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USE OF MEAN ELEMENTS TO CALCULATE THE POSITION OF THE SUN, MOON AND EARTH

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BY

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GODDARD SPACE FLIGHT CENTER

GREENBELT, MD.

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SUMMARY

Formulae for the calculation of the mean orbital elements of the Earth, Moon and Sun are presented in tabular form for the epoch 12 hours ephemeris time, January 0, 1900. The elements are listed in six combinations of units: Three angular units of degrees, radians and revolutions and two time units of days and Julian centuries.

A method is described for updating the numerical coefficients to any arbitrary epoch in order to eliminate double precision arithmetic in computer usage of these expressions. An algorithm is presented for the calculation of the selenocentric position vectors of the Earth and Sun neglecting periodic perturbations.

NOTATION

Orbit Parameters

a	semi-major axis
e	eccentricity
i	inclination of orbit plane
Ω	longitude of ascending node
ω	argument of pericenter
g	mean anomaly
E	eccentric anomaly
f	true anomaly
C	equation of the center
F	mean argument of latitude
u	true argument of latitude
r	magnitude of radius vector
L	mean longitude, ecliptic and mean equinox of date
Γ	mean longitude of pericenter, ecliptic and mean equinox of date

Subscripts and Superscripts

(When used with notation for orbit parameters)

Superscript – indicates center of coordinate system

$'$ = geocentric

$''$ = selenocentric

none = heliocentric

Subscript – indicates body whose motion is being described by the orbit parameters

$_{\circ}$ = Sun

$_{3}$ = Earth

none = Moon

NOTATION

ϵ_M mean obliquity of date

I' inclination of mean lunar equator to the ecliptic

θ'_o Greenwich mean sidereal time

θ' Lunar mean sideral time

$$R_1(\alpha) \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_2(\beta) \equiv \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_3(\gamma) \equiv \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

USE OF MEAN ELEMENTS TO CALCULATE THE POSITION OF THE SUN, MOON AND EARTH

INTRODUCTION

The usage of the position vectors of the Sun, Moon, and Earth in digital computer programs designed to solve problems related to artificial satellites may be roughly divided into three categories:

1. Orbital
   a. Position of central body
   b. Position of disturbing bodies
2. Aspect and Orientation
3. Shadow and Occultation

The accuracy requirements placed upon the computer program in the calculation of these position vectors will usually depend upon two factors:

1. Accuracy of Observations
2. Experimental Requirements

The method most commonly employed in the calculation of the position vectors of these bodies consists of a two step process. First, it is necessary to prepare a magnetic tape containing tables of the positions of the bodies as a function of the independent variable, time. The positions are usually tabulated at fixed intervals of time, the time interval being dependent upon the rates of change of the position of the particular body. The position of the Moon may require position entries every hour, the position of the Sun every 24 hours, the position of the outer planets every 10 days, etc. The magnetic tape containing such information is referred to as an ephemeris tape. Secondly, to utilize this ephemeris tape, it is necessary to prepare two separate sets of computer instructions. The first set of instructions physically positions the magnetic tape to an appropriate initial entry and then reads off a number of tabulated entries from the tables. The second set of instructions interpolates between the tabulated entries to obtain the positions of the bodies for the desired calculation time.

The ephemeris tape so prepared usually consists of values of the position of the body to the highest available accuracy. In most cases, this accuracy is much greater than that required by experimental or observational considerations.

This method has only one feature in its favor – it does not require any thinking on the part of the user. The user usually justifies the use of the method by establishing a requirement for the most accurate available positions. In most cases this requirement will not stand up under a careful analysis of the problem. The disadvantages of the method include the usage of every known bad computer practice, not the least of which is the use of an input/output device to supply data which can be calculated by the computer.

An alternate method is to design subprograms for the computer which will supply the required position vectors by direct calculation from available solutions to the equations of motion of the desired body.[1] These solutions usually have the attractive feature of simplicity in two respects:

1. Only one variable, time, is required to solve for the necessary elements of the orbit in order to obtain a position vector of the body.

2. The solution for any one of the orbital elements is usually of the form $e = e_0 + e_1 t + e_2 t^2 + e_3 t^3$ + periodic terms.

The following section will investigate various means of implementing this alternate method with emphasis upon the use of the "mean elements" of the orbit to obtain the position of the body.

ALGORITHM FOR TWO BODY ELLIPTIC MOTION

From the point of view of computer usage, the most useful solutions to the equations of motion of the Earth, Moon, and planets start on the assumption of "two body motion."

To start the solution, it is assumed that there are no forces acting on the two bodies (e.g. the Earth in orbit around the Sun) which cause a deviation from "two body motion". On the assumption of the existence of the two bodies only, either as point masses or as homogeneous spherical masses, plus know initial

[1] This procedure was suggested in 1961 by R.L. Duncombe, currently Director of the Nautical Almanac Office, U.S. Naval Observatory (see page 453 "Transactions of the International Astronomical Union, Vol XI-B, Proceedings of the Eleventh General Assembly, Berkeley 1961"). To the best of the author's knowledge, this approach has not yet gained widespread acceptance despite the many advantages it offers over other methods.

conditions in the form of a position and velocity vector of one body relative to the other at a specific reference time, t_0, or their equivalent, it can be proven in a straightforward manner that the orbit of one body relative to the second body describes a conic section. The position and velocity vector at any other time can be computed rigorously in terms of a set of six constant elements corresponding to the reference time t_0 and the time elapsed from t_0. These six constant elements can be computed rigorously from the initial conditions. In the event that the relative conic section is an ellipse, the corresponding constant elements are called "elliptic elements". One useful set of elliptic elements is the following set:

a = Semi-major axis

e = Eccentricity

g_0 = Mean anomaly at time t_0

i = Inclination

Ω = Longitude of node

ω = Argument of pericenter.

The elements a and e describe the size and shape of the ellipse and g_0 describes the position of the second body at the epoch time t_0 relative to the first or central body which is located at one focus of the ellipse. The three elements i, Ω, ω describe the orientation of the ellipse relative to an arbitrary rectangular coordinate system centered at the central body. For two body motion all of the above elements are constant in time. These elements, or any equivalent set of six independent elements, can be used to find the position and velocity of the second body relative to the first body by means of the following algorithm for elliptic motion.

Algorithm for Elliptic Motion

1. Compute the mean angular motion

$$n = \sqrt{\frac{\mu}{a^3}}$$

where

$\mu = GM$

G = universal gravitational constant

M = sum of the masses of the two bodies.

2. Compute the mean anomaly for any arbitrary time t

$$g = g_0 + n\,(t - t_0)$$

3. Compute the auxiliary angle E (eccentric anomaly) iteratively by means of the transcendental equation (Kepler's equation)

$$E - e \sin E = g$$

4. Compute position and velocity components relative to a rectangular coordinate system in the orbit plane, centered at the central body, with first principal axis directed towards pericenter, the second principal axis in the orbit plane, 90° away from the first principal axis in the direction of the motion of the second body, and the third principal axis normal to the orbit plane and directed in the right-handed sense:

$$\bar{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{bmatrix} a\,(\cos E - e) \\ a\sqrt{1-e^2}\,\sin E \\ 0 \end{bmatrix}$$

$$r = a\,(1 - e\cos E)$$

$$\dot{\bar{q}} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \frac{\sqrt{\mu a}}{r} \begin{pmatrix} -\sin E \\ \sqrt{1-e^2}\,\cos E \\ 0 \end{pmatrix}$$

where r is the magnitude of the radius vector.

5. Rotate this position and velocity vector from the coordinate system described in 4 to the arbitrary cartesian coordinate system.

$$\bar{r} = R_3\,(-\Omega)\,R_1\,(-i)\,R_3\,(-\omega)\,\bar{q}$$

$$= R\bar{q}$$

where

$$R = R_3(-\Omega) R_1(-i) R_3(-\omega)$$

and

$$\dot{\bar{r}} = R \dot{\bar{q}}$$

MEAN ELEMENTS

If perturbations due to third bodies, non-sphericity of the two bodies, aerodynamic drag, or other causes are now taken into consideration, the above assumption of elliptic elements of constant value are no longer valid. If it is desired to have a solution of the equations of motion in terms of "elliptic elements" which will yield the position and velocity of the second body relative to the first body by means of the above formulae for elliptic motion, the desired set of "elliptic elements" are no longer constant in time. Letting the symbol e denote in general a, e, g_0, i, Ω, ω or any other equivalent "elliptic element," the solution now takes the form:

$$e = e_0 + e_1 d + e_2 d^2 + e_3 d^3 + P$$

where P denotes periodic terms, e_0, e_1, e_2 and e_3 are constant numerical coefficients, and d is the time elapsed from t_0, the epoch or reference time.

This can also be written

$$e = e_m + P$$

where $e_m = e_0 + e_1 d + e_2 d^2 + e_3 d^3$. e_m is commonly called a mean element[2] and e is called an osculating element, or an element of the osculating ellipse.

The geometrical significance of osculating elements and the osculating ellipse may be described as follows. Assume that at a given time t all forces acting upon the two bodies in such a manner as to cause deviation from "two-body motion" are removed. From that time on, the two bodies will then describe simple elliptic motion. The elements which would then define this elliptic motion are called osculating elements. However, since perturbations are continually

[2]There are various other definitions of the term "mean element" according to the method of solution of the equations of motion. The one described here is the most straightforward and useful one for computer applications.

acting on the two bodies, this ellipse, or osculating ellipse, is continually changing shape, size, and orientation in space. From a computational point of view, the important point is that if we have some solution which will yield a set of osculating elements, $e = e(t)$, for any time t, the position and velocity of the second body relative to the first may be easily found for that instant of time by means of the above algorithm for elliptic motion.

Referring again to the general expression for the calculation of osculating elements

$$e = e_0 + e_1 d + e_2 d^2 + e_3 d^3 + P$$

$$= e_m + P$$

there are five items worthy of note:

1. If there are no perturbations, i.e. if we have simple two body motion, then

$$e = e_0 \text{ or } e_1 = e_2 = e_3 = 0 \text{ and } P = 0.$$

2. The expression for P, the periodic terms, usually takes the form of a sine and cosine series with small numerical coefficients. The arguments of the sine and cosine terms in turn usually consist of mean elements, also, so that the entire expression for e may be evaluated with only one variable, d, known, i.e. $e = e(d)$ only.

3. Since the coefficients of the individual terms of P are usually very small, the osculating value of any orbital element may be approximated to a high degree of accuracy by its mean element.[3,4,5]

$$e = e_m = e_0 + e_1 d + e_2 d^2 + e_3 d^3$$

---

[3]One exception to this is the periodic term called evection in the calculation of the longitude of the Moon. The evection has an amplitude of approximately 1°.3 and a period of about 32 days. Acknowledgments are due to Mr. L. Werner of IBM who brought this to the attention of the author as a result of his computer program and discussions with Dr. P. Musen.

[4]On page 113 of the "Explanatory Supplement to the Ephemeris", an estimate of an accuracy 1′ in the position of the inner planets is given using their mean elements.

[5]Clemence, G.M., "On the Elements of Jupiter", Astronomical Journal November, 1946, Vol. 52 Number 4, page 89.

4. With proper analysis of the problem, prerequisite accuracies may be obtained by inspection of the coefficients of the periodic terms of the elements, retaining only a sufficient number of the larger coefficients to meet the requirement of the problem.

5. It is entirely feasible with current digital computer configurations and storage capacities to store the entire solutions of the equations of motion for the Earth, Moon, and planets in the memory of the computer. If these solutions are programmed in the form of a subroutine, the usage becomes extremely simple since the only input requirement is time, and the only limitation on the value of the time used is the limitation imposed by the solution itself. Any storage or computing time problems which may occasionally arise can be overcome by very simple programming techniques.[6]

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[6] An interesting compromise between the practice of having ephemeris on tape and computing internally from the equations of motion has been developed for the planets by Mr. Lloyd Carpenter of the Theoretical Division. Mr. Carpenter's technique consists of fitting a Chebyshev polynominal over discrete time periods.

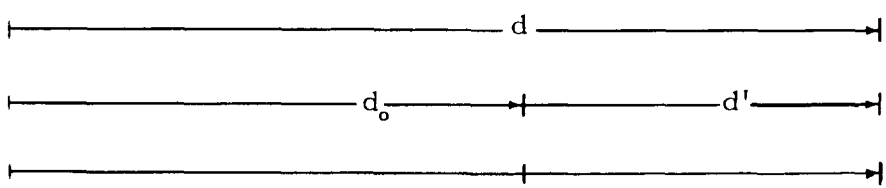
METHOD OF UPDATING THE EPOCH OF MEAN ORBITAL ELEMENTS IN ORDER TO AVOID THE USE OF DOUBLE PRECISION ARITHMETIC

The general expression for the calculation of the mean orbital elements of the Earth, Moon, and planets has the form

$$e_m = e_o + e_1 d + e_2 d^2 + e_3 d^3$$

where e_m is the numerical value of the mean element on any given Julian Date, JD; e_0, e_1, e_2, and e_3 are numerical coefficients valid only for a specific epoch Julian Date, JD_o; and $d = JD - JD_o$. In the enclosed tables, the coefficients are valid for JD_o = 2415020. 0, the Julian Date equivalent to the calendar date, 12 hours ephemeris time, January 0, 1900. In order to use these formulae for the calculation of mean orbital elements during the 1960's, double precision arithmetic would be required in the calculation of the $e_1 d$ term in order to preserve accuracy. Extensive use of double precision arithmetic may be avoided by updating the coefficients to be valid for a new epoch Julian Date, JD_o', equivalent to some convenient calendar date. The term "calendar date" is used in the sense of year, month, day, and time of day.

The required transformation is performed in the following manner.



JD_o	JD_o'	JD
old epoch (1900)	new epoch (arbitrary-to be chosen for convenience and accuracy of calculations)	date for which mean element is to be calculated

JD_o = 2415020. 0 = Julian Date corresponding to calendar date, 12 hours ephemeris time, January 0, 1900

JD_o' = Julian Date of new epoch

JD = Julian Date for which element is to be calculated

$d_o = JD'_o - JD_o$ = number of days from old epoch to new epoch
$d' = JD - JD'_o$ = " " " " new " " calculation date
$d = JD - JD_o = d_o + d'$ = " " " " old " " " "

An equivalent expression

$$e_m = e'_o + e'_1 + e'_2 d'^2 + e'_3 d'^3$$

where d' is the number of days elapsed from the new epoch JD'_o, can be derived from the original expression by making the substitution $d = d_o + d'$.

$$\begin{aligned} e_m &= e_o + e_1(d_o + d') + e_2(d_o + d')^2 + e_3(d_o + d')^3 \\ &= e_o + e_1 d_o + e_1 d' + e_2\left(d_o^2 + d'^2 + 2d_o d'\right) + e_3\left(d_o^3 + d'^3 + 3d_o^2 d' + 3d_o d'^2\right) \\ &= \left(e_o + e_1 d_o + e_2 d_o^2 + e_3 d_o^3\right) + \left(e_1 + 2e_2 d_o + 3e_3 d_o^2\right)d' + \left(e_2 + 3e_3 d_o\right)d'^2 + e_3 d'^3 \\ &= e'_o + e'_1 d' + e'_2 d'^2 + e'_3 d'^3 \end{aligned}$$

where

$e'_o = e_o + e_1 d_o + e_2 d_o^2 + e_3 d_o^3$ new constant term

$e'_1 = e_1 + 2e_2 d_o + 3e_3 d_o^2$ new coefficient of the 1st power of new time variable d'

$e'_2 = e_2 + 3e_3 d_o$

$e'_3 = e_3$

e'_o should be reduced by modulus 2π (= 6.283 185 307 179 586) using double precision arithmetic when angular units of radians are used. For short periods of time, the contributions of the d'^2 and d'^3 terms are usually insignificant, so that the expression for the calculation of any mean element reduces to the simple form

$$e_m = e'_o + e'_1 d'$$ d' = days elapsed from new epoch.

In most cases, single precision arithmetic may be used for a period of one year from the new epoch without significant loss of accuracy. The fastest moving body for which mean elements will normally be required for current space work is the Moon, whose mean orbital motion is approximately 13.1 degrees per day. Letting e'_1 be the mean motion of the Moon, the error associated with the use of single precision arithmetic may be calculated as follows:

$$(e'_1 \pm \Delta e'_1) \times (d' \pm \Delta d') = e'_1 \, d' \pm d' \Delta e'_1 \pm e'_1 \Delta d'$$

neglecting the $\Delta e'_1 \; \Delta d'$ term.

For single precision accuracy (approximately 8 decimal digits on the IBM 7094),

$$\Delta e_1 = \pm (1 \times 10^{-6}) \text{ degrees/day for the mean motion of the Moon}$$

$$\begin{aligned} \Delta d' &= \pm (1 \times 10^{-7}) \text{ days for } d' = 1\text{–}\ \ 9 \text{ days} \\ &= \pm (1 \times 10^{-6}) \quad '' \quad '' \quad '' = 10\text{–}\ 99 \quad '' \\ &= \pm (1 \times 10^{-5}) \quad '' \quad '' \quad '' = 100\text{–}999 \quad '' \end{aligned}$$

so that

$$(e'_1 \pm \Delta e'_1) \times (d' \pm \Delta d') = e'_1 \, d' \pm (1 \times 10^{-6})^{\circ} \; d' \quad \pm 13\overset{\circ}{.}1 \;\; \Delta d'$$

$$= e'_1 d' \pm \begin{cases} (1 \times 10^{-6}) \quad (9)^{\circ} \pm 13\overset{\circ}{.}1 \; (1 \times 10^{-7}) & 1\text{–}\ \ 9 \text{ day} \\ (1 \times 10^{-6}) \quad (99)^{\circ} \pm 13\overset{\circ}{.}1 \; (1 \times 10^{-6}) & 10\text{–}\ 99 \quad '' \\ (1 \times 10^{-6}) \; (999)^{\circ} \pm 13\overset{\circ}{.}1 \; (1 \times 10^{-5}) & 100\text{–}999 \quad '' \end{cases}$$

$$= e_1'd' \pm \begin{cases} 9^{\circ}\!.00 \times 10^{-6} \pm 1^{\circ}\!.3 \times 10^{-6} \\ 9^{\circ}\!.90 \times 10^{-5} \pm 1^{\circ}\!.3 \times 10^{-5} \\ 9^{\circ}\!.99 \times 10^{-4} \pm 1^{\circ}\!.3 \times 10^{-4} \end{cases} = \quad e_1'd' \pm \begin{cases} 10.30 \times 10^{-6} \\ 11.20 \times 10^{-5} \\ 11.22 \times 10^{-4} \end{cases}$$

$$\cong e_1' d' \pm \begin{cases} ''\!.05 & \text{for} \quad 1-\ 9 \text{ days} \\ ''\!.5 & \text{"} \quad 10-\ 99 \text{ "} \\ 5''\!.0 & \text{"} \quad 100-999 \text{ "} \end{cases}$$

Tables of the mean elements of the Sun and Moon appear at the end of this report. These mean elements are listed in 6 combinations of units:

Angular Units : Degrees, radians, revolutions

Time Units : Days, Julian Centuries

The tables are derived strictly from the expressions listed in the American Ephemeris and The Explanatory Supplement. These two references in turn list the elements as given by Newcomb[7] for the theory of the Sun and by Brown[8] for the theory of the Moon. In each case, the mean elements listed by all of these volumes are in units which are awkward and inefficient for computer usage.

The enclosed tables use the same epoch, January 0.5, 1900, as used by the American Ephemeris and The Explanatory Supplement. Sufficient accuracy is carried in the coefficients listed in the tables to maintain a numerical accuracy of $''\!.01$ over a time span of one century from the epoch.

The above method of updating mean elements was programmed for the IBM 7094 computer. In order to compare the method with the formulae used by the Jet Propulsion Laboratory in their trajectory programs, new coefficients were computed for the epoch January 1.0, 1950. Significant discrepancies were found in the following 3 elements:[9],[10]

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(7) Newcomb, "Astronomical Papers Prepared for the use of the American Ephemeris," Vol. 6, Part 1.
(8) Brown, E.W., "Tables of the Motion of the Moon."
(9) Kalensher, B.E., "Selenographic Coordinates," JPL Technical Report No. 32-41, page 23.
(10) Holdridge, D.B., "Space Trajectories Program for the IBM 7090," JPL Technical Report 32-223, page 69.

Moon – Mean Anomaly[11]

(1)	215°.531 46	+	13°.064 993	d	(Sandifer)
(2)	215°.540 13	+	13°.064 992	d	(JPL)
	+ °.008 67	−	°.000 001	d	difference (2-1)

Moon – Argument of Perigee[11]

(1)	196°.731 199	+	0°.164 357 7	d	(Sandifer)
(2)	196°.745 632	+	0°.164 358 6	d	(JPL)
	+ °.014 433	+	°.000 000 9	d	difference (2-1)

Sun – Mean Anomaly[11]

(1a)	358°.000 682[12]	+	0.985 600 3	d	(Sandifer)
(1b)	358.000 670[13]	+	0.985 600 3	d	(Sandifer)
(2)	358°.009 067	+	0.985 600 5	d	(JPL)
	+ °.008 397	+	°.000 000 2	d	difference (2-1a)

Minor discrepancies were found in the other elements listed by JPL. The elements listed above are used by JPL in the calculation of the lunar libration terms for conversion to Selenographic coordinates. The discrepancies noted are of the order of 1×10^{-2} degrees in the constant term or 2×10^{-4} radians. A further source of discrepancy will be encountered when using the JPL formulae for the year 1966. During this year, the value of d is approximately 365 days × 16 years or 5×10^{3} days. Since the coefficients of the d terms carried by JPL in the above 3 cases is of the order of 10^{-6} to 10^{-7} degrees, the error accumulated after 5×10^{3} days is 5×10^{-3} degrees or approximately 10^{-4} to 10^{-5} radians. It is therefore possible that a transformation to a Selenographic coordinate system using the JPL values for the mean elements could reduce the number of

(11) There is also an internal inconsistency in JPL Technical Report No. 32-223 on page 69. The mean anomalies of the moon and sun are defined as $g = ℂ - \Gamma'$ and $g' = L - \Gamma$ and the argument of perigee of the moon is defined as $\omega = \Gamma' - \Omega$. However, if the numerical values of ℂ, L, Γ, Γ′ and Ω listed on page 68 are substituted into these equations, the results are not the same as the numerical expressions for g, g′ and ω listed on page 69.

(12) Corresponds to the value 358° 28′ 33″.04 updated from epoch Jan. 0.5, 1900.

(13) " " " " 358° 28′ 33″.00 updated from epoch Jan. 0.5, 1900.

significant figures in a position vector from 8 significant digits to 3-4 significant digits, due to these causes only. As a double check, both the JPL formulae and the formulae computed by the 7094 program were used to calculate the 3 elements for a date in 1966, using double precision arithmetic in both cases. These values were then compared with the values tabulated in the American Ephemeris. In each case the JPL values were off by the same order of magnitude noted above, whereas the values predicted by the method described in this report, using a January 1.0, 1950 epoch, agreed exactly with the tabulated values.* It is therefore recommended that the expressions for the mean elements for the Sun and Moon, when required, be taken from the tables of this report.

These above mentioned discrepancies are significant in that they illustrate a very bad computing procedure commonly used by many reputable organizations and individuals. There exist today a proliferation of epochs which have been chosen for various reasons. One of the reasons commonly advanced is that the use of double precision arithmetic is avoided by using an epoch close to the computing date. However, the method normally employed is to precompute the updated coefficients of the mean elements of the Sun and Moon on a desk calculator, and then enter these updated coefficients as constants in the computer. This method invariably leads to the kind of confusion encountered in the referenced JPL reports. This error is usually compounded in that other organizations and individuals adopt these erroneous values as published and thus perpetuate the error.

As a case in illustration, the IBM group currently engaged in preparing the AIMP real-time program are using values of the coefficients of the mean elements of the Sun and Moon which appeared in a report by Carson and Clark of the Data Operations Branch. These values, in turn, appear to have been adopted by Carson and Clark in good faith from the previously referenced JPL reports.

In addition, the argument advanced – that the use of double precision arithmetic is avoided with the use of precomputed updated coefficients – is quickly outdated in a few years when the coefficients of the first power of time multiply into an increasingly larger number.

It would be preferable to have the various national observatories responsible for maintaining ephemerides publish the elements in a form suitable for computers. If it is desired to minimize the use of double precision arithmetic, it is a fairly routine and simple procedure to program the computer to update the coefficients automatically and internally, thereby avoiding the type of errors encountered in precomputing the coefficients on a desk calculator.

CALCULATION OF THE SELENOCENTRIC POSITION VECTORS OF THE EARTH AND SUN NEGLECTING PERIODIC PERTURBATIONS

Initialization

1. Choose the Julian Date of 4th stage injection ($= JD_o'$) as the new epoch to replace the epoch $JD_o' = 2415020.0$ for the calculation of the mean orbital elements of the Earth and Sun.* (See note below.)

2. Compute

   A. $d_o = JD_o' - JD_o$ in double precision. (JD_o' must be stored as a double precision word. JD_o may be stored as a single precision word.)

   B. d_o^2, d_o^3 in single precision.

3. Compute for JD_o' the numerical values of the following mean elements:

$$e_o' \qquad \Omega_3''$$

$$\omega_o' \qquad \omega_3'' \qquad \text{(Tables 1e, 3)}$$

$$g_3''$$

$$F_3'' = \omega_3'' + g_3''$$

   Reduce all angles by modulus 2π using double precision arithmetic (2π = 6.283 185 307 179 586)

4. Compute the matrices

$$R' = R_3(-\Omega_3'')R_1(-i_3'') \qquad \text{(Table 3 for } i_3'',\ \sin i_3'',\ \cos i_3''\text{)}$$

$$R'' = R_3(u_3'')R_1(i_3'')R_3(\Omega_3'') \text{ (NOTATION for definitions of } R_1,\ R_3\text{)}$$

5. Compute the coefficients of the sin ng and cos ng terms of the expansions of C_o' and r_o', using the algorithm in Table 4, to the 4th power of e_o'.

---

*Any convenient epoch may be chosen. Time of 4th stage injection is chosen here as one possible convenient epoch for AIMP.

6. Compute the new numerical coefficients of the expressions for g_o', g_3'', ω_o', and ω_3'' valid for the new epoch JD_o', according to the method described in the previous section.

Calculation For Any Time JD After Initialization

1. Compute

$$d' = JD - JD_o' \doteq \text{number of days elapsed since 4th stage injection (or other new epoch)}$$

2. Compute Geocentric position vector of the Sun, referred to the ecliptic and mean equinox of date

Compute:

a. g_o'; reduce by modulus 2π

b. C_o', r_o' using algorithm of Table 4

c. $u_o' = \omega_o' + g_o' + C_o'$; reduce by modulus 2π

d.

$$\bar{r}_o' = r_o' \begin{bmatrix} \cos u_o' \\ \sin u_o' \\ 0 \end{bmatrix}$$ Geocentric position vector of the Sun, ecliptic and mean equinox of date.

3. Compute Selenocentric position vector of the Earth, referred to the ecliptic and mean equinox of date.

Compute:

a. g_3''; reduce by modulus 2π

b. C_3'', r_3'' (Table 3)*

c. $u_3'' = \omega_3'' + g_3'' + C_3''$*; reduce by modulus 2π

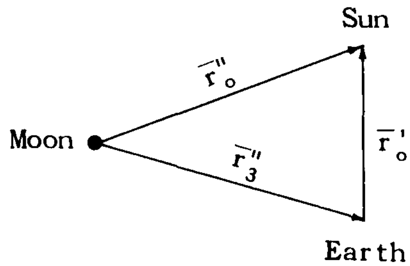
d.

$$\bar{q}_3'' = r_3'' \begin{bmatrix} \cos u_3'' \\ \sin u_3'' \\ 0 \end{bmatrix}$$

e. $\bar{r}_3'' = R'\bar{q}_3''$ Selenocentric position vector of the Earth, referred to the ecliptic and mean equinox of date

4. Compute Selenocentric position vector of the Sun, referred to the ecliptic and mean equinox of date.

$$\bar{r}_o'' = \bar{r}_3'' + \bar{r}_o'$$



5. Transform $\bar{r}_o''$ and $\bar{r}_3''$ from ecliptic and mean equinox of date to a coordinate system formed by using the mean Earth-Moon orbit plane as the fundamental plane and the mean Moon to Earth line at JD_o' as the X axis (JD_o' = 4th stage injection).

$$\bar{r}_o''' = R''\bar{r}_o''$$

$$\bar{r}_3''' = R''\bar{r}_3''$$

TABLE 1a

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{R} = revolutions, ° = degrees, ' = minutes, " = seconds
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = 279°41'48''.04 + 100^R\ 0°46'08''.13\ T + \begin{cases} 1''.089\ T^2 \\ 1''.090\ T^2 \end{cases}$$

$$\begin{bmatrix} \tau'^{*}_o = 279°41'27''.54 + 100^R\ 0°46'08''.13\ T + 1''.3935\ T^2 \\ \theta'_o = \tau'_o + 180° = 99°41'27''.54 + 100^R\ 0°46'08''.13\ T + 1''.3935\ T^2 \end{bmatrix}$$

$$\Gamma'_o = 281°13'15''.00 + 1°43'09''.03\ T + 1''.63\ T^2 + ''.012\ T^3$$

$$\Omega'_o = 0°$$

$$g'_o = L'_o - \Gamma'_o = \begin{cases} 358°28'33''.00^{\dagger} \\ 358°28'33''.04 \end{cases} + 99^R 359°02'59''.10\ T - \begin{cases} ''.541\ T^2 \\ ''.540\ T^2 \end{cases} - ''.012\ T^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o$$

$$= \Gamma'_o = 281°13'15''.00 + 1°43'09''.03\ T + 1''.63\ T^2 + ''.012\ T^3$$

$$F'_o = L'_o - \Omega'_o$$

$$= L'_o = 279°41'48''.04 + 100^R\ 0°46'08''.13\ T + \begin{cases} 1''.089\ T^2 \\ 1''.090\ T^2 \end{cases}$$

$$\epsilon_m = 23°27'08''.26 - 46''.845\ T - ''.0059\ T^2 + ''.00181\ T^3$$

$$e'_o = .016\ 751\ 04 - .000\ 041\ 80\ T - .000\ 000\ 126\ T^2$$

*This is equivalent to $\tau'_o = 18^H\ 38^M\ 45^S.836 + 100^D\ 00^H\ 03^M\ 04^S.542\ T + {}^S.0929\ T^2$

†Constant term according to Newcomb is 358°28'33''.0.

TABLE 1a (Continued)

L'_o, τ'_o	100^R 0°46' 08".13 = 129 602 768".13 = 8 640 184^s.542
Γ'_o	1°43' 09".03 = 6 189".03
g'_o	99^R359°02' 59".10 = 129 596 579".10

Conversion of τ'_o to °, ', "

Constant Term			T Term		
18^H	=	270°	100^D	=	100^R
38^M	=	9° 30'	00^H	=	00°
45^S	=	11' 15"	03^M	=	0° 45'
^{s}.836	=	12".54	04^s.542	=	1' 08".13
18^H 38^M 45^s.836	=	279° 41' 27".54	100^D 00^H 03^M 04^s.542	=	100^R 00° 46' 08".13

TABLE 1b

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	° = degrees
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = 279^{\circ}.696\,677\,778 + 36\,000^{\circ}.768\,925\,000\,T + \left\{\begin{array}{l} ^{\circ}.000\,302\,500\,T^2 \\ ^{\circ}.000\,302\,778\,T^2 \end{array}\right.$$

$$\left[\begin{array}{ll} \tau'_o & = 279^{\circ}.690\,983\,333 + 36\,000^{\circ}.768\,925\,000\,T + {}^{\circ}.000\,387\,083\,T^2 \\ \theta'_o = \tau'_o + 180^{\circ} & = 99^{\circ}.690\,983\,333 + 36\,000^{\circ}.768\,925\,000\,T + {}^{\circ}.000\,387\,083\,T^2 \end{array}\right]$$

$$\Gamma'_o = 281^{\circ}.220\,833\,333 + 1^{\circ}.719\,175\,000\,T + {}^{\circ}.000\,452\,778\,T^2 + {}^{\circ}.000\,003\,333\,T^3$$

$$\Omega'_o = 0^{\circ}.0$$

$$g'_o = L'_o - \Gamma'_o = \left\{\begin{array}{l} 358^{\circ}.475\,833\,333 \\ 358^{\circ}.475\,844\,444 \end{array}\right. + 35\,999^{\circ}.049\,750\,000\,T - \left\{\begin{array}{l} ^{\circ}.000\,150\,278\,T^2 \\ ^{\circ}.000\,150\,000\,T^2 \end{array}\right. - {}^{\circ}.000\,003\,333\,T^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o$$

$$= \Gamma'_o = 281^{\circ}.220\,833\,333 + 1^{\circ}.719\,175\,000\,T + {}^{\circ}.000\,452\,778\,T^2 + {}^{\circ}.000\,003\,333\,T^3$$

$$F'_o = L'_o - \Omega'_o$$

$$= L'_o = 279^{\circ}.696\,677\,778 + 36\,000^{\circ}.768\,925\,000\,T + \left\{\begin{array}{l} ^{\circ}.000\,302\,500\,T^2 \\ ^{\circ}.000\,302\,778\,T^2 \end{array}\right.$$

$$\epsilon_m = 23^{\circ}.452\,294\,444 - {}^{\circ}.013\,012\,500\,T - {}^{\circ}.000\,001\,639\,T^2 + {}^{\circ}.000\,000\,503\,T^3$$

$$e'_o = .016\,751\,04 - .000\,041\,80\,T - .000\,000\,126\,T^2$$

TABLE 1c

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	° = degrees
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = 279^\circ.696\,677\,778 + {}^\circ.985\,647\,335\,387\ d + \left\{\begin{array}{l} {}^\circ.226\,749 \times 10^{-12}\ d^2 \\ {}^\circ.226\,957 \times 10^{-12}\ d^2 \end{array}\right.$$

$$\left[\begin{array}{l} \tau'_o = 279^\circ.690\,983\,333 + {}^\circ.985\,647\,335\,387\ d + {}^\circ.290\,151 \times 10^{-12}\ d^2 \\ \theta'_o = \tau'_o + 180^\circ = 99^\circ.690\,983\,333 + {}^\circ.985\,647\,335\,387\ d + {}^\circ.290\,151 \times 10^{-12}\ d^2 \end{array}\right]$$

$$\Gamma'_o = 281^\circ.220\,833\,333 + {}^\circ.000\,047\,068\,446\ d + {}^\circ.339\,394 \times 10^{-12}\ d^2 + {}^\circ.068\,408 \times 10^{-18}\ d^3$$

$$\Omega'_o = 0^\circ.0$$

$$g'_o = L'_o - \Gamma'_o = \left\{\begin{array}{l} 358^\circ.475\,833\,333 \\ 358^\circ.475\,844\,444 \end{array}\right. + {}^\circ.985\,600\,266\,940\ d - \left\{\begin{array}{l} {}^\circ.112\,646 \times 10^{-12}\ d^2 \\ {}^\circ.112\,437 \times 10^{-12}\ d^2 \end{array}\right. - {}^\circ.068\,408 \times 10^{-18}\ d^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o = \Gamma'_o = 281^\circ.220\,833\,333 + {}^\circ.000\,047\,068\,446\ d + {}^\circ.339\,394 \times 10^{-12}\ d^2 + {}^\circ.068\,408 \times 10^{-18}\ d^3$$

$$F'_o = L'_o - \Omega'_o = L'_o = 279^\circ.696\,677\,778 + {}^\circ.985\,647\,335\,387\ d + \left\{\begin{array}{l} {}^\circ.226\,749 \times 10^{-12}\ d^2 \\ {}^\circ.226\,957 \times 10^{-12}\ d^2 \end{array}\right.$$

$$\epsilon_m = 23^\circ.452\,294\,444 - {}^\circ.000\,000\,356\,263\ d - {}^\circ.001\,228 \times 10^{-12}\ d^2 + {}^\circ.010\,318 \times 10^{-18}\ d^3$$

$$e'_o = .016\,751\,04 - .000\,000\,001\,144\,421\ d - .000\,094\,447 \times 10^{-12}\ d^2$$

TABLE 1d

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{r} = radians
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = 4^{r}881\ 627\ 934\ 112 + 628^{r}331\ 950\ 990\ 910\ T + \left\{\begin{matrix} 5^{r}279\ 621 \times 10^{-6}\ T^2 \\ 5^{r}284\ 469 \times 10^{-6}\ T^2 \end{matrix}\right.$$

$$\tau'_o = 4^{r}881\ 528\ 547\ 307 + 628^{r}331\ 950\ 990\ 910\ T + 6^{r}755\ 879 \times 10^{-6}\ T^2$$

$$\theta'_o = \tau'_o + \pi^{r} = 1^{r}739\ 935\ 893\ 717 + 628^{r}331\ 950\ 990\ 910\ T + 6^{r}755\ 879 \times 10^{-6}\ T^2$$

$$\Gamma'_o = 4^{r}908\ 229\ 466\ 869 + {}^{r}030\ 005\ 264\ 168\ T + 7^{r}902\ 463 \times 10^{-6}\ T^2 + {}^{r}058\ 178 \times 10^{-6}\ T^3$$

$$\Omega'_o = 0^{r}0$$

$$g'_o = L'_o - \Gamma'_o = \left\{\begin{matrix} 6^{r}256\ 583\ 580\ 497 \\ 6^{r}256\ 583\ 774\ 423 \end{matrix}\right. + 628^{r}301\ 945\ 726\ 742\ T - \left\{\begin{matrix} 2^{r}622\ 842 \times 10^{-6}\ T^2 \\ 2^{r}617\ 994 \times 10^{-6}\ T^2 \end{matrix}\right. - {}^{r}058\ 178 \times 10^{-6}\ T^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o$$

$$= \Gamma'_o = 4^{r}908\ 229\ 466\ 869 + {}^{r}030\ 005\ 264\ 168\ T + 7^{r}902\ 463 \times 10^{-6}\ T^2 + {}^{r}058\ 178 \times 10^{-6}\ T^3$$

$$F'_o = L'_o - \Omega'_o$$

$$= L'_o = 4^{r}881\ 627\ 934\ 112 + 628^{r}331\ 950\ 990\ 910\ T + \left\{\begin{matrix} 5^{r}279\ 621 \times 10^{-6}\ T^2 \\ 5^{r}284\ 469 \times 10^{-6}\ T^2 \end{matrix}\right.$$

$$\epsilon_m = {}^{r}409\ 319\ 755\ 203 - {}^{r}000\ 227\ 110\ 969\ T - {}^{r}028\ 604 \times 10^{-6}\ T^2 + {}^{r}008\ 775 \times 10^{-6}\ T^3$$

$$e'_o = .016\ 751\ 04 - .000\ 041\ 80\ T - .000\ 000\ 126\ T^2$$

TABLE 1e

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{r} = radians
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = 4^r.881\ 627\ 934\ 112 + {}^r.017\ 202\ 791\ 266\ 007\ d + \begin{cases} {}^r.003\ 957\ 513 \times 10^{-12}\ d^2 \\ {}^r.003\ 961\ 147 \times 10^{-12}\ d^2 \end{cases}$$

$$\left[\begin{array}{ll} \nu'_o & = 4^r.881\ 528\ 547\ 307 + {}^r.017\ 202\ 791\ 266\ 007\ d + {}^r.005\ 064\ 090 \times 10^{-12}\ d^2 \\ \theta'_o = \nu'_o + \pi^r & = 1^r.739\ 935\ 893\ 717 + {}^r.017\ 202\ 791\ 266\ 007\ d + {}^r.005\ 064\ 090 \times 10^{-12}\ d^2 \end{array}\right]$$

$$\Gamma'_o = 4^r.908\ 229\ 466\ 869 + {}^r.000\ 000\ 821\ 499\ 361\ d + {}^r.005\ 923\ 550 \times 10^{-12}\ d^2 + 1^r.193\ 948 \times 10^{-21}\ d^3$$

$$\Omega'_o = 0^r.0$$

$$g'_o = L'_o - \Gamma'_o = \begin{cases} 6^r.256\ 583\ 580\ 497 \\ 6^r.256\ 583\ 774\ 423 \end{cases} + {}^r.017\ 201\ 969\ 766\ 646\ d - \begin{cases} {}^r.001\ 966\ 037 \times 10^{-12}\ d^2 \\ {}^r.001\ 962\ 403 \times 10^{-12}\ d^2 \end{cases} - 1^r.193\ 948 \times 10^{-21}\ d^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o$$

$$= \Gamma'_o = 4^r.908\ 229\ 466\ 869 + {}^r.000\ 000\ 821\ 499\ 361\ d + {}^r.005\ 923\ 550 \times 10^{-12}\ d^2 + 1^r.193\ 948 \times 10^{-21}\ d^3$$

$$F'_o = L'_o - \Omega'_o$$

$$= L'_o = 4^r.881\ 627\ 934\ 112 + {}^r.017\ 202\ 791\ 266\ 007\ d + \begin{cases} {}^r.003\ 957\ 513 \times 10^{-12}\ d^2 \\ {}^r.003\ 961\ 147 \times 10^{-12}\ d^2 \end{cases}$$

$$\epsilon_m = {}^r.409\ 319\ 755\ 203 - {}^r.000\ 000\ 006\ 217\ 959\ d - {}^r.000\ 021\ 441 \times 10^{-12}\ d^2 + {}^r.180\ 087 \times 10^{-21}\ d^3$$

$$e'_o = .016\ 751\ 04 - .000\ 000\ 001\ 144\ 421\ d - .000\ 094\ 447 \times 10^{-12}\ d^2$$

TABLE 1f

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{R} = revolutions
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = {}^R.776\,935\,216\,049 + 100^R.002\,135\,902\,778\,T + \begin{cases} {}^R.840\,278 \times 10^{-6}\,T^2 \\ {}^R.841\,049 \times 10^{-6}\,T^2 \end{cases}$$

$$\left[\begin{array}{l} \gamma'_o = {}^R.776\,919\,398\,148 + 100^R.002\,135\,902\,778\,T + 1^R.075\,231 \times 10^{-6}\,T^2 \\ \theta'_o = \gamma'_o + {}^R.5 = {}^R.276\,919\,398\,148 + 100^R.002\,135\,902\,778\,T + 1^R.075\,231 \times 10^{-6}\,T^2 \end{array}\right]$$

$$\Gamma'_o = {}^R.781\,168\,981\,481 + {}^R.004\,775\,486\,111\,T + 1^R.257\,716 \times 10^{-6}\,T^2 + 9^R.259 \times 10^{-9}\,T^3$$

$$\Omega'_o = {}^R.0$$

$$g'_o = L'_o - \Gamma'_o = \begin{cases} {}^R.995\,766\,203\,704 \\ {}^R.995\,766\,234\,568 \end{cases} + 99^R.997\,360\,416\,667\,T - \begin{cases} {}^R.417\,438 \times 10^{-6}\,T^2 \\ {}^R.416\,667 \times 10^{-6}\,T^2 \end{cases} - 9^R.259 \times 10^{-9}\,T^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o$$
$$= \Gamma'_o = {}^R.781\,168\,981\,481 + {}^R.004\,775\,486\,111\,T + 1^R.257\,716 \times 10^{-6}\,T^2 + 9^R.259 \times 10^{-9}\,T^3$$

$$F'_o = L'_o - \Omega'_o$$
$$= L'_o = {}^R.776\,935\,216\,049 + 100^R.002\,135\,902\,778\,T + \begin{cases} {}^R.840\,278 \times 10^{-6}\,T^2 \\ {}^R.841\,049 \times 10^{-6}\,T^2 \end{cases}$$

$$\epsilon_m = {}^R.065\,145\,262\,346 - {}^R.000\,036\,145\,833\,T - {}^R.004\,552 \times 10^{-6}\,T^2 + 1^R.397 \times 10^{-9}\,T^3$$

$$e'_o = .016\,751\,04 - .000\,041\,80\,T - .000\,000\,126\,T^2$$

TABLE 1g

MEAN ORBITAL ELEMENTS OF THE SUN

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{R} = revolutions
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L'_o = {}^R.776\ 935\ 216\ 049 + {}^R.002\ 737\ 909\ 264\ 963\ d + \begin{cases} {}^R.629\ 858 \times 10^{-15}\ d^2 \\ {}^R.630\ 436 \times 10^{-15}\ d^2 \end{cases}$$

$$\left[\begin{array}{lll} \tau'_o & = & {}^R.776\ 919\ 398\ 148 + {}^R.002\ 737\ 909\ 264\ 963\ d + {}^R.805\ 975 \times 10^{-15}\ d^2 \\ \theta'_o = \tau'_o + {}^R.5 & = & {}^R.276\ 919\ 398\ 148 + {}^R.002\ 737\ 909\ 264\ 963\ d + {}^R.805\ 975 \times 10^{-15}\ d^2 \end{array}\right]$$

$$\Gamma'_o = {}^R.781\ 168\ 981\ 481 + {}^R.000\ 000\ 130\ 745\ 684\ d + {}^R.942\ 762 \times 10^{-15}\ d^2 + {}^R.190\ 023 \times 10^{-21}\ d^3$$

$$\Omega'_o = {}^R.0$$

$$g'_o = L'_o - \Gamma'_o = \begin{cases} {}^R.995\ 766\ 203\ 704 \\ {}^R.995\ 766\ 234\ 568 \end{cases} + {}^R.002\ 737\ 778\ 519\ 279\ d - \begin{cases} {}^R.312\ 905 \times 10^{-15}\ d^2 \\ {}^R.312\ 326 \times 10^{-15}\ d^2 \end{cases} - {}^R.190\ 023 \times 10^{-21}\ d^3$$

$$\omega'_o = \Gamma'_o - \Omega'_o$$
$$= \Gamma'_o = {}^R.781\ 168\ 981\ 481 + {}^R.000\ 000\ 130\ 745\ 684\ d + {}^R.942\ 762 \times 10^{-15}\ d^2 + {}^R.190\ 023 \times 10^{-21}\ d^3$$

$$F'_o = L'_o - \Omega'_o$$
$$= L'_o = {}^R.776\ 935\ 216\ 049 + {}^R.002\ 737\ 909\ 264\ 963\ d + \begin{cases} {}^R.629\ 858 \times 10^{-15}\ d^2 \\ {}^R.630\ 436 \times 10^{-15}\ d^2 \end{cases}$$

$$\epsilon_m = {}^R.065\ 145\ 262\ 346 - {}^R.000\ 000\ 000\ 989\ 619\ d - {}^R.003\ 412 \times 10^{-15}\ d^2 + {}^R.028\ 662 \times 10^{-21}\ d^3$$

$$e'_o = .016\ 751\ 04 - .000\ 000\ 001\ 144\ 421\ d - .094\ 447 \times 10^{-15}\ d^2$$

TABLE 2a

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{R} = revolutions, ° = degrees, ' = minutes, " = seconds
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

L'	=	270°26'02".99	+	1336^R307°52'59".31 T	−	4".08 T^2	+	".0068 T^3
Γ'	=	334°19 46".40	+	11^R109°02'02".52 T	−	37".17 T^2	−	".045 T^3
Ω'	=	259°10'59".79	−	5^R134°08'31".23 T	+	7".48 T^2	+	".008 T^3
$g' = L' - \Gamma'$	=	295°06'16".59	+	1325^R198°50'56".79 T	+	33".09 T^2	+	".0518 T^3
$\omega' = \Gamma' - \Omega'$	=	75°08'46".61	+	16^R243°10'33".75 T	−	44".65 T^2	−	".053 T^3
$\omega' + 180°$	=	255°08'46".61	+	16^R243°10'33".75 T	−	44".65 T^2	−	".053 T^3
$F = L' - \Omega'$	=	11°15'03".20	+	1342^R 82°01'30".54 T	−	11".56 T^2	−	".0012 T^3
$\theta' = L' - \Omega' + 180°$	=	191°15'03".20	+	1342^R 82°01'30".54 T	−	11".56 T^2	−	".0012 T^3
$D' = L' - L'_o$	=	350°44'14".95	+	1236^R307°06'51".18 T	−	5".17 T^2	+	".0068 T^3
i'	=	5°08'43".428						
I'	=	1°32'06"						
e'	=	.054 900 489						
a'	=	60.2665 Earth radii						
	=	384,399.3537 km (using 1 Earth radius = 6378.3255 km.)						
sin i'	=	+.089 683 453						
cos i'	=	+.995 970 322						
sin I'	=	+.026 787 60						
cos I'	=	+.999 641 15						

TABLE 2b

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	° = degrees
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

L'	=	270°.434 163 889	+ 481 267°.883 141 667 T	− °.001 133 333 T^2	+ °.000 001 889 T^3
Γ'	=	334°.329 555 556	+ 4 069°.034 033 333 T	− °.010 325 000 T^2	− °.000 012 500 T^3
Ω'	=	259°.183 275 000	− 1 934°.142 008 333 T	+ °.002 077 778 T^2	+ °.000 002 222 T^3
$g' = L' - \Gamma'$	=	296°.104 608 333	+ 477 198°.849 108 333 T	+ °.009 191 667 T^2	+ °.000 014 389 T^3
$\omega' = \Gamma' - \Omega'$	=	75°.146 280 556	+ 6 003°.176 041 667 T	− °.012 402 778 T^2	− °.000 014 722 T^3
$\omega' + 180°$	=	255°.146 280 556	+ 6 003°.176 041 667 T	− °.012 402 778 T^2	− °.000 014 722 T^3
$F' = L' - \Omega'$	=	11°.250 888 889	+ 483 202°.025 150 000 T	− °.003 211 111 T^2	− °.000 000 333 T^3
$\theta' = L' - \Omega' + 180°$	=	191°.250 888 889	+ 483 202°.025 150 000 T	− °.003 211 111 T^2	− °.000 000 333 T^3
$D' = L' - L'_o$	=	350°.737 486 111	+ 445 267°.114 216 667 T	− °.001 436 111 T^2	+ °.000 001 889 T^3
i'	=	5°.145 396 667			
I'	=	1°.535 000			
e'	=	.054 900 489			

a' = 60.2665 Earth radii
= 384,399.3537 km. (using 1 Earth radius = 6378.3255 km.)

sin i'	=	+.089 683 453
cos i'	=	+.995 970 322

sin I'	=	+.026 787 60
cos I'	=	+.999 641 15

TABLE 2c

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	° = degrees
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

L'	=	$270^\circ.434\,163\,889 + 13^\circ.176\,396\,526\,808\,d - {}^\circ.849\,527 \times 10^{-12}\,d^2 + {}^\circ.038\,765 \times 10^{-18}\,d^3$
Γ'	=	$334^\circ.329\,555\,556 + {}^\circ.111\,404\,080\,310\,d - 7^\circ.739\,441 \times 10^{-12}\,d^2 - {}^\circ.256\,531 \times 10^{-18}\,d^3$
Ω'	=	$259^\circ.183\,275\,000 - {}^\circ.052\,953\,922\,199\,d + 1^\circ.557\,466 \times 10^{-12}\,d^2 + {}^\circ.045\,605 \times 10^{-18}\,d^3$
$g' = L' - \Gamma'$	=	$296^\circ.104\,608\,333 + 13^\circ.064\,992\,446\,498\,d + 6^\circ.889\,914 \times 10^{-12}\,d^2 + {}^\circ.295\,295 \times 10^{-18}\,d^3$
$\omega' = \Gamma' - \Omega'$	=	$75^\circ.146\,280\,556 + {}^\circ.164\,358\,002\,510\,d - 9^\circ.296\,908 \times 10^{-12}\,d^2 - {}^\circ.302\,136 \times 10^{-18}\,d^3$
$\omega' + 180^\circ$	=	$255^\circ.146\,280\,556 + {}^\circ.164\,358\,002\,510\,d - 9^\circ.296\,908 \times 10^{-12}\,d^2 - {}^\circ.302\,136 \times 10^{-18}\,d^3$
$F' = L' - \Omega'$	=	$11^\circ.250\,888\,889 + 13^\circ.229\,350\,449\,008\,d - 2^\circ.406\,993 \times 10^{-12}\,d^2 - {}^\circ.006\,841 \times 10^{-18}\,d^3$
$\theta' = L' - \Omega' + 180^\circ$	=	$191^\circ.250\,888\,889 + 13^\circ.229\,350\,449\,008\,d - 2^\circ.406\,993 \times 10^{-12}\,d^2 - {}^\circ.006\,841 \times 10^{-18}\,d^3$
$D' = L' - L'_o$	=	$350^\circ.737\,486\,111 + 12^\circ.190\,749\,191\,421\,d - 1^\circ.076\,484 \times 10^{-12}\,d^2 + {}^\circ.038\,765 \times 10^{-18}\,d^3$
i'	=	$5^\circ.145\,396\,667$
I'	=	$1^\circ.535\,000$
e'	=	$.054\,900\,489$
a'	=	60.2665 Earth radii
	=	384,399.3537 km. (using 1 Earth radius = 6378.3255 km.)
$\sin i'$	=	$+.089\,683\,453$
$\cos i'$	=	$+.995\,970\,322$
$\sin I'$	=	$+.026\,787\,60$
$\cos I'$	=	$+.999\,641\,15$

TABLE 2d

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{r} = radians
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L' = 4\overset{r}{.}719\,966\,569\,739 + 8\,399\overset{r}{.}709\,144\,925\,395\,T - \overset{r}{.}000\,019\,780\,398\,T^2 + \overset{r}{.}032\,967 \times 10^{-6}\,T^3$$

$$\Gamma' = 5\overset{r}{.}835\,151\,531\,174 + 71\overset{r}{.}018\,041\,257\,371\,T - \overset{r}{.}000\,180\,205\,245\,T^2 - \overset{r}{.}218\,166 \times 10^{-6}\,T^3$$

$$\Omega' = 4\overset{r}{.}523\,601\,514\,852 - 33\overset{r}{.}757\,146\,246\,552\,T + \overset{r}{.}000\,036\,264\,063\,T^2 + \overset{r}{.}038\,785 \times 10^{-6}\,T^3$$

$$g' = L' - \Gamma' = 5\overset{r}{.}168\,000\,345\,745 + 8\,328\overset{r}{.}691\,103\,668\,024\,T + \overset{r}{.}000\,160\,424\,847\,T^2 + \overset{r}{.}251\,133 \times 10^{-6}\,T^3$$

$$\omega' = \Gamma' - \Omega' = 1\overset{r}{.}311\,550\,016\,322 + 104\overset{r}{.}775\,187\,503\,924\,T - \overset{r}{.}000\,216\,469\,309\,T^2 - \overset{r}{.}256\,951 \times 10^{-6}\,T^3$$

$$\omega' + \pi^{r} = 4\overset{r}{.}453\,142\,669\,912 + 104\overset{r}{.}775\,187\,503\,924\,T - \overset{r}{.}000\,216\,469\,309\,T^2 - \overset{r}{.}256\,951 \times 10^{-6}\,T^3$$

$$F' = L' - \Omega' = \overset{r}{.}196\,365\,054\,887 + 8\,433\overset{r}{.}466\,291\,171\,947\,T - \overset{r}{.}000\,056\,044\,462\,T^2 - \overset{r}{.}005\,818 \times 10^{-6}\,T^3$$

$$\theta' = L' - \Omega' + \pi^{r} = 3\overset{r}{.}337\,957\,708\,477 + 8\,433\overset{r}{.}466\,291\,171\,947\,T - \overset{r}{.}000\,056\,044\,462\,T^2 - \overset{r}{.}005\,818 \times 10^{-6}\,T^3$$

$$D' = L' - L'_o = 6\overset{r}{.}121\,523\,942\,807 + 7\,771\overset{r}{.}377\,193\,934\,485\,T - \overset{r}{.}000\,025\,064\,867\,T^2 + \overset{r}{.}032\,967 \times 10^{-6}\,T^3$$

$$i' = \overset{r}{.}089\,804\,114$$

$$I' = \overset{r}{.}026\,790\,804$$

$$e' = .054\,900\,489$$

$$a' = 60.2665 \text{ Earth radii}$$

$$= 384{,}399.3537 \text{ km. (using 1 Earth radius} = 6378.3255 \text{ km.)}$$

$$\sin i' = +.089\,683\,453$$
$$\cos i' = +.995\,970\,322$$

$$\sin I' = +.026\,787\,60$$
$$\cos I' = +.999\,641\,15$$

TABLE 2e

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{r} = radians
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L' = 4^r.719\ 966\ 569\ 739 + {}^r.229\ 971\ 502\ 941\ 147\ d - {}^r.014\ 827\ 044 \times 10^{-12}\ d^2 + {}^r.676\ 571 \times 10^{-21}\ d^3$$

$$\Gamma' = 5^r.835\ 151\ 531\ 174 + {}^r.001\ 944\ 368\ 001\ 571\ d - {}^r.135\ 078\ 733 \times 10^{-12}\ d^2 - 4^r.477\ 305 \times 10^{-21}\ d^3$$

$$\Omega' = 4^r.523\ 601\ 514\ 852 - {}^r.000\ 924\ 220\ 294\ 225\ d + {}^r.027\ 182\ 914 \times 10^{-12}\ d^2 + {}^r.795\ 965 \times 10^{-21}\ d^3$$

$$g' = L' - \Gamma' = 5^r.168\ 000\ 345\ 745 + {}^r.228\ 027\ 134\ 939\ 576\ d + {}^r.120\ 251\ 689 \times 10^{-12}\ d^2 + 5^r.153\ 876 \times 10^{-21}\ d^3$$

$$\omega' = \Gamma' - \Omega' = 1^r.311\ 550\ 016\ 322 + {}^r.002\ 868\ 588\ 295\ 795\ d - {}^r.162\ 261\ 647 \times 10^{-12}\ d^2 - 5^r.273\ 271 \times 10^{-21}\ d^3$$

$$\omega' + \pi^r = 4^r.453\ 142\ 669\ 912 + {}^r.002\ 868\ 588\ 295\ 795\ d - {}^r.162\ 261\ 647 \times 10^{-12}\ d^2 - 5^r.273\ 271 \times 10^{-21}\ d^3$$

$$F' = L' - \Omega' = {}^r.196\ 365\ 054\ 887 + {}^r.230\ 895\ 723\ 235\ 372\ d - {}^r.042\ 009\ 958 \times 10^{-12}\ d^2 - {}^r.119\ 395 \times 10^{-21}\ d^3$$

$$\theta' = L' - \Omega' + \pi^r = 3^r.337\ 957\ 708\ 477 + {}^r.230\ 895\ 723\ 235\ 372\ d - {}^r.042\ 009\ 958 \times 10^{-12}\ d^2 - {}^r.119\ 395 \times 10^{-21}\ d^3$$

$$D' = L' - L'_o = 6^r.121\ 523\ 942\ 807 + {}^r.212\ 768\ 711\ 675\ 140\ d - {}^r.018\ 788\ 191 \times 10^{-12}\ d^2 + {}^r.676\ 571 \times 10^{-21}\ d^3$$

$$i' = {}^r.089\ 804\ 114$$

$$I' = {}^r.026\ 790\ 804$$

$$e' = .054\ 900\ 489$$

a' = 60.2665 Earth radii
= 384,399.3537 km. (using 1 Earth radius = 6378.3255 km.)

sin i' = +.089 683 453
cos i' +.995 970 322

sin I' = +.026 787 60
cos I' = +.999 641 15

TABLE 2f

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{R} = revolutions, $^\circ$ = degrees, ' = minutes, " = seconds
TIME	:	T = number of Julian centuries elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L' = {}^R.751\ 206\ 010\ 802 + 1\ 336^R.855\ 230\ 949\ 074\ T - 3^R.148\ 148 \times 10^{-6}\ T^2 + 5^R.247 \times 10^{-9}\ T^3$$

$$\Gamma' = {}^R.928\ 693\ 209\ 877 + 11^R.302\ 872\ 314\ 815\ T - 28^R.680\ 556 \times 10^{-6}\ T^2 - 34^R.722 \times 10^{-9}\ T^3$$

$$\Omega' = {}^R.719\ 953\ 541\ 667 - 5^R.372\ 616\ 689\ 815\ T + 5^R.771\ 605 \times 10^{-6}\ T^2 + 6^R.173 \times 10^{-9}\ T^3$$

$$g' = L' - \Gamma' = {}^R.822\ 512\ 800\ 926 + 1\ 325^R.552\ 358\ 634\ 259\ T + 25^R.532\ 407 \times 10^{-6}\ T^2 + 39^R.969 \times 10^{-9}\ T^3$$

$$\omega' = \Gamma' - \Omega' = {}^R.208\ 739\ 668\ 210 + 16^R.675\ 489\ 004\ 630\ T - 34^R.452\ 161 \times 10^{-6}\ T^2 - 40^R.895 \times 10^{-9}\ T^3$$

$$\omega' + {}^R.5 = {}^R.708\ 739\ 668\ 210 + 16^R.675\ 489\ 004\ 630\ T - 34^R.452\ 161 \times 10^{-6}\ T^2 - 40^R.895 \times 10^{-9}\ T^3$$

$$F' = L' - \Omega' = {}^R.031\ 252\ 469\ 136 + 1\ 342^R.227\ 847\ 638\ 889\ T - 8^R.919\ 753 \times 10^{-6}\ T^2 - {}^R.926 \times 10^{-9}\ T^3$$

$$\theta' = L' - \Omega' + {}^R.5 = {}^R.531\ 252\ 469\ 136 + 1\ 342^R.227\ 847\ 638\ 889\ T - 8^R.919\ 753 \times 10^{-6}\ T^2 - {}^R.926 \times 10^{-9}\ T^3$$

$$D' = L' - L_o = {}^R.974\ 270\ 794\ 753 + 1\ 236^R.853\ 095\ 046\ 296\ T - 3^R.989\ 198 \times 10^{-6}\ T^2 + 5^R.247 \times 10^{-9}\ T^3$$

$$i' = {}^R.014\ 292\ 769$$

$$I' = {}^R.004\ 263\ 889$$

$$e' = .054\ 900\ 489$$

$$a' = 60.2665 \text{ Earth radii}$$

$$= 384{,}399.3537 \text{ km. (using 1 Earth radius} = 6378.3255 \text{ km.)}$$

$$\sin i' = +.089\ 683\ 453$$

$$\cos i' = +.995\ 970\ 322$$

$$\sin I' = +.026\ 787\ 60$$

$$\cos I' = +.999\ 641\ 15$$

TABLE 2g

MEAN ORBITAL ELEMENTS OF THE MOON

Coordinate System

CENTER	:	Geocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{R} = revolutions
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$L' = {}^R.751\ 206\ 010\ 802 + {}^R.036\ 601\ 101\ 463\ 356\ d - 2^R.359\ 797 \times 10^{-15}\ d^2 + {}^R.107\ 680 \times 10^{-21}\ d^3$$

$$\Gamma' = {}^R.928\ 693\ 209\ 877 + {}^R.000\ 309\ 455\ 778\ 640\ d - 21^R.498\ 448 \times 10^{-15}\ d^2 - {}^R.712\ 585 \times 10^{-21}\ d^3$$

$$\Omega' = {}^R.719\ 953\ 541\ 667 - {}^R.000\ 147\ 094\ 228\ 332\ d + 4^R.326\ 295 \times 10^{-15}\ d^2 + {}^R.126\ 682 \times 10^{-21}\ d^3$$

$$g' = L' - \Gamma' = {}^R.822\ 512\ 800\ 926 + {}^R.036\ 291\ 645\ 684\ 716\ d + 19^R.138\ 651 \times 10^{-15}\ d^2 + {}^R.820\ 265 \times 10^{-21}\ d^3$$

$$\omega' = \Gamma' - \Omega' = {}^R.208\ 739\ 668\ 210 + {}^R.000\ 456\ 550\ 006\ 971\ d - 25^R.824\ 743 \times 10^{-15}\ d^2 - {}^R.839\ 267 \times 10^{-21}\ d^3$$

$$\omega' + {}^R.5 = {}^R.708\ 739\ 668\ 210 + {}^R.000\ 456\ 550\ 006\ 971\ d - 25^R.824\ 743 \times 10^{-15}\ d^2 - {}^R.839\ 267 \times 10^{-21}\ d^3$$

$$F' = L' - \Omega' = {}^R.031\ 252\ 469\ 136 + {}^R.036\ 748\ 195\ 691\ 688\ d - 6^R.686\ 093 \times 10^{-15}\ d^2 - {}^R.019\ 002 \times 10^{-21}\ d^3$$

$$\theta' = L' - \Omega' + {}^R.5 = {}^R.531\ 252\ 469\ 136 + {}^R.036\ 748\ 195\ 691\ 688\ d - 6^R.686\ 093 \times 10^{-15}\ d^2 - {}^R.019\ 002 \times 10^{-21}\ d^3$$

$$D' = L' - L'_o = {}^R.974\ 270\ 794\ 753 + {}^R.033\ 863\ 192\ 198\ 393\ d - 2^R.990\ 233 \times 10^{-15}\ d^2 + {}^R.107\ 680 \times 10^{-21}\ d^3$$

$$i' = {}^R.014\ 292\ 769$$

$$I' = {}^R.004\ 263\ 889$$

$$e' = .054\ 900\ 489$$

$$a' = 60.2665 \text{ Earth radii}$$

$$= 384{,}399.3537 \text{ km. (using 1 Earth radius} = 6378.3255 \text{ km.)}$$

$$\sin i' = +.089\ 683\ 453$$

$$\cos i' = +.995\ 970\ 322$$

$$\sin I' = +.026\ 787\ 60$$

$$\cos I' = +.999\ 641\ 15$$

TABLE 3

MEAN ORBITAL ELEMENTS OF THE EARTH*

Coordinate System

CENTER	:	Selenocentric
X-Y PLANE	:	Ecliptic of date
X AXIS	:	Mean equinox of date

Units

ANGULAR	:	^{r} = radians
TIME	:	d = number of days elapsed since 12 hours ephemeris time, January 0, 1900 (Julian Date = 2415020.0). One Julian century = 36525 days exactly.

Numerical Values

$$\Omega''_3 = \Omega' = 4^{r}.523\ 601\ 514\ 852 - {}^{r}.000\ 924\ 220\ 294\ 225\ d + {}^{r}.027\ 182\ 914 \times 10^{-12}\ d^2 + {}^{r}.795\ 965 \times 10^{-21}\ d^3$$

$$g''_3 = g' = 5^{r}.168\ 000\ 345\ 745 + {}^{r}.228\ 027\ 134\ 939\ 576\ d + {}^{r}.120\ 251\ 689 \times 10^{-12}\ d^2 + 5^{r}.153\ 876 \times 10^{-21}\ d^3$$

$$\omega''_3 = \omega' + \pi^{r} = 4^{r}.453\ 142\ 669\ 912 + {}^{r}.002\ 868\ 588\ 295\ 795\ d - {}^{r}.162\ 261\ 647 \times 10^{-12}\ d^2 - 5^{r}.273\ 271 \times 10^{-21}\ d^3$$

$$F''_3 = \omega''_3 + g''_3 = 3^{r}.337\ 957\ 708\ 477 + {}^{r}.230\ 895\ 723\ 235\ 372\ d - {}^{r}.042\ 009\ 958 \times 10^{-12}\ d^2 - {}^{r}.119\ 395 \times 10^{-21}\ d^3$$

$$i''_3 = i' = {}^{r}.089\ 804\ 114$$

$$e''_3 = e' = .054\ 900\ 489$$

$$a''_3 = a' = 60.2665 \text{ Earth radii}$$

$$= 384{,}399.3537 \text{ km. (using 1 Earth radius} = 6378.3255)$$

$$\sin i''_3 = \sin i' = +.089\ 683\ 453$$

$$\cos i''_3 = \cos i' = +.995\ 970\ 322$$

$$r''_3 = a''_3 \Big[1.001\ 507\ 032 - .054\ 838\ 449 \cos g''_3 - .001\ 504\ 006 \cos 2g''_3 - .000\ 061\ 878 \cos 3g''_3 - .000\ 003\ 017 \cos 4g''_3 - .000\ 000\ 161 \cos 5g''_3 - .000\ 000\ 009 \cos 6g''_3 - .000\ 000\ 001 \cos 7g''_3\Big]$$

$$C''_3 = +{}^{r}.109\ 759\ 636 \sin g''_3 + {}^{r}.003\ 763\ 418 \sin 2g''_3 + {}^{r}.000\ 178\ 928 \sin 3g''_3 + {}^{r}.000\ 009\ 721 \sin 4g''_3 + {}^{r}.000\ 000\ 568 \sin 5g''_3 + {}^{r}.000\ 000\ 035 \sin 6g''_3 + {}^{r}.000\ 000\ 002 \sin 7g''_3$$

$$f''_3 = g''_3 + C''_3$$

$$u''_3 = F''_3 + C''_3 - g''_3 + \omega''_3 + C''_3$$

*This table derived from Table 2e.

TABLE 4

EXPANSIONS IN ELLIPTIC MOTION OF THE EQUATION OF THE CENTER AND THE MAGNITUDE OF THE RADIUS VECTOR*

$$
\begin{aligned}
C = &+\left[2e - \frac{1}{4}e^3 + \frac{5}{96}e^5 + \frac{107}{4608}e^7\right]\sin g \\
&+\left[\frac{5}{4}e^2 - \frac{11}{24}e^4 + \frac{17}{192}e^6\right]\sin 2g \\
&+\left[\frac{13}{12}e^3 - \frac{43}{64}e^5 + \frac{95}{512}e^7\right]\sin 3g \\
&+\left[\frac{103}{96}e^4 - \frac{451}{480}e^6\right]\sin 4g \\
&+\left[\frac{1097}{960}e^5 - \frac{5957}{4608}e^7\right]\sin 5g \\
&+\left[\frac{1223}{960}e^6\right]\sin 6g \\
&+\left[\frac{47273}{32256}e^7\right]\sin 7g
\end{aligned}
$$

$$
\begin{aligned}
r = a\Bigg\{1 + \frac{1}{2}e^2 &+\left[-e + \frac{3}{8}e^3 - \frac{5}{192}e^5 + \frac{7}{9216}e^7\right]\cos g \\
&+\left[-\frac{1}{2}e^2 + \frac{1}{3}e^4 - \frac{1}{16}e^6\right]\cos 2g \\
&+\left[-\frac{3}{8}e^3 + \frac{45}{128}e^5 - \frac{567}{5120}e^7\right]\cos 3g \\
&+\left[-\frac{1}{3}e^4 + \frac{2}{5}e^6\right]\cos 4g \\
&+\left[-\frac{125}{384}e^5 + \frac{4375}{9216}e^7\right]\cos 5g \\
&+\left[-\frac{27}{80}e^6\right]\cos 6g \\
&+\left[-\frac{16807}{46080}e^7\right]\cos 7g\Bigg\}
\end{aligned}
$$

*For example, refer to "Methods of Celestial Mechanics," page 76-77, by Brouwer and Clemence.

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The tables of the mean orbital elements of the Sun, Moon and Earth constitute the major effort of this publication. In order to ensure the full double precision accuracy of the coefficients and reduce the possibility of introducing accidental errors in the conversion of units, the conversion was performed independently on a desk calculator and by means of a Fortran IV computer program. These results were compared and the final tabulated values of the two methods agree within one unit of the last recorded digit. The author gratefully acknowledges the long and tedious hours devoted by Mrs. Harriet Shub formerly of Litton Systems and now with Bell Telephone Laboratories and Mrs. Adrienne Sussman formerly of Litton Systems and now with Bendix Field Engineering Corp. in the process of programming, desk calculation, and proofreading in order to provide reliable tables.

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