INTERIM TECHNICAL REPORT

OCULOMETER

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by

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by

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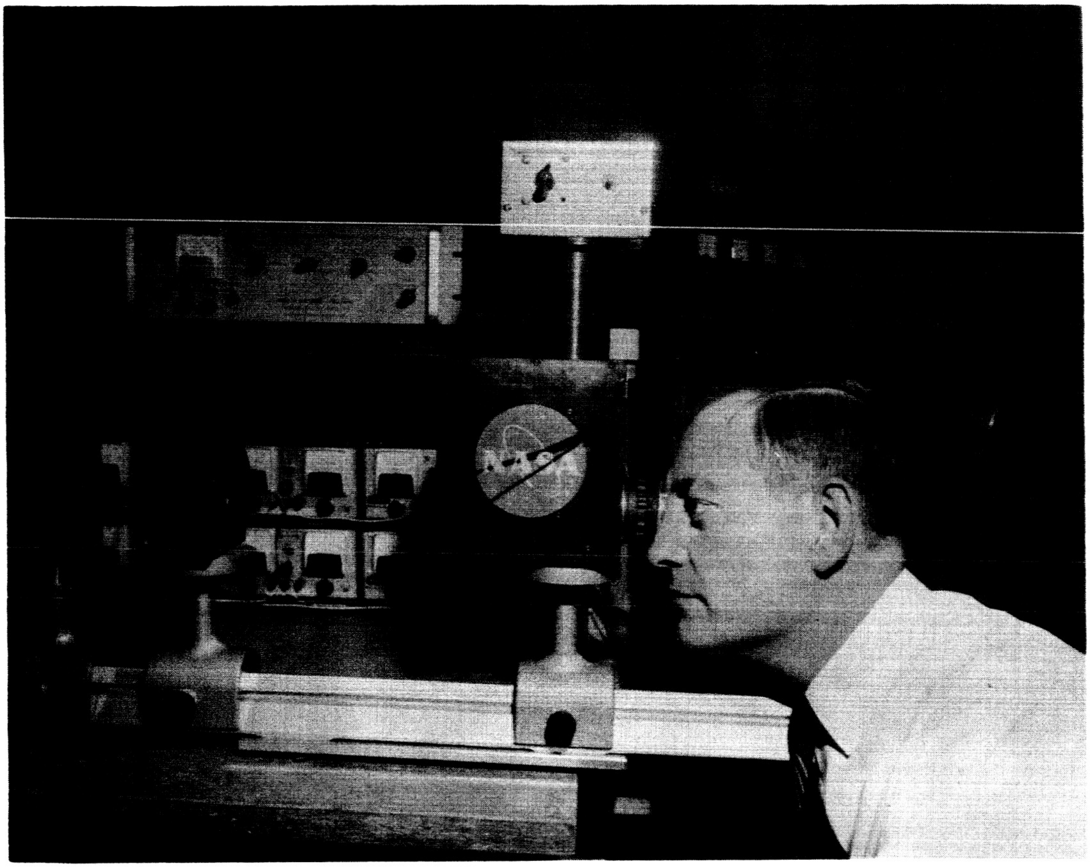


NASA/HONEYWELL OCULOMETER

1.0 INTRODUCTION

The oculometer is an instrument to measure the direction of pointing of the human eye. The instrument described in this report may be considered a telescopic oculometer, because the subject looks through an eyepiece of a conventional monocular telescope. The general principle of operation of the instrument is that the angular direction of the eye can be determined independently of small lateral motions of the head by measuring the position of a corneal reflection relative to the center of the pupil of the eye.

This report covers:

a. The original oculometer concept mechanized in the first phases of the program.

b. Preliminary results obtained.

c. System modification incorporated as a result of item b.

d. Present system performance.

e. A detailed description of the oculometer in the present form.

f. Summary of results obtained to date with recommendations for further work.

2.0 ORIGINAL OCULOMETER CONCEPT

2.1 System Description

In the original system concept, the eye looks through a standard telescopic system, consisting of an eyepiece and an objective lens, while at the same time it is irradiated with light from a bright spot on a cathode ray tube. The light from the cathode ray tube is reflected into the eyepiece and onto the eye via two beam splitters. As the spot of light moves over the surface of the cathode ray tube, the direction of light incident on the eye varies over a range of ±20 degrees in two dimensions. For a given position of the spot on the cathode ray tube, all the light incident on the eye is parallel.

The corneal reflection of the light from the cathode ray tube (CRT) will appear somewhere within the pupil area of the eye. The pupil area of the eye is imaged onto the photo cathode of a scanning photomultiplier (PM) tube (which is essentially an image dissector). The electronics system senses the pupil-iris boundary of the eye and the corneal reflection.

The position of the corneal reflection relative to the center of the pupil is determined. If the reflection is not at the center of the pupil, an error signal is generated and applied to the deflection circuits of the CRT to cause the spot (i.e. corneal reflection) to move into a different position in such a way that the corneal reflection is driven to the center of the pupil. Eye direction is then proportional, to first order, to the direction of the CRT light incident at the eye relative to the axis - i.e., proportional to the position of the spot of light on the cathode ray tube. (See Appendix A)

2.2 Optical Design

The optical system of the oculometer must perform three distinct functions. First, it must provide normal monocular telescopic vision to the subject using the oculometer. Second, it must direct light onto the eye of the subject, as he is looking through the eyepiece, from an external source to form a corneal reflection within the pupil area of his eye. Third, an image of the pupil area of the eye must be formed on the photocathode of the PM tube.

In the normal monocular function of the oculometer, light from the viewed scene is directed onto the eye from the eyepiece as illustrated in Figure 1. It is important to note the position of the exit pupil in this diagram. Light from all parts of the viewed scene passes through the exit pupil. However, at other points of the optical axis, light from various points in the viewed scene passes through different positions in the space near the eyepiece. It is evident, therefore, that if the human eye is to see easily over the wide field of view of the eyepiece, the entrance pupil of the eyeball must be located close to the exit pupil of the eyepiece, as illustrated in Figure 2. The size of the exit pupil chosen for the oculometer is approximately 5 millimeters in diameter (a standard exit pupil size for daylight viewing). This size is larger than the entrance pupil of the eye for normal levels of illumination which would be encountered in the use of this oculometer. The size of the monocular exit pupil is sufficient to permit the eye to see the viewed scene clearly and brightly if the eye is located fairly accurately relative to the axis of the eyepiece. In other words, the restricted exit pupil of the telescopic function of the oculometer constrains the position of the eye somewhat near the optical axis of the eyepiece. It is emphasized, however, that this constraint is usual and is imposed by any conventional monocular viewer.

Whenever the eye is looking through the eyepiece, the remaining parts of the oculometer optical system must insure that:

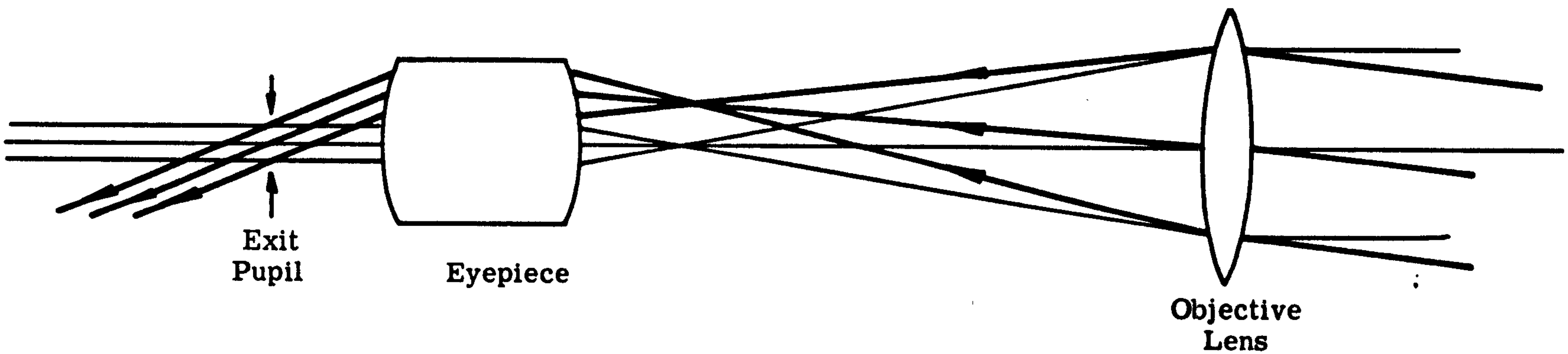


Figure 1 MONCULAR SYSTEM

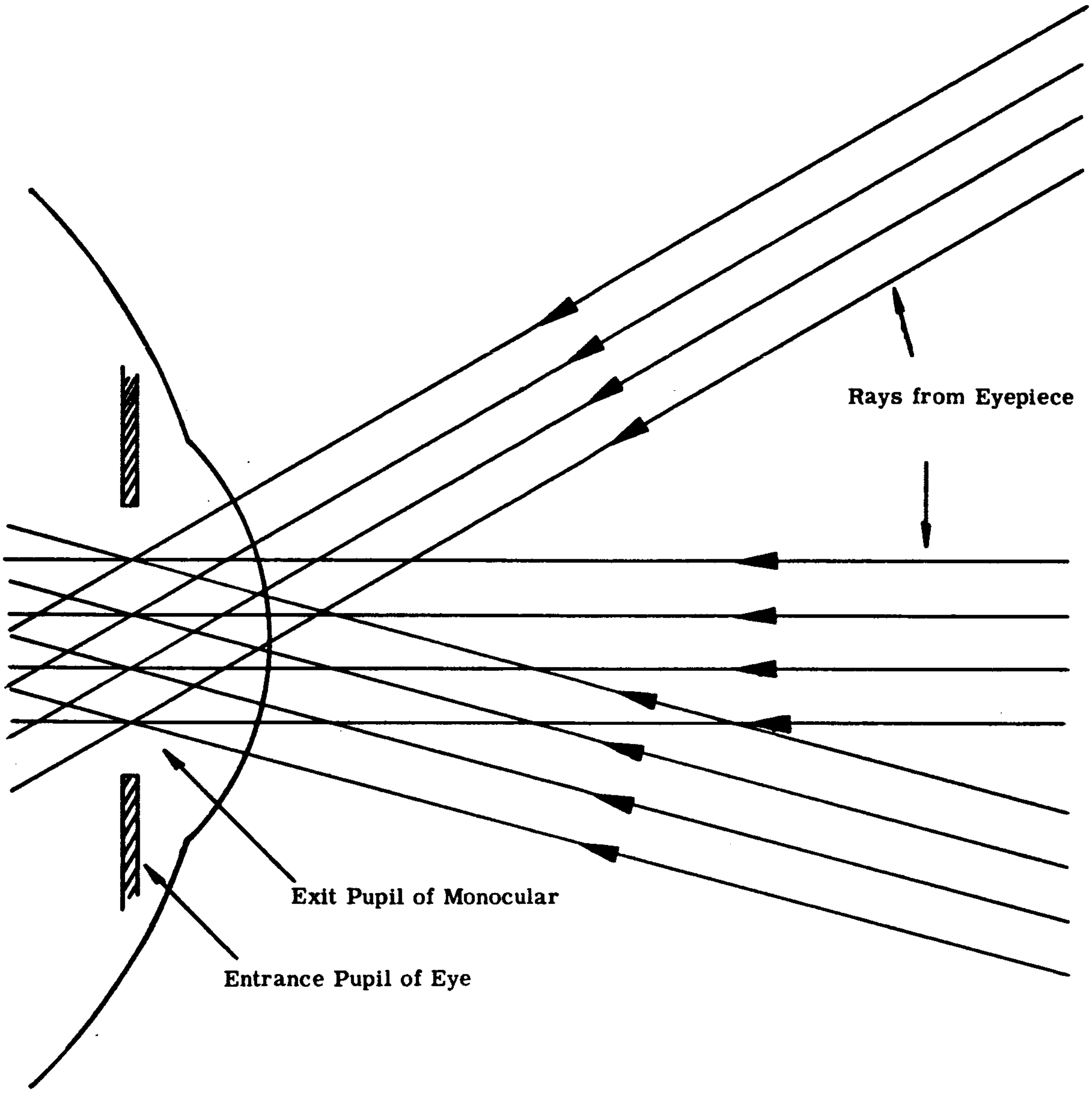


Figure 2 POSITION OF EYE RELATIVE TO MONOCULAR EXIT PUPIL

a. the eye is adequately illuminated from an external source to form a corneal reflection within the eye.

b. that the pupil area of the eye for all realistic positions of the eye, consistent with seeing through the telescope, shall be imaged onto the sensitive screen of the scanning photomultiplier tube (the nominal range of eye motions was designed for ±20 degrees angular, in two dimensions, and ±0.1 inch lateral in three dimensions).

The basic optical design that was adopted is shown in Figure 3.

In the initial concept of the oculometer, the external source of light used to form a corneal reflection was a spot of light on the screen of a cathode ray tube. This spot could be moved around in such a way as to cause the light incident on the eye to vary in angle over a range of ±20 degrees, in two dimensions, from the optical axis of the oculometer. For any given position of the spot on the cathode ray tube, a parallel beam of light is directed at the eye. The exit pupil for the corneal reflection light is shown at position 0.91 inch along the optical axis in Figure 3. This location of the exit pupil for the corneal reflection light insures that the cornea of the eye will be adequately illuminated for all possible positions of the eye as it looks over a field of view of ±20 degrees. The effective collecting aperture for the optical system that images the eye onto the photocathode of the PM tube is shown at a position 1.2 inches along the optical axis. A relatively small collecting aperture is used in order to provide good depth of focus for the optical system of the PM tube. This depth of focus is necessary because the eye is permitted an axial motion toward and away from the eyepiece of approximately ±0.1 inch. For all positions of the eye within this tolerance, a reasonably sharp image of the eye should be formed by the optical system onto the photocathode of the photomultiplier tube.

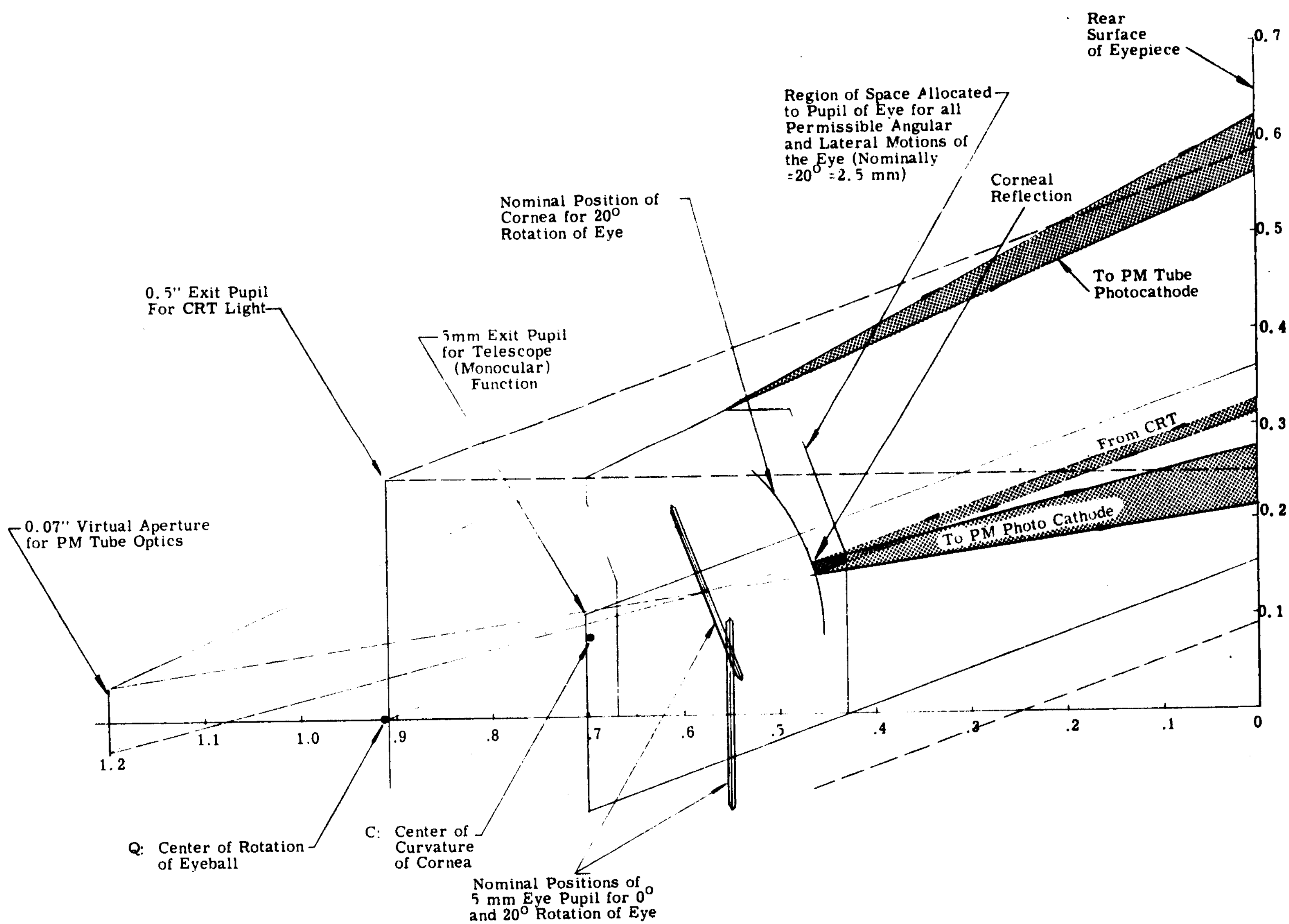


Figure 3 BASIC OPTICAL DESIGN

Figure 3 shows the region of space at the rear part of the eyepiece over which the pupil of the eye may be positioned so that the optical systems of the oculometer can perform properly. Illustrative ray traces show how various parts of the pupil area are imaged onto the photocathode of the PM tube and also how light from the cathode ray tube is directed onto the eye, reflected at the cornea, and then collected by the optical system of the tube to form an image of the corneal reflection on the photocathode.

The three optical functions illustrated in Figure 3 are essentially defined by the size and position of the exit pupil of the telescopic function, and the exit pupil for the incident light being directed onto the eye from the cathode ray tube, and also by the size and position of the virtual aperture for the collecting PM tube optics, shown at position 1.2 inches along the optical axis of Figure 3. These three functions are accomplished as illustrated in Figure 4 by appropriate positioning of optical elements on the far side of the eyepiece. The monocular exit pupil is the image, in the eyepiece, of the defining aperture of the monocular optical system. The exit pupil for the CRT light is, correspondingly, the image in the eyepiece of the auxiliary CRT lens as shown in Figure 4. The virtual aperture for the PM tube optics is the image, in the eyepiece, of the auxiliary PM tube lens used to focus an image of the eye on the photocathode of the PM tube. (Figure 4)

The final consideration in the basic design of the oculometer is the location of two beam splitters on the far side of the eyepiece to separate the axes of the three optical systems previously described. The general positioning of the beam splitters in relation to the eyepiece in the auxiliary lenses is shown in Figure 5. More exact information about the size and position of the beam splitters is contained in Drawing No. SK 87378.

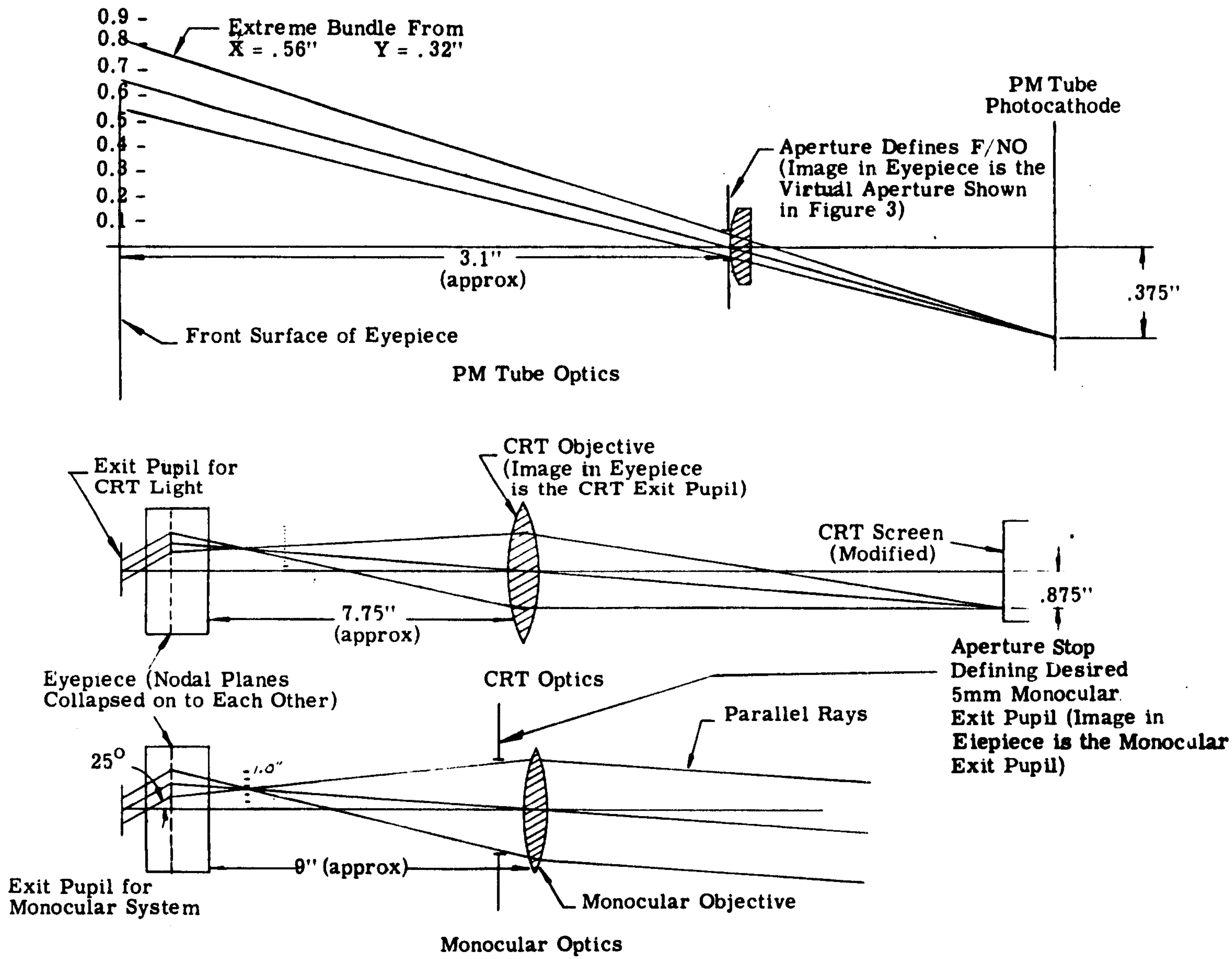


Figure 4 THREE OPTICAL FUNCTIONS

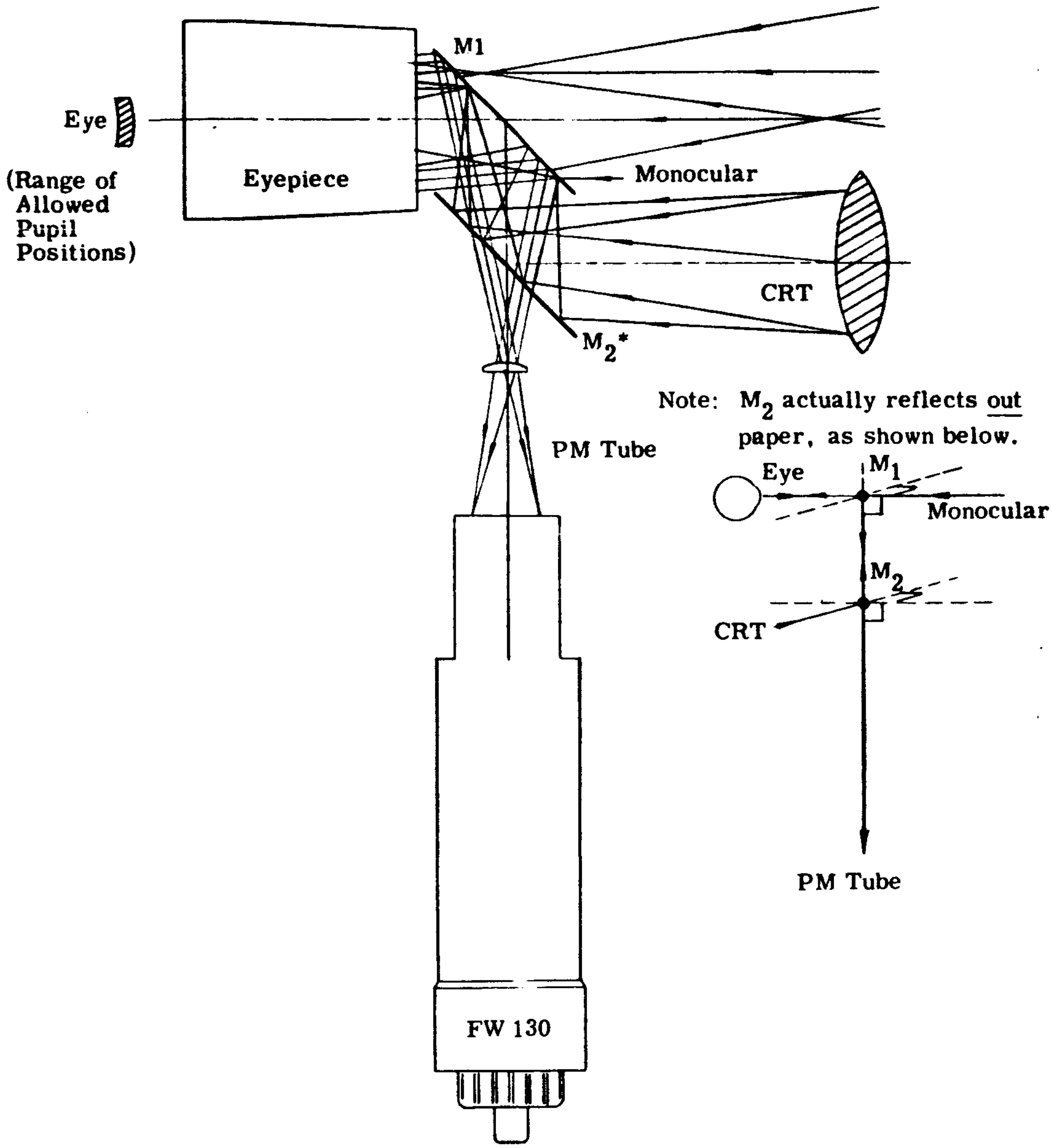


Figure 5 BEAM SPLITTER ARRANGEMENT

A government surplus wide angle Erfle eyepiece was selected for the oculometer optical system. Exact ray traces were performed, by digital computation, using detailed design data for the eyepiece, for the three optical system functions described above. The auxiliary lenses were designed, and their performance evaluated in the optical system, by ray tracing with the computer. The detailed optical design information, relating to the eyepiece and the auxiliary lenses, is given below.

2.2.1 Erfle Eyepiece

Refer to Figure 6 for nomenclature (units are inches):

Surface	Clear Aperture	Radius	Curvature	Thickness	Glass Type
1	1.711	-6.347	-.1575548	.067	617366
2		1.790	.5586592	.572	517645
3		-1.790	-.5586592	.020	Air
4	1.720	2.138	.4677268	.611	517645
5		-1.422	-.7032349	.109	649338
6		-3.492	-.2863688	.016	Air
7	1.414	1.707	.5858231	.565	517645
8		-1.140	-.8771930	.087	649338
9		-2.803	-.3567606	.657	Air

2.2.2 Corneal Reflection Lens (Honeywell #1040)

Refer to Figure 7 for nomenclature (units are inches).

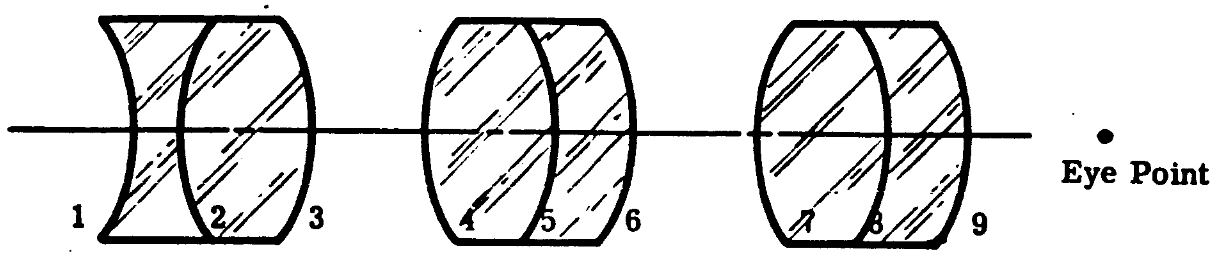


Figure 6 ERFLE EYEPIECE

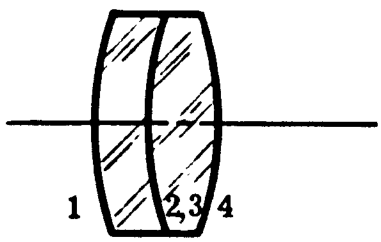


Figure 7 CORNEAL DEFLECTION LENS

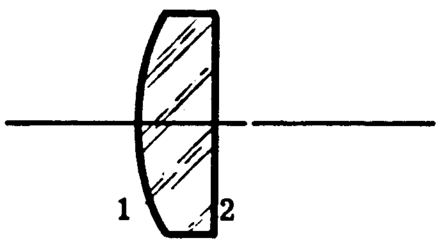


Figure 8 PM TUBE LENS

Surface	Radius	Thickness	Clear Aperture	Material
1	4.086 ±.001	.3060 ±.001	1.5	649338
2	1.838 ±.001	.000	1.5	Cement
3	1.838 ±.001	.7141 ±.001	1.5	517645
4	-2.653 ±.001		1.5	

E.F.L. = 3.688"

Object distance from surface 1 vertex = 5.072"

Image distance from surface 4 vertex = 10.927"

2.2.3 PM Tube Lens (Honeywell #1041)

Refer to Figure 8 for nomenclature (units are inches)

Surface	Radius	Thickness	Clear Aperture	Material
1	1.0041 ±.001	.15 ±.001	.5	620603
2	∞		.5	

Distance to PM tube 1.627"

Distance from eyepiece 3.096"

2.3 Mechanical Design

To accomplish the program objectives at a minimum cost, the simplest possible mechanical design concepts were utilized. The optical system was mounted on low-cost Ealing optical benches. The general arrangement is illustrated in Figure 9. The heart of the oculometer is the black box, shown in Figure 10, mounted on a table at the intersection of the two

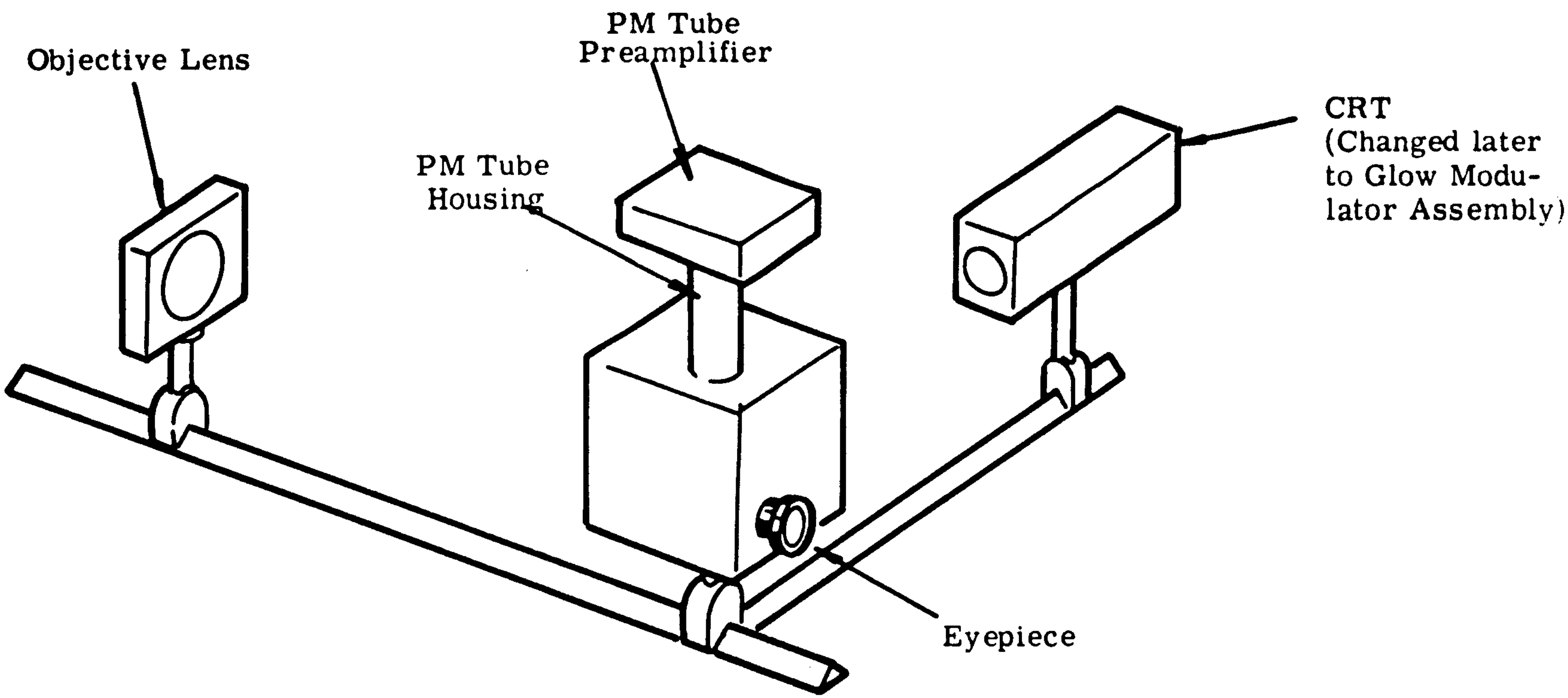


Figure 9 GENERAL VIEW OF OCULOMETER

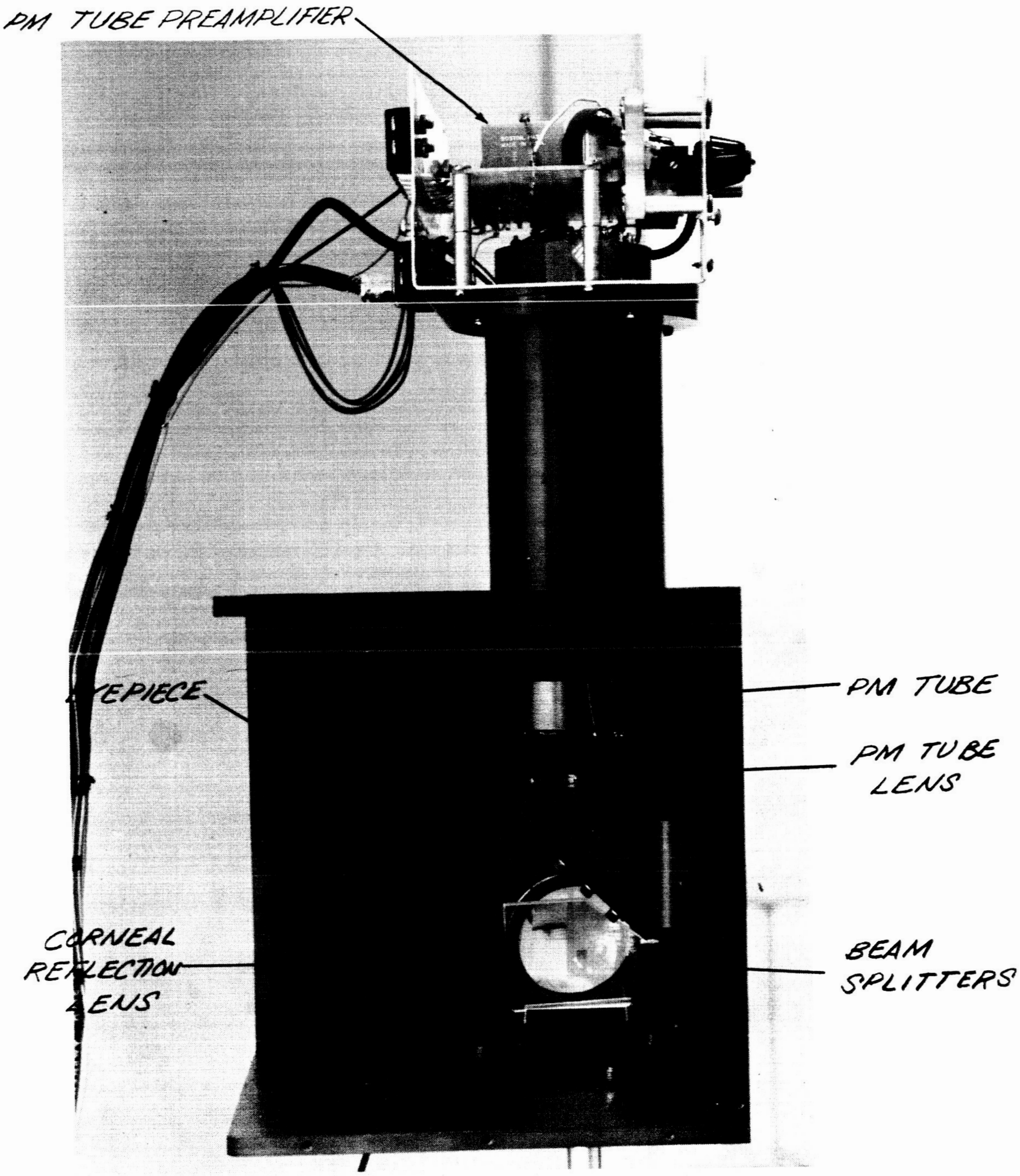


Figure 10 OCULOMETER, MAIN OPTICAL UNIT

optical benches. This box contains the eyepiece, the beam splitters, the auxiliary lenses and the scanning photomultiplier tube with its preamplifier. The objective lens for the monocular system is mounted on a carriage at one end of one of the optical benches. The CRT in the original concept was mounted on a table at the end of the other optical bench. The electronics were mounted on a separate console with connections running from the PM tube and to the CRT. Honeywell general laboratory power supplies were used to power the oculometer electronics and the PM tube.

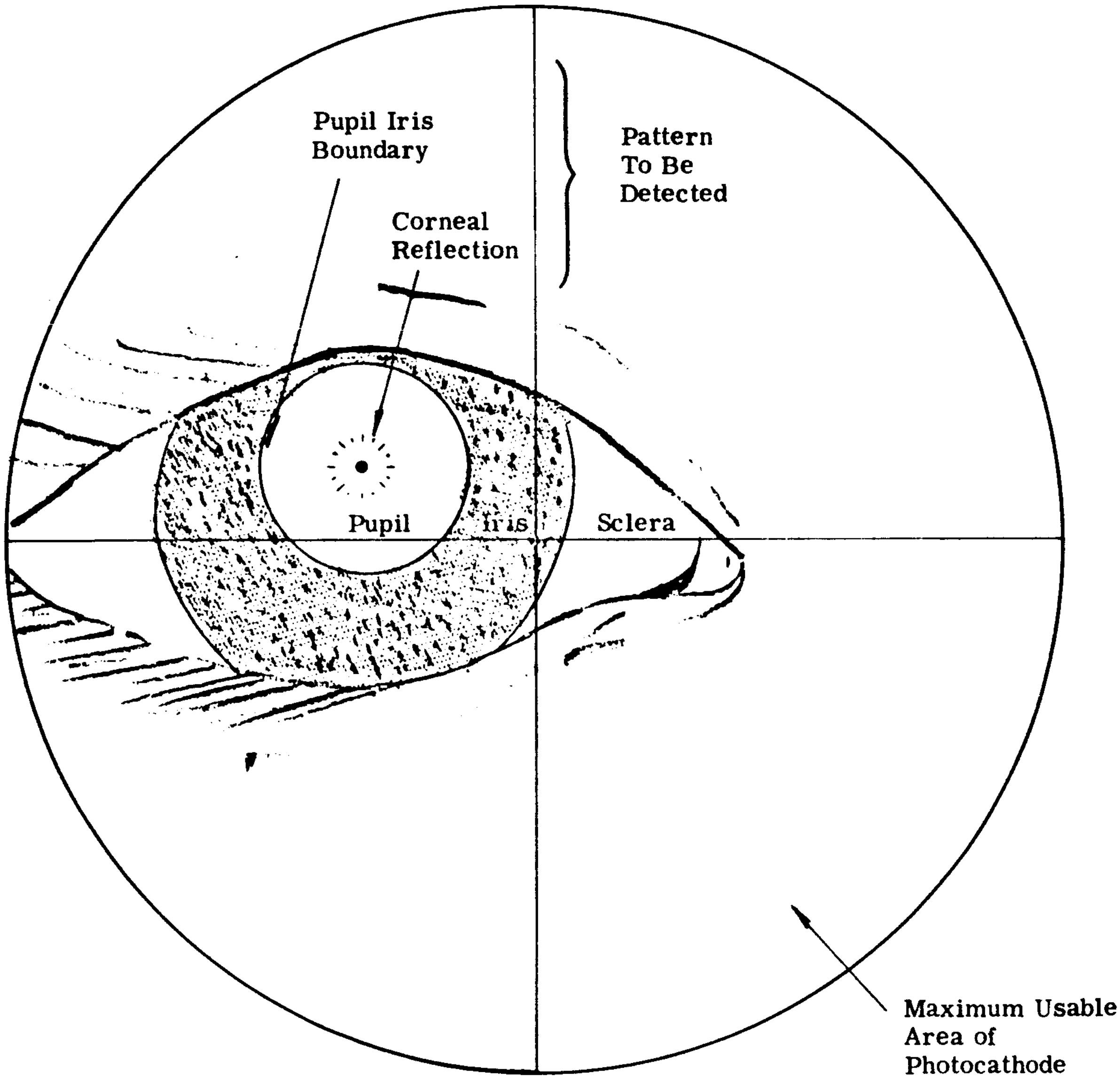
2.4 Electronic System

The primary function of the electronics is to measure the position of the corneal reflection relative to the pupil and to generate an error signal to cause the CRT spot to move until this corneal reflection appears at the center of the pupil. Eye direction may then be read out as a function of the position of the spot on the CRT and the position of the pupil image on the photomultiplier photocathode.

The electronic tracking task can be described entirely in terms of the optical image formed on the photocathode of a PM tube. The pattern in this image that will be used in the tracking system is:

1. the boundary between the black, circular, pupil area and the bright iris.
2. the bright corneal reflection spot, within the pupil.

This pattern may be located anywhere on the photocathode area (Figure 11).



Figue 11 IMAGE OF EYE ON PHOTOCATHODE

The electrons accelerated from the photocathode are focused by an electrostatic lens system onto an aperture plate. This plate has only a small clear open area - of special design - through which electrons may pass onto a multiplier section of the tube (Figure 12).

The electron image may be laterally deflected, in two dimensions, relative to the aperture plate, by a magnetic deflection yoke on the outside of the tube. Thus, selected portions of the electron image can be moved onto the clear portion of the aperture plate. This action may be most easily described in terms of (virtual) motion of the clear part of the aperture plate over the pupil area of the eye.

Figure 13 shows an aperture scan pattern, rotating at 120 rps, designed to locate the pupil position. If the pupil is not in the nominal position, a 120 cps sinusoidal component will appear in the output of the PM tube as a result of the scan. The amplitude and phase of this 120 cps component will define the required vector position. Pupil position information is used to center the aperture scan right over the image of the pupil.

Figure 14 shows a scan pattern designed to locate the position of the corneal reflection spot. If the corneal spot is not at the center of the aperture scan pattern, a 120 cps component will appear in the output of the PM tube. The amplitude and phase of this component will define the required vector position.

To accomplish both of the above tracking tasks simultaneously, time division multiplex is employed. The actual scan pattern is shown in Figure 15. Six samples of the pupil position and six samples of the spot position are made every cycle of the 120 cps rotating scan.

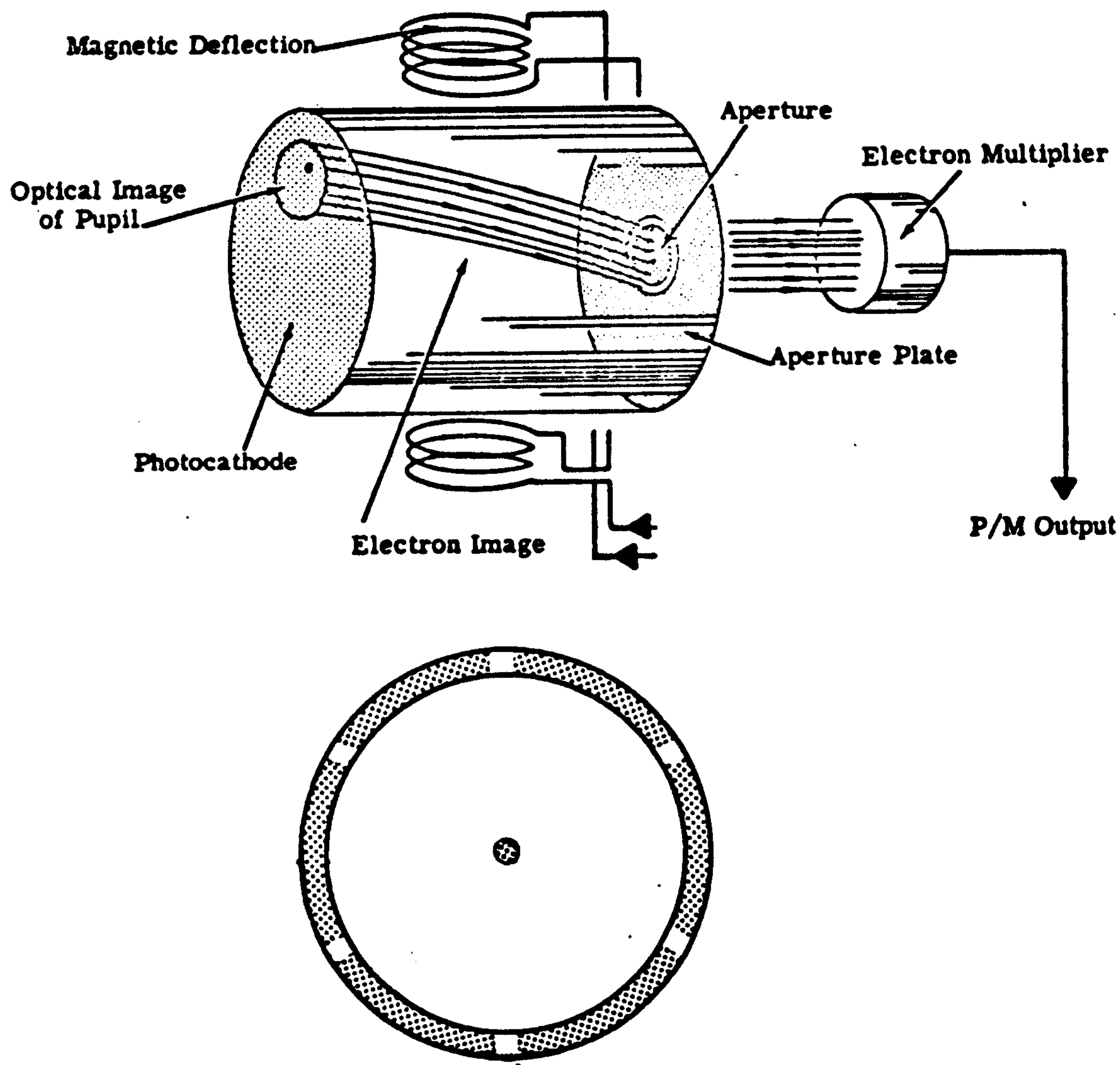


Figure 12 SCANNING PHOTOMULTIPLIER

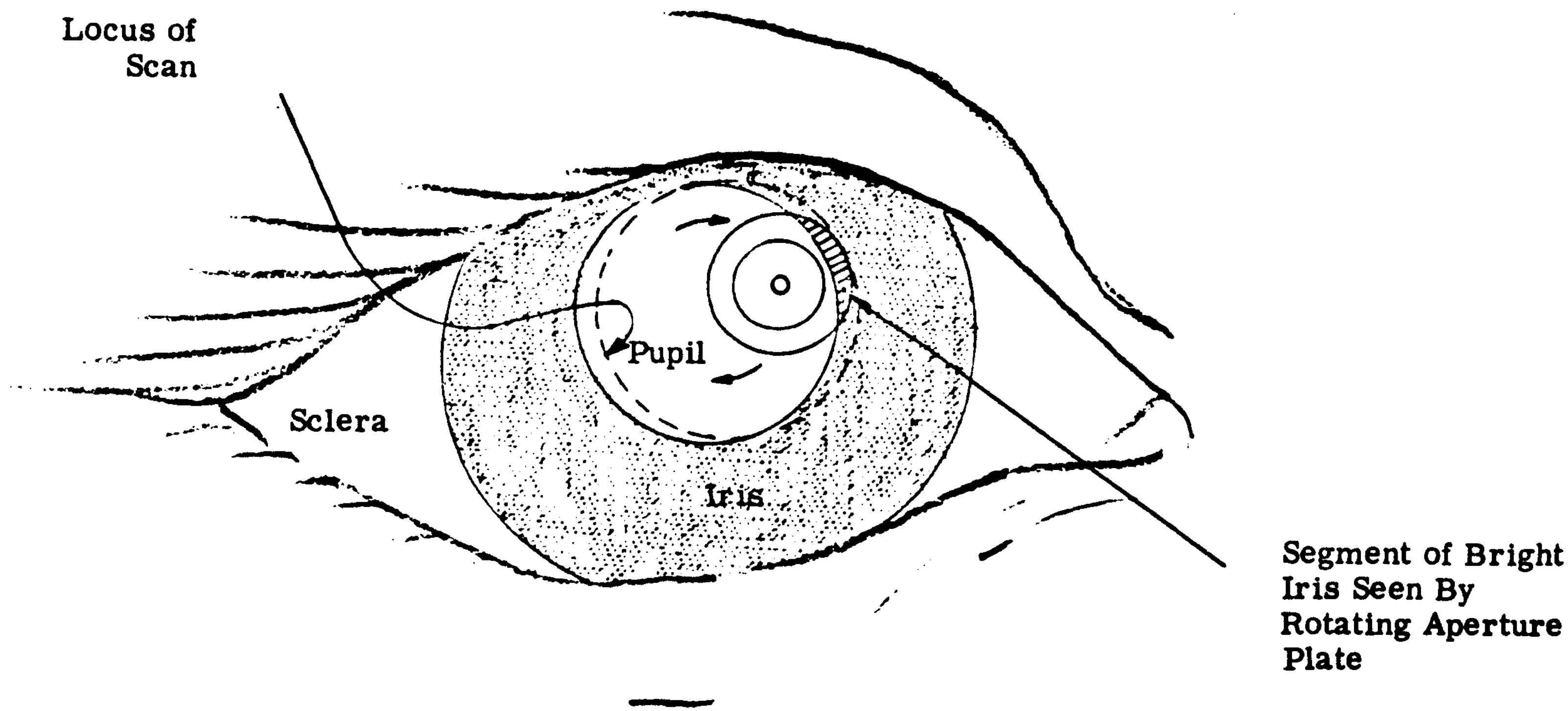


Figure 13 PUPIL SCAN OF APERTURE PATTERN

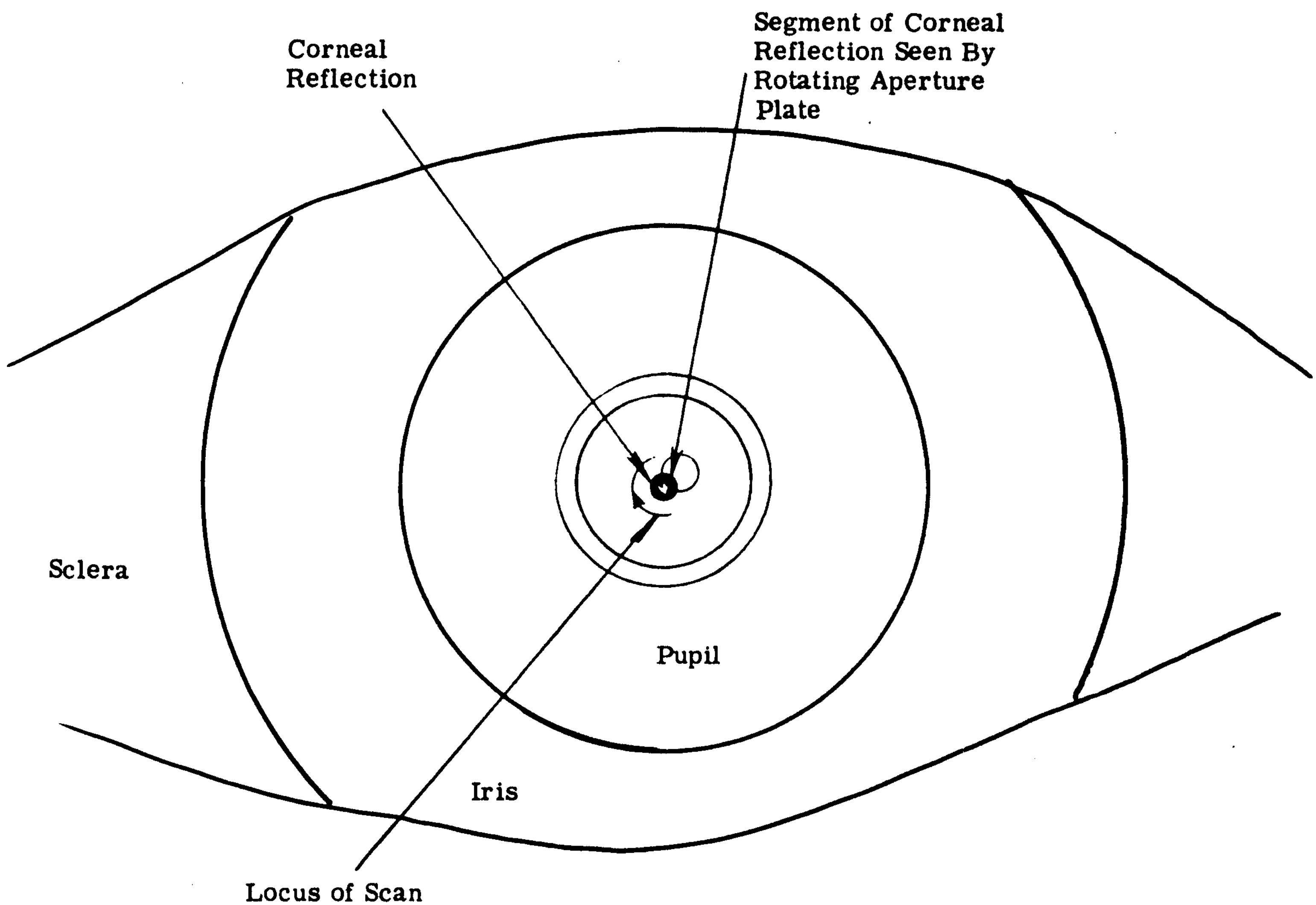


Figure 14 CORNEAL SPOT SCAN OF APERTURE PLATE

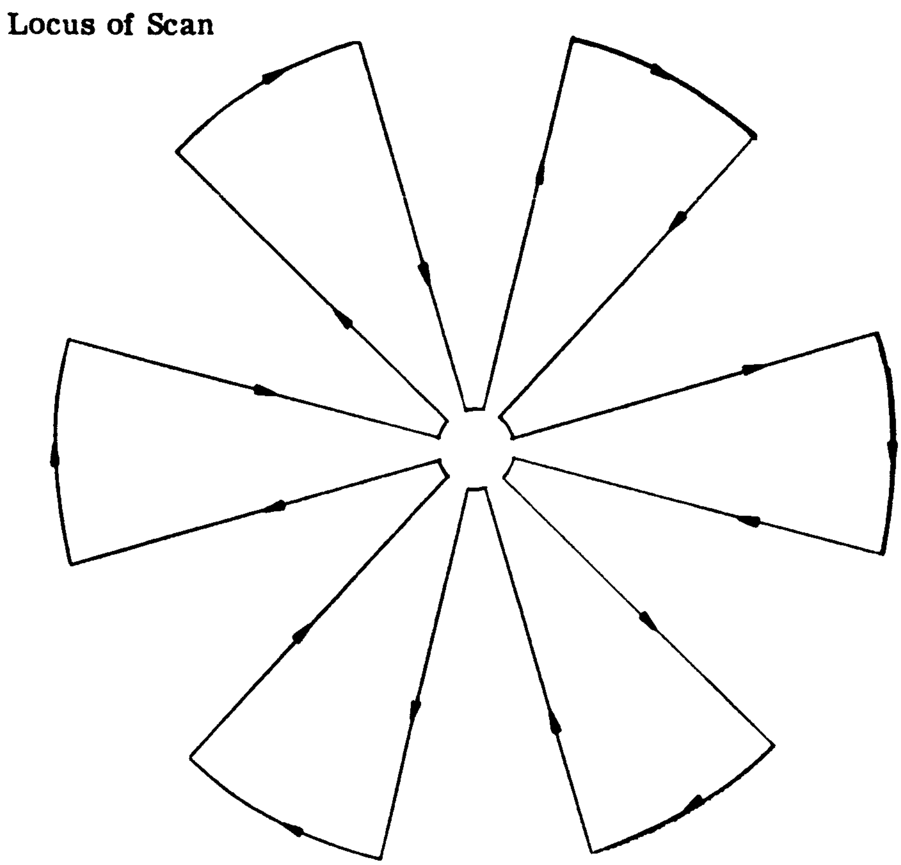


Figure 15 APERTURE SCAN PATTERN

The resulting output of the PM tube will, typically, be as shown in Figure 16. It may be noted that a 720 cps component is present due to a difference between the d-c level of the pupil 120 cps waveform and the d-c level of the spot 120 csp waveform. The magnitude of this difference in d-c level is proportional to the amplitude (radius) of the pupil tracking scan relative to the radius of the eye pupil, and can be used to control this amplitude. Such control is necessary because of changes in the radius of the subject's pupil (2.25 mm to 5.0 mm).

The two time division multiplexed components of 120 cps in the output of the PM tube are separated by switching the output, between two channels, at a rate of 720 cps. Each channel is separately demodulated and the in-phase and out-of phase components are applied to the respective x and y deflection circuits.

There are three system control loops (Figure 17).

1. pupil amplitude
2. pupil position
3. spot position

Each loop contains a single integrator in the feedback path and thus loop response will be essentially that of a simple RC network.

When all the control loops have performed their function, the corneal reflection will be exactly at the center of the pupil and eye direction may then be computed from the position of the spot on the CRT screen and the position of the pupil image on the photocathode of the PM tube.

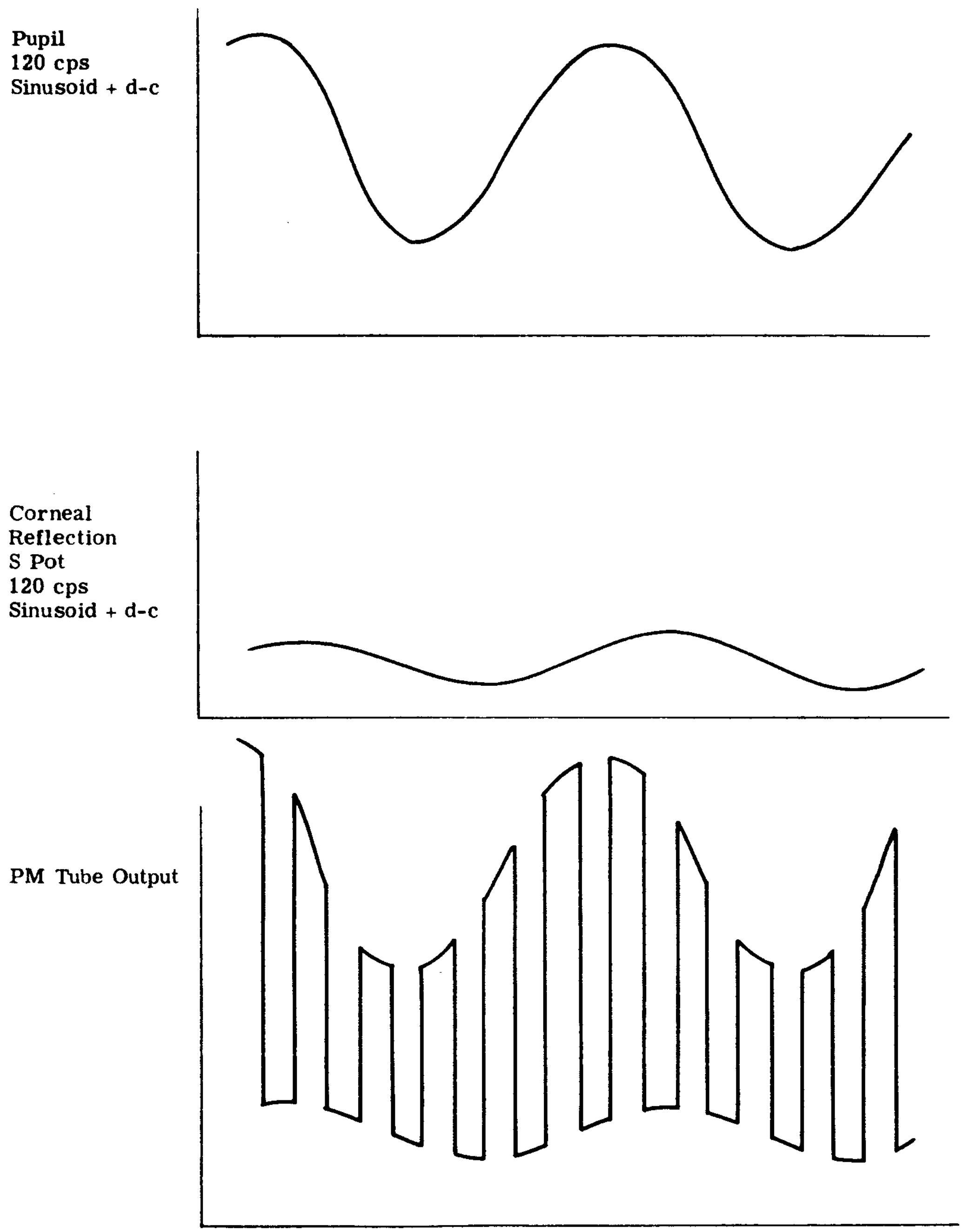


Figure 16 PM TUBE OUTPUT SIGNAL

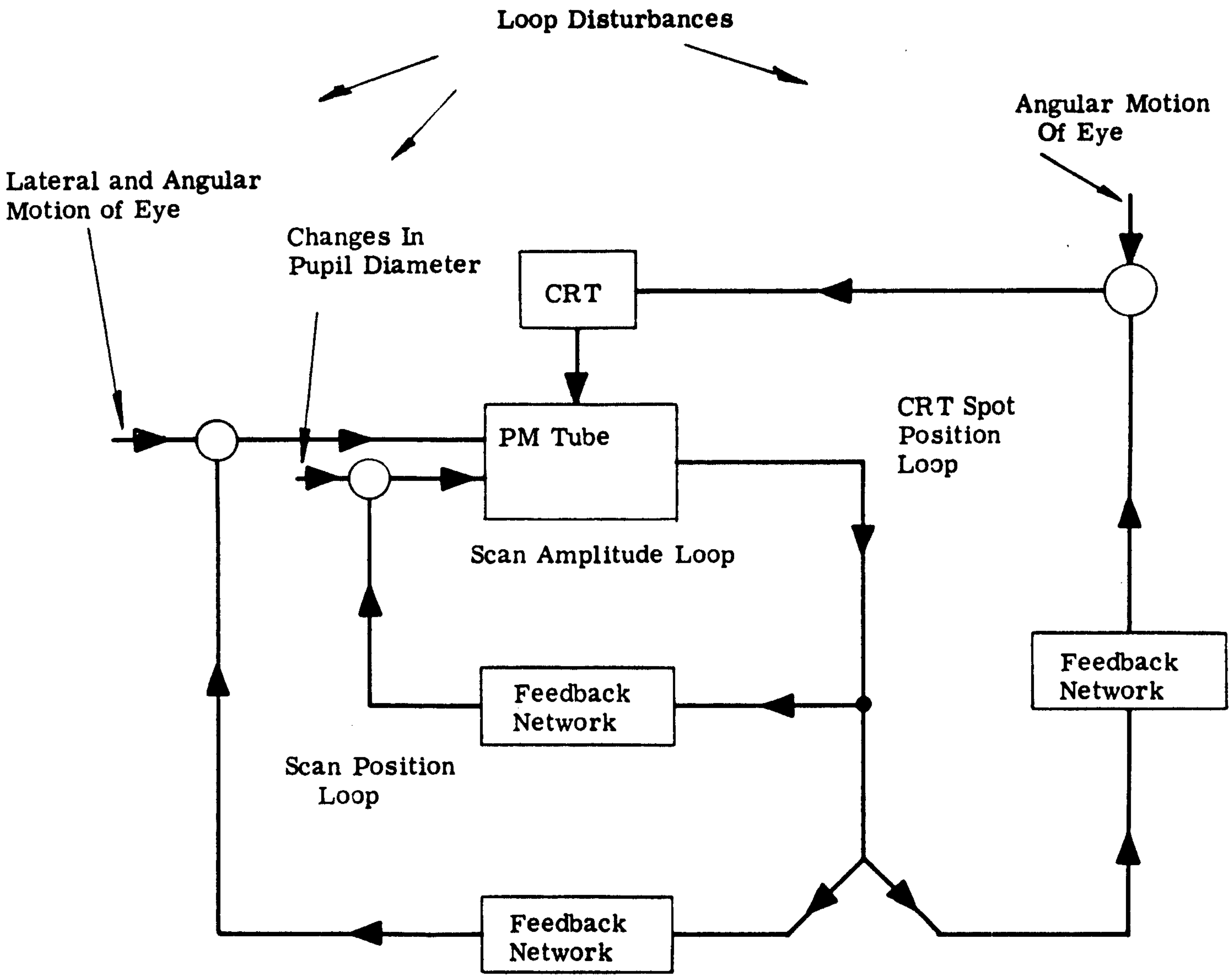


Figure 17 THE THREE OCULOMETER CONTROL LOOPS

3.0 PRELIMINARY RESULTS

Fabrication of the breadboard, in its original form, was essentially completed within six months. The pupil/iris boundary of an artifical eyeball was tracked.

Preliminary evaluation indicated two technical problems:

First, the useful size of the PM tube photocathode proved to be smaller than expected.

The ITT specification for the PM tube refers to a maximum usable photocathode diameter of 0.75 inch. However, tests with the PM tube showed that photocathode sensitivity falls off rapidly beyond 0.20 inch from the center. (See Appendix B) According to ITT, this is normal for the tube. Thus, the useful photocathode diameter is less than 0.50 inch.

The effect, in the present design, of the smaller than expected photocathode area is to restrict the area over which the eye can move and still be tracked. No corrective action was taken during this report phase of the program because the system limitation involved does not directly affect the primary purpose of the program. The limitation can be completely overcome by redesigning the associated optical system to give increased magnification, and/or by using an image dissector instead of the PM tube (Appendix B).

Second, spurious reflections of the light from the CRT screen are formed by the various surfaces in the eyepiece.

The optical image formed at the position of the PM tube photocathode was observed visually. For certain positions of the CRT spot on the screen, the image of its corneal reflection is obscured by spurious reflections of the spot in the various air/glass surfaces within the eyepiece, and also by scattering at the surface of one of the beam splitters. These reflections interfere with the tracking of the corneal reflection.

Antireflection coatings were applied to the eyepiece lenses, but failed to solve the problem. The eyepiece is government surplus, and the lenses already had a coating on them, but not the type of coating required in the present case. Because the old coating could not be removed, there exists doubt as to the actual antireflection performance of the coatings that were applied on top of the old coatings.

4.0 SYSTEM MODIFICATION

The preliminary results showed that the major technical problem was that of spurious reflections of the CRT light formed by the air-glass surfaces of the eyepiece and beam splitters (Figure 18). This problem persisted, in spite of antireflection coatings applied to the lenses. In view of the magnitude of the reflection problem, it was decided to overcome it by providing a means of discriminating true from false reflections, rather than make further attempts to reduce the intensity of the spurious reflections.

Various discrimination techniques were considered. Basically, they all involve chopping the true corneal reflection so that it can be discriminated from false reflections by electronic filtering.

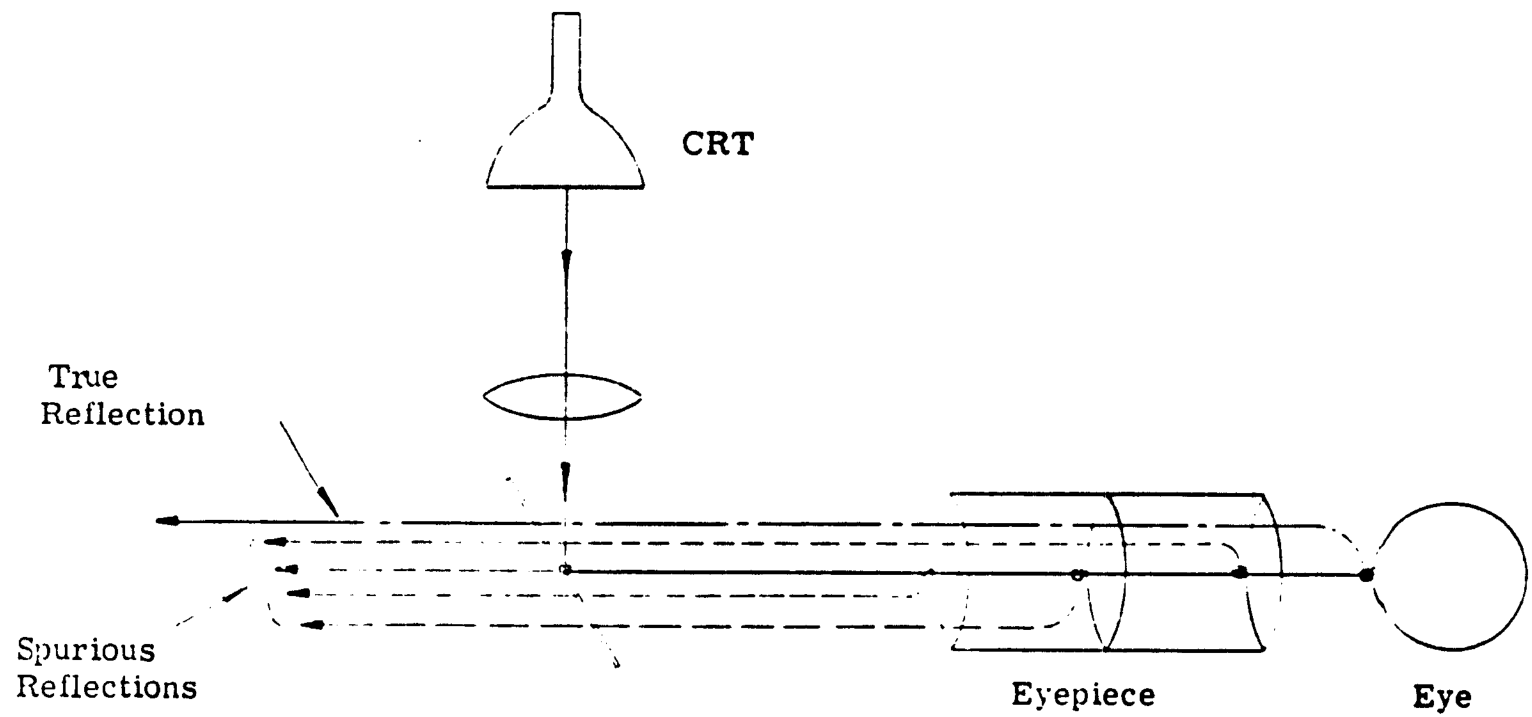


Figure 18 REFLECTION PROBLEM

The simplest chopping method - a rotating chopping wheel - could not be used on the oculometer because the wheel would have to be located between the eye and the eyepiece. There is no space available to safely locate the wheel in this position.

A novel, no-moving-parts, chopping system has been devised for the oculometer. The CRT spot is replaced by two glow modulator tubes which are arranged, with a beam splitter, so that the images of the two bright spots of light produced by these tubes are projected coincidently into the oculometer. Blue and red transmitting filters are placed near the tubes as shown in Figure 19. Each tube is switched on and off at a 14.4 kc rate in such a way that when one is on, the other is off. The result is that the color of the light being projected onto the oculometer (to form a corneal reflection) is switched from red to blue 14,400 times per second. The relative intensity of the red and blue tubes is adjusted so that the spurious reflections are of equal intensity for both lamps. This causes the 14.4 kc component in the PM tube output, due to spurious reflections, to be zero. A blue stopping filter is placed between the eye and the eyepiece. This filter causes the corneal reflection to be modulated at 14.4 kc because only light from the red lamp can reach the eye and return to the PM tube.

The true corneal reflection signal can thus be discriminated from the spurious reflections by a bandpass filter centered at 14.4 kc.

The actual spectrum used, and the electronic filtering circuitry are shown in more detail in Figures 20 and 21.

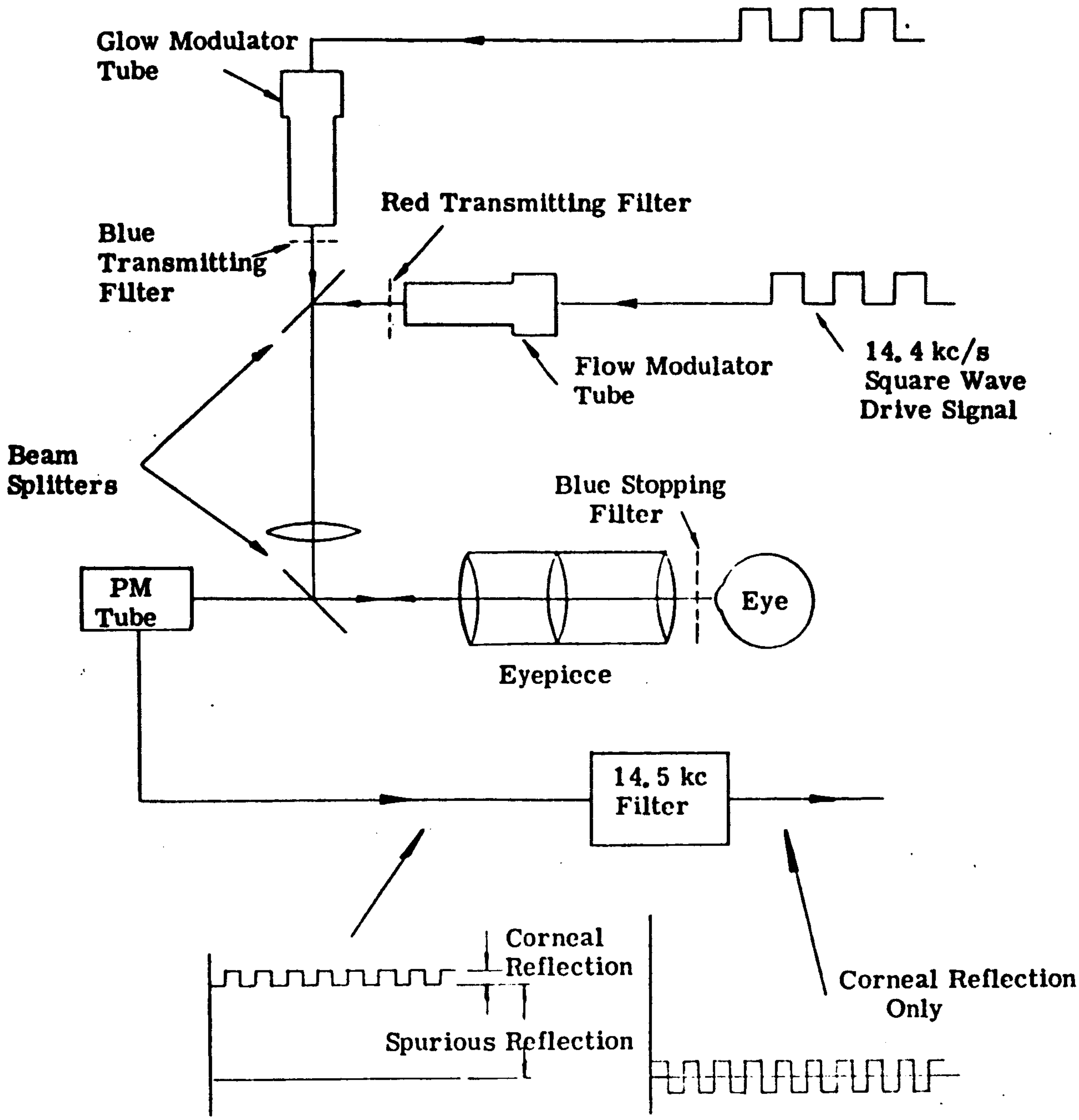


Figure 19 COLOR CHOPPER

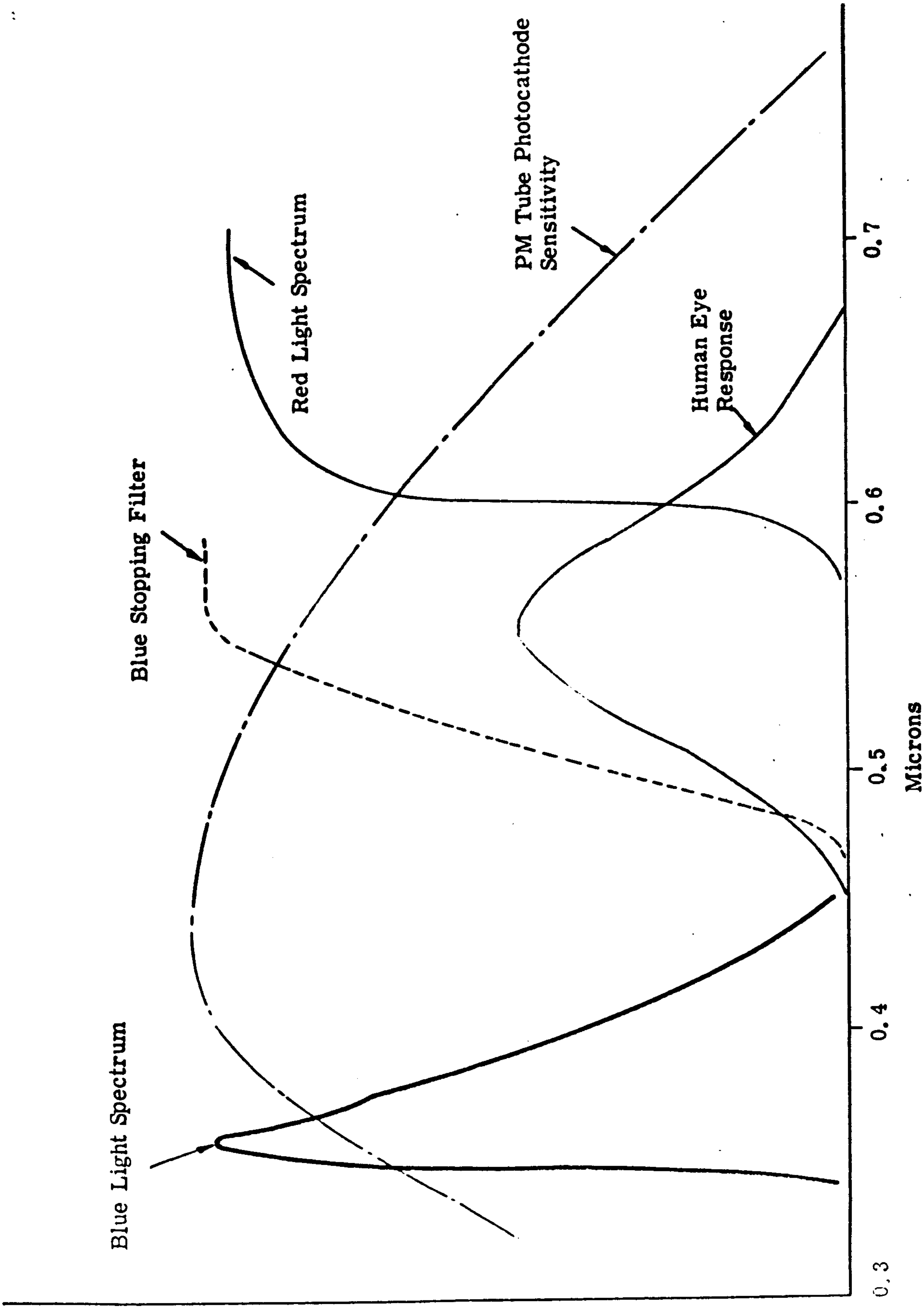


Figure 20 SPECTRA USED IN COLOR CHOPPER

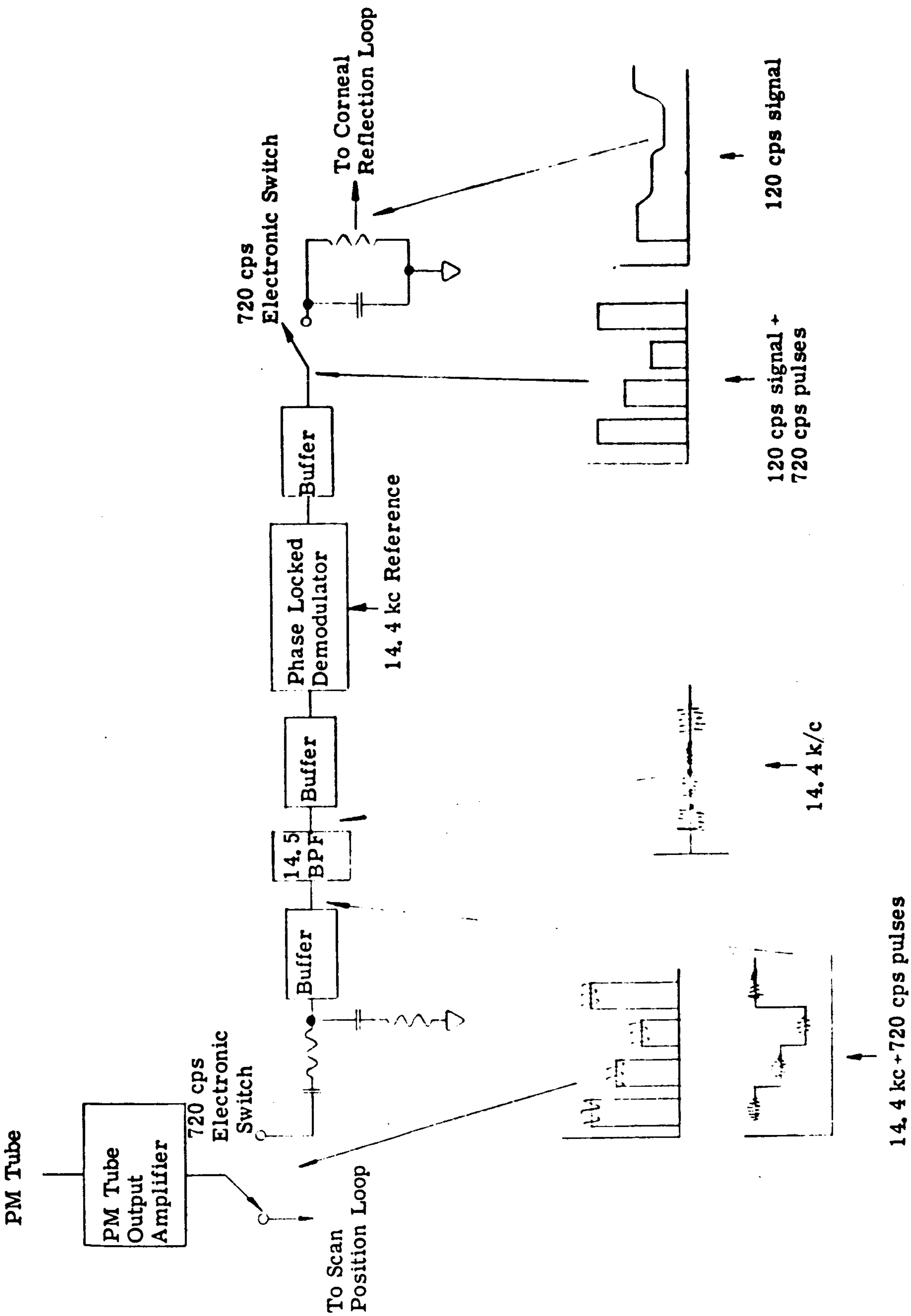


Figure 21 ELECTRONIC SYSTEM COLOR CHOPPER

It should be noted that a non-linear filtering technique is used in the electronics to reduce the intensity of the 720 cps switching pulse (an inevitable consequence of the time division multiplex system that is used to encode two tracking channels on one signal channel). The RC networks following the electronic switches charge up rapidly through the low source impedance feeding the input to the switch, but they discharge very slowly when the switch is opened and the low impedance is removed. (Further suppression of the 720 cps square wave is provided by conventional linear electronic filtering - not shown in Figure 21.) Also, the choice of the chopping frequency, 14.4 kc, is not entirely arbitrary. 14.4 kc is the 20th harmonic of 720 cps. A symmetrical square wave has no even harmonics, so the spectrum around 14.4 kc is free of the effects of the 720 cps switching pulses. The 19th and 21st harmonics will be passed by the 14.5 kc* bandpass filter because the filter pass band must be made of the order of 1 kc to avoid introducing harmful phase shift into the 120 cps information encoded into the signal.

The 19th and 21st harmonics will appear as 720 cps, after the phase locked demodulator. A narrow bandstop filter (not shown in Figure 21) centered at 720 cps will remove these frequencies.

The smoothed output of the second electronic switch, shown in Figure 21, is mainly the 120 cps modulation which contains the desired corneal reflection position information.

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*14.5 kc nearest available stock filter to 14.4 kc

The non-mechanical chopping system discriminates against all spurious reflections except those caused by the last surface of the blue stopping filter placed at the eye. In the present system, this filter will be slightly tilted so that the reflections from it will not be seen by the PM tube.

A consequence of the system modification described above is that only a fixed direction of light can be beamed onto the cornea to form a corneal reflection. In the original concept, this direction was to be adjusted (by moving the spot on the CRT screen) in order to bring the corneal reflection to the center of the pupil. With a fixed spot, the corneal reflection will move around, within the pupil, as the eye rotates. The oculometer system has been modified to track the corneal reflection with the pupil and compute eye direction as a function of the position of the reflection relative to the pupil.

5.0 PRESENT PERFORMANCE

Using the moderate degree of peripheral eye illumination produced by three miniature lamps around the edge of the eyepiece, the pupil/iris boundary of the human eye can be tracked. The range of lateral and angular motions for which tracking can be maintained is limited due to the poor characteristics of the PM tube photocathode (see Appendix B). Two typical eye tracking records (based on tracking of the pupil/iris boundary) are shown in Figure 22 and 23. These illustrate the excellent time response, low noise level, and repeatability, of the pupil tracking system.

By placing the glow modulator tube assembly directly in front of the eyepiece together with a suitable small aperture, it has been possible to test the corneal reflection demodulation and tracking electronics.

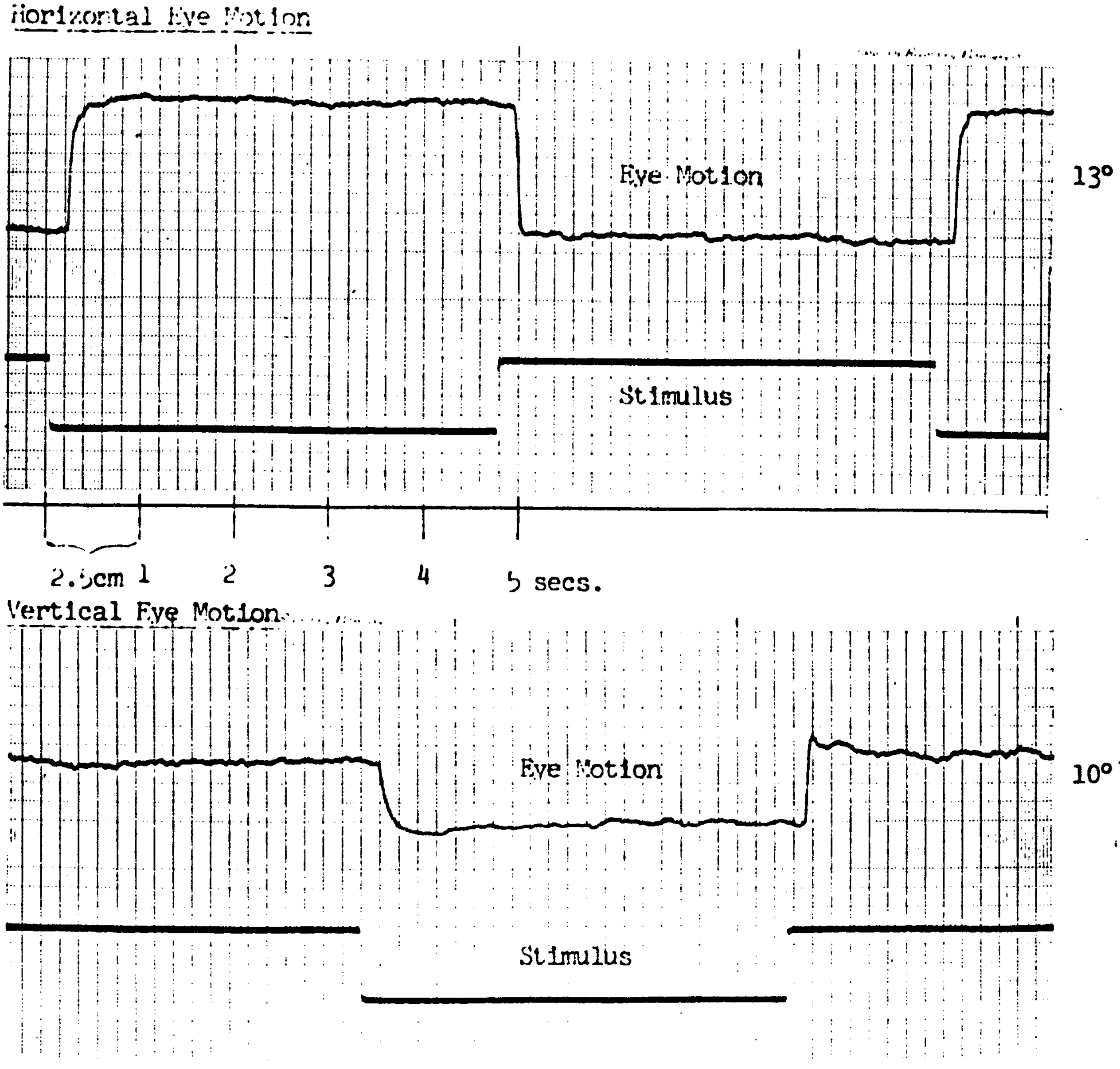

Figure 22 VERTICAL AND HORIZONTAL TRACKING OF THE PUPIL/IRIS BOUNDARY

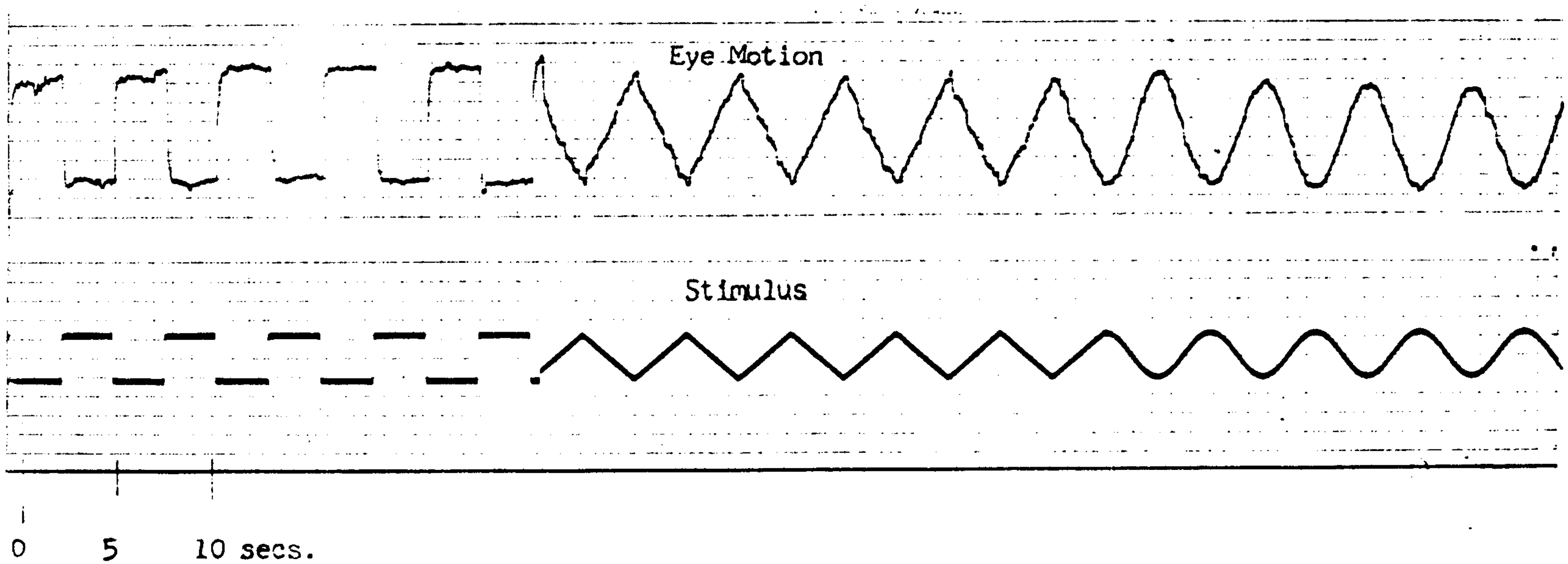

Figure 23 80 SECS. CONTINUOUS TRACKING OF THE PUPIL/IRIS BOUNDARY OF THE HUMAN EYE

Excellent tracking is achieved. Reliable tracking of the true corneal reflection has not been achieved due to insufficient signal, excessive noise and imperfect color cancellation of the spurious red and blue chopped light. (Corneal reflection tracking has been achieved using both an artificial and human eyeball but is noisy and unstable. Track is easily lost and the system tends then to lock on to false signals.)

A more detailed discussion of the corneal reflection tracking system is presented in Appendix C.

6.0 DETAILED BREADBOARD DESCRIPTION

6.1 Mechanical

6.1.1 Electronics Packaging

The oculometer electronics and controls are packaged in a standard open vertical Bud cabinet frame (Figure 24). The controls, which determine the mode of operation and initial conditions, are mounted on the front panel. The electronic circuits are assembled on plug-in epoxy vector boards housed in three Varipac card rack sections. The signal, and power input/output functions, are connected to the oculometer electronics assembly through terminal strips at the rear of the cabinet.

The three vector Varipac racks contain a total of sixteen vector cards; namely, five vector cards located in the first Varipac rack, VR1 (located at the top of the cabinet frame); seven cards located in the second card rack, VR2, and four vector cards located in the third card rack, VR3. The vector cards are numbered 1 through 14, right to left, as viewed from the front and progress from VR1 to VR3. The vector card in the upper right corner of the cabinet is vector card 1. Card 14 is second in from the right on the bottom vector rack, VR3. Vector cards 15 and 16 are numbered from top to bottom respectively and located in the left side of VR3.

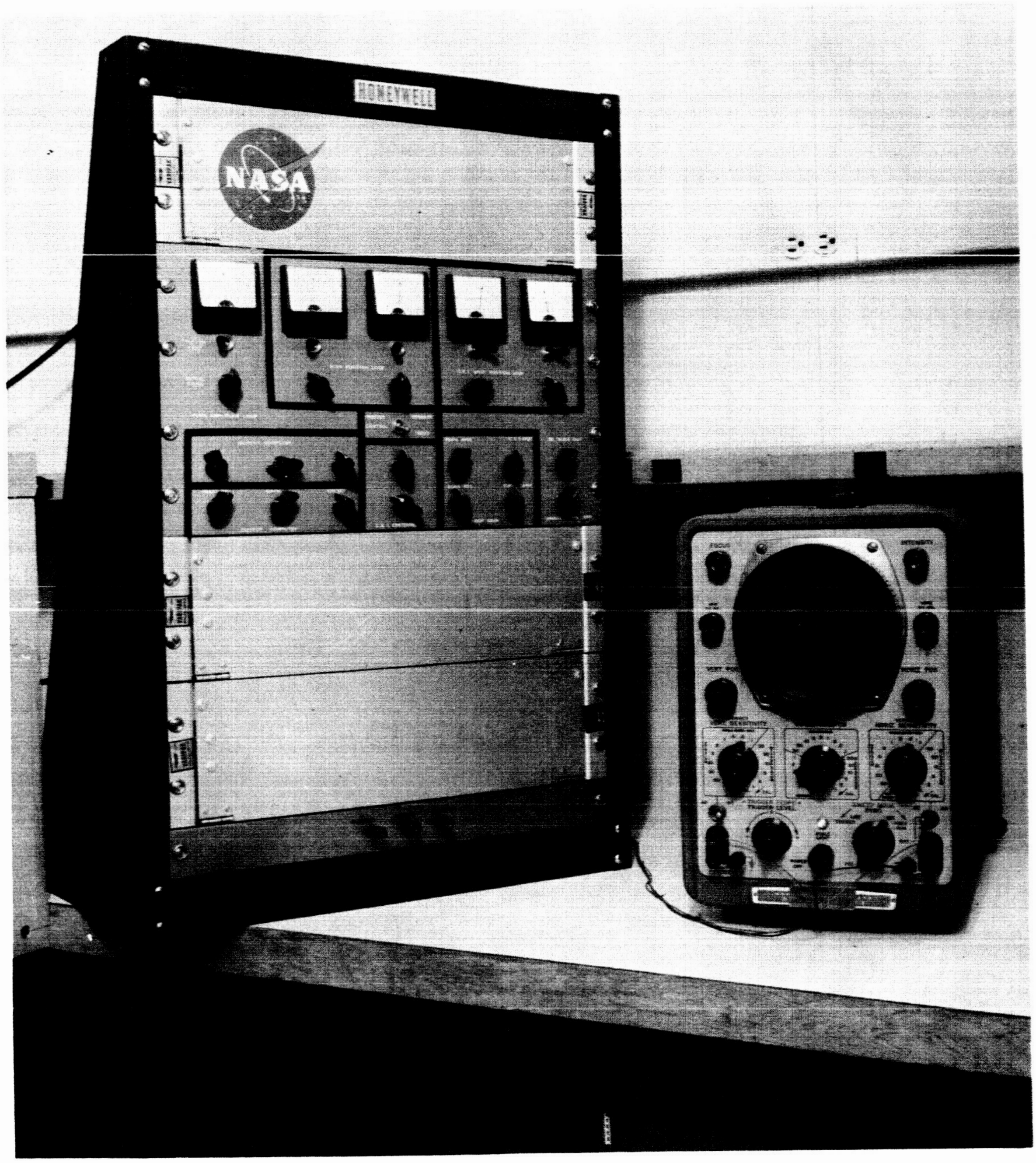

Figure 24 OCULOMETER ELECTRONICS PANEL

The remaining front panel controls and meters are grouped and labeled according to function and are shown in Figures 25, 26 and 27. The galvanometers and top row of switches and potentiometers are numbered one through five from right to left and correspond to the M, R and S designations on Figure 25. The remaining controls do not follow an ordered numbering sequence; function names will therefore be used.

Two low pass filters (720Hz) are attached to the rear of VR2 on a phenolic board. The centering adjustments for the yoke of the PM tube are mounted on a phenolic board attached to the rear of VR3, directly behind the fan, which is fastened to the bottom of the cabinet.

The d-c voltage applied to the relay is derived from a line rectifier circuit mounted on a phenolic board fastened to the left side of the cabinet as viewed from the rear.

The external master control switch is wired to the master control switch which is located at the center of the front control panel.

6.1.2 Optics

The optical system is mounted on two Ealing optical benches as shown in Figure 9. The main unit is the box mounted at the intersection of the benches (Figure 10). It contains:

a. PM tube ITT FW130
b. PM deflection coil ITT F4501
c. Magnetic screen for PM tube
d. PM tube preamplifier
e. Eyepiece (Government surplus, identifying #7638223 A. Jaegers Co. IE 2670)
f. Neutral density beam splitters
g. PM tube auxiliary lens
h. CRT (glow modulator) auxiliary lens
i. Peripheral eye illuminator (three miniature lamps, Edmund Scientific Co. #40,691, set in a plastic ring)

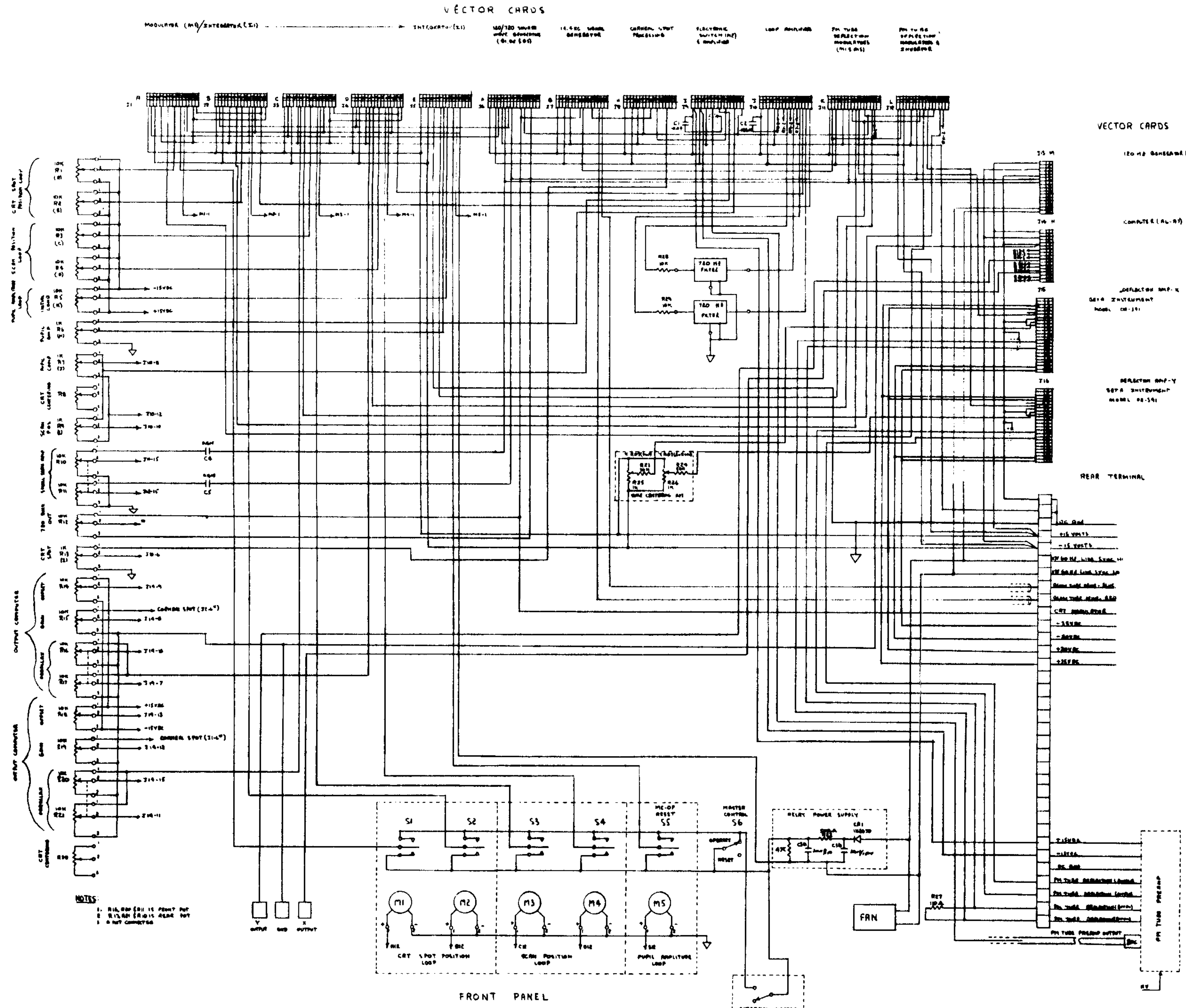

Figure 25 OCULOMETER ELECTRONICS ASSEMBLY

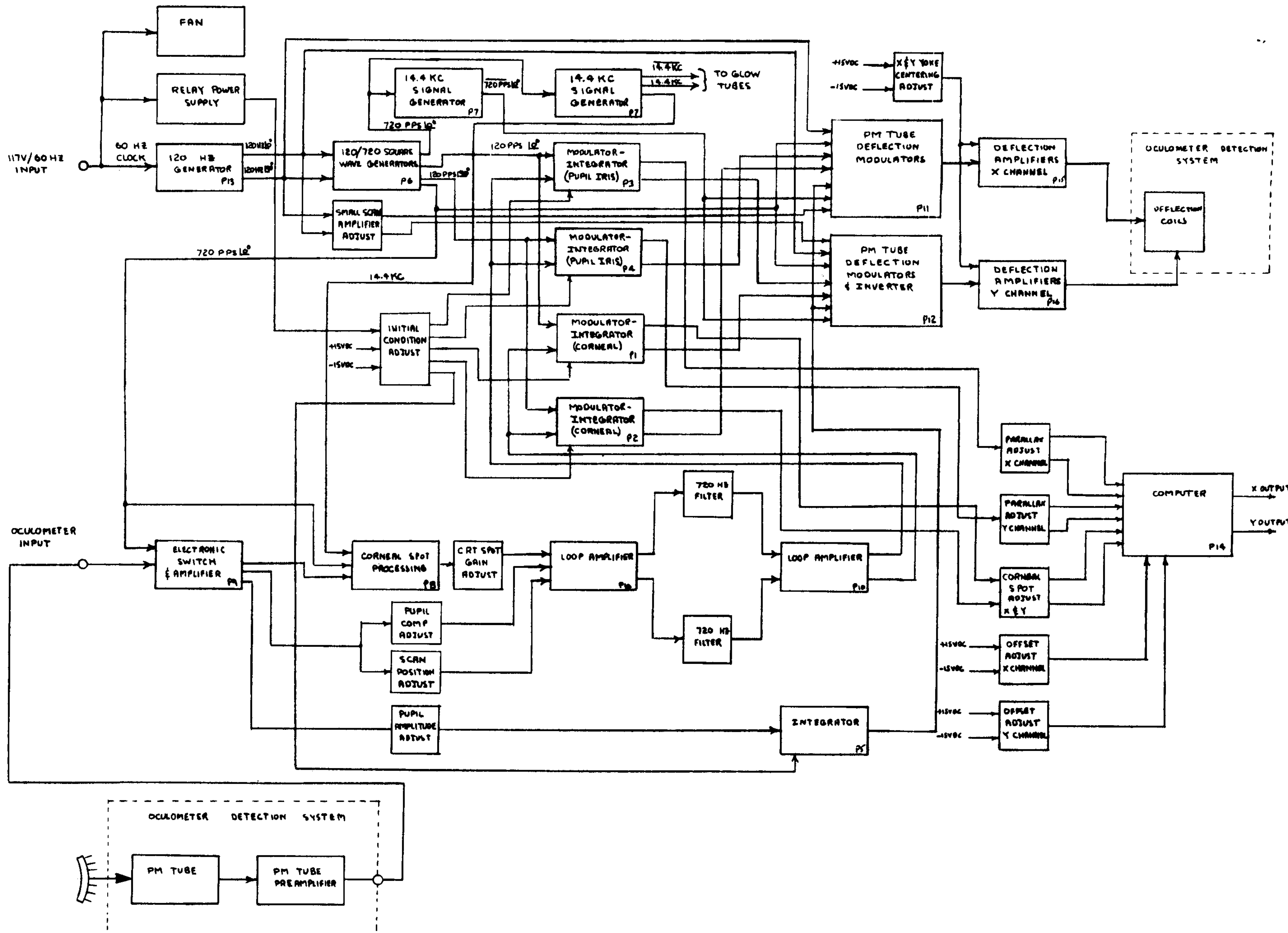

Figure 26 OCULOMETER BLOCK DIAGRAM

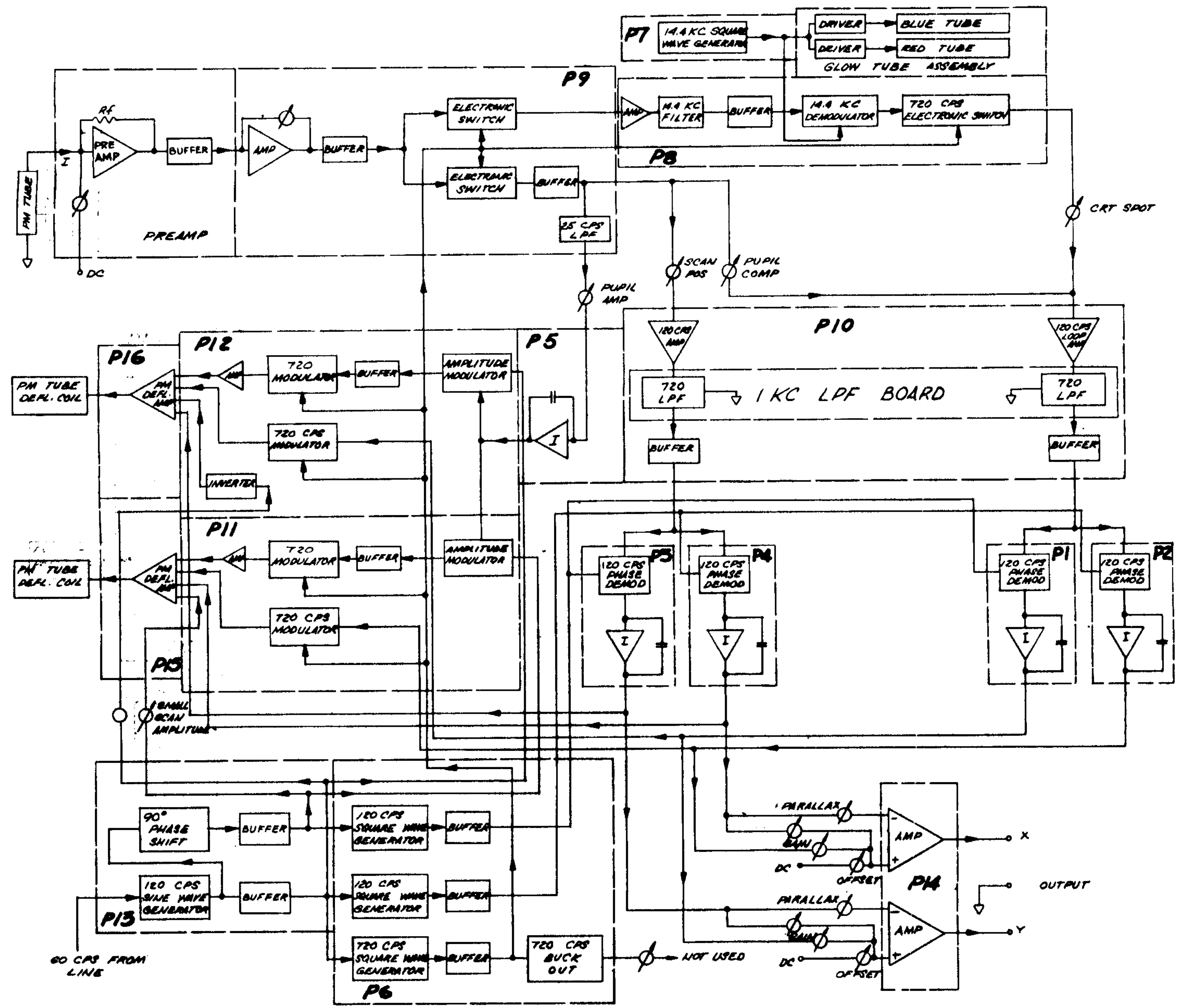

Figure 27 OCULOMETER FUNCTION DIAGRAM

The objective lens for the monocular function is mounted on the main optical bench. The glow modulator assembly is mounted on the bench which is at right angles to the main optical bench.

The objective lens is a government surplus f/5, 12-inch focal length aerostigmat. (See Sky + Telescope Dec. 1965, p. 373.)

The glow modulator tubes (Sylvania type, Red: IB 59R-1130B, Blue: R1169) are mounted in a separate subassembly as shown in Figure 28.

6.2 Electronic Circuits

6.2.1 Power Supplies

The general electronics power supply is ±15 vdc. The PM tube deflection amplifiers require ±20 vdc and ±35 vdc and the PM tube power supply is -1800 vdc. The glow modulator drive circuit required +300 vdc.

To minimize noise problems, and interaction between circuit boards, Kepco power supplies were used for ±15v, ±20v and ±35v. These supplies have a very low source impedance (0.1 ohm). To further ensure against interaction and noise, each circuit board and the PM tube preamplifier has its own decoupling network located on the board itself. The ±15 vdc and ground for each board have a common pin allocation throughout. (Pin 1 = +15 vdc, Pin 3 = -15 vdc, and Pin 5 = GND.)

6.2.2 Preamplifier

The preamplifier (Figure 29) amplifies the PM output signal above the noise level of the general electronics. The heart of the preamplifier is the Philbrick PP25A operational amplifier. This very low noise amplifier gives a signal-to-noise ratio of 5 to 1 at the (theoretical) minimum PM tube output. The amplifier operates as a current to voltage transformer. The PM tube anode is connected

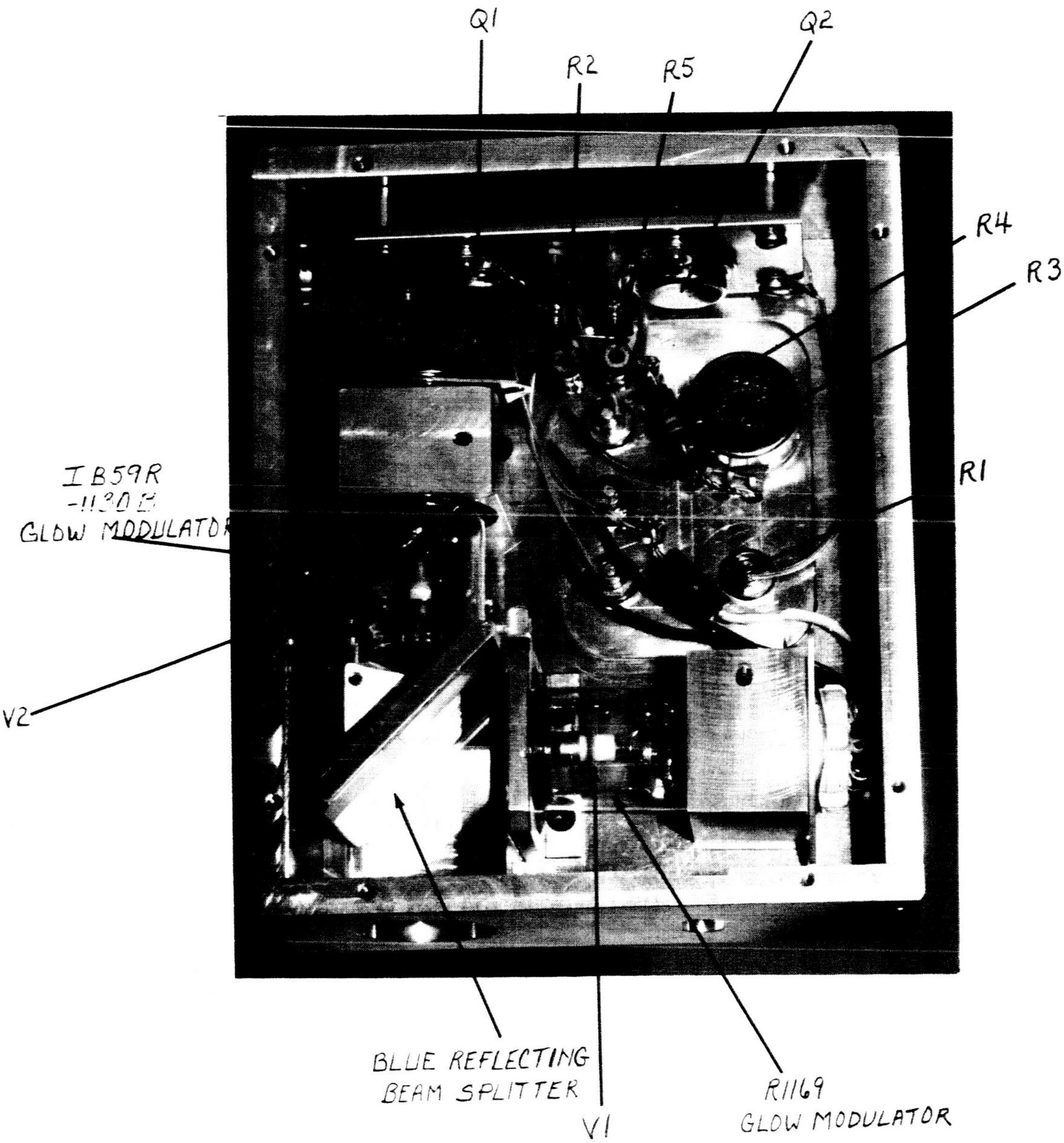

Figure 28 GLOW TUBE ASSEMBLY

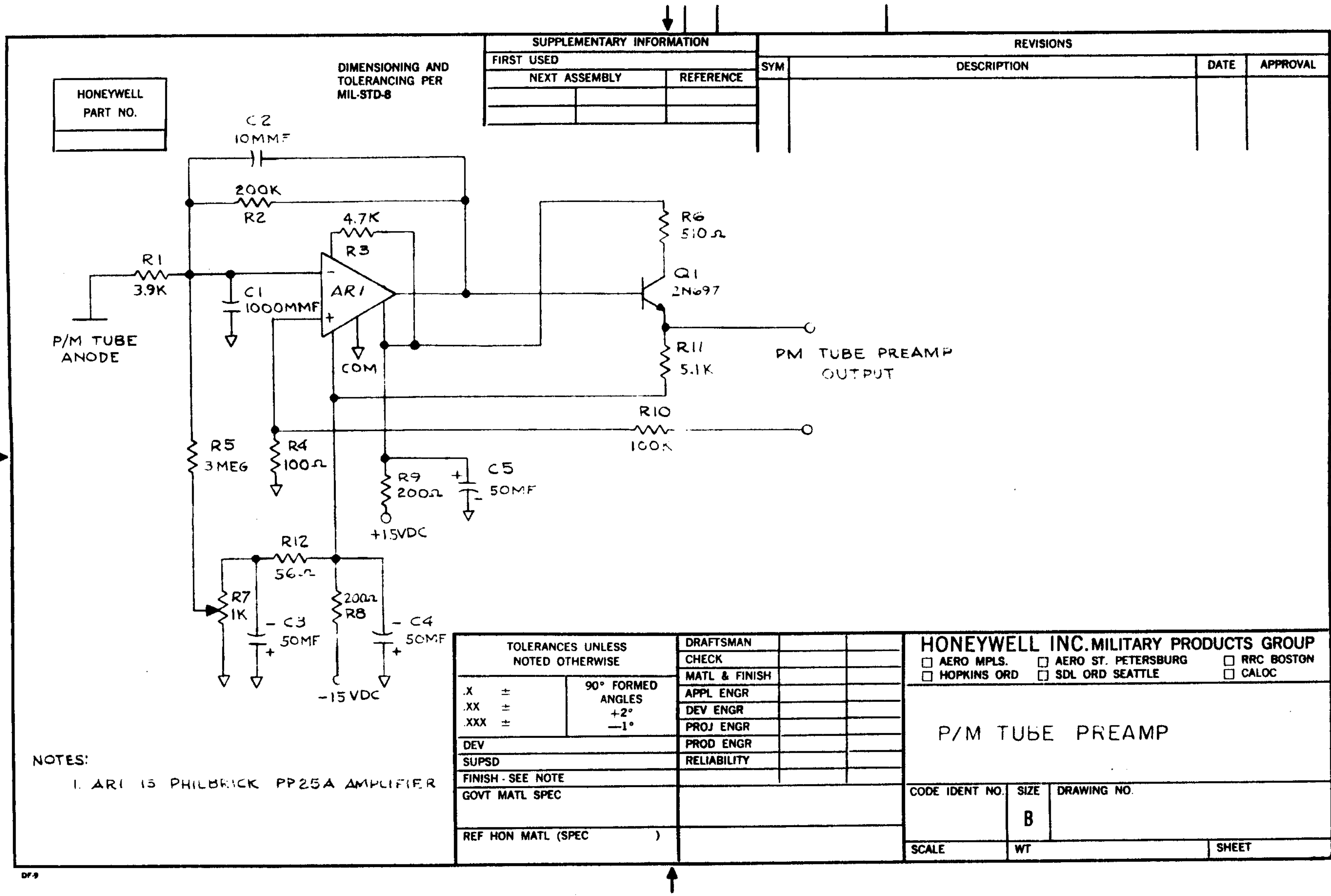

Figure 29 PM TUBE PREAMPLIFIER

directly, with no input resistor, to the summing junction of the amplifier. The transfer function is R_f, where R_f is the value of the feedback resistor. That is, $E_{out} = R_f I_{in}$. The input current (I_{in}) is in the range of 0.6 to 50 nanoamperes. The frequency response of the preamplifier is d-c to 20 k Hz. The output of the preamplifier is buffered by an emitter follower to reduce the load on the Philbrick amplifier, and also for impedance matching.

A d-c current (bias) is fed to the summing junction of the operational amplifier from a potentiometer (R7) to provide adjustment of d-c levels in the system. The preamplifier is located in a minibox mounted on the base of the PM tube.

6.2.3 Electronic Switch Board (P9)

The electronic switch board (Figures 30 and 31) contains the PM tube signal amplifier, buffers, and the electronic switch. The purpose of the PM tube signal amplifier is to further amplify the signal output of the PM tube (input on Pin 7). A Philbrick PP65AU operational amplifier, having a gain between 1 and 50 which can be adjusted by a trimpot feedback resistor (R3), is used. The frequency response of the amplifier is d-c to 20 k Hz. Following the amplifier is an emitter follower buffer to drive the electronic switch.

The electronic switch demodulates the time division multiplexing. The amplifier output is fed through one channel and then the other by alternately opening and closing series transistor switches (Q2, Q3) 720 times per second. One output channel of the electronic switch feeds the corneal spot tracking loop (output on Pin 11). The other output feeds the pupil-iris tracking loop via a 720 Hz RC filter and a buffer with unity voltage gain (output on Pin 9). There is also a 25 Hz LP filter feeding the pupil amplitude loop (Pin 14) from the output of the pupil-iris channel.

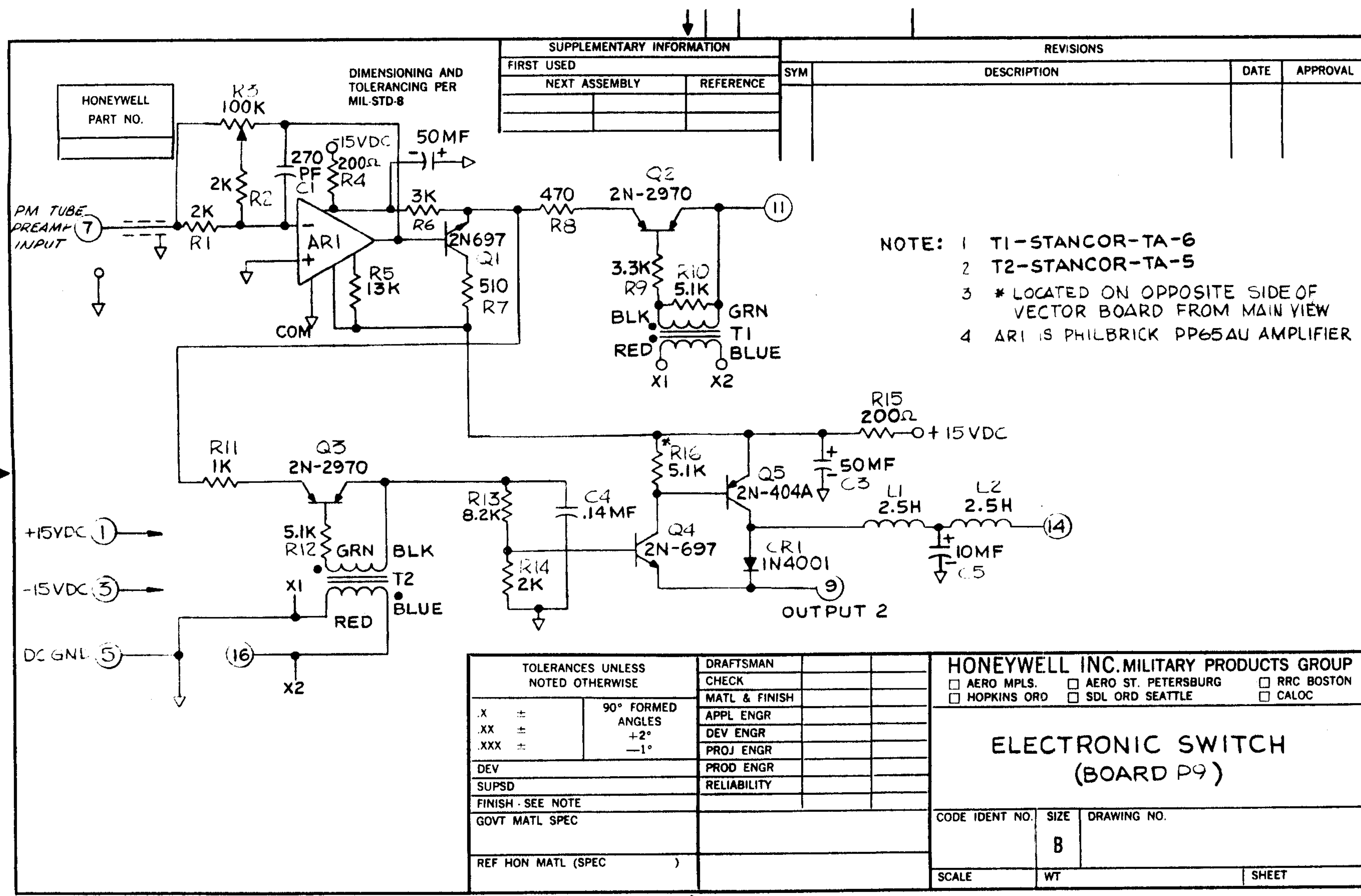

Figure 30 ELECTRONIC SWITCH SCHEMATIC DIAGRAM

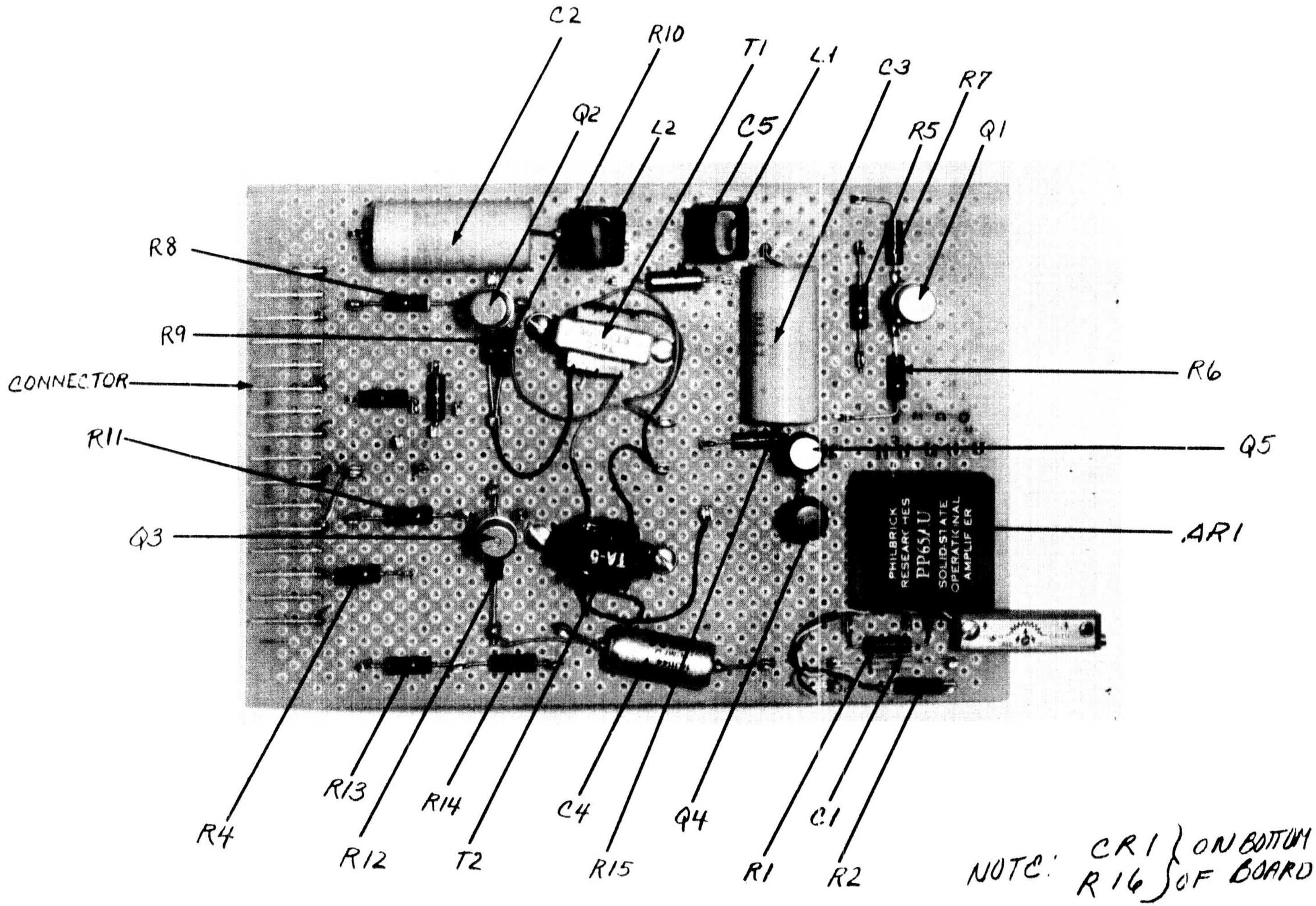

Figure 31 ELECTRONIC SWITCH AND AMPLIFIER (P9)

6.2.4 Corneal Spot Signal Processing Board (P8)

The purpose of the corneal spot signal processing board (Figures 32 and 33) is to amplify, filter, and demodulate the 14.4 k Hz carrier information in the corneal spot tracking loop.

The input to the corneal spot board (pin 11) feeds a Middlebrook amplifier having a gain of seven at 14.4 k Hz with 3 db points at 11 k Hz and 17 k Hz. The output of the amplifier feeds at 14.5 k Hz passband filter (FL1) with 3 db points at ±7.5% of center frequency. The output of the filter feeds an emitter follower used for impedance matching. The emitter follower drives the 14.4 k Hz phase demodulator. This demodulator is a PP65AU operational amplifier, with switches connected across the plus and minus inputs to feed the signal in through one or the other input at a 14.4 k Hz rate. The demodulator feeds another transistor switch (Q8) operating at 720 Hz. This switch provides a 720 Hz alternating source impedance for the 120 Hz output information (on Pin 14).

6.2.5 Loop Amplifiers (P10)

The loop amplifiers (Figures 34 and 35) provide additional gain for the pupil-iris and corneal spot tracking loops and also to filter out (select) the signal information over the band 120 ±100 Hz.

The amplifiers for both channels are PP65AU Philbrick operational amplifiers. The frequency response of these amplifiers, determined by the input and feedback impedances, is 20 to 220 Hz. There is a series resonant circuit in each input to the operational amplifiers, which is resonant at 720 Hz. This circuit rejects the 720 Hz component in the input signal due to the action of electronic switch. The gain of each operational amplifier is 20 at

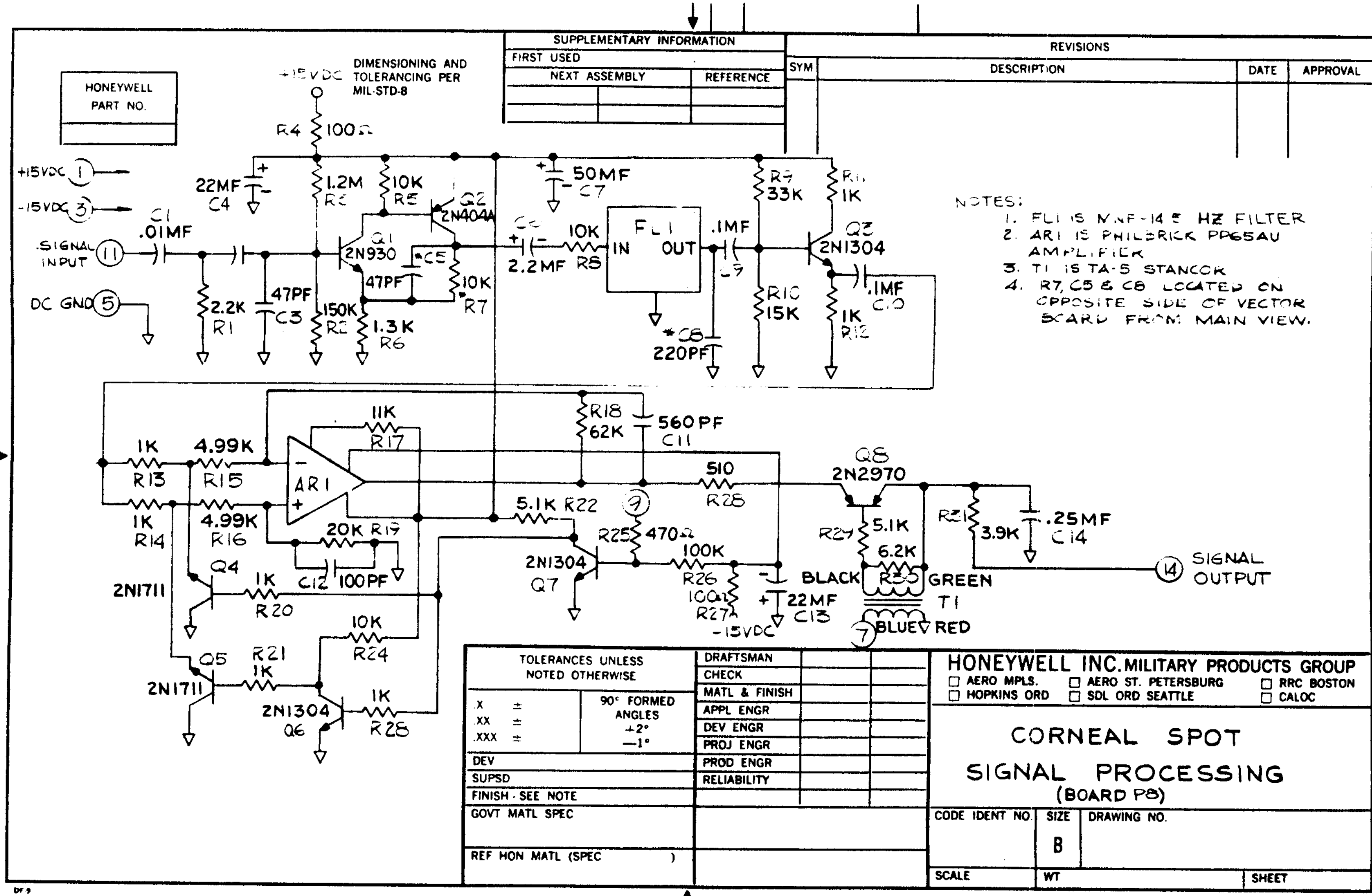

Figure 32 CORNEAL SPOT PROCESSING SCHEMATIC DIAGRAM

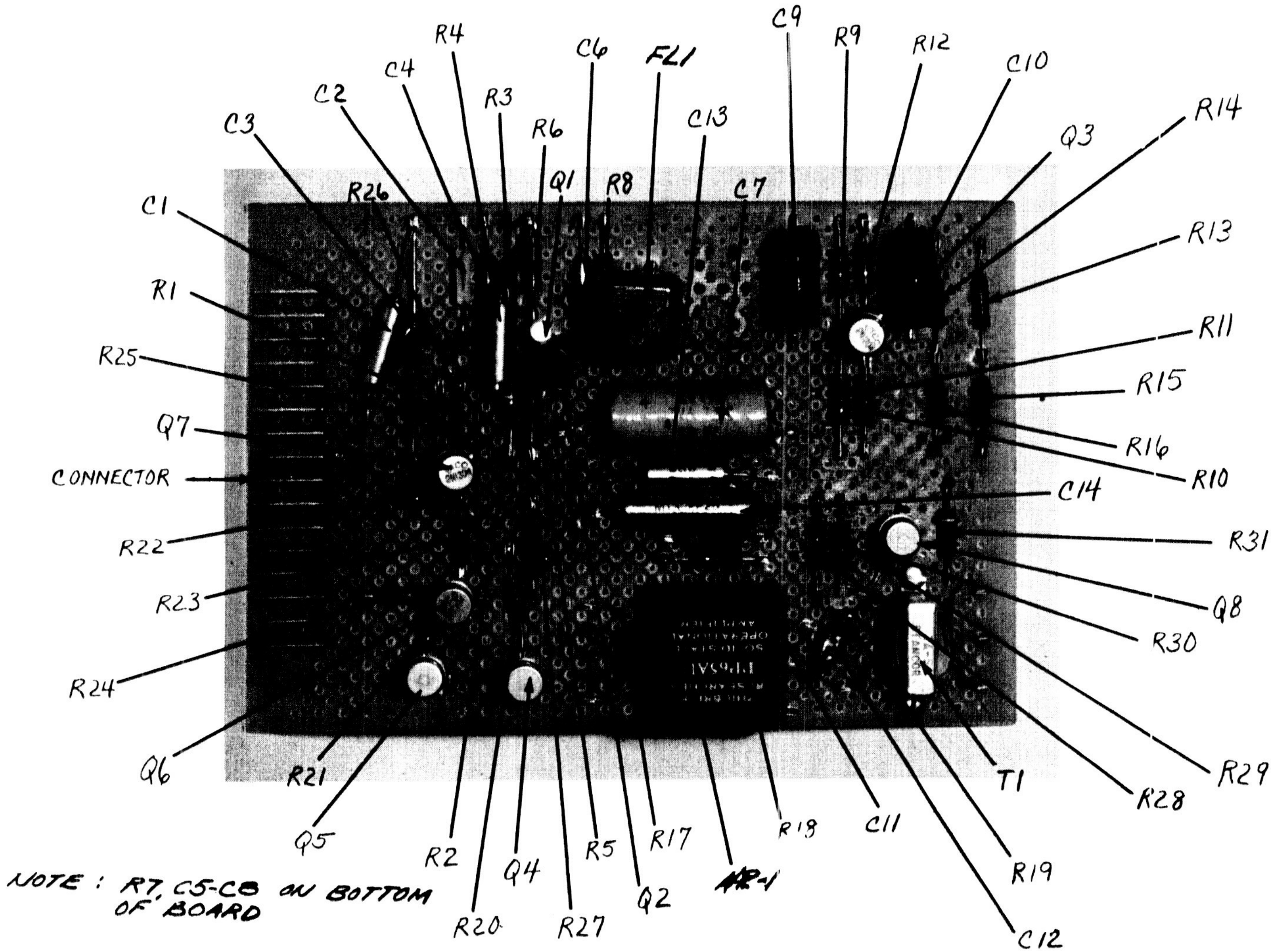

Figure 33 CORNEAL SPOT PROCESSING (P8)

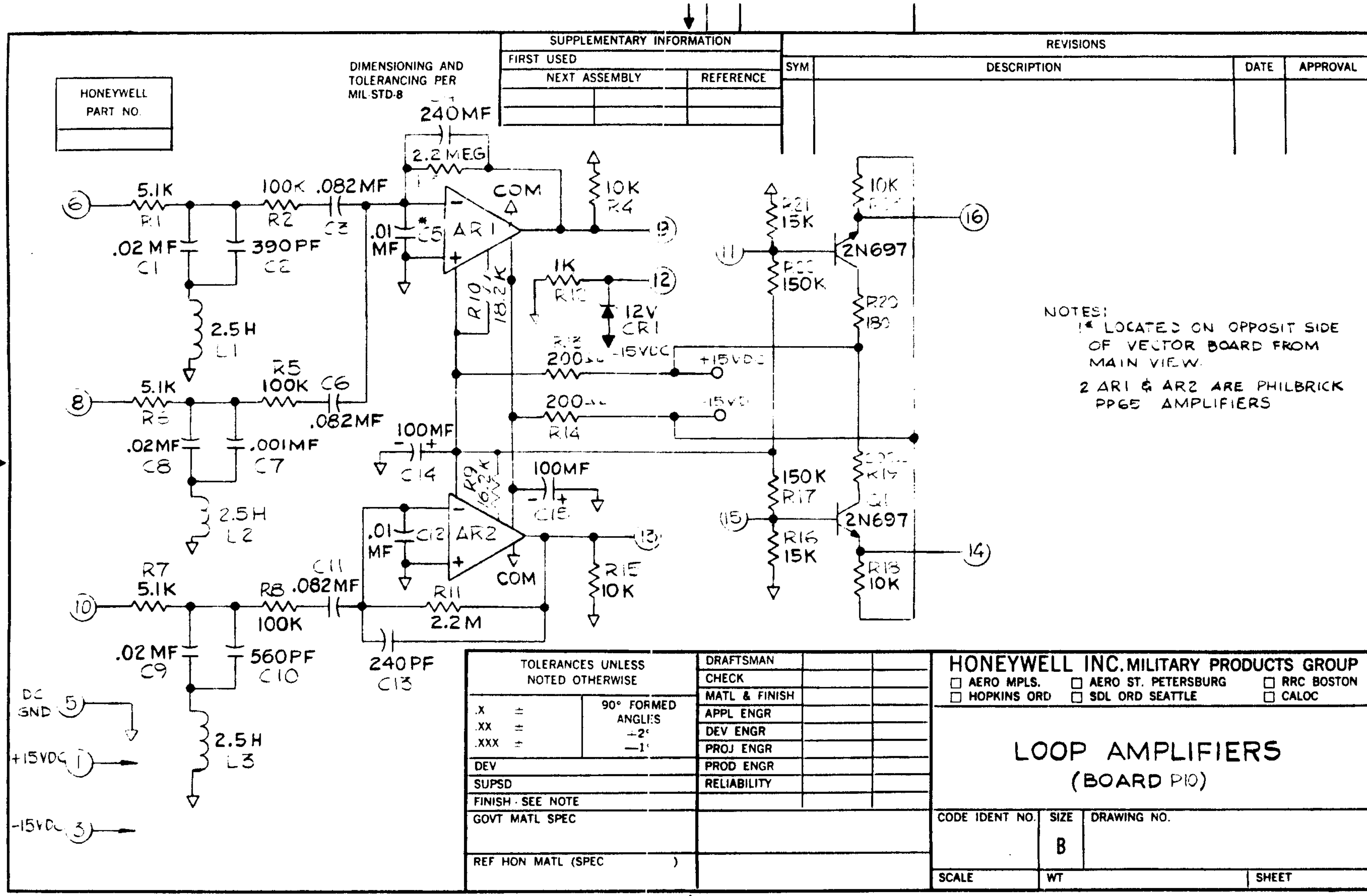

Figure 34 LOOP AMPLIFIER SCHEMATIC DIAGRAM

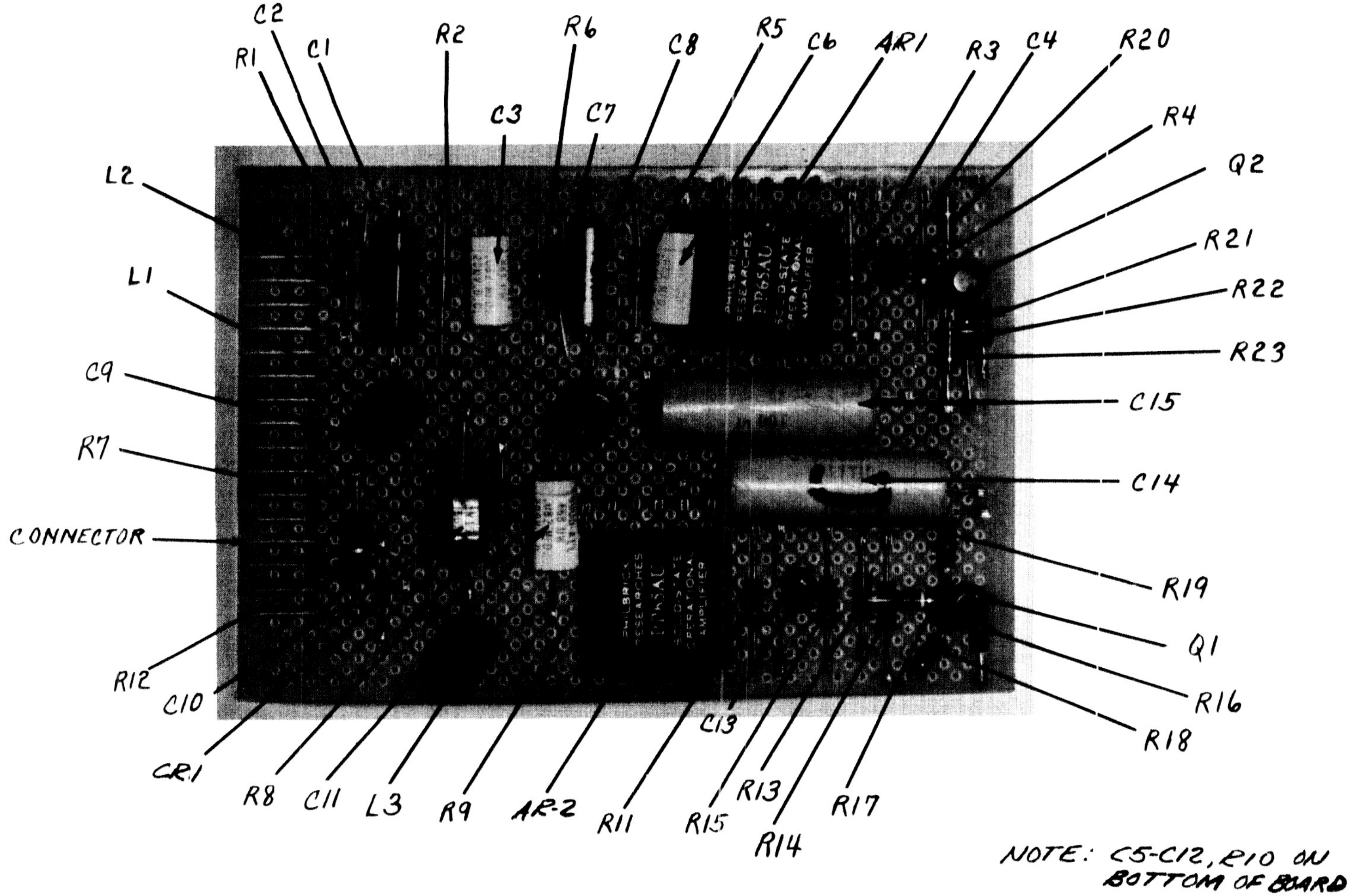

Figure 35 LOOP AMPLIFIERS (P10)

120 Hz. The output of each operational amplifier is filtered by a 1 k Hz low pass filter. This rejects harmonics of 720 Hz due to switching. An emitter follower buffer is used for impedance matching between the 1 kc filter and the output (Pins 14 and 16) to the quadrature demodulators.

6.2.6 Modulator/Integrator (P1, P2, P3, P4)

The modulator/integrator board (Figures 36 and 37) provides quadrature demodulators and integrators for x-position and y-position in the pupil-iris loop and corneal spot loops. There are four of these boards which function identically.

The input (Pins 15, 16) from the loop amplifiers is fed into the phase sensitive quadrature demodulators. A 120 Hz square wave reference signal is used to modulate (multiply) the input signal, which is in the 20 to 200 Hz band. The output of each phase sensitive detector is a varying (0-100 Hz) level which is fed into the corresponding integrator.

The integrator is a standard circuit using a PP65AU Philbrick operational amplifier. The time constant of the integrator is $R_{in} C_f = (10K)(0.33\ \mu f) =$ 3.3 millisceonds. The operational amplifier can be switched, by a relay, either as an amplifier (reset mode) or as an integrator (operate mode). An adjustable d-c input (adjustable by a potentiometer on the front panel) provides a method of adjusting the x and y of each loop as an initial condition in the reset mode.

6.2.7 PM Tube Deflection Modulators (P11, P12)

The purpose of these boards, one for x deflection, the other for y deflection,

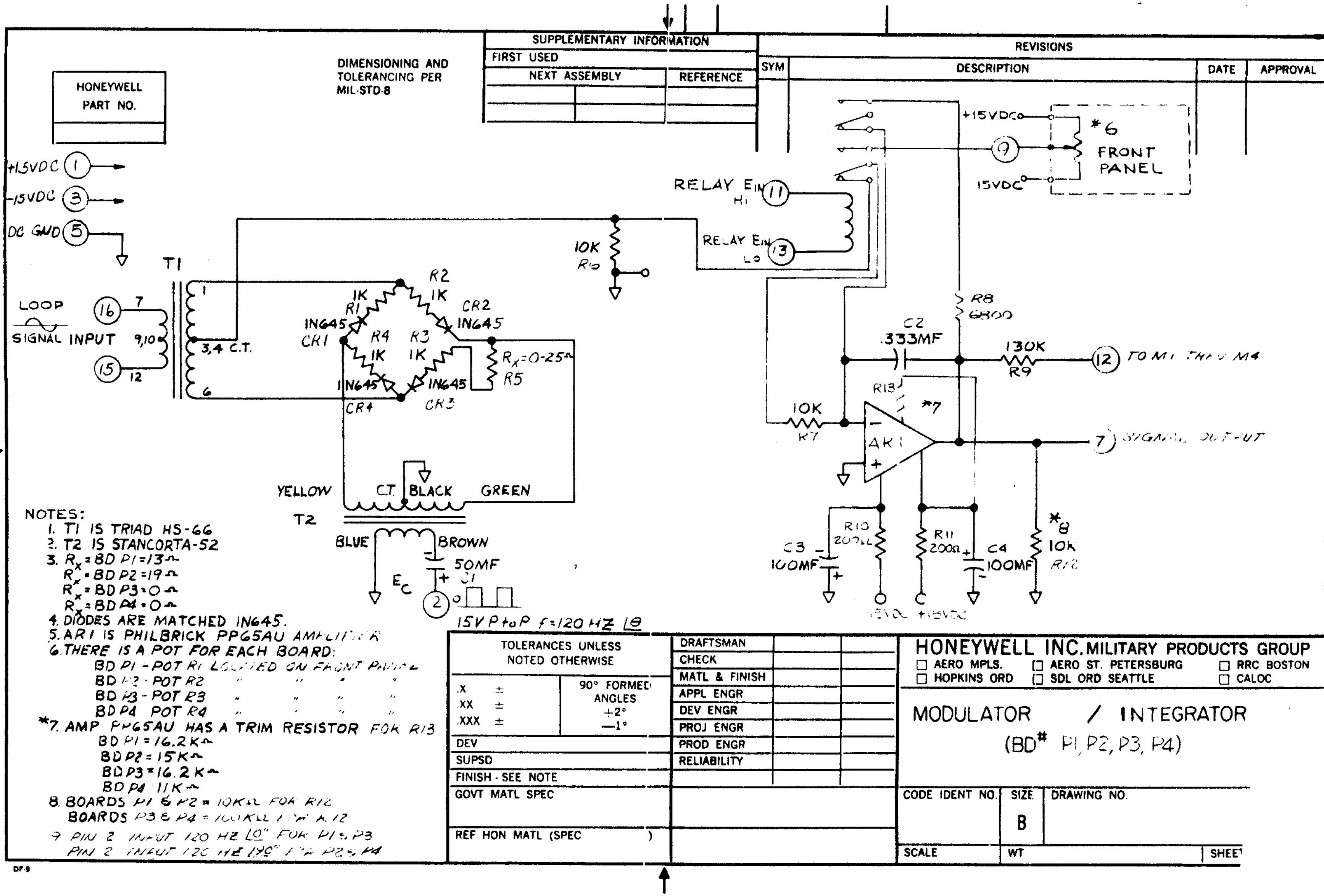

Figure 36 MODULATOR/INTEGRATOR SCHEMATIC DIAGRAM

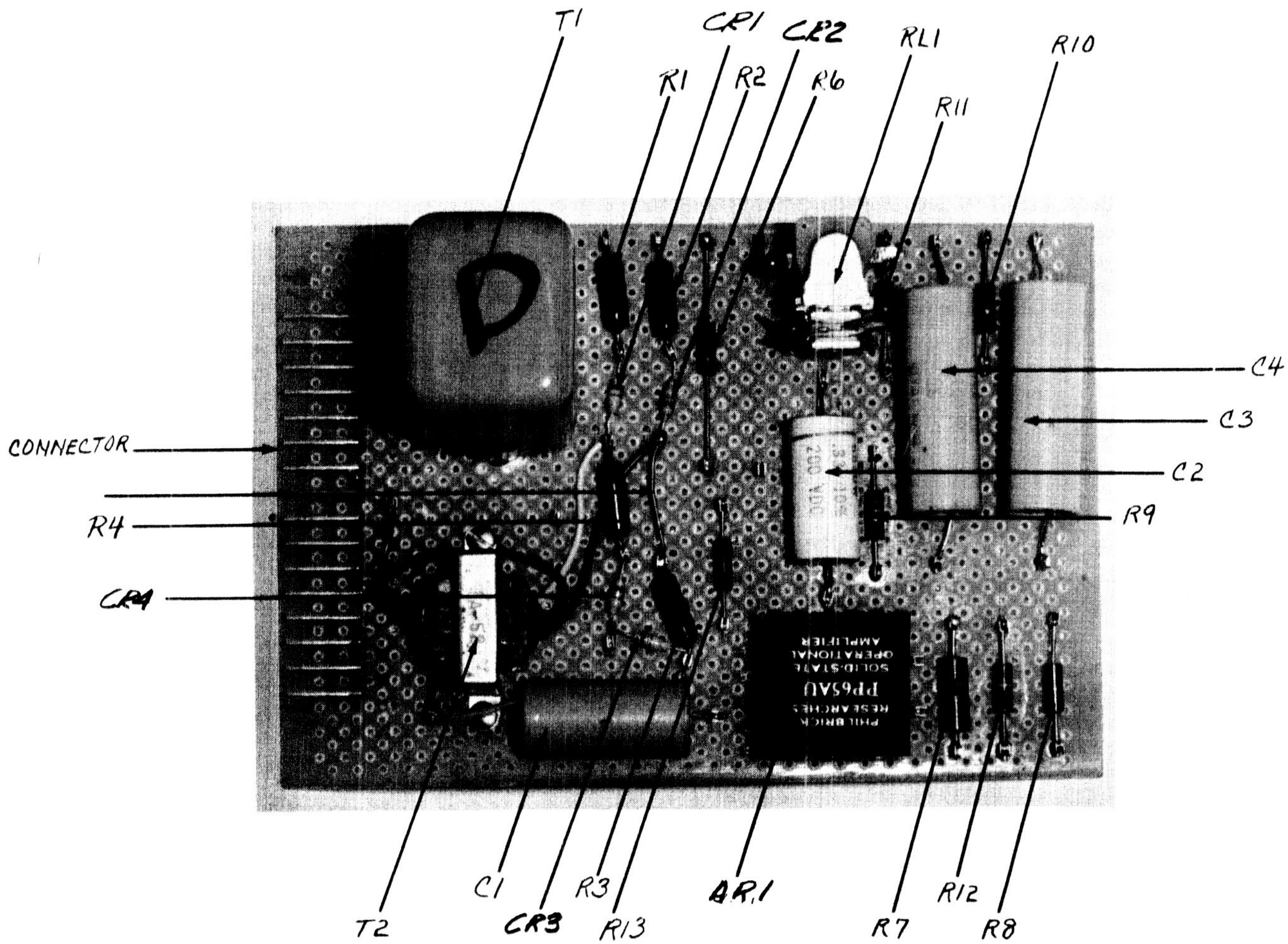

Figure 37 MODULATOR INTEGRATOR

(Figures 38 to 41) is to provide amplitude modulation and chopping of the 120 Hz sine wave signals used to generate the rotating circular scan of the PM tube.

The 120 Hz sine wave input (Pin 7) is fed into a four-element field effect transistor which acts as an amplitude modulator. This device operates as a variable transconductance amplifier, with the input to the second gate being the output of the pupil amplitude loop integrator. This varies the size of the pupil-iris scan as the pupil of the eye expands and contracts. (The scan amplitude is proportional to the amplitude of the scan sine waves.) The output of the amplitude modulator is fed through an emitter-follower buffer into the chopper. The chopper is a series switch which chops the 120 Hz sine wave to zero amplitude at a 720 Hz rate. This means that the pupil-iris scan is chopped into six equally-spaced and equal segments, each one-twelfth of a revolution. The output of the chopper feeds an amplifier with a gain of four which also acts as a buffer. The output of this amplifier feeds the summing junction of the Beta Instrument Deflection Amplifiers.

There are a number of other PM tube deflection input signals, besides the circular pupil-iris scan, discussed above. The pupil-iris loop x and y position information (integrator outputs), and the corneal spot circular scan signals, come in directly to the summing junction of the deflection amplifiers. The corneal spot x and y position information (integrator outputs) feeds into the summing junctions through a chopper. The chopper is a parallel switch using field-effect transistors (Q3) driven at a 720 Hz rate. The corneal spot x and y choppers are driven out of phase with the pupil-iris circular scan choppers.

The phase inverter on board P12 inverts the sine wave input for one axis of

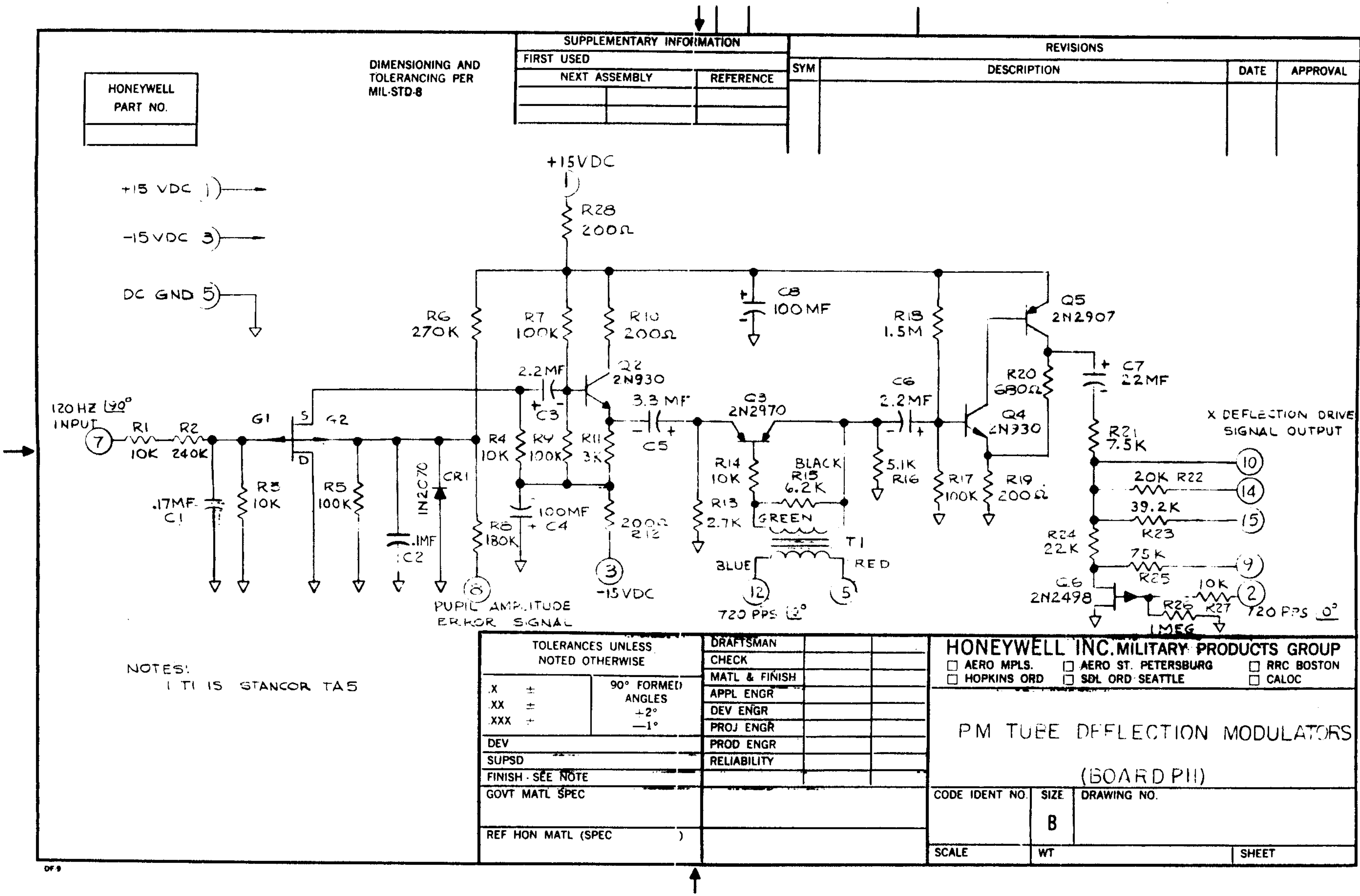

Figure 38 PM TUBE DEFLECTION MODULATORS SCHEMATIC DIAGRAM

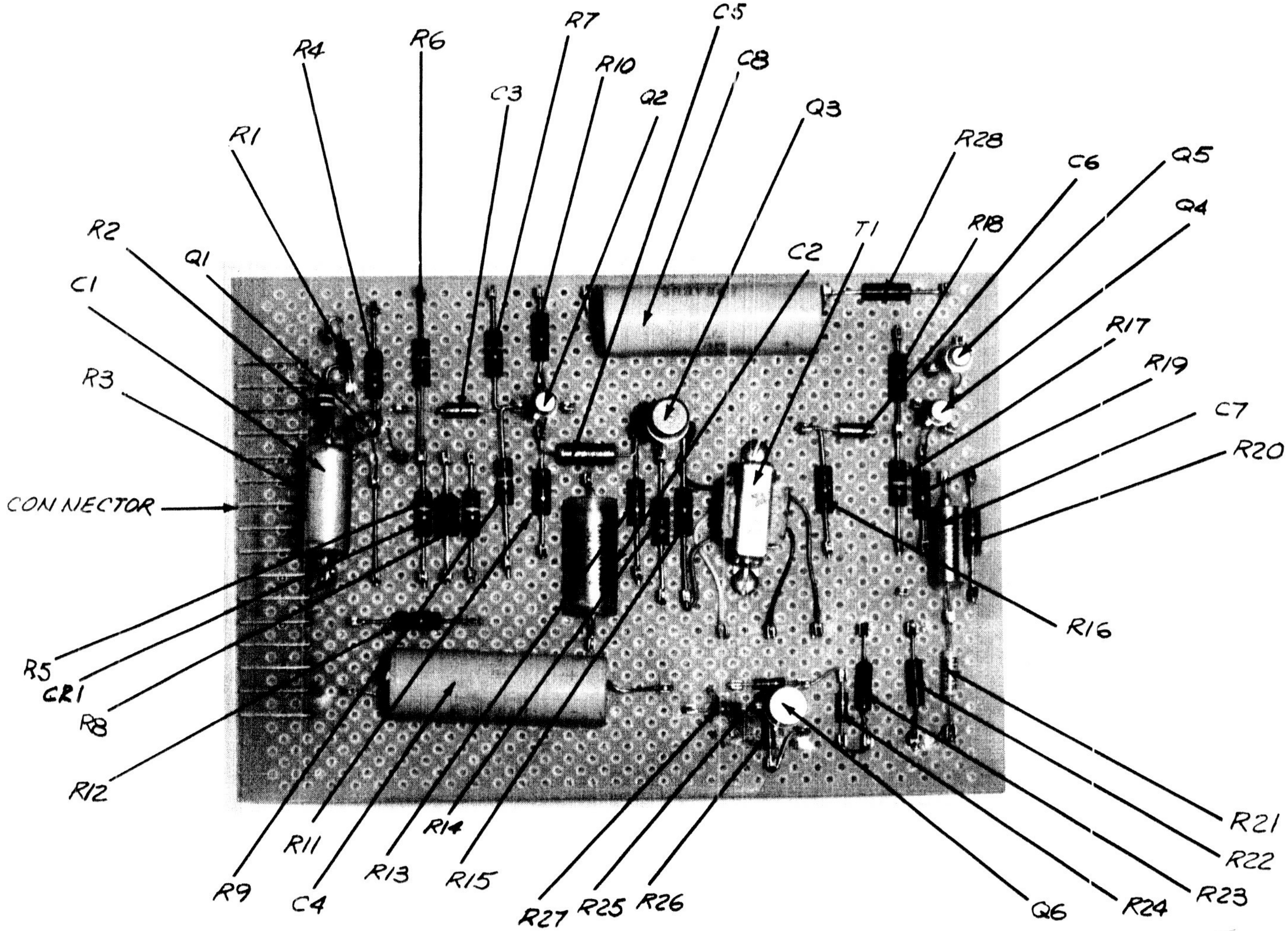

Figure 39 PM TUBE DEFLECTION MODULATORS (P11)

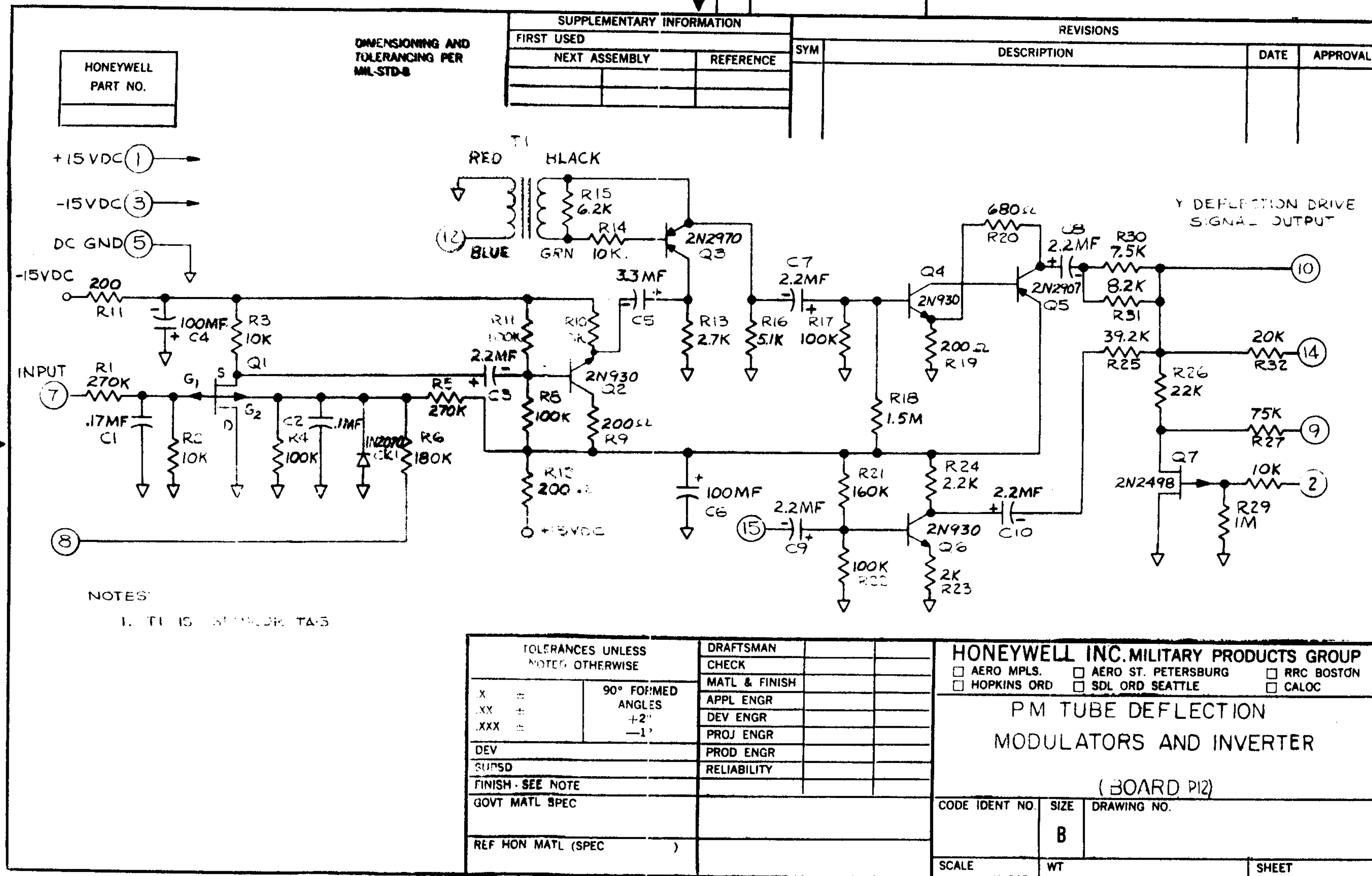

Figure 40 PM TUBE DEFLECTION MODULATORS AND INVERTER SCHEMATIC DIAGRAM

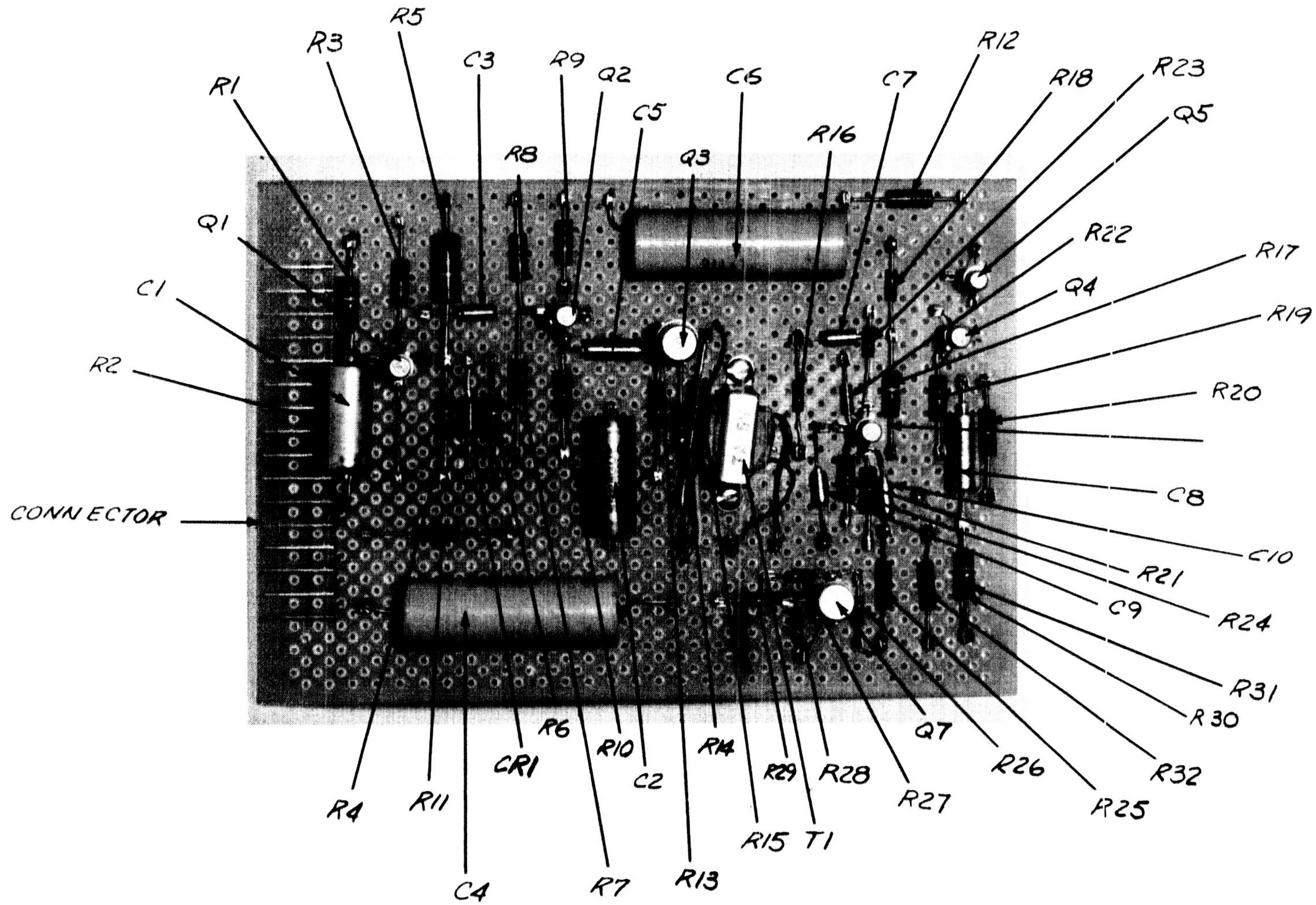

Figure 41 DEFLECTION MODULATORS AND INVERTER (P12)

the corneal spot circular scan. This is done to make the corneal spot circular scan, and the pupil-iris circular scan, 180 degrees out of phase with respect to each other.

6.2.8 PM Tube Deflection Amplifiers (P15, P16)

The PM tube deflection amplifiers (Figure 42) provide x and y deflection signals (current) for the magnetically deflected PM tube.

The deflection amplifiers are purchased items (Beta Instruments Corp, Model DA-341) which provide a current output proportional to the input voltage. They are operational amplifiers, with the input fed directly into the summing junction. The output current range is ±200 ma which is more than sufficient to deflect the PM tube over the full photocathode area. The response of the amplifiers, with the coils supplied with the PM tube, is approximately 5 microamperes per microsecond.

6.2.9 Pupil Amplitude Loop Integrator (P5)

The pupil amplitude integrator (Figures 43 and 44) integrates the average d-c level coming through the pupil-iris loop in order to generate a control signal for the amplitude modulator on cards P11 and P12.

The integrator is a Philbrick PP65AU operational amplifier. The time constant is $(R_{in})(C_f) = (10K)(.33\mu f) = 3.3$ ms. This board has a relay circuit, to switch the integrator as an amplifier in the reset mode, and an adjustable (on the front panel) d-c input with which to set the pupil amplitude loop initial conditions.

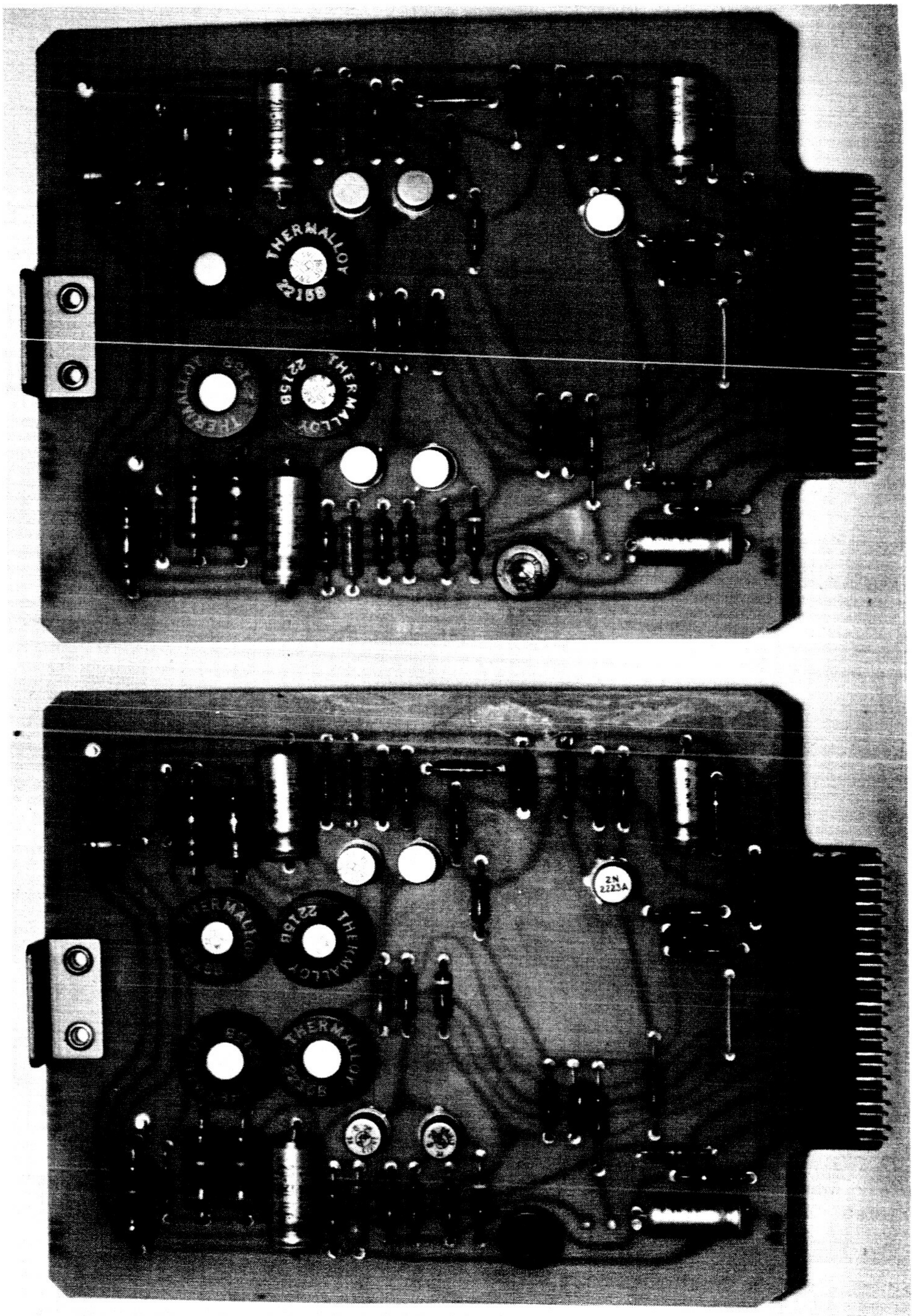

Figure 42 DEFLECTION AMPLIFIERS

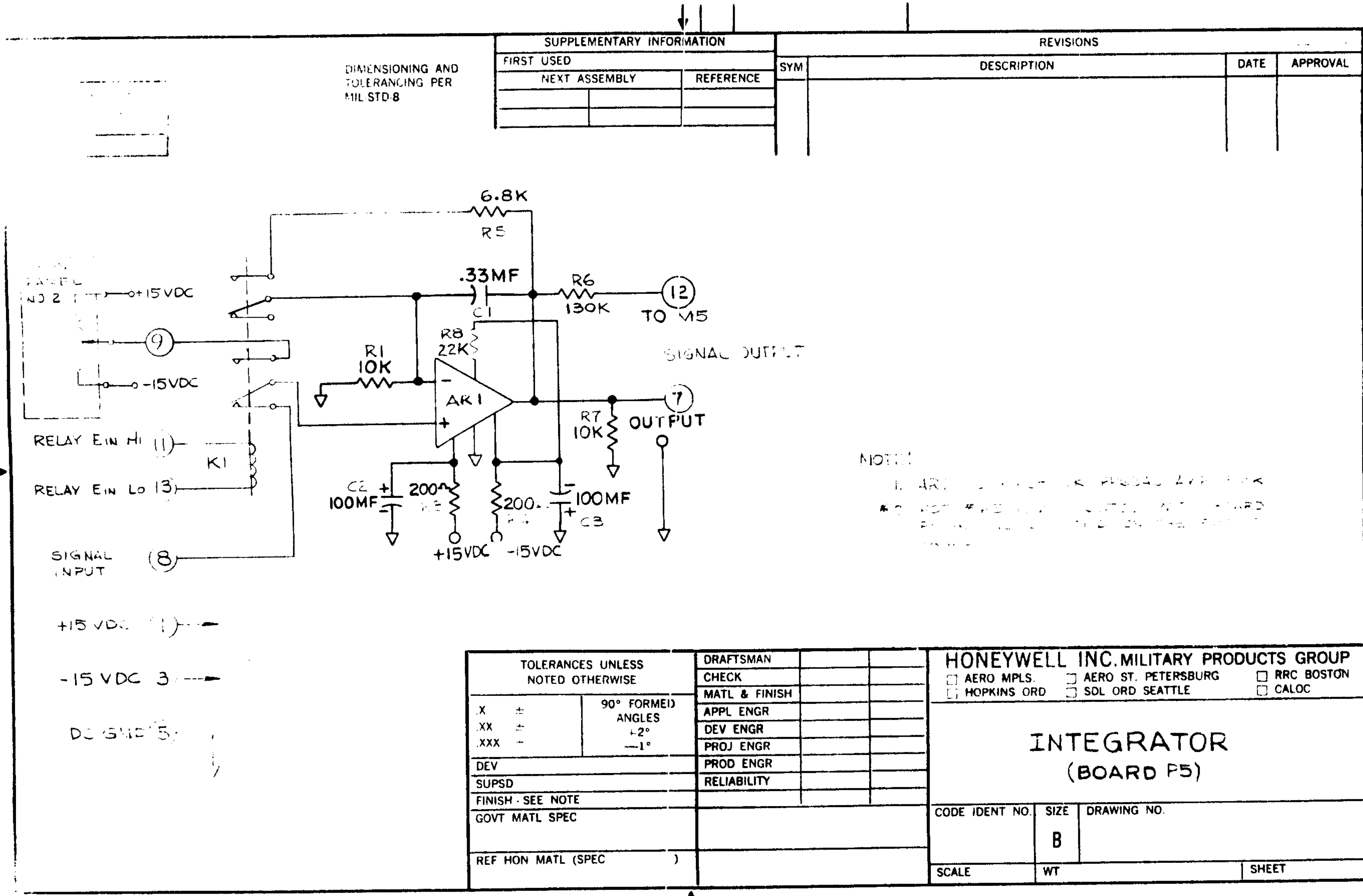

Figure 43 PUPIL AMPLITUDE LOOP INTEGRATOR SCHEMATIC DIAGRAM

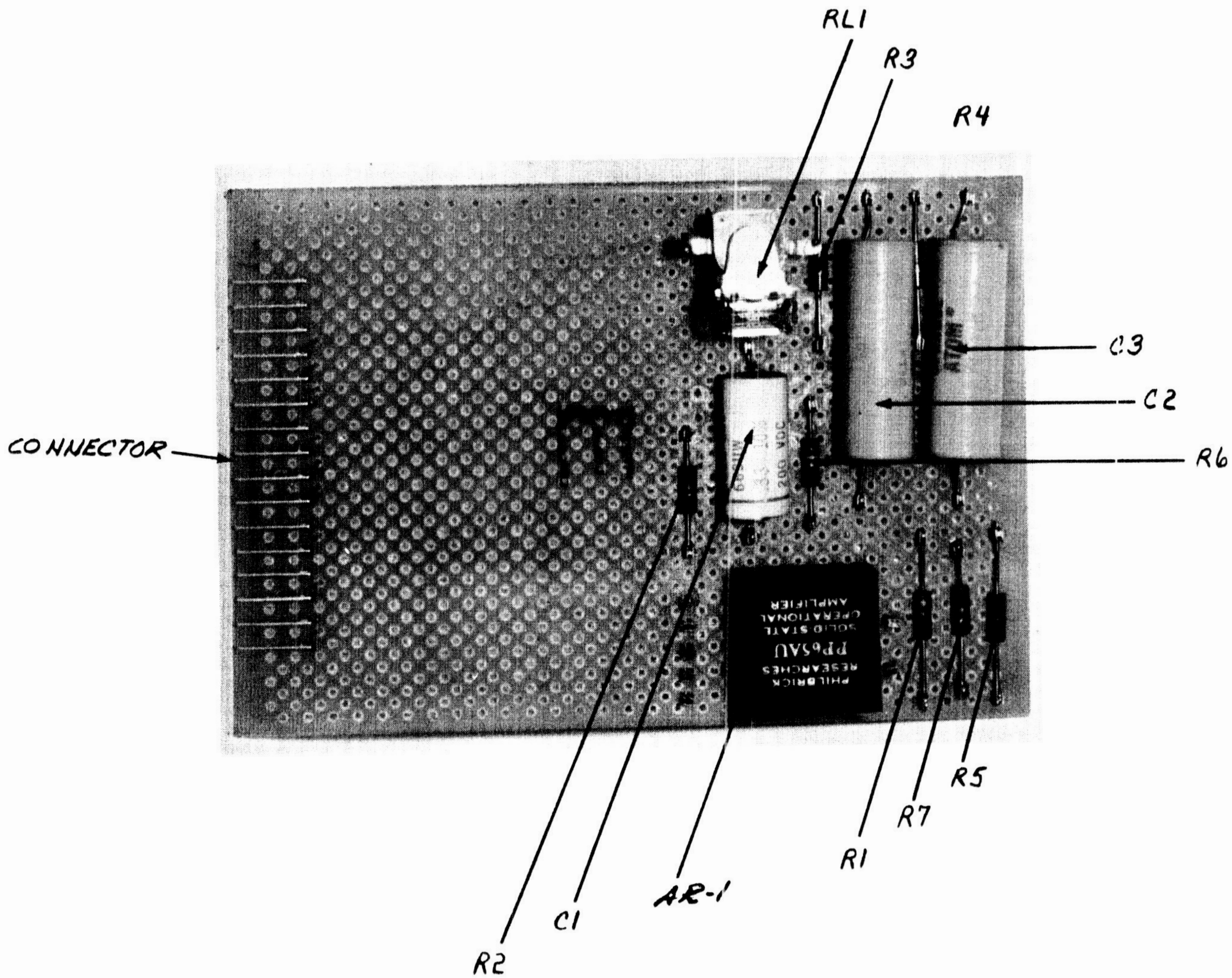

Figure 44 INTEGRATOR I5 (P5)

6.2.10 Glow Tube Assembly

The glow tube assembly (Figures 28 and 45) provides alternately red and blue flashing lights of equal intensity. The assembly has red and blue glow modulator tubes which flash alternately at a 14.4k Hz rate. They are switched on and off by high voltage transistor switches driven by the 14.4 k Hz generator. The red tube has a potentiometer in series with it to vary the light output in order that it may be adjusted to be effectively equal to the blue light.

6.2.11 Computer Board (P14)

The computer board (Figures 46 and 47) perform the computations necessary to derive x and y position information from the outputs of the pupil-iris loop and corneal spot loop.

The computer consists of two identical operational amplifiers (PP65AU), one for x-position and one for y-position. The gain of each amplifier is twenty. There are three non-inverting inputs; d-c offset, corneal spot position, and pupil-iris position. There is also an inverting input of pupil-iris position information. The two pupil-iris inputs are fed into the computer by a dual potentiometer called the parallax control. In this way, a small fraction of the pupil position can either be added, or subtracted, from the corneal spot position ($\frac{r_2 - r_1}{r_1} \psi$ in Appendix A).

6.2.12 120 Hz Sine Wave Generator (P13)

The 120 Hz sine wave generator (Figures 48 and 49) generates two sine waves 90 degrees out of phase and synchronized to the 60 Hz line frequency.

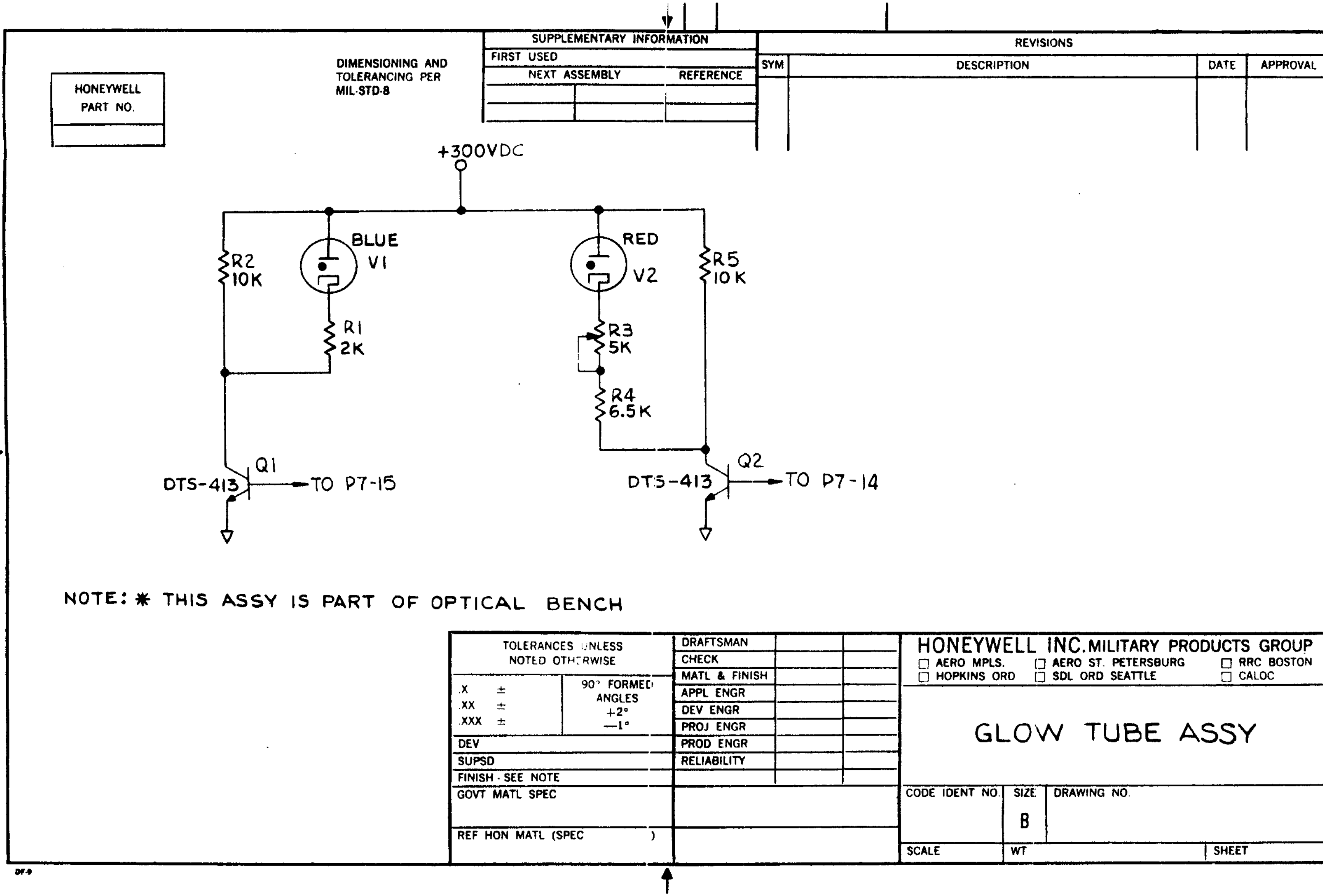

Figure 45 GLOW TUBE ASSEMBLY SCHEMATIC DIAGRAM

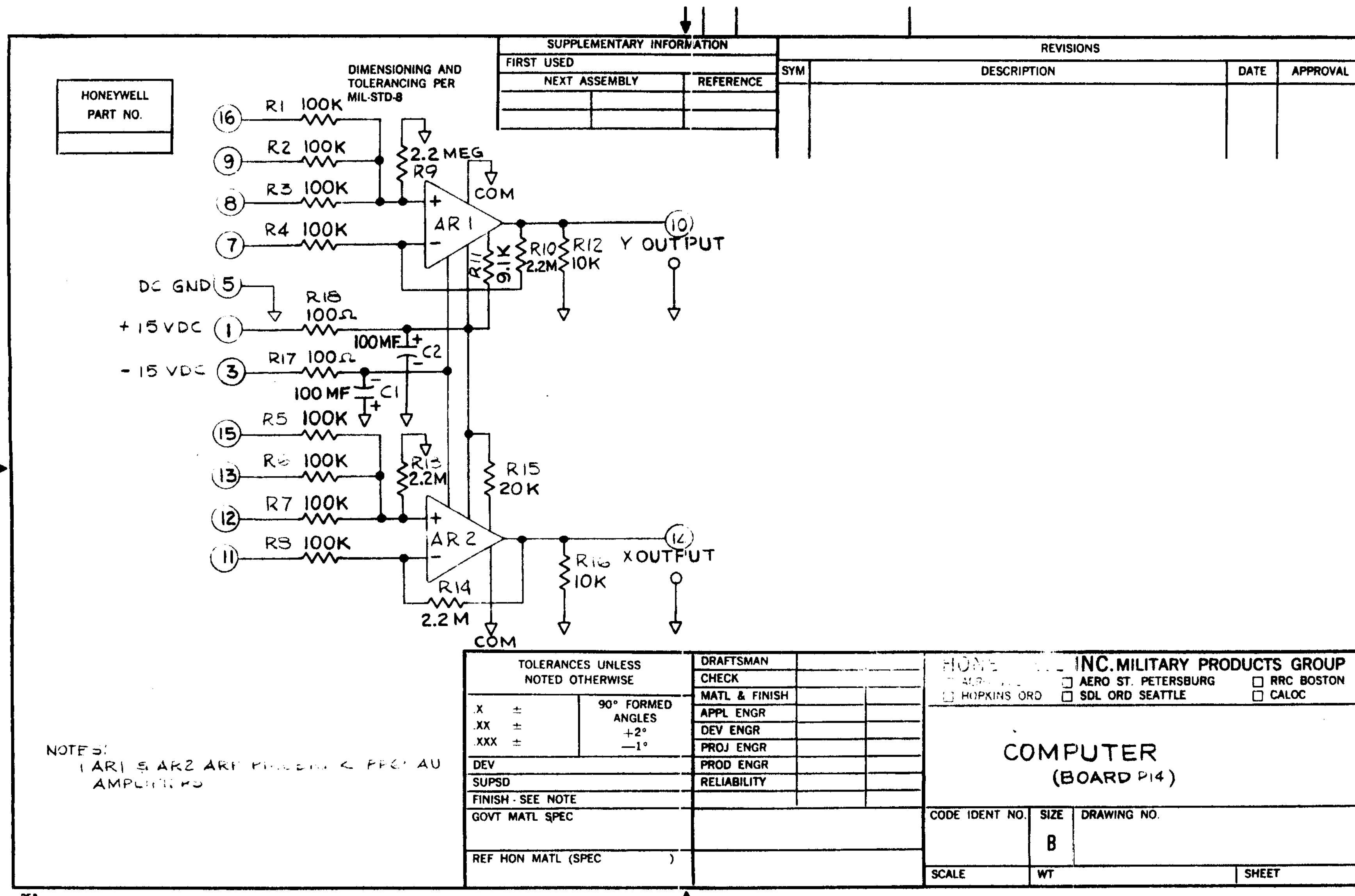

Figure 46 COMPUTER SCHEMATIC DIAGRAM

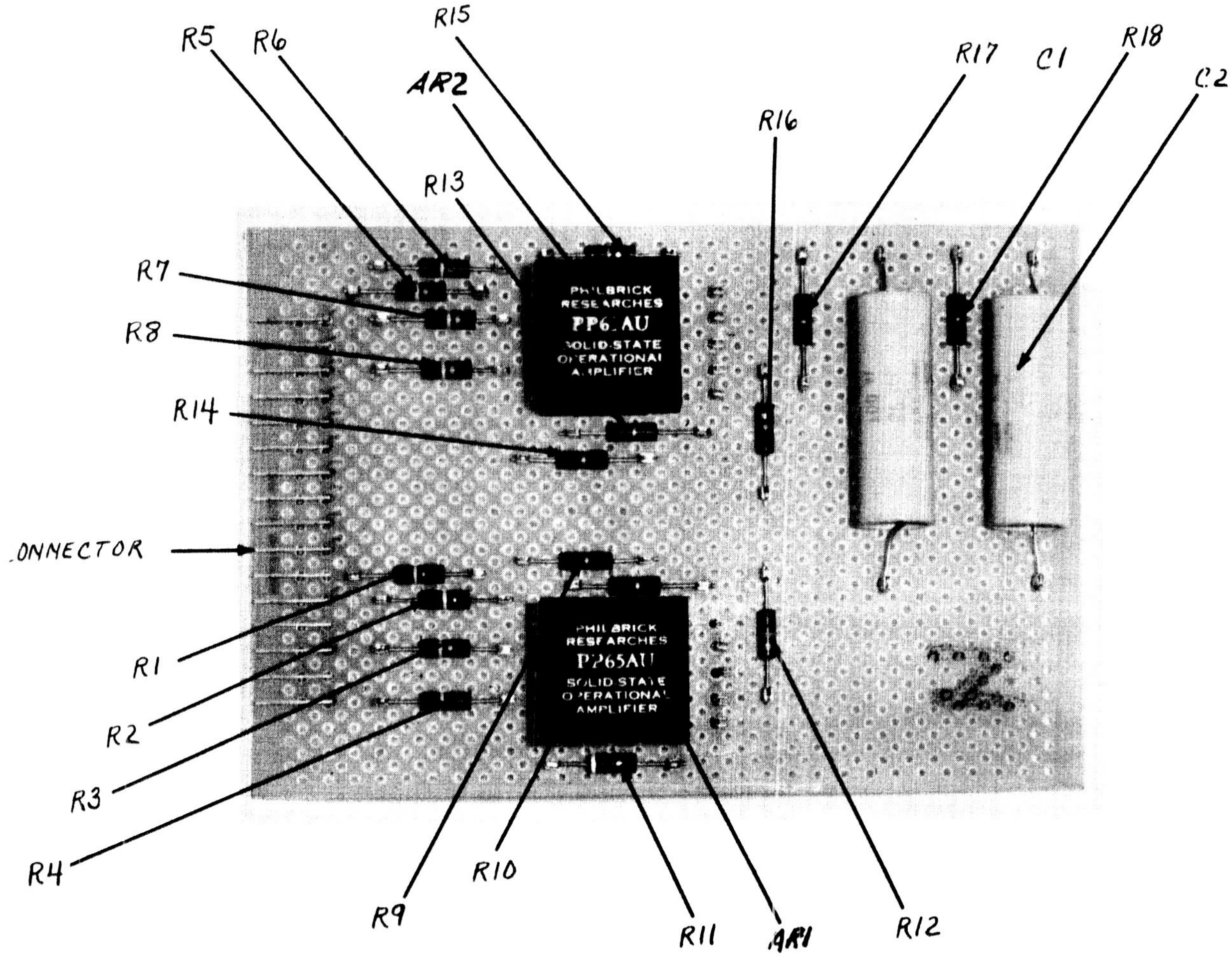

Figure 47 COMPUTER (P14)

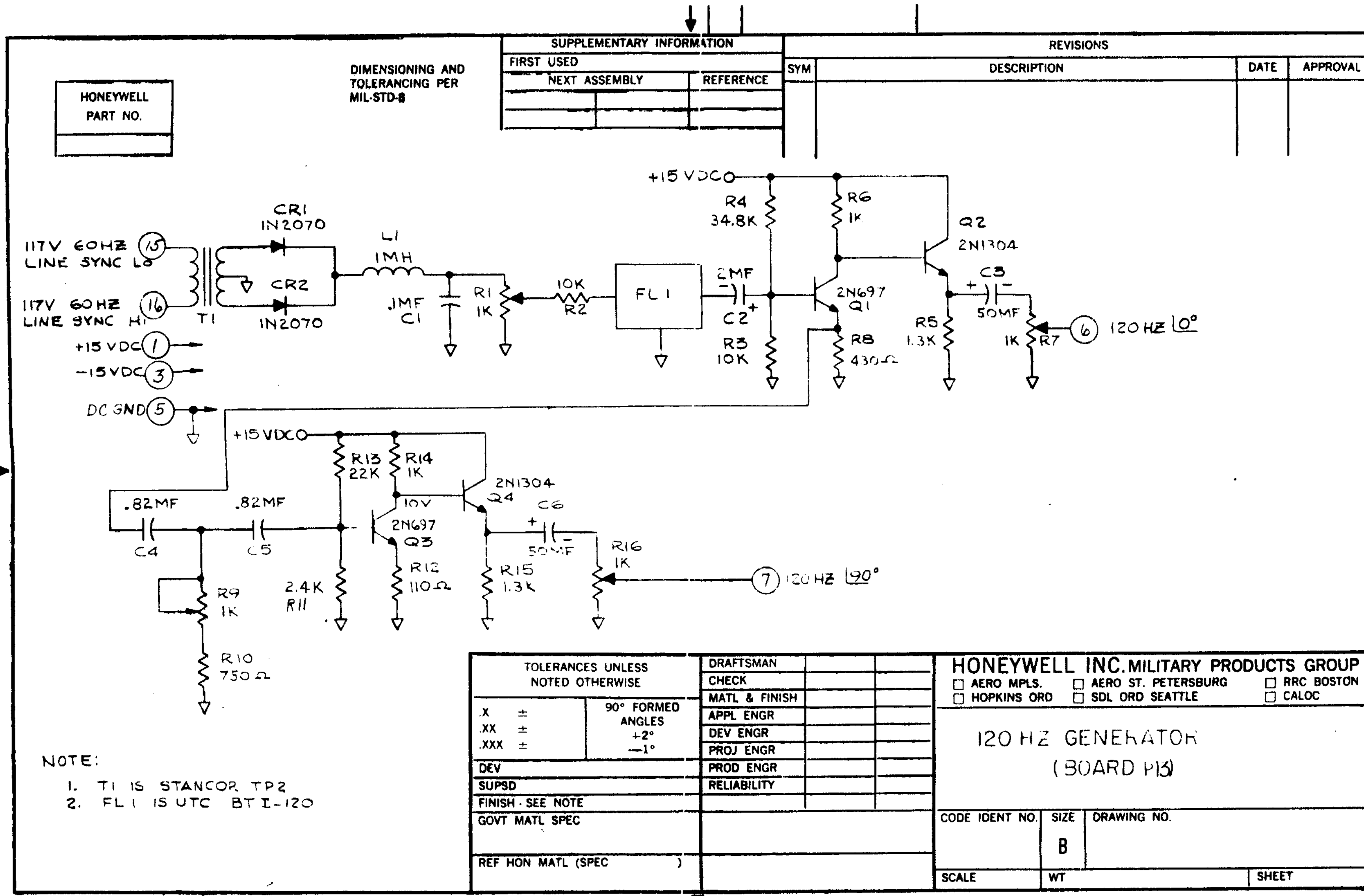

Figure 48 120 Hz SINE WAVE GENERATOR SCHEMATIC DIAGRAM

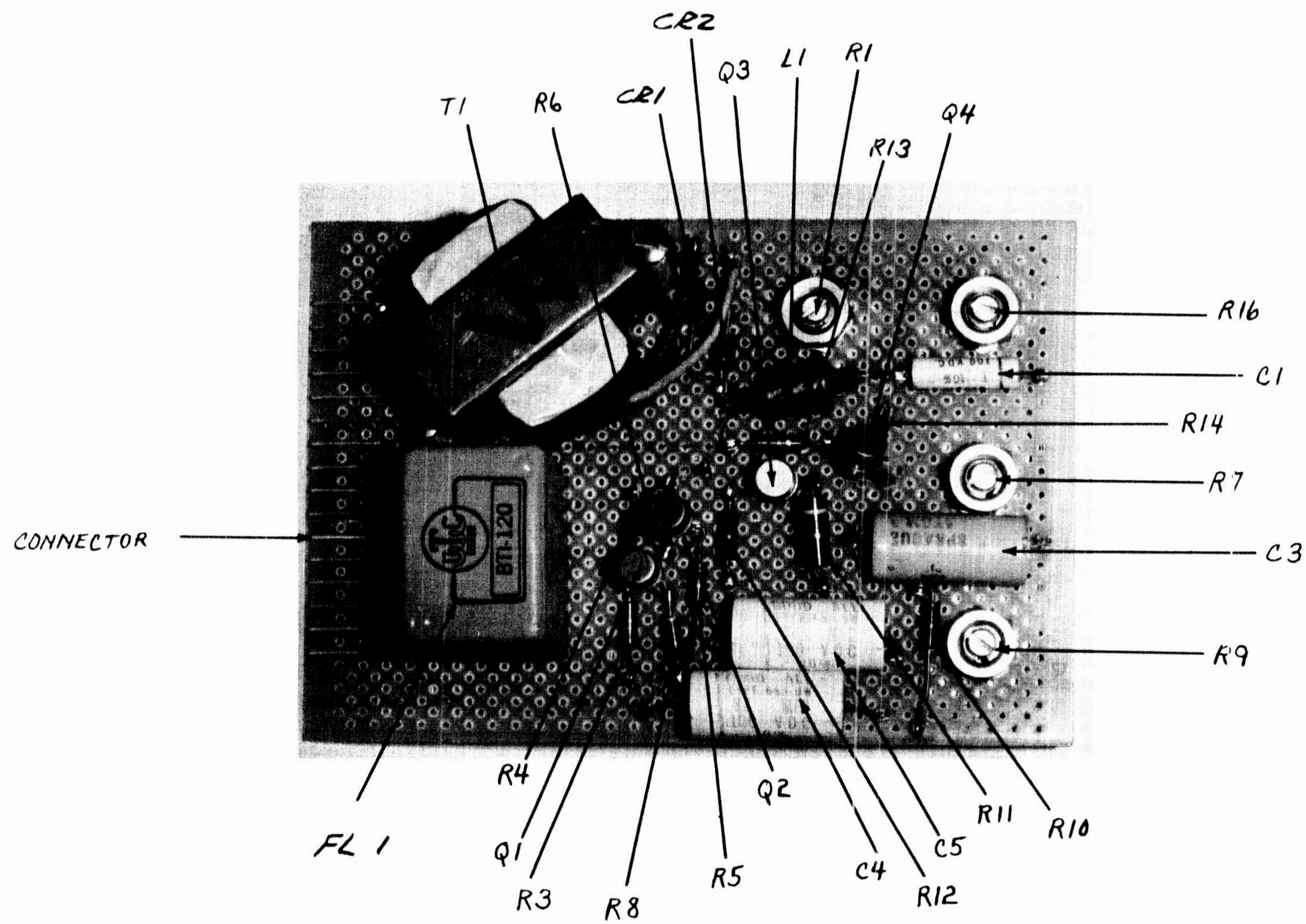

Figure 49   120 Hz GENERATOR

The 120 Hz sine wave is generated by full-wave rectifying the 60 Hz line and feeding it through a 120 Hz passband filter (UTC BTI-120). The output of the filter is fed into an amplifier-buffer with a voltage gain of two. The 90 degree phas shifted sine wave is generated by a phase shift network adjusted for exactly 90 degree phase shift. The amplitude of both outputs is adjusted for 4 volts, peak to peak, by the respective preset potentiometers.

6.2.13 120/720 Square Wave Generators (P6)

The 120/720 square wave generators (Figure 50 and 51) generate two 120 Hz square waves, synchronized to the 120 master line reference, and one 720 Hz square wave synchronized to the in-phase 120 Hz square wave generator.

The two 120 Hz square wave generators are free-running multivibrators. Each is synchronized, both in frequency and phase, to a sine wave generator. They therefore generate two 120 Hz square waves 90 degrees apart in phase. Each generator has a buffer at its output which provides a low source impedance, and 15 volt peak to peak signal level.

There is also a 720 Hz square wave pulse amplifier which provides the 720 Hz buck-out pulse for the preamplifier (not now used).

6.2.14 14.4 k Hz Signal Generator (P7)

The 14.4 K Hz signal generator (Figures 52 and 53) generates a 14.4 k Hz square wave input to the glow tube drive ciruit and to the 14.4 k Hz phase demodulator.

The generator is a free-running multivibrator tuned to run at 14.4 k Hz. Each output is followed by an emitter follower buffer. Both outputs are

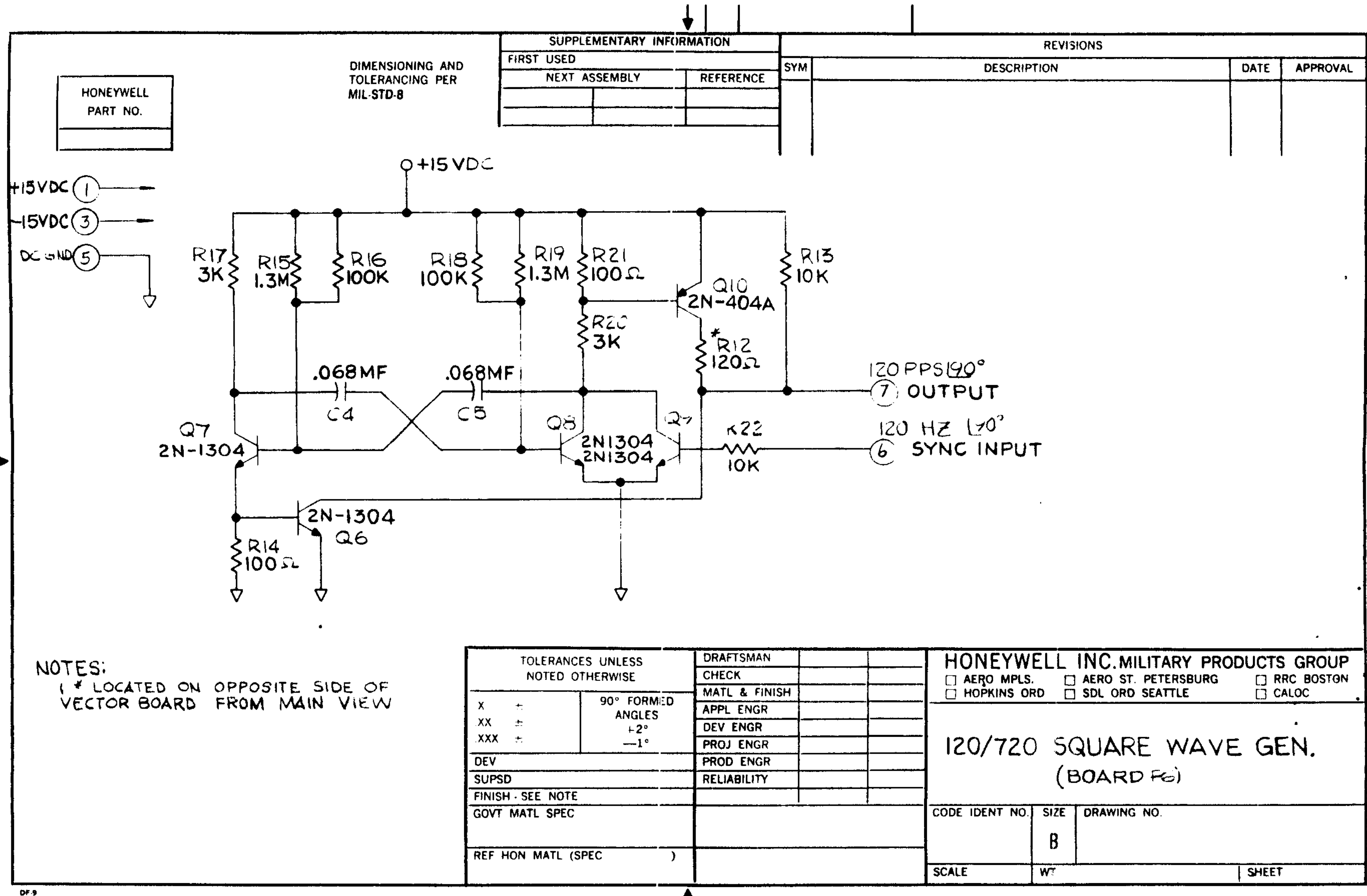

Figure 50 120/720 SQUARE WAVE GENERATOR

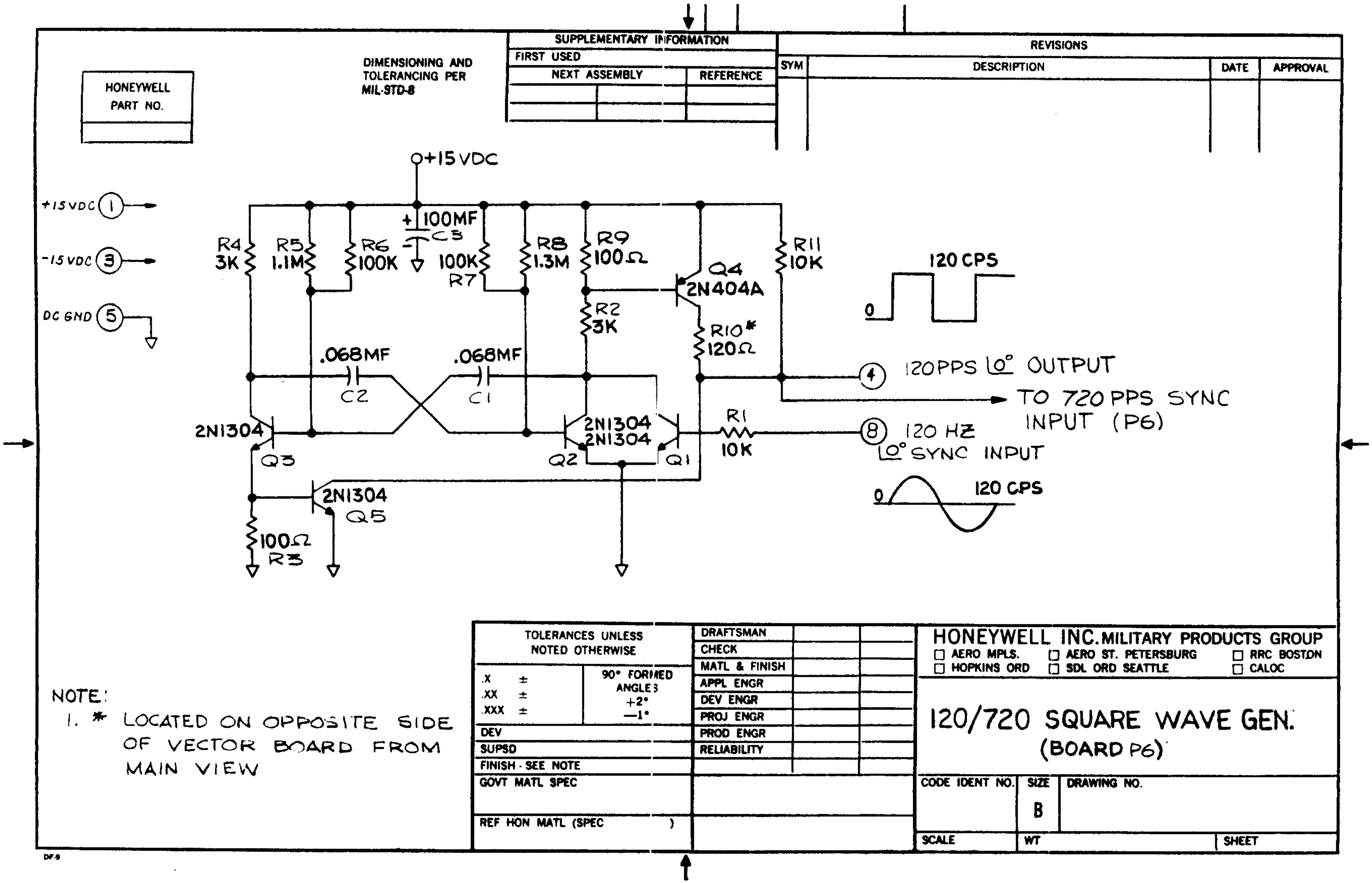

Figure 50a 120/720 SQUARE WAVE GENERATOR

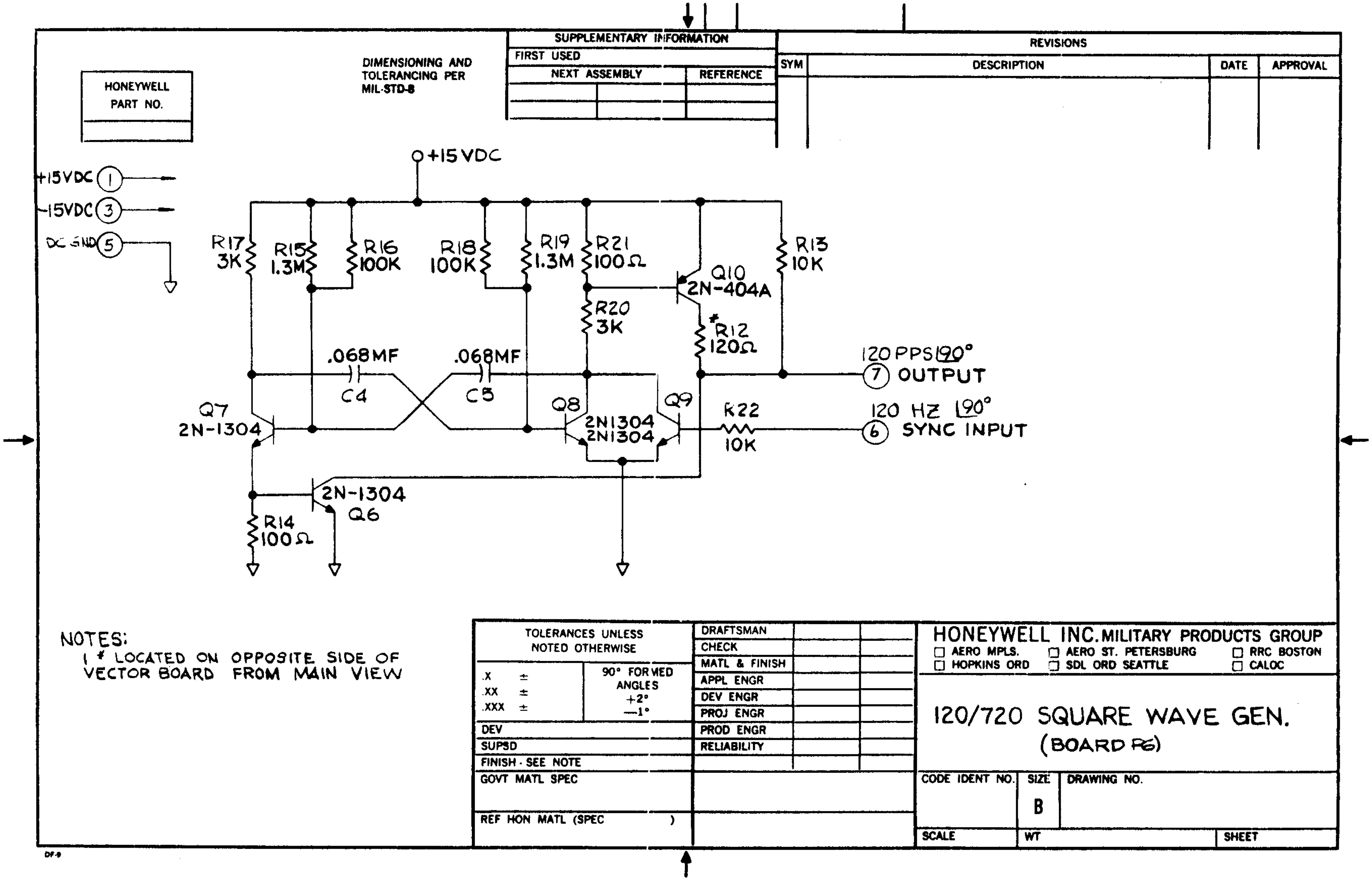

Figure 50b 120/720 SQUARE WAVE GENERATOR

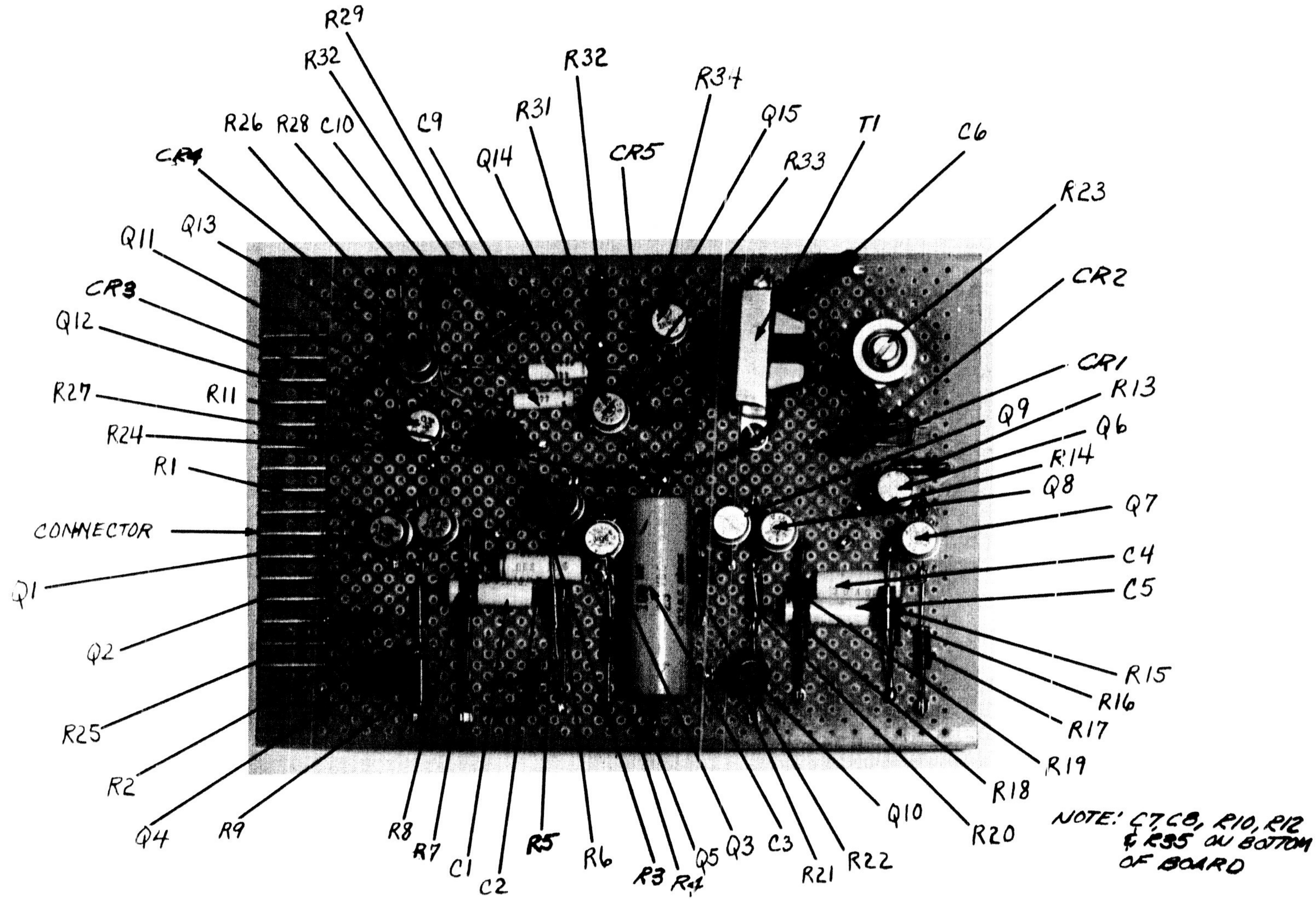

Figure 51 120/720 SQUARE WAVE GENERATORS (P6)

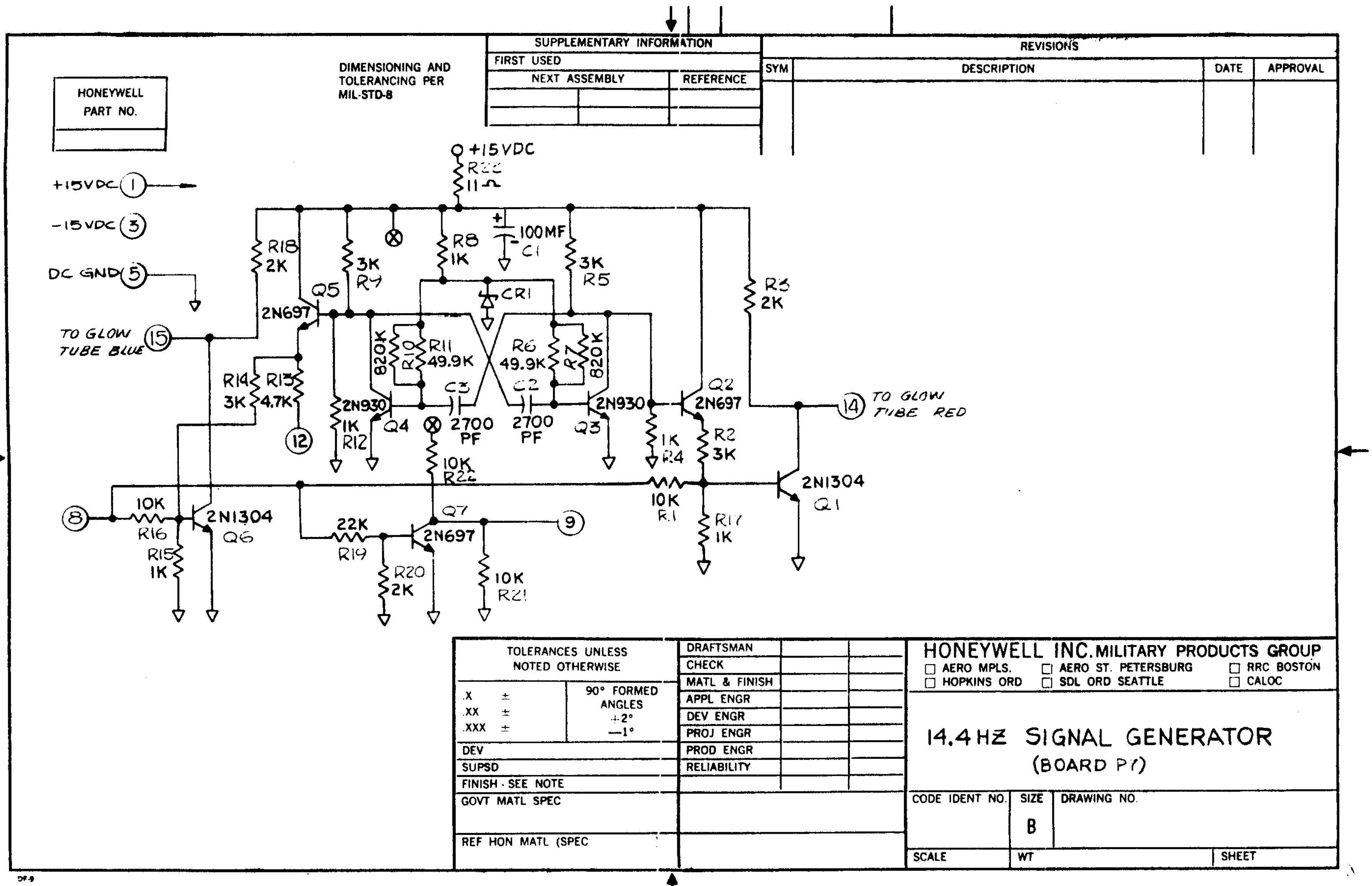

Figure 52 14.4 Hz SIGNAL GENERATOR

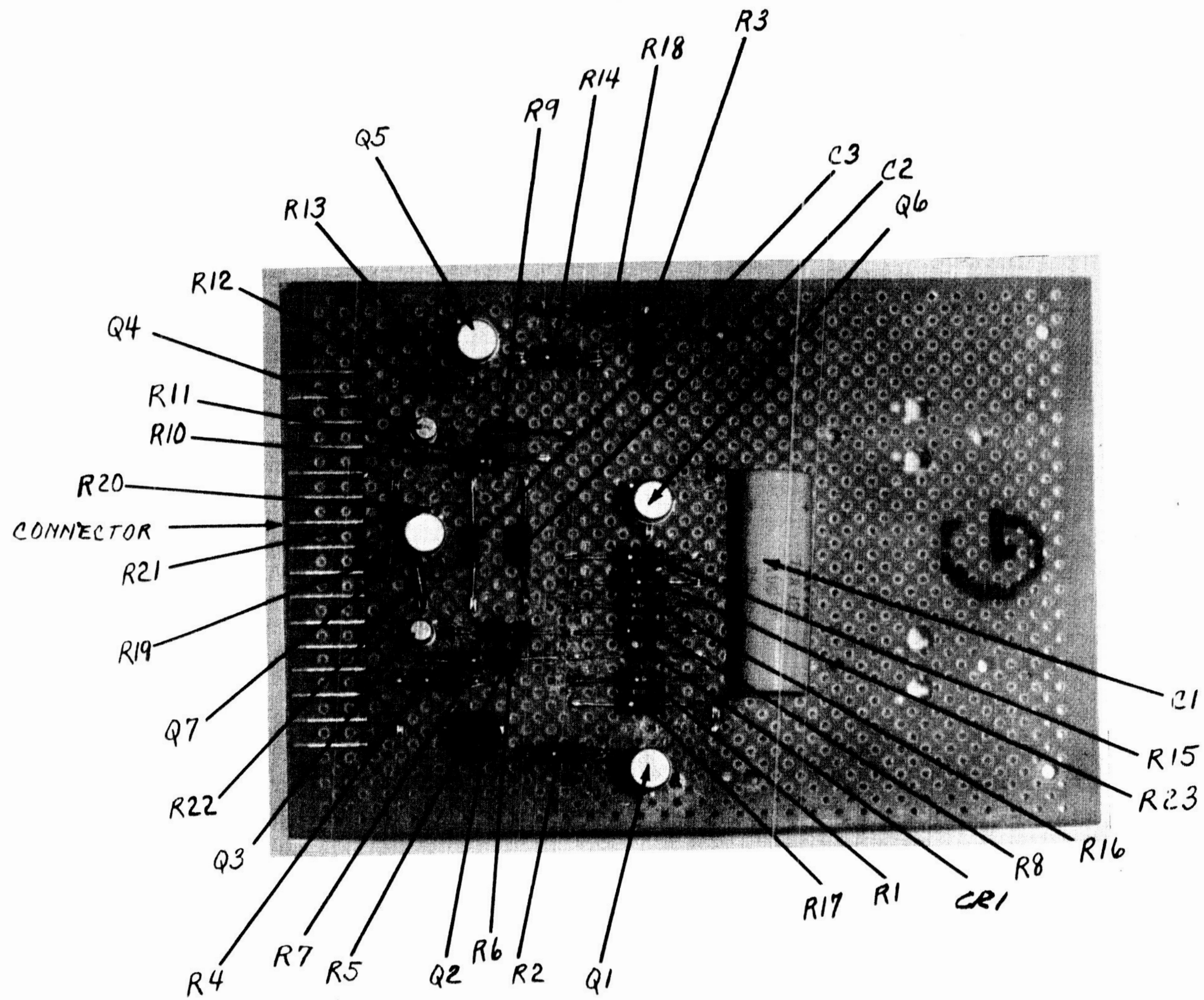

Figure 53 14.4 Kc SIGNAL GENERATOR

switched on and off simultaneously at a 720 k Hz rate by two saturating switches. Also present on this board is a 720 k Hz drive which operates the corneal-spot-loop x and y information choppers.

7.0 SUMMARY OF RESULTS AND RECOMMENDATIONS

The feasibility of tracking the pupil/iris boundary of the human eye, as it looks through a conventional eyepiece, has been established. (See Figures 22, 23) The basic system, electronic and optical design adopted to accomplish the program objectives is sound.

The most important technical problem encountered has been that of satisfactorily tracking the corneal reflection within the eye. The source of this problem has been identified and the appropriate corrective action defined (discussed in Appendix C).

Two dimensional tracking of the pupil/iris boundary has never before, as far as is known, been accomplished. This function is essential to the proposed eye measuring technique. Corneal reflections in the eye have previously been tracked. The difficulties encountered in corneal tracking in the present program are of an incidental nature and will certainly be overcome. The basic relationship between eye direction (independent of position) and the location of a corneal reflection within the pupil is readily established by direct observation. Consequently, the results achieved to date are sufficient to establish the basic feasibility of the new eye measuring technique, and additionally have contributed greatly to the solution of the practical engineering problems involved. What remains to be done now, is to develop the equipment into a more operational form, and then to evaluate the actual limitations of performance of the measuring technique.

Further work is now recommended as follows:

1. Integration of additional features, i. e., blink suppression, and automatic acquisition.

2. Substitution of an image dissector tube for the PM tube.

3. Re-introduction of the moving corneal spot feature by using a servo controlled moving mirror system.

4. Change in the electron aperture plate design to improve signal/noise ratio in corneal tracking (See Appendix C).

5. Redesign of the beam splitter system to improve overall transmission factor and reduce the generation of spurious signals by scattering (See Appendix C).

6. Redesign of the color chopping system to give increased signal and improved color cancellation (See Appendix C).

APPENDIX A

The principle of the measuring technique can be readily demonstrated by direct observation of the eye. The apparent position, to an observer, of the corneal reflection with the eye, relative to the center of the pupil, is independent (to first order) of the lateral position of the eye of the subject relative to the observer, but is directly proportional to the angular direction of the subject's eye relative to the (distant) source reflected in his cornea.

The following is a simple geometric proof of this phenomena. Referring to Figure A1, the coordinates of R (the virtual image of the collimated light incident at the eye, at angle θ to the axis of the eye) relative to axes through P, parallel to PC, are $(r_1 - r_2 \cos \theta)$, $r_2 \sin \theta$.

Let the "y" coordinate of R relative to axes through P, parallel to PO, be δ; then,

$$\delta = r_2 \sin \theta \cos (\psi - \theta) - (r_1 - r_2 \cos \theta) \sin (\psi - \theta)$$

where ψ is the angle of observation of the eye, relative to the direction of the incident light.

$$\therefore \delta = r_2 \sin \psi - r_1 \sin (\psi - \theta)$$

$$= 2r_1 \cos (\psi - \theta/2) \sin \theta/2 + (r_2 - r_1) \sin \psi$$

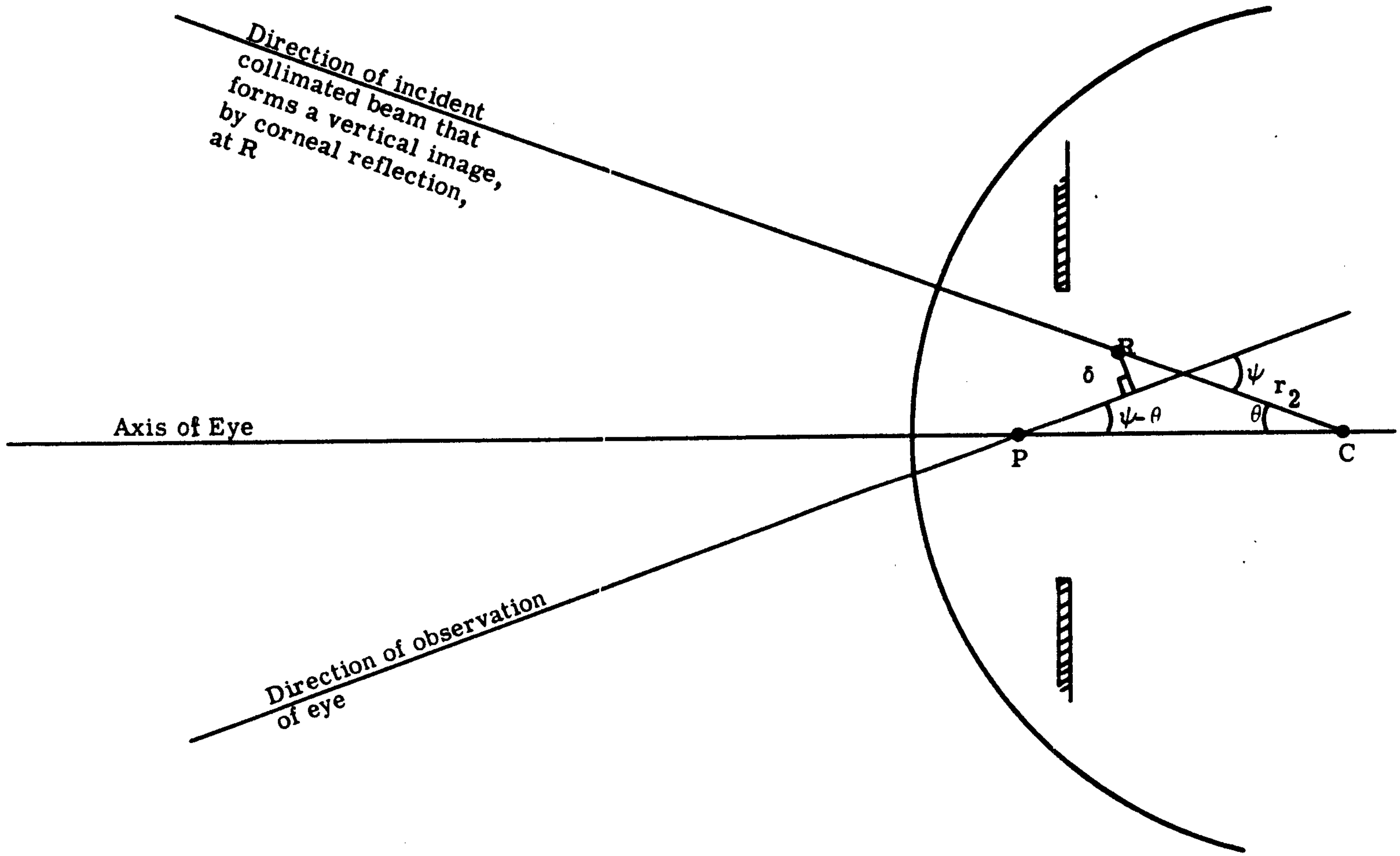

Figure A1 GEOMETRY OF THE EYEBALL

There are three particular cases of interest:

(1) the direction of the incident light is adjusted so that the corneal reflection appears to be at the center of the pupil (i. e. $\delta = 0$) Then $\delta = 0$ and, assuming ψ is not large,

$$2r_1 \sin \theta/2 \approx -(r_2 - r_1) \sin \psi$$

$$\theta \approx -\frac{(r_2 - r_1)}{r_1} \psi$$

That is, the axis of the eye is approximately parallel to the direction of the incident collimated light and eye direction can then be measured by measuring the direction of the incident light (a small correction, $\theta \approx \frac{(r_2 - r_1)}{r_1} \psi$, must be included).

This is the case conceived in the original concept of the system. The direction of the incident light was to be varied by varying the position of a spot on the screen of a CRT until its corneal reflection appeared at the center of the pupil. Eye direction was the proportional, to first order, to the position of the spot on the CRT. The small correction, $\frac{(r_2 - r_1)}{r_1} \psi$, was to be computed by measuring the position of the image of the pupil on the photocathode of the PM tube.

(2) The direction of the eye is determined by measuring δ and ψ.

If ψ is not large, and also $\theta/2$ is not large, then

$$\delta \approx r_1\theta + (r_2 - r_1)\ \psi$$

$$\theta \approx \frac{\delta}{r_1} - \frac{(r_2 - r_1)}{r_1}\ \psi$$

This is the basis of the system concept, as modified during the program.

(3) It is possible to arrange that, at all times, $\psi = 0$, then

$$\delta = r_1 \sin \theta$$

In other words, eye direction relative to the direction of the incident light, is directly proportional to δ, and totally independent of lateral position of the eye.

One method of arranging for $\psi = 0$ in all cases is to use a fixed source of external light (for corneal reflection) and to make the virtual collecting aperture for the PM tube optical system appear at position ∞ on the optical axis. (See Figure 3)

APPENDIX B

The relative sensitivity of the PM tube, across to the photocathode, was measured by projecting a small spot of light onto the photocathode with a travelling microscope.

The relative electrical signal was measured for different positions along a straight line across the photocathode. Two runs, at right angles, were made (as shown in Figure B1). The relative response in each case (I and II) is shown plotted in Figure B2.

In addition to measuring linear position, along the scan line, with the travelling microscope scale, the current in the PM tube deflection coils was also measured. In this way it was possible to relate the two scan lines to their actual positions on the photocathode, relative to electrical center.

The results of these measurements indicate that the region of the photocathode over which reasonably uniform response can be obtained is about 0.4 inch in diameter.

In the design, phase, it was assumed that satisfactory performance out to 0.75 inch diameter could be realized. The poorer than expected performance of the tube resulted in a reduced area at the eye over which the pupil could be tracked.

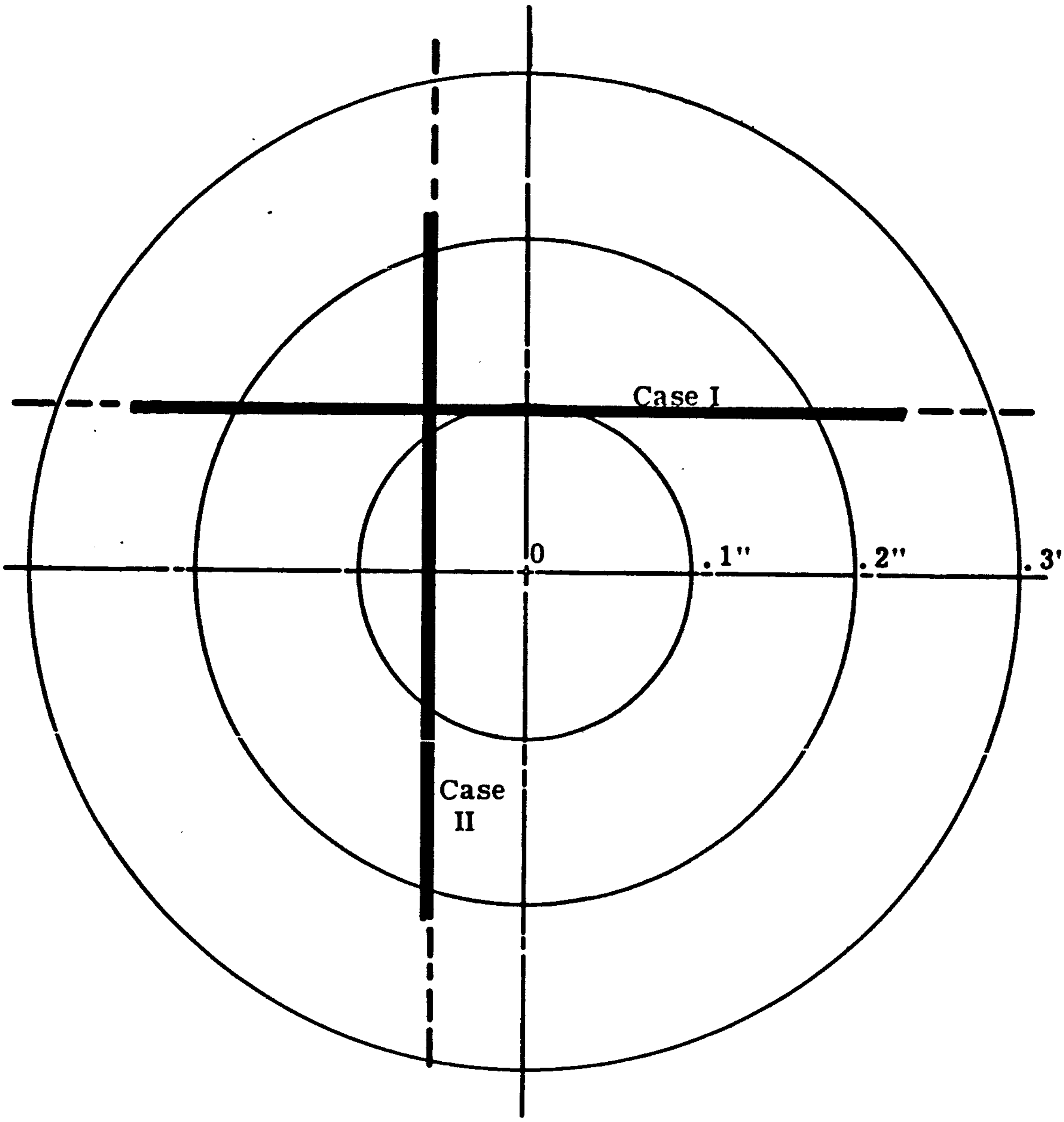

Figure B1 TWO DIMENSIONAL PLOT OF CASES I, II - SCANS ACROSS PM PHOTOCATHODE (FW 130)

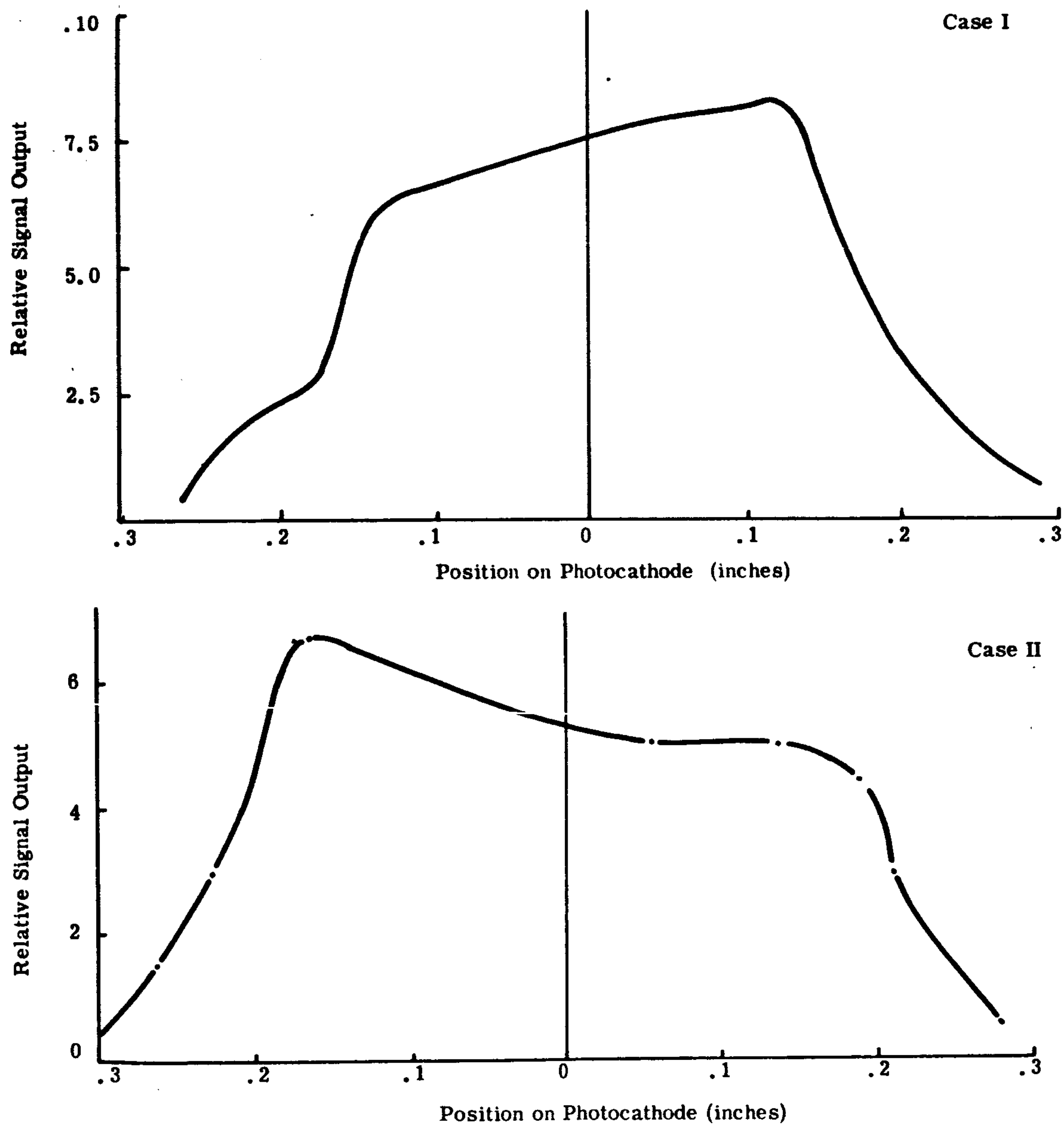

Figure B2 RELATIVE SENSITIVITY OF FW 130 PM TUBE AS A FUNCTION OF POSITION ON THE PHOTOCATHODE

APPENDIX C

CORNEAL TRACKING

Satisfactory corneal reflection tracking has not yet been achieved. There are three basic factors involved.

1. Insufficient signal is obtained from the true corneal reflection (chopped red light).
2. The spurious reflections produce a constant intensity (chopped red and chopped blue = constant) which has an appreciable shot noise component.
3. The relative intensity of the red and blue components, in the spurious reflections, is not constant over the photocathode.

Items 1 and 2 result in an unsatisfactory signal to noise ratio, and false sisnals are produced because of item 3.

The special demodulation and tracking electronics incorporated as a system modification have proven completely satisfactory and do not contribute in any way to the problems detailed above.

The source of the problem has been identified, and is described below.

C.1 Insufficient Signal

(a) Beam Splitters

Light from the corneal reflector must pass four times, by reflection or transmission, through beam splitters, which are now all neutral density.

Assuming a transmission/reflection factor of 40% in each case, the overall transmission is only 2.5%. Thus the effective brightness of the true corneal reflection is only about 0.1% of the glow tube source (the reflection factor of the cornea is about 3%).

(b) Source Intensity

The glow modulator tubes have a relatively high brightness. The specifications quote a figure of 43 candles/per square inch. However, the size of the bright source is small (approx 0.05 inch diameter) and a high f-number optical system is used to collect the glow modulator light and divert it onto the eyepiece. An equivalent statement is that the corneal reflection image in the eye is very small (theoretically about 0.003 inch). Thus, although the corneal reflection is moderately bright, it is very small and hence delivers only a small amount of total light to the PM tube.

C.2 Noise Level

The primary noise source in the system is optical shot noise, arising from the discrete, quantum, nature of light. If, in one time interval, a total of N photons are produced, there is an inherent statistical fluctuation of $\sqrt{N}$ photoelectrons associated with this signal.

Shot noise arises in two ways - from the signal itself (shot noise in signal) - and from the steady (unchopped) background of the combination of red and blue spurious reflections (shot noise in background). This latter source is particularly significant because while the signal is being picked up only by the small central hole in the aperture plate (Figure 12), the steady background can be picked up by the larger area of the central hole plus surrounding annulus.

C.3 Non-Uniform Spectral Reflection

The cancellation of spurious reflections is based upon the assumption that the red and blue light is equally reflected, and that the spurious reflection image as detected by the PM tube is correspondingly unchopped. Limitations of this assumption are as follows:

1. The eyepiece surfaces are coated, and the spectral property of these coatings is a function of angle of incidence.

   The spurious reflections arriving at the various points on the photocathode each have a different angle of incidence at the eyepiece lenses. Thus, the reflected "color" of the lenses depends on position on the photocathode of the PM tube.

2. In the present arrangement, the bright areas of the two glow tubes are of different sizes, resulting in a non-uniform color of their reflections.

3. A major source of spurious light is by scattering at one of the beam splitters. This reflecting mechanism has a different spectral characteristic to that of reflection at the eyepiece lenses.

The following action is thus indicated:

1. Redesign of beam splitter system to improve the overall transmission factor.

2. Redesign of the glow modulator tube optics system to permit the collection of more light from the tubes.

3. Redesign of PM tube aperture plate to give:

   a. increase in corneal reflection signal

   b. decrease in steady (unchopped) background

4. Redesign of beam splitters system to avoid the detection by the PM tube of glow tube light by scattering.
5. Use of specially coated, or non-coated, eyepiece lenses to:
   a. yield minimum red reflection
   b. uniformity of the ratio of the red to blue reflection factor.
6. Use of red and blue glow modulator tubes with luminous craters of equal size.

All of these factors must be considered in the next phase system design.

APPENDIX D

SYSTEM ANALYSIS

The primary physical limitation of the electro-optical tracking system is the shot noise in the light received by the PM tube from the eye. In order, therefore, to ensure satisfactory performance, the light level must be high enough so that, during each sample period, the number (N) of photoelectrons detected is large relative to the statistical fluctuation (shot noise) which is of magnitude $\sqrt{N}$ (i.e., $N/\sqrt{N} = \sqrt{N} \gg 1$).

An exact accuracy analysis involves the detailed geometry of overlapping circles, blur circle of the optics, etc. The following semi-quantitative analysis, in which the basic detectability of the detail within the eye is considered, is more appropriate in the present context.

D.1 Corneal Reflection

Let B (watts/seradian/cm^2) be the brightness of the crater on the glow modulator tube. Let the active area of the crater be A (cm^2). Let k_1 be the overall loss factor of the beam splitters and other optics (glow tube light in, to corneal reflection light out). Then the effective brightness of the corneal reflection is $0.03B_1k_1$ (corneal reflection factor is approximately 3%). The area of the corneal reflection is approximately

$$a_1 \left(\frac{1}{2}\right)^2 \left(\frac{1}{10}\right)^2 = a_1 2.5\,10^{-3}$$

(The 1/2 factor arises from the imaging of the glow tube at the focal plane of the tyepiece - the factor 1/10 is the ratio of the focal length of the eyepiece to the focal length of the corneal mirror.)

Let F be the effective number of the PM tube optics. Then the illumination, E, at the PM tube photocathode, due to the corneal reflection, is

$$E_1 = \frac{\pi 0.03 B_1 k_1}{4F^2 (1 + m)^2} \text{ watts/cm}^2$$

where m is the magnification. In the present case $F \approx 5$* and $m \approx 1$

The area of the PM tube image of the corneal reflection is $2.5 a_1 m^2 10^{-3}$

Thus the total light flux at the PM tube due to corneal reflection is

$$P_1 = \frac{\pi 0.03 B_1 k_1 \, 2.5 a_1 10^{-3}}{4 \quad 10^2} \quad \text{watts}$$

The fundamental sampling time may be considered as 1/240 sec (1/2f, where f is the frequency of the rotary scan). The flux must be decreased by two because of the time division multiplex between corneal and pupil tracking.

*A relatively high f-number is necessary in order to provide sufficient depth of focus to meet the ±0.1" axial positioning tolerance.

Thus the total quantity of energy received per the sample time is

$$Q_1 = \frac{\pi\, 0.03 B_1 k_1\, 2.5 a_1 10^{-3}}{4 \quad 10^2 \quad 480} \quad \text{watt seconds}$$

To convert to photoelectrons, multiply by (approximately) 3×10^{18} (to convert watt seconds to photons) and also by the quantum efficiency (approximately 0.1) of the photocathode, i.e.,

$$Q_1 = \frac{\pi\, 0.03 B_1 k_1\, 2.5 a_1 10^{-3}\; 3\; 10^{17}}{4 \quad 10^2 \quad 480} \quad \text{photo electrons/sample time}$$

$$\approx 3.8 \times 10^8 B_1 k_1 a_1$$

There are four intersections with neutral density beam splitters involved in the optical path. Thus k_1 will be given approximately by

$$k_1 = 0.7 \; (0.4)^4$$

(70% transmission factor of the optics)

$$\therefore\; k_1 = 1.8 \times 10^{-2}$$

B_1 is given as

$$B_1 = 40 \text{ candles/in}^2 = 6.7 \times 10^2 \text{ watts/stradian/in}^2$$

$$a_1 = 2.5 \times 10^{-3} \text{ in}^2$$

$$\therefore Q_1 = 3.8 \times 10^8 \; 6.7 \; 10^{-2} \; 2.5 \; 10^{-3} \; 1.8 \; 10^{-2} \approx 10^3$$

Finally this figure must be divided by 2 to account for the chopping of the light in the corneal reflection. The statistical fluctuation within this number (i.e. 500) is about 22, or about 5%. In practice, performance will be significantly poorer because

1. tracking depends on detecting a fraction (e.g. 10%), rather than the entire, light content of the spot.
2. the limited sharpness of the optical system (and of the electron optics) will result in less signal being collected than as predicted by theory.
3. the existence of shot noise in the steady background as detected by the annulus (which has a larger area than the central hole).

Taking the above factors into consideration, and also realizing the approximate nature of the analysis, the numerical result obtained is consistent with the observed poor corneal tracking capability of the system.

D.2 Pupil Iris Boundary

Let E_2 (watts/cm^2) be the illumination at the periphery of the eye. Assuming a 10% diffusivity of the iris, the brightness of the iris is $\frac{0.1 \, E_2}{\pi}$.

The illumination at the PM tube is

$$\frac{\pi \, 0.1 \, E_2 \, 0.1}{4 \, \pi \, 10^2}$$

where the factor 0.1 accounts for the losses in the optical system and beam splitters, and the 10^2 factor for the f-number (5) and magnification (unity).

For the present purpose, the total flux received per sample time from an area of 10^{-2} square millimeters will be computed. As before,

$$P_2 = \frac{\pi \, 0.1 \, E_2 \, 0.1 \, 10^{-4}}{4 \quad \pi \quad 10^2} \quad \text{watts}$$

$$Q_2 = \frac{\pi \, 0.1 \, E_2 \, 0.1 \, 10^{-4}}{4 \, \pi \, 10^2 \, 480} \quad \text{watt seconds}$$

$$= \frac{\pi \, 0.1 \, E_2 \, 0.1 \, 10^{-4} \, 3 \times 10^{17}}{4 \, \pi \, 10^2 \quad 480} \quad \text{photoelectrons/sample time}$$

$$= 1.5 \, 10^6 \quad E_2$$

Assuming

$$E_2 = 30 \text{ ft candles}$$

$$= \frac{30}{600} \times 10^{-3} \text{ watts/cm}^2$$

$$= 5 \times 10^{-5}$$

$$Q_2 = 75 \text{ photoelectrons/sec}$$

This figure indicates that detail down to 10^{-4} cm^2 can be reliably "seen" during the 1/240 sampling period.