Models Demonstrating Instability of Nonconservative Mechanical Systems

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MODELS DEMONSTRATING INSTABILITY OF NONCONSERVATIVE MECHANICAL SYSTEMS*

by

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Abstract

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Nine different models designed and constructed at the Structural Mechanics Laboratory of Northwestern University are described, which serve the purpose of demonstrating instability of equilibrium of mechanical systems subjected to follower forces. Such forces are non-conservative, nondissipative and are also called circulatory forces. Instability is observed to occur by either divergence (static instability, attainment of another equilibrium position) or flutter (dynamic instability, oscillations with increasing amplitudes). The connection between some of these models and related nonconservative problems of elastic stability considered analytically in the past is indicated.

Introduction

Beginning with the pioneering studies of E. L. Nikolai [1]* on the stability analysis of elastic systems subjected to forces which do not possess a potential but are nondissipative, a new class of stability problems has come to the fore with the attribute "nonconservative". These forces have also been termed "circulatory" by Ziegler [2,3].

The nonconservative problems of structural stability may be classified, following Bolotin [4], as (1) those of flexible shafts and related objects in automatic control systems, (2) those of elastic bodies moving with respect to a surrounding fluid and (3) those of elastic systems subjected to "follower" forces. In this last category follower forces are defined as being dependent on the deformation of the system. In recent years a vast body of literature has grown, devoted to the analysis of stability of such problems [4-25]. Whereas in categories (1) and (2) both analytical and experimental work brought balanced progress, it was a particular feature of category (3) that all the efforts were devoted entirely to analytical investigations, in many cases of elastic systems subjected to fictitious follower forces. The only exception appears to be the work of Benjamin [15-16]. Indeed, this aspect of the state of development was the cause of some concern, as expressed in the following quote from reference [26] with regard to the problem of a cantilevered column subjected at its free end to a tangential follower force: "No definite conclusion can be made (as yet) regarding the practical value of

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*Numbers in brackets designate References at end of report.

this result, since no method has been devised for applying a tangential force to a column during bending." The authors of a more recent analytical study [10] phrase their reservations with regard to their own work as: "The physical interpretation of the « supertangential » loading has not so far been known to us, but the theoretical analysis of this range leads to interesting conclusions."

Thus it appeared desirable to make attempts at realizing several systems subjected to such follower forces and to demonstrate the two principal types of instability which have been described analytically. The two types were found to be: (1) Static loss of stability (another equilibrium position is attained), called divergence, (2) Dynamic loss of stability (oscillations with increasing amplitudes), called flutter.

A series of structural models embodying these features has been designed and constructed at the Structural Mechanics Laboratory of Northwestern University and it is the purpose of this report to describe qualitatively these models and their behavior. Since it is not intended to present here any quantitative data associated with these models, no numerical values of dimensions, weights, etc., are given. Such data will be included in following studies devoted to quantitative measurements. Likewise, certain significant effects which influence flutter, such as velocity-dependent forces (e.g., Coriolis forces or viscous damping forces), and post-critical behavior, which have been studied analytically [17-25,27], are not discussed here but will be examined experimentally in later studies.

Model 1

The model consists of two like rigid rods (Fig. 1-1). One rod is elastically hinged to a fixed base while the other is elastically hinged to the first rod and free at the other end. The system is constrained to move in a horizontal plane, being suspended by long, light strings. Various rigid attachments can be placed at the free end of the second rod. These attachments are essentially in the form of rigid flat plates, to be described in detail below, which are rigidly fixed and normal to the axis of the second rod. In the absence of any disturbance the system is in equilibrium when the two rods are colinear (undisturbed configuration).

A fixed nozzle is placed along the equilibrium axis of the system some distance away and a fluid (air) jet is made to impinge upon the attachment. The flow rate can be varied by means of a valve.

It is observed that as the flow rate is increased and passes a certain (critical) value, the system does not remain in the undisturbed configuration (loss of stability). This loss occurs either by divergence or by flutter, depending upon the type of the attachment. If the attachment is a smooth plate (center of Fig. 1-2), stability is lost by divergence. By contrast, if the attachment is a plate with a screen of certain mesh size (right in Fig. 1-2), or a smooth plate with a circular rim (left in Fig. 1-2), stability will be lost by flutter.

Fig. 1-3 illustrates divergence and shows the buckled equilibrium configuration. The sequence of photographs in Fig. 1-4 illustrates flutter-type motion.

The problem of a cantilevered bar with a rigid plate at its free end and subjected to a force which is always directed along the initial undeformed axis was first posed by Reut in 1939 and solved by B. L. Nikolai in the same year, as indicated by Bolotin [4]. In this problem stability is lost by flutter. Bolotin [4] suggested that such a force may be produced by the pressure from a jet of absolutely inelastic particles, or by the pressure of a jet of liquid or gas, the inclination of which, as the bar deforms, can be ignored. To the authors' knowledge no attempt was ever made to follow up these suggestions or to realize Reut's problem in any other way.

It is seen that the model described represents a realization of a class of problems in which Reut's problem is a special case. This special case is realized when a plate with a screen is attached. By using different mesh sizes of the screen the direction of the resultant aerodynamic force and thus the degree of nonconservativeness can be controlled. As a result the loss of stability by divergence and by flutter, as described in [6], can be realized. In the case when a plate with a rim is attached, it is likely that not only a resultant nonconservative force is acting, but also a nonconservative resultant couple.

Model 2

The model consists of two like rigid pipe-segments (Fig. 2-1). The first is elastically hinged to a fixed base, while the other is elastically hinged to the first and carries a nozzle at the free end. In addition to the elastic hinges, the stiffness of the system can be varied by means of lateral, spiral springs. The system is constrained to move in a horizontal plane, being suspended by long, light strings. A fluid can be conveyed through the pipes, entering at the fixed end and leaving through the nozzle. In the absence of the fluid, or for small rate of discharge, the pipes are at rest and colinear, defining the equilibrium configuration. Two symmetrically placed strings in the horizontal plane are attached to the free end of the pipe and pulled toward the fixed base at a small angle relative to the pipe axis.

It is observed that as the flow rate is increased, and passes a certain (critical) value, the pipe system does not remain in the undisturbed configuration. The loss of stability occurs either by divergence or by flutter, depending upon the stiffness of the auxiliary coil springs at the free end and the tension in the wires. If the coil spring at the free end is sufficiently soft, or is removed, and the tension in the wires small, then the loss of stability occurs by flutter-type motion. By contrast, for sufficiently stiff coil springs, or for large enough tension in the wires, the system loses stability by divergence (Fig. 2-2).

In experimenting with this system, it was found that the system can admit two distinct critical flutter flow rates. One is associated with relatively large initial disturbances and the other corresponds to small initial perturbations. That is, for a certain range of flow rates, the system is asymptotically stable when disturbed by sufficiently small initial input of energy, while it oscillates with increasing amplitude about the undeformed axis for sufficiently large initial perturbations (loss of stability in the large). Above this range the system loses stability by flutter for any initial disturbances (loss of stability in the small).

A thorough and systematic investigation (both analytical and experimental) of articulated pipes conveying fluid was presented by Benjamin [15,16]. The model described here represents a generalization of Benjamin's system by including a nozzle to control Coriolis forces, lateral springs to control effective constraints, and tension wires to control the direction of the resultant forces acting at the free end. The Coriolis forces are known [21-25] to have a pronounced effect on critical flutter flow rates. It appears that the existence of loss of stability in the large was not observed before in such systems.

Model 3

This model consists essentially of a piece of a rubber tube, fixed at one end and elastically restrained in the axial direction at the other end, at which rotation is prevented, Fig. 3-1. The tube is confined to move in the horizontal plane, being suspended by means of long, light strings. A fluid can be conveyed through the tube, entering at the fixed end. The other end being closed, the fluid is ejected through two nozzles, placed at a certain distance from the fixed end symmetrically with respect to the tube in the direction parallel to the tangent to the tube at that section. The nozzles are mounted in a fixture which is made to slide on an air cushion. The sleeve providing the sliding support at the elastically constrained end is also supported by an air bearing. In Fig. 3-1 the tubes supplying air for the bearings are seen on the left part of the photograph.

It is observed that the straight equilibrium configuration may be lost if the flow rate of the air passing through the tube exceeds a certain critical value. Loss of stability can occur by either flutter or divergence, depending upon the distance between the nozzles and the fixed end. It may be remarked that by attaching a series of pairs of nozzles along the tube, the problem of a bar subjected to distributed tangential follower forces may be realized.

Model 4

This model consists of a cantilevered thin elastic strip at whose free end a circular rigid plate is attached in a plane normal to the axis, Fig. 4-1. The surface of the plate can be varied by placing screens of different mesh sizes. A nozzle whose axis is parallel to the axis of the strip can be made to discharge fluid at a constant rate which impinges upon the plate.

It is observed that as a certain critical flow rate is exceeded, the cantilever may lose stability by either flutter or divergence, depending upon the mesh size of the screen attached to the plate. Both torsional and bending deformation are observed to occur for both types of loss of stability, with torsional deformations becoming more pronounced with increased eccentricity.

The problem of a cantilevered bar with narrow cross-section subjected to an eccentric compressive follower force has been formulated and solved in reference [28].

Model 5

This model consists of a cantilevered thin elastic strip at whose two longitudinal edges flexible tubes are attached through one of which fluid at constant rate can be conveyed, entering at the fixed end and leaving through the open end, Fig. 5-1. The other tube does not convey any fluid and is provided solely to decrease the asymmetry of the cross-section.

It is observed that as the flow rate exceeds a certain critical value, the cantilever loses stability by bending-torsional flutter, Fig. 5-2. It is also observed that a certain range of flow rates restores the original undeformed equilibrium configuration which may have been lost by lateral buckling caused by attaching a given weight at the free end. Fig. 5-3 shows the buckled configuration at zero flow rate and Fig. 5-4 shows the restored original equilibrium position, achieved with a certain flow rate. As the flow rate is increased further beyond a certain value, stability is lost by flutter.

Model 6

This model consists, as in the previous two cases, of a cantilevered elastic strip at whose two longitudinal edges flexible tubes are attached. A rigid pipe is placed along the transverse free edge and connected to the longitudinal tubes, Fig. 6-1. Fluid is conveyed at a constant rate through the longitudinal tubes, entering at the fixed end of the cantilever, and is discharged through an end opening in the rigid pipe, whose other end is closed.

It is observed that as the flow rate is increased beyond a certain critical value, stability is lost by bending-torsional flutter. The system may be considered as model of an aircraft wing with a jet engine at the free end.

The problem of a cantilevered strip subjected to a transverse follower force at the free end was first posed by Bolotin [4] and analyzed for a special case by Como [29].

Model 7

This model consists of a rigid closed cylinder which can roll on a horizontal plane. A piece of a rigid pipe is attached to the cylinder by means of an elastic hinge, which carries a nozzle at the free end, Fig. 7-1. Fluid can be conveyed into the cylinder by means of a flexible tube, which then enters the pipe and is discharged through the nozzle.

It is observed that as the rate of discharge is increased beyond a certain value, the system acquires a (stable) equilibrium position such that the pipe is vertical and its axis passes through the center of the cylinder, Fig. 7-2. As the rate of discharge is increased further, another definite (critical) value is reached, beyond which the system begins to execute oscillations with increasing amplitudes about the preceding equilibrium state (flutter).

Model 8

This model consists of a rigid cylinder, as in the previous model, which can roll on a convex rigid cylindrical segment which in turn is fixed in a concave rigid cylindrical segment, this latter being free to roll on a horizontal plane, Fig. 8-1. The rigid cylinder is closed at the end planes and is provided with an opening and a nozzle on the lateral surface, the axis of the nozzle passing through the center of the cylinder. Fluid can be conveyed through a flexible tube to the cylinder and is discharged through the nozzle.

It is observed that as the rate of discharge is increased beyond a certain value, the system acquires a (stable) equilibrium configuration such that the axis of the nozzle is vertical, Fig. 8-2. As the rate of discharge is increased further, another definite (critical) value is reached, beyond which the system begins to oscillate with increasing amplitudes about the preceding equilibrium state (flutter), Fig. 8-3. If the convex cylinder segment on which the cylinder rolls is replaced by a flat plate, Fig. 8-4, no flutter is observed.

Model 9

This model consists of a rigid pipe segment suspended by means of a flexible tube and hanging in the vertical position, Fig. 9-1. The lower end of the rigid pipe carries an attachment, the essential part of which consists of two nozzles placed in a plane normal to the axis of the pipe segment, parallel to each other. The flexible tube is connected to a fixed base. Fluid can be conveyed through the flexible tube, entering the rigid pipe segment and discharging through the nozzles in opposite directions.

It is observed that for any constant flow rate above a certain minimum value, the rigid pipe begins to move like a spherical pendulum with monotonically increasing amplitude, which will approach a limiting value for a sufficiently small flow rate. The minimum value of the constant flow rate which produces the onset of the pipe motion is not sharply defined. It is further observed that the same motion is initiated if the rigid pipe segment is made very short as compared to the flexible tube, and vice versa.

The problem of a cantilevered bar subjected at the free end to a twisting moment which rotates with the end cross-section of the bar was first considered by E. L. Nikolai [1]. He found that the undeformed rectilinear equilibrium configuration is unstable for any nonvanishing magnitude of the twisting moment.

It should be remarked that Nikolai's problem [1], and the model which is associated with it, as described above, fall into a different class of nonconservative systems than all those discussed in this report.

In the case of Nikolai's problem the applied follower torque produces essentially a gyroscopic effect which induces precessional motion. By contrast, in all other models which did not lose stability by divergence, the resultant forces, beyond a certain value, produced an oscillatory motion with increasing amplitudes, i.e. flutter.

Acknowledgment

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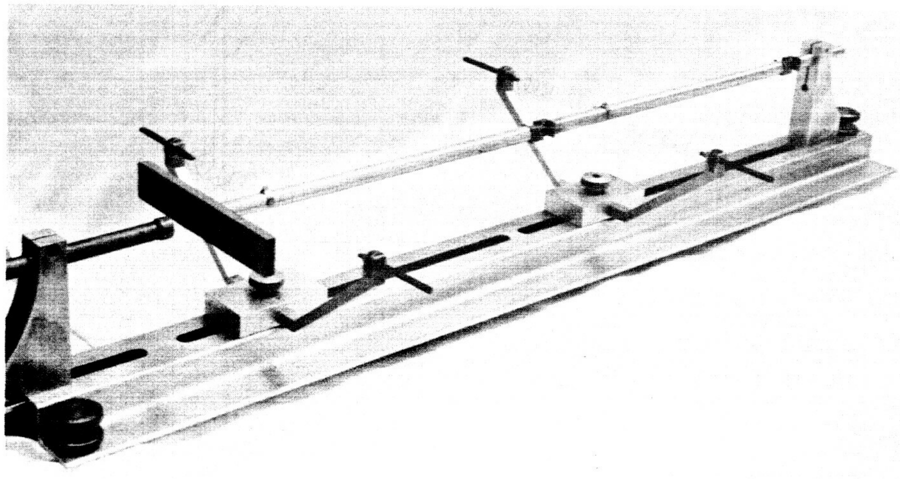

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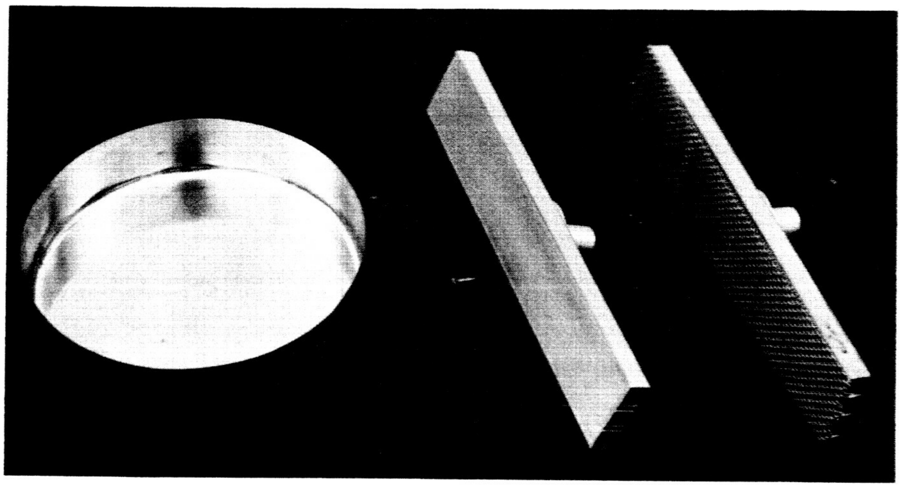

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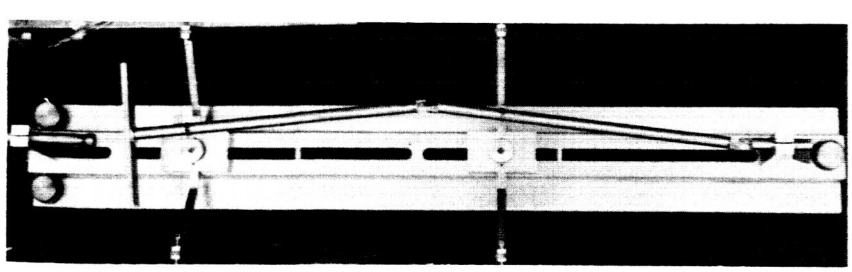

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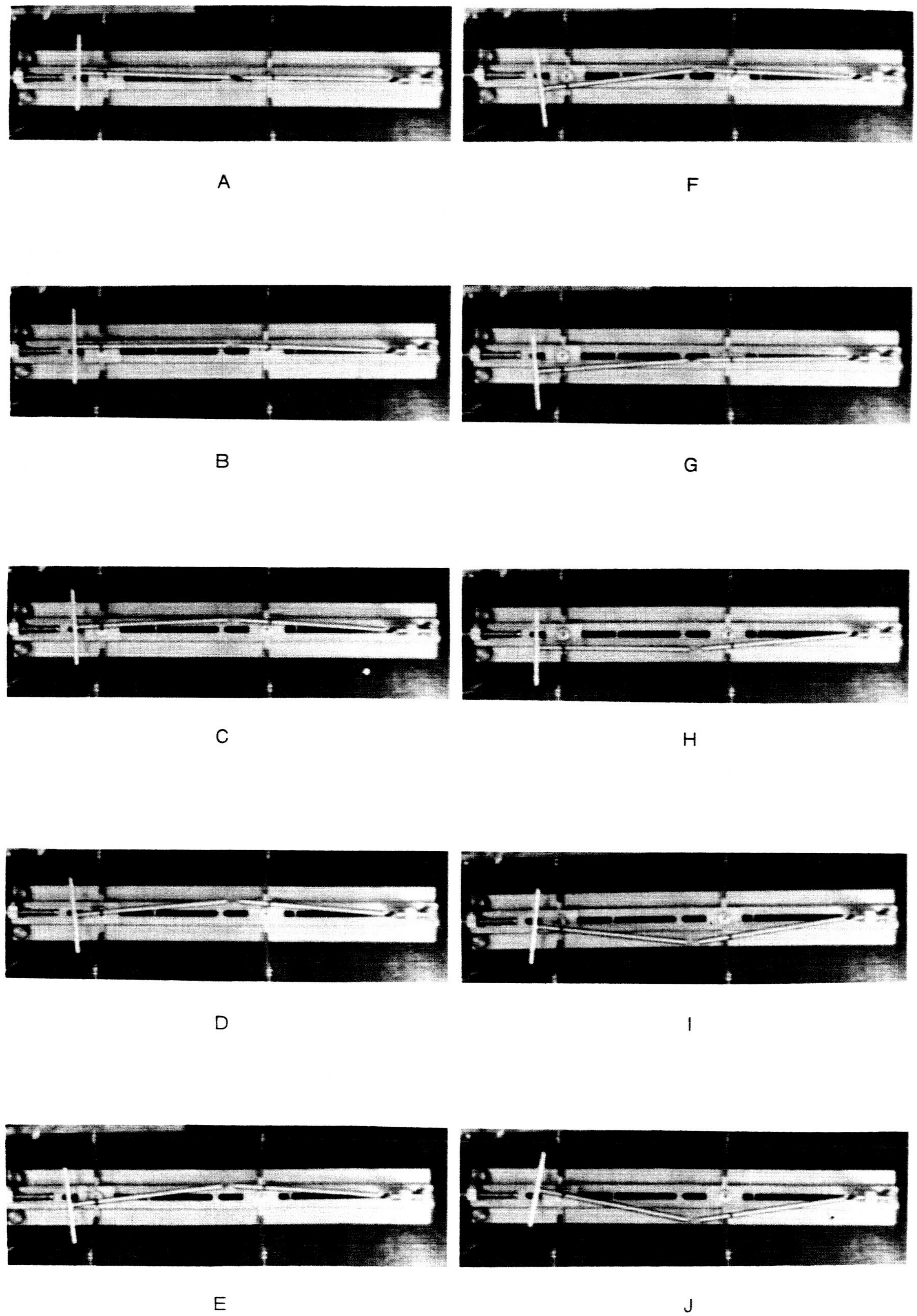

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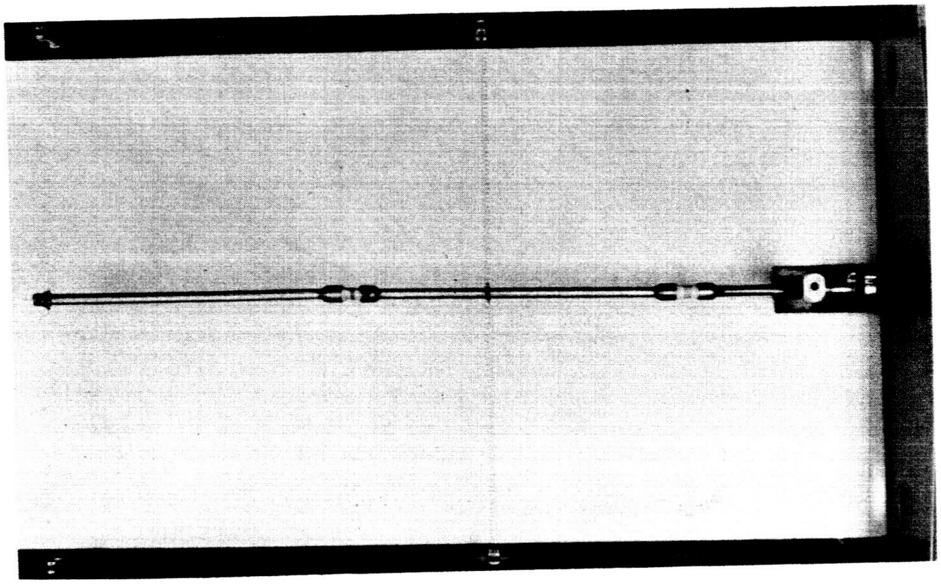

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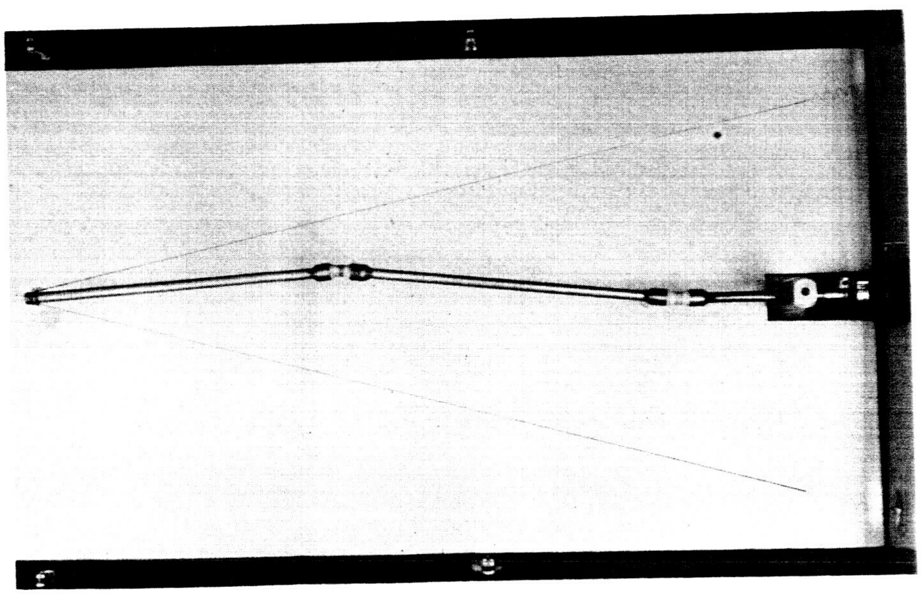

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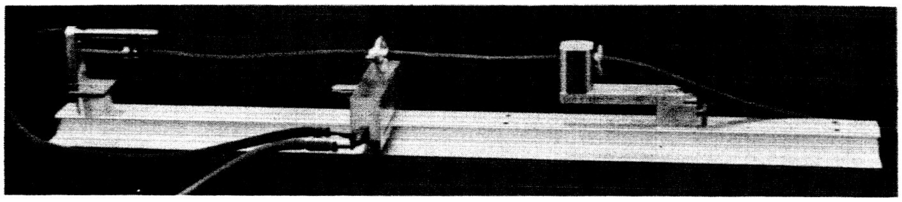

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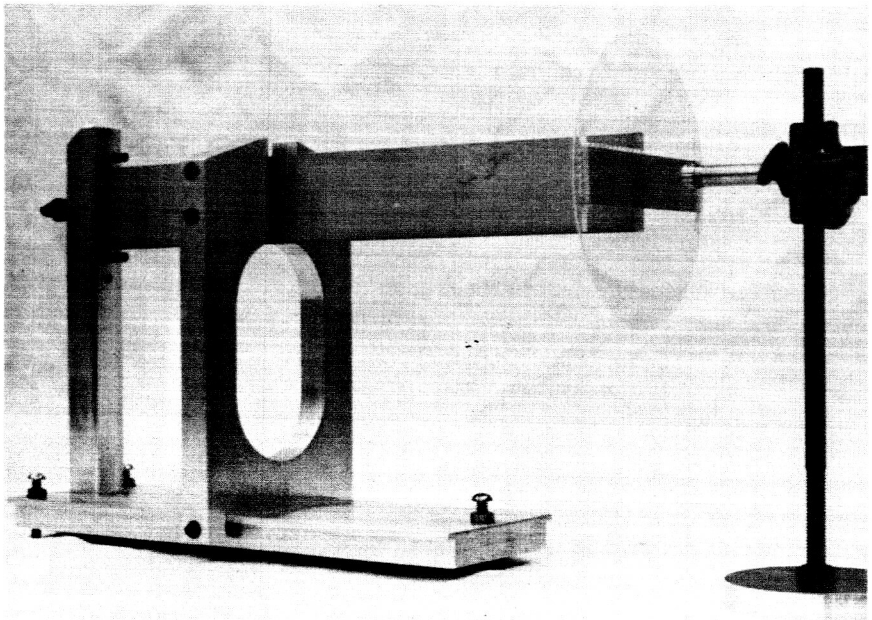

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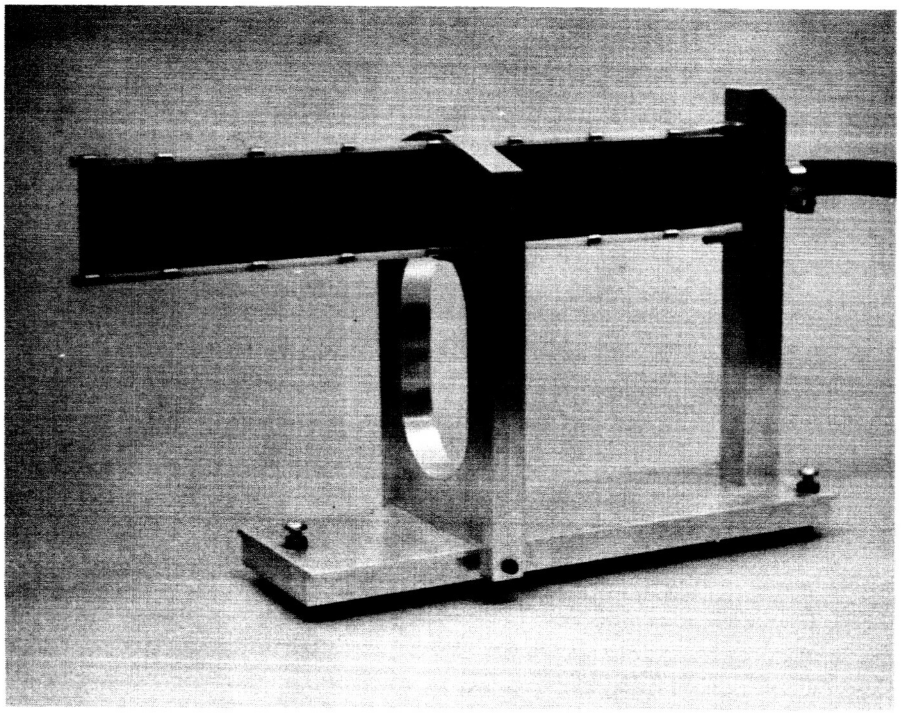

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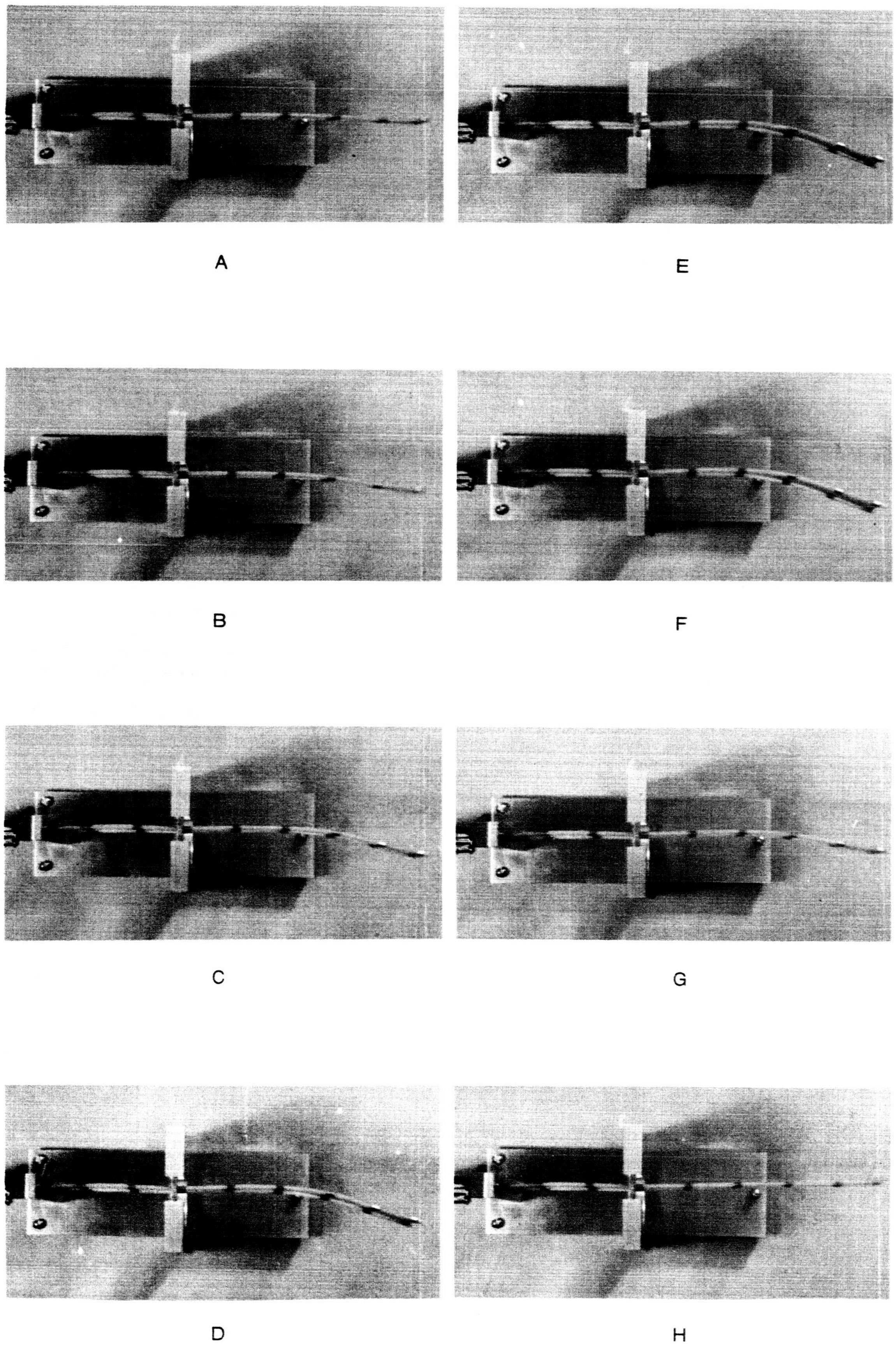

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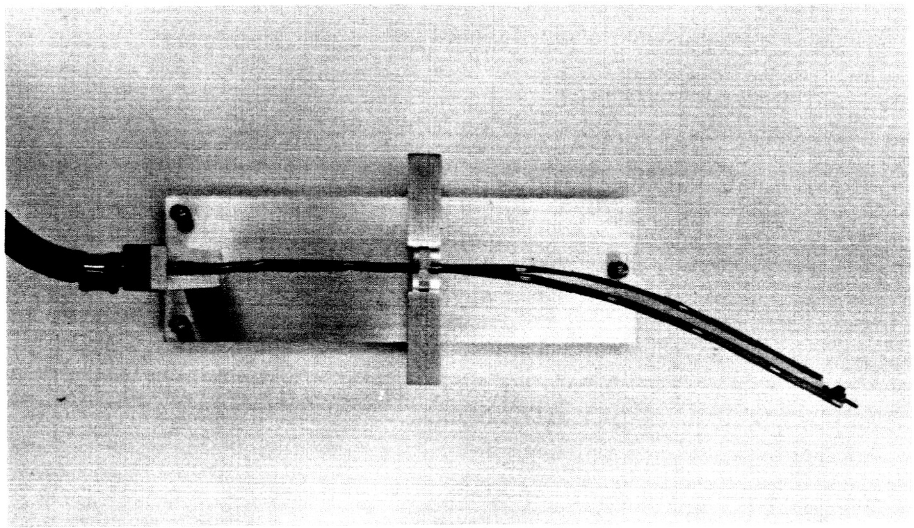

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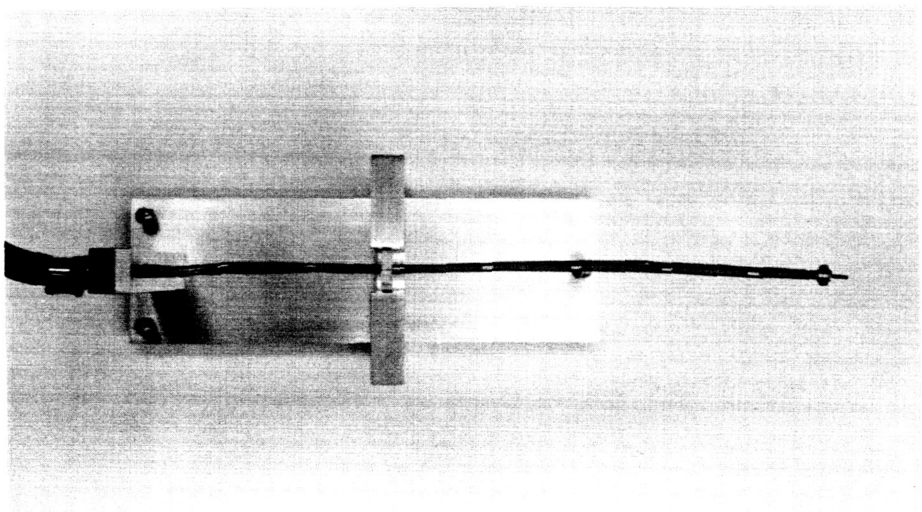

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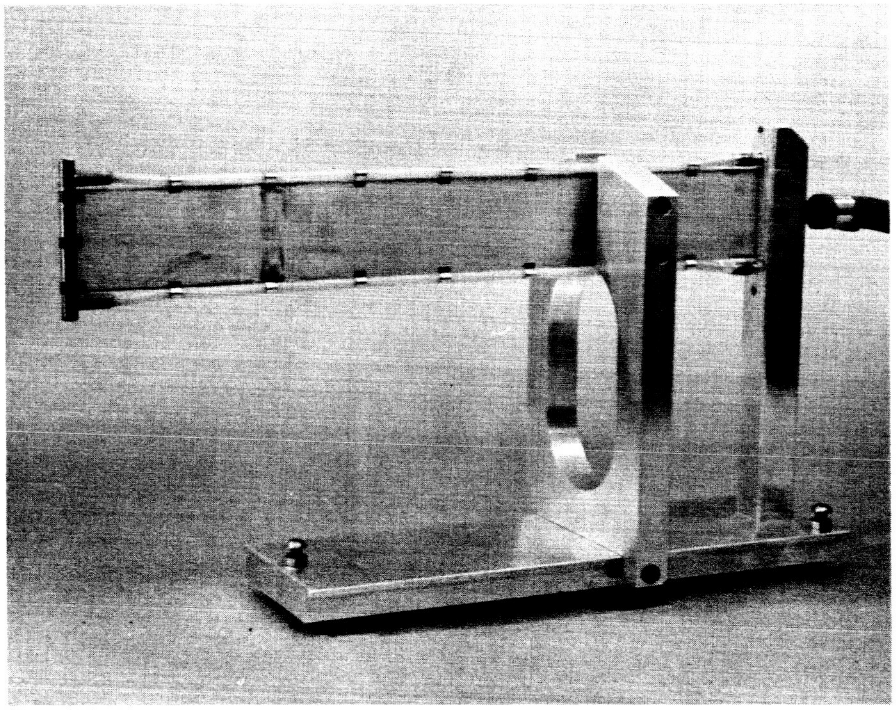

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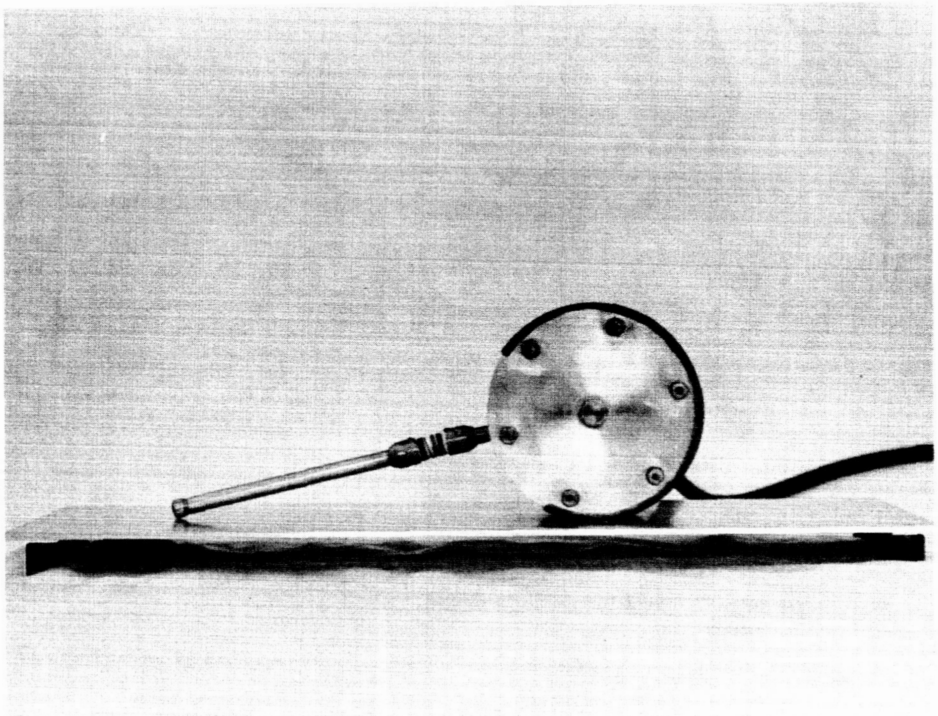
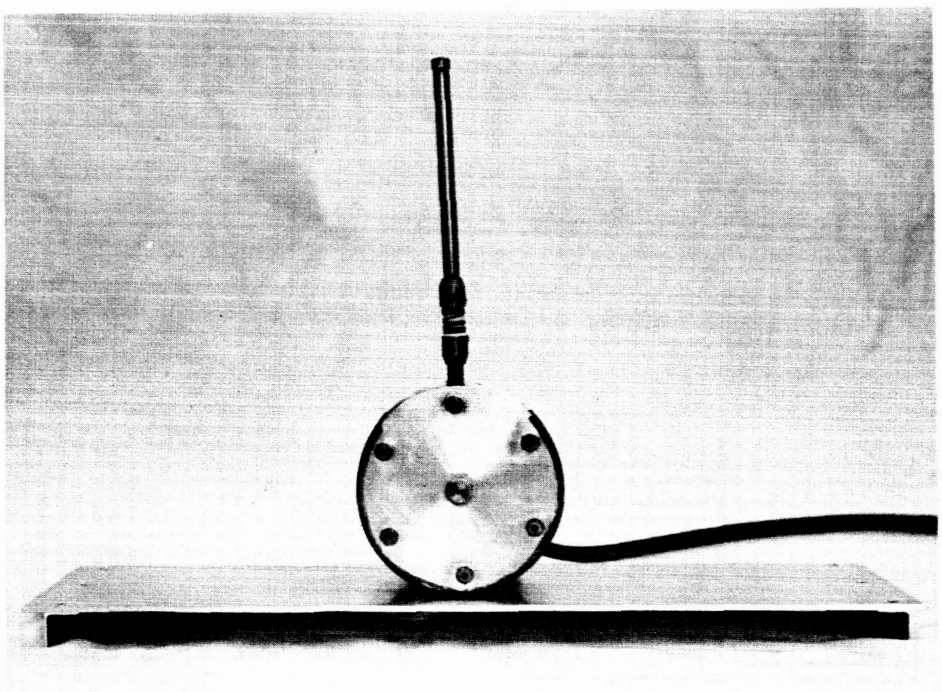


Fig. 7-1



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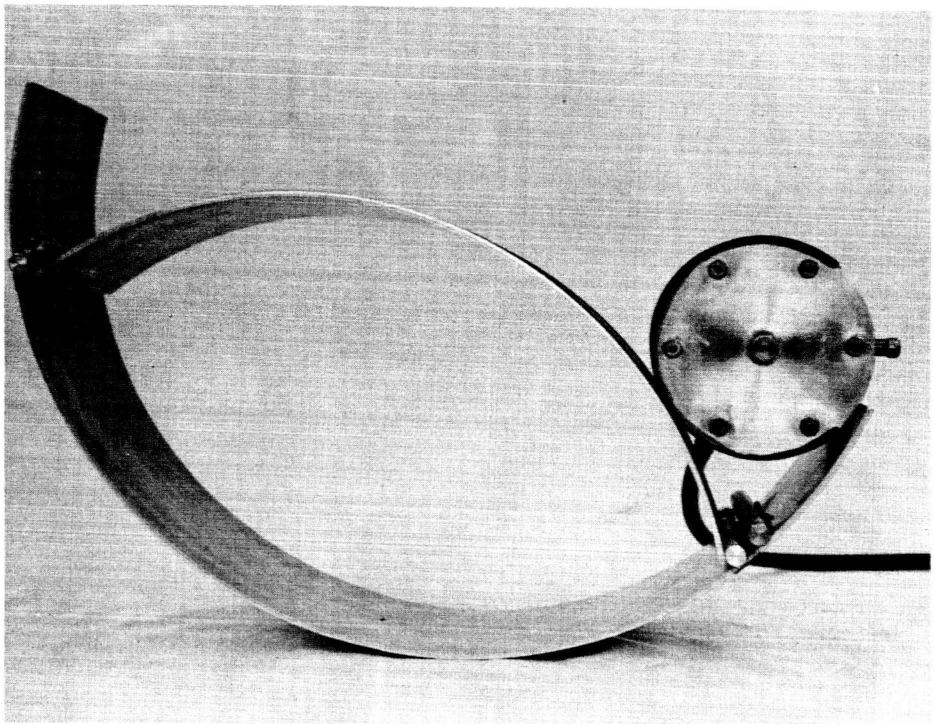

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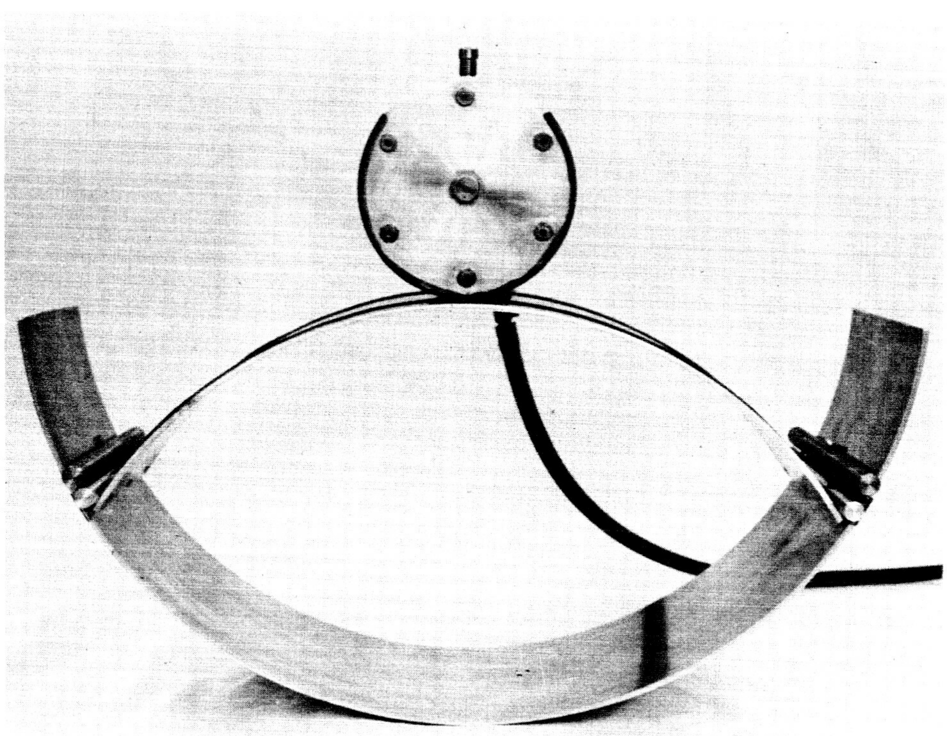

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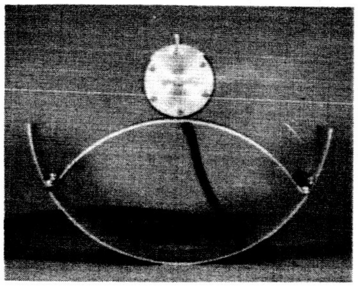

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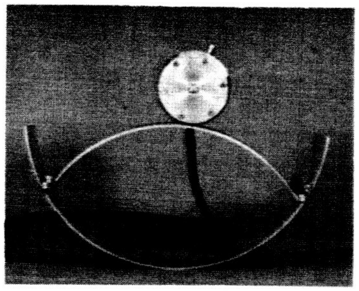

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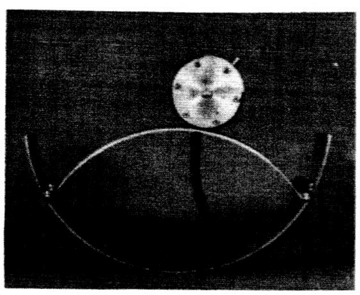

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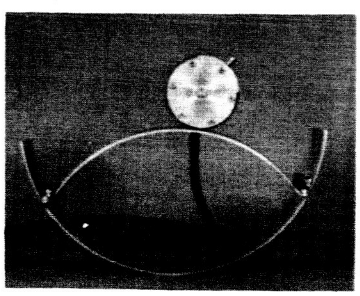

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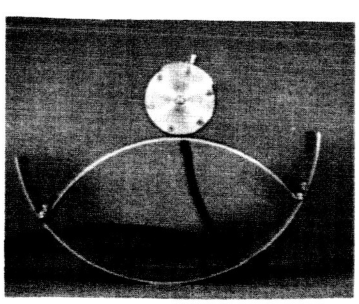

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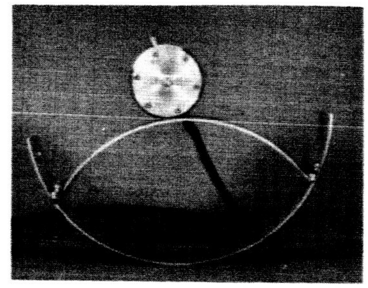

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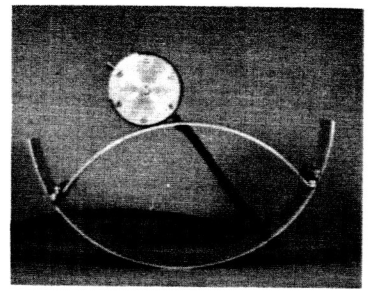
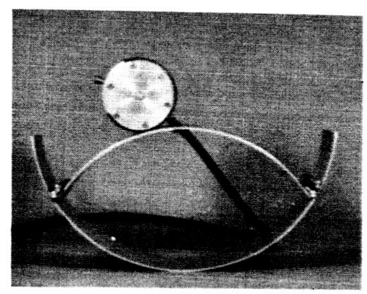


G



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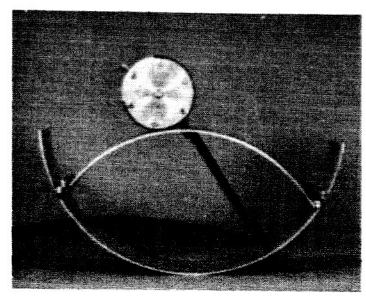

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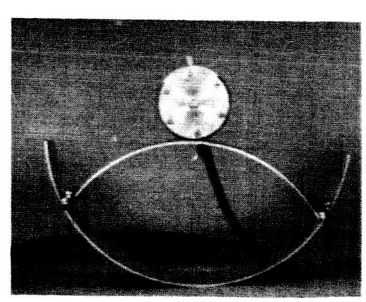

J

Fig. 8-3

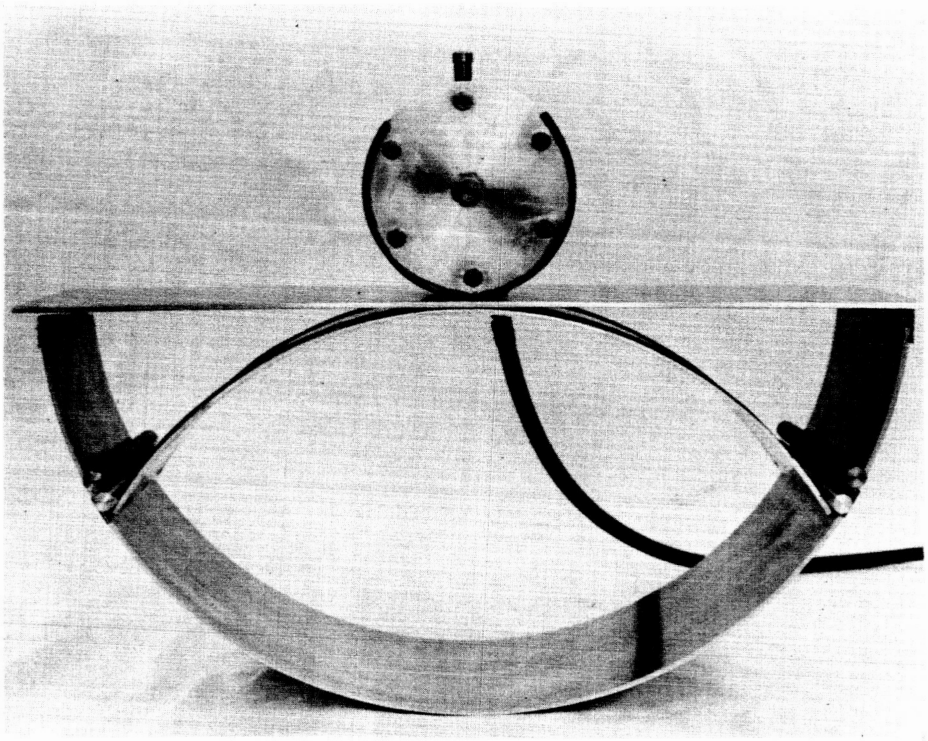

Fig. 8-4

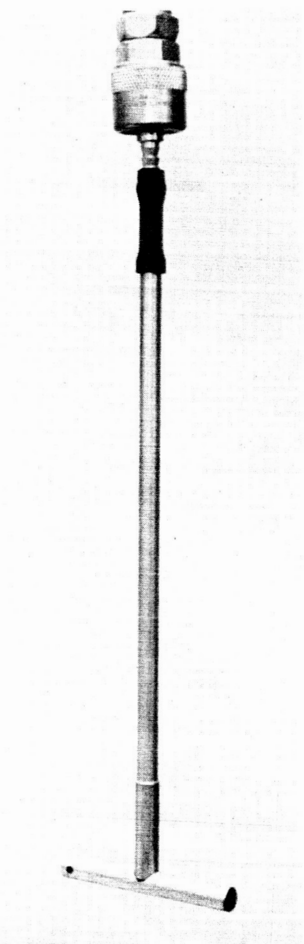

Fig. 9-1