# A NOTE ON THE THERMODYNAMICS OF LASERS

By R. L. Longini

Prepared under Contract No. NAS 8-5269 by
Carnegie Institute of Technology
Pittsburgh, Pennsylvania
for

National Aeronautics and Space Administration

# A NOTE ON THE THERMODYNAMICS OF LASERS

R. L. Longini

## Introduction

The very high efficiency of injection lasers has resulted in the suggestions that these devices may remove heat from their environment and convert it into light, at least if the efficiency were increased somewhat.[1] The thermodynamics have previously been discussed.[2,3,4,5,6] It is the purpose of this note to show the limits which must be achieved before cooling can result. Various experiments have been done which leave things pretty much in limbo though the authors claim more. These experiments and interpretations will not be discussed.

If any cooling is accomplished the laser system obviously would be acting as a refrigeration mechanism of sorts. As such it must fall within the limits of thermodynamic efficiency. A question of thermodynamic interest is "How much of the energy of the photon system can be supplied by the phonons?" In order to answer this it is necessary to define the critical temperatures involved. It is a principal purpose of this note to clarify this point.

Temperatures are also involved in the field of noise and communication. Presumably the same temperatures are involved and therefore an understanding of the defining temperatures is important from a point of understanding the fundamental limits of communication using lasers. In the case of noise, however, the type of statistics may be important. Usual noise expressions are based on a Boltzmann statistics approximation.

## Analysis

We will show that there are equivalent temperatures which can be put into thermodynamic terms of the laser structure and the heat sink. When a real heat engine, such as a steam turbine, is compared with a Carnot engine the temperatures of importance are the highest temperature (the steam inlet temperature if only the turbine efficiency is involved)

---

[1] G. C. Dousmanis, C. W. Mueller, H. Nelson, & K.G. Petzinger, "Evidence of Refrigeration Action by Means of Photon Emission in Semiconductor Diodes", Phys Rev v133, pp. A316-A318 (1964). Other footnotes from this article.

and the lowest temperature of the exhaust steam. In a refrigerator the critical temperatures are the lowest temperature of the refrigerant after it expands and does work, and the highest temperature of the refrigerant as it leaves the compressor where work was done on it. In identical manner the laser refrigerator has an exhaust "fluid", namely the coherent light "output". The temperature of this "fluid" is therefore vital to thermodynamic understanding of the laser action. In the case of a normal chemical refrigerant or of steam one takes a thermometer of sorts and determines the temperature. Usually such measurements involve some systematic errors - deviations of the determination from the true value toward "room" temperature, e.g., high temperature readings must be corrected for cooling of the thermocouple junction by conduction down the leads. For the laser exhaust fluid (the light generated) we have similar difficulties but can in principal show how to at least assign a lower limit to the light temperature.

Let us assume bodies one and two at temperatures $T_1$ and $T_2$ ($T_2 > T_1$); both are in contact with infinite heat reservoirs at their respective temperatures. Body one at $T_1$ will be put in thermal contact with an injection laser which is to act as the refrigerating machine and is to pump heat to the heat-sink at temperature $T_2$. The heat energy is to be received by body two as light which is emitted by the laser. The heat sink (at temperature $T_2$) is isolated from the world except 1) by radiation coupling to the laser, 2) by thermal contact to a heat reservoir at temperature $T_2$ as stated, though this contact is initially left open, and 3) by thermal contact to the hot side of a perfect heat engine. The radiation coupling is through a port which acts as a perfect mirror except to a narrow band of radiation which is directed along the laser-black body line. Wavelengths of this band include the radiation peak of the laser. In this narrow band the port permits transmission and is lossless, i.e., non-absorbing to radiation passing between the laser and body #2.

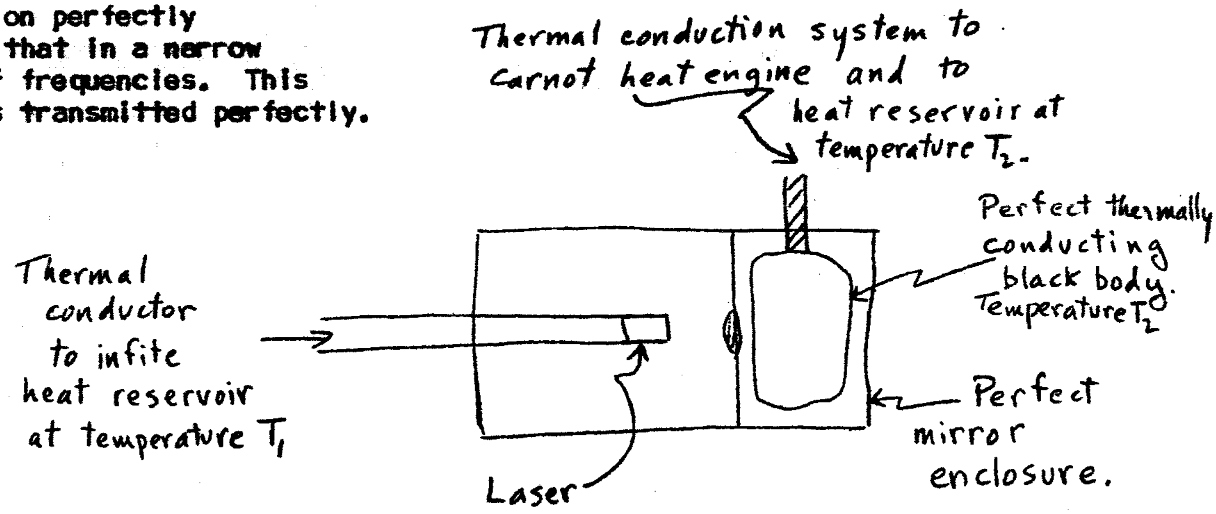
All light coming from the laser through the port will be absorbed by body two, which is a black body, and body two will in turn radiate energy[1]. However, only that energy in the radiation band with a wavelength near that of the laser can get out. It will be radiated back to the laser by

---

[1] A system something like this has been suggested by M. G. A. Bernard and G. Duraffourg. Physica Status Solidi, I, pp. 699-703 (1961). The purpose and conclusions differ somewhat.

the directional port. The heat energy of body two which was received as light and was not reradiated is conducted to the heat engine. The heat fluxes and power output related to this engine can be measured. Only a small heat flow is allowed to go to the heat engine so that most of the heat received will be reradiated and the amount sent to the heat engine will not cool body two significantly. If a steady state is assumed, the heat sink then reradiates energy at almost the same rate that it receives it from the laser. This arrangement is shown schematically in Figure I. Since body #2 receives all its energy from the laser "exhaust fluid" it must be at a temperature less than that of the fluid. That is, the port properties can always be improved to reduce radiation losses without interferring significantly with the laser light. Such "improvements" would result in a higher value for $T_2$. $T_2$ is thus a lower bound to the "light temperature".

Port reflects all radiation perfectly except that in a narrow band of frequencies. This band is transmitted perfectly.




In order for a steady-state to be reached, the radiation from the black body out of the port plus that going to the heat engine must equal the energy received from the laser through the port. Under these conditions the laser is pumping energy into the light (exhaust fluid) at a presumably known rate and the light will be at or above the temperature ($T_2$) which also can be defined. Under these conditions we have a refrigerator carrying heat from a low temperature body to a high temperature medium. To complete the cycle for thermodynamic argument we add extra heat at temperature $T_2$ to the heat engine such that the power output will just maintain the laser operation, no other power being needed. Now the efficiency of the refrigerator cannot exceed the Carnot efficiency of a thermodynamic engine (just as in any other system an engine could otherwise be conceived which would result in perpetual motion).

If we consider the laser refrigeration as carrying heat from body #1 at $T_1$ to body #2 at $T_2$ it must have an efficiency less than that of a Carnot refrigeration operating between these temperatures as the temperature $T_2$ is certainly somewhat less than that of the light exhaust fluid. The active electrons of the laser presumably get their energy both from the Carnot heat engine (which is assumed to convert its output with 100% efficiency into the desired electric form) and the phonon system of the solid of which the laser is composed. The hot electrons and perhaps the phonons have their energy carried away by the photon stream, the temperature of which can be measured in the separate but optically coupled enclosure.

It is clear that the black body heat sink must indeed be very hot in the steady state condition. An injection laser may have an output of one tenth watt steady state when held at liquid nitrogen temperature. This radiation output is in a narrow band of energy much less than one tenth Angstrom wide at, e.g., 8400 Angstroms. The black body must re-radiate this energy. It will be assumed to have a total port area available to it one tenth square centimeter. $T_2$ must be such that it can re-radiate say 99% of the power received, the remaining one per cent going to the heat engine. The one tenth watt which is radiated must fall in the one tenth angstrom band at 8400°A and must be directed so as to go through the port. It is clear that the temperature of the black body must be many thousands of degree Kelvin in order to emit radiation in the nearly visible region with such high intensity (for one tenth watt in one tenth angstrom band for one tenth centimeter square, the temperature must be 45,000°K). Therefore we are involved with an apparatus which cools a body at 77°K and pumps the heat to another body at 45,000°K. The maximum possible "efficiency" of such an apparatus acting as a refrigerator is approximately 1/6%, i.e., at most 1/6% of the photon energy could be supplied by the phonons. Thus, for the unit to act as a refrigerator, the system must have an efficiency of greater than 99.8%. It should be pointed out that this is a very conservative estimate as the area might be $10^{-6}$ square centimeters, the spectral line width $10^{-4}$ Angstroms, and the power might be watts. Furthermore the ports could involve collimators requiring great directionality. This latter characteristic alone would increase the temperature by orders of magnitude. Each of these factors increases the temperature linearly so that temperatures should be measured in giga degrees and efficiencies of the order of $(100 - 10^{-6})$% or $(100 - 10^{-12})$% are needed for any cooling. Interestingly enough, an increase of the power decreases the permitted cooling per watt transmitted by just the amount of the increase of power.

The validity of the above remarks may be questioned on several grounds. First, the thermodynamic engine is not completely cyclic and therefore doesn't meet certain thermodynamic requirements in comparison with Carnot's cycle. Second, the receiver might be considered by some as the relevant hot body and this can be made cool thereby apparently increasing the laser "efficiency". This is not a reasonable conclusion as the receiver might be sufficiently remote that it could not interact with the laser if the latter were pulsed. In case the heat sink is closely coupled with the laser there may be some interaction but this is presumably of no great consequence. Thus the heat sink temperature in normal laser operation is not the temperature of thermodynamic importance.

## Laser Light Temperature

When a laser is in the "coherent" region or at least in the "super-radiant" region, its light is generated by electrons which fall from one level to another. In such non-equilibrium electronic distribution, terms such as negative temperature have been invented to describe the inversion of electron population between various states. A real temperature can be ascribed to the radiation itself, however, in spite of its coherence and of the negative temperature assigned to the electron population.

In order to avoid semantic difficulties we will define some terms for use here. We will first define "stimulated radiation". Light photon states can be occupied by a number of quanta simultaneously. In such states all of the quanta have identical phase, direction, etc., in fact

act as one photon except for the energy which will have an integral multiple of the single photon energy. It differs from a single photon in its behavior only in that part of it can be absorbed or scattered. Now in such a multiply occupied state we define the distribution between spontaneous and stimulated events as follows: one quanta in such a photon state is spontaneous, all the rest are stimulated. Since they are identical they cannot be sorted out on this basis. These quanta are all coherent in the sense that they are phase related. Of course the relationship is closer than that.

We will, however, define "coherence": the quanta in a single photon state are coherent. The optical system involved, whether Fabry-Perot, mirrors on two or four sides, lenses, etc., are means for detecting this coherence or perhaps even of inducing it but the essence of coherence is that the quanta are all in one photon state.

Ordinary black body radiation obeys Planck's distribution law. The form of the denominator comes about by virtue of Einstein's coefficient of spontaneous and stimulated radiation. It can be shown that, for a black body, the ratio of spontaneous radiation to stimulated radiation is given by

$$\frac{(\text{spon})}{(\text{stim})} = e^{h\nu/kT} - 1$$

$$e^{h\nu/kT} = 1 + \frac{(\text{spon})}{(\text{stim})} = \frac{(\text{stim}) + (\text{spon})}{(\text{stim})} = \frac{(\text{total})}{(\text{stim})}$$

$$\frac{h\nu}{kT} = \ln \frac{(\text{total})}{(\text{stim})}$$

$$\frac{1}{T} = \frac{k}{h\nu} \ln \frac{(\text{total})}{(\text{stim})}$$

It is clear that this is a proper temperature for light when considering that the radiation should be in equilibrium with a black body at the temperature of the radiation. For the light from a laser the temperature should therefore be defined by this relationship between (stimulated) radiation and total radiation. The laser noise process involves spontaneous variation in the intensity of the "coherent" light. This variation is, then, a direct indication of some lack of coherence and, therefore, of a finite temperature. The radiation from an ordinary laser changes modes and thus has a limited coherence as indicated by the finite length of time of operation in a given mode.

In laser operation there may be some modes which are noisier than others. It is necessary to consider the temperature of the radiation of each of these modes separately. Each one should be considered in the sense that it can be isolated by suitable filters and put individually in equilibrium with a black body suitably surrounded as in the example for the refrigerator given above. Thus no unique temperature can be given to all the radiation from the laser, but rather a temperature is suitable for each mode. The more coherent the radiation of a given mode, the higher its temperature will be.

There is no sharp break-over in this analysis between the almost purely spontaneous, the "super-radiant" region, or the "coherent" region where the radiation must be in a Fabry-Perot mode. In order to use the definition of temperature involved here, spontaneous and stimulated radiation must be used as defined earlier.

Any change of phase or of wavelength indicates a new spontaneous event. If we have a system in which there are 100 quanta, on the average, per photon state, we will have a ratio of 100 to 99 which has a natural log of 0.01. Thus, a kT of energy would equal 100 $h\nu$ for such a system. When $h\nu$ is approximately one electron volt this implies a temperature of about 1,000,000°K for such a radiation field. At the extreme of zero stimulation the temperature becomes zero by this definition.

For the case of a laser in the super-radiant range, where presumably it acts as a light amplifier (by stimulated emission of radiation, of course) we can make more definite statements. If we assume spontaneous events take place uniformly along the length of the laser and assume its amplification per unit length is constant and without noise, (undoubtedly a bad assumption) we find, for no incident light, that:

Spontaneous quanta of light out = $gL$

where $g$ is the number of spontaneous events per unit length per unit time and $L$ is the length of the laser. The spontaneous light is amplified by $\alpha$ stimulated emissions per unit length. If this is integrated over the entire length we find:

$$\text{Total light out} = \frac{g}{\alpha}\left(e^{\alpha L} - 1\right)$$

It is easily shown that for incident light at $x = 0$ the laser amplifies it before it leaves at $L$ by an amplification $A = e^{\alpha L}$.

Then, since $\alpha L = \ln A$, we have $\frac{(\text{stim})}{(\text{total})} = 1 - \frac{\ln A}{A-1}$ . For A large, since $\ln(1-\epsilon) = -\epsilon$,

$$\ln \frac{(\text{total})}{(\text{stim})} = \frac{\ln A}{A}$$

From which we find

$$T = \frac{h\nu}{k} \frac{A}{\ln A}$$

For a green light amplifier having a gain of ten the light output has a temperature of about $10^5$ degrees kelvin.

In using this definition of temperature a proper averaging of amplification over the frequency band involved must be made. The band will certainly involve the range over which the amplification peak shifts during operation.

The effect of a neutral filter is to reduce the energy density. It therefore clearly must be reducing the temperature of the radiation. This in turn implies an increase of the spontaneous/stimulated ratio in the light. We must conclude, then, that a neutral filter reduces the number of quanta in a given photon state. This is exactly what is expected from a quantum mechanical view of such absorption.