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ON THE PROBABLE INFLUENCE OF HIGHER ORDER EARTH GRAVITY IN THE DETERMINATION OF THE EQUATORIAL ELLIPTICITY OF THE EARTH FROM THE DRIFT OF SYNCOM II OVER BRAZIL

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ABSTRACT

This document gives calculations from the best available (summer 1964) geodetic sources showing that the Syncom II - Brazil second-order earth gravity experiment reported earlier by this author probably contained a small but significant bias due to higher order earth gravity. This bias is estimated by "extrapolation" from these sources.

Bounds on the true ellipticity of the earth's equator are derived by combining the best estimate of this bias with the actual Syncom II drift measurements. These bounds, though somewhat wider than measured by Syncom II (over Brazil) alone, should be absolute and the target for all future investigations into longitude-dependent earth gravity.

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INTRODUCTION

Purpose

This report makes use of lower altitude gravity data to evaluate the likely bias due to higher order earth effects in the Syncom II - Brazil second-order gravity-drift experiment.

The Syncom II - Brazil drift, previously analyzed for second-order longitude gravity only (i.e., the ellipticity of the earth's equator) sampled a limited portion of that longitude field. From this limited sampling, no direct evidence of higher order earth gravity effects at 24-hour satellite altitudes was available. Nevertheless, *preliminary* calculations using the evidence of a number of "full field" geoids derived from lower altitude gravity observations appeared to indicate that the assumption (in the Syncom II gravity experiment) of small higher order gravity effects at 24-hour altitudes was justified. The reason for this study is to present *detailed* calculations "extrapolating" the evidence from lower altitude gravity surveys in support or contradiction of this judgment.*

If the effects of higher order earth gravity at 24-hour altitudes are large and not well determined from the lower altitude evidence, then the isolated accelerations measured by Syncom II drift over Brazil cannot fairly be extrapolated to provide comprehensive information about the likely 24-hour longitude gravity accelerations at other longitudes. However, if, as assumed in the previous study, Syncom II drift accelerations over Brazil were likely to be fairly representative of the second-order longitude gravity field only, then these isolated but well-measured accelerations tell us significant information about the probable drift accelerations on 24-hour satellites at all longitudes.

Background on the External Gravity Field of the Earth

For over 100 years scientists have represented the earth's actual external gravity field as an infinite series of spherical harmonic components (Appendix A, Table A-1), each representing a

*Subsequent 24 hour satellite data has confirmed the minor influence of higher order gravity on the Syncom-Brazil measurements, as predicted in this report.

small distortion of the dominant spherical field first calculated by Isaac Newton in the 17th century (Reference 1). It can be appreciated that every measurement of this field at a single point in geographical space involves all the terms in this series. Thus, theoretically, one can never determine each term of the series with complete certainty unless one has an infinite number of exact point measurements to deal with. However, as a practical matter, the earth proves to be so nearly spherically homogeneous that the first-order term of the gravity series has changed but little since Newton's time in the 17th century. Nor is there any reason to suppose it will drastically alter in the future when even more gravity data will become available. The same hopeful remarks can be made for the dominant second-order gravity component that nearly perfectly describes the polar flattening of the earth due to the relatively strong centrifugal action of its rotation. For evidence of this the reader is referred to the many recent studies of the gravity field in the literature which report values for this "oblate" term, particularly those which contain sufficient data to allow for a simultaneous determination of a large number of higher order "oblate" terms (Reference 2, for example). A more complete discussion of this problem with respect to the oblate and zonal terms of the gravity field is found in Reference 3. With respect to the very small longitude components of the field, the situation is far more difficult to resolve. This is so, essentially because the earth does not seem to be able to support mechanically large mass inhomogeneities in the upper mantle and crust. One can avoid the problem of finding approximations to the true longitude field valid everywhere by reporting effectively the point force measurements themselves and leaving the stud of what may appear to be longitude gravity "noise" to some future date. Nevertheless, there *are* theoretical expectations (associated with the deep earth convection currents that are supposed to be the cause of the external magnetic field) for a dominant longitude gravity field distortion of low order. But the search for a well-defined, lowest (second) order longitude field, represented by an earth with an elliptical equator, had been disappointing and inconclusive until the advent of Syncom II, the first operational 24-hour satellite (Reference 4). The great height of this satellite makes it sensitive mainly to gravity terms of a low order associated with mass distortions distributed over wide areas of the earth's surface.

Method of Analysis

Anomalous long term accelerations of Syncom II were heretofore assumed by this author (Reference 4) to be caused by the second-order longitude gravity field of the earth only. Since essentially only two independent point measurements of the field were obtained from the limited extent of drift data in Reference 4, no information on higher order distorting effects can be obtained without recourse to gravity data from other sources.

This study selects eight recent and largely independently derived "full" fourth-order geoids as the basis for assessing the likely influence of higher than second-order earth gravity on the Syncom II - Brazil drift accelerations. Over each geoid the full field drift accelerations on the two Syncom II - Brazil orbits are calculated. As in the actual Syncom II - Brazil experiment, these calculated full field accelerations for each geoid are then assumed to arise from a simple second-order field only. This "two point" second-order longitude field (essentially giving the magnitude and longitude orientation of the earth's elliptical equator) is then calculated. Direct comparison is then made of this "two point" second-order field with the "actual" second-order field for each geoid. This comparison gives the bias implicit in the "two point" field measured under Syncom II - Brazil orbit conditions for each fourth-order geoid. It is assumed that earth gravity effects higher than the fourth order have negligible effect on the 24-hour satellite.

A SYNCOM II ORBIT GRAVITY EXPERIMENT SIMULATED IN A FOURTH-ORDER FIELD

Imagine the Syncom II longitude sampling over Brazil (Reference 4) to have taken place in the earth gravity fields of Appendix A. (See also Figure 1.) From Reference 5 it is shown that the orbit-averaged longitude acceleration of the 24-hour satellite *figure eight* ground track (or ascending equator crossing of the satellite) is given as

$$\ddot{\lambda} = -12\pi^2 \left(\frac{R_0}{a_s}\right)^2 \left\{ 6J_{22}\, F(i)_{22} \sin 2(\lambda - \lambda_{22}) - \frac{3}{2}\left(\frac{R_0}{a_s}\right) J_{31}\, F(i)_{31} \sin(\lambda - \lambda_{31}) \right.$$

$$+ 45 \left(\frac{R_0}{a_s}\right) J_{33}\, F(i)_{33} \sin 3(\lambda - \lambda_{33}) - 15 \left(\frac{R_0}{a_s}\right)^2 J_{42}\, F(i)_{42} \sin 2(\lambda - \lambda_{42})$$

$$\left. + 420 \left(\frac{R_0}{a_s}\right)^2 J_{44}\, F(i)_{44} \sin 4(\lambda - \lambda_{44}) \right\} \frac{\text{rad}}{(\text{sid day})^2}, \tag{1}$$

where i is the inclination of the satellite and

$$F(i)_{22} = \frac{1}{4}(1 + \cos i)^2, \tag{2}$$

$$F(i)_{31} = \frac{1}{2}(1 + \cos i) - \frac{5}{8}(1 + 3\cos i)\sin^2 i, \tag{3}$$

$$F(i)_{33} = \frac{1}{8}(1 + \cos i)^3, \tag{4}$$

$$F(i)_{42} = \frac{1}{4}(1 + \cos i)^2 - \frac{7}{4}(1 + \cos i)\sin^2 i \cos i, \tag{5}$$

$$F(i)_{44} = \frac{1}{16}(1 + \cos i)^4. \tag{6}$$

Equation 1, with only the J_{22} term, was first derived in Reference 4. The right-hand side is essentially the orbit-averaged tangential (energy-changing) force on the 24-hour inclined-orbit satellite. The left-hand side is the longitude acceleration caused by the continual period change effected by this force.

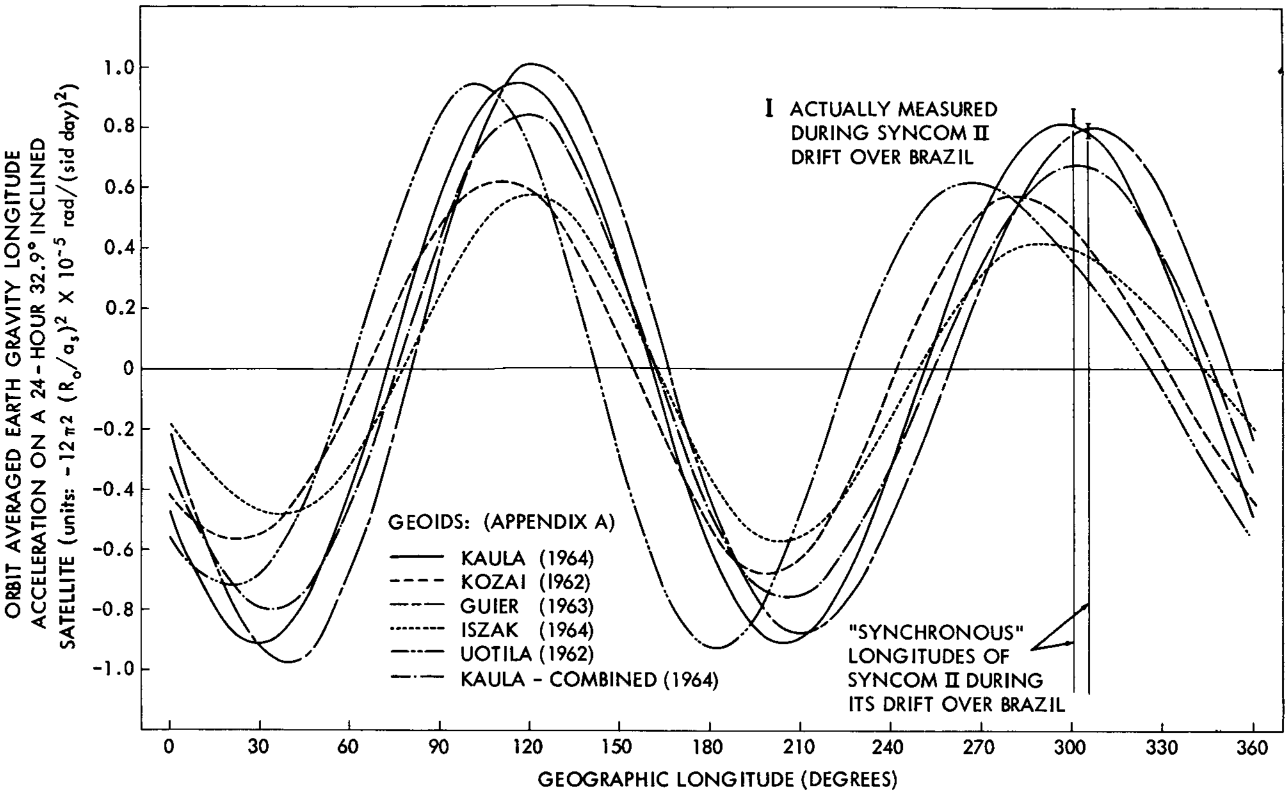

Figure 1—Full-field orbit-averaged longitude accelerations on Syncom II as predicted by six recent geoids and as actually measured in a Syncom II gravity experiment.

The inclination of Syncom II during the excursions over Brazil reported in Reference 4 was $i \doteq 32.9°$. With this value in Equations 2-6 the "inclination factors" for Syncom II in Aug. - Feb. 1963-64 are:

$$F(i)_{22} = 0.845$$

$$F(i)_{31} = 0.267$$

$$F(i)_{33} = 0.775$$

$$F(i)_{42} = 0.046$$

$$F(i)_{44} = 0.710 \quad (7)$$

From Reference 4, $(R_0/a_s) = (6378/42166) = 0.1513$ and $(R_0/a_s)^2 = 0.0229$. Equation 1 then gives

$$\ddot{\lambda} = \Big\{5.06\,J_{22}\sin 2(\lambda-\lambda_{22}) - 0.0605\,J_{31}\sin(\lambda-\lambda_{31}) + 5.26\,J_{33}\sin 3(\lambda-\lambda_{33}) - 0.0158\,J_{42}\sin 2(\lambda-\lambda_{42}) + 6.83\,J_{44}\sin 4(\lambda-\lambda_{44})\Big\}, \tag{8}$$

which is in units of $-12\pi^2 (R_0/a_s)^2$ rad/day^2. Figure 1 gives a graph of Equation 8 for six recent geoids. For the first "synchronous" longitude location of Syncom II at $\lambda = -54.8°$, Equation 8 gives

$$\ddot{\lambda}_{-54.8°} = \Big\{5.06\,J_{22}\sin 2(-54.8° - \lambda_{22}) - 0.0605\,J_{31}\sin(-54.8° - \lambda_{31}) + 5.26\,J_{33}\sin 3(-54.8° - \lambda_{33}) - 0.0158\,J_{42}\sin 2(-54.8° - \lambda_{42}) + 6.83\,J_{44}\sin 4(-54.8° - \lambda_{44})\Big\}. \tag{9a}$$

For the second "synchronous" longitude location of Syncom II at $\lambda = -59.2°$ Equation 8 is

$$\ddot{\lambda}_{-59.2°} = \Big\{5.06\,J_{22}\sin 2(-59.2° - \lambda_{22}) - 0.0605\,J_{31}\sin(-59.2° - \lambda_{31}) + 5.26\,J_{33}\sin 3(-59.2° - \lambda_{33}) - 0.0158\,J_{42}\sin 2(-59.2° - \lambda_{42}) + 6.83\,J_{44}\sin 4(-59.2° - \lambda_{44})\Big\}. \tag{9b}$$

In the actual gravity experiment on Syncom II in Reference 4 it was assumed that Equations 9a and 9b could be truncated at the first right-hand terms; or that longitude gravity terms higher than second-order had negligible net effect on the longitude acceleration. We can test this assumption by performing the identical experiment with the "fourth-order field" accelerations of Equations 9a and 9b (as assessed by the geoids of Appendix A) under the same assumption.

For each geoid a J_{22} and λ_{22} is listed in Table 1. These values are assumed to be best for a certain set of gravity data taken over a large zone of geographic space. If one assumes in each case that the high-order geoid is perfect, then apparent J_{22} and λ_{22} values can be computed from a limited two-point measurement of the field of that geoid as in the actual Syncom II experiment over Brazil. Comparing the two sets of coefficients gives a direct measure of the distortion in J_{22} and λ_{22} as it is computed in the Syncom II - Brazil experiment which *would be* expected if that particular geoid were perfect (see DISCUSSION).

For each geoid, a J_{22} apparent (J_{22}') and a λ_{22} apparent (λ_{22}') may be computed from the fourth-order field accelerations of Equations 9a and 9b under the simplifying assumption of Reference 4. Thus $\ddot{\lambda}(-59.2°)$, as it is calculated from Equation 9a, is the fourth-order field acceleration which would have been measured by Syncom II at -59.2° for the geoid whose gravity constants are J_{nm} and λ_{nm}. Under the simplifying assumption of the actual Syncom II - Brazil Experiment (Reference 4), this higher order acceleration (actually measured) was assigned to only the second order gravity field, giving apparent earth constants J_{22}' and λ_{22}'.

Table 1

Higher Order Gravity Effects on Equatorial Ellipticity as Seen by a Syncom II - Orbit at Two Longitudes, for Eight High-Order Geoids.

Geoid	J_{22} (reported)	λ_{22} (reported) (degrees)	Full Field Acceleration on Syncom II, Orbit Averaged (see Figure 1) (units of $-12\pi^2 (R_0/a_s)^2 \times 10^{-6}$ rad/(sid day)^2) $\lambda = -54.8°$	$\lambda = -59.2°$	$R_{22} = \frac{J_{22}\text{ (reported)}}{J_{22}\text{ (apparent)***}}$ ($\lambda = -54.8°$ and $\lambda = -59.2°$)	$\Delta\lambda_{22} = \lambda_{22}$ (reported) $-\lambda_{22}$ (apparent)***
Kaula-Comb. (1964)	-1.51×10^{-6}	-15.5°	6.793	6.815	1.12	-2.9°
Izsak (1964	-1.00	-17.0°	3.876	4.099	1.19	+5.0°
Kaula (1964)	-1.77	-18.2°	7.962	8.194	1.085	-0.9°
Guier (1963)	-1.80	-10.4°	8.115	7.944	1.12	-2.3°
Uotila (1962)	-1.52	-36.5°	3.130	3.682	1.55	-1.2°
Kozai (1962)	-1.2	-26.4°	4.287	4.809	1.07	+3.9°
Zhongolovitch (1957)	-5.95	-7.7°	28.684	27.436	1.025	-3.8°
Jeffreys (1942)	-4.1	0.0	18.184	16.888	1.06	-0.8°
Actually Measured in Syncom II - Brazil Experiment			8.1*	8.45**		
					Mean = 1.095 (excluding Uotila (1962))	Mean = -0.4° sample root mean square deviation is 2.9°

*From Reference 4, $\ddot{\lambda}(-54.8°) = -2.20 \times 10^{-5}$ rad/(sid day)^2

**From Reference 4, $\ddot{\lambda}(-59.2°) = -2.29 \times 10^{-5}$ rad/(sid day)^2

***Calculated values from the full field accelerations of the $\lambda = -54.8°$ and $\lambda = -59.2°$ columns, assuming these accelerations arise from a J_{22} field only.

Duplicating the experimental procedure for each of the geoids in Appendix A, we set Equation 9a to

$$\ddot{\lambda}_{-59.2°} = 5.06\, J'_{22} \sin 2\left(-59.2° - \lambda'_{22}\right)$$

$$= 5.06 \sin(-118.4°) \left[J'_{22} \cos 2\lambda'_{22}\right] + 5.06 \cos(-118.4°) \left[-J'_{22} \sin 2\lambda'_{22}\right] . \qquad (10a)$$

Similarly, we simulate the actual Syncom II - Brazil experiment at $\lambda = -54.8°$ by setting Equation 9b to

$$\ddot{\lambda}_{-54.8°} = 5.06\, J'_{22} \sin 2\left(-54.8° - \lambda'_{22}\right)$$

$$= 5.06 \sin(-109.6°) \left[J'_{22} \cos 2\lambda'_{22}\right] + 5.06 \cos(-109.6°) \left[-J'_{22} \sin 2\lambda'_{22}\right] \tag{10b}$$

Solving Equations 10a and 10b for $\left(J'_{22} \cos 2\lambda'_{22}\right)$ and $\left(-J'_{22} \sin 2\lambda'_{22}\right)$ in terms of $\ddot{\lambda}(-54.8°)$ and $\ddot{\lambda}(-59.2°)$ for a fourth-order field will give

$$J'_{22} \cos 2\lambda'_{22} = \frac{\begin{vmatrix} \ddot{\lambda}(-59.2°) & 5.06 \cos(-118.4°) \\ \ddot{\lambda}(-54.8°) & 5.06 \cos(-109.6°) \end{vmatrix}}{\begin{vmatrix} 5.06 \sin(-118.4°) & 5.06 \cos(-118.4°) \\ 5.06 \sin(-109.6°) & 5.06 \cos(-109.6°) \end{vmatrix}}$$

$$= \frac{\left\{\ddot{\lambda}(-59.2°) \cos(-109.6°) - \ddot{\lambda}(-54.8°) \cos(-118.4°)\right\}}{\left[5.06 \quad \sin(-8.8°)\right]}, \tag{11a}$$

and

$$-J'_{22} \sin 2\lambda'_{22} = \frac{\begin{vmatrix} 5.06 \sin(-118.4°) & \ddot{\lambda}(-59.2°) \\ 5.06 \sin(-109.6°) & \ddot{\lambda}(-54.8°) \end{vmatrix}}{(5.06)^2 \left[\sin(-8.8°)\right]}$$

$$= \frac{\left\{\ddot{\lambda}(-54.8°) \sin(-118.4°) - \ddot{\lambda}(-59.2°) \sin(-109.6°)\right\}}{5.06 \left[\sin(-8.8°)\right]}. \tag{11b}$$

When Equation 11a is divided by Equation 11b, λ_{22} (apparent) may be calculated as in the actual Syncom II - Brazil experiment, from

$$\lambda'_{22} = \frac{1}{2} \tan^{-1} \left[\frac{-\left[\ddot{\lambda}(-54.8°) \sin(-118.4°) - \ddot{\lambda}(-59.2°) \sin(-109.6°)\right]}{\ddot{\lambda}(-59.2°) \cos(-109.6°) - \ddot{\lambda}(-54.8°) \cos(-118.4°)}\right]. \tag{12a}$$

Similarly, when Equations 11a and 11b are squared, J_{22} (apparent) may be calculated, as in the actual Syncom II experiment, from

$$J'_{22} = \frac{-10^{-6}}{5.06 \sin(8.8°)} \left\{\left[\ddot{\lambda}(-54.8°) \sin(-118.4°) - \ddot{\lambda}(-59.2°) \sin(-109.6°)\right]^2\right.$$

$$\left. + \left[\ddot{\lambda}(-59.2°) \cos(-109.6°) - \ddot{\lambda}(-54.8°) \cos(-118.4°)\right]^2\right\}^{1/2} . \qquad (12b)$$

When the following values are used:

$$\sin(-8.8°) = -0.15299$$

$$\sin(-118.4°) = -0.87965$$

$$\cos(-118.4°) = -0.47562$$

$$\sin(-109.6°) = -0.94206$$

$$\cos(-109.6°) = -0.33545 ,$$

Equations 12a and 12b become

$$\lambda'_{22} = \frac{1}{2} \tan^{-1} \left\{ \frac{-\left[-0.87965\ddot{\lambda}(-54.8°) + 0.94206\ddot{\lambda}(-59.2°)\right]}{\left[-0.33545\ddot{\lambda}(-59.2°) + 0.47562\ddot{\lambda}(-54.8°)\right]} \right\} \qquad (13a)$$

and

$$J'_{22} = -1.292 \times 10^{-6} \left\{ \begin{array}{l} \left[-0.87965\ddot{\lambda}(-54.8) - 0.94206\ddot{\lambda}(-59.2)\right]^2 \\ + \left[-0.33545\ddot{\lambda}(-59.2) + 0.47562\ddot{\lambda}(-54.8)\right]^2 \end{array} \right\}^{1/2} . \qquad (13b)$$

For each geoid in Appendix A one can calculate a ratio $R_{22} = J_{22}/J'_{22}$ from Equation 13b, giving the gross higher order effect on the magnitude of equatorial ellipticity which would have been seen by Syncom II if that geoid were correct. The cumulative divergence of these geoids from "reality" at the two Syncom II longitudes can best be seen in Figure 1. The cumulative gravity forces of the Syncom II - Brazil geoid ($J_{22} = -1.7 \times 10^{-6}$, $\lambda_{22} = -19°$) at those longitudes are exact to within about 3%, since the full field forces were those actually sensed in the experiment of Reference 4.

Similarly, for each geoid in Appendix A one can calculate a major axis change, $\Delta\lambda_{22} = \lambda_{22}$ (for that geoid) - λ'_{22} (from Equation 13a), giving the gross higher order effect on the location of the equatorial major axis which would have been seen by Syncom II if that geoid were correct.

In Appendix B this calculation, which is just a simulation of the Syncom II experiment in eight recent gravity fields reporting *individual* higher order tesseral coefficients, is carried through for the geoid of Kaula-Combined (1964). The results for this higher order geoid and the others of Appendix A are presented in Table 1.

PROBABLE RANGE OF HIGHER ORDER DISTORTIONS TO EQUATORIAL ELLIPTICITY AS SEEN BY SYNCOM II OVER BRAZIL

From this simulated study, reasonable bounds on the bias to be applied to the actual Syncom II determination of earth ellipticity appear to be

$$\left.\begin{aligned} R_{22} &= 1.1 \pm .1 \\ &\text{and} \\ \Delta\lambda_{22} &= -(1 \pm 3)^\circ . \end{aligned}\right\} \tag{14}$$

The "mean" values of the bias factors in Equations 14 are, together, close to the factors computed for Kaula (1964) and Guier (1963) which give the full field accelerations for Syncom II closest to those actually observed (Table 1 and Figure 1).

The Syncom II reduction of the observed full field accelerations, assuming only the second order effect, gave (Reference 4)

$$\left.\begin{aligned} J_{22}\ (\text{apparent}) &= -(1.70 \pm .05) \times 10^{-6} \\ &\text{and} \\ \lambda_{22}\ (\text{apparent}) &= -(19 \pm 6)^\circ . \end{aligned}\right\} \tag{15}$$

When the bias factors of Equations 14 are applied to Equations 15, the J_{22} and λ_{22} values from the Syncom II - Brazil experiment corrected for higher order earth effects become

$$J_{22} = -(1.70 \pm 0.05)(1.1 \pm 0.1) \times 10^{-6} \simeq -(1.87 \pm 0.18) \times 10^{-6} \tag{16a}$$

$$\lambda_{22} = -\left[(19 \pm 6)^\circ - (1 \pm 3)^\circ\right] = -(20 \pm 6.5)^\circ . \tag{16b}$$

It is to be especially noted that though the full field forces (or accelerations) at the two Syncom II longitudes are widely different for the eight geoids, the distorting effects on the simulated J_{22} reductions, assuming a second-order field only, are small and consistent. For all geoids tested, the result of a minimum sampling of two longitudes of the field at 54.8° West and 59.2° West was to yield values of J_{22}' or J_{22} (apparent) up to 20% less than the reported J_{22} value for that geoid. The anomalous result from Uotila (1962) is due to an overemphasis of the third-order harmonics in that geoid which is not shown by either the recent satellite geoids or the older gravimetric geoids. The distortions in λ_{22} are all less than 5.0° and appear to have a westward bias of the order of 1°.

DISCUSSION

The calculation of probable higher order distortion of true equatorial ellipticity as seen by Syncom II over Brazil given in the first two sections of this paper, did not utilize directly the constraints of the observed accelerations of the satellite. It was felt that, except in two cases, the observed accelerations were so far from those predicted by the lower altitude geoids that to attempt to force fit observed accelerations with those predicted from the lower altitude data would be no more meaningful than the straightforward simulation actually calculated.

These simulations at least have the virtue of assessing precisely and directly the distortions of "reality" we are interested in finding. Of course the "real earth" in each simulation is only an estimate, good or poor as the case may be. Yet there is encouraging consistency in the "internal" distortion results calculated for such widely differing geoids. By way of comparison, Kaula (Reference 7) has performed a calculation for the second-order gravity field constrained by the Syncom II accelerations directly. In this calculation all higher order effects were accumulated as a single third-order effect whose relationship with the second-order one was established by an averaging of the results of recent geoids. The conclusion was similar to the one obtained here. Kaula obtained revised Syncom II values of

$$J_{22} = -1.87 \times 10^{-6}$$

and

$$\lambda_{22} = -22.3^\circ ,$$

giving a small strengthening and westward shifting of equatorial ellipticity over that found by ignoring higher order effects altogether.

The method of estimation employed here is felt to be more comprehensive, to include a wider range of effects, and to be free from any preassignment of the relationship between them.

The geoids chosen are felt to be fairly representative, without duplication, of the great amount and variety of longitude gravity data that has been accumulated over the years (Table A-1). The geoid selection in each case but one was made on the basis of its being the latest and presumably the most comprehensive higher than second-order geoid by that author. The geoid Kaula-Combined (1964) was chosen as an example of a weighted estimate based on recent satellite-camera, satellite-Doppler and surface gravimeter geoids. Of the eight geoids, three deal with satellite-camera data (Izsak (1964), Kaula (1964) and Kozai (1962)) reduced from the arcs of up to ten medium altitude and medium inclination satellite orbits. Izsak (1964) and Kaula (1964) include reductions from a number of high inclination orbits as well. Guier (1963) is an example of a recent geoid reduced from satellite-Doppler data on a number of "TRANSIT" satellites.

The three surface gravimeter geoids are the most recent (Uotila (1962)); a recent Russian geoid (Zhongolovitch (1957)) which was chosen for comparison because both the data and the statistical reduction techniques behind it are significantly different from those used as the basis for the other two gravimetric geoids; and the oldest geoid reporting terms higher than second order, which is by a highly regarded authority in the field (Jeffreys (1942)).

In any case, it can be confidently expected that further precise Syncom II drift data at other longitudes will tell geodesists much more about the true extent of higher order longitude gravity than the rough bounds which have been established here by dealing with imprecise geoids derived from lower altitude data. The important fact to be gleaned from this study is that the bias in the simple second-order determination from Syncom II observations due to higher order effects is, from all indications, small.

In Equation 16b the estimated total deviation in λ_{22} is taken as the square root of the sums of the squares of the component deviations, as is customary in the theory of errors for the sum of two random variables. In Equation 16a the estimated total deviation in J_{22} is taken as the square root of the sum of the cross products of squared means and deviations (Reference 6). Rounding off these results upward to maintain a conservative estimate, we estimate the realistic bounds on the ellipticity of the equator as sensed by Syncom II in a seven month drift period over Brazil to be

$$J_{22} = -(1.9 \pm .2) \times 10^{-6} \qquad (17a)$$

or

$$a_0 - b_0 = 73 \pm 8 \text{ meters} ,$$

which is the difference in major and minor equatorial radii of the earth (Reference 1), and

$$\lambda_{22} = -(20 \pm 7)^\circ \text{ with respect to Greenwich} , \qquad (17b)$$

which locates the major axis of the elliptical equator of the earth.

CONCLUSIONS

From the evidence of many recent geoids based on different kinds of gravity data the following conclusions are drawn:

1. Earth gravity reductions of long term drift accelerations of 24-hour satellites can, at favorable longitudes, ignore all but the second-order effect without incurring large errors.

2. Estimates of earth equatorial ellipticity based on Syncom II drift over Brazil sensitive to *only* the second order effect were:

$$J_{22} = -(1.70 \pm 0.05) \times 10^{-6} ;$$

$$\lambda_{22} = -(19 \pm 6)^\circ .$$

When one accounts for the probable range of all higher order gravity effects on the 24-hour satellite, *on the basis of lower altitude data only*, these figures should be

$$J_{22} = -(1.9 \pm 0.2) \times 10^{-6}$$

and

$$\lambda_{22} = -(20 \pm 7)^\circ .$$

3. Further data from Syncom II should clarify the relatively small uncertainty left in exact knowledge of the earth's elliptical equator.

4. A very conservative estimate of the absolute maximum tangential velocity requirements for station keeping of 24-hour equatorial satellites is (see Appendix C)

$$\Delta V_t = 7.5 \text{ ft./sec./yr.}$$

(Manuscript received June 3, 1965)

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Appendix A

The Longitude Gravity Field of the Earth According to Investigations Since 1915

Table A-1 lists 30 sets of longitude coefficients for the earth's gravity potential as reported by geodesists since 1915. The significance of the gravity constants with respect to the shape of the sea-level geoid is discussed in Reference 1 and illustrated in Reference 8.

Table A-1

Longitude Coefficients in the Earth's Gravity Potential $\left\{V_E = \frac{\mu}{r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left[1 - (R_0/r)^n P_n^m(\sin\phi) J_{nm} \cos m(\lambda - \lambda_{nm})\right]\right\}^1$, as Reported 1915-1964[2].

LONGITUDE GEOID[7]	J_{22}	λ_{22}	J_{31}	λ_{31}	J_{32}	λ_{32}	J_{33}	λ_{33}	J_{41}	λ_{41}	J_{42}	λ_{42}	J_{43}	λ_{43}	J_{44}	λ_{44}
(1) Wagner (1964)[3]	-1.90×10^{-6}	-20.0°														
*(2) Kaula-Combined (1964)[6]	-1.51	-15.5	-1.51×10^{-6}	0.0°	$-.102 \times 10^{-6}$	0.0°	$-.149 \times 10^{-6}$	22.8°	$-.465 \times 10^{-6}$	-136.0°	$-.163 \times 10^{-6}$	37.0°	$-.061 \times 10^{-6}$	-1.9°	$-.0053 \times 10^{-6}$	35.8°
*(3) Izsak (1964)[4]	-1.00	-17.0	-.934	-15.5	-.116	19.0	-.173	38.0	-.949	-146.0	-.074	47.5	-.024	-3.9	-.0206	25.3
*(4) Kaula (1964)[4]	-1.77	-18.2	-2.12	-5.4	-.379	10.5	-.105	23.1	-.263	-239.0	-.117	42.3	-.0473	15.0	-.0104	14.5
(5) Anderle and Oesterwinter (1963)[3]	-2.09	-14.1							-.773	-140.6	-.287	34.7	-.163	-4.3		
*(6) Guier (1963)[3]	-1.80	-10.4	-1.77	6.3	-.286	-2.6	-.204	24.1	-.73	-141.0	-.273	38.6	-.0791	-.7	-.0102	35.0
(7) Kaula (Sept. 1963)[4]	-1.51	-18.1	-1.65	5.3	-.144	46.4	-.145	15.8	-.471	-228.0	-.078	44.2	-.0265	22.6	-.0038	23.3
(8) Izsak (July 1963)[4]	-1.05	-11.2	-1.1	3.2	-.20	-21.8	-.14	20.0	-.43	-132.1	-.13	37.0	-.026	11.5	-.019	14.8
(9) Kaula (May 1963)[4]	-1.4	-21.5	-1.6	-1.9	-.15	35.8	-.156	18.5	-.53	-233.7	-.12	44.5	-.019	10.7	-.0038	23.3
(10) Cohen (May 1963)[3]	-2.08	-14.1							-.775	-159.0	-.288	34.6	-.162	-4.3		
(11) Kaula (Jan 1963)[4]	-1.62	-21.4	-1.81	-3.57	-.145	6.6	-.112	37.6	-.479	-245.5	-.072	47.7	-.0988	5.9	-.0132	28.4
*(12) Uotila (1962)[5]	-1.52	-36.5	-.685	-81.0	-.409	-5.2	-.398	19.5	-.238	-127.0	-.211	14.6	-.082	-9.3	-.0142	-2.6
*(13) Kozai (Oct 1962)[4]	-1.2	-26.4	-1.9	4.6	-.14	-16.8	-.10	42.6	-.52	-122.5	-.062	65.2	-.035	0.5	-.031	14.9
(14) Newton (April 1962)[3]	-2.15	-10.9							-2.53	-189.1						
(15) Newton (Jan 1962)[3]	-4.16	-11.0														
(16) Kozai (June 1961)[4]	-2.32	-37.5	-3.21	22.0	-.41	31.0	-1.91	51.3	-.262	-196.5	-.168	54.0	-.044	-13.0	-.054	50.3
(17) Kaula (June 1961)[6]	-.55	-13.3	-1.19	20.6	-.33	-.9	-.21	22.6	-.617	-166.0	-.14	21.1	-.031	-.5	-.008	26.4
(18) Izsak (Jan 1961)[4]	-5.35	-33.2														
(19) Kaula (1961)[5]	-1.68	-38.5							-1.15	-13.0						
(20) Krassowski (1961?)	-5.53	15.0														
(21) Kaula (1959)[5]	-.62	-20.9	-.98	55.4	-.11	13.3	-.19	14.3	-.46	-132.3	-.081	48.6	-.01	-30.0	-.02	22.5
(22) Jeffreys (1959)[5]	-4.17	0.0														
(23) Uotila (1957)[5]	-3.5	-6.0														
*(24) Zhongolovitch (1957)[5]	-5.95	-7.7	-2.21	-25.7	-.628	-26.4	-.54	13.0	-.78	-149.1	-.080	45.0	-.051	-3.8	-.0224	15.9
(25) Subbotin (1949)[5]	-5.5															
(26) Niskanen (1945)[5]	-7.67	-4.0														
*(27) Jeffreys (1942)[5]	-4.1	0.0	-2.1	0.0	-.66	0.0	-.24	33.3								
(28) Heiskanen (1928)[5]	-6.34	0.0														
(29) Heiskanen (1924)[5]	-9.0	18.0														
(30) Helmert (1915)[5]	-6.0	-17.0														

[1] r is the radial distance of the field point to the center of mass of the earth, μ the earth's Gaussian gravity constant $\doteq 3.9860 \times 10^{20}$ cm^3/sec^2, R_0 the mean equatorial radius of the earth $\doteq$ 6378.2 km. ϕ is the geocentric latitude of the field point. λ is the geographic longitude of the field point. $J_{21} \doteq 0$, since the polar axis is very nearly a principal axis of inertia for the earth. $P_n^m(\sin\phi) = \cos^m\phi \sum_{t=0}^{K} T_{nmt} \sin^{n-m-2t}\phi$, where K is the integer part of $(n-m)/2$ and $T_{nmt} = \frac{(-1)^t (2n-2t)!}{2^n t!(n-t)!(n-m-2t)!}$ (See Kaula, 1964). The longitude coefficients are those for which $m \neq 0$.

[2] The J_{nm}'s and λ_{nm}'s in this table, except in one or two instances, have been converted from the original author's set of gravity coefficients. The blanks indicate the author did not consider that particular harmonic in fitting an earth potential to the observed data. In one or two instances, noted below, the author reported tesseral coefficients to higher order than the fourth.

[3] Satellite – Doppler geoid.

[4] Satellite-camera geoid.

[5] Surface-gravimetric geoid.

[6] Combined astro-geodetic, gravimetric and satellite geoid.

[7] Detailed information on references for the geoids listed below given on the following page.

*Geoids used in the data reductions reported in Table 1.

Geoid	References
(1)	"Determination of the Ellipticity of the Earth's Equator from Observations on the Drift of the Satellite" NASA Technical Note D-2759, May 1965.
(2)	Private communication from W. M. Kaula (July 1964).
(3), (4)	Private communication from W. M. Kaula (July 1964).
(5)	Private communication from W. M. Kaula (July 1964).
(6), (10), (14)	Guier, W. H., and Newton, R. R., "Non-Zonal Harmonic Coefficients of the Geopotential from Satellite Doppler Data," Silver Spring, Maryland: Johns Hopkins University Applied Physics Laboratory, TG-520, November 1963.
(7)	Kaula, W. M., "Improved Geodetic Results from Camera Observations of Satellites," *J. Geophys. Res.* 68(18):5183-5190, September 15, 1963.
(8)	Izsak, I. G., "Tesseral Harmonics in the Geopotential," *Nature*, 199(4889):137-139, July 13, 1963.
(9)	Private communication from W. M. Kaula: (May 1963).
(11)	Kaula, W. M., "Tesseral Harmonics of the Gravitational Field and Geodetic Datum Shifts Derived from Camera Observations of Satellites," *J. Geophys. Res.* 68(2):473-484, January 15, 1963.
(12)	Private communication from W. M. Kaula (July 1964).
(13)	Private communication from Y. Kozai: (Oct. 1962).
(14)	Private communication from Newton: (April 1962).
(15)	Newton, R. R., "Ellipticity of the Equator Deduced from the Motion of Transit 4A," *J. Geophys. Res.* 67(1):415-416, January 1962.
(16)	Kozai, Y., "Tesseral Harmonics of the Gravitational Potential of the Earth as Derived from Satellite Motions," *Astron. J.* 66(7):355-358, September 1961.
(17)	Kaula, W. M., "A Geoid and World Geodetic System Based on a Combination of Gravimetric, Astrogeodetic and Satellite Data," *J. Geophys. Res.* 66(6):1799-1811, June 1961.
(18)	Izsak, I. G., "A Determination of the Ellipticity of the Earth's Equator from the Motion of Two Satellites," *Astron. J.* 66(5):226-229, June 1961.

(19) Kaula, W. M., "Analysis of Satellite Observations for Longitudinal Variations of the Gravitational Field," In: *Space Research II*, ed. by H. C. van de Hulst, C. de Jager, and A. F. Moore: 360-372, Amsterdam: North-Holland Publishing Co., 1961.

(20) Nicolaides, J. D. and Macomber, M. M., "Passive Dynamics in Space Flight," Bureau of Naval Weapons Paper, 1962.

(21) Kaula, W. M., "Statistical and Harmonic Analysis of Gravity," *J. Geophys. Res.* 64(12):2401-2421, December 1959.

(22) Jeffreys, H., *The Earth: its Origin, History, and Physical Constitution*, 4th ed., London: Cambridge University Press, 1959:127-194.

(23), (26), (28), (29), (30) Heiskanen, W. A., and Vening-Meinesz, F. A., *The Earth and its Gravity Field*, New York: McGraw-Hill Book Co., 1958:79.

(24) Zhongolovitch, I. D., "Potential of the Terrestrial Attraction," *Akad. Nauk SSSR Inst. Theoret. Astron.* 6(8):505-523, 1957 (In Russian).

(25) Subbotin, M. F., *Kurs Nebesnoi Mekhaniki*, Vol. 3, 2nd ed., Moscow: Gostekhizdat, 1949.

(27) In: *Monthly Notices Royal Astronom. Soc. Geophys. Suppl.*, 5(55):1942.

Appendix B

Calculation of Apparent Equatorial Ellipticity From Syncom II - Orbit Drift Over Brazil, in the Gravity Field of Kaula - Combined (1964)

From Appendix A the gravity constants of the geoid of Kaula - Combined (1964) are

$$J_{22} = -1.51 \times 10^{-6} \qquad \lambda_{33} = 22.8^\circ$$

$$\lambda_{22} = -15.5^\circ \qquad J_{42} = -0.163 \times 10^{-6}$$

$$J_{31} = -1.51 \times 10^{-6} \qquad \lambda_{42} = 37.0^\circ$$

$$\lambda_{31} = 0.0^\circ \qquad J_{44} = -0.0053 \times 10^{-6}$$

$$J_{33} = -0.149 \times 10^{-6} \qquad \lambda_{44} = 35.8^\circ .$$

Thus, from these values in Equation 9a, if Syncom II had been drifting in the gravity field of this geoid, at $\lambda = -54.8^\circ$ its acceleration would have been

$$\begin{aligned} \ddot{\lambda}(-54.8^\circ)_{\text{Kaula-Comb. 1964}} = {} & 5.06\left(-1.51\times 10^{-6}\right)\sin 2(-54.8^\circ + 15.5^\circ) \\ & -0.0605\left(-1.51\times 10^{-6}\right)\sin(-54.8^\circ - 0.0^\circ) + 5.26\left(-0.149\times 10^{-6}\right)\sin 3(-54.8^\circ - 22.8^\circ) \\ & -0.0158\left(-0.163\times 10^{-6}\right)\sin 2(-54.8^\circ - 37.0^\circ) + 6.83\left(-0.0053\times 10^{-6}\right)\sin 4(-54.8^\circ - 35.8^\circ) \\ = {} & 6.793 \times 10^{-6} , \end{aligned} \qquad \text{(B-1)}$$

in units of $-12\pi^2 (R_0/a_s)^2$ rad/day^2. Similarly, at -59.2°, the acceleration would have been

$$\begin{aligned} \ddot{\lambda}(-59.2^\circ)_{\text{Kaula-Comb. 1964}} = {} & 5.06\left(-1.51\times 10^{-6}\right)\sin 2(-59.2^\circ + 15.5^\circ) \\ & -0.0605\left(-1.51\times 10^{-6}\right)\sin(-59.2^\circ - 0.0^\circ) + 5.26\left(-0.149\times 10^{-6}\right)\sin 3(-59.2^\circ - 22.8^\circ) \\ & -0.0158\left(-0.163\times 10^{-6}\right)\sin 2(-59.2^\circ - 37.0^\circ) + 6.83\left(-0.0053\times 10^{-6}\right)\sin 4(-59.2^\circ - 35.8^\circ) \\ = {} & 6.815\times 10^{-6} , \end{aligned} \qquad \text{(B-2)}$$

in units of $-12\pi^2 (R_0/a_s)^2$ rad/day^2.

Under the assumption that these accelerations arise from the second order field only, (the assumption made in Reference 4) from Equations 13a, B-1, and B-2, the apparent equatorial major axis location is calculated at

$$\lambda'_{22} = \frac{1}{2}\tan^{-1}\left\{\frac{-(-0.87965\times 6.793+0.94206\times 6.815)}{(-0.33545\times 6.815+0.47562\times 6.793)}\right\}$$

$$= -12.6^\circ \, . \tag{B-3}$$

It is seen that the error in λ_{22} made under the simplifying assumption for this geoid is

$$\Delta\lambda_{22} = \lambda_{22}\,(\text{actual}) - \lambda'_{22}\,(\text{apparent})$$

$$= -15.5 - (-12.6) = -2.9^\circ \tag{B-4}$$

(see Table 1). Under the same simplifying assumption, from Equations 13b, B-1, and B-2, an apparent magnitude of equatorial ellipticity is calculated from Syncom II drift above this geoid, as

$$J'_{22} = -1.292\times 10^{-6}\left\{\begin{matrix}(-0.87965\times 6.793+0.94206\times 6.815)^2 \\ +\,(-0.33545\times 6.815+0.47562\times 6.793)^2\end{matrix}\right\}^{1/2}$$

$$= -1.35\times 10^{-6} \, . \tag{B-5}$$

Expressed as a ratio, the error in J_{22} made under the simplifying assumption for Syncom II drift above this geoid, is

$$R_{22} = \frac{J_{22}}{J'_{22}} = \frac{-1.51\times 10^{-6}}{-1.35\times 10^{-6}} = 1.12 \, . \tag{B-6}$$

See Table 1 for the results of this simulated drift calculation for seven other recent geoids based on different kinds of gravity data (as listed in Table A-1).

Appendix C

Probable Maximum East-West Station Keeping Velocity for 24-Hour Equatorial Satellites

On the basis of an equatorial ellipticity with a gravity constant given by

$$J_{22} = -1.7 \times 10^{-6} , \tag{C-1}$$

it was calculated in Reference 4 that the maximum tangential station keeping requirement for 24-hour equatorial satellites for positions midway between the major and minor equatorial axes (i.e., at about 26°, 116°, -154° and -64°) is

$$\Delta V_t = 5.36 \text{ ft/sec-yr} , \tag{C-2}$$

which is proportional to J_{22}. This result was predicted on the basis of an entirely negligible influence of higher than second-order earth longitude gravity on 24-hour satellites. From the section of this report covering the probable range of higher order distortions to equatorial ellipticity the most conservative estimate of the probable distortion in the above Brazil - Syncom II J_{22} determination due to higher order effects is seen to be

$$J_{22} = -(1.9+0.2) \times 10^{-6} = -2.1 \times 10^{-6} . \tag{C-3}$$

It is pointed out in Reference 7 that absolute maximum accelerations on 24-hour satellites in a fourth order field are likely to be as much as 12% higher at certain longitudes than the maximum accelerations in a simple second order field.

A conservative 15% margin on the velocity from the above maximum value of J_{22} gives a very conservative value of

$$\Delta V_t \text{ (max.)} = \frac{1.15 \times 2.1 \times 5.36}{1.7} = 7.5 \text{ ft/sec-yr} \tag{C-4}$$

that may be necessary for station keeping of 24-hour satellites. From Figure 1 it is seen that the longitude location for this absolute maximum requirement is most likely to be over Indonesia in the vicinity at 116°East.

Appendix D

List of Symbols

a_0, b_0 Major and minor radii of the elliptical equator of the earth (according to the convention in Reference 1, $R_0 = (a_0 + b_0)/2$)

a_s Semimajor axis of the synchronous, or 24-hour satellite

$F(i)_{nm}$ Inclination factor for the nm gravity component expressing the reduction in mean long term longitude acceleration of an inclined 24-hour satellite over that for an equatorial 24-hour satellite

J_{nm}, λ_{nm} Gravity constants of the earth's field; magnitude and phase angle of the nm spherical gravity harmonic (Reference 1)

J'_{22}, λ'_{22} Apparent values of the second-order earth longitude gravity constants computed in the Syncom II - Brazil experiment under the simplifying assumption that the higher order longitude gravity field is negligible (See text)

R_0 Mean equatorial radius of the earth ($\sim$ 6378.2 km)

$R_{22}, \Delta\lambda_{22}$ Bias factors of J_{22} and λ_{22} resulting from the simplifying assumption that the higher order longitude gravity field is negligible (See text)

ΔV_T Tangential velocity requirement for perfect station keeping of an equatorial "synchronous" satellite

λ Geographic longitude east of Greenwich

μ_E Earth's Gaussian gravity constant ($\sim 3.986 \times 10^5$ km^3/sec^2)