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IN THE INTERPLANETARY PLASMA

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Evidence for the rather general existence of ionized gases and magnetic fields in interplanetary space and in interstellar space, and evidence concerning the likely properties of these gases and magnetic fields, have led to the assumption that magnetohydrodynamic waves would propagate through these regions (c.f. Alfven[1]). This note is a preliminary report on evidence for the existence of these waves in interplanetary space. The measurements which provided this evidence were obtained with the fluxgate magnetometer and ion spectrometer, or plasma probe of the electrostatic analyzer type, on board the spacecraft, Mariner 2. These measurements were obtained on a trajectory between the earth and Venus during the period 29 August through 15 November, 1962. Measurements of three orthogonal vector components of the field and of the vector component of velocity of the interplanetary plasma in the direction radially outward from the sun were obtained during the four-month flight of Mariner 2 between the earth and Venus. Our purpose here is to show that the properties of the variations observed in the magnetic field and plasma velocity satisfy certain necessary conditions of hydromagnetic wave behavior.

It will be convenient to employ heliocentric spherical polar coordinates with the polar axis coincident with the sun's axis of rotation. Thus, the positive r direction is radially outward from the sun, the φ direction is parallel to the solar equatorial plane and positive in the direction of planetary motion, and the θ direction completes the usual right-handed system.

In this system, the variables measured by the magnetometer are B_r, B_θ, and B_φ and the variable measured by the plasma probe is V_r. As the first step in a statistical analysis of the data, amplitude distributions of these variables were examined. It was found that the distributions were roughly Gaussian.

Next, the records were examined for correlations between the various pairs of these variables. The procedure employed was the usual one of obtaining estimates of the cross spectra of each pair of variables for several sections of the record. It was found that each pair of variables exhibited significant coherence on the average, and that the coherence was particularly strong for the pair V_r, B_r. Further, the phase differences were usually found to be either 0° or 180° and roughly independent of frequency. These results from the cross spectra provided evidence for wave-like motions in the interplanetary plasma, and led to the somewhat more quantitative comparison between the observed properties and those expected for idealized hydromagnetic waves. The results of this comparison will be summarized below.

We will be concerned with data obtained during four 10-day periods and two 6-day periods indicated in Table 1. These periods were selected in order to exclude the major reversals of the direction of the average interplanetary field that occurred twice during each period of solar rotation.[2] Thus, during each of these periods, the direction of the interplanetary field was nearly always that listed in Table 1, wherein a positive polarity is assigned to a field outward from the sun and a negative polarity is assigned to a field inward toward the sun.

We will consider the frequency range from 1 to 50 cycles per day (cpd). The lower limit was selected in order to limit the computer time required for the analysis. An upper limit of 100 cpd was imposed by the sampling rate of the plasma probe. Indications of significant aliasing between 50 and 100 cpd in the power spectra of V_r prompted the selection of 50 cpd as the upper limit for the frequency range of interest here.

However, the magnetic field measurements provided information in the frequency range 0-1000 cpd. Thus, although we cannot be certain about the behavior of V_p at frequencies above 50 cpd, we know that the properties of the field variations in the range 50-500 cpd are the same as those in the range from 1-50 cpd, except that the absolute power densities decreased with increasing frequency, reaching the noise level of the magnetometer at about 500 cpd. This result suggests that the relationships between V_r and $\vec{B}$ at frequencies in the range 50-500 cpd are the same as those observed in the range 1-50 cpd.

In the frequency range from 1-50 cpd, then, the auto spectra of the measured variables provided estimates of the power densities, P, as functions of the frequency. In Fig. 1 are plotted the ratios of the spectral densities $P(B_r)/P(V_r)$, $P(B_\theta)/P(V_r)$, and $P(B_\varphi)/P(V_r)$ as functions of the frequency, f, for the six sets of data. Note that for each period the values are roughly independent of frequency.

The cross spectra provided estimates of the magnitude of the square of the coherence, $|R|$, as a function of frequency and estimates of the phase difference, Φ, as a function of frequency. It was found that $|R|$ and Φ, obtained from the cross spectra for the various pairs of the measured variables, were for the most part roughly independent of the frequency. On the average, $|R(V_r, B_r)|$ was higher, by factors in the range from 5 to 8, than $|R|$ for the other pairs, although all pairs, on the average, exhibited significant coherences. Thus, only the average values of $|R(V_r, B_r)|$ and $\Phi(V_r, B_r)$ are listed in Table 1. (Here the average is taken over frequency.)

Rather than $|R|$ the quantity tabulated is R/R_{min} where R_{min} is the minimum value of R that corresponds to a meaningful estimate of the phase difference. Thus, if $R = R_{min}$, the rms uncertainty in the phase difference is 45°.

From Table 1, it is apparent that the coherence of V_r and B_r was relatively high and that these two variables were nearly 180° out of phase when the average field was directed from the sun (+)

and nearly in phase when the average field was directed toward the sun (-). (The apparent differences of -10° to -15° from 0° and 180° is a result of the manner in which the time was assigned to the measurements of V_r rather than the result of any real phase differences.)

In comparing the measured values of the ratios of the power densities and the phase differences to those expected on theoretical bases, only the simplest model has been employed. Following Thompson[3], we considered plane waves in the plasma displacement, $\vec{d}$, moving through a uniform, perfectly conducting, isentropic, ideal gas in a uniform magnetic field $\vec{B}_o$ for the case in which the gas is moving with a bulk velocity $\vec{V}_o$. For such a system, the linearized equations in the reference frame moving with the plasma are

$$\partial\rho/\partial t + \rho_o \nabla \cdot \vec{v} = 0 \qquad (1)$$

$$\rho_o(\partial\vec{v}/\partial t) = -\nabla p + (1/4\pi)(\nabla \times \vec{b}) \times \vec{B}_o \qquad (2)$$

$$\partial p/\partial t - (\gamma p_o/\rho_o)(\partial\rho/\partial t) = 0 \qquad (3)$$

$$\vec{E} + (\vec{v}/c) \times \vec{B}_o = 0 \qquad (4)$$

$$(1/c)\,\partial\vec{b}/\partial t = -\nabla \times \vec{E} \qquad (5)$$

where ρ_o is the mean density and ρ is the density perturbation,

$\vec{V} = \vec{V}_o + \vec{v}$ is the velocity, $\vec{B} = \vec{B}_o + \vec{b}$ is the magnetic field, p_o is the mean pressure and p is the pressure perturbation, $\vec{E}$ is the electric field, γ is the ratio of specific heats for the ideal gas, and c is the velocity of light. It was assumed that $\vec{V}_o$ and $\vec{B}_o$ are constant and uniform.

During the flight of Mariner 2, the average value of the field strength $|\vec{B}|$ was $5\gamma = 5 \cdot 10^{-5}$ gauss. According to Neugebauer and Snyder[4,5], the average value of V_r was 500 km/sec, the average number density for protons was 5 cm^{-3}, and the average ion temperature was about $1.7 \cdot 10^{5}$ °K.

Now from the equations given above, it can be shown that there are three modes in which plane waves can propagate. Still following Thompson[3], we will designate these modes as the Alfven mode, the fast mode, and the slow mode. For any particular ideal plane wave, the power density ratios and the coherences and phase differences for the various pairs of variables depend upon the average properties of the plasma and the field as well as the geometry orientation of the field and the direction of the wave propagation.

We will let $\alpha = \tan^{-1} (-B_\varphi/B_r)$ and $\beta = \tan^{-1} B_\theta/(B_r{}^2 + B_\varphi{}^2)$. During the flight α was typically 40° or $40^\circ + 180^\circ$, and β was typically in the range $\pm 25^\circ$.

We will let θ_k be the angle from the line of force corresponding to $\vec{B}_o$ to the propagation vector $\vec{k}$ for a plane. However, this angle is always measured from the direction <u>outward</u> along this line of force. In the calculations for each mode, we will consider the case of a superposition of many plane waves with random phases and with the orientations of the planes of polarization distributed uniformly over the allowed range. Thus, in computing the average power densities, P, the average coherences, R, and the phase relationships, Φ, for waves at a particular frequency, we computed values for a random distribution of plane waves or, in effect, values for circularly polarized waves.

If we assume that all the waves originate within a finite region of space centered at the sun, then on the average, disturbances produced by waves propagating outward from the sun will predominate. However, since a point source in a uniform field will produce a spherically spreading disturbance in the fast mode and disturbances moving along the field lines in both directions in the transverse and slow mode and since there are reasons to expect disturbances in interplanetary space, it is likely that waves moving back toward the sun also contribute to the observed variations. Now the phase relationships between $\vec{B}$ and $\vec{V}$ for a wave moving back toward the sun, differ by 180° from those for a wave moving outward. Thus, any contribution to $\vec{B}$ from inwardly traveling waves will reduce the coherences, indicated by R, between V_r and B_i, $i = r, \theta, \varphi$, but will not change the phase relationships so long as the contribution from outwardly traveling waves predominates. Accordingly, the values of R calculated for the ideal waves are those for the outwardly traveling waves, or $\cos \theta_k \geq 0$. Thus, the various cases for which P, R, and Φ have been computed are as follows:

Modes:	Alfven, Fast, Slow
θ_k:	0, 15, 30, 45, 60, 75, 90°
β:	±25, 0°
α:	40°
Polarization:	Circular
B_o:	5.0γ
n_p:	5 cm^{-3}
T:	10^{5}°K

The results of these calculations of P, R, and Φ are listed in Table 2. Values of R and Φ are given only for the cross spectra of V_r and B_r, since the equations indicate that in this case the phase relationship and the coherence are least dependent upon the sign and magnitude of β, and since this pair of variables was actually observed to exhibit the highest measured coherence.

For our purposes, the most significant quantities listed in Table 2 are the ratios $P(B_i)/P(V_r)$, i = r, θ, φ, since these quantities are directly measurable from the power spectra of V_r and B_i and since they are less affected by the geometry of the system and the direction of propagation than are R and Φ. The theoretical values for the idealized waves depend only upon $|B_o|$, n_p, T, the absolute value of cos θ_k, and the orientation of the line of force of $\vec{B}_o$, without regard to the sense or polarity of $\vec{B}_o$.

The theoretical values of these ratios, as well as those of R and Φ, are also independent of frequency according to our model. Since we expect the model to be more realistic at frequencies well below the ion gyro frequency, 0.5 cps in a field of 5γ, the frequency independence of our measured ratios is expected for hydromagnetic waves over the frequency range considered here. Further, the measured values of these ratios are generally within or close to the ranges given for the idealized waves.

Note also that in the ideal case, variations in V_r and B_r are 180° out of phase regardless of the type of waves, except for fast waves with $\theta_k > 75°$. These phase relationships hold when, on the average, $\vec{k} \cdot \vec{B}_o > 0$, i.e., when the component of $\vec{k}$ along $\vec{B}_o$ is parallel, rather than antiparallel, to $\vec{B}_o$. Since we have assumed that the predominant waves move outward from the sun, we have $\vec{k} \cdot \vec{B}_o > 0$ when the field is outward from the sun. Thus, on the basis of the model and this assumption, we would expect the predominant coherent variations in V_r and B_r to be 180° out of phase when the direction of the field is (+). This is the phase difference that was observed when the field was (+).

When the field is (-), $\vec{k} \cdot \vec{B}_o < 0$ and we would expect the coherent variations in V_r and B_r to be in phase. This phase difference of 0° was observed when the field was (-) or directed back toward the sun.

Acknowledgements

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Table Captions

Table 1. Ratios of the power densities, coherences squared, and phase differences obtained from six sets of auto and cross spectra for the measured variables B_r, B_θ, B_ϕ, and V_r. The power densities, P, the coherences squared, R, and the phase differences, Φ, are averages taken over the frequency range 1-50 cycles per day. The quantity R_{min} is the value of R required for a significant measurement of Φ, i.e., for a measurement in which the rms uncertainty of Φ is 45°. A (+) polarity is assigned if the average field was directed outward from the sun; a (-) polarity is assigned if it was inward toward the sun.

Table 2. Results of calculations of power-density ratios, coherences squared, and phase differences for idealized hydromagnetic waves. In these calculations, $B_o = 5 \cdot 10^{-5}$ gauss, $n = 5$ protons/cm^3, $T = 10^{5}$ $^\circ$K. The phase difference, Φ, between a pair of variables is given by the sign of R, the square of the coherence, for the pair. Thus, the pair is in phase for $R > 0$ and 180° out of phase for $R < 0$. Where two signs are given, the upper sign corresponds to the case of $\beta = 25^\circ$ and the lower sign is for $\beta = -25^\circ$. The quantity P for a particular variable is just the average power that would appear in a record of the variable that was one period of oscillation in length. In these calculations, it was assumed that the variations would be produced by a superposition of randomly polarized waves. For transverse waves the

Table Captions
(continued)

range of θ_k indicated by 'all' is $0 \leq \theta_k \leq 90^o$. The results given here are for the case $\vec{K} \cdot \vec{B}_o > 0$. For $\vec{k} \cdot \vec{B}_o < 0$, all phase differences change by 180^o.

Table 1

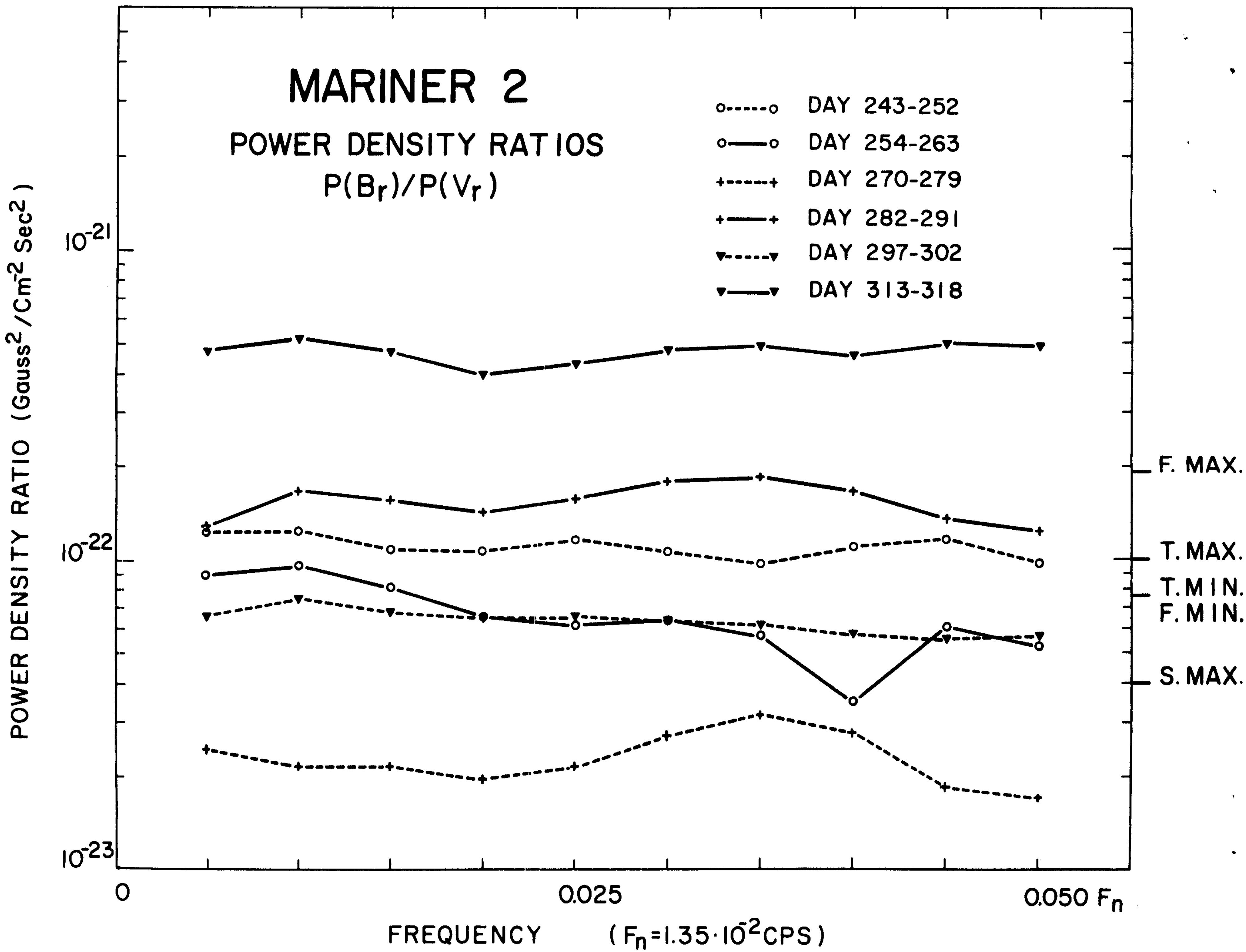
Period Days	Polarity	$P(B_r)/P(V_r)$	$P(B_\theta)/P(V_r)$	$P(B_\varphi)/P(V_r)$	$R(V_r, B_r)/R_{min}$	$\Phi(V_r, B_r)$
		10^{-23} gauss^2 cm^{-2} sec^2				Deg.
243-252	+	11.8	16.5	14.5	10.60	169 ± 7
254-263	-	7.8	21.2	13.4	2.18	- 16 ±14
270-279	+	2.3	5.3	4.2	2.05	160 ± 8
282-291	-	14.4	31.7	29.3	16.40	- 8 ± 4
297-303	+	6.6	16.4	14.6	16.80	167 ± 4
313-318	-	46.1	32.3	32.3	2.76	- 6 ± 8
Average	+	6.9	12.7	11.1	9.82	165
Average	-	22.8	28.4	25.0	7.11	- 10
Average	Both	14.5	20,6	18.1	8.46	

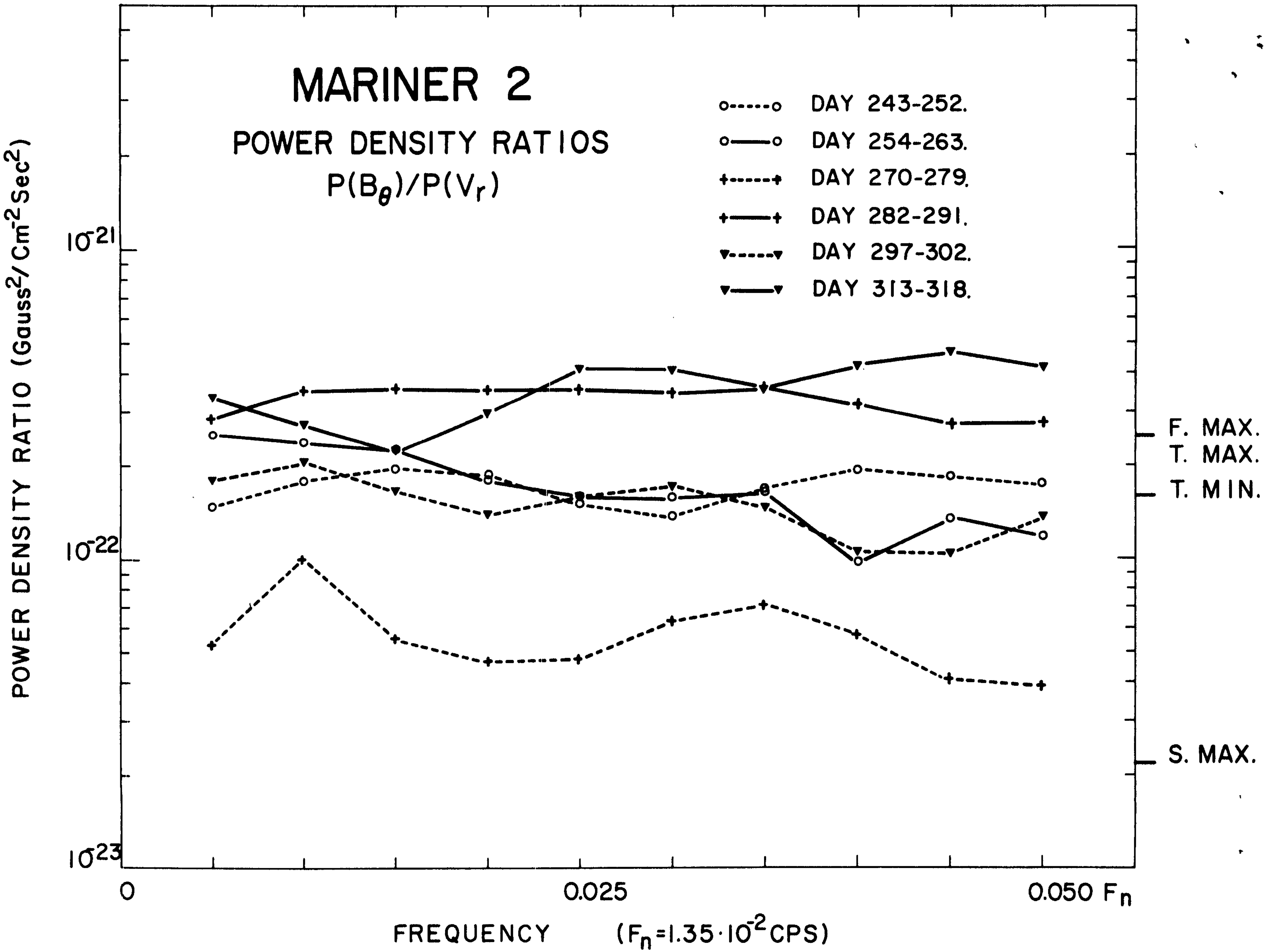
Table 2

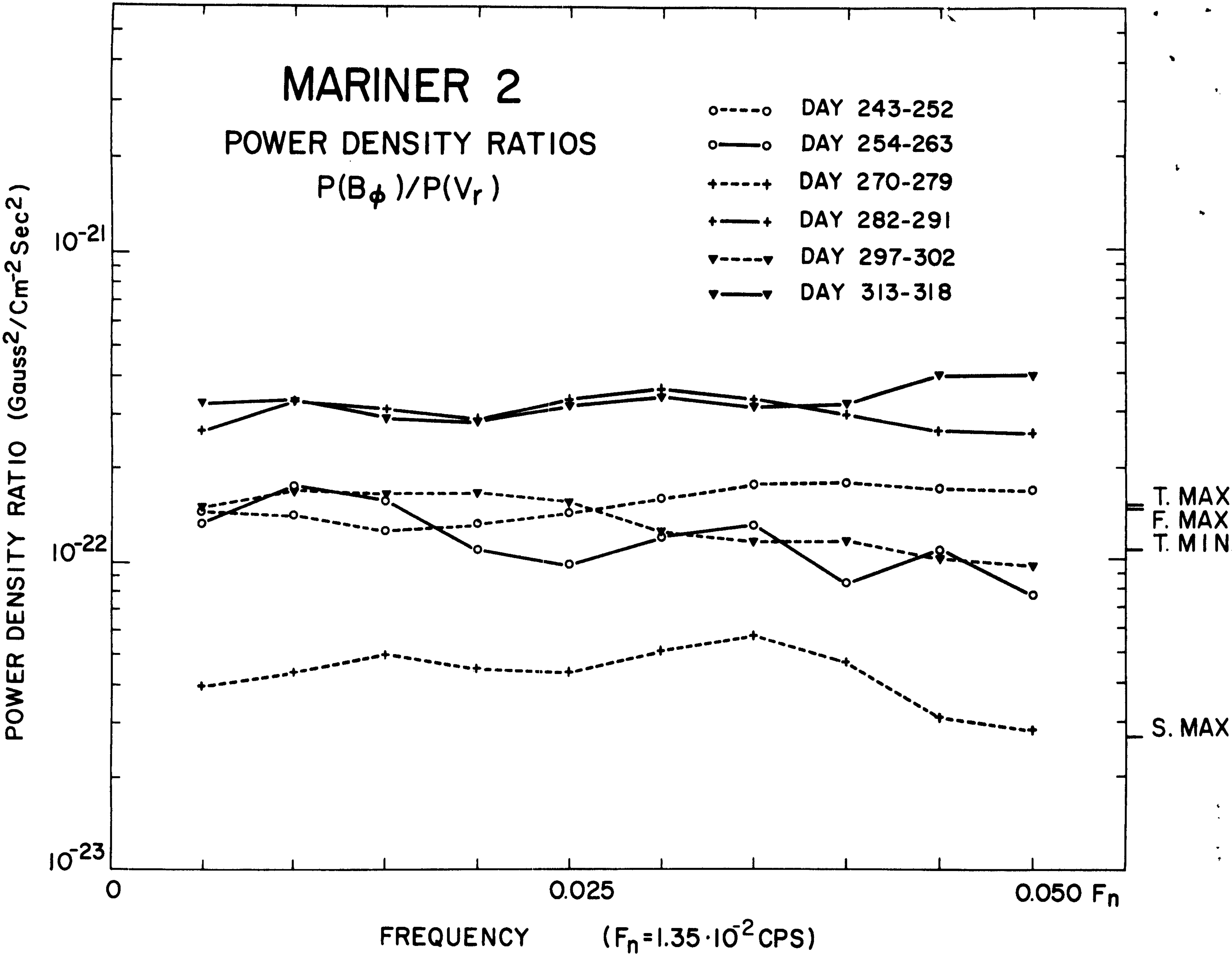
θ_k	$P(B_r)/P(V_r)$	$P(B_\theta)/P(V_r)$	$P(B_\varphi)/P(V_r)$	$R(V_r, B_r)$	$\Phi(V_r, B_r)$
Deg.	10^{-23} gauss^2 cm^{-2} sec^2				Deg.
Slow Mode, $\beta = 0^{\circ}$					
0	0.00	0.00	0.00	0.00	---
15	0.58	1.17	0.76	-0.55	180°
30	1.48	1.83	1.58	-0.82	180°
45	2.35	1.48	2.10	-0.93	180°
60	3.12	0.79	2.43	-0.98	180°
75	3.65	0.22	2.64	-0.99	180°
90	3.84	0.00	2.71	-1.00	180°
Slow Mode, $\beta = \pm 25^{\circ}$					
0	0.00	0.00	0.00	0.00	---
15	0.83	1.19	1.00	-0.52	180°
30	1.84	2.06	1.95	-0.78	180°
45	2.64	2.10	2.39	-0.91	180°
60	3.28	1.82	2.59	-0.97	180°
75	3.70	1.54	2.68	-0.99	180°
90	3.84	1.42	2.71	-1.00	180°
Fast Mode, $\beta = 0^{\circ}$					
0	10.50	25.40	14.90	-1.00	180°
15	9.08	18.30	11.80	-0.66	180°
30	9.74	12.10	10.40	-0.30	180°
45	12.20	7.71	10.90	-0.11	180°
60	15.30	3.90	12.00	-0.02	180°
75	17.90	1.07	12.90	0.00	---
90	18.90	0.00	13.30	0.00	---
Fast Mode, $\beta = \pm 25^{\circ}$					
0	10.50	16.60	13.40	-1.00	180°
15	9.08	13.10	11.00	-0.76	180°
30	8.78	9.84	9.28	-0.48	180°
45	9.69	7.70	8.75	-0.28	180°
60	10.90	6.08	8.66	-0.15	180°
75	12.00	4.98	8.69	-0.06	180°
90	12.40	4.59	8.71	0.00	---
Transverse Mode, $\beta = 0^{\circ}$					
All	10.00	25.00	15.10	-1.00	180°
Transverse Mode, $\beta = \pm 25^{\circ}$					
All	7.70	16.00	10.80	-1.00	180°

Figure Captions

Fig. 1. Ratios of power densities. The ratios of the power density in the auto spectrum of each vector field component, B_r, B_θ, and B_φ, to the power density in the auto spectrum of the radial component of the plasma velocity, V_r, is plotted versus frequency for six sets of auto spectra. The theoretically allowed ranges for each ratio are also indicated on the right-hand side of the corresponding plot. Separate ranges are shown for each mode of propagation, fast (F), slow (S), and Alfven or transverse (T).


MARINER 2
POWER DENSITY RATIOS
P(Br)/P(Vr)
DAY 243-252
DAY 254-263
DAY 270-279
DAY 282-291
DAY 297-302
DAY 313-318
POWER DENSITY RATIO (Gauss2/Cm-2 Sec2)
10-21
10-22
10-23
F. MAX.
T. MAX.
T. MIN.
F. MIN.
S. MAX.
0
0.025
0.050 Fn
FREQUENCY (Fn=1.35·10-2CPS)


MARINER 2
POWER DENSITY RATIOS
$P(B_\theta)/P(V_r)$
DAY 243-252.
DAY 254-263.
DAY 270-279.
DAY 282-291.
DAY 297-302.
DAY 313-318.
POWER DENSITY RATIO (Gauss^2/Cm^{-2}Sec^2)
10^{-21}
10^{-22}
10^{-23}
0
0.025
0.050 F_n
FREQUENCY (F_n=1.35·10^{-2}CPS)
F. MAX.
T. MAX.
T. MIN.
S. MAX.


MARINER 2
POWER DENSITY RATIOS
P(Bϕ)/P(Vr)
DAY 243-252
DAY 254-263
DAY 270-279
DAY 282-291
DAY 297-302
DAY 313-318
POWER DENSITY RATIO (Gauss2/Cm-2Sec2)
10-21
10-22
10-23
0
0.025
0.050 Fn
FREQUENCY (Fn=1.35·10-2CPS)
T. MAX.
F. MAX.
T. MIN.
S. MAX.