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# 852 "S-D" Exchange Model -- Zero-Bias Tunneling Anomalies<sup>†</sup>

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Anomalies in the tunneling conductance centered at zero bias have been found in several experiments.<sup>1</sup> These studies were performed on a large class of p-n junctions, as well as on junctions composed of normal metals separated by an insulating oxide layer.

In particular, Wyatt has observed<sup>2</sup> a peak in the conductance,  $G(V)$ , centered at zero bias in tunneling junctions where Ta or Nb was separated from Al by a thin oxide layer. Wyatt found that  $G(V)$  could be divided into a temperature-independent part  $G_0(V)$ , and a strongly temperature-dependent part  $\Delta G(V) = G(V) - G_0(V)$ .  $\Delta G(V)/G_0(V)$  varied as  $\ln|eV/kT|$  for  $eV > kT$ , while  $\Delta G(0)/G_0(0)$  varied with temperature as  $\ln T$ . The effect persisted both above and below the superconducting transition temperature when care

was taken to quench the superconductivity with an applied magnetic field of 9 kG. The effect of varying the magnetic field between 4 kG and 20 kG was observable only at 1.5°K, where a 10% broadening of  $\Delta G(V)$  was observed. Wyatt assumed that the zero bias anomaly in  $G(V)$  was due to a logarithmic singularity in the density of states at the Fermi surface.

Anderson has suggested<sup>3</sup> that the Wyatt anomalies may be caused by magnetic impurities, and recent experiments<sup>4</sup> appear to corroborate his idea. Stimulated by Anderson's suggestion, we have analyzed several microscopic mechanisms; one of these is closely related to the scattering singularities discussed by Kondo.<sup>5</sup>

We begin by remarking that near zero bias and at low temperatures one expects localized states to contribute to the tunneling current by serving as a momentum reservoir for the tunneling electrons. Such a reservoir is obviously essential if the initial and final states of the tunneling electron have wave vectors differing by a substantial fraction of a reciprocal lattice vector. This is the case for p-n junctions of Si and Ge and is probably the case for the junctions studied by Wyatt, considering the nature of the Fermi surfaces of the metals involved.

We shall assume that the localized states (which may be associated specifically with impurities or with the metal-oxide

interface) are paramagnetic and are coupled to the conduction electrons by an exchange interaction of the following form:

$$H = H_1 + H_2, \quad (1)$$

$$\begin{aligned} H_1 = & T_J \sum_{\underline{k}, \underline{k}'} S_Z [(a_{\underline{k}+}^* b_{\underline{k}'+} - a_{\underline{k}-}^* b_{\underline{k}'-} + (b_{\underline{k}'+} a_{\underline{k}+} - b_{\underline{k}'-} a_{\underline{k}-})] \\ & + T_J \sum_{\underline{k}, \underline{k}'} S^+ (a_{\underline{k}-}^* b_{\underline{k}'+} + b_{\underline{k}'-}^* a_{\underline{k}+}) \\ & + T_J \sum_{\underline{k}, \underline{k}'} S^- (a_{\underline{k}+}^* b_{\underline{k}'-} + b_{\underline{k}'+}^* a_{\underline{k}-}), \end{aligned} \quad (2)$$

$$\begin{aligned} H_2 = & +J \sum_{\underline{k}, \underline{k}'} S_Z [(a_{\underline{k}+}^* a_{\underline{k}'+} - a_{\underline{k}-}^* a_{\underline{k}'-}) + (b_{\underline{k}+}^* b_{\underline{k}'+} - b_{\underline{k}-}^* b_{\underline{k}'-})] \\ & + J \sum_{\underline{k}, \underline{k}'} S^+ (a_{\underline{k}-}^* a_{\underline{k}'+} + b_{\underline{k}-}^* b_{\underline{k}'+}) \\ & + J \sum_{\underline{k}, \underline{k}'} S^- (a_{\underline{k}+}^* a_{\underline{k}'-} + b_{\underline{k}+}^* b_{\underline{k}'-}), \end{aligned} \quad (3)$$

where  $S_Z$ ,  $S^+$  and  $S^-$  are the spin operators of the localized states. Electrons on the left (a) side of the junction which have momentum  $\underline{k}$  and spin  $\sigma$  are described by the creation and annihilation operators  $a_{\underline{k}\sigma}^+$ ,  $a_{\underline{k}\sigma}$ , respectively, while the operators  $b_{\underline{k}'\sigma'}^*$ ,  $b_{\underline{k}'\sigma'}$  have similar meanings for electrons on the right (b) side of the junction.

$J$  is an exchange coupling for electrons which remain on the same side of the junction after scattering off the localized spin.  $T_J$  is similarly an exchange coupling, except it

is defined in terms of electrons on opposite sides of the junction. It will, consequently, be smaller than  $J$  by a factor, the order of the overlap of the exponential tails of the wavefunctions on opposite sides of the junction. The two terms,  $H_1$  and  $H_2$ , represent respectively, the two possible outcomes of an electron scattering off a localized spin: either the electron scatters to the other side of the junction or remains on the same side. Thus, the interaction described by  $H$  is confined to within a mean free path of the junction.

$H$  contains the interaction of conduction electrons with a single localized spin. We assume that we may neglect interference effects between localized spin states. To obtain the total current  $J_{ab}$  between sides  $a$  and  $b$  we multiply  $j_{ab}$ , the current calculated from  $H$  by  $N$ , the number of localized spins.

We obtain  $j_{ab}$  from

$$j_{ab} = e \sum_{\substack{\underline{k}\sigma \\ \underline{k}'\sigma'}} \left[ W_{\underline{k}\sigma; \underline{k}'\sigma'} f(\epsilon_{\underline{k}\sigma}) (1 - f(\epsilon_{\underline{k}'\sigma'})) - W_{\underline{k}'\sigma'; \underline{k}\sigma} f(\epsilon_{\underline{k}'\sigma'}) (1 - f(\epsilon_{\underline{k}\sigma})) \right], \quad (4)$$

where  $f(\epsilon_{\underline{k}\sigma})$  is the Fermi-Dirac distribution function and  $W_{\underline{k}\sigma; \underline{k}'\sigma'}$  is the transition probability for an electron scattering from state  $(\underline{k}, \sigma)$  on side  $a$  to  $(\underline{k}', \sigma')$  on side  $b$ .

The third order in the exchange coupling  $W$  is given by:

$$W_{i;j} = \frac{2\pi}{\hbar} \left[ \sum_{\underline{k} \neq i} \frac{H_{i\underline{k}} H_{\underline{k}j} H_{1j}}{E_i - E_{\underline{k}}} + \text{complex conj.} + |H_{1j}|^2 \right] \delta(E_i - E_j), \quad (5)$$

where  $E_i$  and  $E_f$  are the energies of the initial and final states of the electron-localized spin system. We shall be interested in only those terms of  $W_{i,j}$  which are first order<sup>6,7</sup> in  $T_J$ . We assume there is a magnetic field,  $H$  present, so that the localized spins have a Zeeman energy,  $\Delta$  equal to  $g|\mu_B|H$ ,  $g$  is the  $g$ -factor of the localized spin,  $\mu_B$  the Bohr magneton. The Zeeman energies of the conduction electrons were found to have no effect on the current.

Typical scattering processes which contribute to  $\Delta G$  are shown in Fig. 1. Following an analysis similar to that of Kondo,<sup>5</sup> one obtains for the processes shown in Fig. 1 the following:

$$\begin{aligned}
 W_{\underline{k}_+ M \rightarrow \underline{k}_+', M+1} &= -2T_J^2 J(S(S+1) - M(M+1)) \\
 &\times [g^{(a)}(\epsilon_{\underline{k}_+ - \Delta}) + g^{(a)}(\epsilon_{\underline{k}_+}) + g^{(b)}(\epsilon_{\underline{k}_-}) + g^{(b)}(\epsilon_{\underline{k}_- + \Delta})] \\
 &\times \delta(\epsilon_{\underline{k}_+} + eV - \epsilon_{\underline{k}_+ - \Delta})
 \end{aligned} \quad (6)$$

where  $M$  is the component of localized spin along the magnetic field and  $S$  is the total spin of the localized state. We assume the Fermi energy on the  $a$  side has been raised by an energy  $eV$ , the applied potential.  $g^{(n)}(\omega)$  is defined by:

$$g^{(n)}(\omega) = \sum_{\underline{k}^{(n)}} \frac{f(\epsilon_{\underline{k}})}{\epsilon_{\underline{k}} - \omega} = \int_{\epsilon_F - E_0}^{\epsilon_F + E_0} \frac{\rho^{(n)}(\epsilon) f(\epsilon) d\epsilon}{\epsilon - \omega}; \quad n=a, b. \quad (7)$$

where the index  $n$  specifies whether the above sum is carried out over states from the a or b sides.

$\epsilon_{\underline{k}\sigma}$  is the energy of an electron with momentum  $\underline{k}$  and spin  $\sigma$  before the external electric field is applied, and the Fermi energy appearing in  $f(\epsilon_{\underline{k}})$  is the common Fermi energy of sides a and b when  $V=0$ .

We have limited the sum over states to a narrow energy region of width  $2E_0$  centered at the Fermi energy. This reflects the fact that only in a narrow energy is our assumption valid that the exchange couplings  $J$  and  $T_J$  appearing in Eq. (1)-(3) are constant. It is difficult to estimate  $E_0$  and it will be taken as an adjustable parameter.

When all terms which contribute to  $W_{i,f}$  are evaluated, the  $\underline{k}$ -space sums in Eq. (4) performed, and the derivatives with respect to  $V$  taken, one obtains<sup>8-11</sup> for  $G(V)$ :

$$G(V) = G_0(V) + \Delta G(V) \quad (8)$$

where

$$G_0(V) = C(1 + \frac{\langle M \rangle}{2S(S+1)}) \times (\tanh \frac{eV+\Delta}{2k_B T} - \tanh \frac{eV-\Delta}{2k_B T}) \quad (9a)$$

$$\Delta G(V) = -J(\rho^a(\epsilon_F) + \rho^b(\epsilon_F))C[\Delta G_1(V) + \Delta G_2(V) + \Delta G_3(V)] \quad (9b)$$

and

$$\Delta G_1(V) = 2[1 - \frac{\langle M^2 \rangle}{S(S+1)} + \frac{\langle M \rangle}{2S(S+1)}(\tanh \frac{\Delta-eV}{2k_B T} + \tanh \frac{\Delta+eV}{2k_B T})] \times$$

$$\ln \frac{|eV| + k_B T}{E_0} \quad (10a)$$

$$\Delta G_2(V) = [1 + \frac{\langle M^2 \rangle}{S(S+1)} + \frac{\langle M \rangle}{S(S+1)} \tanh \frac{\Delta - eV}{2k_B T}] \ln \frac{|eV - \Delta| + k_B T}{E_0} \quad (10b)$$

$$\Delta G_3(V) = [1 + \frac{\langle M^2 \rangle}{S(S+1)} + \frac{\langle M \rangle}{S(S+1)} \tanh \frac{\Delta + eV}{2k_B T}] \ln \frac{|eV + \Delta| + k_B T}{E_0} \quad (10c)$$

$$C = \frac{4\pi e^2}{h} T_J^2 \rho^a(\epsilon_F) \rho^b(\epsilon_F) S(S+1)$$

To order  $J T_J^2$ , only terms with a strong temperature and voltage dependence have been retained.  $\langle M^2 \rangle$  and  $\langle M \rangle$  refer to appropriate statistical averages of  $S_z^2$  and  $S_z$ , respectively. Terms of the form  $\ln \frac{k_B T + |\omega|}{E_0}$  which appear above are interpolative approximations to the function  $F(\omega)$ , where

$$\rho^n(\epsilon_F) F(\omega) = \int_0^\infty d\epsilon g^{(n)}(\epsilon) \frac{\partial}{\partial \epsilon} f(\epsilon + \omega). \quad (11)$$

We have assumed throughout that  $\rho^{(n)}(\epsilon)$ , the density of states, is a slowly varying function of energy and where it has appeared in integrals we have replaced it by its value at  $\epsilon_F$ , the Fermi energy.

For  $H = 0$ , ( $\Delta = 0$ )  $G(V)$  simplifies to:

$$G(V) = C(1 - 4J(\rho^a(\epsilon_F) + \rho^b(\epsilon_F))) \ln \frac{|eV| + k_B T}{E_0} \quad (12)$$

whence

$$\frac{\Delta G(0)}{G(0)} = -4J(\rho^a(\epsilon_F) + \rho^b(\epsilon_F)) \ln \frac{k_B T}{E_0} \quad (13)$$

This is precisely the temperature and voltage dependence found by Wyatt. Fitting his data we obtain  $E_0 = 10.6$  meV and  $J(\rho^a + \rho^b) = 0.012$ . Little significance should be attached to the small values of  $E_0$  and  $J$  obtained from the data, as these parameters will be renormalized when, e.g., the current from non-magnetic localized states, is considered. Notice also that  $J$  is positive, which implies antiferromagnetic coupling between the conduction electrons and the localized spins, and that ferromagnetic coupling in Eq. (12) implies a dip in  $G(V)$  instead of a peak.

For  $\underline{H} \neq 0$ ,  $G(V)$  assumes the rather complicated form given in Eqs. (8)-(10). For weak magnetic fields (those for which  $\Delta/2k_B T \ll 1$ ),  $\underline{H}$  should have no noticeable effect on  $G(V)$ . This is expected to be the case for all temperatures studied by Wyatt except  $T = 1.5^\circ\text{K}$ . (We have assumed  $g = 1$  in evaluating  $\Delta$ .) At this temperature the essential effect of  $\underline{H}$  is to broaden the peak.<sup>12</sup> This can be seen from (10), where the single logarithmic peak centered on zero bias for  $\underline{H} = 0$  is split into three peaks, two of which are displaced by  $\Delta$  to either side of zero bias. We see, therefore, that the magnetic field dependence of  $G(V)$  is consistent with that found by Wyatt.

In the limit  $eV, k_B T \ll \Delta$  we expect  $H$  to have a more noticeable effect on  $\frac{\Delta G(0)}{G_0(0)}$ . This effect will be different; depending on whether the main contribution to  $G_0$  comes from magnetic or nonmagnetic scattering. In the former case, Eq.(13)



is replaced by

$$\frac{\Delta G(0)}{G(0)} = -4J(\rho^a(\epsilon_F) + \rho^b(\epsilon_F)) \ln \frac{k_B T + \Delta}{E_0}, \quad (14)$$

while in the latter, one should use instead of Eq. (13)

$$\frac{\Delta G(0)}{G(0)} = -4J(\rho^a(\epsilon_F) + \rho^b(\epsilon_F)) \left(\frac{S}{S+1}\right) \ln \frac{k_B T + \Delta}{E_0} \quad (15)$$

in the high field limit.

Experimental studies in the above regime should serve to test the proposed exchange model.

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## REFERENCES

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1. R. A. Logan and J. M. Rowell, Phys. Rev. Letters 13, 404 (1964); A.F.G. Wyatt, Phys. Rev. Letters 13, 401 (1964); John Shewchun and Robert M. Williams, Phys. Rev. Letters 15, 160 (1965); J. M. Rowell, private communication.
2. A.F.G. Wyatt, see reference 1.
3. P. W. Anderson, private communication (1964) to J. C. Phillips.
4. L.Y.L. Shen and J. M. Rowell, Bul. Am. Phys. Soc. 11, 224 (1966).
5. J. Kondo, Prog. Theor. Phys. 32, 37 (1964); 34, 204 (1965); 34, 523 (1965).
6. D. J. Kim, Phys. Letters 18, 215 (1965) has considered a transfer mechanism based on electron-electron exchange in which electrons spin flip scatter from one side of the junction to the other with conduction electrons on either side of the junction taking up the spin recoil. Kim obtains a singular term in the current, but only in third

order in the tunneling coupling constant  $T$ . His results are open to question, however, on a number of points, not the least of which is his use of a tunneling "transfer" Hamiltonian to calculate lifetimes to third order (a procedure we feel of doubtful validity); currents are then obtained by a semiclassical transport argument. Using Kim's Hamiltonian we have calculated currents using the equation-of-motion method (ref. 7) and have found that singular terms appear first in fourth order in  $T$ . These terms are probably too small to be observable.

7. M. H. Cohen, L. M. Falicov and J. C. Phillips, Phys. Rev. Letters 8, 316 (1962).
8. As noted by Abrikosov (ref.9), Nagaoka (ref.10) and Suhl (ref.11), the logarithmic singularity appearing in third order in the coupling constants  $J$  and  $T_J$  is removed at sufficiently low temperatures by summing to all orders in perturbation theory. In our analysis this will also be the case, but because  $T_J$ , the relevant coupling constant in this analysis, is very much smaller than  $J$ , we expect the temperature at which this occurs to be very much smaller than in the Kondo effect. Thus, higher order terms in  $T_J$  are probably unimportant in the temperature region we are studying.

9. A. A. Abrikosov, Physics 2, 5, 61 (1965).
10. Y. Nagaoka, Phys. Rev. 138, A1112 (1965).
11. H. Suhl, Phys. Rev. 138, A515 (1965).

## FIGURE CAPTIONS

Fig. 1 - Diagram I represents a second order scattering process in which an electron in state  $\underline{k}+$  scatters into the virtual state  $\underline{q}-$ . The localized spin state, represented in the diagram by a circle, changes its  $\underline{z}$ -component of spin by one unit. The virtual state  $\underline{q}-$  then scatters into the final state  $\underline{k}!$ . Such a process may also occur in the reverse order: e.g.,  $\underline{q}-$  first scattering  $\underline{k}-$ , then  $\underline{k}+$  scattering  $\underline{q}-$ . The double vertical lines represent the junction interface. Diagrams II-IV are similarly interpreted.

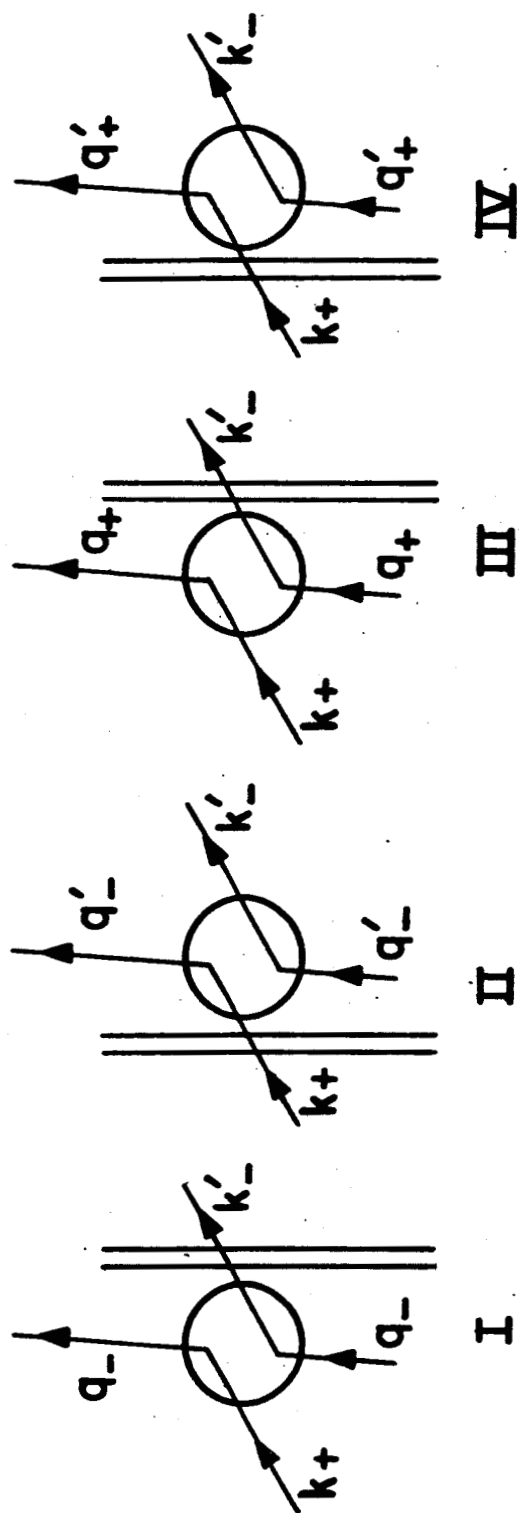


Fig.1