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Optimization of System Operating Parameters for Heat Sterilizable Liquid Propulsion Systems

Howard D. Curtis

Allen D. Harper

Duane F. Dipprey, Manager
Liquid Propulsion Section

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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ABSTRACT

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As a means of attaining a sterile spacecraft, design information is required for liquid propulsion systems which can be heat sterilized in the loaded condition without venting. An analysis was performed to determine the values of the system operating parameters which minimize the system mass. Results are presented for both internally and externally pressurized tankage systems. The tankage mass for sterilizable systems is approximately twice that for nonsterilizable systems. This will mean an increase of 5 to 10% in propulsion system mass for typical applications. The tankage mass is minimized when the tank is 60 to 70% filled with propellants (prior to heating) in the externally pressurized case and approximately 40% filled in the internally pressurized case.

I. INTRODUCTION

A prime objective of future unmanned missions is the search for life on other planets. In order to conduct such experiments meaningfully, any spacecraft landing on the planet or entering its atmosphere must be void of Earth organisms. If not, the possibility will exist that "hitchhiking" organisms might find a climate favorable for growth, perhaps even growth on a large scale; therefore, it has become national policy that objects which enter the atmospheres of other planets must be sterilized. Currently it appears that the requisite degree of decontamination may be achieved only by extended exposure to high temperatures (dry heat sterilization). A typical requirement would be 24 hr at 293°F (145°C).

At present, there are two applications for sterilizable liquid propellant supply systems under consideration. One is the touchdown rocket for a soft lander. This mission would probably not be attempted until several successful "hard" landings had been made. Since throttling is a required characteristic of the touchdown system, only liquid propulsion systems are being considered. The other possible use is for the propellant supply system of a turbine-driven turbo-alternator power supply system. Ref. 1 has shown that this type of power supply may have desirable characteristics for a limited-lifetime lander.

While the sterilization requirement has large implications on many spacecraft subsystems, we are concerned herein with the effects on propulsion systems, and on liquid propulsion systems in particular. A cursory study of typical pressure-fed monopropellant and bipropellant systems indicated that certain components could probably withstand the high temperature environment without modification provided that:

1. Materials were used which were not incompatible with the propellants at the higher temperature.

2. The system would not be required to function while exposed to the elevated temperature.

Thrust chambers and pumping system components (valves, regulators, etc.) are examples of nonaffected systems, the former because they must withstand high temperatures during operation, and the latter as a consequence of Provision 2 above.

The remaining major system elements are the pressurization sources and the propellant tankage and supply system.

If consideration is limited to cold-gas pressurized systems, these may be eliminated for most cases of interest by the following considerations. Normally, personnel safety considerations require that pressure vessels for such systems be designed with a yield stress safety factor of 2.2. If it can be assumed that the sterilization may be carried out remotely, the required safety factor would be lowered to 1.1. Since the ratio of the safety factors, 2.2/1.1 = 2, is greater than the ratio of the tank pressure at sterilization to that at ambient conditions, the tanks would not have to be strengthened. [The tank pressure rise is directly proportional to the increase in absolute temperature. Therefore, this ratio would be (293° + 460°)/(70° + 460°) = 1.4. The difference in these ratios (2 versus 1.4) is large enough to handle easily such secondary effects as lowered allowable stress at the higher temperature.]

Elimination of the pressurization system leaves the propellant tankage and supply subsystem as the one most affected by the sterilization requirement. This occurs because the vapor pressure of many propellants is a very strong function of the temperature. While the pressurization gas tank pressure would only increase by 40% as the temperature is raised from 70 to 293°F, the propellant vapor pressure for some propellants increases by an order of magnitude. Therefore, an analysis was undertaken to determine ways to minimize the penalties which arise from this increased pressure.

II. EXTERNALLY PRESSURIZED TANKAGE SYSTEMS

A. Statement of Problem

Find the mass and wall thickness of a spherical propellant tank parametrically as a function of the thermodynamic and spatial variables of the propellant-tank system. The following are given:

1. Operating pressure.
2. Tank material.
3. Weld factor and safety factors.
4. Propellant mass and properties.
5. The ullage space above the propellant is prepressurized to some fraction of the nominal operating pressure with an inert gas in equilibrium with the propellant vapor.
6. The system is heat sterilized by raising the temperature from T_1 to T_2 without venting.

B. Analysis

1. Sterilizable Systems

For an initial tank of volume V_t containing a propellant of volume V_p, the ullage fraction π_u is defined by

$$\pi_u = 1 - \frac{V_{p_1}}{V_{t_{1u}}}. \tag{1}$$

The tank, filled to a certain π_u, is pressurized to a fraction, π_p, of the operating pressure P_{op}. The radius of the tank will enlarge due to the pressurization and attain the value

$$R_{op} = R_{1u}\left[1 + \frac{\sigma_2(1-\nu)}{E}\right], \tag{2}$$

where

$$R_{1u} = \left[\frac{3}{4\pi}V_{t_{1u}}\right]^{1/3} = \left[\frac{3}{4\pi}\frac{m_p}{\rho_{p1}(1-\pi_u)}\right]^{1/3}. \tag{3}$$

Therefore, the prepressurized tank volume is

$$V_{t_{1p}} = \frac{4}{3}\pi R_{1p}^3 = \frac{m_p}{\rho_{p1}(1 - \pi_u)}\left[1 + \sigma_1 \frac{(1-\nu)}{E}\right]^3 .$$

Since the term

$$\sigma_1 \frac{(1-\nu)}{E}$$

is very much smaller than unity (on the order of 10^{-3}), we linearize to obtain

$$V_{t_{1p}} = \frac{m_p}{\rho_{p1}(1 - \pi_u)}\left[1 + 3\sigma_1 \frac{(1-\nu)}{E}\right]. \tag{4}$$

If the temperature is raised from T_1 to T_2 during a sterilization cycle, a volumetric expansion of the tank occurs due to thermal effects and to the concomitant increased pressure. The propellant expands due to increased temperature, but its volume change due to increased pressure of the order to be considered can easily be shown to be negligible. The change in radius due to thermal expansion is given by

$$(R_{2t} - R_{1p}) = R_{1p}\,\alpha\,(T_2 - T_1). \tag{5}$$

The increased pressure inside the tank gives rise to a new stress in the tank, σ_2. Had the tank expanded from a condition of zero stress to σ_2, the change in radius due to pressure effects would be

$$(R_{2p} - R_{1u}) = R_{1u}\,\sigma_2 \frac{(1-\nu)}{E}. \tag{6}$$

However, the tank actually expands from the prepressurized condition at T_1, where the wall stress is σ_1. Hence, the change in radius due to the pressure rise is

$$(R_{2p} - R_{1p}) = (R_{2p} - R_{1u}) - (R_{1p} - R_{1u}).$$

Substituting Eqs. (2) and (6)

$$(R_{2p} - R_{1p}) = R_{1u}\frac{(1-\nu)}{E}(\sigma_2 - \sigma_1). \tag{7}$$

The first-order total change in radius due to both the thermal and pressure effects is

$$(R_2 - R_{1p}) = (R_{2p} - R_{1p}) + (R_{2t} - R_{1p}),$$

or, from Eqs. (2), (5), and (7)

$$R_2 = R_{1u}\left[1 + \sigma_2 \frac{(1-\nu)}{E} + \alpha(T_2 - T_1)\right], \tag{8}$$

where the higher-order terms have been neglected. Now we have the new tank volume, namely

$$V_{t_2} = \frac{4}{3}\pi R_2^3 . \tag{9}$$

The increased pressure at T_2 is due to increased vapor pressure of the propellant and to the heating of the prepressurization gas. In order to calculate the partial pressure, p_2, of the pressurizing gas at T_2, we use the relation

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}. \tag{10}$$

The difference between the tank volume and propellant volume at either temperature is the volume occupied by the gas. We write Eq. (10) as

$$p_2 = p_1 \frac{T_2}{T_1}\frac{[V_{t_{1p}} - V_{p1}]}{[V_{t_2} - V_{p2}]},$$

or

$$p_2 = (\pi_p P_{op} - p_{v1})\frac{T_2}{T_1}\frac{[V_{t_{1p}} - V_{p1}]}{[V_{t_2} - V_{p2}]}. \tag{11}$$

Since the propellant volume is known at each temperature from volume = mass/density, substitution of Eqs. (4) and (9) into (11) yields

$$p_2 = (\pi_p P_{op} - p_{v1})(T_2/T_1) \times \frac{\left[3\sigma_1 \frac{(1-\nu)}{E} + \pi_u\right]}{\left[1 + \frac{3\sigma_2}{E}(1-\nu) + 3\alpha(T_2 - T_1) - \frac{\rho_{p1}}{\rho_{p2}}(1 - \pi_u)\right]}. \tag{12}$$

To arrive at Eq. (12), linearizations of the following kind were made:

$$(1 + \epsilon + \beta)^3 = 1 + 3\epsilon + 3\beta,$$

where $\epsilon, \beta \ll 1$.

The total pressure within the tank at T_2 is

$$P_2 = p_2 + p_{v2}.$$

Note, however, that the pressure which stresses the tank walls is the gage pressure, since the system under consideration will not be sterilized in a vacuum.

The mass of the tank is found from

$$M_t = 4\pi \rho_2 R_2^2\, t_2 W,$$

where

$$t_2 = \frac{P_2' R_2}{2\sigma_2}. \tag{13}$$

and

$$P_2' = P_2 - P_{atm} . \tag{14}$$

Thus, we have

$$M_t = \frac{4\pi\rho_2 R_2^3 P_2' W}{2\sigma_2} . \tag{15}$$

In order to obtain the desired expression for the tank mass, we require an expression relating the density of the tank material at the sterilization condition to the density at T_1, before prepressurization. Consider a volume element of the tank wall with edges consisting of two orthogonal meridians and the tank radius. Let the lengths along the two meridional directions be x and y, and the thickness be t. The volume of the element is

$$V = txy . \tag{16}$$

Due to the internal pressure, there is a stress, σ, in the tank which gives rise to the following strains in the meridional directions:

$$\frac{\Delta x}{x} = \frac{\Delta y}{y} = \sigma \frac{(1 - \nu)}{E} . \tag{17}$$

The strain in the radial direction is

$$\frac{\Delta t}{t} = - \frac{2\sigma\nu}{E} . \tag{18}$$

Taking the differential of Eq. (16), the change in volume of the element due to the strain may be written

$$\frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta t}{t} . \tag{19}$$

Thus, from Eqs. (17) and (18), we have the change in volume due to the internal pressure:

$$\left(\frac{\Delta V}{V}\right)_{press} = \frac{\sigma}{E} [2 - 4\nu] . \tag{20}$$

Since

$$V = \frac{M}{\rho} ,$$

it follows that

$$\frac{\Delta V}{V} = - \frac{\Delta \rho}{\rho} . \tag{21}$$

The thermal expansion due to the heating of the tank gives rise to the following strains:

$$\frac{\Delta x}{x} = \frac{\Delta y}{y} = \frac{\Delta t}{t} = \alpha \Delta T .$$

The change in volume due to the heating is, therefore

$$\left(\frac{\Delta V}{V}\right)_{temp} = 3\alpha \Delta T . \tag{22}$$

From Eqs. (20), (21), and (22), the density change, due to the combined effects of the internal pressure and thermal expansion, is

$$-\frac{\Delta \rho}{\rho} = \frac{\rho_1 - \rho_2}{\rho_1} = \frac{\sigma}{E} [2 - 4\nu] + 3\alpha \Delta T ,$$

or

$$\frac{\rho_2}{\rho_1} = 1 - \frac{\sigma}{E} [2 - 4\nu] - 3\alpha \Delta T . \tag{23}$$

Substituting Eqs. (23), (14), (12), (8), and (3) into (15), the tank mass can be expressed nondimensionally:

$$\frac{M_t}{M_p} = \frac{3}{2} \frac{W}{\sigma_2} \frac{\rho_1}{\rho_{p1}} \frac{C}{(1 - \pi_u)}$$

$$\times \left\{ \frac{(\pi_p P_{op} - p_{v1}) \left(\frac{T_2}{T_1}\right) \left[\frac{3\sigma_1 (1 - \nu)}{E} + \pi_u \right]}{\left[A - \frac{\rho_{p1}}{\rho_{p2}} (1 - \pi_u) \right]} + p_{v2} - P_{atm} \right\} , \tag{24}$$

where

$$A = 1 + 3\sigma_2 \frac{(1 - \nu)}{E} + 3\alpha (T_2 - T_1)$$

and

$$C = 1 + \frac{\sigma_2}{E} (1 + \nu) .$$

Products of terms small compared to unity are neglected.

In order to obtain an explicit expression for the tank wall thickness, note that

$$t_2 = t \left[1 + \alpha (T_2 - T_1) - \frac{2\sigma_2 \nu}{E} \right] . \tag{25}$$

Substituting Eqs. (8), (12), (14), and (25) into (13), we get

$$\frac{t}{R_{1u}} = \frac{C}{2\sigma_2}$$

$$\times \left\{ (\pi_p P_{op} - p_{v1}) \left(\frac{T_2}{T_1}\right) \frac{\left[3\sigma_1 \frac{(1 - \nu)}{E} + \pi_u \right]}{\left[A - \frac{\rho_{p1}}{\rho_{p2}} (1 - \pi_u) \right]} + p_{v2} - P_{atm} \right\} . \tag{26}$$

This expression is of interest because t is the thickness to which the tank would be built.

To obtain a functional relationship between σ_2 and σ_1, we again use Eq. (13), applying it at both T_1 (pressurized) and T_2, obtaining

$$\frac{\sigma_1}{\sigma_2} = \frac{P'_1}{P'_2}\,\frac{t_2}{t_{1p}}\,\frac{R_{1p}}{R_2} = \frac{(\pi_p P_{op} - P_{atm})}{P'_2}$$

$$\times \frac{\left[1 + \alpha (T_2 - T_1) - \frac{2\sigma_2 \nu}{E}\right]\left[1 + \sigma_1 \frac{(1 - \nu)}{E}\right]}{\left[1 - \frac{2\sigma_1 \nu}{E}\right]\left[1 + \frac{\sigma_2}{E}(1 - \nu) + \alpha (T_2 - T_1)\right]},$$

$$\frac{\sigma_1}{\sigma_2} = \frac{(\pi_p P_{op} - P_{atm})}{P'_2}$$

$$\times \frac{\left[1 + \alpha (T_2 - T_1) - \frac{2\sigma_2 \nu}{E} + \sigma_1 \frac{(1 - \nu)}{E}\right]}{\left[1 + \frac{\sigma_2}{E}(1 - \nu) + \alpha (T_2 - T_1) - \frac{2\sigma_1 \nu}{E}\right]},$$

or

$$\frac{\sigma_1}{\sigma_2} = \frac{(\pi_p P_{op} - P_{atm})}{P'_2}\left[1 - \frac{(\sigma_2 - \sigma_1)(1 + \nu)}{E}\right], \qquad (27)$$

where only first-order terms are retained.

Several practical considerations enter into the evaluation of the foregoing equations. These affect both the stress term and the pressure term in equations like (15) or (24). From the form of these equations, it may be observed that the tank must be designed for the highest ratio of p/σ which must be accommodated. For the current problem, three conditions must be considered: (1) sterilization, (2) ground handling, and (3) flight. A discussion of the design requirements for these conditions follows.

The allowable design stress at *sterilization* is

$$\sigma'_2 = f_{s2} f_t \sigma . \qquad (28)$$

σ is the nominal or "handbook" room temperature yield stress of the tank material. The factor f_t accounts for the higher-than-room temperature at sterilization. ($f_t = 0.8$ has been used herein.) f_{s2} is the required safety factor at sterilization, which is set at 1/1.1 by the assumption that sterilization may be accomplished remotely. The pressure at T_2 is given by Eqs. (12) and (14).

For *ground handling*, the allowable stress is

$$\sigma'_1 = f_{s1} \sigma , \qquad (29)$$

where $f_{s1} = 1/2.2$. The higher safety factor is required for personnel safety when the tank is prepressurized. The pressure in this condition is

$$P'_1 = \pi_p P_{op} - P_{atm} . \qquad (30)$$

In *flight*, the factor of safety may be reduced to 1.1 since the vehicle is (hopefully) a long way from people. Thus

$$\sigma'_f = f_f \sigma ,$$

where $f_f = f_{s2} = 1/1.1$. The pressure that must be withstood is the full P_{op}.

The in-flight and ground handling conditions may be compared by examining the p/σ ratios for the two conditions. For the in-flight condition

$$\frac{p}{\sigma} = \frac{P_{op}}{\sigma/1.1} , \qquad (31)$$

while for ground handling

$$\frac{p}{\sigma} = \frac{\pi_p P_{op} - P_{atm}}{\sigma/2.2} . \qquad (32)$$

Equating (31) and (32) and substituting $P_{op} = 220$ psia and $P_{atm} = 14.7$ psia, one obtains

$$\pi_p = 0.567 .$$

Thus, for $\pi_p > 0.567$, the ground handling condition will be more severe than flight.

Now if the tank is designed such that $\sigma_2 = \sigma'_2$, then σ_1 will have a value which can be calculated by means of Eqs. (27) and (12). This value of σ_1 must be less than σ'_1 or σ'_f. If such is not the case, then the tank must be designed to satisfy the more restrictive condition, namely $\sigma_1 = \sigma'_1$ or σ'_f. The value of σ_2 may then be calculated via the iterative procedure and must be less than σ'_2.

Six propellants were analyzed with the tanks designed to the sterilization criterion. For most cases, the constraint

$$\sigma_1 < \sigma'_1 \qquad (33)$$

remained satisfied. The exceptions were the propellants with very low vapor pressures at T_2, namely H_2O_2 and

RP-1. For these propellants, it would be necessary to design the tanks on the ground handling criterion for certain values of π_p, as shown below, or to the flight criterion if $\pi_p < 0.567$.

Although it appears that in most cases propellant tanks will be designed to satisfy the high temperature criterion, an analysis was made to arrive at a relationship which would enable one, given a particular propellant and prepressurization level, to establish the range of π_p for which the high-temperature condition is not restrictive. This analysis and the results appear in Appendix A.

2. Nonsterilizable Systems

For comparison purposes, computations were desired for nonsterilizable systems. The analysis for this case, while it is a rather trivial version of the foregoing, is given below to obviate ambiguity. We may rewrite Eq. (15) in this case as

$$M_t = \frac{4\pi \rho_1 R_{1p} W P_1'}{2\sigma_1} . \tag{34}$$

Substituting Eqs. (3) and (28) into (34), we obtain after linearization

$$\frac{M_t}{M_p} = \frac{3}{2} \frac{W}{\sigma_1} \frac{\rho_1}{\rho_p} \left[1 + \frac{3\sigma_1 (1-\nu)}{E} \right] \frac{(\pi_p P_{op} - P_{atm})}{(1-\pi_p)} . \tag{35}$$

As in the case of the sterilizable tank, there are two conditions which the design must satisfy: operations on the ground ($f_s = 2.2$) and in flight ($f_s = 1.1$). As shown previously, the ground handling constraint is applicable for $\pi_p > 0.567$. For $\pi_p \leq 0.567$, the flight condition is applicable, and only one curve is obtained since all tanks are stressed to σ_1' at a tank pressure of 220 psia. Therefore, this curve is a lower bound on the tank mass.

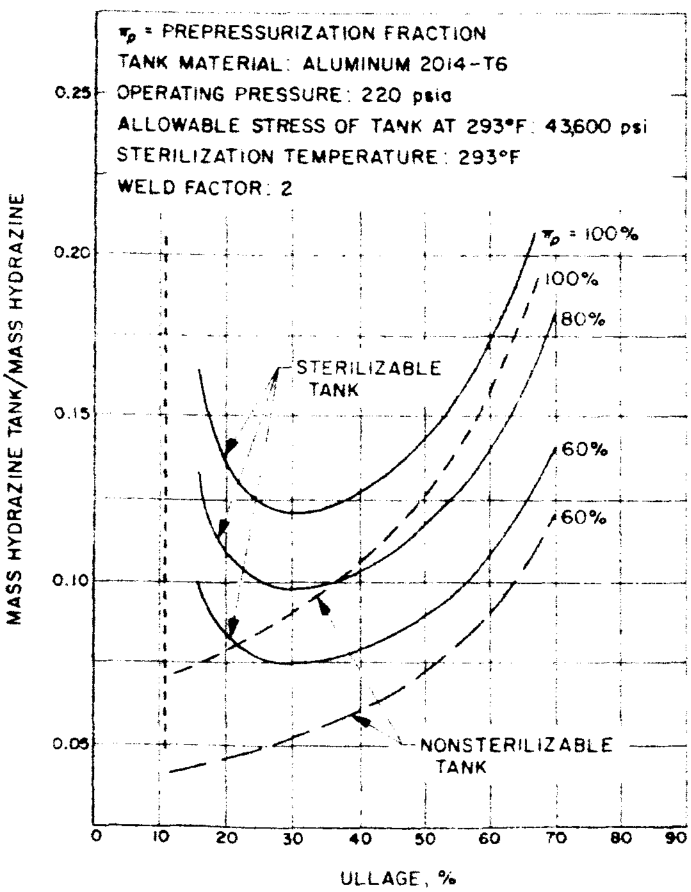

Fig. 1. Ratio of tank mass to propellant mass for different prepressurization levels in a hydrazine system

C. *Results*

The relationships presented previously have been evaluated to obtain tankage mass and thickness for several

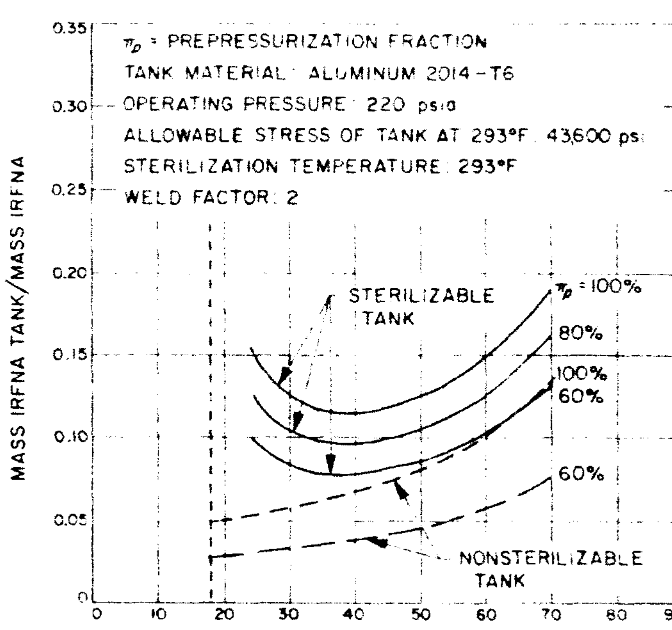

Fig. 2. Ratio of tank mass to propellant mass for different prepressurization levels in a IRFNA system

propellants of interest. The tanks were assumed to be made of aluminum, 2014-T6 with a weld factor W of 2.0. Calculations have been made for N_2H_4, IRFNA, N_2O_4, UDMH, RP-1 and H_2O_2; results are presented in Figs. 1 through 20.

Plots of Eq. (24) for the sterilizable system and Eq. (34) for the nonsterilizable system are given in Figs. 1 through 6. The sterilizable system curves were computed on the basis that $\sigma_2 = \sigma_2'$. An indication is given of the limiting value of π_p for which this is a valid procedure. (See Appendix A for method of calculation.) Table 1 contains values of $\pi_{u,opt}$ determined from Figs. 1 through 6. $\pi_{u,opt}$ is the value of π_u which yields the minimum tank mass. For those cases in which a range of $\pi_{u,opt}$ is given, the higher values of $\pi_{u,opt}$ correspond to higher π_p.

Table 1. Optimum ullage fractions

Propellant	$\pi_{u,opt}$, %
Hydrazine	30 to 31
IRFNA	35.5 to 37.5
N_2O_4	44 to 48
UDMH	36 to 40
RP-1	27
H_2O_2	30

If only the value of $\pi_{u,opt}$ is required, it may be computed by differentiation of Eq. (24), since inspection of this equation shows that only π_u remains as a free variable. This computation has been carried out; the details are given in Appendix B.

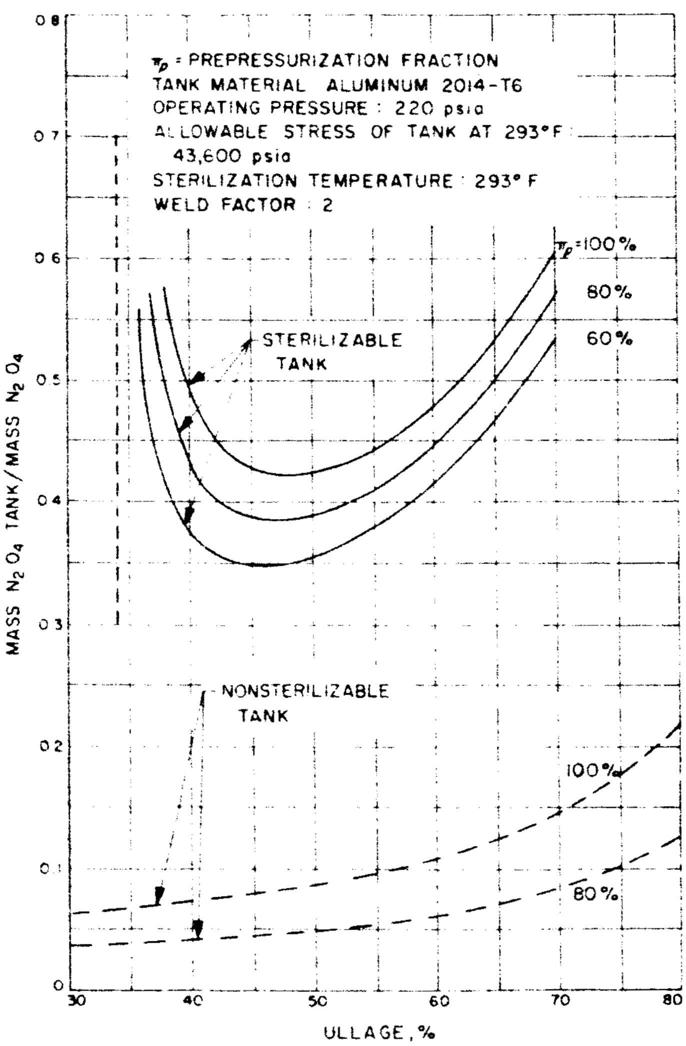

Fig. 3. Ratio of tank mass to propellant mass for different prepressurization levels in a N_2O_4 system

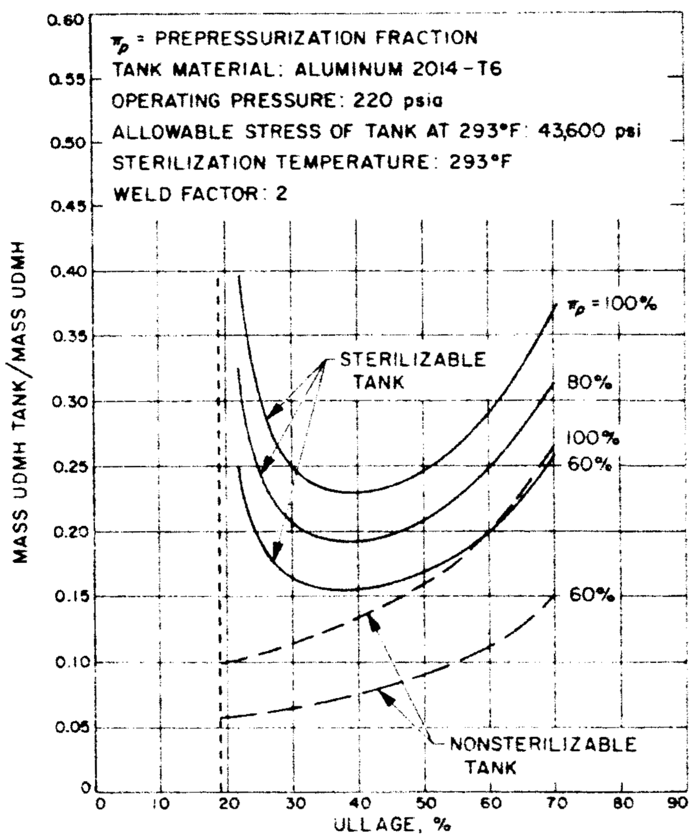

Fig. 4. Ratio of tank mass to propellant mass for different prepressurization levels in a UDMH system

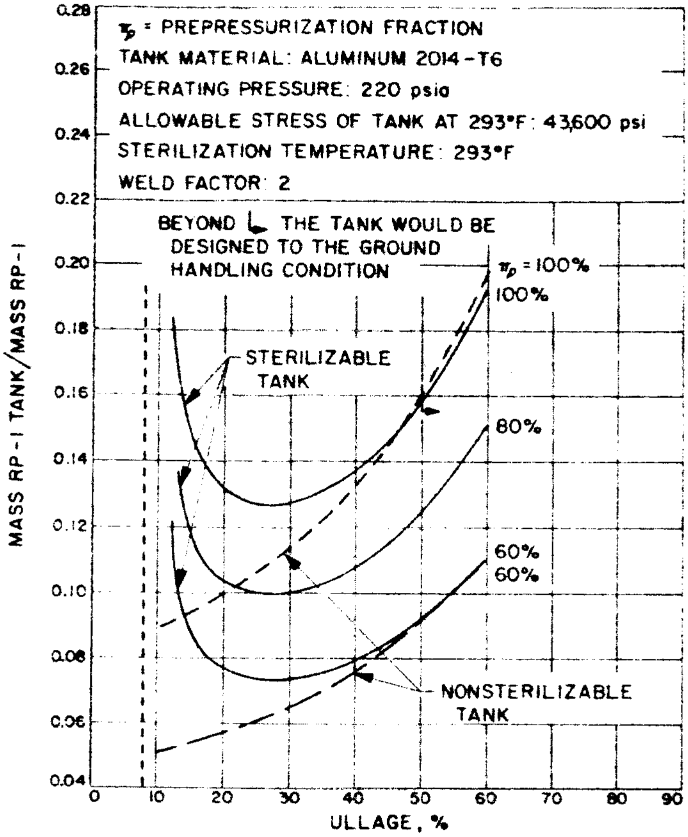

Fig. 5. Ratio of tank mass to propellant mass for different prepressurization levels in a RP-1 system

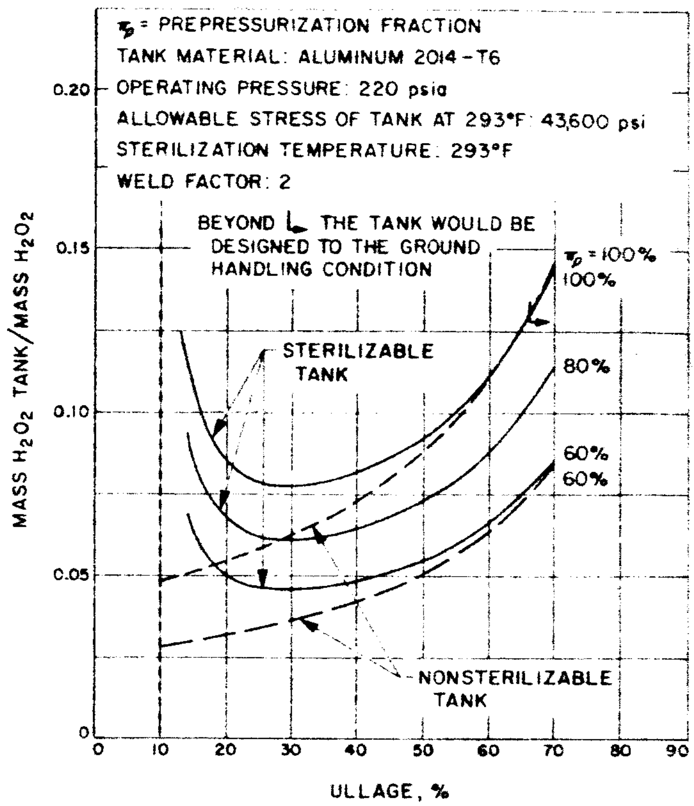

Fig. 6. Ratio of tank mass to propellant mass for different prepressurization levels in a H_2O_2 system

The important conclusion to be drawn from Figs. 1 through 6 concerns the relationship of the tank mass requirements for sterilized and nonsterilized systems. For the sterilized system, it is clear that the design condition would be chosen close to $\pi_{u\,opt}$. For nonsterilized systems, the choice is less obvious. In this study, the idealized assumption has been made that both the sterilized and nonsterilized systems would be loaded and fired at a uniform temperature. In reality, a range of temperatures would have to be accommodated, since tight control of spacecraft system temperatures during flight is quite difficult. Therefore, an analysis similar to that of the sterilizable system is required even for nonsterilized systems in the real case. While such is beyond our scope here, it is sufficient to note that values of 5 to 20% for π_u have been used in nonsterilized systems flown to date. Therefore, it is reasonable to use π_u's in this range as base points for the unsterilized system in the system mass comparisons.

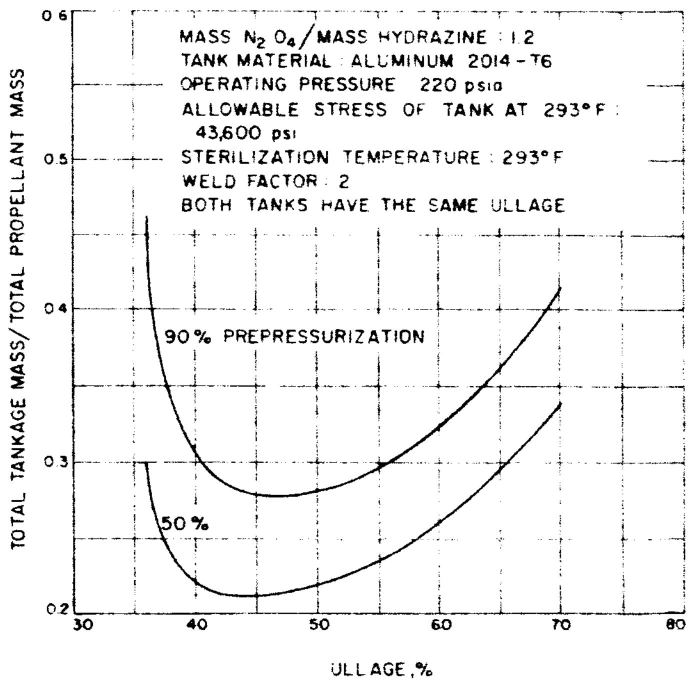

Fig. 7. Ratio of total tank mass to total propellant mass in a N_2O_4–N_2H_4 system

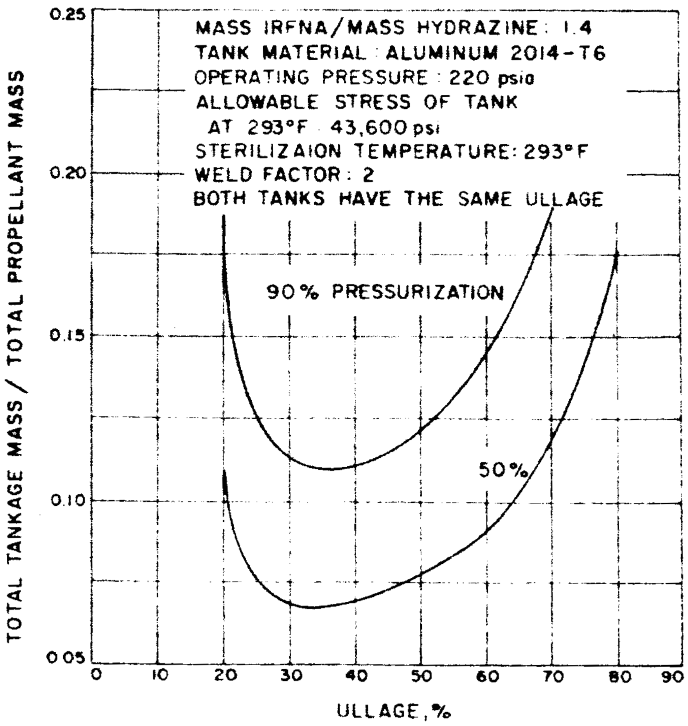

Fig. 8. Ratio of total tank mass to total propellant mass in a IRFNA–N_2H_4 system

Table 2. Ratio of system mass for sterilizable and nonsterilizable systems

Prepressurization level	60 %	100 %
Propellant	Mass ratio	Mass ratio
N_2H_4	1.8	1.7
IRFNA	2.7	2.3
N_2O_4	9.4	6.7
UDMH	2.7	2.3
RP-1	1.4	1.4
H_2O_2	1.6	1.6

A comparison of the masses for sterilized and nonsterilized systems is given in Table 2. One may note that the ratio varies from about 1.5 to 2.5 for propellants which look at least fairly suitable for use in sterilized systems. This comparison is based on the optimum ullage fraction values given in Table 1.

If it were desirable to fill both the fuel and oxidizer tanks to the same ullage, the value of π_u which minimizes the sum of the propellant tank masses may be

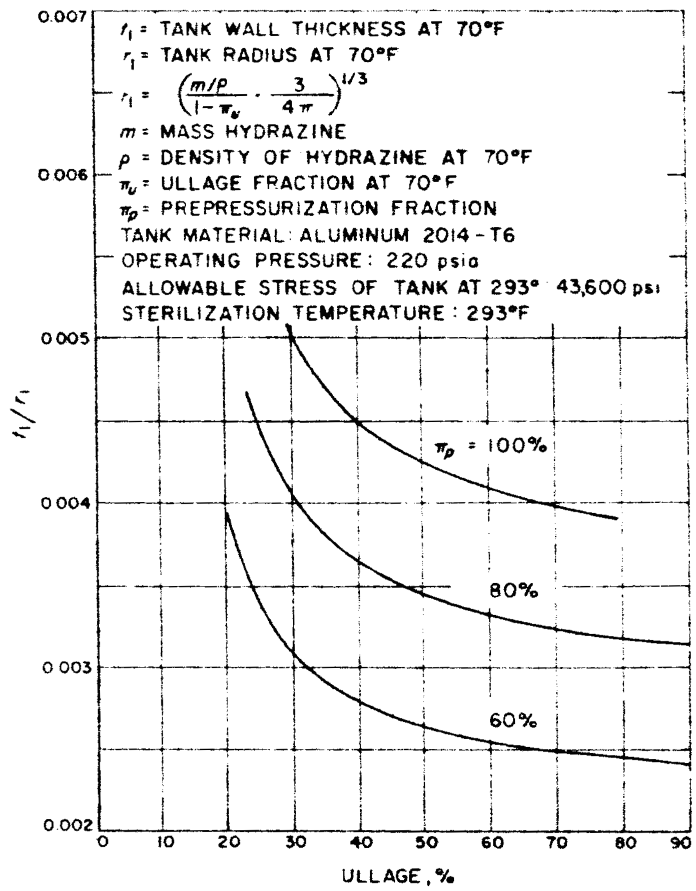

Fig. 9. Ratio of tank thickness to tank radius for different prepressurization levels in a hydrazine system

determined from Figs. 7 and 8, for the N_2O_4–N_2H_4 and IRFNA–hydrazine propellant combinations, respectively. The optimum ullages are:

N_2O_4 + Hydrazine: 43.5 to 46.2%

IRFNA + Hydrazine: 34 to 35%.

At π_p = 90%, with both the hydrazine and IRFNA tanks filled to the above ullage, the optimum total tank-to-propellant mass ratio is 0.109, compared to 0.108 for the case where the π_u's were separately optimized. For the N_2O_4–hydrazine combination, however, the equal-ullage ratio (π_p = 90%) is 0.278, compared with 0.269 for the separately optimized tankage. The optimum ullage ratio for each tank would most likely be used in this case.

Figs. 9 through 14 are plots of the ratio of tank wall thickness to tank radius at 70°F (not prepressurized) as

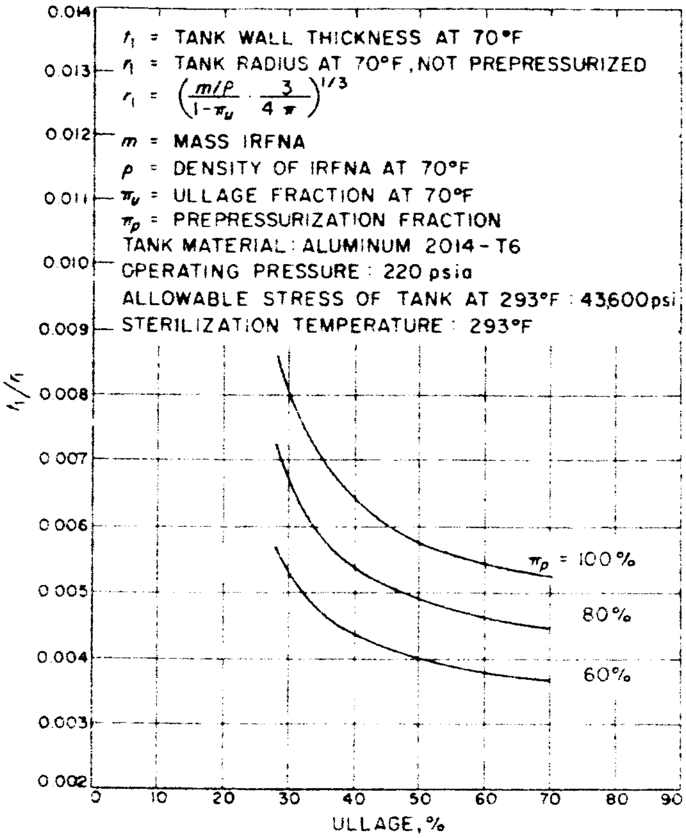

Fig. 10. Ratio of tank thickness to tank radius for different prepressurization levels in a IRFNA system

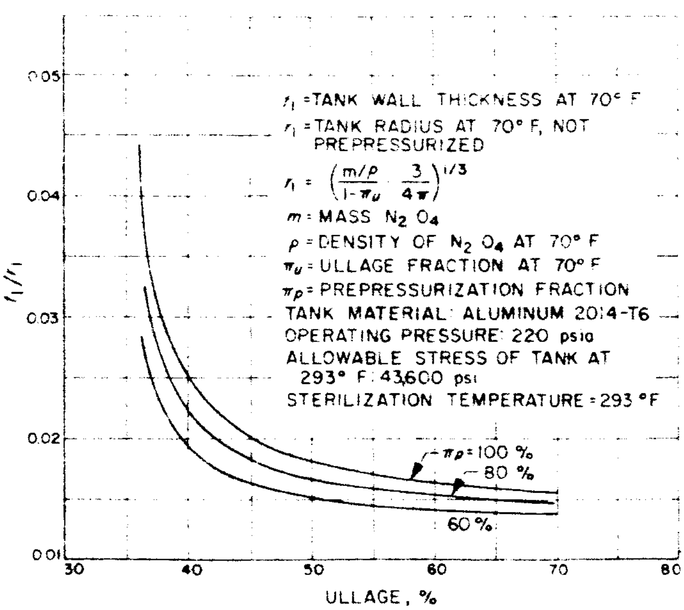

Fig. 11. Ratio of tank thickness to tank radius for different prepressurization levels in a N_2O_4 system

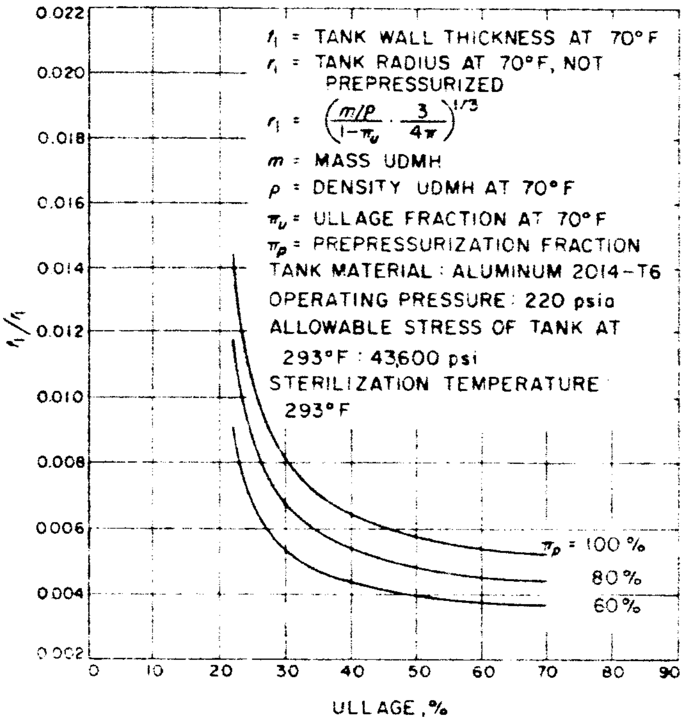

Fig. 12. Ratio of tank thickness to tank radius for different prepressurization levels in a UDMH system

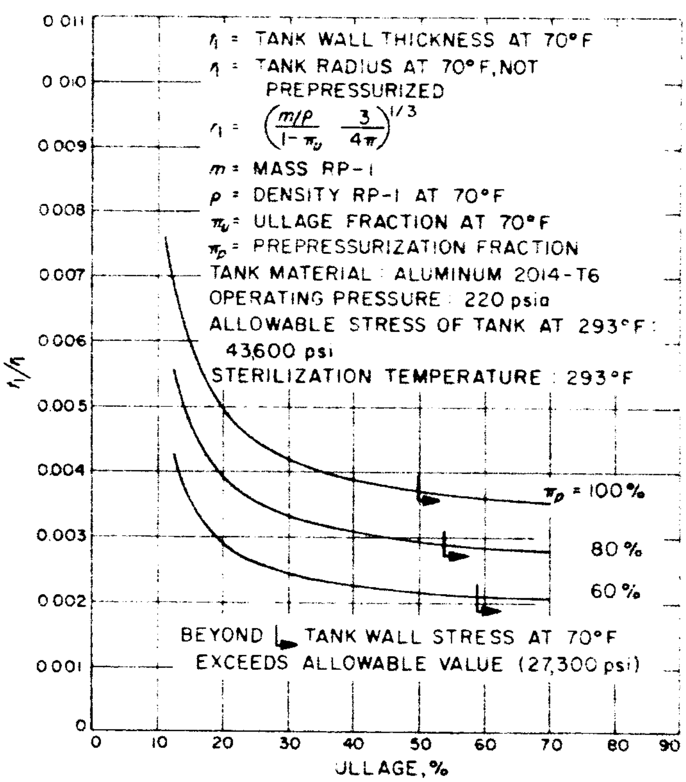

Fig. 13. Ratio of tank thickness to tank radius for different prepressurization levels in a RP-1 system

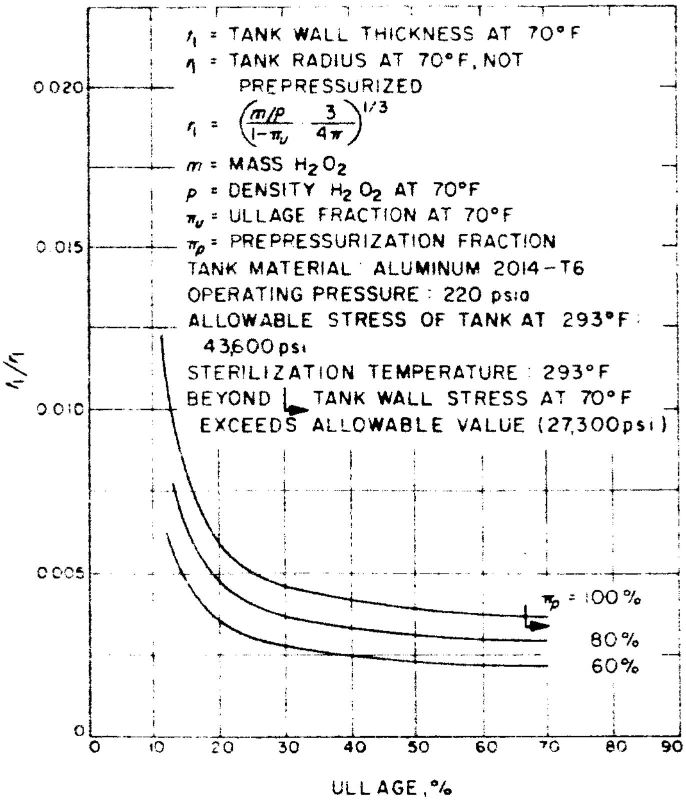

Fig. 14. Ratio of tank thickness to tank radius for different prepressurization levels in a H_2O_2 system

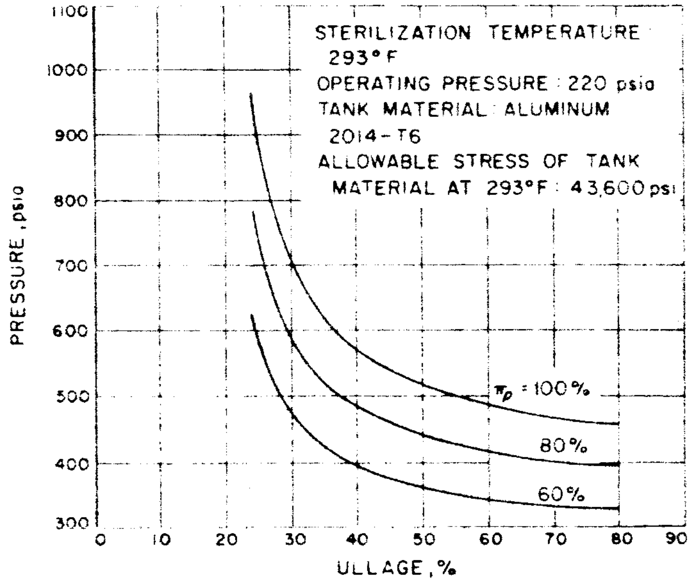

Fig. 16. Propellant tank pressure at sterilization temperature versus percent ullage for different prepressurization levels in a IRFNA system

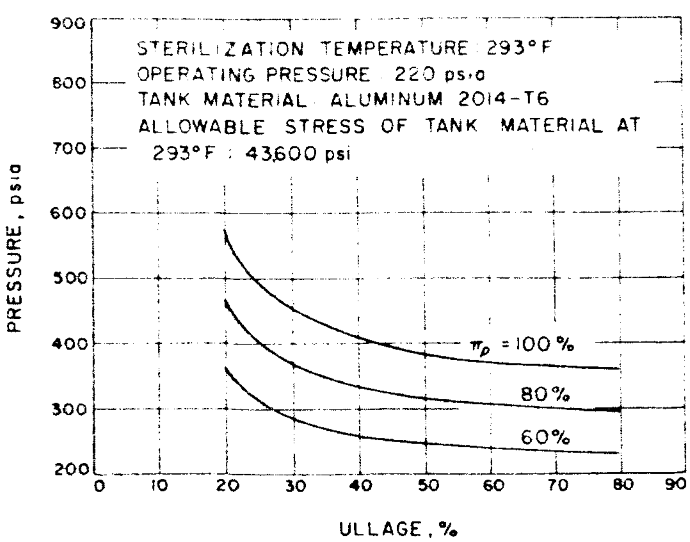

Fig. 15. Propellant tank pressure at sterilization temperature versus percent ullage for different prepressurization levels in a hydrazine system

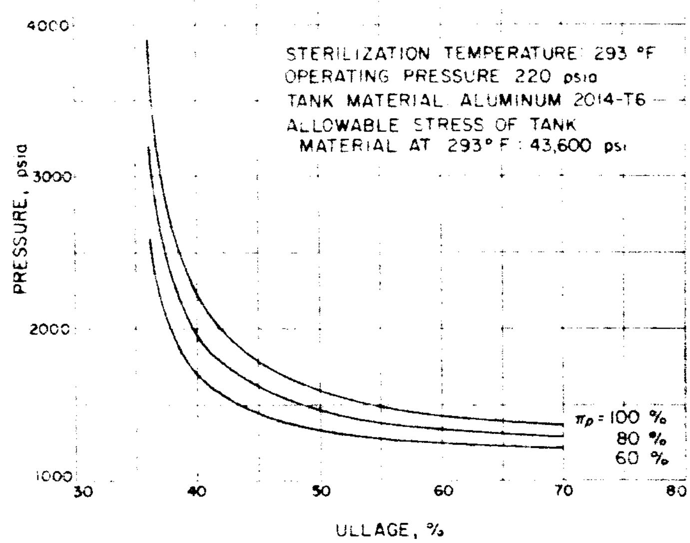

Fig. 17. Propellant tank pressure at sterilization temperature versus percent ullage for different prepressurization levels in a N_2O_4 system

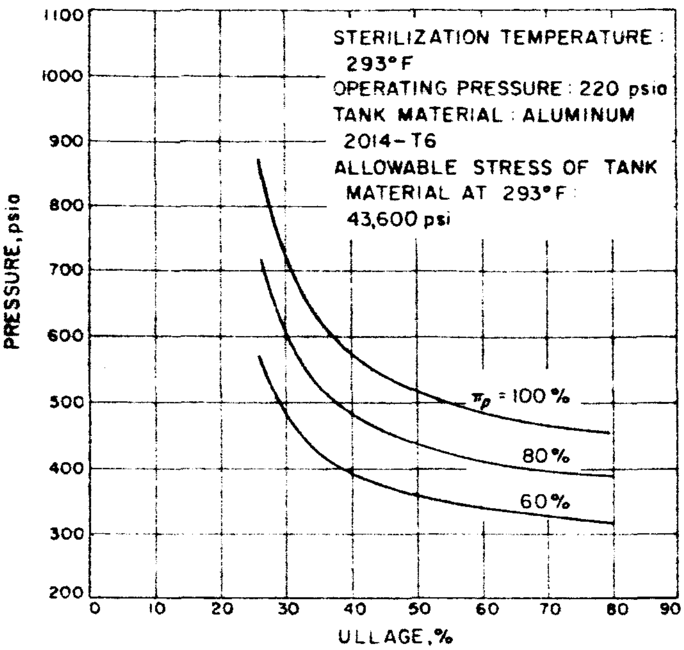

Fig. 18. Propellant tank pressure at sterilization temperature versus percent ullage for different prepressurization levels in a UDMH system

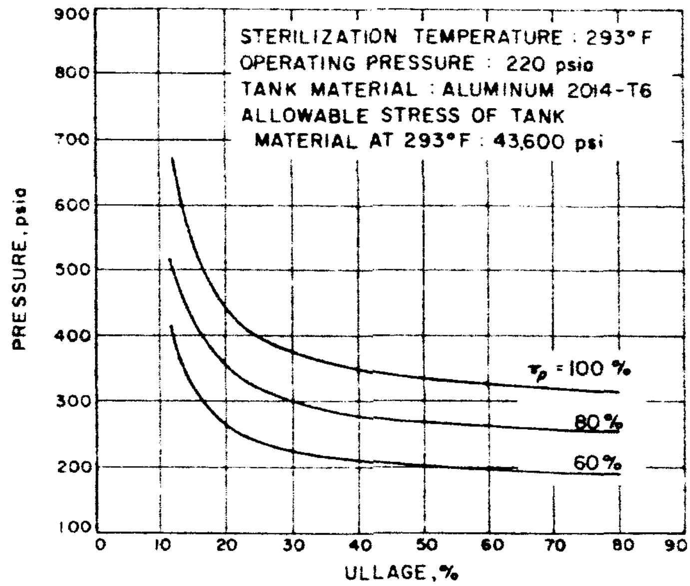

Fig. 19. Propellant tank pressure at sterilization temperature versus percent ullage for different prepressurization levels in a RP-1 system

a function of ullage fraction [Eq. (26)]. The tank thickness may be obtained by multiplying the plotted tank thickness-to-radius ratio by the radius of the unpressurized tank. The latter may be determined from Eq. (3) for any propellant mass of interest. It must be noted that, for certain ranges of mass of propellant, a wall thickness may be computed by means of Eq. (26) which will be less than the minimum thickness which could be manufactured. This will happen, depending on the propellant, at the lower prepressurization levels; in these cases, the tank wall thickness must be increased to the minimum thickness allowed by fabrication methods. This value is typically about 0.015 in.; however, no allowance for this minimum thickness effect has been made in Figs. 1 through 14.

The propellant tank pressures at the sterilization temperature are plotted as a function of the ullage fraction in Figs. 15 through 20.

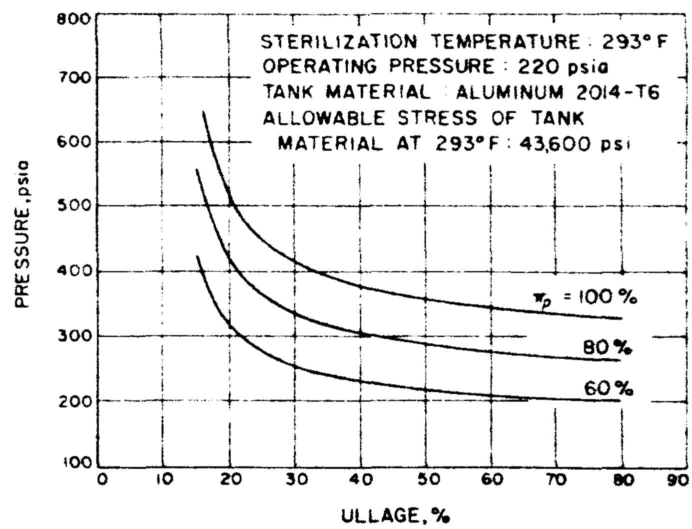

Fig. 20. Propellant tank pressure at sterilization temperature versus percent ullage for different prepressurization levels in a H_2O_2 system

III. INTERNALLY PRESSURIZED TANKAGE SYSTEMS

A. Statement of Problem

The internally pressurized propellant supply system contains, after loading, both the propellant and the pressurization gas required for propellant expulsion. Monopropellant systems only are considered in this analysis. The propellant and gas are assumed to be separated by a thin flexible membrane of negligible mass which separates the propellant and the pressurant. That part of the total tank volume occupied by the gas is defined as the *pressurization fraction.*

During operation, the tank "blows-down" from its initial pressure to a minimum pressure, which is attained at completion of the propellant expulsion. For a given pressurization fraction, the minimum pressure and the temperature at which this minimum pressure is reached determine the amount of pressurization gas which must be loaded. After loading the tank with propellant and gas, the system is heat-sterilized without venting.

The problem under consideration is to determine the dependence of the tank mass and wall thickness as a function of the minimum tank pressure and the flight temperature of the system.

No detailed consideration is given the loading process; the system, after loading, is assumed to be at a nominal temperature of 70°F. Helium is assumed as the pressurization gas, and spherical propellant tanks are employed.

B. Analysis

1. Sterilizable Systems

There are four states at which information is required regarding the thermodynamic and spatial variables of the closed system (propellant tank, propellant, and pressurizing gas). Subscripts will indicate the state to which the variables refer, as follows:

Subscript	Definition
1u	System at loading temperature and not pressurized
1p	System at loading temperature and pressurized
2	System at sterilization temperature
3	System at flight temperature
4	System at complete expulsion of propellant

We define the pressurization fraction in the same way ullage fraction has been previously defined:

$$f_p = 1 - \frac{V_{p1}}{V_{t_{1u}}} . \tag{36}$$

Changes in the propellant volume due to pressure can be shown to be negligible.

From Eq. (36), we obtain

$$V_{t_{1u}} = \frac{V_{p1}}{1 - f_p} = \frac{M_p/\rho_{p1}}{1 - f_p} . \tag{37}$$

The tank is loaded to a pressure, p_1. The gas then cools to temperature of the surroundings, T_1. There must be enough gas loaded so that, at complete expulsion of the propellant which occurs at T_4, the pressure in the tank will be the required minimum, p_{min}.

In order to calculate T_4, we assume a polytropic expansion process:

$$p\rho^{k} = \text{constant}.$$

Included in determination of the value of k are such considerations as the mass flow rate of propellant; heat conduction and radiation from the spacecraft to the tank and from the tank to the gas; the tank mass, specific heat and conductivity; and the blow-down pressure ratio.

From the definition of a polytropic process, we have

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} , \tag{38}$$

or

$$\frac{T_4}{T_3} = \left(\frac{V_{t3} - V_{p3}}{V_{t4}}\right)^{k-1} . \tag{39}$$

The radius of the tank at T_4 will have changed slightly from its value at T_1 before pressurization. From Eq. (8),

$$R_4 = R_{1u}\left[1 + \frac{\sigma_4 (1 - \nu)}{E} + \alpha (T_4 - T_1)\right] . \tag{40}$$

The volume of the tank at T_4 is, therefore,

$$V_{t_4} = \frac{4\pi}{3} R_4^3$$

$$= \frac{4\pi}{3} R_{1a}^3 \left[1 + \frac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1) \right]. \quad (41)$$

The linearization is possible since the last two of the bracketed terms in Eq. (41) are much smaller than unity.

We may write Eq. (41) as

$$V_{t_4} = V_{t_{1a}} \left[1 + \frac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1) \right]. \quad (42)$$

Similarly,

$$V_{t_3} = V_{t_{1a}} \left[1 + \frac{3\sigma_3 (1-\nu)}{E} + 3\alpha (T_3 - T_1) \right]. \quad (43)$$

Substituting Eqs. (42) and (43) into (39),

$$\frac{T_4}{T_3} = \left[\frac{1 + \dfrac{3\sigma_3 (1-\nu)}{E} + 3\alpha (T_3 - T_1) - \dfrac{V_{p_3}}{V_{t_{1a}}}}{1 + \dfrac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1)} \right]^{k-1}. \quad (44)$$

Into Eq. (44), we substitute (37) to get

$$T_4 = T_3 \left[\frac{1 + \dfrac{3\sigma_3 (1-\nu)}{E} + 3\alpha (T_3 - T_1) - \dfrac{\rho_{p1}}{\rho_{p3}} (1 - f_p)}{1 + \dfrac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1)} \right]^{k-1}. \quad (45)$$

The mass of gas in the tank is:

$$M = \frac{p_{min} V_{t_4}}{R T_4}. \quad (46)$$

Substituting Eqs. (45) and (42), Eq. (46) becomes

$$M = \frac{p_{min}}{R T_3} \left(\frac{M_p/\rho_{p1}}{1 - f_p} \right) \times \frac{\left[1 + \dfrac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1) \right]}{\left[\dfrac{1 + \dfrac{3\sigma_3 (1-\nu)}{E} + 3\alpha (T_3 - T_1) - \dfrac{\rho_{p1}}{\rho_{p3}} (1 - f_p)}{1 + \dfrac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1)} \right]^{k-1}}. \quad (47)$$

Also, at loading,

$$M = \frac{p_1 (V_{t_{1p}} - V_{p1})}{R T_1} = \frac{V_{t_{1a}} \left[1 + \dfrac{3\sigma_1 (1-\nu)}{E} \right] - V_{p1}}{R T_1}, \quad (48)$$

where, from Eq. (4), we have used

$$V_{t_{1p}} = V_{t_{1a}} \left[1 + \frac{3\sigma_1 (1-\nu)}{E} \right].$$

Substituting Eq. (37), we have

$$M = \frac{p_1 M_p}{R \rho_{p1} T_1} \left[\frac{f_p + \dfrac{3\sigma_1 (1-\nu)}{E}}{1 - f_p} \right]. \quad (49)$$

Equating Eqs. (47) and (49) and solving for p_1,

$$p_1 = p_{min} \frac{T_1}{T_3} \times \frac{\left[1 + \dfrac{3\sigma_4 (1-\nu)}{E} + 3\alpha (T_4 - T_1) \right]^k}{\left[1 + \dfrac{3\sigma_3 (1-\nu)}{E} + 3\alpha (T_3 - T_1) - \dfrac{\rho_{p1}}{\rho_{p3}} (1 - f_p) \right]^{k-1}} \times \frac{1}{\left[f_p + \dfrac{3\sigma_1 (1-\nu)}{E} \right]}. \quad (50)$$

At the sterilization temperature, the pressure in the tank becomes, in analogy to relation (12),

$$p_2 = \frac{p_1 \dfrac{T_2}{T_1} \left[\dfrac{3\sigma_1 (1-\nu)}{E} + f_p \right]}{\left[1 + \dfrac{3\sigma_2 (1-\nu)}{E} + 3\alpha (T_2 - T_1) - \dfrac{\rho_{p1}}{\rho_{p2}} (1 - f_p) \right]}. \quad (51)$$

From Eq. (15), the mass of the tank is

$$M_t = \frac{4\pi \rho_t R_2^3 (p_2 - P_{atm}) W}{2\sigma_2}. \quad (52)$$

It was found in Eq. (23) that

$$\rho_2 = \rho_1 \left[1 - \frac{\sigma_2 (2 - 4\nu)}{E} - 3\alpha (T_2 - T_1) \right]. \quad (53)$$

By analogy to Eq. (40), given previously herein

$$R_2 = R_{1a} \left[1 + \frac{\sigma_2 (1-\nu)}{E} + \alpha (T_2 - T_1) \right]. \quad (54)$$

where, using Eq. (37)

$$R_{1u} = \left[\frac{3}{4\pi} V_{t1u}\right]^{1/3} = \left[\frac{3}{4\pi}\left(\frac{M_p/\rho_{p1}}{1-f_p}\right)\right]^{1/3}. \qquad (55)$$

Substituting Eqs. (53), (54), and (55) into (52), and neglecting products of terms which are small compared with unity

$$\frac{M_t}{M_p} = \frac{3}{2}\frac{W}{\sigma_2}\frac{\rho_1}{\rho_{p1}}\left[\frac{1+\dfrac{\sigma_2(1+\nu)}{E}}{(1-f_p)}\right](p_2 - P_{atm}). \qquad (56)$$

The wall thickness to which the tank would be built is given by relations (26) and (12),

$$\frac{t_2}{R_{1u}} = \left[\frac{1+\dfrac{\sigma_2(1+\nu)}{E}}{2\sigma_2}\right](p_2 - P_{atm}). \qquad (57)$$

Relationships between the wall stresses $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ may be obtained by analogy from Eq. (27). Thus, for any two tank gage pressures, p_1 and p_2, the corresponding wall stresses are related by

$$\frac{\sigma_1}{\sigma_2} = \frac{p_1}{p_2}\left[1 - \frac{(\sigma_2 - \sigma_1)(1+\nu)}{E}\right]. \qquad (58)$$

The total system mass is the sum of the tank mass and the mass of the pressurization gas. Adding Eqs. (49) and (56) yields

$$\frac{M_t + M}{M_p} = \frac{p_2}{RT_2\rho_{p1}}\left[\frac{f_p + \dfrac{3\sigma_2(1-\nu)}{E}}{(1-f_p)}\right]$$

$$+ \frac{3}{2}\frac{W}{\sigma_2}\frac{\rho_1}{\rho_{p1}}\left[\frac{1+\dfrac{\sigma_2(1+\nu)}{E}}{(1-f_p)}\right](p_2 - P_{atm}). \qquad (59)$$

2. Nonsterilizable Systems

By analogy from Eq. (56), the equation for tank mass, in this case, may be written as

$$\frac{M_t}{M_p} = \frac{3}{2}\frac{W}{\sigma_1}\frac{\rho_1}{\rho_{p1}}\left[\frac{1+\dfrac{3\sigma_1(1-\nu)}{E}}{(1-f_p)}\right](p_1 - P_{atm}), \qquad (60)$$

with p_1 given by Eq. (50). Addition of M from Eq. (49) gives the following version of Eq. (59) for the nonsterilizable case

$$\frac{M_t + M}{M_p} = \frac{p_1}{RT_1}\left[\frac{f_p - \dfrac{3\sigma_1(1-\nu)}{E}}{(1-f_p)}\right]$$

$$+ \frac{3}{2}\frac{W}{\sigma_1}\frac{\rho_1}{\rho_{p1}}\left[\frac{1+\dfrac{3\sigma_1(1-\nu)}{E}}{(1-f_p)}\right](p_1 - P_{atm}). \qquad (61)$$

C. Results

Using the given mathematical relations, a computer program was written; the results contained in Figs. 21 through 27 were calculated. Hydrazine and hydrogen peroxide propellants were considered with titanium 6Al-4V as the propellant tank material. For a conservative estimate, $k = 1.4$ was used as the polytropic expansion coefficient in Figs. 21 through 26. For comparison, $k = 1.2$ was used to produce Fig. 27. (Note: for isentropic expansion of helium, $k = 1.66$.)

Plots of Eqs. (59) and (61) are given in Figs. 21 and 24. Curves are given in Fig. 21 for both sterilizable and nonsterilizable hydrazine systems for tank pressures at complete expulsion of 200, 400, and 600 psi. The ordinate for these figures is the sum of tank and helium masses divided by the propellant mass. The same information for hydrogen peroxide is given in Fig. 24. Additionally, two curves are given in Fig. 21 for a sterilizable system that would be loaded at 70 F and fired at 0 or 125°F (p_{min} = 400 psia). Figs. 21 and 24 show that the optimum pressurization fractions are near 60% and that the sterilizable systems are approximately twice as heavy as nonsterilized systems operating under comparable conditions.

The tank pressures at sterilization and loading are given in Figs. 22 and 25 for the hydrazine and hydrogen peroxide systems, respectively [Eqs. (50), (51)]; Figs. 23 and 26 contain the tank thickness-to-radius ratio [Eq. (57)].

Curves of all of the resultant variables for $k = 1.2$ and hydrazine (p_{min} = 600 psi) are shown in Fig. 27. Thus, the effects of changes in k may be determined by comparison of Fig. 27 with the appropriate curves of Figs. 21, 22, and 23. For example, one may observe that the value of the pressurization fraction which yields the lowest mass system increases as k increases. In Fig. 27, the minimum mass point occurs at $f_p = 0.54$, while for $k = 1.4$ (Fig. 21), the optimum occurs at $f_p = 0.57$.

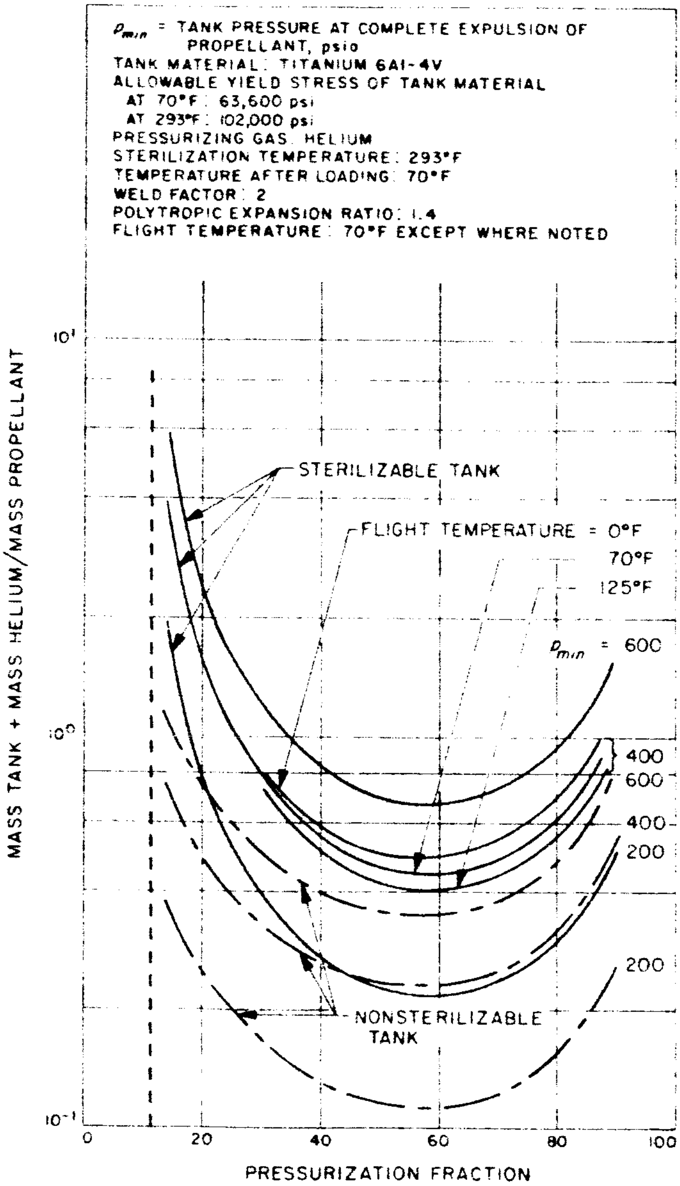

Fig. 21. Ratio of system mass to propellant mass for an internally pressurized hydrazine system

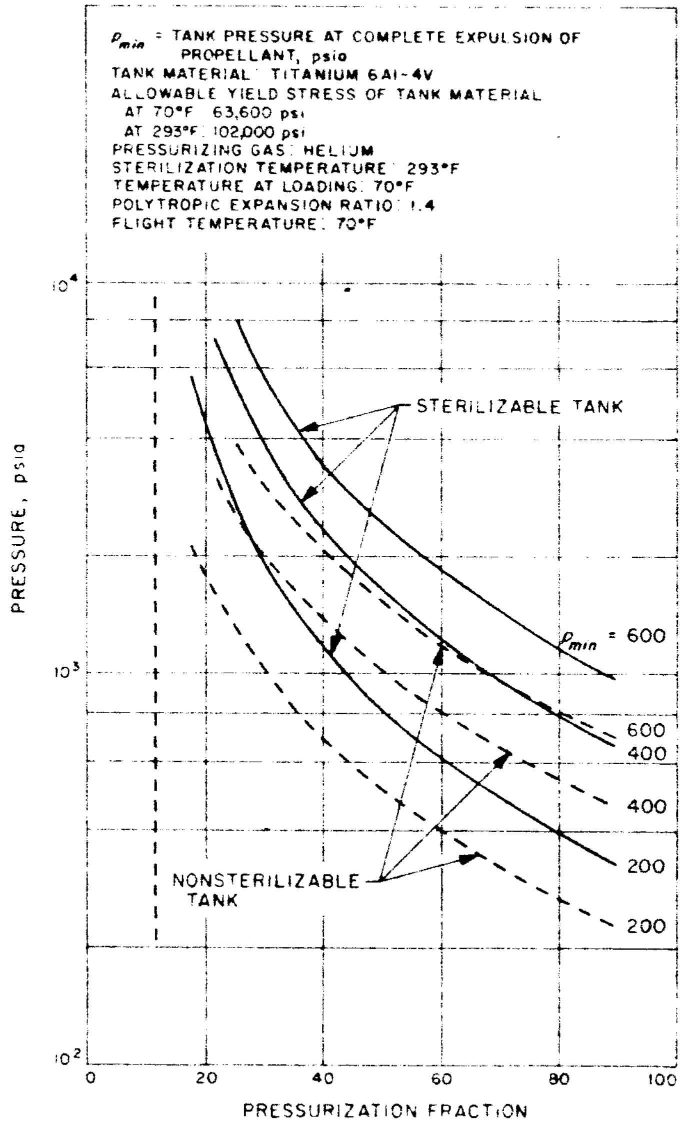

Fig. 22. Tank pressure versus pressurization fraction for a hydrazine system

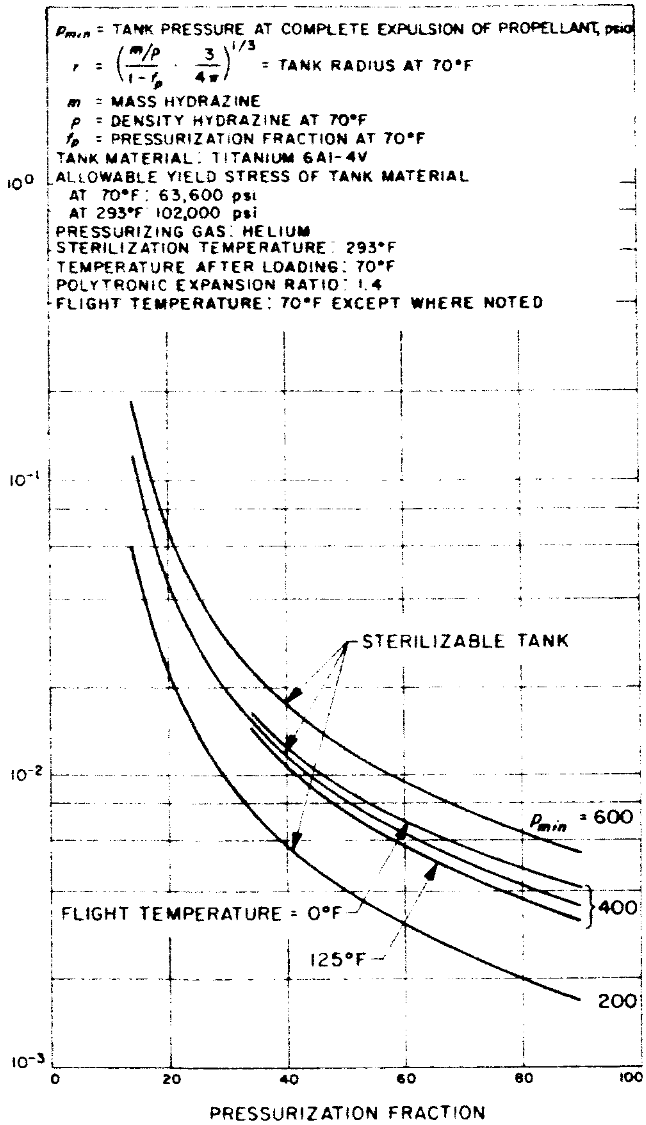

Fig. 23. Ratio of wall thickness to tank radius for an internally pressurized hydrazine system

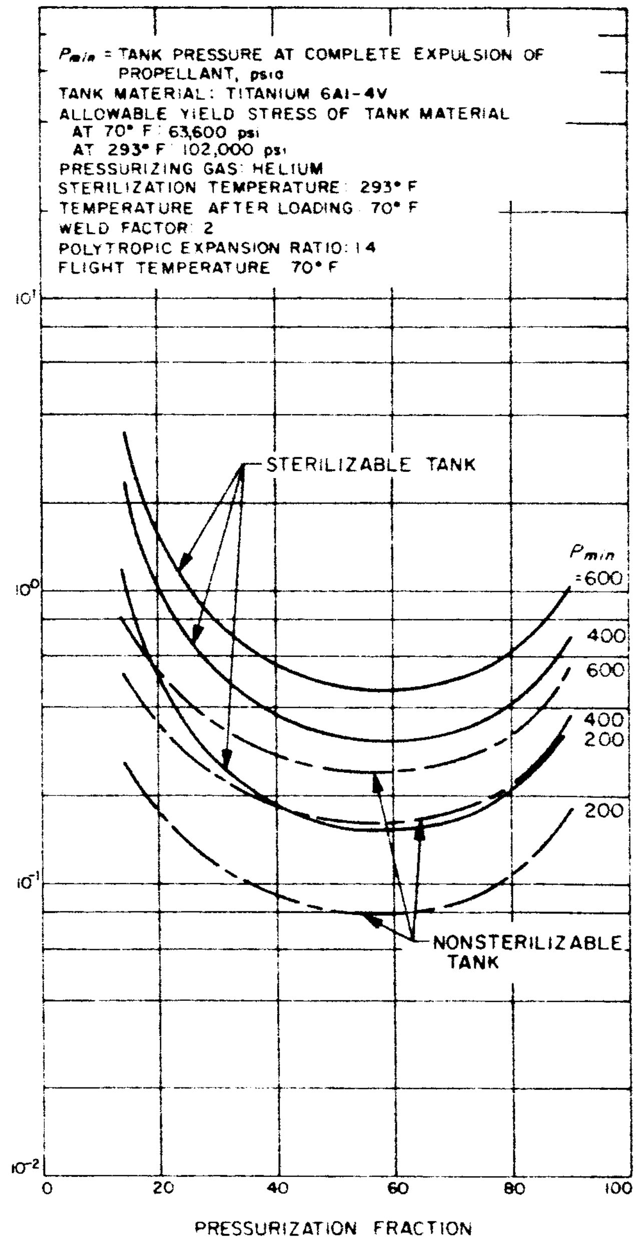

Fig. 24. Ratio of system mass to propellant mass for an internally pressurized hydrogen peroxide system

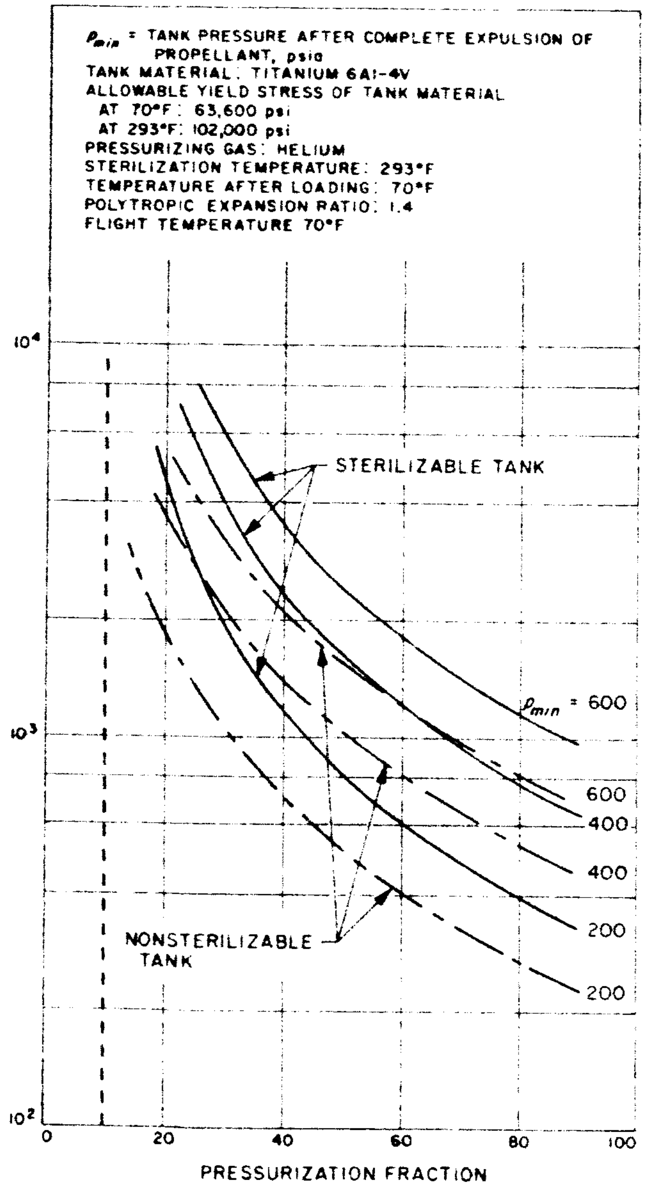

Fig. 25. Tank pressure versus pressurization fraction for a hydrogen peroxide system

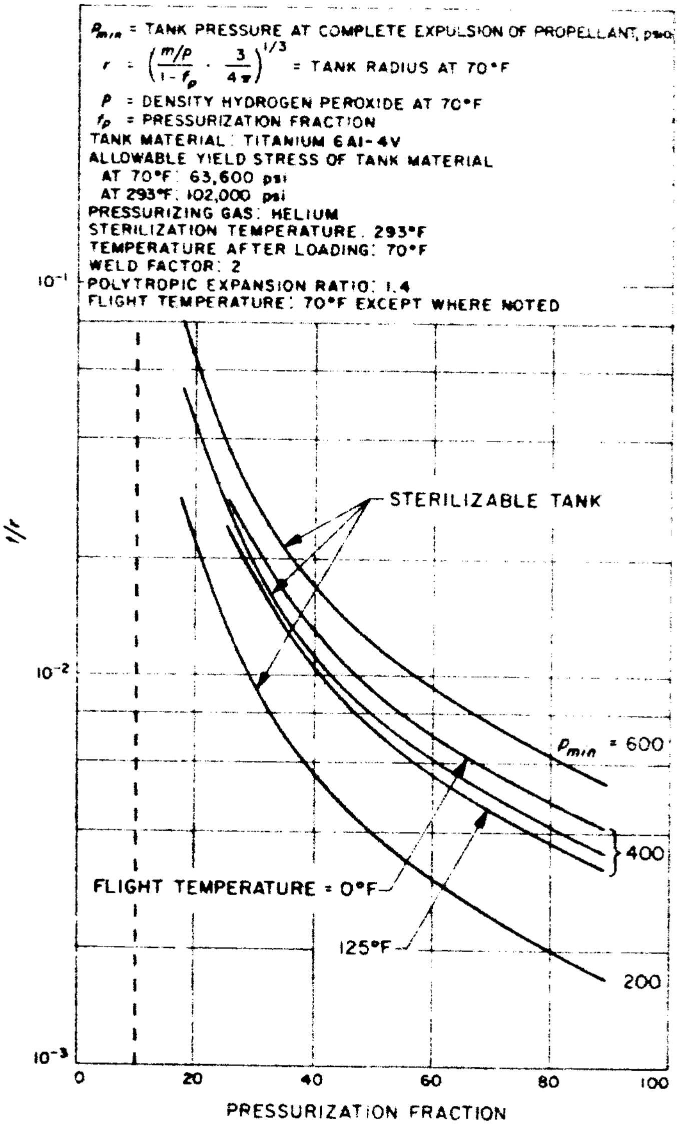

Fig. 26. Ratio of wall thickness to tank thickness for an internally pressurized hydrogen peroxide system

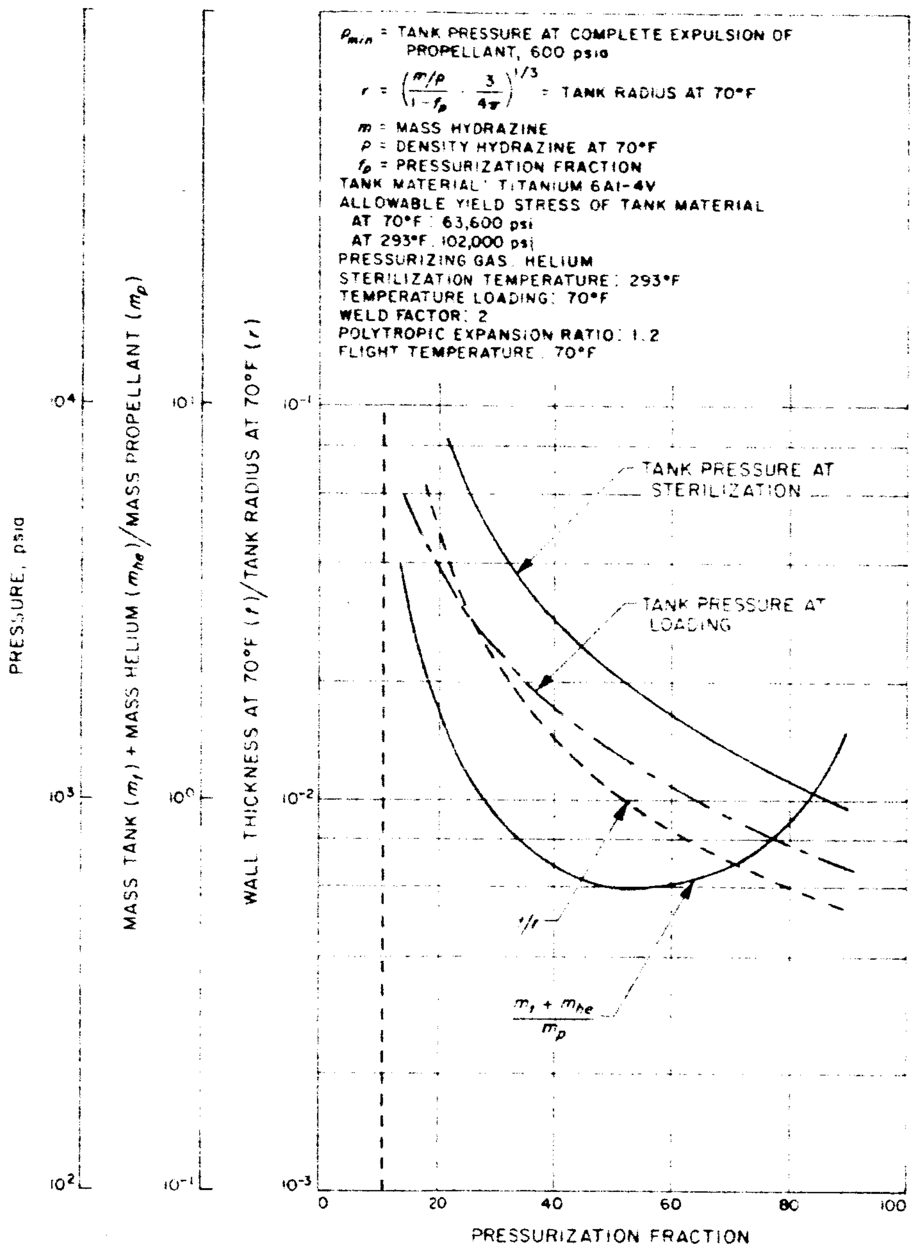

Fig. 27. Composite plot of tank mass, thickness, and pressures for a hydrazine system

APPENDIX A

Selection of Design Stress Condition

Assuming that the propellant tank is designed so that $\sigma_2 = \sigma_2'$, with σ_1 determined via relationship (27), under what conditions is $\sigma_1 < \sigma_1'$?

The correction factors for pressure and thermal effects in Eq. (27) will be set equal to unity for simplicity, although some error is thereby introduced into the analysis. Substituting from Eq. (14), we have

$$\frac{p_2 + p_{v2} - P_{atm}}{(\pi_p P_{op} - P_{atm})} = \frac{\sigma_2'}{\sigma_1} . \qquad \text{(A-1)}$$

Using the constraint (33) and Eqs. (28) and (29), we arrive at the following

$$\frac{p_2 + p_{v2} - P_{atm}}{\pi_p P_{op} - P_{atm}} > \frac{f_{s2} f_t}{f_{s1}} = f .$$

If we substitute Eq. (12), neglect the correction factors, and rearrange terms, we obtain

$$\frac{\pi_u}{1 - \frac{\rho_{p1}}{\rho_{p2}}(1 - \pi_u)} > \frac{f(\pi_p P_{op} - P_{atm}) - (p_{v2} - P_{atm})}{(T_2/T_1)(\pi_p P_{op} - p_{v1})} . \qquad \text{(A-2)}$$

The inequality is valid so long as both $(\pi_p P_{op} - P_{atm})$ and $(\pi_p P_{op} - p_{v1})$ are positive. Furthermore

$$\pi_u > \left[1 - \frac{\rho_{p1}}{\rho_{p2}}(1 - \pi_u)\right] \frac{f(\pi_p P_{op} - P_{atm}) - (p_{v2} - P_{atm})}{(T_2/T_1)(\pi_p P_{op} - P_{v1})} , \qquad \text{(A-3)}$$

if the inequality

$$1 - \frac{\rho_{p1}}{\rho_{p2}}(1 - \pi_u) \geq 0$$

holds, that is, if

$$\pi_u \geq 1 - \frac{\rho_{p2}}{\rho_{p1}} . \qquad \text{(A-4)}$$

The range of π_u indicated by Ineq. (A-4) includes those values which are used herein.

Letting

$$K \equiv \frac{f(\pi_u p_{op} - P_{atm}) - (p_{v2} - P_{atm})}{(T_2/T_1)(\pi_p P_{op} - p_{v1})} ,$$

Ineq. (A-3) can be rearranged to the following:

$$\pi_u\left(1 - \frac{\rho_{p1}}{\rho_{p2}} K\right) > \left(1 - \frac{\rho_{p1}}{\rho_{p2}}\right) K . \qquad \text{(A-5)}$$

Given a propellant, the initial and final temperatures, the operating pressure, π_p, and tank safety factors, this relationship tells us for what π_u the constraint is satisfied.

In this analysis, the following are assumed:

$$f_t = 0.8$$

$$f_{s1} = \frac{1}{2.2}$$

$$f_{s2} = \frac{1}{1.1}$$

Thus

$$f = 1.6$$

The propellants analyzed along with the constraints of relation (A-4) are listed for $\pi_p = 0.9$ and $P_{op} = 220$ psia:

Propellant	K	ρ_{p1}/ρ_{p2}
IRFNA	0.624	1.220
Hydrazine	0.975	1.125
N_2O_4	−2.65	1.530
UDMH	0.641	1.230
RP-1	1.100	1.090
Hydrogen peroxide	1.050	1.111

Using these values in Ineq. (A-5) gives the following range of π_u, in which Ineq. (33) is satisfied, for the selected pressurization level:

IRFNA	$\pi_u > -0.57$
Hydrazine	$\pi_u < 1.22$
N_2O_4	$\pi_u > 0.216$
UDMH	$\pi_u > -0.70$
RP-1	$\pi_u < 0.52$
H_2O_2	$\pi_u < 0.69$

The physically significant range of π_u is defined using Ineq. (A-4) as follows:

$$1 - \frac{\rho_{p2}}{\rho_{p1}} < \pi_u < 1.0 . \qquad \text{(A-6)}$$

Specifically, Ineqs. (A-5) and (A-6) are satisfied for the following values of π_u:

IRFNA	$0.18 < \pi_u < 1$
Hydrazine	$0.11 < \pi_u < 1$
N_2O_4	$0.34 < \pi_u < 1$
UDMH	$0.19 < \pi_u < 1$
RP-1	$0.08 < \pi_u < 0.52$
H_2O_2	$0.1 < \pi_u < 0.69$

These lower limits on ullage fraction (indicated by vertical dotted lines on Figs. 1 through 6) are, respectively, the values for which pressure, and therefore tank mass, go on to infinity.

Thus, it is seen that for only RP-1 and H_2O_2 would it be necessary to design the tank to the low-temperature criterion within a portion of the physically realizable range of π_u. However, the optimum values of π_u for RP-1 and H_2O_2 are 0.27 and 0.30, respectively. Therefore, it may be concluded that the high-temperature criterion will be the limiting one for most cases of practical interest.

APPENDIX B

Determination of Optimum Pressurization Fractions

The problem under consideration is to determine the optimum (lowest mass) ullage levels analytically.

If we assume σ_2 fixed and therefore controlling the value of σ_1, then from Eq. (24), we have

$$M_t = f_1 \{\pi_u, \sigma_1\} \; . \tag{B-1}$$

From Eq. (27),

$$\sigma_1 = f_2 \{P'_2\} \; ,$$

and from Eq. (12),

$$P'_2 = f_3 \{\pi_u\} \; .$$

Differentiating Eq. (B-1) with respect to π_u gives

$$\frac{dM_t}{d\pi_u} = \frac{\partial f_1}{\partial \pi_u} + \frac{\partial f_1}{\partial \sigma_1}\frac{\partial f_2}{\partial P'_2}\frac{df_3}{d\pi_u} \; . \tag{B-2}$$

Explicitly, considering first-order terms only

$$\frac{\partial f_1}{\partial \pi_u} = \frac{3}{2}\frac{W}{\sigma_2}\frac{\rho_1}{\rho_{p1}}\frac{1}{(1-\pi_u)\left[A - \frac{\rho_{p1}}{\rho_{p2}}(1-\pi_u)\right]}$$

$$\times \left\{\frac{(\pi_p P_{op} - p_{r1})\left(\frac{T_2}{T_1}\right)\left\{A - \frac{\rho_{p1}}{\rho_{p2}}\left[1 + \frac{3\sigma_1(1-\nu)}{E}\right]\right\}}{A - \frac{\rho_{p1}}{\rho_{p2}}(1-\pi_u)}\right.$$

$$+ \frac{1}{(1-\pi_u)}\left\{(\pi_p P_{op} - p_{r1})\left(\frac{T_2}{T_1}\right)\left[\frac{3\sigma_1(1-\nu)}{E} + \pi_u\right]\right.$$

$$\left.\left. + (p_{r2} - P_{atm})\left[A - \frac{\rho_{p1}}{\rho_{p2}}(1-\pi_u)\right]\right\}\right\} ,$$

$$\frac{\partial f_1}{\partial \sigma_1} = \frac{3W\rho_{p1}C}{2\sigma_2\rho_{p1}(1-\pi_u)\left[A - \frac{\rho_{p1}}{\rho_{p2}}(1-\pi_u)\right]}$$

$$\times (\pi_p P_{op} - p_{r1})\left(\frac{T_2}{T_1}\right)\left[\frac{3(1-\nu)}{E}\right] ,$$

$$\frac{\partial f_2}{\partial P'_2} = \frac{d}{dP'_2}\frac{(\pi_p P_{op} - P_{atm})\left[1 - \frac{(1+\nu)\sigma_2}{E}\right]}{\left\{1 - \left[\frac{(1+\nu)}{E}\frac{(\pi_p P_{op} - P_{atm})\sigma_2}{P'_2}\right]\right\}}\sigma_2$$

$$= \frac{(\pi_p P_{op} - P_{atm})\left[1 - \frac{(1+\nu)\sigma_2}{E}\right]\left[P'_2 - \frac{(1+\nu)(\pi_p P_{op} - P_{atm})\sigma_2}{E}\right]}{P'_2\left[P'_2 - \frac{(1+\nu)(\pi_p P_{op} - P_{atm})\sigma_2}{E}\right]^2}$$

$$- \frac{\frac{(\pi_p P_{op} - P_{atm})^2\sigma_2(1+\nu)}{E} ,}{P'_2\left[P'_2 - \frac{(1+\nu)(\pi_p P_{op} - P_{atm})\sigma_2}{E}\right]^2} \; .$$

$$\frac{df_2}{d\pi_u} = (\pi_p P_{op} - p_{v1})\left(\frac{T_2}{T_1}\right)$$

$$\times \frac{\left\{1 + \frac{3\sigma_2}{E}(1-\nu) + 3\alpha(T_2 - T_1) - \frac{\rho_{p1}}{\rho_{p2}}\left[1 + \frac{3\sigma_1(1-\nu)}{E}\right]\right\}}{\left[1 + \frac{3\sigma_2}{E}(1-\nu) + 3\alpha(T_2 - T_1) - \frac{\rho_{p1}}{\rho_{p2}}(1 - \pi_u)\right]^2}.$$

An order of magnitude analysis of these derivatives shows that

$$\frac{\partial f_1}{\partial \pi_u} \sim 1$$

$$\frac{\partial f_1}{\partial \sigma_1} \sim 10^{-8}$$

$$\frac{\partial f_2}{\partial P'_2} \sim 10$$

$$\frac{df_2}{d\pi_u} \sim 10^3.$$

Therefore, we neglect the second term on the right in Eq. (B-2) and set $\partial f_1/\partial \pi_u = 0$ to obtain the extremum. The same conclusion is reached if σ_2 is considered variable with σ_1 fixed. Thus, we have the following quadratic equation in π_u:

$$\frac{\rho_{p1}}{\rho_{p2}}\left[(\pi_p P_{op} - p_{v1})\left(\frac{T_2}{T_1}\right) + (p_{v2} - P_{atm})\left(\frac{\rho_{p1}}{\rho_{p2}}\right)\right]\pi_u^2$$

$$+ 2\frac{\rho_{p1}}{\rho_{p2}}\left\{(\pi_p P_{op} - p_{v1})\left(\frac{T_2}{T_1}\right)\left[\frac{3\sigma_1(1-\nu)}{E}\right]\right.$$

$$\left. + \left(A - \frac{\rho_{p1}}{\rho_{p2}}\right)(p_{v2} - P_{atm})\right\}\pi_u + (\pi_p P_{op} - p_{v1})\left(\frac{T_2}{T_1}\right)$$

$$\times \left\{\left[\frac{3\sigma_1(1-\nu)}{E}\right]\left(A - \frac{2\rho_{p1}}{\rho_{p2}}\right) + A - \frac{\rho_{p1}}{\rho_{p2}}\right\}$$

$$+ \left(A - \frac{\rho_{p1}}{\rho_{p2}}\right)^2 (p_{v2} - P_{atm}) = 0.$$

Solving for $(\pi_u)_{opt}$ (the optimum ullage point):

$$(\pi_u)_{opt} = -C_1 \pm (C_1^2 - C_2)^{1/2}, \qquad \text{(B-3)}$$

where

$C_1 \equiv$

$$\frac{(\pi_p P_{op} - p_{v1})(T_2/T_1)(B) + (A - \rho_{p1}/\rho_{p2})(p_{v2} - P_{atm})}{(\pi_p P_{op} - p_{v1})(T_2/T_1) + (p_{v2} - P_{atm})(\rho_{p1}/\rho_{p2})},$$

$C_2 \equiv$

$$\frac{(\pi_p P_{op} - p_{v1})(T_2/T_1)\left[(B)\left(A - \frac{2\rho_{p1}}{\rho_{p2}}\right)\right]}{\rho_{p1}/\rho_{p2}\left[(\pi_p P_{op} - p_{v1})(T_2/T_1) + (p_{v2} - P_{atm})(\rho_{p1}/\rho_{p2})\right]}$$

$$\frac{+\left(A - \frac{\rho_{p1}}{\rho_{p2}}\right)^2 (p_{v2} - P_{atm})}{\rho_{p1}/\rho_{p2}\left[(\pi_p P_{op} - p_{v1})(T_2/T_1) + (p_{v2} - P_{atm})(\rho_{p1}/\rho_{p2})\right]}$$

$$A \equiv 1 + \frac{3\sigma_2(1-\nu)}{E} + 3\alpha(T_2 - T_1),$$

$$\beta \equiv \frac{3\sigma_2(1-\nu)}{E}.$$

If Eq. (B-3) indicates two positive values for $(\pi_u)_{opt}$, then relation (A-6) in Appendix A may be used to determine which value is the physically significant one.

To use Eq. (B-3), a value for σ_1 must be known. The terms involving σ_1 may be neglected (i.e., set $B = 1$) with the loss of accuracy in $(\pi_u)_{opt}$ depending on the propellant under consideration. In all cases considered to date, the error induced in $(\pi_u)_{opt}$ by setting $B = 1$ would be less than 1%. However, to include σ_1, an empirical relation has been plotted to give σ_1 as a function of p_{v2}, a readily available propellant property. This curve was derived from the results for the six propellants which have been analyzed. The computer program, which gave the data for plotting Figs. 1 through 20, also gave σ_1 at each ullage level. For each propellant, the value of σ_1 at $(\pi_u)_{opt}$ was obtained, and this point is plotted against the vapor pressure on Fig. B-1.

An illustration of the use of Eq. (B-3) follows for the IRFNA propellant tank. Assume $\pi_p = 0.9$. The parameters are:

$$\frac{T_2}{T_1} = \frac{753°\text{R}}{530°\text{R}} = 1.42$$

$$(\pi_p P_{op} - p_{v1}) = (0.9)(220) - 2.3 = 195.7 \text{ psi}$$

$$\frac{\rho_{p1}}{\rho_{p2}} = \frac{0.0561 \text{ lb-in.}^{-3}}{0.046 \text{ lb-in.}^{-3}} = 1.22$$

$$p_{v2} = P_{atm} = 135 - 15 = 120 \text{ psi}$$

$$A = \left[1 + 3\frac{(43600)(0.67)}{10.6 \times 10^6} + 3 \times 12.5 \times 10^{-6}(223)\right] = 1.0167.$$

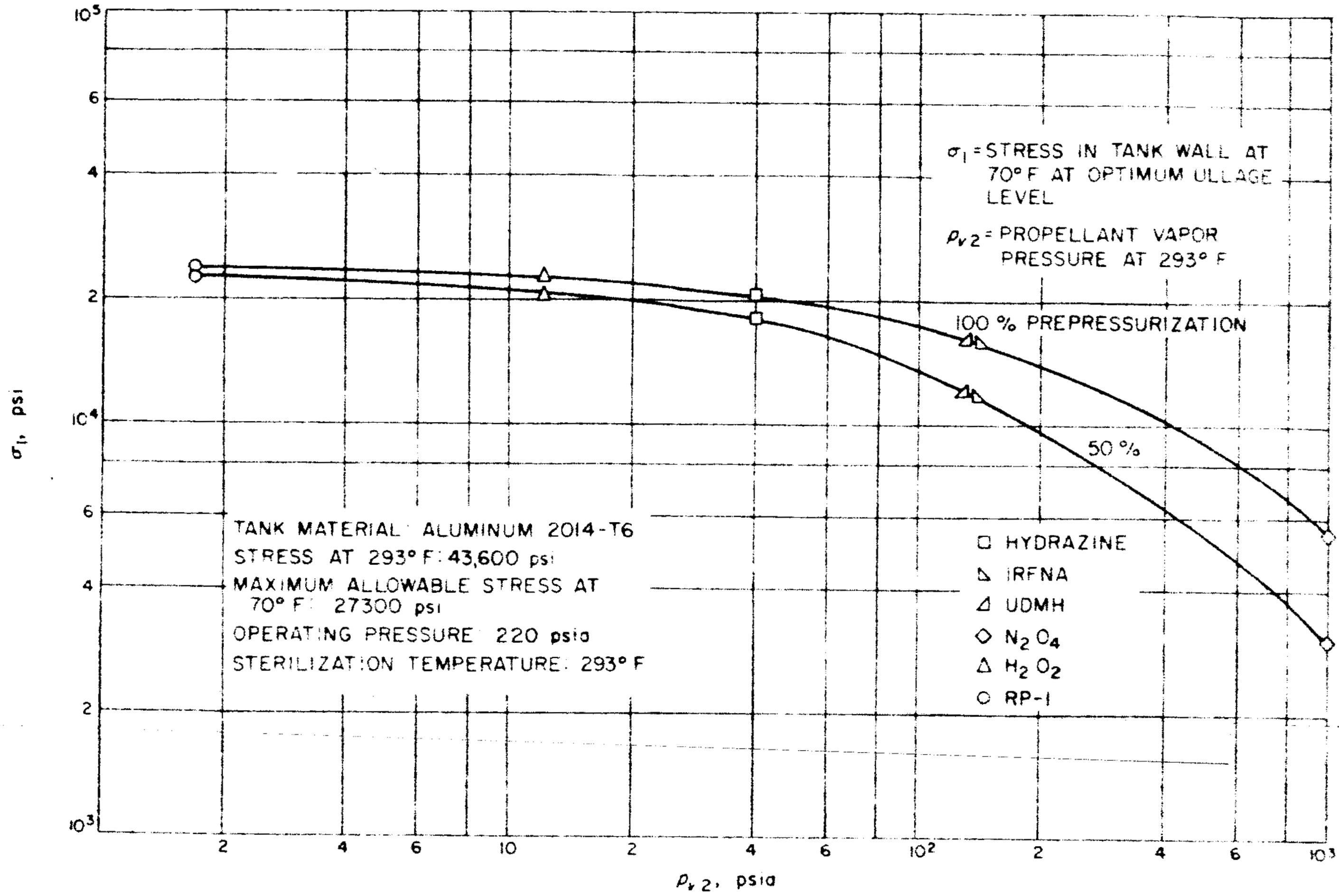

Fig. B-1. Room temperature tank wall stress versus vapor pressure at sterilization

From Fig. B-1, $\sigma_1 = 16{,}000$ at $p_{v2} = 135$, $\pi_p = 0.9$

$$B = \left[3 \frac{(16000)(0.67)}{10.6 \times 10^6} \right] = 0.003$$

$$C_1 = -0.056$$

$$C_2 = -0.105.$$

Substituting into Eq. (B-3),

$$(\pi_u)_{opt} = 0.056 \pm (0.105)^{1/2}$$

$$= 0.056 \pm 0.324.$$

The physically meaningful value is the positive one.

$$(\pi_u)_{opt} = 0.380\,.$$

This value may be compared with that of the optimum π_u for 90% prepressurization on Fig. 2. Also, if B were to have been set equal to 0, $(\pi_u)_{opt}$ would have been 0.378.

NOMENCLATURE

E	modulus of elasticity, lbf/in.^{2}
f_f	safety factor on tank yield stress during flight
f_p	pressurization fraction
f_{s1}	safety factor on tank yield stress at T_1
f_{s2}	safety factor on tank yield stress at T_2
f_t	correction factor on nominal yield strength due to high temperature (T_2)
k	polytropic expansion ratio
M	mass pressurization gas, lbm
M_p	mass of propellant, lbm
M_t	mass of propellant tank, lbm
P_{atm}	atmospheric pressure, lbf/in.^{2}
p_{min}	tank pressure after complete expulsion of propellant, lbf/in.^{2}
P_{op}	nominal operating pressure, lbf/in.^{2}
P_1, P_2	total tank pressure at T_1, T_2, lbf/in.^{2}
P_1', P_2'	tank gage pressure at T_1, T_2, lbf/in.^{2}
p_1, p_2	partial pressures of the pressurizing gas at T_1, T_2, lbf/in.^{2}
p_{v1}, p_{v2}	vapor pressure of propellant at T_1, T_2, lbf/in.^{2}
R	gas constant, lbf-in./lbm-°R
R_{1p}	radius of tank at T_1, prepressurized, in.
R_{1u}	radius of tank at T_1, not prepressurized, in.
R_{2p}	radius of tank at T_2 (including pressure effects only), in.
R_{2t}	radius of tank at T_2 (including temperature effects only), in.
R_2	radius of tank at T_2 (including both temperature and pressure effects), in.
R_4	tank radius at time of complete expulsion of propellant, in.
t	wall thickness at T_1 and $\sigma = 0$, in.
t_{1p}	wall thickness at T_1, prepressurized, in.
t_2	wall thickness at T_2, in.
t_3	tank wall thickness at T_3, in.
T	temperature, °R
T_1	loading temperature, °R
T_2	sterilization temperature ($> T_1$), °R
T_3	flight temperature, °R
T_4	temperature of gas at complete expulsion of propellant, °R
V_{p1}, V_{p2}	volume of propellant at T_1, T_2, in.^{3}
V_{t1u}	volume of tank at T_1, unpressurized, in.^{3}
V_{t1p}	volume of prepressurized tank at T_1, in.^{3}
V_{t2}	volume of pressurized tank at T_2, in.^{3}
V_{t3}	volume of tank at T_3, in.^{3}
V_{t4}	volume of tank at complete expulsion of propellant, in.^{3}
V_1, V_2	ullage volume at T_1, T_2, in.^{3}
V_3	volume of pressurizing gas at T_3, in.^{3}
V_4	volume of pressurizing gas at T_4, in.^{3}
W	weld factor
α	coefficient of thermal expansion, (°F)$^{-1}$
ν	Poisson's ratio of tank material
π_p	fraction of the operating pressure
π_u	ullage fraction at the unpressurized condition
$(\pi_u)_{opt}$	optimum ullage fraction
ρ_1	density of tank material at T_1 ($\sigma_1 = 0$), lbm/in.^{3}
ρ_2	density of tank material at T_2 and σ_2, lbm/in.^{3}
ρ_{p1}, ρ_{p2}	propellant density at T_1, T_2, lbm/in.^{3}
σ	stress of tank material, lbf/in.^{2}
σ_1	stress at P_1, lbf/in.^{2}
σ_2	stress at P_2, lbf/in.^{2}
σ_1'	allowable design stress at T_1 with safety factor of 2.2 included, lbf/in.^{2}
σ_2'	allowable design stress at T_2 with safety factor of 1.1 included, lbf/in.^{2}
σ_3	wall stress at T_3, lbf/in.^{2}
σ_4	wall stress at T_4, lbf/in.^{2}
σ_f	stress during flight, lbf/in.^{2}

REFERENCE

1. Pedersen, E. S., "Heat-Sterilizable Power Source Study for Advanced *Mariner* Missions," Technical Memorandum No. 33-180, Jet Propulsion Laboratory, Pasadena, California, July 1, 1964.