Plasma Resonance Radiation and Type III Bursts

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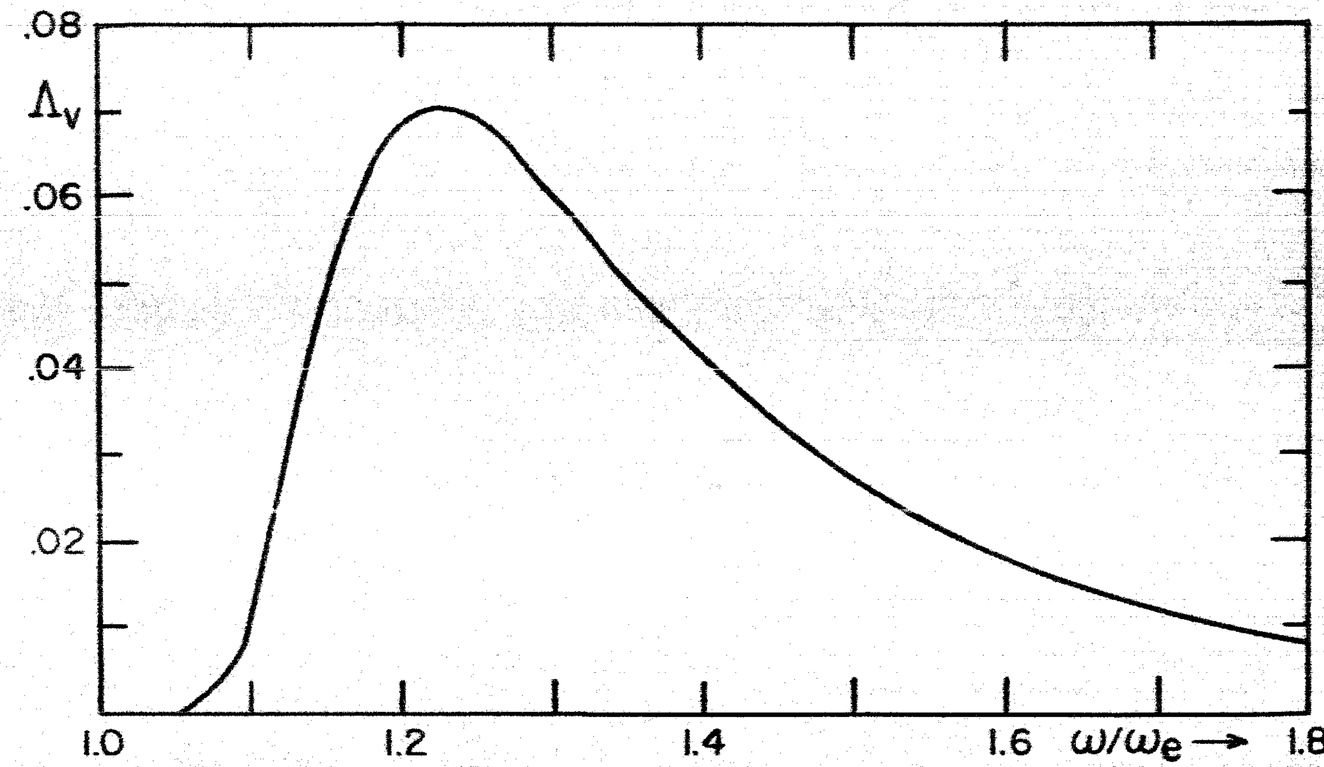
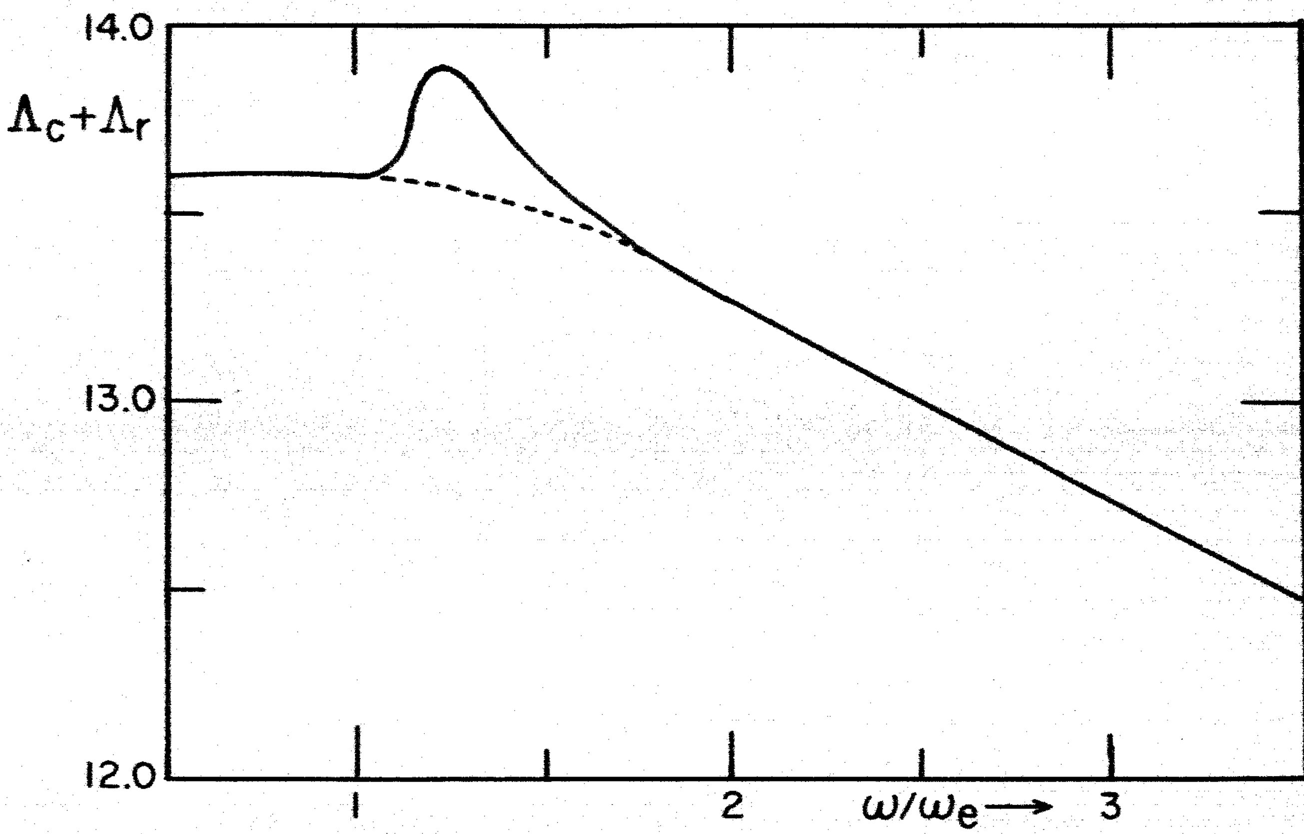

.08
Λ_v
.06
.04
.02
1.0
1.2
1.4
1.6
$\omega/\omega_e \rightarrow$
1.8

FIG. 1


14.0
$\Lambda_c+\Lambda_r$
13.0
12.0
1
2
$\omega/\omega_e \rightarrow$
3

ABSTRACT

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Altschuler's hydrodynamic theory of plasma resonance radiation induced by single test particles is extended to particle ensembles. It is shown that his expressions agree with the known results of kinetic theory in the case of thermal equilibrium. Next, the situation of a high temperature plasma permeated by fast particle clouds is considered, spectrum and total power emitted are calculated, and the results compared with type III bursts. It is found that the observed values can be represented by assuming a total of 10^{32} cloud particles in units of 10^7 particles, with each cloud occupying about 10^3 cm^3. The number of fast particles in the source area is thus less than 10^{-4} of the coronal electron density.

1. INTRODUCTION

There appears to be a consensus that type III events are physically to be understood as a resonance response of the coronal electron plasma to the passage of clouds of fast particles whose velocity is a significant fraction of the speed of light, although relativistic effects proper are not responsible for the major characteristics of the radiation pattern.

This radiation pattern consists essentially of a broad spectral line whose center frequency coincides with the electron plasma frequency in the corona at the position from which the emission is originating. Since the exciting source moves outward in the corona from higher to lower density regions, the characteristic frequency decreases as a function of time, although some events, known as U-bursts, show a reversal of the trend and in this framework require a source of excitation that falls back towards the photosphere. In any case, the radiation is emitted by the coronal electrons and not by the fast-moving stream of particles, as can be seen from the relation between the characteristic frequency of the type III bursts and the electron plasma frequency of the corona proper.

Recently, a series of papers[1,2,3,4] appeared whose applications aimed at the related problem of type II emission, in particular, the appearance of the second harmonic, and was based on kinetic theory considerations. We have chosen the framework of the hydrodynamic equations for two reasons: 1. The basic system of equations is comparatively simple and thus permits a continuous control of the physical implications. 2. The results can be presented in closed form that makes the computation of radiation patterns in a straight forward manner possible even for quite complicated source systems.

Our fundamental equations were obtained in a form suitable for application to the type III problem by Altschuler[5], based on previous work by Majumdar[6] and Cohen[7]. Altschuler's expressions are written for an unspecified "test particle" passing through a background of plasma. Resonance radiation appears if the test particle's relative speed relative to the radiating field electron is supersonic.

After reviewing in Section 2 Altschuler's results as far as they concern the resonance radiation, we summarize the general problem of finding the resonance radiation from an ensemble of field electrons (Section 3). Specifically, the emission of a Maxwellian plasma in the neighborhood of the plasma frequency is obtained in Section 4. We then turn to the situation of particle clouds moving through the corona (Section 5), and summarize our conclusions for type III bursts in Section 6.

2. EMISSION BY A TEST PARTICLE - FIELD ELECTRON PAIR

Altschuler's procedure to arrive at the resonance radiation is as follows[5]: he starts from the self-consistent set of equation of motion and Maxwell's equations and derives the electric field E due to test particle and background plasma electrons, acting on an arbitrary field electron at a specified distance b (impact Parameter). If $E(b,\omega)$ is the Fourier transform in time, we obtain for the electromagnetic radiation

$$Q(b,\omega) = (4e^{2}/3c^{3})\,(eq^{2}/m)^{2}\,|E(b,\omega)|\,. \qquad (1)$$

From $Q(b,\omega)$ a physically meaningful quantity is obtained considering the radiation emitted by a field electron in an "average distance" from the test particle, i.e., by defining the so-called "radiation probability"

$$\chi(\omega) = 2\pi\int Q(\omega,b)\, b\,db. \tag{2}$$

Altschuler showed that both $Q(b,\omega)$ and $\chi(\omega)$ have a singularity at the "resonance frequency"

$$\omega_r = \omega_e/\sqrt{(1 - V^2/u^2)}, \tag{3}$$

where ω_e is the plasma frequency of the field electrons, V is the thermal speed, u the relative speed of the test particle. Mathematically, the resonance behavior is expressed by the fact that Q and are proportional to $Y_o\,(b\zeta_1)$, where Y_o is the Bessel function in conventional notation. However,

$$\zeta_1{}^2 = \omega^2[V^{-2} - u^{-2}] - \omega_e{}^2/V^2 \tag{4}$$

vanishes at $\omega = \omega_r$.

To dominant order, the spectrum function reads

$$Q_r(b,\omega,u) = (8e^2/3\pi c^3)\ (qe^2/m)^2\ (\omega_e{}^4/\omega_r{}^2V^4)\ (\pi^2/4)\ Y_o{}^2(b\zeta_1), \tag{5}$$

where e, m, and c are the natural constants, q is the charge of the test particle, while the radiation probability becomes

$$\chi(\omega,u) = (16e^2/3c^3)\ (qe^2/m)^2(\pi^2/8u^2V^2)\ [\omega_e{}^4b_m{}^2/(\omega^2-\omega_e{}^2)]\ Y_o{}^2(b_m\zeta_1). \tag{6}$$

b_m is identified with the Debye length, viz.,

$$b_m = V/\omega_e\ . \tag{7}$$

The total amount radiated in the resonance is finite. From Eq. (5) or from Eq. (6), accurate to order $\ln(b_m/b_o)$, where

$$b_o = qe^2/mu^2 , \tag{8}$$

we find

$$\langle\chi_r\rangle_\omega \equiv \int\chi_r(\omega,u)\, d\omega = (16e^2/3c^3)\, (qe^2/m)^2\, (\omega_e/2\gamma u)\, [\sqrt{(u^2-V^2)}/V^2]. \tag{9}$$

$\gamma = 1.78...$ is Euler's constnat.

3. RESONANCE RADIATION FROM A PARTICLE ENSEMBLE

Let us consider an ensemble of field electrons with a normalized distribution function of relative speeds u such that

$$\int_0^\infty f(u)\, du = 1. \tag{10}$$

The considerations to follow are written down for spherically symmetric distributions in phase space, but are equally applicable to anisotropic distributions by remembering that

$$u^2 = u_x{}^2 + u_y{}^2 + u_z{}^2, \tag{11}$$

while under non-relativistic conditions the limits of integration run from $-\infty$ to $+\infty$.

So long as f(u) is smooth and not of the form of a δ - function, f(u) is a slowly varying function across the singularity $\zeta_1 \to 0$, so that

$$\langle u\chi_r\rangle_u = (16e^2/3c^3)\, (qe^2/m)^2\, (\pi^2/8u_o{}^2V^2)\, [\omega_e{}^4 b_m{}^2/(\omega^2-\omega_e{}^2)] \times$$

$$\times\, u_o\cdot f(u_o)\cdot\int Y_o{}^2(b_m\zeta_1(u))\, du, \tag{12}$$

with u_o defined by $\zeta_1 \to 0$, i.e.,

$$u_o{}^2 = V^2[\omega^2/(\omega^2-\omega_e{}^2)]. \tag{13}$$

It has been shown in Reference 5 that the resonance is symmetric with respect to $\omega = \omega_r$, so that the integration in (12) results in twice the value of the integral between u_o and ∞.

Close to the resonance, Y_o can be expanded. To dominant order,

$$Y_o{}^2(b_m\zeta_1) \approx (4/\pi^2)\ \ln^2(\gamma b_m\zeta_1/2). \tag{14}$$

The integration in (12) is then extended from u_o to some value $u_o + \varepsilon$, and the integral becomes

$$\pi^{-2} \int \ln^2[(\gamma^2 b_m{}^2/4)\ \{(\omega^2-\omega_e{}^2/V^2) - (\omega^2/u^2)\}]\ du. \tag{15}$$

Now,

$$\int_0^{z_1} \ln^2 z\ dz = z\ (\ln^2 z - 2\ln z + 2)\Big|_0^{z_1} \quad . \tag{16}$$

The upper limit of integration is to good accuracy given by the condition that $u_o + \varepsilon$ reduces to $z_1 = 1$. In this approximation

$$\int Y_o{}^2(b_m\zeta_1)\ du = (4/\pi^2)\ (\omega V^3/\gamma b_m{}^2)\ (\omega^2 - \omega_e{}^2)^{-3/2}, \tag{17}$$

and

$$\langle u\chi_r\rangle_u = (8e^2/3\pi c^3)\ (qe^2/m)^2\ [\omega_e{}^2/(\omega^2-\omega_e{}^2)^2]\ f(u_o). \tag{18}$$

The total radiation emitted in the resonance is then found from (18) by integrating over ω, viz.,

$$\langle u\chi_r\rangle_{u,\omega} = (8e^2/3\pi c^3)\ (qe^2/m)^2\omega_e{}^4 \int f[u_o(\omega)]/(\omega^2-\omega_e{}^2)^2\ d\omega. \tag{19}$$

4. MAXWELLIAN ENSEMBLE

As an illustrationof the above relations, we compute the resonance spectrum and the total radiation in the case of a Maxwellian ensemble, defined by

$$f_1(u) = 4\pi(m/2\pi KT)^{3/2}\, u^2 \exp(-mu^2/2KT). \qquad (20)$$

The functional behavior in the neighborhood of the plasma frequency becomes, leaving numerical factors of order unity aside,

$$\{u\chi_r\}_u \approx (q^2e^6/c^3m^2)\,(m/KT)^{1/2}\,[\omega_e{}^2/(\omega^2-\omega_e{}^2)]\,\exp[-3\omega_e{}^2/2(\omega^2-\omega_e{}^2)], \qquad (21)$$

where the curly brackets stand for the Maxwellian average, and $\omega \approx \omega_e$.

A more accurate calculation was carried out numerically with the aid of Eq. (6) and the Bessel function Y_o. The result is plotted in Fig. 1 as a function of frequency in units of the plasma frequency. The normalization is made with the aid of the bremsstrahlung continuum whose intensity for $\omega >> \omega_e$ is given by[8]

$$4\pi\varepsilon_\omega d\omega = [32q^2e^6/3\surd(2\pi)m^2c^3]\,(m/KT)^{1/2}\,\Lambda_c, \qquad (22)$$

where

$$\Lambda_c = \ln\,[(2KT/\gamma m)^{3/2}(2m/\gamma qe^2\omega)]. \qquad (23)$$

The absolute value of the resonance emission is then obtained by multiplying Λ_r from Fig. 1 with the factor in front of Λ_c in Eq. (22). In the neighborhood of the plasma frequency, Λ_c is not accurate and has to be replaced by the numerical value obtained elsewhere[9], whereas for very low frequencies

$$\Lambda_c(\omega\to 0) \approx \ln(b_m/\bar{b}_o) - 1/2 \ . \tag{24}$$

b_m is the Debye length of Eq. (7), $\bar{b}_o$ the 90° deflection parameter as in Eq. (8), but with u^2 replaced by $2KT/m$.

If Λ_c is added to Λ_r, the spectrum of Fig. 2 is obtained. The numerical values correspond to a ratio $b_m/\bar{b}_o = 10^6$, conforming to the plot in figure 1 of Reference 9. It is readily seen that the resonance emission from a Maxwellian ensemble amounts to an insignificant contribution of approximately $0.3\omega_e$ half width and of some 0.5 % peak intensity at about $1.2\omega_e$. Preciesely this result was found by Dawson and Oberman[10] with the aid of kinetic theory.

The total radiation in the resonance, finally, can be found by integrating the spectrum of Fig. 1. The qualitative behavior is seen from Eq. (19) which, aside from the numerical factors, gives for a Maxwellian distribution

$$\{u\chi_r\}_{u,\omega} \approx (q^2e^6/m^2c^3)\ (m/KT)^{1/2}\ \omega_e \ . \tag{25}$$

This is Tidman and Dupree's result[3] for an equilibrium plasma.

5. NON - THERMAL DISTRIBUTIONS

We now turn to field electron distributions that differ from the Maxwellian by the existence of a high energy tail or, more specifically, by the presence of a secondary, one-dimensional distribution around a velocity $W \gg V$. Such a distribution can be represented by

$$f_2(u) = (\pi^{1/2}U_o)^{-1}\ \exp[-(u-W)^2/U_o{}^2]. \tag{26}$$

We assume that $U_o \ll W$, i.e., that the spread in velocities of the secondary distribution is much smaller than the average speed of order W.

The spectral behavior is found from Eq. (18) which yields

$$\langle u\chi_r\rangle_u \simeq (q^2e^6/m^2c^3)\ [\omega_e^2/(\omega^2-\omega_e^2)]^2\ U_o^{-1}\ \exp\{-[V\omega_e/\sqrt{(\omega^2-\omega_e^2)} - W]^2/U_o^2\}. \quad (27)$$

The width of the resonance peak is determined by the exponent, and since we assume that $U_o \ll W$, we have approximately

$$(\omega_{1/2} - \omega_e)/\omega_e \simeq V^2/W^2\ , \quad (28)$$

As one would expect, the resonance emission is the narrower, the higher the mean intensity of the secondary distribution is. This follows from the fact that for $u \to \infty$ the resonance frequency $\omega_r \to \omega_e$.

The total radiation is obtained by carrying out the integration (19) over the distribution (26). This is conveniently done with

$$x^2 = \omega_e^2/(\omega^2-\omega_e^2) \quad (29)$$

as variable. The quantity of interest then becomes

$$U_o^{-1} \int x^2 \cdot \exp\{-V^2(x - W/V)^2/U_o^2\}dx\ , \quad (30)$$

which for $U_o \ll W$ leads to

$$\langle u\chi_r\rangle_{u,\omega} \simeq (q^2e^6/m^2c^3)\ (\omega_e/V)\ (W/V)^2\ . \quad (31)$$

Eq. (31) is the desired result. It is independent of U_o. As compared with an equilibrium plasma, distribution (26) yields an increase in the resonance radiation by a factor $(W/V)^2$. Similar values were obtained by Tidman and Dupree[3] for non-thermal distributions.

6. TYPE III EMISSION

The derivations outlined in the last Section can now be applied to the problem of type III emission by giving the distribution (26) a physical meaning. For this purpose, we postulate clouds of fast particles traversing the coronal plasma with a constant speed W, and interacting with the thermal electrons in the corona. Eq. (26) then represents the thermal electrons in the rest frame of the particle cloud with $U_o = V$. The two coordinates perpendicular to the direction of W are integrated out and absorbed in the normalization. Note that the total radiation (31) does not depend on U_o, so long as U_o is much higher than W.

If there are N_1 electrons per cm^3 (and, for that matter, N_1 ions) present in the thermal distribution, and if a particle cloud consists of N_2 uncorrelated particles, the emission per cm^3 and sec is found by multiplying Eq. (31) by N_1N_2. If, however, the cloud of fast particles consists of N_3 subgroups of correlated particles, so that each subgroup has a total charge of Q, we have for the emission per cm^3 and sec in all directions

$$4\pi\langle\varepsilon_r\rangle_\omega \simeq N_1N_3Q^2(e^6/m^2c^3)\ (\omega_e/V)\ (W/V)^2\ . \qquad (32)$$

Eq. (32) can serve as a basis of estimating the parameters of physical interest to the type III problem, namely, N_3, Q, and W/V. deJager and van't Veer[11] estimated the energy emitted by one burst to be of the order 10^{16} to 10^{17} erg/sec. Slightly higher values are quoted by Wild, Sheridan and Neylan[12]. Taking as representative frequency $\omega_e/2\pi$ = 200 Mc, we have N_1 = = 5 10^8. A coronal temperature of 2 10^6°K results in V = 5.5 10^8 cm/sec. W/V is typically about 20. Finally, assuming a source volume $L^3 = 10^{28}$ cm^3, we have

$$4\pi\langle\varepsilon_r\rangle L^3 = 2.2\cdot10^6\cdot N_3\cdot Q^2, \tag{33}$$

or

$$N_3 Q^2 \simeq 10^{11}. \tag{34}$$

Obviously, any number of combinations of N_3 and Q^2 can fulfill Eq. (34), however, since the Debye length is just under 1 cm, and a coherence length much in excess of the Debye length is physically not reasonable, we take $N_3 \simeq 10^{-3}$ as the optimal case. Then, $Q^2 \simeq 10^{14}$, and the number of cloud particles per cm^3 becomes 10^{4}, that is, a small fraction of the density of thermal electrons. The total number of fast particles required is thus 10^{32}, in good agreement with other estimates, for instance Sturrock's[13].

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