

It was further noted that, if one attempts to compute a theoretical value for A_1, one finds that such a value is quite sensitive to the number used for J_4. Let the secular disturbing function be

$$F_S = -\frac{J_2}{4L^3G^3}\left(1 - 3\frac{H^2}{G^2}\right) + \frac{J_2^2}{L^{10}}\left[\frac{3}{128}\frac{L^5}{G^5}\left(5 - 18\frac{H^2}{G^2} + 5\frac{H^4}{G^4}\right)\right.$$

$$\left. + \frac{3}{32}\frac{L^6}{G^6}\left(1 - 6\frac{H^2}{G^2} + 9\frac{H^4}{G^4}\right) - \frac{15}{128}\frac{L^7}{G^7}\left(1 - 2\frac{H^2}{G^2} - 7\frac{H^4}{G^4}\right)\right]$$

$$-\frac{3}{128}\frac{J_4}{L^{10}}\left(5\frac{L^7}{G^7} - 3\frac{L^5}{G^5}\right)\left(3 - 30\frac{H^2}{G^2} + 35\frac{H^4}{G^4}\right)$$

$$-\frac{1}{32}\left[n_\odot^2 m_\odot (2 - 3\sin^2 i_\odot) + n_☾^2 m_☾ (2 - 3\sin^2 i_☾)\right] L^4 \left(5 - 3\frac{G^2}{L^2}\right)\left(1 - 3\frac{H^2}{G^2}\right). \tag{6}$$

which includes the contributions from the earth's zonal harmonics through J_4 and the principal contributions from the sun and moon. Here, L, G, H are the coordinate variables in the Delaunay set (L, G, H, ℓ, g, h). Under the canonical transformation

$$
\begin{aligned}
L &= L' & \ell &= \ell' \\
G &= G' & g &= g' - h' \\
H &= G' + H' & h &= h',
\end{aligned}
$$

one obtains

$$
\begin{aligned}
F &= F_S + F_R \\
&= \frac{1}{4}\frac{J_2}{L'^3 G'^3}\left(2 + 6\frac{H'}{G'} + 3\frac{H'^2}{G'^2}\right) \\
&+ \frac{J_2^2}{L'^{10}}\left[-\frac{3}{128}\frac{L'^5}{G'^5}\left(8 + 16\frac{H'}{G'} - 12\frac{H'^2}{G'^2} - 20\frac{H'^3}{G'^3} - 5\frac{H'^4}{G'^4}\right)\right. \\
&+ \frac{3}{32}\frac{L'^6}{G'^6}\left(4 + 24\frac{H'}{G'} + 48\frac{H'^2}{G'^2} + 36\frac{H'^3}{G'^3} + 9\frac{H'^4}{G'^4}\right) \\
&+ \left.\frac{15}{128}\frac{L'^7}{G'^7}\left(8 + 32\frac{H'}{G'} + 44\frac{H'^2}{G'^2} + 28\frac{H'^3}{G'^3} + 7\frac{H'^4}{G'^4}\right)\right] \\
&- \frac{3}{128}\frac{J_4}{L'^{10}}\left(5\frac{L'^7}{G'^7} - 3\frac{L'^5}{G'^5}\right)\left(8 + 80\frac{H'}{G'} + 180\frac{H'^2}{G'^2} + 140\frac{H'^3}{G'^3} + 35\frac{H'^4}{G'^4}\right) \\
&+ \frac{1}{32}\left[n_\odot^2 m_\odot (2 - 3\sin^2 i_\odot) + n_☾^2 m_☾ (2 - 3\sin^2 i_☾)\right] L'^4\left(5 - 3\frac{G'^2}{L'^2}\right)\left(2 + 6\frac{H'}{G'} + 3\frac{H'^2}{G'^2}\right) \\
&+ \frac{15}{64} L'^4\left(1 - \frac{G'^2}{L'^2}\right)\left(2 + \frac{H'}{G'}\right)^2 \left[n_\odot^2 m_\odot \sin^2 i_\odot \cos 2(\Omega_\odot - g')\right. \\
&\qquad \left. + n_☾^2 m_☾ \sin^2 i_☾ \cos 2(\Omega_☾ - g')\right].
\end{aligned}
$$

Then

$$\dot{g}' = -\frac{\partial F}{\partial G'}$$

$$= \frac{3}{4}\frac{J_2}{L'^3 G'^4}\left(2 + 8\frac{H'}{G'} + 5\frac{H'^2}{G'^2}\right) + \frac{3}{128}\frac{J_2^2}{L'^{10} G'}\left[-\frac{L'^5}{G'^5}\left(40 + 96\frac{H'}{G'} - 84\frac{H'^2}{G'^2}\right.\right.$$

$$\left. - 160\frac{H'^3}{G'^3} - 45\frac{H'^4}{G'^4}\right) + 24\frac{L'^6}{G'^6}\left(4 + 28\frac{H'}{G'} + 64\frac{H'^2}{G'^2} + 54\frac{H'^3}{G'^3} + 15\frac{H'^4}{G'^4}\right)$$

$$\left. + 5\frac{L'^7}{G'^7}\left(56 + 256\frac{H'}{G'} + 396\frac{H'^2}{G'^2} + 280\frac{H'^3}{G'^3} + 77\frac{H'^4}{G'^4}\right)\right]$$

$$- \frac{15}{128}\frac{J_4}{L'^{10} G'}\left[\frac{L'^7}{G'^7}\left(56 + 640\frac{H'}{G'} + 1620\frac{H'^2}{G'^2} + 1400\frac{H'^3}{G'^3} + 385\frac{H'^4}{G'^4}\right)\right.$$

$$\left. - 3\frac{L'^5}{G'^5}\left(8 + 96\frac{H'}{G'} + 252\frac{H'^2}{G'^2} + 224\frac{H'^3}{G'^3} + 63\frac{H'^4}{G'^4}\right)\right]$$

$$+ \frac{3}{16}\left[n_\odot^2 m_\odot (2 - 3\sin^2 i_\odot) + n_☾^2 m_☾ (2 - 3\sin^2 i_☾)\right] \cdot$$

$$\cdot \frac{L'^4}{G'}\left[\frac{G'^2}{L'^2}\left(2 + 3\frac{H'}{G'}\right) + 5\left(\frac{H'}{G'} + \frac{H'^2}{G'^2}\right)\right]$$

$$+ \frac{15}{32}\frac{L'^4}{G'}\left[2\frac{G'^2}{L'^2}\left(2 + \frac{H'}{G'}\right) + \left(2\frac{H'}{G'} + \frac{H'^2}{G'^2}\right)\right]\left[n_\odot^2 m_\odot \sin^2 i_\odot \cos 2(\Omega_\odot - g')\right.$$

$$\left. + n_☾^2 m_☾ \sin^2 i_☾ \cos 2(\Omega_☾ - g')\right].$$

Choosing values of the orbital elements for about the middle of the 654 day period, and using the values for J_2 and J_4 adopted by Goddard (see Appendix A), the result is

$$\dot{g}' = (2.4683208 \times 10^{-3} + 2.9319824 \times 10^{-4} - 1.3199611 \times 10^{-3} - 6.2660326 \times 10^{-6}$$

$$- 1.2970913 \times 10^{-5} + 4.0201082 \times 10^{-5}) \text{ deg/day}$$

$$= 1^{\circ}.4625221 \times 10^{-3} \text{ /day;}$$

the various contributions are due, respectively, to the J_2, J_2^2, J_4, solar secular, lunar secular, and near-resonant parts. This value is about 16% off from that determined by least squares. However, use of the number -1.672×10^{-6} for J_4 will give the value $1^{\circ}.7428435 \times 10^{-3}$/day for $\dot{g}'$.. Such a J_4 value is in line with more recent determinations (see Reference 3).

ECCENTRICITY AND INCLINATION

The number $1^{\circ}.7428435 \times 10^{-3}$/day was used for $\dot{g}' = \dot{g} + \dot{h}$ in Equations 5. Values for $(\delta e)_R$ and $(\delta i)_R$ are listed in Tables 1 and 2, respectively, in addition to values for $e_c - (\delta e)_R$ and $i_c - (\delta i)_R$.

Figures 1 and 4 show that the principal periodic variations in e and i have been accounted for without taking the near-resonant terms into consideration. Moreover, it is clear that an apparent "secular" increase in eccentricity remains, in addition to a substantial perturbation of 300-plus day period. The same variations remain in the inclination, although they are not as apparent from Figure 4. However, Figure 1 also shows that the "secular" increase in e was due to the two near resonant terms discussed previously, inasmuch as the values of $e_c - (\delta e)_R$ do not indicate a steady increase. A cursory examination of the $(\delta i)_R$ values indicates that such an increase was present in the inclination, also.

It was noted that the remaining 300-plus day period variation in e was in phase with the argument of perigee, which has a period of about 325 days. Hence, a least squares fit to the expression

$$e_0 + B_1 \sin g_c + B_2 \cos g_c$$

was made for the values of $e_c - (\delta e)_R$ with the following result:

$e_0 = .23778226 \pm .00000136$

$B_1 = .00004119 \pm .00000196$

$B_2 = -.00000832 \pm .00000189.$

Values of $e_c - (\delta e)_R - B_1 \sin g_c - B_2 \cos g_c$ are listed in Table 1 and plotted in Figure 3. Figure 2 illustrates the closeness of the fit.

Hence, the remaining variation in e can be satisfactorily explained by a trigonometric term of 325 days period and amplitude $\sqrt{B_1^2 + B_2^2} = .00004202$. Moreover, the perturbation formula for e due to the third earth zonal harmonic contains such a term:

$$\delta e = -\frac{1}{2}\frac{J_3 \sin i}{J_2\, a} \sin g.$$

Using mean values for a and i and solving for a ΔJ_3 (assuming an amplitude of .00004202), one obtains

$$\Delta J_3 = -\frac{2(.00004202)\,(1.08219 \times 10^{-3})\,(1.745)}{.7233}$$

$$= -.219 \times 10^{-6};$$

this indicates a value for J_3 of -2.504×10^{-6}, which again is in line with recent determinations (Reference 3).

A similar least squares fit was made to the values of $i_c - (\delta i)_R$. However, the results were somewhat inconclusive since the amplitude of the trigonometric term was about the same magnitude as the mean error in i. A predicted amplitude can be computed from the amplitude for e, using the relation

$$\delta i = -\frac{e}{(1 - e^2)\tan i}\,\delta e,$$

with $\delta e = .00004202 \sin g$. Hence,

$$\delta i = -.00058 \sin g.$$

ARGUMENT OF PERIGEE AND LONGITUDE OF ASCENDING NODE

Tables 3 and 4 give the mean and corrected values of g and h, and also list residuals from least squares fits to expressions of the type

$$C_0 + C_1 (t - t_0).$$

The results of the least squares analyses:

$$g = 184^\circ.70999 + (1^\circ.1063884/\text{day}) (t - t_0)$$

$$g_c = 184^\circ.68544 + (1^\circ.1063814/\text{day}) (t - t_0)$$

$$h = 223^\circ.62537 - (1^\circ.1046913/\text{day}) (t - t_0)$$

$$h_c = 223^\circ.63397 - (1^\circ.1046373/\text{day}) (t - t_0).$$

Figures 5 and 6 indicate that the major periodic variations have been accounted for. However, it would appear that there remains an unexplained variation of quite long period in both g and h.

A SECOND METHOD FOR HANDLING THE NEAR-RESONANT TERMS

It will be recalled that the perturbations $(\delta e)_R$ and $(\delta i)_R$ arising from the near-resonant terms were able to be computed only after the expressions

$$\sin^2 i_{☾} \cos 2\Omega_{☾}, \quad \sin^2 i_{☾} \sin 2\Omega_{☾}$$

were written, using elementary spherical trigonometrical relations, as functions of parameters having essentially constant secular motions. However, it is quite conceivable that this cannot be achieved in the case of terms for which $i_{☾}$ appears in a function other than $\sin^2 i_{☾}$. In such instances, an empirical approach could be employed. To illustrate one such approach, the terms $\sin^2 i_{☾} \cos 2\Omega_{☾}$ and $\sin^2 i_{☾} \sin 2\Omega_{☾}$ are analyzed.

Figures 7 and 8 are plots of $\sin^2 i_{☾} \cos 2\Omega_{☾}$ and $\sin^2 i_{☾} \sin 2\Omega_{☾}$, respectively, vs. time, beginning January 1.0, 1964. These figures indicate a basic period of approximately 6800 days (18.6 years), which is also the period of Ω''; the appearance of such a period is not surprising if one has studied the geometry of the situation. It is highly probable that other terms will possess the same basic period. Therefore, a harmonic analysis fit to an expression of the type

$$Q_0 + \sum_{j=1}^{n} P_j \sin jx + \sum_{j=1}^{n} Q_j \cos jx$$

is indicated, where Q_0 is a constant and x has a period of 6800 days. Such fits were made, by means of least squares, for $\sin^2 i_{☾} \cos 2\Omega_{☾}$ and $\sin^2 i_{☾} \sin 2\Omega_{☾}$, and for $n = 10$ - data spanning a period of 6800 days at 100-day intervals were used. The results:

$$\begin{aligned}
\sin^2 i_{☾} \cos 2\Omega_{☾} = {} & .15636214 + .06393639 \sin x - .01284302 \cos x - .00285589 \sin 2x \\
& - .00683245 \cos 2x + .00000318 \sin 3x - .00000024 \cos 3x \\
& + .00000237 \sin 4x - .00000013 \cos 4x + .00000188 \sin 5x \\
& - .00000009 \cos 5x + .00000155 \sin 6x - .00000006 \cos 6x \\
& + .00000132 \sin 7x - .00000005 \cos 7x + .00000114 \sin 8x \\
& - .00000004 \cos 8x + .00000101 \sin 9x - .00000003 \cos 9x \\
& - .00000003 \cos 10x
\end{aligned}$$

$$\begin{aligned}
\sin^2 i_{☾} \sin 2\Omega_{☾} = {} & .00000011 + .01399949 \sin x + .06967810 \cos x + .00680809 \sin 2x \\
& - .00285014 \cos 2x + .00000087 \sin 3x - .00000001 \cos 3x \\
& + .00000060 \sin 4x - .00000002 \cos 4x + .00000046 \sin 5x \\
& - .00000001 \cos 5x + .00000039 \sin 6x + .00000032 \sin 7x \\
& - .00000001 \cos 7x + .00000027 \sin 8x + .00000025 \sin 9x.
\end{aligned}$$

Five-place accuracy was obtained.

Using these expressions, then, and designating the resulting perturbations in e and i by $(\delta e)_H$ and $(\delta i)_H$, one obtains

$$(\delta e)_H = -\frac{15}{32} a\sqrt{a}\, e\sqrt{1-e^2}\,(1+\cos i)^2 \left\{ \frac{n_\odot^2 m_\odot \sin^2 i_\odot}{2\dot{g}'} \cos 2(\Omega_\odot - g') \right.$$

$$+ \frac{1}{2} n_☾^2 m_☾ \left[\frac{.00000022}{2\dot{g}'} \sin 2g' + \frac{.31272428}{2\dot{g}'} \cos 2g' + \frac{.13361449}{\dot{z}+2\dot{g}'} \sin(z+2g') \right.$$

$$- \frac{.02684251}{\dot{z}+2\dot{g}'} \cos(z+2g') + \frac{.00574171}{\dot{z}-2\dot{g}'} \sin(z-2g')$$

$$- \frac{.00115647}{\dot{z}-2\dot{g}'} \cos(z-2g') - \frac{.00570603}{2(\dot{z}+\dot{g}')} \sin 2(z+g')$$

$$- \frac{.01364054}{2(\dot{z}+\dot{g}')} \cos 2(z+g') + \frac{.00000574}{2(\dot{z}-\dot{g}')} \sin 2(z-g')$$

$$+ \frac{.00002436}{2(\dot{z}+\dot{g}')} \cos 2(z-g') + \frac{.00000317}{3\dot{z}+2\dot{g}'} \sin(3z+2g')$$

$$- \frac{.00000110}{3\dot{z}+2\dot{g}'} \cos(3z+2g') - \frac{.00000319}{3\dot{z}-2\dot{g}'} \sin(3z-2g')$$

$$- \frac{.00000063}{3\dot{z}-2\dot{g}'} \cos(3z-2g') + \frac{.00000235}{4\dot{z}+2\dot{g}'} \sin(4z+2g')$$

$$- \frac{.00000074}{4\dot{z}+2\dot{g}'} \cos(4z+2g') - \frac{.00000239}{4\dot{z}-2\dot{g}'} \sin(4z-2g')$$

$$- \frac{.00000047}{4\dot{z} - 2\dot{g}'} \cos(4z - 2g') + \frac{.00000188}{5\dot{z} + 2\dot{g}'} \sin(5z + 2g')$$

$$- \frac{.00000055}{5\dot{z} + 2\dot{g}'} \cos(5z + 2g') - \frac{.00000189}{5\dot{z} - 2\dot{g}'} \sin(5z - 2g')$$

$$- \frac{.00000037}{5\dot{z} - 2\dot{g}'} \cos(5z - 2g') + \frac{.00000155}{6\dot{z} + 2\dot{g}'} \sin(6z + 2g')$$

$$- \frac{.00000044}{6\dot{z} + 2\dot{g}'} \cos(6z + 2g') - \frac{.00000156}{6\dot{z} - 2\dot{g}'} \sin(6z - 2g')$$

$$- \frac{.00000033}{6\dot{z} - 2\dot{g}'} \cos(6z - 2g') + \frac{.00000131}{7\dot{z} + 2\dot{g}'} \sin(7z + 2g')$$

$$- \frac{.00000037}{7\dot{z} + 2\dot{g}'} \cos(7z + 2g') - \frac{.00000133}{7\dot{z} - 2\dot{g}'} \sin(7z - 2g')$$

$$- \frac{.00000027}{7\dot{z} - 2\dot{g}'} \cos(7z - 2g') + \frac{.00000114}{8\dot{z} + 2\dot{g}'} \sin(8z + 2g')$$

$$- \frac{.00000031}{8\dot{z} + 2\dot{g}'} \cos(8z + 2g') - \frac{.00000114}{8\dot{z} - 2\dot{g}'} \sin(8z - 2g')$$

$$- \frac{.00000023}{8\dot{z} - 2\dot{g}'} \cos(8z - 2g') + \frac{.00000100}{9\dot{z} + 2\dot{g}'} \sin(9z + 2g')$$

$$- \frac{.00000028}{9\dot{z} + 2\dot{g}'} \cos(9z + 2g') - \frac{.00000101}{9\dot{z} - 2\dot{g}'} \sin(9z - 2g')$$

$$- \frac{.00000021}{9\dot{z} - 2\dot{g}'} \cos(9z - 2g') - \frac{.00000003}{10\dot{z} + 2\dot{g}'} \cos(10z + 2g')$$

$$\left. \left. + \frac{.00000003}{10\dot{z} - 2\dot{g}'} \cos(10z - 2g') \right] \right\}$$

$$(\delta i)_H = \frac{e(1 - \cos i)}{(1 - e^2)\sin i} (\delta e)_H ,$$

where $z = x + 1^\circ\!.10692$.

Values of $(\delta e)_H$ and $(\delta i)_H$ are given in Tables 1 and 2, respectively. They agree quite closely with the values of $(\delta e)_R$ and $(\delta i)_R$.

References

1. Murphy, J. P., and Felsentreger, T. L., "An Analysis of Lunar and Solar Effects on the Motion of Close Earth Satellites", GSFC X-547-65-251, June 1965.

2. Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag," A. J. 64(9), pp. 378-397, November 1959.

3. Kozai, Y., "New Determination of Zonal Harmonics Coefficients of the Earth's Gravitational Potential," SIAO Special Rept. No. 165, November 2, 1964.

APPENDIX A

SYMBOLS

a = semi-major axis of satellite's orbit

e = eccentricity of satellite's orbit

i = inclination of satellite's orbital plane to earth's equatorial plane

ℓ = mean anomaly of satellite

g = argument of perigee of satellite's orbit

h = longitude of ascending node of satellite's orbit

$L = \sqrt{a}$

$G = L\sqrt{1 - e^2}$

$H = G \cos i$

$L' = L$

$G' = G$

$H' = H - G$

$\ell' = \ell$

$g' = g + h$

$h' = h$

$t - t_0$ = days since Jan. 21, 1964, 21 hrs. 41 min. U.T.

e_c, i_c, etc. = element corrected for long period luni-solar effects (excluding near-resonant terms)

$(\delta e)_R$, $(\delta e)_H$, etc. = perturbation arising from two near-resonant terms

J_2, J_3, J_4 = zonal harmonic coefficients in the earth's gravitational potential ($J_2 = 1.08219 \times 10^{-3}$, $J_3 = -2.285 \times 10^{-6}$, $J_4 = -2.123 \times 10^{-6}$)*

F_R = sum of two near-resonant terms in luni-solar disturbing function

F_S = secular disturbing function

$F = F_S + F_R$

$n_{\odot}$, $n_{☾}$ = mean motion of the sun (moon) relative to the earth

$$m_{\odot}, m_{☾} = \frac{\text{mass of sun (moon)}}{\text{mass of sun (moon) + mass of earth}}$$

$i_{\odot}$, $i_{☾}$ = inclination of sun's (moon's) orbital plane to earth's equatorial plane

$\Omega_{\odot}$, $\Omega_{☾}$ = longitude of the mean ascending node of the sun's (moon's) orbit on the earth's equator, measured from the mean equinox of date

i'' = inclination of moon's orbital plane to the ecliptic = $5^{\circ}\!.1453964$

Ω'' = longitude of the mean ascending node of the lunar orbit on the ecliptic, measured from the mean equinox of date

*Values currently in use at Goddard Space Flight Center

APPENDIX B

TABLES

Table 1
Eccentricity of Relay 2

$t - t_0$	e	e_c	$(\delta e)_R$	$(\delta e)_H$	$e_c - (\delta e)_R$	$e_c - (\delta e)_R$ $-.00004119 \sin g_c$ $+.00000832 \cos g_c$
0	.23916879	.23935622	.00157180	.00157163	.23778442	.23777951
3	18664	36475	157239	157222	79236	78987
10	19456	37028	158374	158358	78654	78973
17	22892	35933	158869	158853	77064	77944
24	27068	38142	159863	159846	78279	79705
31	33034	36599	160307	160290	76292	78237
38	40283	38239	161325	161308	76914	79344
45	47408	38471	161784	161768	76687	79556
52	53221	38013	162512	162495	75501	78758
59	59429	37177	163270	163253	73907	77491
66	69019	39528	164220	164203	75308	79155
73	72216	39113	164622	164605	74491	78531
80	78211	40353	165423	165406	74930	79089
87	79752	40424	166238	166221	74186	78388
101	83057	42187	167394	167377	74793	78851
108	85044	43696	168148	168131	75548	79422
115	80577	44389	168857	168839	75532	79152
129	75226	47562	170273	170256	77289	80208
151	62376	49992	172878	172861	77114	78527
158	54862	52256	173593	173575	78663	79530
165	52890	50635	174112	174095	76523	76828
172	48358	55233	174873	174855	80360	80098
186	44387	56988	176182	176165	80806	79434
193	47369	56582	176661	176644	79921	78026
200	45423	60301	177472	177455	82829	80446
207	46782	57534	177928	177910	79606	76778
214	45627	60740	179062	179044	81678	78457
221	47860	58088	179724	179706	78364	74809
228	47284	61688	180228	180211	81460	77635
235	51100	61499	180731	180713	80768	76744
256	48801	63427	183481	183463	79946	75771
263	52075	64334	183685	183667	80649	76576

Table 1 (Continued)
Eccentricity of Relay 2

$t - t_0$	e	e_c	$(\delta e)_R$	$(\delta e)_H$	$e_c - (\delta e)_R$	$e_c - (\delta e)_R$ $-.00004119 \sin g_c$ $+.00000832 \cos g_c$
270	.23951083	.23966644	.00184360	.00184342	.23782284	.23778387
277	53056	66838	185097	185079	81741	78092
284	50738	67467	186026	186008	81441	78106
291	51205	67112	186471	186454	80641	77681
298	50703	68532	187228	187210	81304	78773
305	52989	69374	187697	187679	81677	79621
312	53486	70329	188490	188472	81839	80295
319	54979	70453	188716	188698	81737	80733
326	53316	68729	189703	189685	79026	78582
333	56460	69367	190315	190297	79052	79175
340	58014	68957	191200	191183	77757	78445
347	59645	69252	191730	191712	77522	78762
354	60994	68163	192583	192565	75580	77350
361	64962	69162	193440	193423	75722	77989
368	70272	69060	194260	194242	74800	77523
375	74123	70956	194973	194955	75983	79113
382	78360	68446	195918	195901	72528	76007
389	82901	69786	196781	196764	73005	76770
395.09653	93855	73284	197437	197419	75847	79805
402.09653	93127	70145	198140	198122	72005	76117
409.09653	99572	70252	198861	198843	71391	75582
423.09653	.24009397	73047	200770	200752	72277	76396
430.09653	10479	75061	201467	201450	73594	77564
437.09653	13675	74380	202157	202140	72223	75971
444.09653	13411	77626	202690	202672	74936	78394
451.09653	15930	77426	203581	203563	73845	76950
458.09653	12694	79661	204449	204431	75212	77907
465.09653	15156	80753	205077	205059	75676	77912
472.09653	08553	82203	205883	205865	76320	78056
479.09653	08837	81931	206541	206523	75390	76595
486.09653	04325	85667	207084	207066	78583	79234
493.09653	03848	84643	208024	208006	76619	76705
500.09653	.23999406	89208	208830	208812	80378	79896
507.09653	.24000203	89220	209099	209081	80121	79083
514.09653	.23993788	91126	210134	210116	80992	79413
521.09653	94137	90383	210467	210449	79916	77827

Table 1 (Continued)
Eccentricity of Relay 2

$t - t_0$	e	e_c	$(\delta e)_R$	$(\delta e)_H$	$e_c - (\delta e)_R$	$e_c - (\delta e)_R$ $-.00004119 \sin g_c$ $+.00000832 \cos g_c$
528.09653	.23989607	.23994417	.00211390	.00211372	.23783027	.23780465
535.09653	89052	93111	211743	211725	81368	78381
549.09653	83043	94607	213049	213031	81558	77890
556.09653	76757	96885	214146	214128	82739	78828
563.09653	77087	97563	214366	214349	83197	79115
570.09653	71596	99458	215472	215455	83986	79806
577.09653	70072	97910	215914	215896	81996	77796
584.09653	65666	.24000534	216682	216665	83852	79707
591.09653	63383	.23998347	217008	216990	81339	77326
598.09653	60478	99585	217649	217631	81936	78127
605.09653	61040	98811	218264	218246	80547	77012
612.09653	59903	.24000071	219174	219157	80897	77701
619.09653	61533	.23999951	219886	219868	80065	77266
626.09653	61584	99952	220217	220200	79735	77384
633.09653	65381	.24000257	221249	221231	79008	77148
640.09653	68173	01056	222034	222017	79022	77687
647.09653	72714	01624	222323	222305	79301	78515
654.09653	75218	01208	223442	223424	77766	77544

$t - t_0$ = no. of days since 1/21/64, 21 hrs. 41 min. U.T.

$a \simeq 1.7449$ earth radii

Table 2
Inclination of Relay 2

$t - t_0$	i	i_c	$(\delta i)_R$	$(\delta i)_H$	$i_c - (\delta i)_R$
0	46.315160	46.328030	.009772	.009771	46.318258
3	312764	326980	9776	9775	317204
10	311053	325823	9846	9845	315977
17	312790	327106	9879	9878	317227
24	313675	325747	9943	9942	315804
31	315490	327371	9974	9973	317397
38	317296	325398	.010041	.010040	315357
45	318046	327465	10073	10072	317392
52	318866	325829	10121	10120	315708
59	317083	326734	10171	10170	316563
66	317668	326770	10235	10234	316535
73	315507	326796	10261	10260	316535
80	313306	324774	10313	10312	314461
87	313097	324509	10365	10364	314144
101	318887	327565	10440	10439	317125
108	319382	326894	10488	10487	316406
115	323473	325763	10531	10530	315232
129	331348	327195	10619	10618	316576
151	336591	325796	10776	10775	315020
158	336571	326320	10817	10816	315503
165	335124	323422	10848	10847	312574
172	335835	325270	10893	10892	314377
186	337174	326087	10973	10972	315114
193	338207	326398	11005	11004	315393
200	337973	325818	11054	11053	314764
207	338625	327034	11083	11082	315951
214	338630	326569	11153	11152	315416
221	337093	327126	11195	11194	315931
228	336052	326799	11226	11225	315573
235	333856	327465	11259	11258	316206
256	326897	327532	11427	11426	316105
263	325529	326484	11441	11440	315043
270	325006	327949	11483	11481	316466
277	325105	326448	11530	11528	314918
284	324535	327828	11586	11585	316242
291	326138	327253	11614	11613	315639
298	326310	329336	11661	11660	317675
305	326668	329095	11692	11691	317403

Table 2 (Continued)
Inclination of Relay 2

$t - t_0$	i	i_c	$(\delta i)_R$	$(\delta i)_H$	$i_c - (\delta i)_R$
312	46.323171	46.327870	.011741	.011739	46.316129
319	322183	329584	11755	11754	317829
326	318580	328198	11815	11814	316383
333	316633	330571	11854	11853	318717
340	314014	328017	11909	11908	316108
347	311700	329616	11943	11941	317673
354	313183	328885	11997	11996	316888
361	312732	330319	12052	12051	318267
368	315015	329018	12107	12106	316911
375	316367	329874	12154	12153	317720
382	318188	328697	12216	12215	316481
389	319574	328988	12273	12272	316715
395.09653	321090	329894	12320	12319	317574
402.09653	321938	330187	12364	12363	317823
409.09653	319737	328833	12412	12411	316421
423.09653	319132	329031	12537	12536	316494
430.09653	319328	327263	12581	12580	314682
437.09653	319345	327756	12626	12625	315130
444.09653	321052	325554	12660	12659	312894
451.09653	322959	325167	12718	12716	312449
458.09653	327400	325801	12771	12770	313030
465.09653	332206	327120	12813	12812	314307
472.09653	337249	328922	12861	12860	316061
479.09653	341253	328940	12904	12903	316036
486.09653	341606	328678	12935	12934	315743
493.09653	342541	327435	12994	12993	314441
500.09653	343523	329317	13042	13041	316275
507.09653	343278	328525	13059	13058	315466
514.09653	342996	329407	13120	13119	316287
521.09653	341786	328664	13140	13139	315524
528.09653	341769	329013	13195	13194	315818
535.09653	340391	328836	13216	13215	315620
549.09653	339289	328637	13294	13293	315343
556.09653	338673	328076	13358	13357	314718
563.09653	338493	329826	13372	13371	316454
570.09653	336806	329592	13437	13436	316155
577.09653	334532	329615	13463	13462	316152
584.09653	331526	330144	13507	13506	316637

Table 2 (Continued)
Inclination of Relay 2

$t - t_0$	i	i_c	$(\delta i)_R$	$(\delta i)_H$	$i_c - (\delta i)_R$
591.09653	46.328215	46.328122	.013525	.013524	46.314597
598.09653	325625	329254	13562	13561	315692
605.09653	327280	330650	13601	13600	317049
612.09653	323270	329687	13656	13655	316031
619.09653	324241	329062	13702	13701	315360
626.09653	323782	329433	13722	13721	315711
633.09653	324795	328588	13789	13788	314799
640.09653	325795	329347	13840	13839	315507
647.09653	326590	331122	13861	13860	317261
654.09653	326712	330889	13933	13932	316956

Angles are in degrees.

Table 3
Argument of Perigee (Relay 2)

$t - t_0$	g	Residuals*	g_c	Residuals**
0	184.72550	.01551	184.70789	.02245
3	188.04367	.01451	188.01737	.01278
10	195.79345	.01957	195.76817	.01891
17	203.54360	.02500	203.50802	.01409
24	211.28943	.02611	211.25670	.01810
31	219.03735	.02932	218.99542	.01215
38	226.78167	.02892	226.74395	.01601
45	234.52550	.02803	234.48207	.00946
52	242.26211	.01992	242.22424	.00696
59	250.00924	.02233	249.96655	.00460
66	257.75348	.02185	257.71647	.00985
73	265.49836	.02201	265.45522	.00393
80	273.23801	.01694	273.19929	.00333
87	280.98995	.02416	280.94379	.00316
101	296.47886	.02363	296.42694	-.00303
108	304.22055	.02060	304.17175	-.00289
115	311.97051	.02585	311.91509	-.00422
129	327.46271	.02861	327.40554	-.00311
151	351.80451	.02986	351.75117	.00212
158	359.54759	.02822	359.49669	.00298
165	7.28816	.02407	7.24099	.00261
172	15.02716	.01835	14.98429	.00123
186	30.50252	.00428	30.47291	.00051
193	38.23512	-.00784	38.21487	-.00220
200	45.96571	-.02197	45.95919	-.00255
207	53.69508	-.03732	53.69831	-.00809
214	61.43068	-.04644	61.45084	-.00023
221	69.16163	-.06021	69.19430	-.00145
228	76.88592	-.08064	76.93625	-.00417
235	84.61829	-.09299	84.67520	-.00989
256	107.84057	-.10487	107.91913	.00003
263	115.58490	-.10525	115.65501	-.00876
270	123.32854	-.10633	123.39894	-.00950
277	131.08956	-.09003	131.14434	-.00877
284	138.84171	-.08260	138.89194	-.00584
291	146.60404	-.06499	146.63340	-.00905
298	154.35531	-.05844	154.37863	-.00850
305	162.11990	-.03857	162.11916	-.01263

Table 3 (Continued)
Argument of Perigee (Relay 2)

$t - t_0$	g	Residuals*	g_c	Residuals**	$t - t_0$
					0
312	169.87030	-.03289	169.86319	-.01327	3
319	177.62906	-.01884	177.60116	-.01997	10
326	185.38243	-.01020	185.35114	-.01466	17
333	193.14368	.00633	193.09418	-.01629	24
340	200.89313	.01107	200.84079	-.01435	31
347	208.64566	.01887	208.58284	-.01697	38
354	216.39229	.02079	216.32960	-.01488	45
361	224.14445	.02823	224.07528	-.01387	52
368	231.88913	.02819	231.82169	-.01213	59
375	239.63442	.02876	239.56674	-.01176	66
382	247.37658	.02620	247.31501	-.00816	73
389	255.11624	.02114	255.05853	-.00931	80
395.09653	261.85264	.01241	261.80295	-.00997	87
402.09653	269.59350	.00856	269.54748	-.01011	101
409.09653	277.32858	-.00109	277.29197	-.01029	108
423.09653	292.81632	-.00278	292.78730	-.00431	115
430.09653	300.56079	-.00304	300.53137	-.00491	129
437.09653	308.30538	-.00317	308.27643	-.00451	151
444.09653	316.05042	-.00284	316.01798	-.00768	158
451.09653	323.80102	.00304	323.76408	-.00620	165
458.09653	331.55652	.01382	331.51502	.00006	172
465.09653	339.30257	.01515	339.25586	-.00376	186
472.09653	347.05660	.02446	347.00300	-.00129	193
479.09653	354.80955	.03270	354.74733	-.00163	200
486.09653	2.55633	.03476	2.48986	-.00378	207
493.09653	10.31222	.04593	10.24014	.00184	214
500.09653	18.06564	.05463	17.98895	.00597	221
507.09653	25.81186	.05613	25.72818	.00054	228
514.09653	33.56167	.06122	33.47991	.00759	235
521.09653	41.29995	.05479	41.21820	.00121	256
528.09653	49.04487	.05499	48.96869	.00702	263
535.09653	56.78332	.04871	56.70845	.00212	270
549.09653	72.25135	.02730	72.19796	.00230	277
556.09653	79.99006	.02129	79.95322	.01289	284
563.09653	87.71810	.00462	87.68796	.00295	291
570.09653	95.45270	-.00550	95.44096	.01128	298
577.09653	103.18726	-.01566	103.18256	.00821	

Table 3 (Continued)
Argument of Perigee (Relay 2)

$t - t_0$	g	Residuals*	g_c	Residuals**
584.09653	110.92135	-.02629	110.93150	.01248
591.09653	118.66011	-.03226	118.66928	.00559
598.09653	126.39331	-.04378	126.41341	.00505
605.09653	134.14541	-.03638	134.15917	.00614
612.09653	141.88845	-.03807	141.90650	.00880
619.09653	149.65122	-.02002	149.65494	.01257
626.09653	157.38894	-.02702	157.39415	.00711
633.09653	165.15754	-.00314	165.14465	.01294
640.09653	172.90778	.00238	172.89173	.01535
647.09653	180.66715	.01704	180.63224	.01118
654.09653	188.42346	.02863	188.38548	.01976

Angles are in degrees.

*From least squares fit $g = 184^{\circ}\!.70999 + (1^{\circ}\!.1063884/\text{day})\,(t - t_0)$

**From least squares fit $g_c = 184^{\circ}\!.68544 + (1^{\circ}\!.1063814/\text{day})\,(t - t_0)$

Table 4
Longitude of Ascending Node and Longitude
of Perigee (Relay 2)

h	Residuals*	h_c	Residuals**	$g_c + h_c$
223.60701	-.01836	223.59840	-.03557	48.30629
220.29898	-.01233	220.28989	-.03017	30726
212.56667	-.01179	212.55994	-.02765	32811
204.83617	-.00945	204.82920	-.02594	33722
197.10396	-.00882	197.09847	-.02420	35517
189.37322	-.00673	189.36761	-.02260	36303
181.64265	-.00446	181.63711	-.02064	38106
173.91196	-.00231	173.90689	-.01840	38896
166.18301	.00158	166.17757	-.01525	40181
158.45025	.00166	158.44849	-.01187	41504
150.71539	-.00036	150.71493	-.01297	43140
142.97742	-.00549	142.98326	-.01218	43848
135.24371	-.00636	135.25309	-.00989	45238
127.50468	-.01255	127.52333	-.00719	46712
112.02914	-.02241	112.06132	-.00427	48826
104.29263	-.02608	104.33005	-.00308	50180
96.55755	-.02832	96.60035	-.00032	51544
81.08926	-.03094	81.13666	.00091	54220
56.79327	-.02372	56.83995	.00622	59112
49.06452	-.01962	49.10824	.00698	60493
41.33098	-.02032	41.37348	.00468	61447
33.60310	-.01536	33.64445	.00811	62874
18.14214	-.01064	18.17987	.00845	65278
10.41169	-.00826	10.44613	.00718	66100
2.68330	-.00381	2.71654	.01005	67573
354.95613	.00186	354.98536	.01133	68367
347.22723	.00580	347.25323	.01166	70407
339.49868	.01009	339.52115	.01204	71545
331.76913	.01338	331.78811	.01146	72436
324.03967	.01676	324.05740	.01321	73260
300.84771	.02332	300.86208	.01528	78121
293.11138	.01982	293.12907	.01473	78408
285.38067	.02195	285.39705	.01517	79599
277.64458	.01870	277.66429	.01487	80863
269.91440	.02136	269.93328	.01632	82522
262.18092	.02073	262.19945	.01496	83285
254.45123	.02387	254.46746	.01543	84609

Table 4 (Continued)
Longitude of Ascending Node and Longitude
of Perigee (Relay 2)

$t - t_0$	h	Residuals*	h_c	Residuals**	$g_c + h_c$
305	246.72100	.02649	246.73463	.01506	48.85379
312	238.99201	.03033	239.00414	.01703	86733
319	231.26117	.03233	231.26949	.01484	87065
326	223.52903	.03303	223.53676	.01458	88790
333	215.79731	.03415	215.80365	.01393	89783
340	208.06417	.03385	208.07203	.01477	91282
347	200.32950	.03202	200.33857	.01377	92141
354	192.59569	.03105	192.60639	.01405	93599
361	184.86116	.02936	184.87491	.01503	95019
368	177.12760	.02864	177.14193	.01451	96362
375	169.39156	.02544	169.40854	.01358	97528
382	161.65994	.02666	161.67603	.01354	99104
389	153.92761	.02716	153.94670	.01667	49.00523
395.09653	147.19303	.02737	147.21194	.01636	01489
402.09653	139.45710	.02428	139.47939	.01627	02687
409.09653	131.72231	.02232	131.74600	.01534	03797
423.09653	116.24667	.01236	116.28178	.01605	06908
430.09653	108.50863	.00717	108.54919	.01592	08056
437.09653	100.76753	-.00109	100.81514	.01433	09157
444.09653	93.02939	-.00639	93.08238	.01404	10031
451.09653	85.29051	-.01243	85.35098	.01510	11506
458.09653	77.55292	-.01719	77.61532	.01190	13034
465.09653	69.81813	-.01914	69.88464	.01368	14050
472.09653	62.08605	-.01837	62.15213	.01363	15513
479.09653	54.35198	-.01961	54.41873	.01269	16606
486.09653	46.62128	-.01747	46.68554	.01197	17540
493.09653	38.88872	-.01719	38.95111	.01000	19125
500.09653	31.15534	-.01773	31.21633	.00768	20529
507.09653	23.42223	-.01799	23.48067	.00448	20884
514.09653	15.68942	-.01797	15.74723	.00349	22714
521.09653	7.95819	-.01637	8.01361	.00235	23181
528.09653	0.22347	-.01824	0.27913	.00033	24781
535.09653	352.49063	-.01824	352.54446	-.00188	25291
549.09653	337.02749	-.01570	337.07665	-.00478	27461
556.09653	329.29539	-.01496	329.34064	-.00832	29386
563.09653	321.56520	-.01232	321.60848	-.00802	29644
570.09653	313.83778	-.00690	313.87467	-.00937	31563

Table 4 (Continued)
Longitude of Ascending Node and Longitude of Perigee (Relay 2)

$t - t_0$	h	Residuals*	h_c	Residuals**	$g_c + h_c$
577.09653	306.10446	-.00738	306.13971	-.01186	49.32227
584.09653	298.37394	-.00505	298.40365	-.01546	33515
591.09653	290.64104	-.00512	290.67045	-.01620	33973
598.09653	282.91146	-.00186	282.93666	-.01753	35007
605.09653	275.17442	-.00606	275.20047	-.02126	35964
612.09653	267.44346	-.00418	267.46791	-.02136	37441
619.09653	259.70670	-.00810	259.73051	-.02629	38545
626.09653	251.97322	-.00874	251.99569	-.02865	38984
633.09653	244.24164	-.00748	244.26157	-.03031	40622
640.09653	236.50789	-.00839	236.52661	-.03281	41834
647.09653	228.77489	-.00855	228.78902	-.03794	42126
654.09653	221.04201	-.00859	221.05380	-.04070	43928

Angles are in degrees.

*From least squares fit $h = 223^\circ\!.62537 - (1^\circ\!.1046913/\text{day})\,(t - t_0)$

**From least squares fit $h_c = 223^\circ\!.63397 - (1^\circ\!.1046373/\text{day})\,(t - t_0)$

Appendix C

GRAPHS

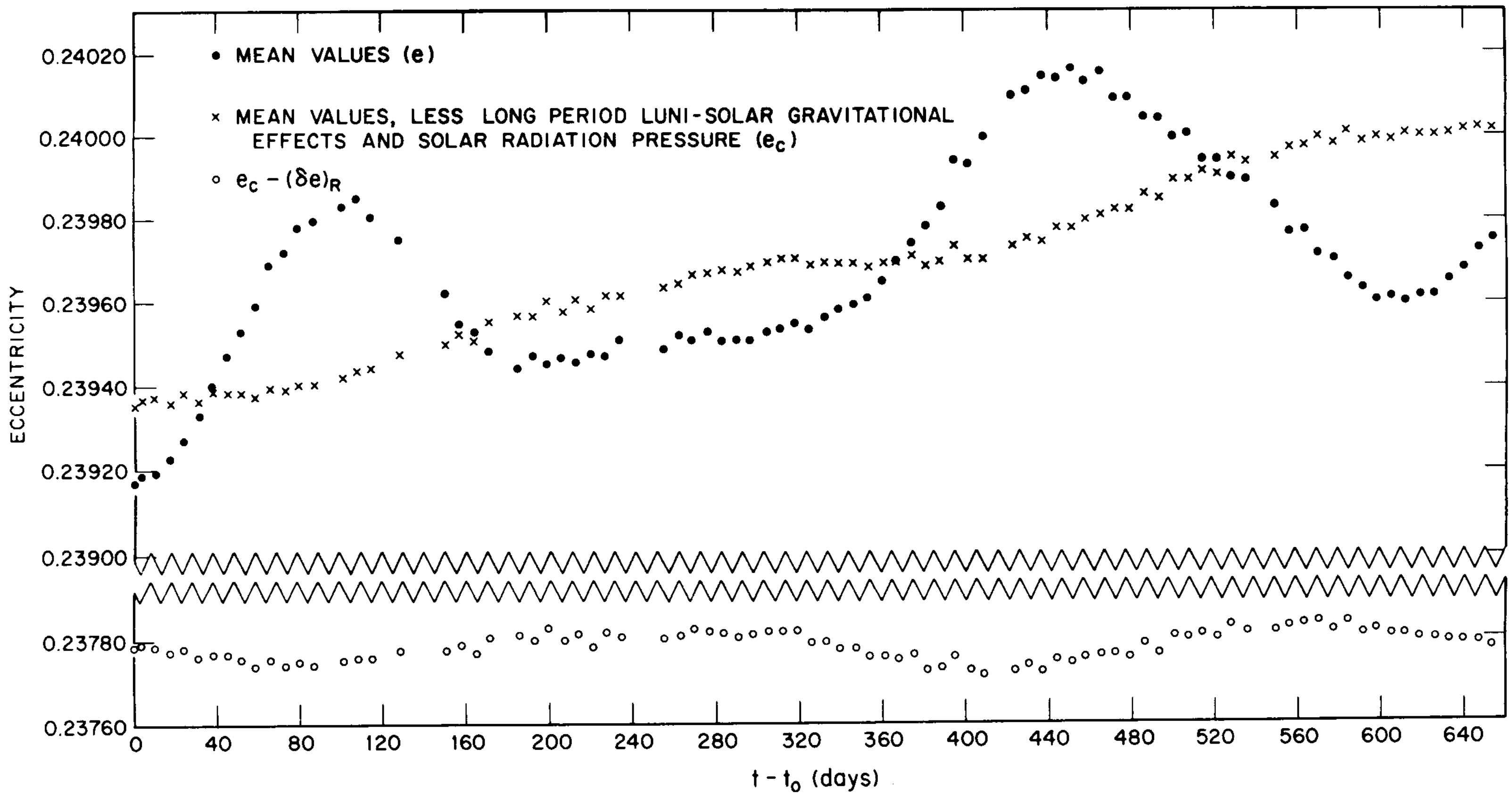


Figure 1–Eccentricity of Relay 2

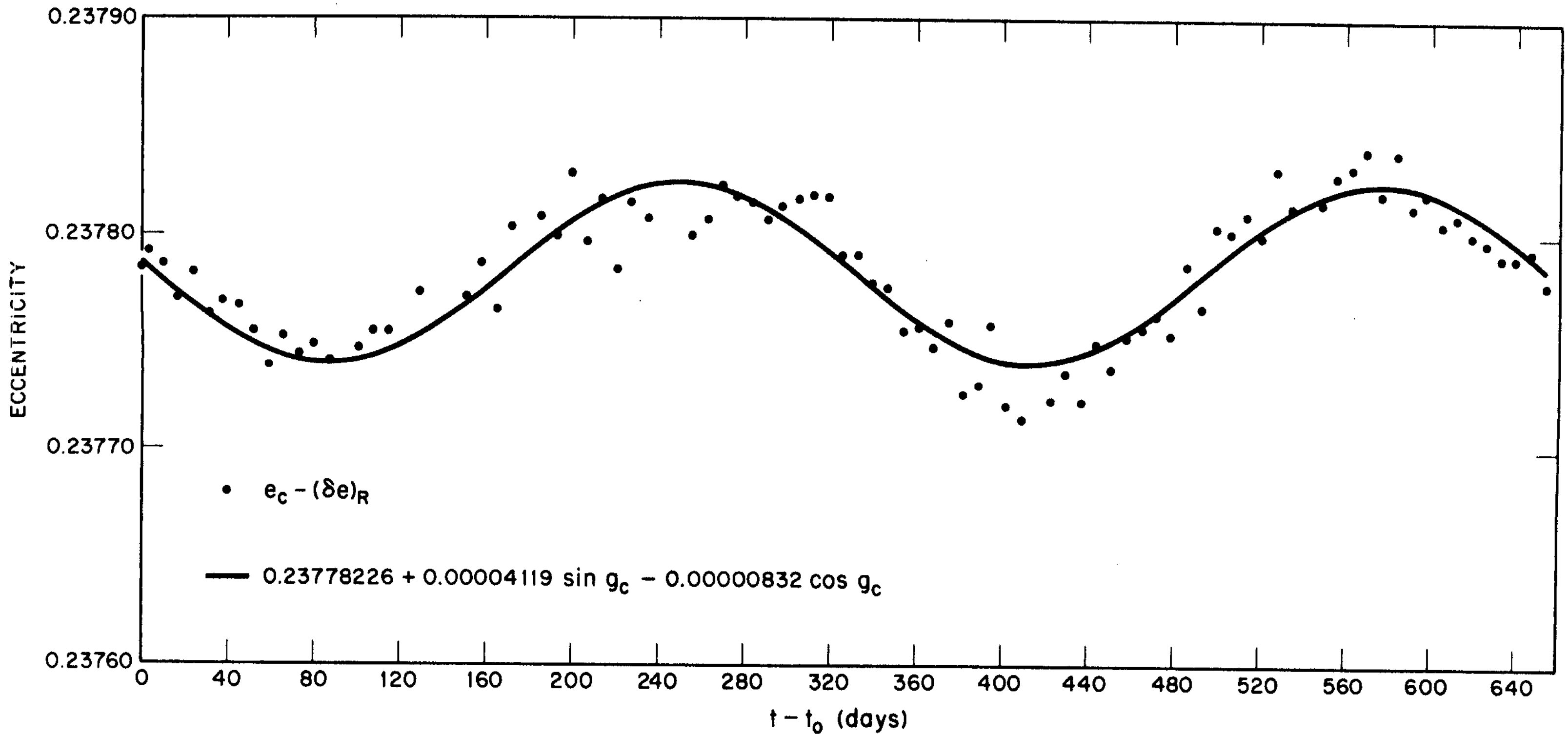


Figure 2–Eccentricity of Relay 2

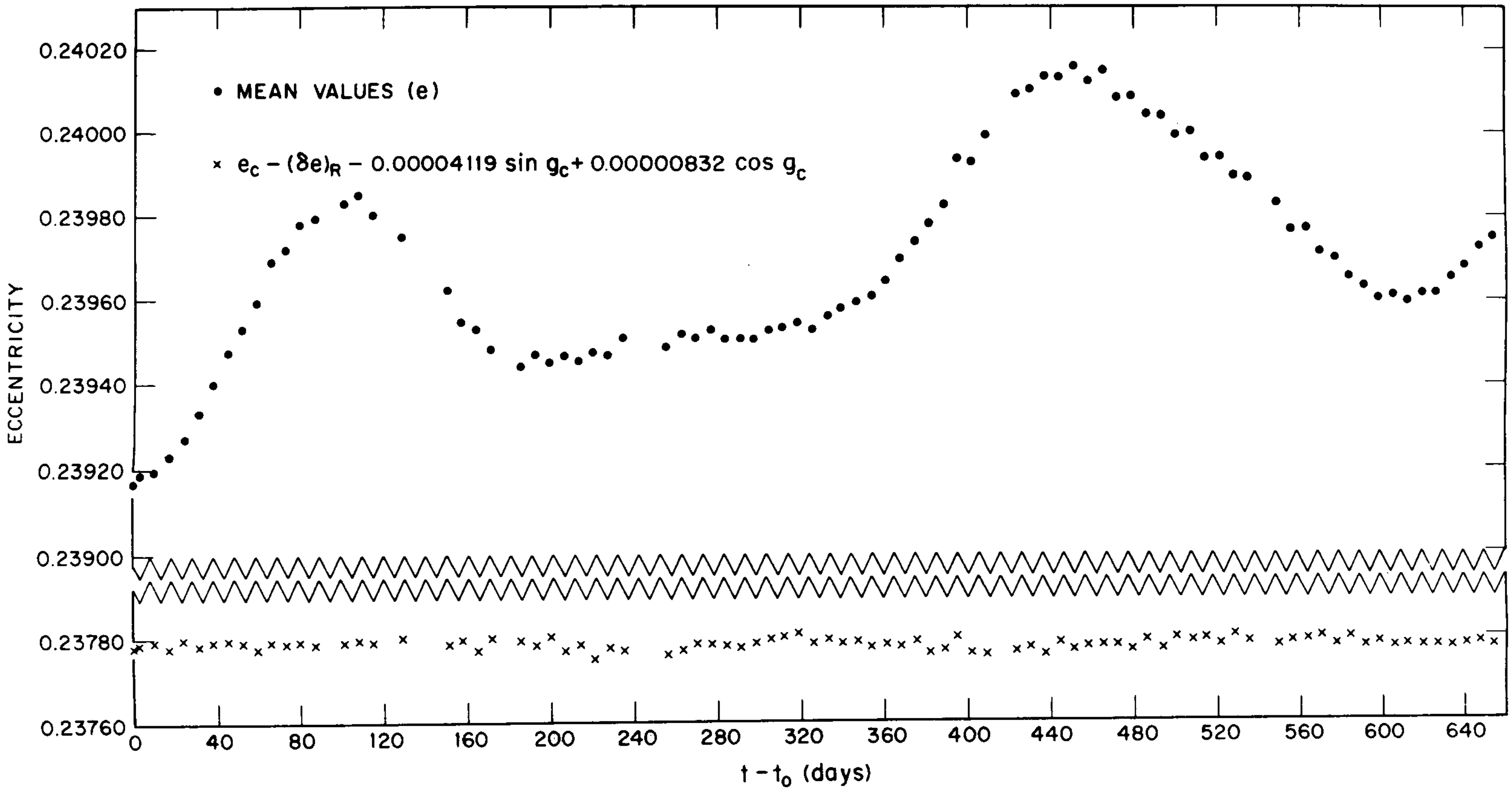


Figure 3–Eccentricity of Relay 2

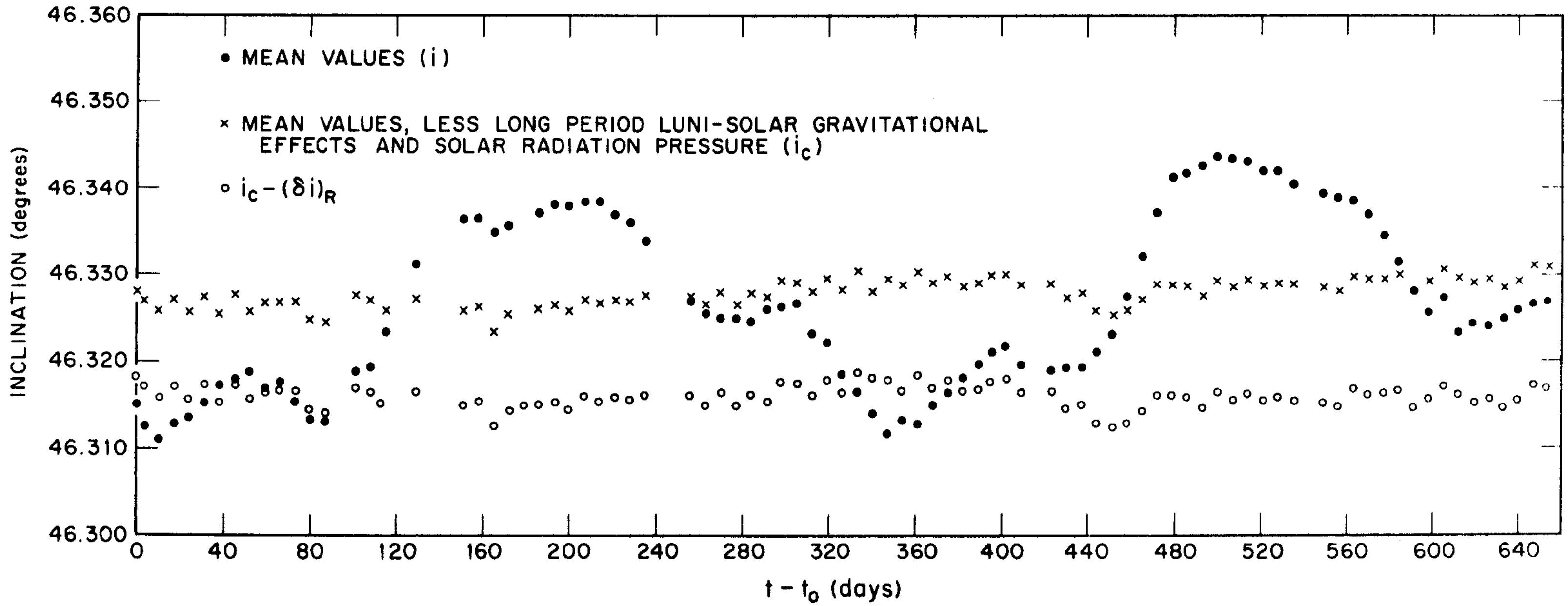


Figure 4—Inclination of Relay 2

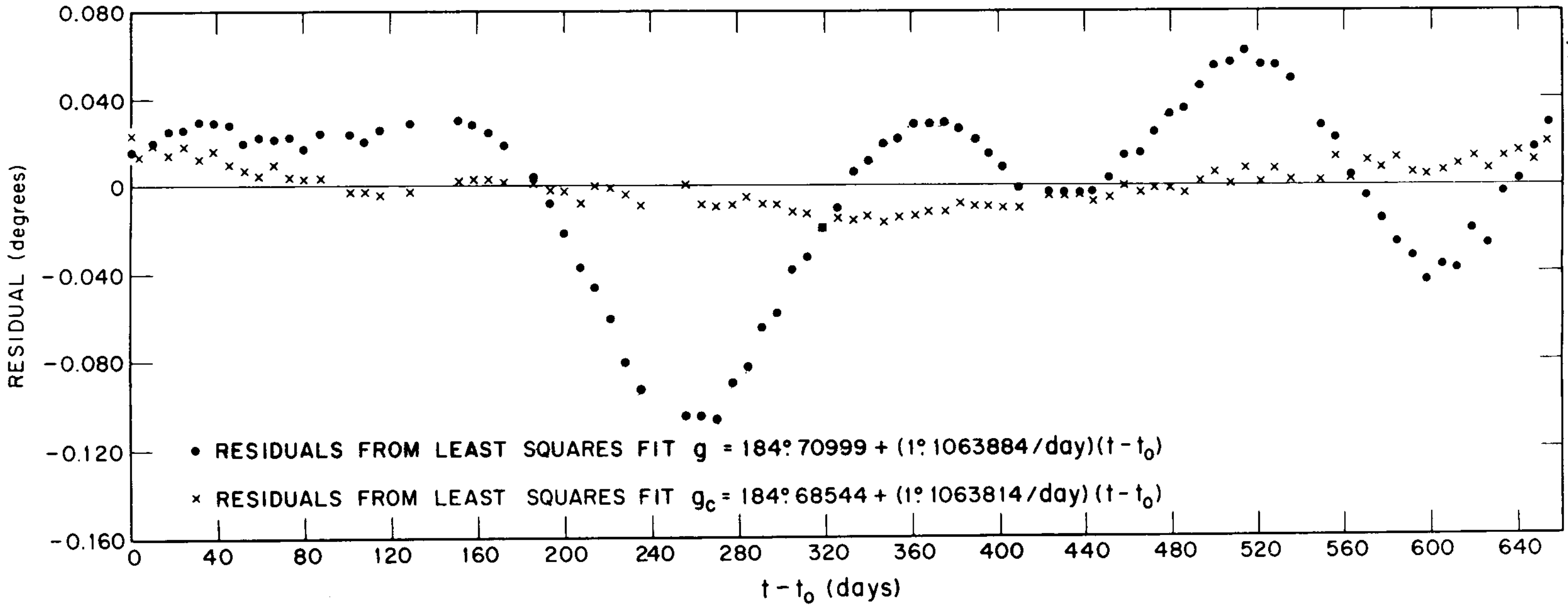


Figure 5—Argument of Perigee (Relay 2)

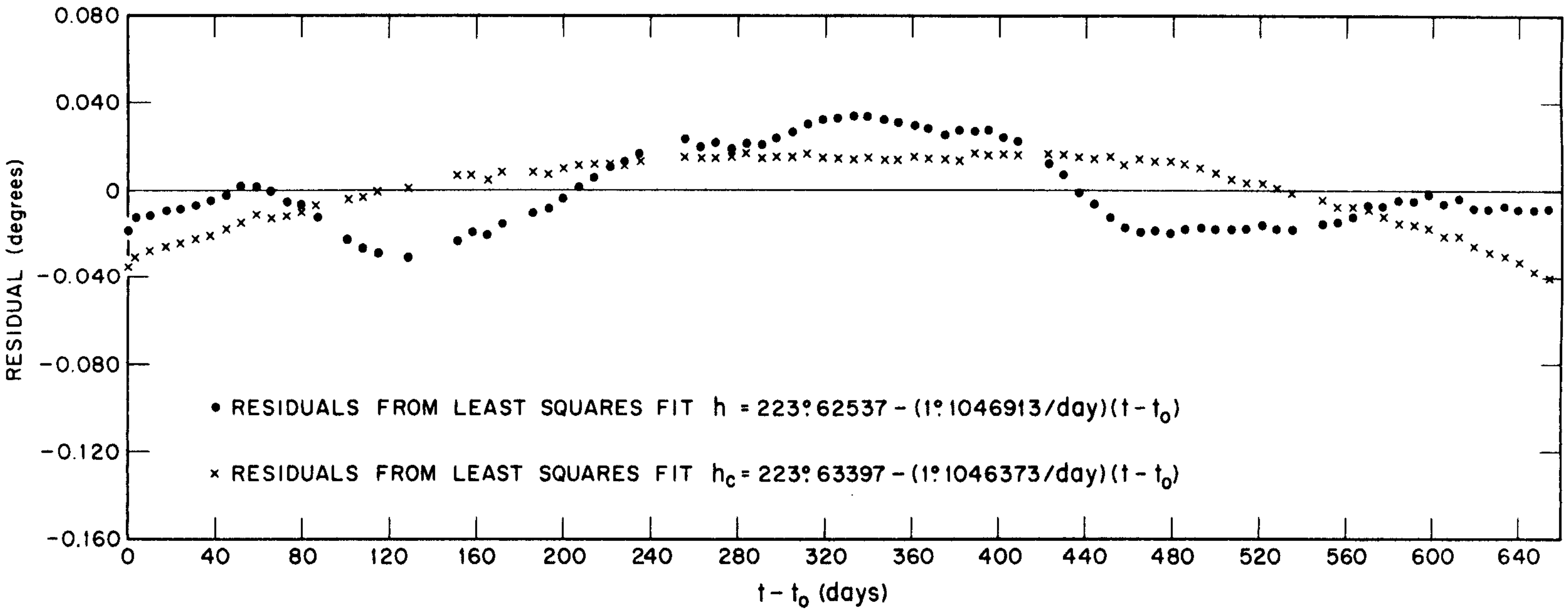


Figure 6—Longitude of Ascending Node (Relay 2)

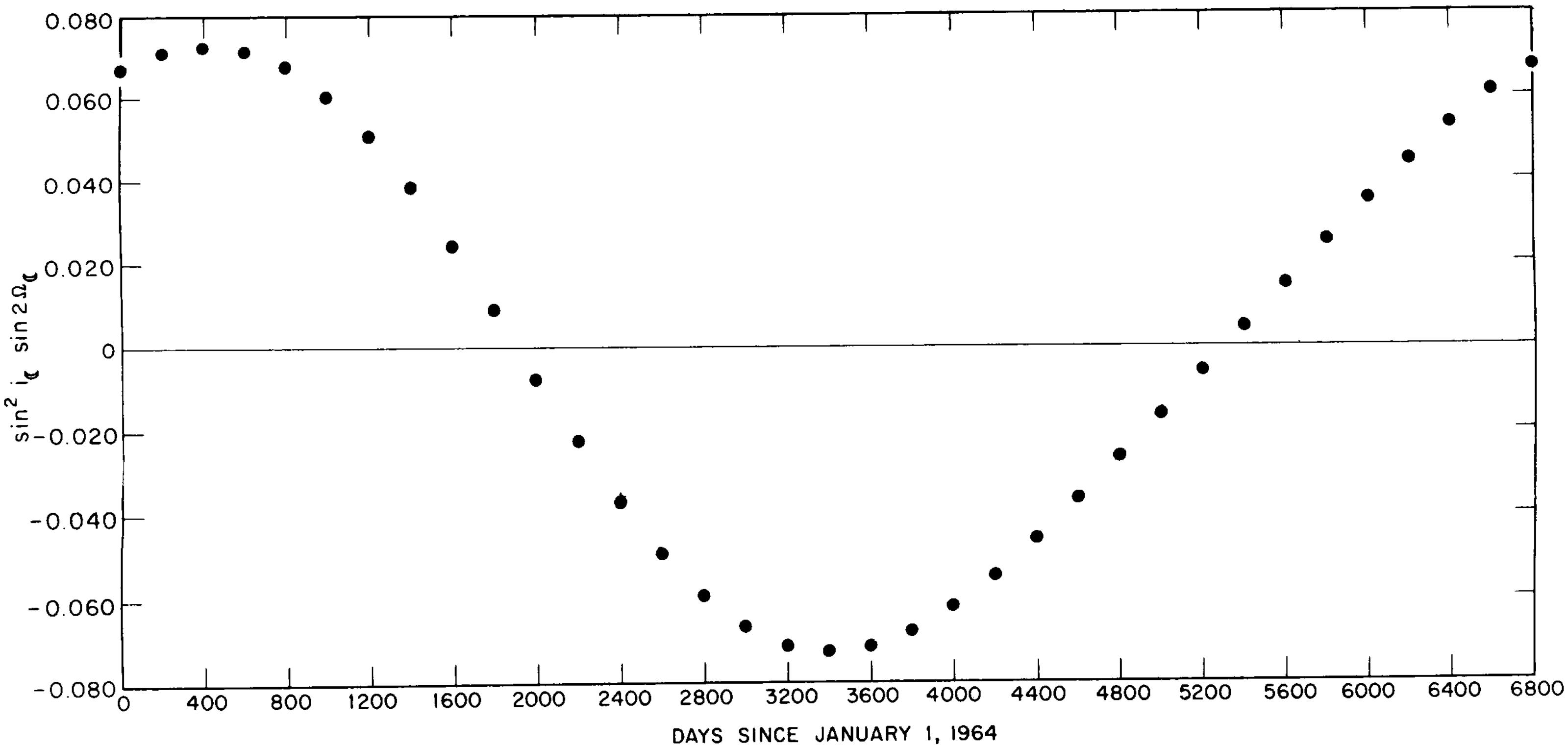


Figure 7–$\sin^2 i_{☾} \sin 2\Omega_{☾}$ vs. Time

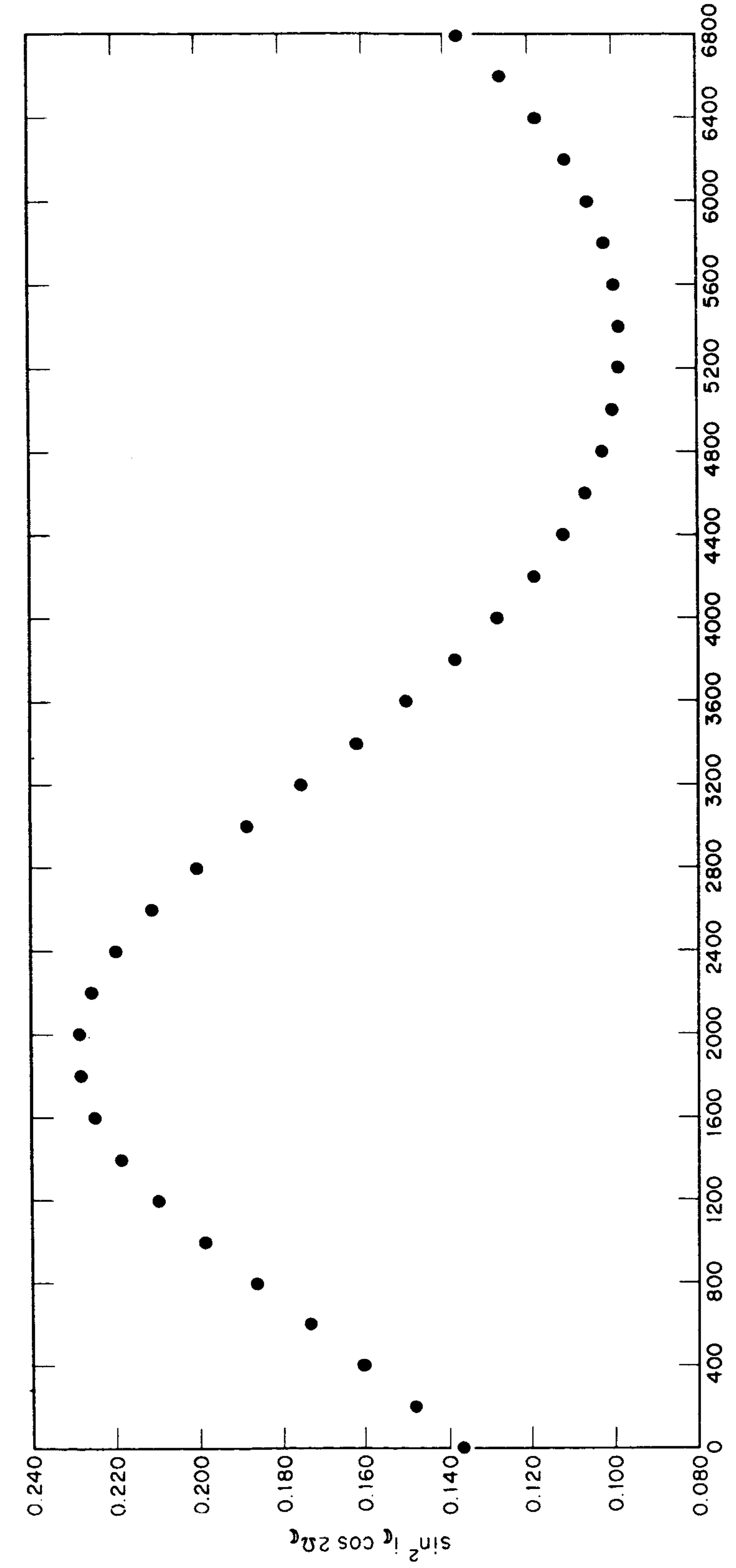


Figure 8—$\sin^2 i_e \sin 2\Omega_e$ vs. Time