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AN INVESTIGATION OF THE DESIGN CRITERION FOR A
PHASE STABILITY STUDY SYSTEM OF VLF PROPAGATION
OVER SHORT DISTANCES

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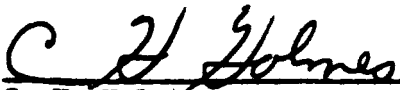
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FOREWORD

This is a technical report of a study conducted by the Electrical Engineering Department of Auburn University under the auspices of the Auburn Research Foundation toward the fulfillment of the requirements prescribed in NASA contract NAS8-5231.

ABSTRACT

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This report describes a system for the measurement of the phase stability of a VLF signal radiated over a short distance; such a system is independent of the incidental phase errors of the associated transmission equipment, and is a function only of the propagation medium. Amplitude stability of the VLF signal can be measured together with the inherent phase errors introduced by transmitting antennas caused by wind loading, temperature, etc.

The results of this investigation indicated that a radiated power of 100 to 200 milliwatts is sufficient for communications over a 320 km path at approximately 20 kc/s with available receiving systems. It was also determined that, if a constant power is radiated from a short vertical radiator, the unattenuated field strength is independent of antenna height. Thus, extremely short vertical radiators can be utilized for VLF communications; however, the antenna efficiency decreases with a decrease in antenna height.

A study of scaled model umbrella antennas indicated that a 300 foot vertical radiator with a 12-rib, 300 foot umbrella is a feasible antenna structure for this work. The measured capacitance is large enough to reduce the antenna base voltage to a reasonable value and also to minimize the tuning inductance requirements.

(over)

An investigation to determine the effect of the ground system on the antenna efficiency indicated that the effective loss resistance caused by the ground conduction currents was much smaller than originally expected for reasonable ground wire lengths. This study is based upon the determination of the equivalent resistance of the ground calculated from the power dissipated in the radial ground wire system, the earth adjacent to the ground wire system, and the power dissipated in the earth beyond the ground system. This study shows that the required ground wire lengths for extremely short vertical radiators are much less than the 0.3λ to 0.5λ lengths recommended for a satisfactory radial ground wire system at higher frequencies.

A survey of the effects of the finite conductivity of the earth on the antenna radiation characteristics is presented only insofar as it is related to short path propagation. The receiving antenna requirements are determined upon the basis of this survey, and upon the characteristics of the VLF propagation medium for the proposed path of communications.

Author

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AN INVESTIGATION OF THE DESIGN CRITERION FOR A
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E. R. Graf and C. E. Smith

I. INTRODUCTION

During the last few years, the interest in the very-low-frequency (VLF) spectrum has steadily increased. This increased interest is a result of the discovery that the stability of VLF propagation over long paths approaches that required for precise navigation and dissemination of world-wide time and frequency standards. In general, the degree of accuracy of phase comparison measurements of a one-way propagation path is inherently limited by the phase stability of the compared frequency standards and associated system components; however, usable precise measurements and interpretations of phase comparison data can be obtained with modern frequency standards that maintain a high degree of frequency stability.

If the phase of a VLF signal received from a high stability transmitter is compared with that received from a high stability local frequency standard, the variation in phase is interpreted as a change of electrical distance between the transmitter and receiver station. This total phase variation can be attributed to phase variations in

either frequency standards, associated interconnecting transmission and receiving equipment, propagation medium, or a combination of these phenomena.

The major source of phase error in the interconnecting transmission and receiving systems is the transmitting antenna. In large antenna structures there are slow variations caused by changes in transmitter impedance, caused in turn by heat dissipation in tuning coils and short time variation resulting from wind loading and related effects.¹

For accurate time comparisons and navigation systems based on VLF transmission, it is desirable to correct for these phase variations in order to obtain the greatest degree of accuracy for a given set of operating conditions. The operational characteristics of the frequency standards and the associated receiving system can be predetermined to any given degree of accuracy to eliminate these related errors. Thus, the stability and predictability of the atmospheric effects on VLF propagation, and the effects of the associated transmission system, are limiting factors in the precise measurement of phase.

Numerous investigations have been made in order to characterize the propagation characteristics of the region in which the ionosphere affects VLF propagation. The ground wave signal, which is reasonably stable, has been utilized as the receiving station frequency reference at distances up to several hundred kilometers in order to determine VLF ionospheric propagation characteristics.² Commercial telephone circuits

have also been utilized to transmit reference signals to receiving stations in order to obtain similar information.³

The objective of this very-low-frequency propagation study is two-fold:

- (1) to provide more accurate information concerning the propagation characteristics and phase stability of very-low-frequency signals; and,
- (2) to investigate the possibilities of using very-low-frequency signals for use in direction finding and in accurate location problems.

The purpose of this report is to describe a system for studying the phase stability of propagated VLF signals, and which will eliminate errors caused by the antenna and frequency standards or the references of the system. The proposed system is a two-way transmission system, in which a signal at frequency f_1 is transmitted to the transponder station, where it is then re-transmitted at a phase-coherent frequency f_2 slightly offset from f_1 . Frequency f_2 is received at the reference station, and, after being phase coherent frequency offset, it is compared with the transmitted frequency f_2 as shown in Figure 1. The total phase shift over the path may be directly attributed to the propagation medium, (See Figure 2). The data obtained from the experiments utilizing this system are to be used to furnish information about phase and amplitude stabilities at very low frequencies resulting from variations in the propagation medium and the associated transmission

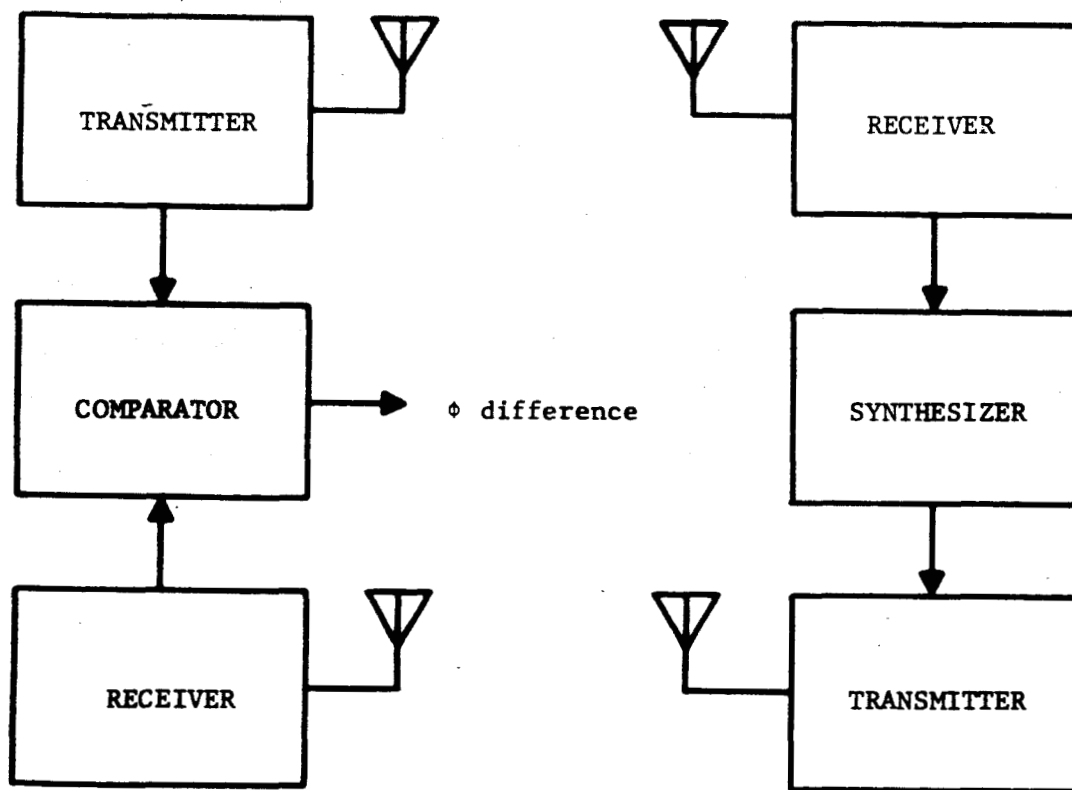


Fig. 1--System block diagram for the proposed VLF phase stability study.

system. It is expected that the data will be useful in determining predictable measures for the short and long term phase stability of the propagation medium, and these measures can be utilized for precise phase comparison measurements over small distances.

At present there is no available data which relates the effect of the phase stability of the propagation medium upon the obtainable accuracy of point-to-point phase comparison measurements at VLF frequencies over short distances. The results of this investigation will indicate a measure of the phase stability of the propagation medium, and also a measure of the associated transmission antenna systems utilized in the measurement system.

II. SYSTEM DESCRIPTION

The proposed phase stability measurement system is to be established over an approximate range of 300 to 400 kilometers, between a control station (located in Auburn, Alabama) and a transponder station (located in Mobile, Alabama). Phase measurements will be made over the two-way path by (1) transmission of a CW signal to the transponder station from the control station, (2) synthesis of a phase-coherent CW signal frequency offset from the control station signal, (3) transmission of the phase coherent signal to the control station, and (4) comparison of the received signal with the reference signal synthesized from the control station signal. A simple block diagram of this system is presented in Figure 1.

A more detailed diagram of the instrumentation required for this system is shown in Figure 2. The control station will consist of two sections, which will be referred to as the transmitter and the receiver sections. The transmitting section will be located several kilometers from the receiving station along the path connecting the receiving section and the transponder station. The transmitted signal f_1 of the control transmitter will be received at the control receiving station and utilized as the reference for phase comparisons. This technique will eliminate phase shifts in the transmitted signal resulting

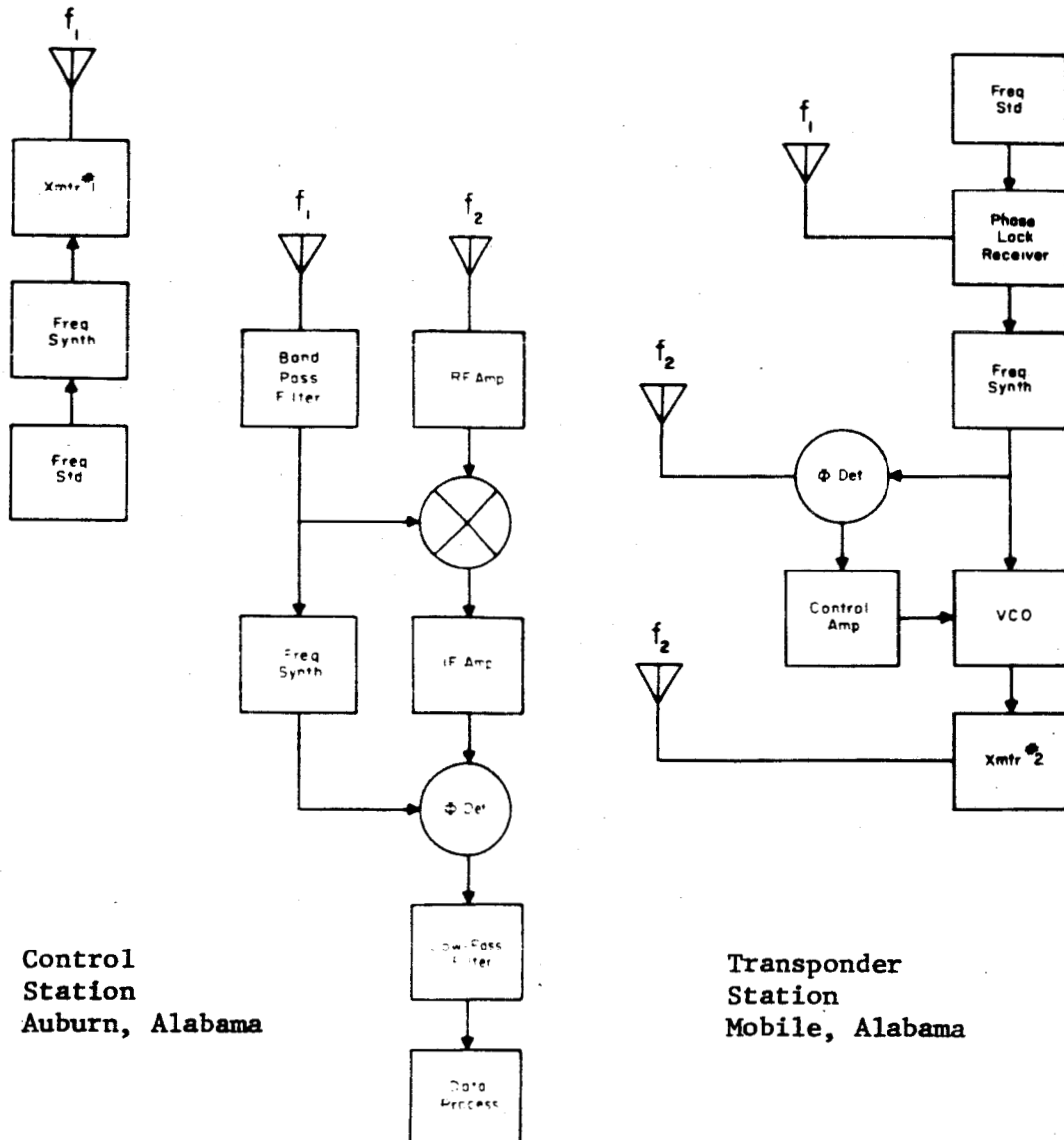


Fig. 2--Instrumentation for the proposed VLF phase stability study.

from phase variations, which in turn are caused by physical variations in the antenna configurations or similar factors. The transmitting section is composed of a frequency standard, (the stability of which is relatively unimportant),* frequency synthesizer, transmitter, and antenna. The transmitted frequency is expected to be close to 20 kc/s, but of necessity between 15 kc/s and 30 kc/s.

At the transponder station the control station signal f_1 is received and synthesized in order to obtain a signal f_2 which is frequency offset from that of signal f_1 . This signal is phase coherent with f_1 and does not depend upon the local frequency standard. The transponder signal f_2 is re-transmitted to the control station receiver. The in-line phase control loop between the synthesizer and transmitter is required to stabilize the phase of the transmitted signal f_2 because phase errors are introduced in the transmitter and antenna.

The transponder signal f_2 will be received at the control station receiver where it will be amplified, filtered, and mixed with signal f_1 , which is approximately 1 kc/s above or below signal f_2 . The phase detector compares the phase between the 1 kc/s signal (synthesized from f_1) and signal f_2 at the 1 kc/s i-f frequency; and, the output is filtered and recorded for data processing. The phase difference obtained is independent of the instrumentation, which is not the case

*As long as the frequency change does not exceed the receiver bandwidth and the time-rate-of-change is small.

in a one-way phase comparison system, and indicates the total two-way phase shift introduced by the propagation medium.

All modern VLF phase tracking receivers are equipped with a facility for securing amplitude variation data. It is proposed that this data be simultaneously recorded and compared for the separate one-way amplitude variations. This data will indicate the magnitude of the amplitude stability, and also will give a measure of the difference, if any, in the amplitude stabilities in the two propagation paths.

In addition, the phase variations introduced by the transponder transmitting antenna will be obtained from a record of the phase error in the in-line phase-lock loop. This data will yield a measure of the phase error introduced by the transmitting antenna type utilized in this phase comparison system.

III. VLF ANTENNA SYSTEM

Transmitting Antenna

General: Any practical antenna for the propagation of electromagnetic energy in the VLF spectrum will be electrically short due to the impossibility of erecting self-resonant structures at these long wavelengths. The main considerations of VLF transmitting antenna design are, in most cases, (1) the amount of power required to maintain communications over the required range, and (2) the value of antenna quality factor Q required to satisfy the bandwidth requirements of the modulated system. In this particular system, the power requirement is the determining factor. The quality of a vertical radiator, with respect to efficiency and power capacity, increases with height. However, at the very lowest frequencies, any practical vertical radiator is in the order of several degrees of electrical length, and hence it is necessary to utilize some form of capacitive top loading in order to obtain even the minimum required performance. The required amount of top loading is usually determined by the maximum allowable antenna voltage for high power installations, although tuning requirements may influence the final choice for low power transmitting installations.

In VLF transmitting antenna design, there is no real optimum design for a particular structural configuration, because additional efficiency or bandwidth can be obtained by increasing the antenna height, top

loading, ground plane size, etc. The cost-performance relationship increases rapidly for improved operation because of the increase in price per foot of erecting and maintaining the antenna structures, and the cost of higher voltage insulators required for large transmitted powers.

A study has been made to determine some antenna configuration which will perform under the required conditions and which will be financially feasible. Two configurations have been considered as possible transmitting antennas. One is a diamond-shaped top-hat antenna fed by one, or multiple, down leads, and the other is an umbrella antenna, which is a simple vertical radiator utilizing the guy wires as loading elements. Since the umbrella antenna has the simplest physical configuration, it, in general, costs less to erect and maintain; thus, the umbrella type antenna was selected for utilization in the system.

Power Requirements: Since the design of the complete system is dependent upon the determination of the required radiated power, a careful study was made to determine an estimate of the required power. This study was based upon the technique for determining the required power for a VLF transmission path presented by J. A. Pierce,⁴ and it was assumed that this data was sufficiently accurate over the range in question (approximately 320 km). Pierce's technique employs the highest measured value of atmospheric noise in the United States, called Kansas noise, as a basis for extracting power requirements for a given signal-to-noise ratio, SNR.

Since the criterion for radiated power is related to receiver sensitivity, it is desirable to base the calculations on a VLF phase tracking receiver system presently available for utilization in the proposed system. A typical receiver having a sensitivity of 0.01 uv for a -20 db signal-to-noise ratio into the tracking filter for positive automatic gain control (AGC) action was selected. The bandwidth for this SNR is 50 c/s, and the tracking filter has a bandwidth of 0.002 c/s. Hence, the SNR existing in the tracking filter would be

$$\text{SNR} = -20 + 10 \log_{10} \left(\frac{50}{0.002} \right) = 24 \text{ db.} \quad (1)$$

From the available information for the determination of required power, a curve indicating the variation of SNR as a function of frequency and power of a 320 km path length has been prepared and is presented in Figure 3. The original calculations are for SNR's for a 1 kc/s bandwidth, and are a function of frequency for several powers at the required distance. The right ordinate of this curve is the SNR in reference to Kansas noise for a tracking filter bandwidth of 0.002 c/s. For a transmission frequency of 20 kc/s the required power is approximately 2.4 watts, as determined from the curve. It must be noted that the determination assumes that no additional noise is added by the preceeding receiver stages and by the antenna. At these frequencies, the assumption is valid because the noise figure of the available equipment is sufficiently small in most cases.

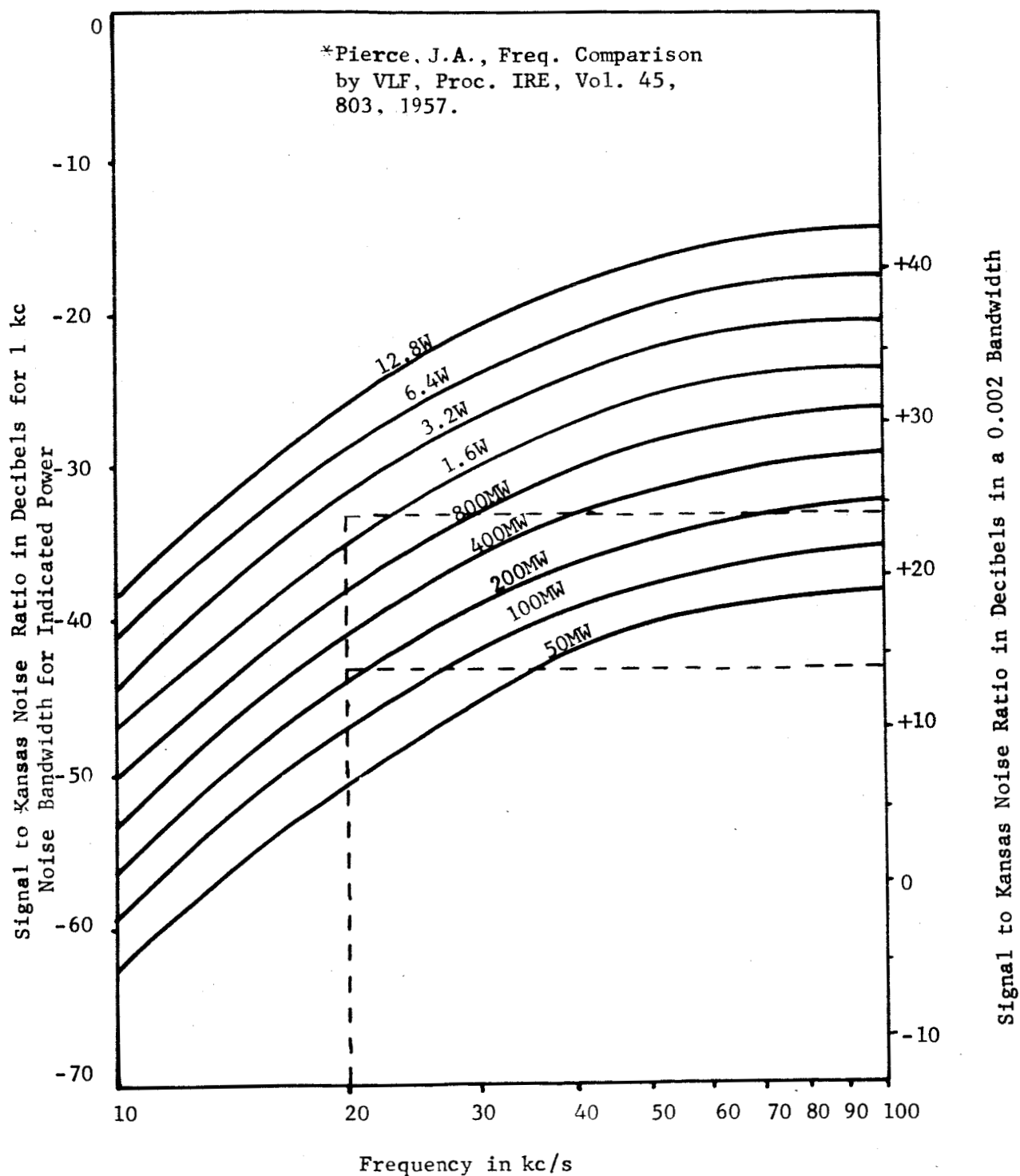


Fig. 3 --Variation of Signal to Noise Ratios As a Function
of Frequency and Power for a 320 KM Path Length*

Additional examples of atmospheric noise, as a function of frequency at various locations, are presented in Figure 4. It is important to note that all values of noise are below the recorded Kansas noise. The median value of noise in this latitude (32°N) as presented in "Reference Data for Engineers, (ITT)," is approximately 26 db below Kansas noise at 20 kc/s. The measured average of noise peaks at 15 kc/s which was measured near Gainesville, Florida (28°N) is approximately 16 db below Kansas noise; this average was obtained from a study performed by the University of Florida Electrical Engineering Department. Because noise levels generally decrease with frequency, the noise as measured in the University of Florida Study is seen to be at least 10 db below Kansas noise. Since all recorded noise data indicate that the existing noise level in this region is at least 10 db below Kansas noise, then a shift of 10 db for the SNR to Kansas noise for the 0.002 bandwidth indicates that a radiated power of 200 mw is required for 320 km at 20 kc/s.

These curves were based upon the required SNR for positive AGC control as specified by the receiver manufacturer; thus, additional receiver sensitivity may be obtained by operating without AGC, which will permit a lower than normal set point for the receiver tracking filter input level. Under specified conditions a radiated power of 100 to 200 mw, should be sufficient for communications over this path during all periods except those in which ambient noise is excessive.

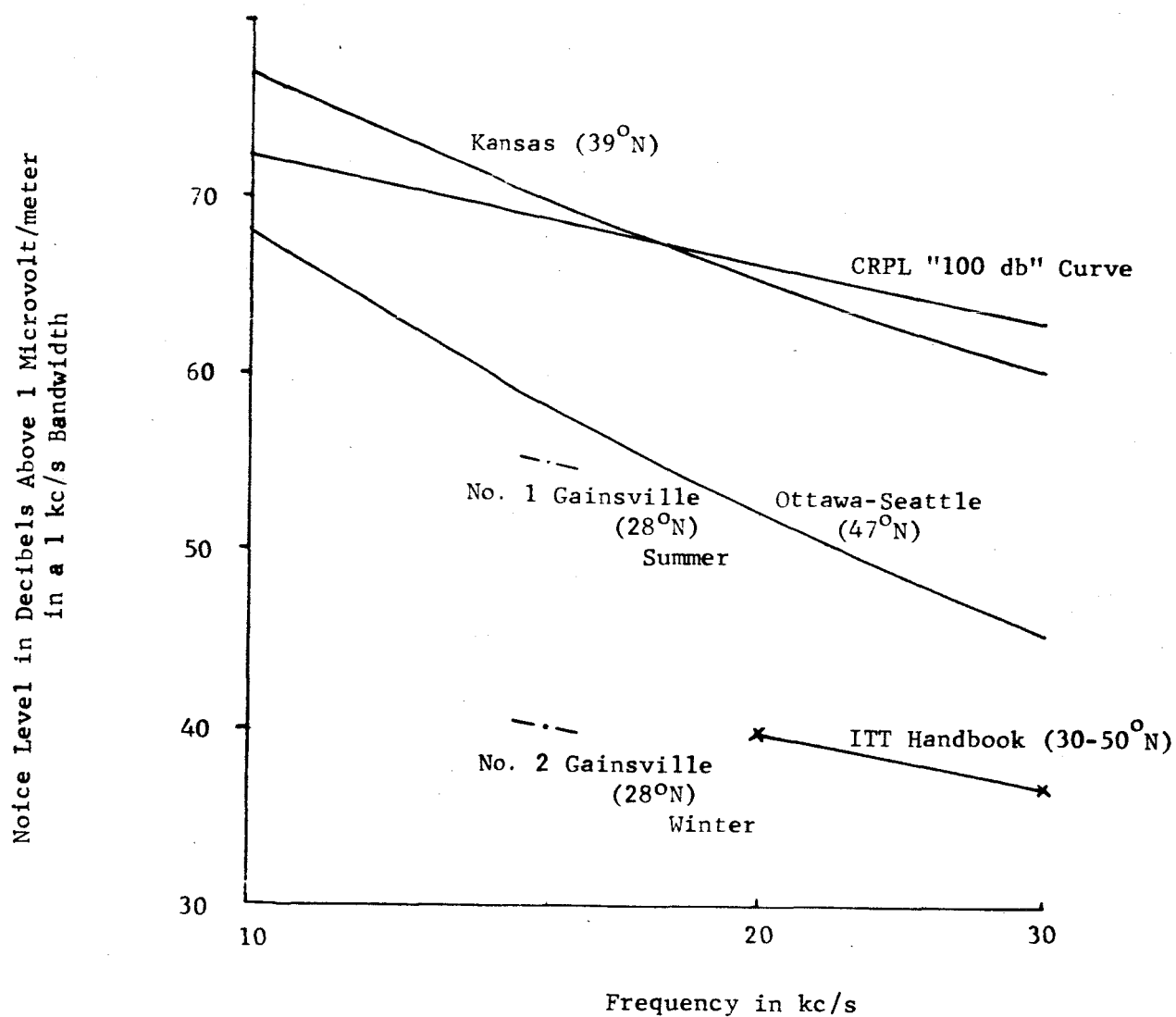


Fig. 4--Examples of the variation of atmospheric noise with frequency at various locations at the most noisy time of day.*

*Pierce, J. A. Frequency Comparison by VLF, Proc. IRE, Vol. 45, 802, 1957.

Vertical Radiator Characteristics

The main disadvantages of VLF propagation are the high cost and the practical difficulty of erecting antenna structures with dimensions appreciable with respect to the operating wavelength. All VLF antennas are usually some form of an electrically short vertical radiator with capacitive top loading. Thus, it is convenient to discuss the performance of such antennas in terms of a single vertical radiator.

The first consideration in the design of a VLF antenna is the determination of the antenna height. It has been shown that the unattenuated field strength⁵ at the surface of a perfectly conducting flat earth one mile from a simple vertical radiator is

$$|\bar{F}_s| = 37.25 I_0 \left[\frac{1 - \cos G}{\sin G} \right] \text{ millivolts/meter} \quad (2)$$

where

$$I_0 = \sqrt{\frac{w}{R_t}}, \text{ antenna base current, or } I_0 = \sqrt{\frac{P_r}{R_r}}$$

w = total antenna power input,

P_r = radiated power,

R_t = total antenna resistance, $R_t = R_r + R_L$,

R_r = radiation resistance,

R_L = effective loss resistance,

h = $360^\circ/\lambda$, angular antenna height,

G = $2\pi a/\lambda$, radian antenna height,

and a = antenna height.

If $G \ll 2\pi$, which is the case for electrically short antennas, and I_0 is expressed in terms of radiated power, the field strength is

$$|\bar{F}_s| = 37.25 \sqrt{\frac{P_r}{R_r}} \left[\frac{1 - (1 - G^2/2)}{G} \right], \quad (3)$$

or

$$|\bar{F}_s| = 37.25 \sqrt{\frac{P_r}{R_r}} (G/2) \text{ mv/meter}, \quad (4)$$

where $\cos G \approx 1 - G^2/2$

and $\sin G \approx G$

which are approximations obtained from series expansions. The established expression for radiation resistance of a short vertical radiator is

$$R_r \approx 10 G^2 \approx h^2/312 \quad (5)$$

for $h < 30^\circ$. The substitution of (5) in (4) yields a field strength of

$$|\bar{F}_s| \approx 5.9 \sqrt{P_r} \text{ mv/meter}, \quad (6)$$

which is independent of antenna height for a constant radiated power.

Equation (6) requires that a specific power be radiated regardless of the radiation resistance, which is a function of antenna height as indicated in Figure 5 for $\frac{I_{top}}{I_{base}} = 0$. A comparison of this result

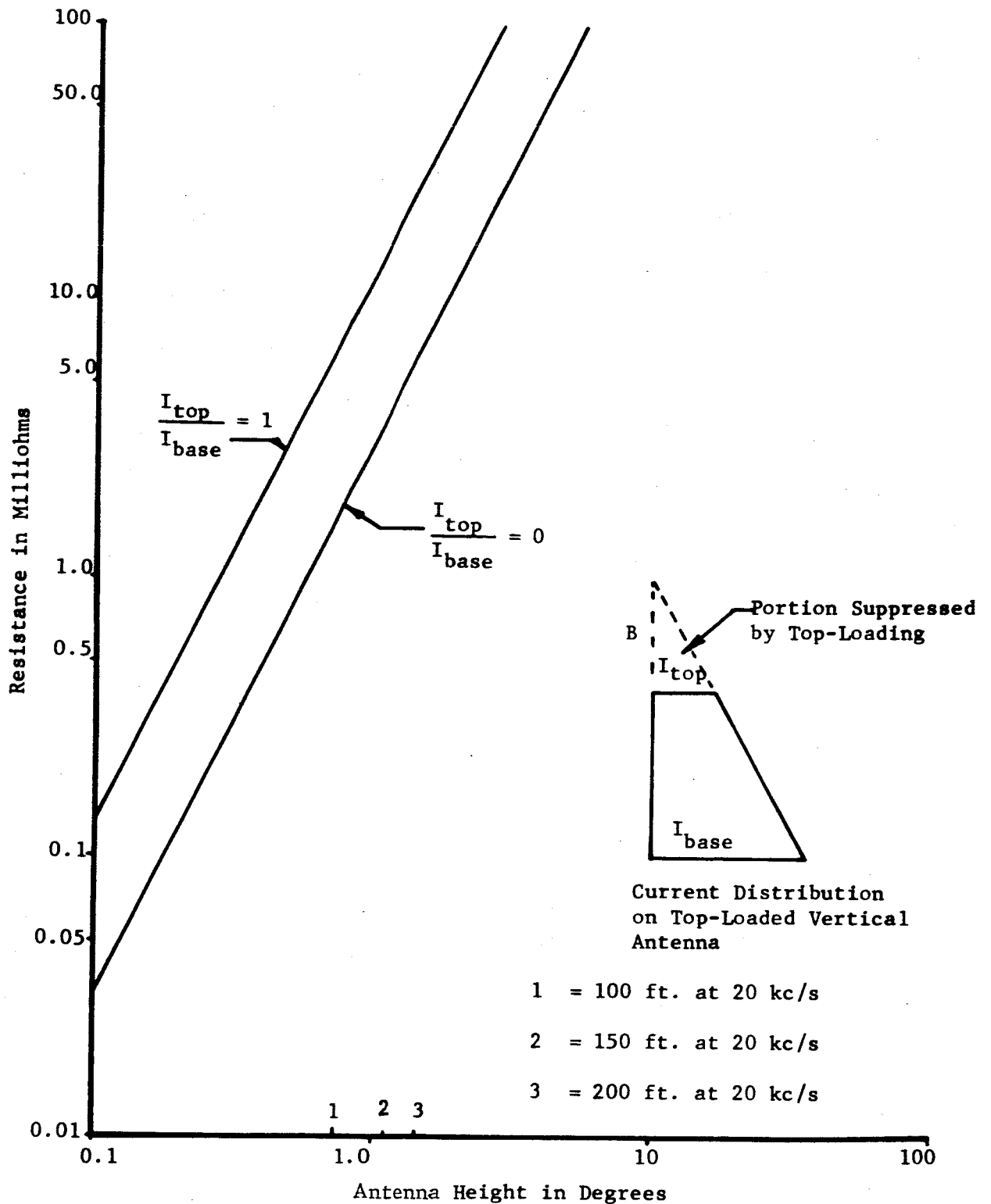


Fig. 5--Theoretical radiation resistance of a vertical antenna for assumed linear current distribution.

with the larger field strength at one mile obtained from a quarter wavelength vertical antenna indicates only an approximate five percent difference. This result is easily deduced from the field strength distribution in the vertical plane around a one-quarter wave length antenna, and a short dipole or infinitesimal antenna above a lossless ground plane⁶. The radiation pattern factor of the short dipole is

$$P_f = \sin \theta, \quad (7)$$

and for the quarter wavelength antenna it is

$$P_{f \lambda/4} = \frac{\cos (\pi/2 \cos \theta)}{\sin \theta} \quad (8)$$

where θ is measured from the vertical. The pattern of the quarter wave antenna is only slightly more directive than the pattern of the short dipole, and the magnitude is equal for $\theta = \pi/2$ (along the ground plane).

A similar, yet different, analysis⁵ has been made by others wherein the input power w is held constant for a perfectly conducting ground. On this assumption, equation (2) becomes

$$|\bar{F}_s| = 37.25 \sqrt{\frac{w}{R_T + R_L}} \quad \frac{G}{2} \text{ mv/meter} \quad (9)$$

for $G \ll 2\pi$. For $R_L = 0$, perfectly conducting earth, and a lossless antenna system,

$$|\bar{F}_s| = 5.9 \sqrt{w} \text{ mv/meter,} \quad (10)$$

which is similar to equation (6), and also indicates that the field strength is independent of antenna height. However, (10) applies for a lossless antenna system. If $R_L \neq 0$ then the field strength decreases with height since

$$\lim_{G \rightarrow 0} |\bar{F}_s| = \lim_{G \rightarrow 0} \left[37.25 \sqrt{\frac{w}{10G^2 + R_L}} \frac{G}{2} \right] \rightarrow 0 \quad (11)$$

for $G \ll 2\pi$. Equation (9) is usually plotted for various values of equivalent loss resistance in more relevant references, and hence it can create some misunderstanding if the power is not clearly specified. This type of analysis is certainly correct, and it is employed to emphasize the effect of radiator height on antenna efficiency for a fixed available power input. But, because only small radiated powers are required in the proposed system, the antenna efficiency is not the main concern, and the field strength can be considered independent of antenna height if power sources are available to maintain a constant radiated power regardless of radiation resistance.

The considerations thus far have applied only for an entirely lossless antenna system. In practice the losses encountered greatly

influence system performance because the radiation resistance is extremely small, as indicated in Figure 5. These losses occur in the tuning inductance, in the vertical radiator, in the insulators, and in the ground resistance as shown in the equivalent circuit given in Figure 6.

The tuning inductance is required in order to reduce the driving voltage for the antenna reactance which is of the form⁷

$$X_A = Z_0 \cot h \quad (12)$$

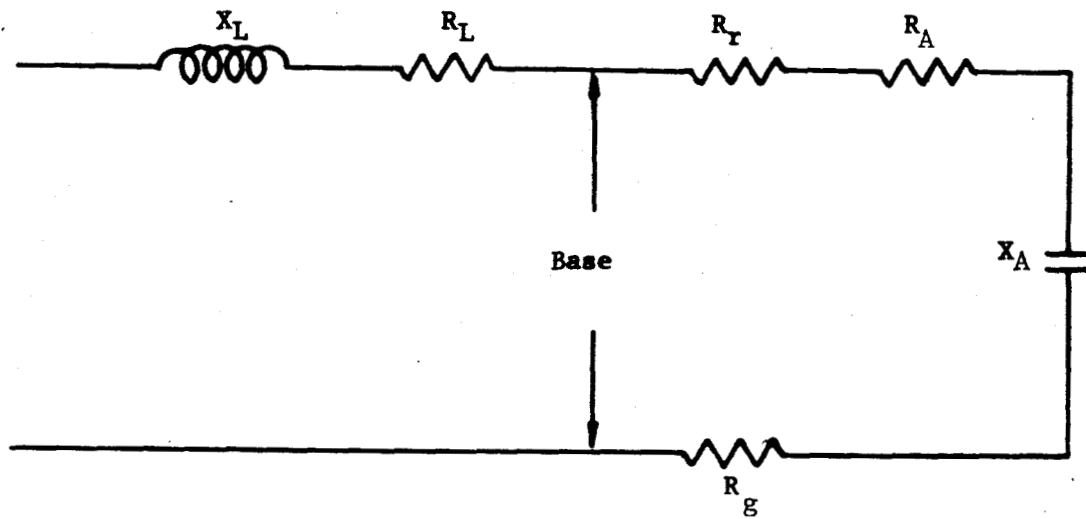
where

$$Z_0 = 60 \ln \left(\frac{h}{r_1} - 1 \right) \quad (13)$$

and

$$r_1 = \text{effective radius of the antenna.}$$

The reactance, capacitance, and required tuning inductance as a function of height for a simple vertical radiator is presented in Figure 7. The tuning inductance is excessively large for unloaded towers, which indicates the need for capacitive top loading; however, even with capacitive top loading, practical tuning inductors usually have loss resistances in the hundreds of milliohms or more. This resistance is much larger than the radiation resistance of an antenna of several electrical degrees of length. The antenna loss resistance



X_L = Inductive Tuning Reactance

R_L = Inductor Loss Resistance

R_T = Radiation Resistance

R_A = Antenna Loss Resistance

R_g = Ground Loss Resistance

X_A = Antenna Capacitive Reactance

Fig. 6--VLF antenna equivalent circuit.

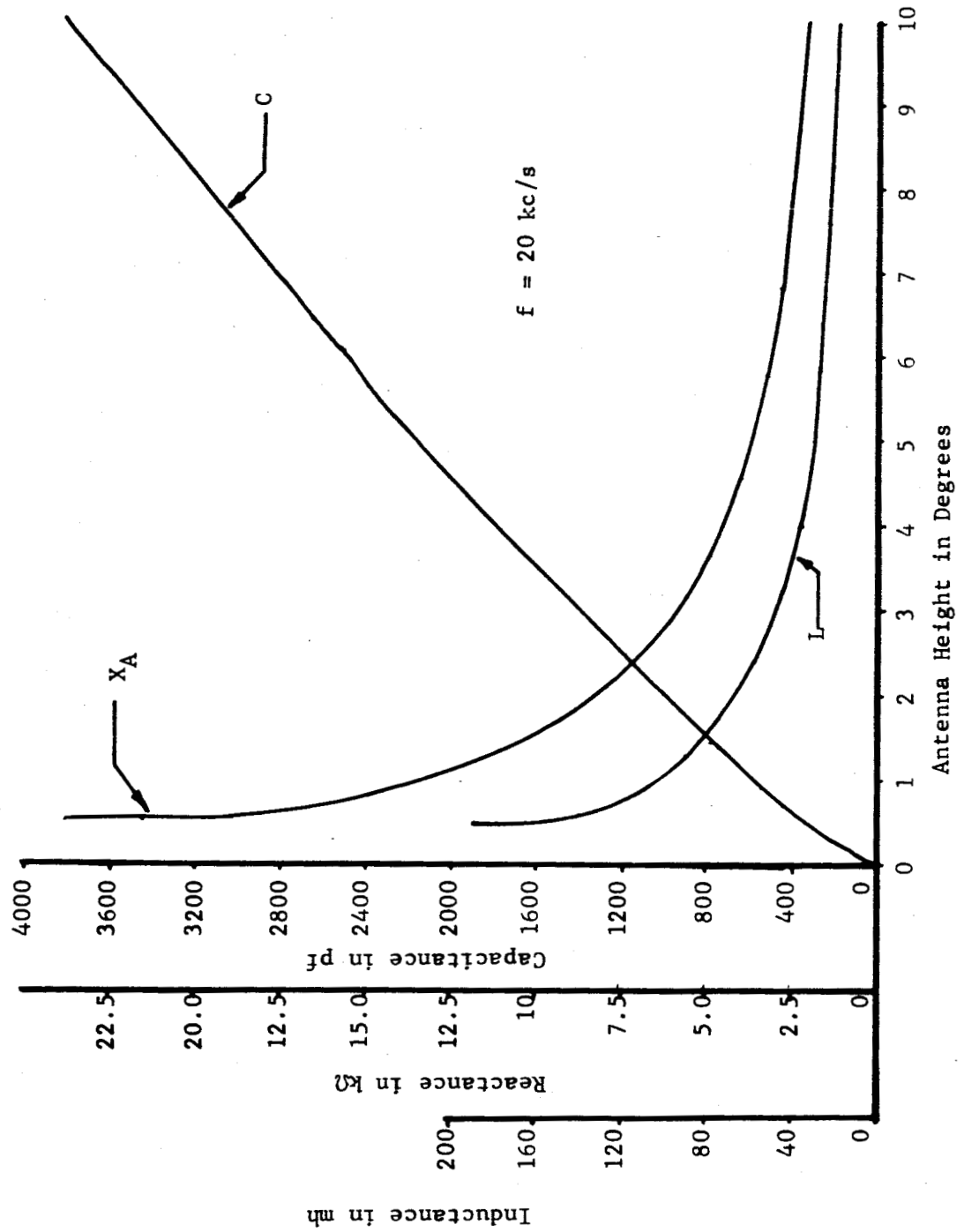


Fig. 7--Capacitance, reactance, and required tuning inductance as a function of Antenna Height.

is usually negligible if the component parts of the vertical radiator are bonded together satisfactorily, and are of sufficient surface area. The ground resistance R_g is an effective resistance referred to the antenna equivalent circuit, which is calculated from

$$R_g = \frac{\text{Power Loss in Ground System}}{(\text{Base Current})^2} = \frac{P_G}{I_o^2} \quad (14)$$

Typically, the ground resistance is not less than several hundred milliohms at these frequencies. The power dissipated in the ground is the result of conduction currents induced in the resistive ground system. Radial ground wires are usually employed to minimize this loss, which makes the total power loss in the ground the sum of the power loss in the wires (P_w), the ground adjacent to the wires (P_e), and the ground beyond the ground wires (P_x). Through the placement of a sufficient number of wires of the proper length and size, the power dissipated in the wires, and in the earth adjacent to the wires, can be reduced to an acceptable value. Most sources of information related to the selection of ground plane size recommend that approximately 120 radial ground wires at least 0.3λ long be utilized for minimum ground resistance. However, this is not at all practical for wavelengths in the order of tens of miles because of the space requirement.

Since this requirement is impractical, a study has been made to determine the effective resistance of the ground with respect to a short vertical radiator and a short radial wire ground plane. This study was based upon the assumption that

$$P_G = P_w + P_e + P_x \quad (15)$$

or

$$P_G = \int_0^l D_w dx + \int_0^l D_e dx + \int_l^\infty D_x dx \quad (16)$$

where D indicates power density in power/unit length in a radial direction from the antenna, and where l is the length of the ground wires.

Thus,

$$R_g = \frac{P_G}{I_o^2} = \frac{\int_0^l (D_w + D_e) dx}{I_o^2} + \frac{\int_l^\infty D_x dx}{I_o^2} \quad (17)$$

or

$$R_g = R_{gp} + \frac{\int_l^\infty D_x dx}{I_o^2}$$

where

$$R_e + R_w = R_{gp} = \int_0^l (D_w + D_e) dx / I_o^2. \quad (18)$$

The ground plane resistance R_{gp} can be calculated if it is assumed that the radial current distribution⁵ over a ground plane of infinite conductivity is

$$|I_X| = \frac{I_o}{\sin(G)} \left[1 + \cos^2(G) - 2\cos(G) \cos[(2\pi/\lambda)(a^2 + x^2 - x)] \right]^{1/2} \quad (19)$$

in the coordinate system of Figure 8 and 9 where

$$|I_x| = |I_e + I_w|, \quad (20)$$

$$I_e/I_w = j\gamma_e \cdot 4\pi^2 \cdot 10^{-9} f c^2 \ln\left[\frac{c}{r} - 0.15\right], \quad (21)$$

γ_e = earth conductivity,

f = frequency,

x = distance from antenna,

n = number of equally spaced radial wires,

c = $\pi x/n$,

and

r = radius of the wire in ground system.

This effective resistance R_{gp} for $a = 100$ ft. has been determined for 120 equally spaced no. 8 wires as a function of length up to twenty times the antenna height, and is less than eighty milliohms for conductivities greater than 10^{-6} mhos/cm and a frequency of 20 kc/s, as shown in Figures 10 and 11. These calculations were made on the assumption that an ideal ground shield of 20 feet in diameter is utilized to minimize dielectric losses in the earth caused by the high voltages at the base of the antenna.

If it is assumed that there is no fringing of current where equally spaced ground wires terminate, and that the change in current distribution is slight, then the power loss outside the radial ground plane is

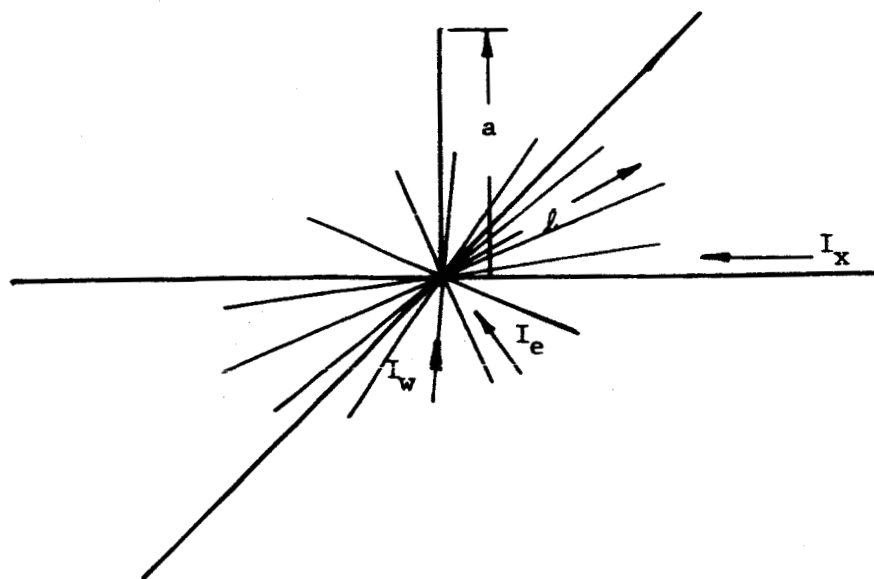


Fig. 8--Conduction current flow about a vertical radiator with radial ground wires.

S = Skin Depth
 $= \frac{1}{(\pi \gamma_e f)}^{1/2}$
 γ_e = Earth Conductivity
 f = Frequency

$$dP_x = I_x^2 dR = \frac{I_x^2 dx}{2\pi x S \gamma_e}$$

$$D_x = \frac{dP_x}{dx}$$

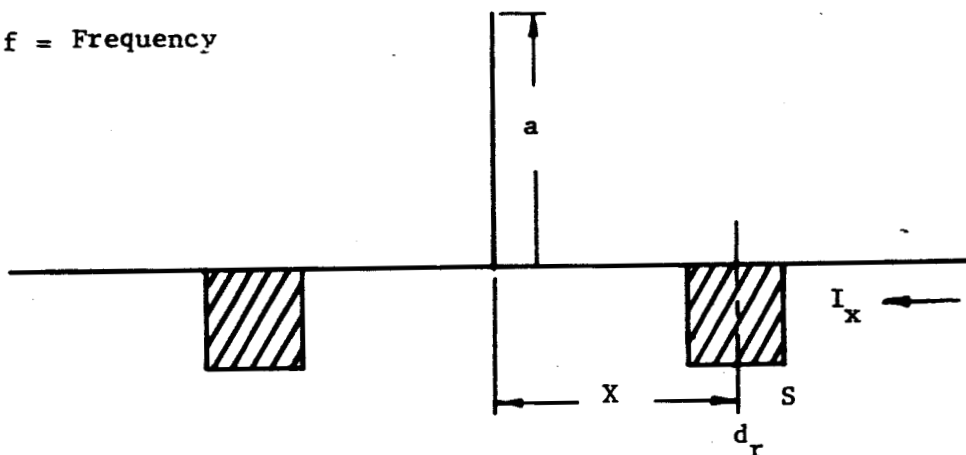


Fig. 9--Diagram of current flow in a ring about a vertical radiator.

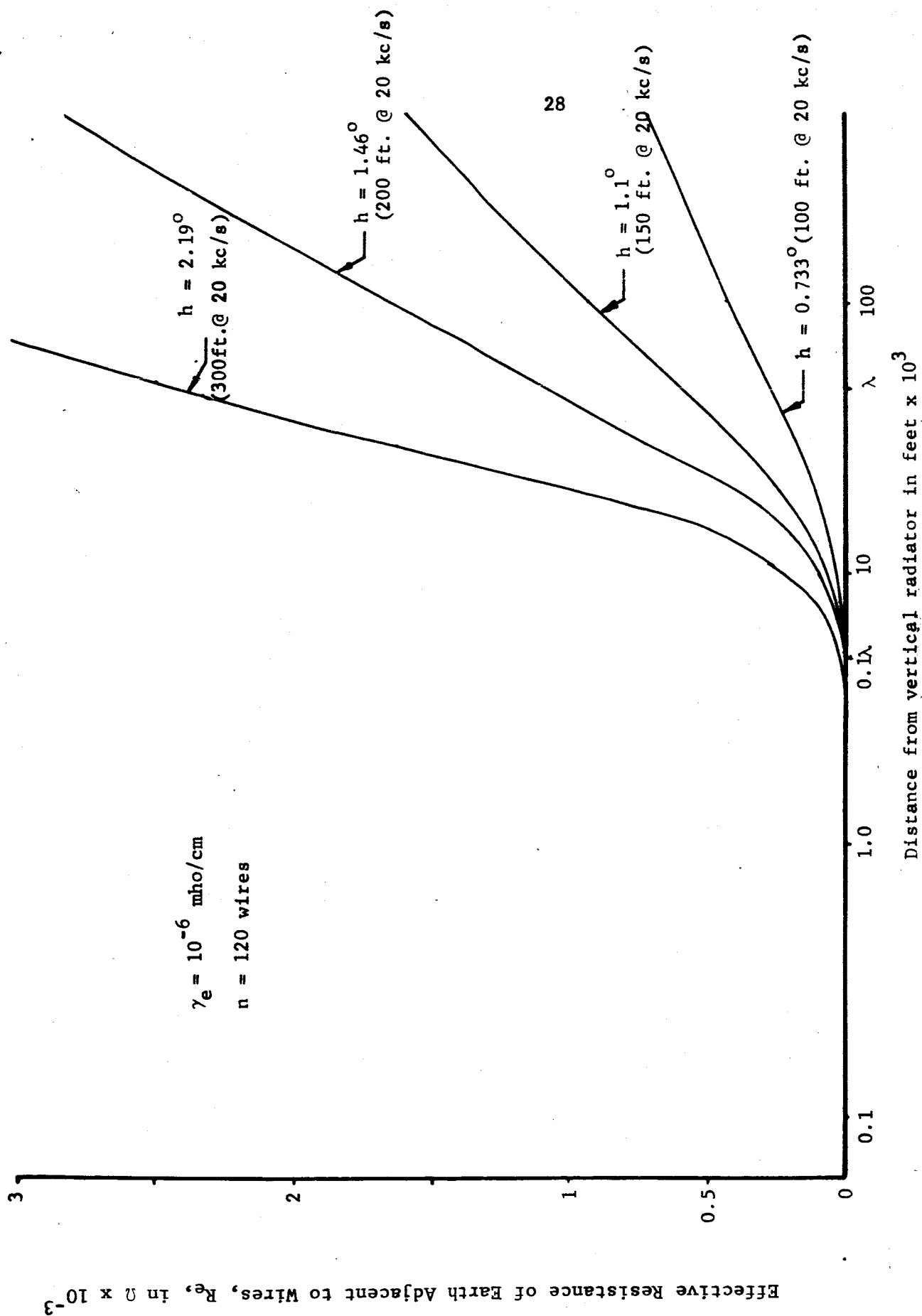


Fig. 10--Effective resistance due to power dissipation in the earth adjacent to ground wires as a function of wire length.

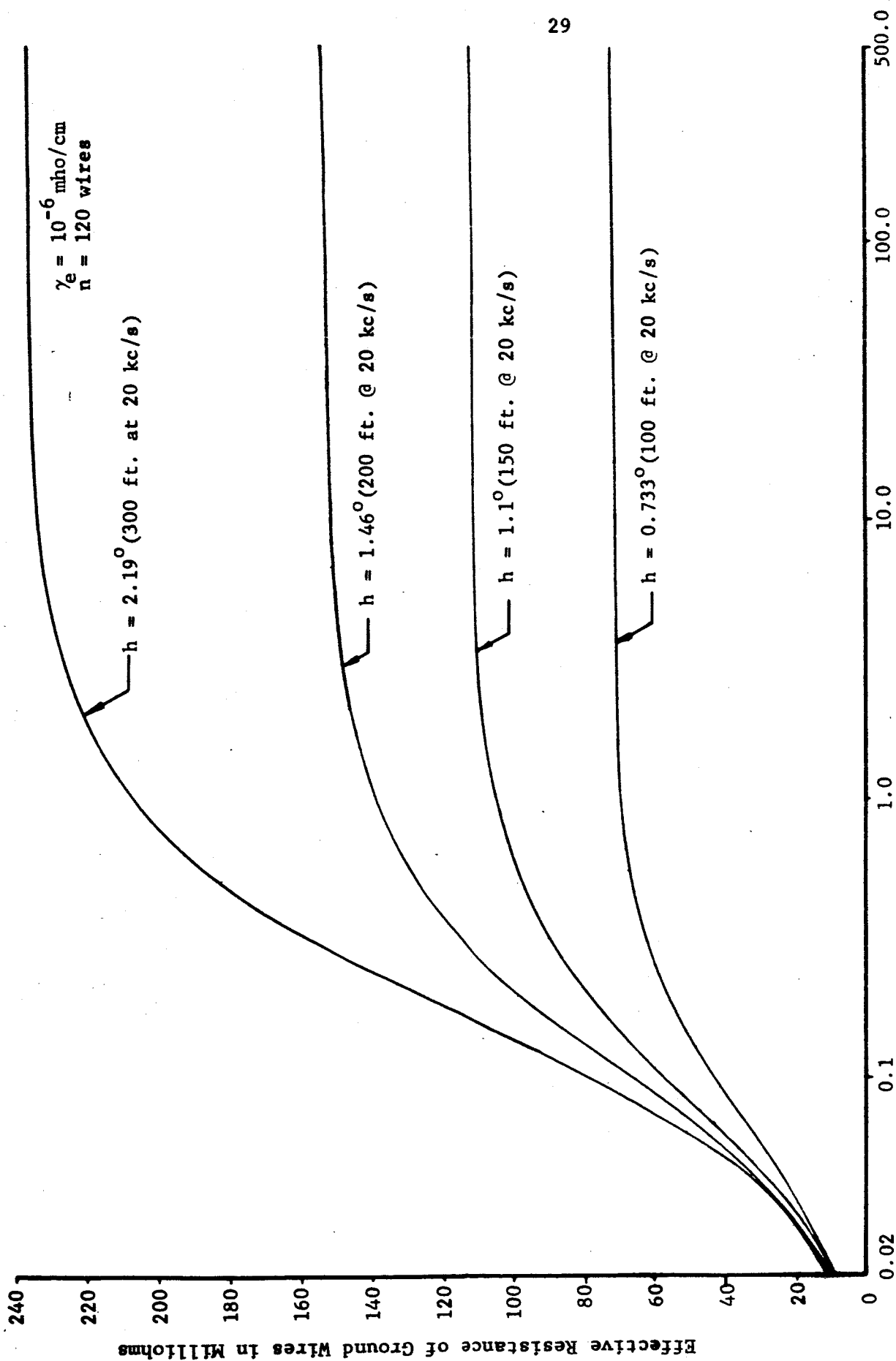


Fig. 11--Effective resistance of the ground wires as a function of wire length.

$$P_x = I_o^2 R_x = \int_l^{\infty} \frac{|I_x|^2 dx}{2\pi x s \gamma_e} \quad (22)$$

where s is skin depth as defined in Figure 9. Now,

$$R_x = \frac{1}{I_o^2} \int_l^{\infty} \frac{|I_x|^2 dx}{2\pi x s \gamma_e} \quad (23)$$

or

$$R_x = \int_l^{\infty} \frac{|I_x'|^2 dx}{2\pi x s \gamma_e} \quad (24)$$

where

$$|I_x'| = \left[1 + \cos^2(G) - 2\cos(G) \cos[(2\pi/\lambda)(x^2 + a^2 - x)] \right]^{1/2} / \sin(G) \quad (25)$$

is the current distribution factor. This integral has been evaluated for antenna heights of 0.733° , 1.10° , 1.46° , 22° , and 45° , which correspond to antenna heights of 100, 150, 200, 3069 and 6138 feet, respectively, at 20 kc/s for several conductivities. These data are presented graphically in Figures 12 and 13 as a function of distance from the antenna. It can be seen that the distance from the antenna, at which the earth beyond the ground plane contributes very little resistance, is a function of antenna height and conductivity, and is not a direct function of wavelength. The theoretical value of ground resistance for a 120-wire radial ground, of 500-feet in length, for a antenna 100 feet tall at 20 kc/s, is less than 100 milliohms for a

* If $\gamma_e \neq 10^{-6}$, multiply the Resistance by $\sqrt{\frac{10^{-6}}{\gamma}}$ where γ is the actual earth conductivity.

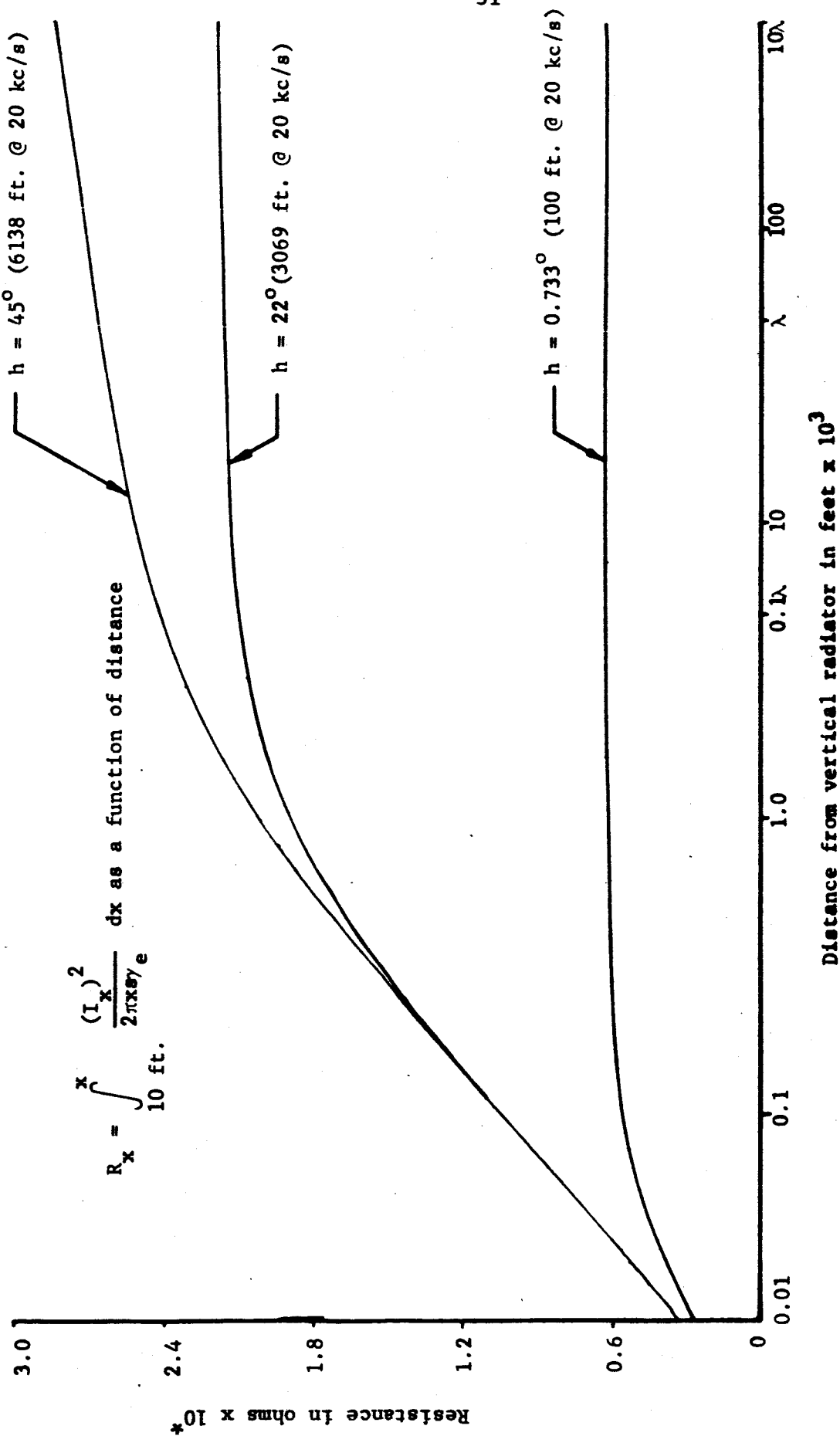


Fig. 12--Magnitude of ground resistance integral as a function of distance.

* If $\gamma_e \neq 10^{-6}$, multiply the Resistance by $\sqrt{\frac{10^{-6}}{\gamma}}$ where γ is the actual earth conductivity.

$$\gamma_e = 10^{-6} \text{ mho/cm}$$

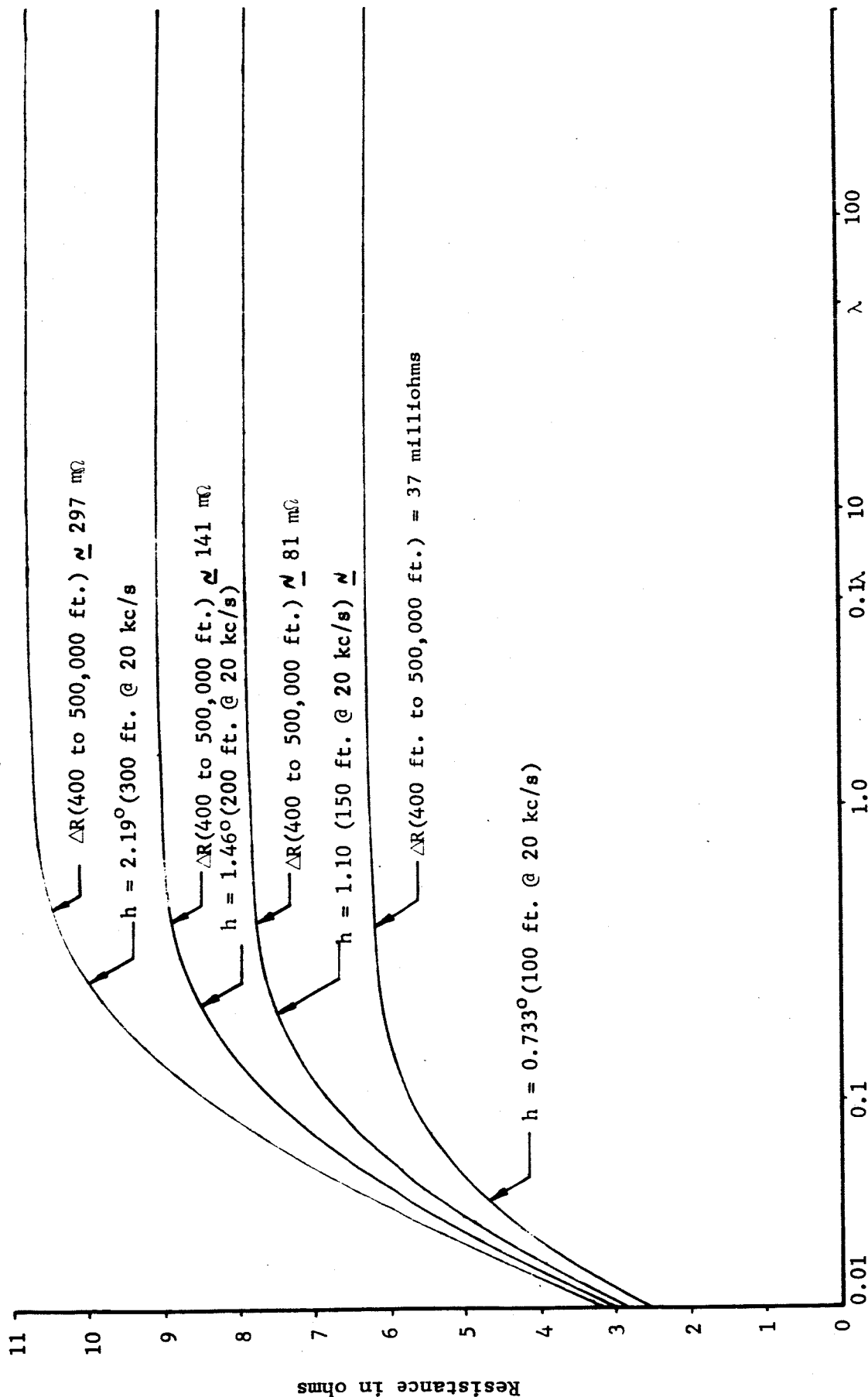


Fig. 13--Expanded magnitude of resistance integral as a function of distance.

conductivity of 1×10^{-6} mhos/cm, which is the minimum conductivity for dry ground. This small resistance is a direct result of the definition of equivalent resistance in terms of power dissipated in an incremental volume of the earth. The resistance is a function of the conductivity, the skin depth, and the current distribution factor. If the current distribution factor is unity, the resistance obtained is the resistance of a ring with the center at radius ℓ , and the outer radius approaching infinity. However, the current distribution is not necessarily constant, and the variation of the quantity, as a function of distance from the antenna, is presented in Figure 14 for various antenna heights. The graphical representation indicates that the current distribution factor is large at a short distance from a short vertical antenna, but that it decreases rapidly as the distance is increased. Because of this fact, the contribution of the resistance integral to the total resistance decreases rapidly as a function of distance and approaches a limit at distances much less than 0.3λ for short antennas. This at least gives an indication that small effective ground resistances for short antennas can be obtained through utilizing radial ground systems of lengths less than three-tenths of a wavelength. Studies of ground resistance are continuing, and a more detailed discussion will be presented in succeeding reports after experimental verification is obtained. Whatever the case, the ground resistance is much greater than the radiation resistance, but the inductor loss resistance will probably be the dominant factor.

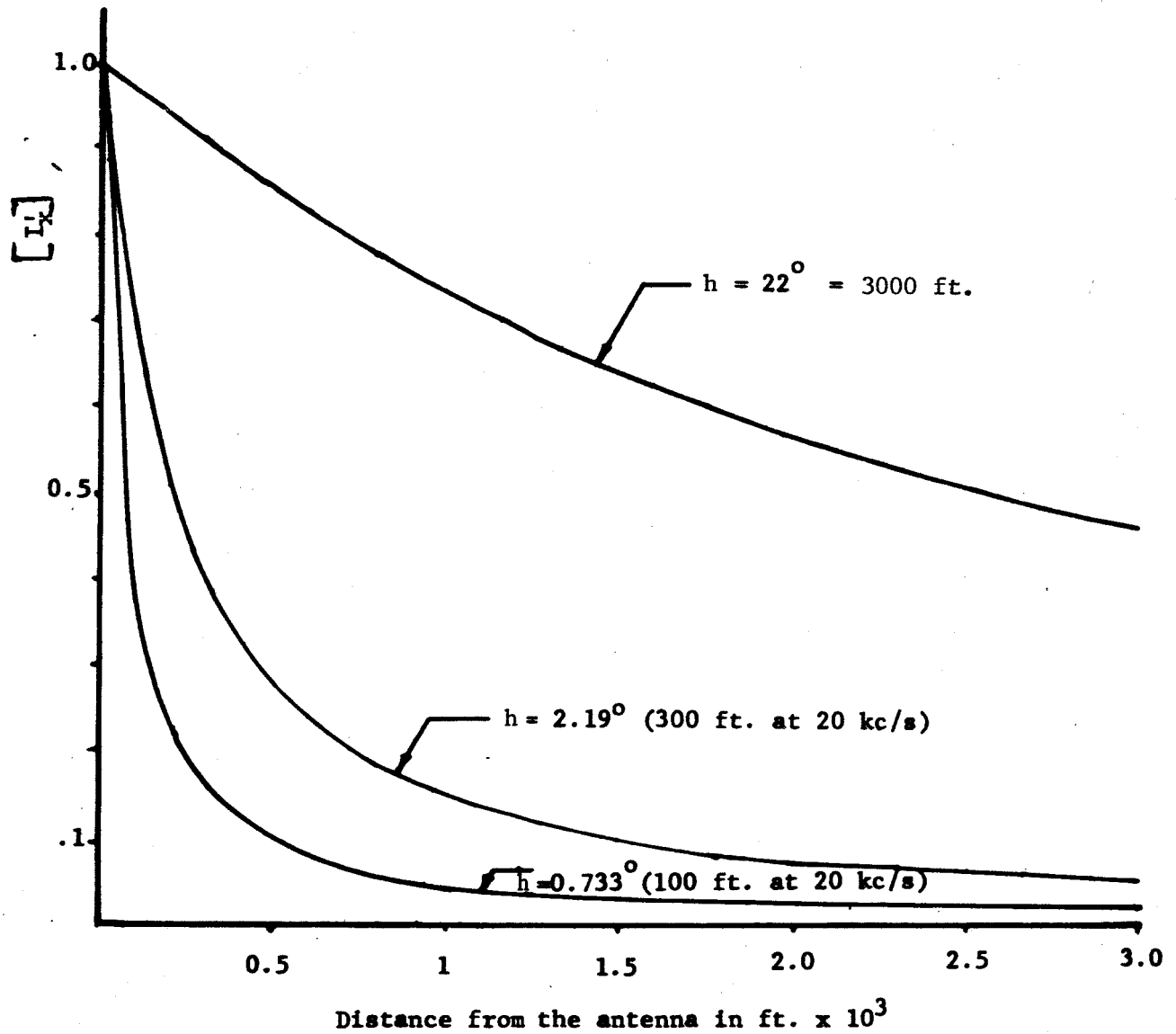
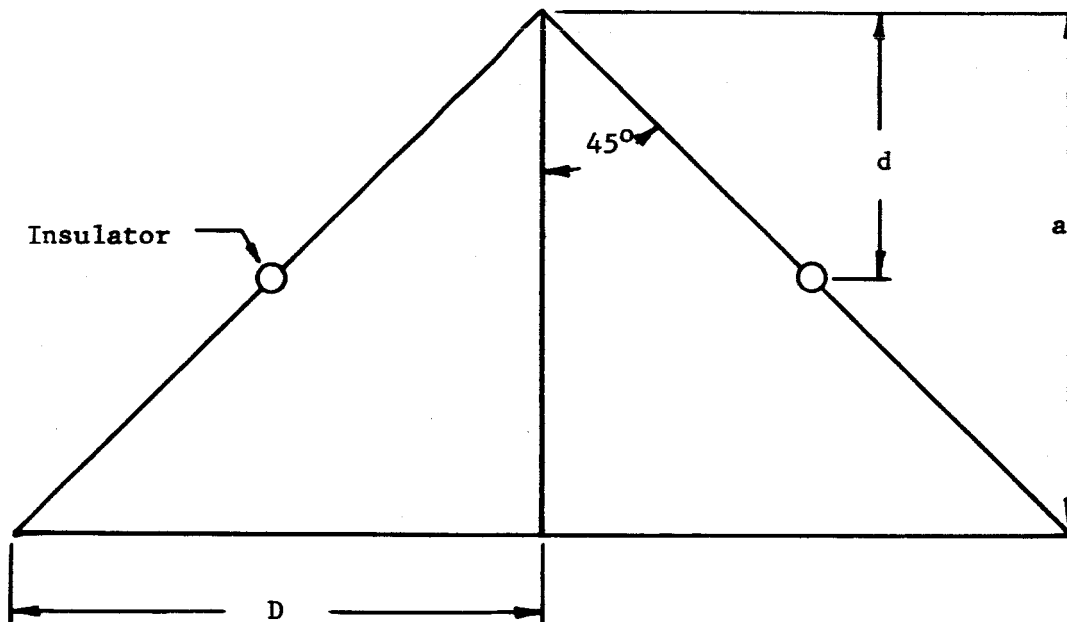


Fig. 14--Current distribution for several antenna heights.

As indicated in Figure 7, the reactance of a short vertical radiator increases rapidly as height decreases. Thus, an extremely large voltage is required to establish the antenna current necessary for a specified power. It is a common practice to employ a series inductance to resonate the antenna so that reasonable driving voltages are obtained. For the short vertical antennas the required tuning inductance is quite large, as shown in Figure 7, which makes it desirable to employ top loading. In this system, top loading, which consists of adding some form of "capacitive hat", serves three desirable purposes: it increases the effective height; it reduces the required tuning inductance; and it lowers the base voltage.

The increase in effective height resulting from top loading is caused by the change in current distribution as shown in Figure 5. The top-loading capacitance prevents the current from decreasing to zero at the top of the antenna such as occurs in the case of a capacitor terminating an open-circuited transmission line. Since the current has a value greater than zero at the top, the average current over the total antenna length is increased. The field strength and effective height of an electrically short antenna is proportional to this average value of current, and any increase in the average current distribution along the length of the antenna tends to increase the measured field parameters. Top loading can increase the effective height by, at the most, a factor of two because of the relationship between the triangular current distribution without top loading and

the limiting value of a constant current distribution along the total length of the antenna. The added capacitance may be obtained in several different ways, but the simplest method of achieving large amounts of top loading is the use of the umbrella antenna. The umbrella antenna is basically a vertical radiator with wires extending away from the top of the radiator in a manner similar to that of an extended umbrella as shown in Figure 15. Additional capacitive top loading may be obtained by connecting the ends of the umbrella rib wires together, as shown in Figure 17 (connecting wires are called antenna skirt wires). Experimental results indicate that a decided increase in antenna capacitance can be obtained by utilizing this technique in the construction of electrically small antennas. This increased capacitance is a function of the area enclosed by the skirt wire and of the height of the wire above ground, or a combination of both d and D . The skirt wire does tend to reduce the effective height of the antenna because the enclosed triangular, conic sections tend to shield the vertical radiator. This construction technique is an approach used to reduce the antenna capacitive reactance, thereby minimizing the required tuning inductance. A compromise must be made between the antenna effective height and the required tuning inductance, depending upon the relative importance of each parameter for a given antenna system. Such an antenna configuration has been examined experimentally by C. E. Smith and E. M. Johnson⁸, and theoretically by L. C. Smeby.⁹



d = Distance from top of radiator to insulator

D = Distance to extremities of umbrella support wires

a = Antenna height

Fig. 15--Diagram of umbrella rib configuration.

From their experimental results, it was concluded that the optimum length of the umbrella wires is not necessarily independent of frequency; however, the optimum radiation resistance is obtained when the value of d is approximately

$$d = \frac{3a}{7} \quad (26)$$

as defined in Figure 15 for electrically short antennas. (See also Figure 17). Experimental results for a circular wire, or skirt, attached to the ends of the umbrella wires indicate that the optimum radiation resistance occurs for a given d slightly less than the specified value. If d is increased beyond the point of optimum radiation for a fixed value of D , or, if D is increased with d fixed, then the capacitive reactance of the system is decreased. Through the varying of d , a compromise can be effected between efficiency and capacitive reactance for the reduction of the tuning inductor requirements, which is, of course, extremely important in this system. Satisfactory results have also been obtained with

$$d = \frac{5a}{7} \quad (27)$$

by others without excessive shielding of the vertical radiator. Their results indicated that radiation resistance increased with an increase in D , and that this dimension should be as large as possible structurally.

The theoretical study by Smeby⁹ showed that, through the use of the optimum length of the umbrella ribs, the vertical radiation characteristics are the same as they would be from the radiator without top loading. Smeby describes a modified skirt umbrella in which the umbrella wires are connected at the outer periphery by a skirt-wire, and in which the alternate rib wires have an insulator at the top of the antenna. This configuration was suggested as an approach to obtain larger radiation resistance. The skirt on the standard umbrella was thought to be mechanically undesirable, in practice, in view of the fact that the same resistance can be obtained with an umbrella of slightly greater radius.

An expression for the base terminal voltage can be developed from the antenna equivalent circuit of Figure 6. For a given antenna height, the base voltage is

$$V_b = I_o Z \quad (28)$$

where

$$|Z| = \left[(R_r + R_A + R_g)^2 + X_A^2 \right]^{1/2}.$$

From the theoretical discussion it can be seen that

$$R_t = R_r + R_L = R_r + R_A + R_g$$

and

$$|X_A| \gg R_t;$$

thus,

$$|V_b| \approx I_o X_A, \quad (30)$$

$$|V_b| \approx \sqrt{\frac{W}{R_t}} X_A = \sqrt{\frac{P_r}{R_t}} X_A, \quad (31)$$

or

$$|V_b| \approx \frac{0.32 \sqrt{P_r}}{G} X_A. \quad (32)$$

If the antenna capacitance is increased by means of top loading for a given antenna height, the base voltage will decrease. The base voltage is of primary interest since it determines the quality of the base insulator required for a specified radiated power. The base voltage varies inversely with antenna height as described by (32), and it is presented in Figure 16 for a radiated power of 100 mw. The base voltages for several umbrella antennas discussed in a later section of this report are also recorded in this figure, and are based upon experimental scale model capacitance measurements.

The voltage distribution over the antenna would tend to follow a cosine law of distribution, with the maximum voltage occurring at a point on the antenna most distant from the base terminals. The voltage distribution is a function of the terminal voltage, of the antenna self-inductance, and of the capacitive fringing effects. The voltage at the most distant point on the antenna¹⁰ is usually approximated as

$$V_f = 1.05 I_o X_A \left[1/\cos 90^\circ(f/f_o) \right] \quad (33)$$

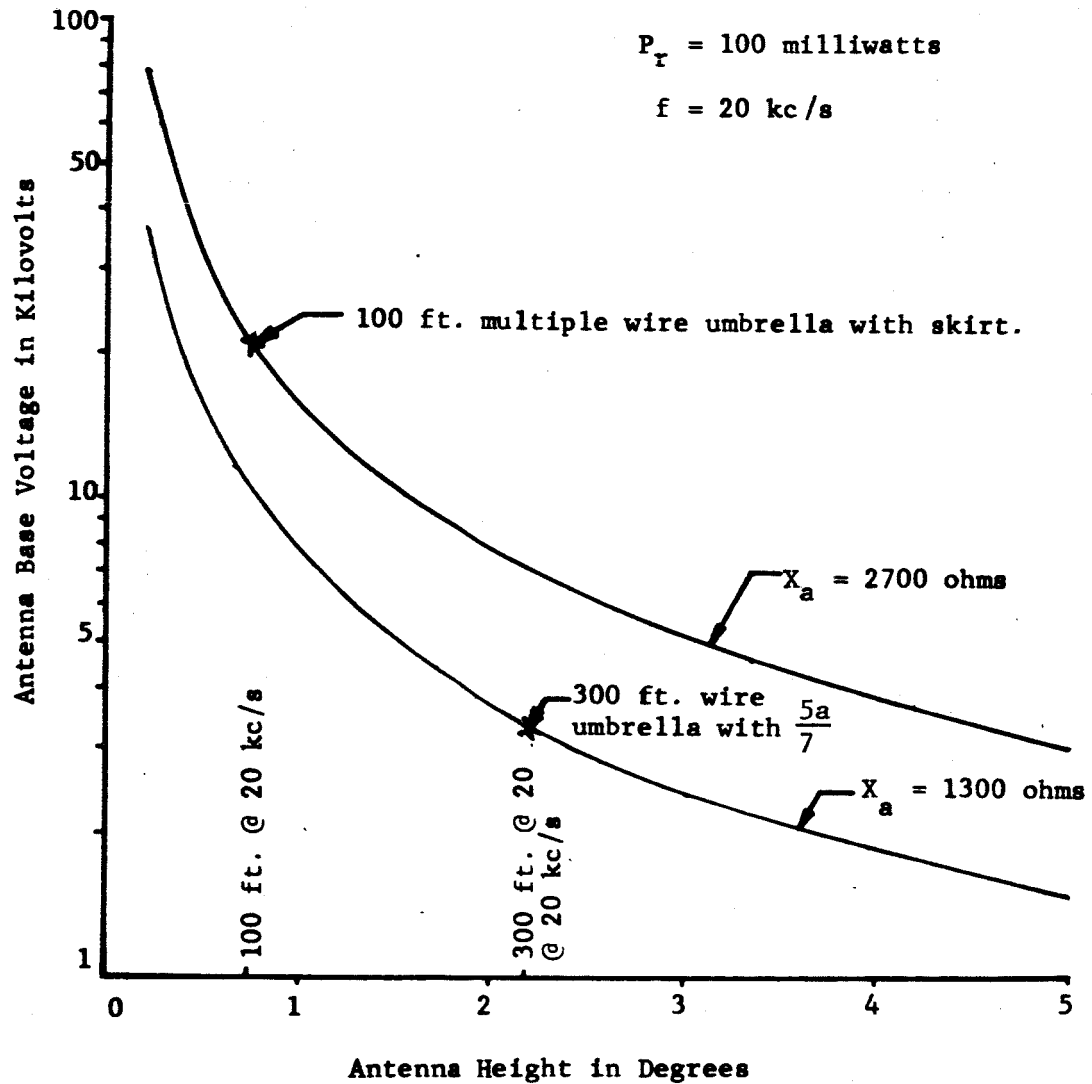


Fig. 16--Maximum antenna base voltage (RMS) for different constant capacitive reactance loads as a function of antenna height.

where f is the operating frequency and f_o is the frequency at which the antenna is electrically $\lambda/4$ in length. In this case

$$f \ll f_o, \quad (34)$$

and hence

$$V_f \approx 1.05 |V_b|. \quad (35)$$

This result should be expected since the entire antenna structure is only a fraction of a wavelength. The voltage at any point on the structure should be approximately equal to the base voltage, where edge effect may cause a variation of up to 10% of this value. The basic efficiency of antennas such as these is

$$\eta = \frac{\text{Power Radiated}}{\text{Power Input}} \times 100\%, \quad (36)$$

which can be expressed in terms of the equivalent circuit parameters of Figure 6,

$$\eta = \frac{R_r}{R_L + R_r + R_A + R_g} \times 100\%, \quad (37)$$

where the inductance X_L with loss resistance R_L is included as a part of the antenna system. This inductance may be replaced in practice by a Pi, T, or an L matching section; however, for the

present purposes, a series inductance with a loss resistance is satisfactory. The radiation resistance (see Figure 5), with top loading will have range values of:

$$\begin{aligned} 1.6 \text{ m}\Omega < R_r < 6.8 \text{ m}\Omega & \text{ for a 100 ft. antenna;} \\ 4 \text{ m}\Omega < R_r < 15 \text{ m}\Omega & \text{ for a 150 ft. antenna;} \\ \text{and, } 7 \text{ m}\Omega < R_r < 27 \text{ m}\Omega & \text{ for a 200 ft. antenna.} \end{aligned} \quad (38)$$

The maximum top loading becomes more difficult to obtain as height increases for umbrella antennas because of the shielding of the vertical radiator; and, furthermore, maximum top loading cannot be obtained with simple structures. A feasible estimate of the system performance can be based upon the radiation resistance of a vertical radiator not having top-loading capacitance.

A recent investigation of electrically short umbrella top-loaded antennas¹¹ has indicated that the static effective height of such structures can be determined by approximating the integral equation describing the antenna charge distribution. This is accomplished by dividing the integral (or antenna configuration) into a finite number of segments over which the charge density is assumed constant, and wherein the integral is represented by a set of simultaneous linear equations. The results of this computation of the integral are presented in the form of nomograms relating the capacitance, the effective height, and the quality factor Q of the umbrella antenna as

a function of height, d , D , and the number of umbrella ribs where

$$r_1/a = 10^{-3},$$

$$r_2/a = 10^{-5},$$

r_1 is the effective radius of the antenna tower,

and r_2 is the effective radius of the umbrella rib wires.

From these nomograms, the effective height h_e of an umbrella antenna wherein

$$N = 12$$

$$d = 5a/7$$

$$D = a$$

was determined to be

$$h_e \approx 0.5a,$$

which is exactly the effective height of an unloaded vertical antenna.

This decrease in the anticipated effective height results from the fact that the rib length exceeds the value for optimum radiation.

This particular choice of an antenna configuration was investigated because maximum static capacitance is desired. The configuration will be discussed in the following section.

Calculations have been made to determine the loss resistance of the required tuning inductance for an antenna with approximately 5000 picofarads equivalent capacitance at 20 kc/s. The calculated inductance for this requirement has the following characteristics:

Inductance	≈ 12.0 millihenries
Number of Turns	96 turns
Length	≈ 6.0 feet
Diameter	6 feet
Wire Diameter	0.5 inches
A.C. Resistance	≈ 6.0 ohms

These characteristics provide a basis for estimating the order of expected magnitudes of the loss resistance and the coil size for an anticipated quality factor of 250 at these frequencies. The minimum radiation efficiency for a 300-foot vertical radiator, which used top-loading to obtain the specified capacitance, is

$$\eta \approx \frac{14 \times 10^{-3}}{6.0 + 0.5 + 14 \times 10^{-3}} \times 100\% = 0.21\%, \quad (39)$$

with the estimated component values for a 500 ft. ground system where $R_G = 0.5$. Equation (39) indicates that the required input power will be in the order of 50 watts (for the specified 300 foot antenna radiating one hundred milliwatts of power), which is certainly reasonable even if the efficiency is poor. Equation (39) further indicates that the efficiency will depend, for the most part, on the inductor loss resistance, which emphasizes the need for decreasing the capacitive reactance of the antenna.

Experimental Antenna Capacitance Measurements

In the design of antennas, considerable information related to the impedance characteristics and the radiation pattern of a given structural configuration can often be obtained from a scaled model mounted on a larger ground plane.¹² Scale factors as large as one-hundred to one (100:1) can be utilized to obtain useful information if the structural design is carefully scrutinized. Intricate details, such as lattice structures, wire configuration, etc., can be idealized through the use of some simple geometric forms wherein the measurements are corrected accordingly.

Three 100:1 scale model VLF umbrella antennas of 100, 150 and 200 feet have been constructed for the determination of the static capacitance of the structures. The value of antenna capacitance is extremely important since it determines the required tuning inductance, and the total base voltage, for a given transmitted power. The 100-foot antenna consists of an ungrounded, triangular, vertical radiator, which is two feet on each side, and an umbrella, consisting of eight ribs with skirt. Each rib is in a cage form of four wires on a four-foot square, as shown in Figures 17 and 18. The 150- and 200-foot antennas consist of ungrounded triangular, vertical radiators with eight single-wire ribs with skirt at a 45° angle from the vertical, as shown in Figure 19. Static capacitance measurements were made at a frequency of one thousand cycles per second, using a simple audio frequency bridge with the antenna mounted on an effective

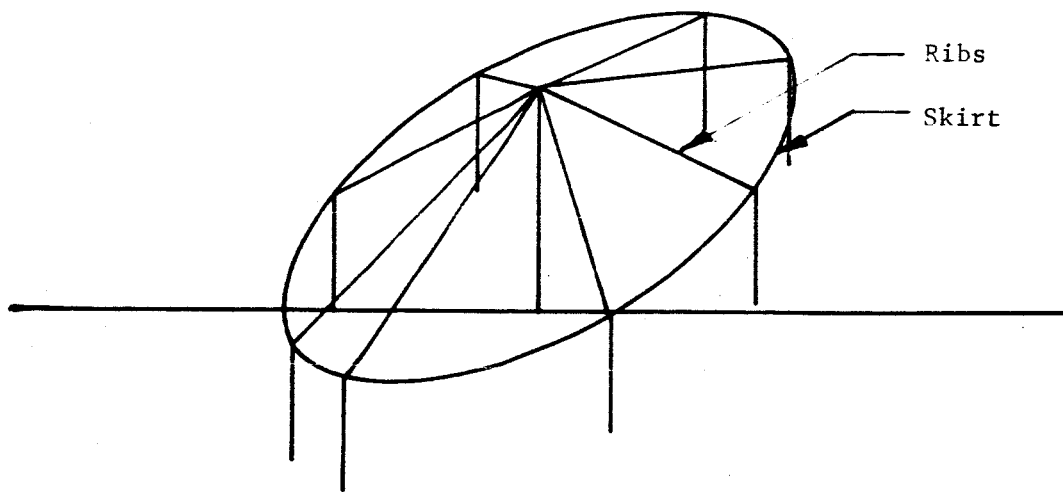


Fig. 17--VLF umbrella antenna design.

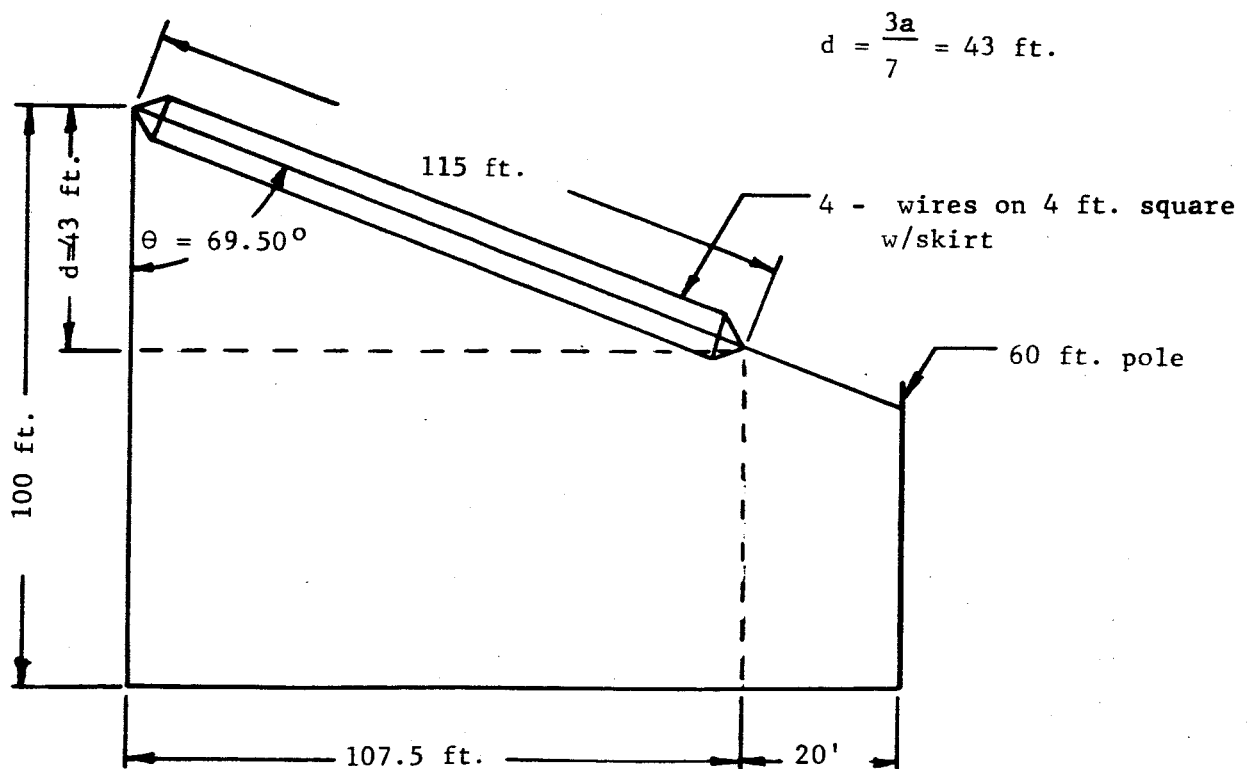
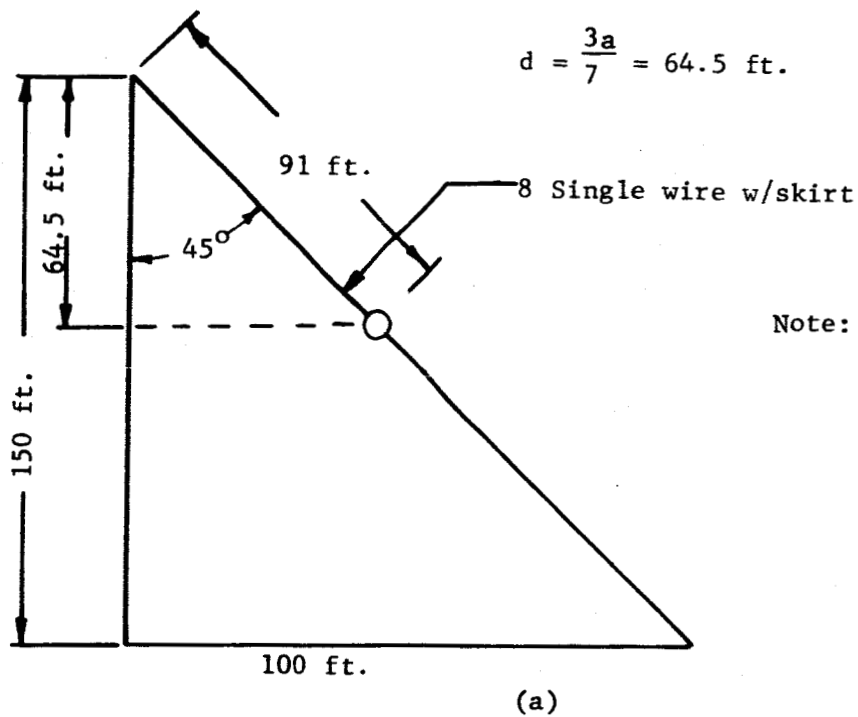


Fig. 18--Rib construction for 100 ft. VLF umbrella antenna.



Note: Multiply
dimensions by
 $\frac{20}{15}$ for 200 ft.
antenna.

Fig. 19a--Rib construction for 150 and 200 ft. VLF umbrella antennas with $d = 3a/7$.

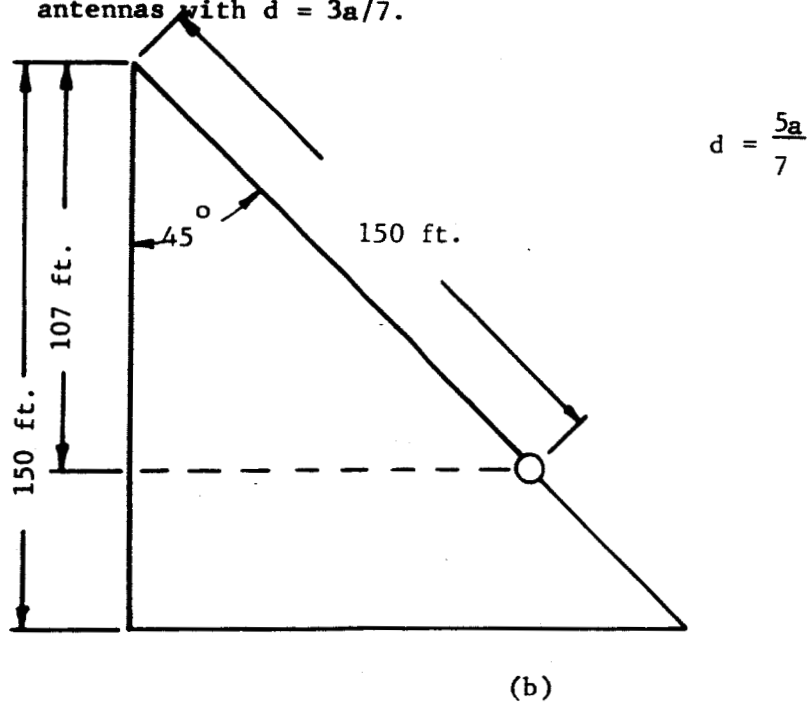


Fig. 19b--Construction for 150 and 200 ft. VLF umbrella antennas with $d = 5a/7$.

2000-foot-square ground plane. The corrected full scale measured capacitance values are recorded in Table 1 for several structural variations of the basic design. In addition, capacitance measurements were made at the proper scaled frequency for these models in an effort to determine the actual operating capacitance. The results are also presented in Table 1. Since the scale factor was extremely large, it was impossible to scale all the physical parameters of the structure so that more information could be obtained (loss and radiation resistances, for example). Although there was some difficulty in modeling the structures, the wire sizes, the tower dimensions and the lattice construction were approximated as closely as possible.

The static capacitance of top-loaded umbrella antennas can be determined using the nomograms developed by Gangi, Sensiper and Dunn¹¹ described in "The Vertical Radiator Characteristics" section of this report. The capacitance of a vertical radiator where

$$a = 300 \text{ feet,}$$

$$d = 5a/7,$$

$$D = a,$$

$$\text{and } N = 12$$

is determined to be

$$C_A \approx 4860 \text{ pf,}$$

for the nomograms given in the referenced report. The value is well within the specified ten percent accuracy when compared to the measured

TABLE 1

CAPACITANCE MEASUREMENTS OF SCALE MODEL
VERTICAL RADIATORS WITH VARIOUS TOP-LOADING CONFIGURATIONS¹

NO.	HEIGHT In Ft.	UMBRELLA					ACTUAL CAPACITANCE		
		Wire Size In Inches	Skirt	No. of Ribs	No. of Wires In Rib	d	D	Static f = 10 c/s	Operating f = 20 kc/s
1	100	0.5	No	0	0	43 ft.	267 ft.	470 pf.	420 pf.
2	100	0.5	No	8	1	43	267	1770	1720
3	100	0.5	Yes	8	1	43	267	2480	2320
4	100	0.5	Modified	8	1	43	267	2440	2320
5	100	0.5	No	8	4	43	267	2490	2320
6	100	0.5	Yes	8	4	43	267	2990	2820
7	150	0.5	No	0	0	64.5	150	690	520
8	150	0.5	Yes	4	1	64.5	150	1050	920
9	150	0.5	No	8	1	64.5	150	1400	1320
10	150	0.5	Yes	8	1	64.5	150	1580	1570
11	150	0.5	Modified	8	1	64.5	150	1580	1570

¹Scale 100:1; Vertical Radiator Scaled for a Triangular Shape 2 feet on each side.

TABLE 1 (Continued)

CAPACITANCE MEASUREMENTS OF SCALE MODEL
VERTICAL RADIATORS WITH VARIOUS TOP-LOADING CONFIGURATIONS¹

NO.	HEIGHT In Ft.	UMBRELLA						ACTUAL CAPACITANCE	
		Wire Size In Inches	Skirt	No. of Ribs	No. of Wires In Rib	d	D	Static f = 10 c/s	Operating f = 20 kc/s
12	150	1.0	No	8	1	107 Ft.	150 Ft.	2100 pf.	NA ²
13	150	1.0	Yes	8	1	107	150	2880	NA
14	150	1.0	No	12	1	64.5	150	1620	NA
15	150	1.0	Yes	12	1	64.5	150	1820	NA
16	150	1.0	No	12	1	107	150	2540	NA
17	150	1.0	Yes	12	1	107	150	3120	NA

¹Scale 100:1; Vertical Radiator Scaled for a Triangular Shape 2 feet on each side.²Data not measured

scale model capacitance (Table 1, No. 16),

$$C_A \approx 2(2540) = 5080 \text{ pf,}$$

for such a structure. The comparison of the model results with the calculated values of full-scale capacitance indicates a high degree of credibility in this design parameter.

The umbrella antenna exhibits, in most cases, a significant improvement in antenna performance over the simple vertical radiator. However, the umbrella has the distinct disadvantage of increasing the mechanical load on the supporting tower. Taller vertical radiators, such as the 150 and 200 foot antennas, are more difficult to load capacitively, with minimum shielding of the radiator, unless additional tall, rib-supporting structures are utilized. However, a strong 100 foot tower can be heavily top loaded, through the use of shorter inexpensive supporting structures, without shielding the vertical radiator. At these frequencies the maximum capacitive loading is required in order to reduce the base voltage and the tuning inductance required for reasonable transmitter output voltages.

Remarks on Antenna Design

Unless electrically large structures are employed, the design of the transmitting antenna to be utilized in this system is electrically very small because of the long wavelengths involved. The design of electrically small structures presents a class of unique problems.

The large antenna capacitive reactance, for antennas of less than two degrees of electrical length, is by far the most important consideration in the design of electrically small antennas. In order to obtain even small radiated powers, the antenna base voltages reach enormous proportions, and large tuning inductances are required in order to reduce the driving voltage of the untuned antenna. Since the loss resistance increases as the inductance increases, it is mandatory that the required tuning inductance be reduced if reasonable antenna efficiencies are to be obtained.

It has been shown that the radiation characteristics, for a specified radiated power, are independent of antenna height. Employing this philosophy, several antenna configurations were constructed, and the reactance characteristics measured. Two basic designs were used. In the first case, d (the length of the ribs) and in the other case, D (the angle from the vertical for a fixed rib length) were each varied in order to obtain the antenna reactance characteristics. A 100-foot umbrella antenna, using multiple wire ribs with $D > a$, has a larger capacitance (2990 pf) than any of the 150 ft. antenna structures investigated. This large static capacitance can be obtained by means of the proposed utilization of inexpensive supporting structures (creosote poles) for the ribs, and D would thereby be made as large as structurally possible. At larger heights a structure using physical supports for the umbrella ribs would be even more expensive than a simple top hat supported by four large towers. By extending the tower

heights to 200 or 300 feet, and by using the maximum length for 12 ribs without skirts ($d = 5a/7$), capacitances of 3390 and 5080 pf., respectively, can be obtained, (See Table 1, and use a linear scale change). The 200-foot antenna, loaded in the specified manner, yields only slightly more capacitance than the heavily loaded 100-foot structure. The 300 foot antenna produces an almost 2:1 increase in capacitance over the 100 foot antenna, which results in a large improvement in efficiency because the loss resistance decreases with required tuning inductance (in the order of 12 mh), and because the radiation resistance increases.

For electrically small umbrella antennas, with a radial ground wire system at 20 kc/s, it was observed that the loss resistance of the tuning inductance is the dominant factor in determining antenna efficiency. But, this is true only if the ground plane is of sufficient length to reduce losses resulting from induced conduction currents. The results shown in Figures 10, 11, 12, and 13 have been combined to obtain the effective ground resistance R_g , as a function of ground wire length as presented in Figure 20. It should be noted that for ground wires longer than 3 to 4 times the antenna height do not reduce the effective resistance to any great degree. In addition, it should be noted that effective ground resistance decreases with decreases in antenna height for a fixed radial ground wire length, and the decrease is a direct result of the current distribution. However, the effective ground resistance is still large when compared to the radiation resistance.

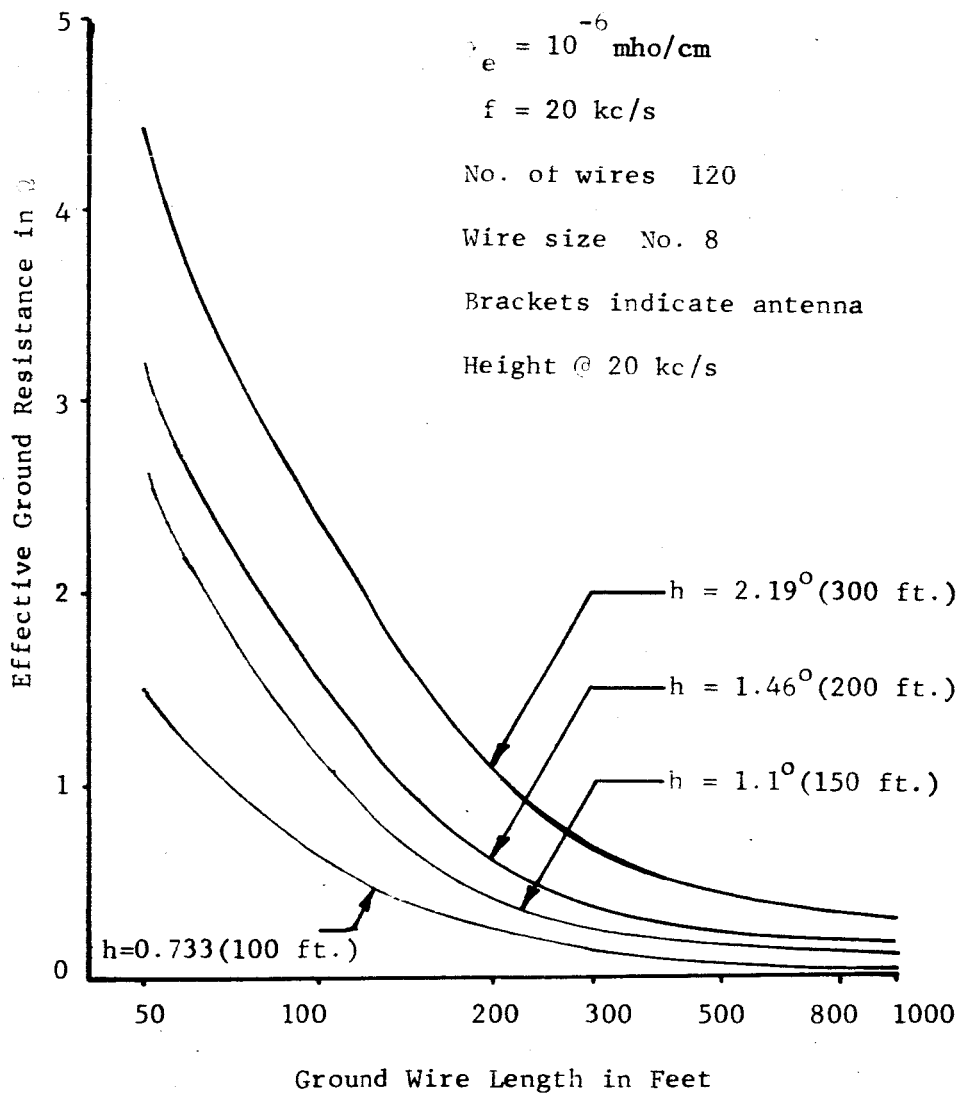


Fig. 20--Effective ground resistance R_g as a function of radial ground wire length.

In order to obtain maximum efficiency, and also a reduced tuning inductance requirement, a 300 ft. umbrella antenna with $d = 5a/7$, and a ground system of 120 radial wires of approximately 1000 feet, should be utilized. However, the radial ground wire length can be decreased to a length of 500 feet without seriously affecting the efficiency because the effective ground resistance is still much smaller than the expected values of inductor loss resistance.

Effect of Finite Ground Conductivity on Antenna Radiation Characteristics⁷

In the foregoing discussion of the radiation characteristics of a short dipole, the earth was considered a perfect conductor. While this assumption may be satisfactory in some cases, the effect of a finite conducting earth upon the radiation characteristics of a short dipole in the VLF frequency range merits further consideration. The major effect of the finite conductivity of the earth upon the radiation pattern of the space wave is that it reduces the magnitude of the field, at low angles, from its value over a perfectly conducting earth, as shown in Figure 21. The functional parameter n is

$$n = \frac{18 \times 10^5 \gamma_e}{f_{mc} \epsilon_r}, \quad (40)$$

where

ϵ_r is the relative dielectric constant,
 f_{mc} is frequency in megacycles, and
 γ_e is the earth conductivity in mhos per cm.

The reduction in field magnitude is caused by the rapid changes of the ground reflection coefficient for angles near the pseudo-Brewster angle, which is a function of earth conductivity. At angles of incidence below the pseudo-Brewster angle, the phase of the reflection coefficients is approximately zero, and at angles above it is 180 degrees. Thus, the reflected wave tends to cancel the propagated wave below the pseudo-Brewster angle, and tends to reinforce the propagated wave above the angle. If an average value is assumed for ϵ_r , and if a frequency of 20 kc/s is considered, with conductivities ranging from 1×10^{-6} mho/cm (poor) to 2×10^{-5} mho/cm (average), then

$$6 \leq n \leq 120, \quad (41)$$

where $\epsilon_r = 15$. The dielectric constant is reasonably stable for different conductivities, and these values of n will not vary appreciably as a function of the dielectric constant. It is apparent that the effect of the finite conductivity is to reduce the signal strength at low angles for these values of n , as indicated in Figure 21. The reduction of field strength at low angles indicates that the selection of the transmitter location is extremely important for long range space wave communications, which require radiation at low elevation angles. Whenever possible, a site having high conductivity in the first few wavelengths from the antenna should be selected in order to minimize this effect.

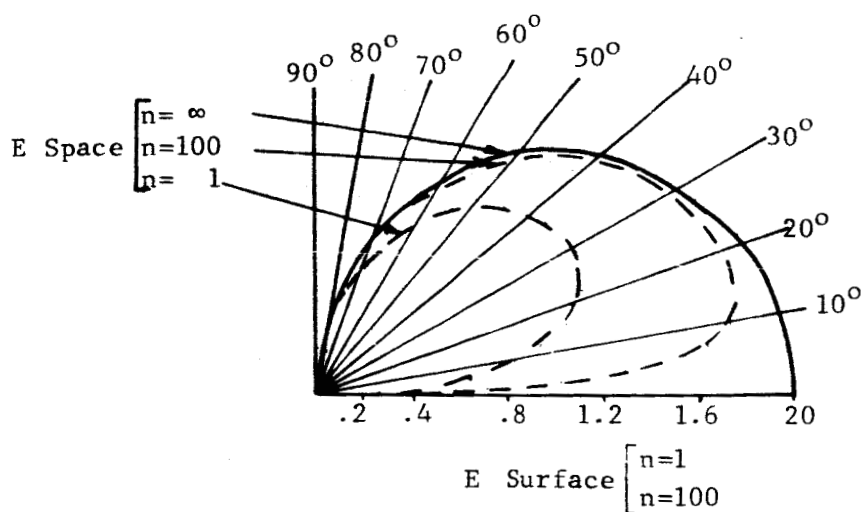


Fig. 21--Vertical radiation pattern for the space wave of a short dipole located on the surface of a flat earth with $\epsilon_r \approx 15$.

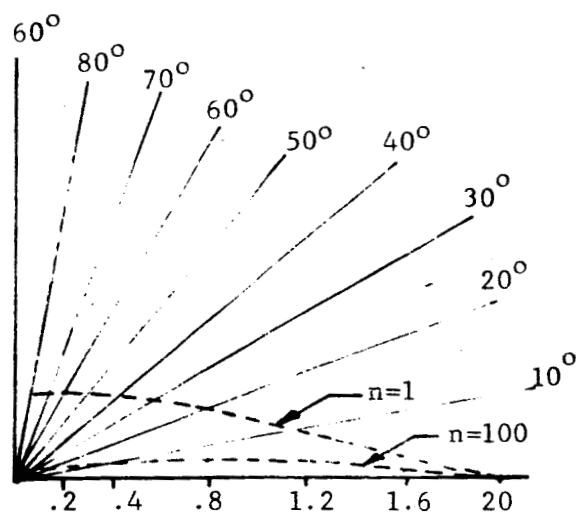


Fig. 22--Vertical radiation pattern for the surface wave of a short dipole located on the surface of a flat earth with $\epsilon_r \approx 15$.

In this particular system, the communications path is approximately 320 km, which is relatively short. The angle of incidence measured from the horizontal is of the order of 25 degrees, if a mean height of 75 km is assumed for the ionosphere. Since the lowest conductivity for any portion of the proposed propagation path is 3×10^{-5} mhos/cm, $(n = 180)^*$, the angle of elevation of the vertical radiation space wave pattern will be less than 25 degrees. This fact indicates that the finite conductivity of the earth will alter the radiated space wave magnitude only a slightly in the proposed propagation path.

The surface wave radiated from the vertical radiator is also dependent on the finite conductivity of the earth. For large values of n (low frequencies and good conductivity), the unattenuated surface wave is very small, except near grazing angles. For smaller values of n , the unattenuated surface wave has an appreciable value at high angles, as shown in Figure 22. However, the attenuation of this wave is also a function of the earth conductivity.

Receiving Antenna Requirements

Propagation of VLF Energy over Small Distances If only the vertical component of the electric field is considered, a received signal at a distance L measured along the earth's surface from the transmitter will consist of contributions both from the space wave and from the surface wave. In the simplified case of a signal consisting of a surface wave field component, and a once-reflected sky wave field component, the signal will be proportional to the total field strength, $\bar{E}_t$, where

*(FCC map)

$$\begin{aligned}\bar{E}_t &= \bar{E}_g + \bar{E}_s, \\ \bar{E}_g &= \text{Amplitude of the surface wave,} \\ \text{and } \bar{E}_s &= \text{Amplitude of the space wave.}\end{aligned}\tag{42}$$

The phase of $\bar{E}_t$ continually changes because there are periodic and non-periodic variations generated in the space wave $\bar{E}_s$, relative to the extremely constant phase of the surface phase of the surface wave. These variations are a result of ionospheric phenomena. The magnitude of the total field strength is a function of the reflection height, the reflection coefficient, and the phase of the ground wave. This phase variation, which is independent of equipment-induced phase error, is the major quantity to be investigated by means of the proposed phase measurement system.

The contribution of the surface wave to the received signal strength can be expressed as

$$|\bar{E}_g| = \left[A \right] \frac{|\bar{F}_s|}{L}, \tag{43}$$

where $|\bar{F}_s|$ is the unattenuated field strength at one mile from the antenna, and $\left[A \right]$ is the ground wave attenuation factor.⁷ The attenuation factor is a function of the frequency, the polarization, the distance with curvature, the effective earth conductivity, and the dielectric constant of the earth. It has been shown that for short numerical distances the attenuation factor varies almost exponentially with distance as

$$\left[A \right] = e^{-0.43p + 0.01p^2}, \quad (44)$$

where

$$P \approx \frac{\pi L}{\lambda x} \cos b; \text{ numerical distance,}$$

$$b \approx \tan^{-1} \frac{\epsilon_r + 1}{x}, \text{ phase constant}$$

and

$$x = n\epsilon_r$$

for $b < 5^\circ$ and $p < 4.5$.

At a frequency of 20 kc/s, and for a propagation path of 320 km, the magnitude of A is

$$0.65 \leq \left[A \right] \leq 0.98. \quad (45)$$

For conductivities between 10^{-6} mhos/cm, and 3×10^{-5} mhos/cm, the use of this attenuation factor and the specified transmitted power respectively produces a calculated surface wave field strength of

$$6.0^{uv}/m < |\overline{E}_g| < 9.3^{uv}/m \quad (46)$$

for $L = 320$ km, for the specified conductivities.

The effective amplitude of the space wave at the receiver location is a function of the distance, the reflection coefficients of the ionosphere and of the earth, and the antenna characteristics. Since the finite conductivity of the earth does not alter the vertical

radiation characteristics appreciably for this propagation path, the amplitude of the once-reflected space wave¹³ for a vertical radiator is

$$|\bar{E}_s| = \frac{|\bar{F}_s|}{L} \sin i_g ||R||^M, \quad (47)$$

where

i_g is the angle of incidence of the space wave with the ground,

$||R||^*$ is the reflection coefficient of the ionosphere,

L is the length of the sky wave path,

and M is a factor introduced by the focusing effect of the spherical ionospheric reflecting surface; however, for these short distance M approaches unity.

The reflection coefficient of the ionosphere depends upon the frequency, the polarization, the angle of incidence, the time of year, etc., and is therefore very difficult to specify. Experimental results obtained at 16 kc/s for comparable distances indicate that a variation¹⁴ of the reflection coefficient, $||R||$, occurs in the range of 0.27 to 0.55, for a summer day and night, respectively. If a reflection coefficient of 0.27 is assumed, then

$$|\bar{E}_s| = 925 \text{ nanovolts/meter}, \quad (48)$$

where

$L \approx 354 \text{ km}$ based on 75 km mean ionosphere height over a flat earth,

*This subscript notation, $||$, denotes that the electric field of the incident wave is parallel to the plane of incidence and the notation, $\perp$, means it is perpendicular to the plane of the incidence.

$$i_g \approx 25^\circ,$$

and

$$D \approx 1.$$

A comparison of the surface wave and the space wave indicates that the surface wave will be the predominant field component where

$$|\bar{E}_t| \leq \sqrt{|E_g|^2 + |E_s|^2}, \quad (49)$$

and further, that the amplitude of the total electric field strength will be approximately that given in (46) where $\|R\| = 0.27$. Even with larger reflection coefficients, the ground wave is the predominant factor in field strength determination. It should be noted that if a study devoted exclusively to as the space wave, reflected by the ionosphere, is performed in the future, the required transmitted power will increase slightly, except for periods of low ambient atmospheric noise.

The values of electric field strengths obtained in this section are approximate values which can be utilized in the planning of the terminal installations. The field strengths calculated from (46), (48), and (49) are in the same range of magnitude, as determined from Pierce's equation,⁴ which was utilized to determine the required transmitted power in Chapter III. A plot of the field strength as a function of distance for Pierce's equation,

$$|\bar{F}_s| = 210 \left(\frac{P_r}{\sin \theta} \right)^{1/2} \epsilon^{0.27f^{3/4\theta}} \text{ uv/m} \quad (50)$$

where

θ is the angular distance in radians,

P_r is in kilowatts,

and

f is in kilocycles/second,

has been made, and is presented in Figure 23 for 100 milliwatts of radiated power.

Receiving Antenna

The receiving antenna requirements are less severe than those of the transmitting antenna at VLF frequencies. Since the ambient atmospheric noise is much greater than the inherent noise introduced by the receiver, the SNR is established on the basis of the atmospheric noise, independent of the receiver characteristics. The efficiency of the receiving antenna is thus of minor importance, since both the noise and the signals are reduced in the same proportion for any given efficiency for non-directional antenna systems. The only requirement of the antenna is that the effective height, or aperture, be large enough to ensure that the received signal is above the receiver sensitivity for a sufficient SNR.

The receiving antenna usually employed is a standard whip antenna of about 3 meters physical length, and about 0.75 to 1.5 meters of effective height. If both the field strength and the receiver sensitivity are known, a compromise can obtain an antenna structure which will satisfy all requirements.

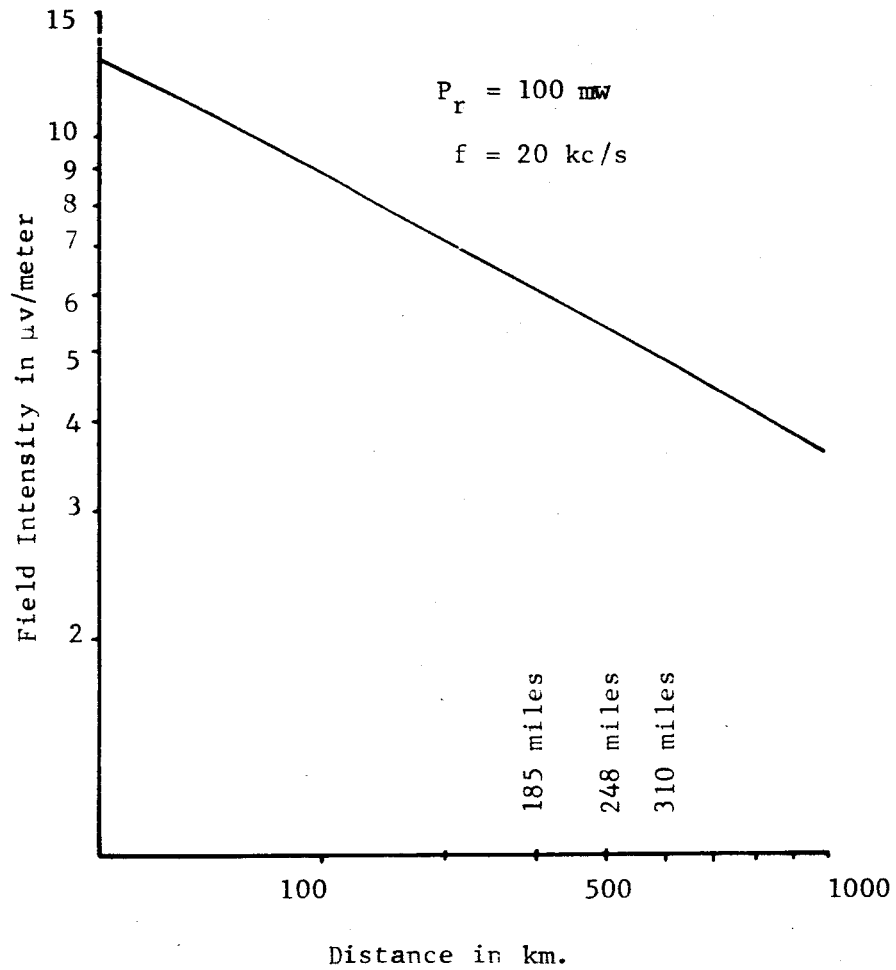


Fig. 23--Field strength as a function of distance for a radiated power of 100 milliwatts at 20 kc/s.

For a distance of 200 to 300 miles, the received signal will be in the range of 5 to 6 microvolts if a whip antenna of 1 meter of effective height is utilized (See Figure 23), and if the required SNR for phase tracking is maintained. This signal strength is certainly adequate since the sensitivities of presently available receivers are in the order of 50 nanovolts for positive AGC control.

Since the SNR is set independent of the receiving system, directive arrays are of considerable importance, because the total noise received is reduced, thereby yielding higher SNR's. Loop and wave antennas have been utilized in certain applications to some advantage; however, neither the small effective height of the loop antenna, nor the large physical dimensions of the wave antenna are desirable for the present system requirements. For future requirements the loop antenna might possibly have some applications, even though the effective height is very small.

IV. CONCLUSIONS AND RECOMMENDATIONS

The results of this investigation clearly indicate that phase comparison of low-power VLF signals over short distances by means of electrically small antennas is feasible. It has been shown that it is desirable to utilize an antenna structure which has a reasonable efficiency and a static capacitance so that the base voltage and the tuning inductance requirements are minimized. A 300-foot vertical antenna with the following characteristics should satisfy these requirements:

Physical Requirements:

Type	Umbrella with multiple wire ribs
Height	300 feet,
Number of ribs	12
Length of ribs	$\approx$ 300 feet
Rib wire size	4 to 6 wires on 4 inch diameter
Rib angle with vertical	45°
Voltage Breakdown-Base insulator	50,000 RMS
Radial Groundwire length	500 to 1000 feet
Number of ground wires	120
Size of ground wires	No. 10
Ground shield at antenna base	20 foot diameter

Electrical Characteristics:

Radiation resistance	$< 20 \text{ m}\Omega$
Efficiency	$< 1 \%$
Base voltage	$< 5 \text{ kv RMS}$
Effective ground resistance	$\approx 0.5 \Omega$
Static capacitance	$\approx 5000 \text{ pf.}$
Required tuning inductance	$\approx 12 \text{ mh.}$
Tuning inductance loss resistance	$< 6 \Omega$

These values represent the expected range of the parameters specified, and cannot be determined more specifically until the final design is completed.

The calculated field strengths at 320 km from the transmitter, where $P_r = 100 \text{ mw}$, should be sufficient for the study of the composite VLF signal, or for certain associated field components, if the proper receiving antenna system is utilized. This capability will in turn facilitate possible phase stability studies of ionospheric propagation of VLF signals over short distances.

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