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by Albert L. Braslow

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Langley Station, Hampton, Va.

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A REVIEW OF FACTORS AFFECTING BOUNDARY-LAYER TRANSITION[1]

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SUMMARY

A brief review is made of the current state of the art of boundary-layer transition. Discussed, in various degrees of detail, are experimentally determined effects on transition of pressure gradients, surface to free-stream temperature ratio, free-stream Mach number, free-stream turbulence, noise, two- and three-dimensional-type surface roughness, and laminar boundary-layer control through suction. Certain aspects of the theoretical approach to transition are discussed and some comparisons with experiment are made. The review is intended primarily for the engineer or scientist desiring a general understanding of boundary-layer transition phenomena rather than for the active researcher in the field of fluid mechanics. Some needs for further research are indicated.

INTRODUCTION

The year 1963 marked the eightieth anniversary of Osborne Reynolds' classical experiments with the water tank in which he demonstrated that under certain conditions, the flow in a tube changes from laminar to turbulent (ref. 1). After all these years, the mechanism of this transition process is still not completely understood, and experimental investigation must be relied upon heavily. A great part of the difficulty involved in an understanding of the transition from laminar to turbulent flow lies in the numerous factors which affect transition, but which are not independent of each other. When more than one of these factors are present at the same time, therefore, their individual effects are not additive. Significant progress has been made, however, in the determination of the individual trends of these factors, and knowledge of these trends is necessary in any attempt either to explain the observed transition or to predict the occurrence of transition. The purpose of this review is to present in broad perspective information on boundary-layer transition that will provide a general understanding of this phenomenon. It is intended therefore for use of scientists and engineers having need for knowledge of

[1]This paper was originally presented at a Graduate Seminar of the Department of Aerospace Engineering, School of Engineering and Applied Science, University of Virginia, Charlottesville, Virginia, on February 17, 1965.

the elementary aspects of the problem rather than for active researchers in the field of fluid mechanics. Thus, the approach is merely to indicate the numerous factors that need to be considered and to describe certain of these factors briefly. In particular, a few details are noted on the effects of surface roughness and laminar boundary-layer control. Recent results from the current laminar-flow control flight program of the U.S. Air Force on the X-21 airplane are included.

Because of the rather general approach to this presentation, no effort is made to provide an exhaustive list of source material. However, references 2 to 8 are reviews and summaries that include more comprehensive bibliographies. The reference material cited herein should therefore be considered as merely representative of the many sources equally pertinent to this subject.

SYMBOLS

d	width or diameter of roughness particle
k	height of roughness particle
K	constant
M	Mach number
r	leading-edge radius
R	Reynolds number
T	temperature
U'	free-stream turbulence velocity
U	free-stream velocity
W	tangential component of free-stream velocity along wing leading edge
α	frequency of two-dimensional disturbance
θ	boundary-layer momentum thickness
Λ	wing leading-edge sweep

ν	coefficient of kinematic viscosity

Subscripts:

δ^*	based on boundary-layer displacement thickness
k	conditions at top of roughness particle
θ	based on boundary-layer momentum thickness
t	conditions at which turbulent spots appear
T	based on distance to transition position
w	conditions at wall
aw	adiabatic wall conditions
o	local conditions outside boundary layer
∞	conditions in undisturbed free stream

DISCUSSION

Stability Theory

The theoretical approach to the transition phenomenon is usually through the stability theory (ref. 2). This theory indicates whether infinitesimally small disturbances present in the laminar boundary layer will amplify or damp out as they travel downstream. If these disturbances, known as Tollmein-Schlichting waves, amplify as they travel downstream, experiment indicates that they eventually break up into turbulent spots which grow in size with further downstream movement and finally merge to form a continuously turbulent region. Figure 1 shows theoretical predictions of stability theory and verifying experimental points over a flat plate (ref. 9). Plotted against the Reynolds number based on the laminar boundary-layer displacement thickness is a dimensionless parameter $\alpha\nu/U^2$ where α is the frequency of the disturbance and U is the free-stream velocity. Any disturbance having a frequency of such value that the ordinate falls within this loop amplifies as it moves downstream. Disturbances that lie outside this loop damp out as they move downstream. These results show that for a boundary-layer Reynolds number up to some maximum value, disturbances of all frequencies are damped. It should be recognized that this stability theory indicates only the initiation of amplified disturbances and not the actual position of transition. Transition takes place at some point downstream after the disturbances have grown to sufficient magnitude.

The particular curve drawn in figure 1 represents the predictions for the laminar flow over a flat plate, that is, for two-dimensional flow with a zero pressure gradient. There are several variables which affect not only this maximum value of boundary-layer Reynolds number for complete stability but also the length of the amplification region, namely, the distance between the position at which the first instability occurs and the position for completely turbulent flow. Some of these factors are longitudinal pressure gradients, lateral pressure gradients resulting from three-dimensional-flow effects, the ratio of surface temperature to free-stream temperature, free-stream Mach number, free-stream turbulence, noise, and boundary-layer control through suction.

Experimental Trends With Smooth Surfaces

Experimental trends of some of the preceding factors from reference 4 are shown in figures 2 to 5. Figure 2 shows the effect of Mach number on transition under conditions of no heat transfer and no pressure gradient. The scatter of the data is attributed to differences in the methods used in observing transition and to differences in wind-tunnel disturbances. The lower curve represents the boundary for the beginning of transition and the upper curve represents that for completely turbulent flow. It is clear that the transition Reynolds number at first decreases with increasing Mach number and then shows a definite increase at Mach numbers above about 4. This increased transition Reynolds number at the higher Mach numbers is discussed further subsequently. In figure 3, the ratio of the transition Reynolds number with variable free-stream turbulence to that with no free-stream turbulence is plotted against the value of the turbulence level. As the disturbances in the free-stream increase, the transition Reynolds number decreases.

The effects of both wall heating and cooling and longitudinal pressure gradient on the transition Reynolds number at $M_\infty = 3.1$ are shown in figure 4. The cone at this Mach number has about zero pressure gradient and the parabolic body has a favorable pressure gradient. The ratio of wall temperature to the adiabatic wall temperature is plotted against transition Reynolds number. Heating the surface above the adiabatic wall temperature decreases the transition Reynolds number; whereas cooling the surface increases the transition Reynolds number. For the conditions considered in this figure, the transition Reynolds number was increased by a factor of 3 by cooling the cone model from the insulated-surface condition to a temperature ratio of about 0.58. The favorable effect on transition of a negative or favorable pressure gradient can be seen from a comparison of the results obtained on the cone and on the parabolic body. Favorable pressure gradients, like surface cooling, increase the laminar boundary-layer stability and delay transition. Unfavorable pressure gradients have large destabilizing effects on the stability and in many cases induce almost immediate transition in boundary layers that are not artificially stabilized. An important point to note from this figure is the fact that as the temperature approaches some value asymptotically, the transition Reynolds number increases very rapidly and appears to be approaching infinity. The small-disturbance stability theory (ref. 2) actually predicts such an occurrence; that is, it predicts that at some value of wall cooling, the boundary layer is completely stable to disturbances of all frequencies at all values of the Reynolds number.

Asymptotic values of wall-temperature ratio obtained from data such as these are compared in figure 5 with the temperatures predicted by small-disturbance theory as necessary for complete stability. For values of wall-temperature ratio above the curve, disturbances will amplify, whereas for wall-temperature ratios below the curve, complete stability is predicted. The asymptotic experimental values are seen to be in good agreement with the predicted trends. Stability theory also predicts well the previously shown favorable effects of negative pressure gradients on transition. In general, the stability theory indicates that any parameter that increases the convexity of the laminar boundary-layer velocity profile will increase the boundary-layer stability. Another factor (to be discussed subsequently) that has a large effect on the boundary-layer stability because of its favorable effect on the profile shape is boundary-layer control through suction. Any parameter which decreases the boundary-layer thickness, such as boundary-layer control by suction or wall cooling, also delays transition because the effective Reynolds number is decreased and, as indicated previously, transition is delayed if the Reynolds number is maintained below the critical value. Leading-edge bluntness at supersonic speeds is another parameter that has a large effect on effective or local Reynolds numbers. In this case, the change in local flow conditions near the surface behind the normal shock results in a decreased local Reynolds number, so that transition is delayed.

Effects of Surface Roughness

It has already been indicated that external disturbances, such as free-stream turbulence or noise, promote premature transition. These small disturbances appear to be compatible with the small-disturbance amplification theory. Some disturbances that are initiated inside the laminar boundary layer, such as those due to two-dimensional-type surface roughness, also appear to be of the Tollmein-Schlichting type and are subject to amplification theories during their movement downstream. Surface roughness such as spanwise ridges or grooves produces effects typical of two-dimensional-type roughness. For two-dimensional-type surface roughness, when some critical value of Reynolds number is reached, spots of turbulence begin to move forward of the natural position of transition. No turbulence spots are noted at forward positions and a further increase in Reynolds number is required to move the transition gradually forward.

The effects of discrete particles of surface roughness (that is, a three-dimensional-type roughness) contrast significantly with the effects of two-dimensional-type roughness. It has been found that for the three-dimensional-type roughness, a critical height exists below which the roughness has no influence on the natural transition and above which the roughness causes premature transition. This result is shown in figure 6 by means of several observations of the variation of the streamwise boundary-layer velocity fluctuations with time. (See ref. 6.) These measurements were made with the use of a hot-wire anemometer on a cone with a 10° apex angle at a free-stream Mach number of 2.01. The vertical location of a trace indicates the corresponding unit Reynolds number of the stream. The traces in the left plot were obtained through a Reynolds number range for a smooth cone. The bottom trace indicates completely laminar flow. The next higher trace indicates occasional bursts of turbulent flow. The trace next higher indicates laminar flow a small part of the time, and the top trace indicates fully turbulent flow. All traces were made with the hot wire at the same location. This change in the character of the boundary layer with changes in Reynolds number is consistent with the concept of transition beginning as turbulent spots that grow as they move downstream. In the upper right of figure 6 are shown hot-wire traces taken behind some roughness grains 0.003 inch high on the cone. This roughness had no effect on the natural transition as can be seen by the fact that the turbulence was initiated at the same value of stream unit Reynolds number. In contrast, the traces in the lower right part of the figure, taken behind roughness grains of larger size, show a large reduction in stream unit Reynolds number for the initiation of turbulence. This comparison clearly indicates that a critical size of this three-dimensional roughness exists. Other hot-wire measurements have also shown that for three-dimensional roughness only slightly smaller than the critical size, the level of the velocity fluctuations in the laminar layer at appreciable distances downstream of the roughness was as low as that measured with the smooth surface. It appears, then, that no upstream movement of the transition region occurs at speeds below the critical speed of the roughness. From measurements of the type shown in figure 6, it seems likely that for three-dimensional roughness, transition results from the formation of discrete eddies or disturbances originating at the roughness particles. It should then be possible to relate the occurrence of these disturbances to the local flow conditions at the roughness. Such a relationship has been provided (see ref. 3) at subsonic speeds on the basis of a critical roughness Reynolds number, formed with the height of the roughness k and the local flow conditions at the top of the particle when the particle began to introduce turbulence spots into the boundary layer. The square root of this critical roughness Reynolds number is equal to a correlation Reynolds number originally proposed by Schiller (ref. 10) on the basis of the roughness height and the friction velocity. (See appendix.)

Numerous data points from several low-speed investigations of three-dimensional roughness particles (ref. 6) are presented in figure 7 in the form of the square root of the roughness Reynolds number for transition R_k plotted against the ratio of the particle width (or diameter) d to the particle height k. These data cover a wide range of particle shape, distribution, number, height, distance from model leading edge, and degree of boundary-layer stability as affected by pressure gradient and boundary-layer control. Differences in symbol templates are indicative of differences in configuration. In spite of large differences in roughness configuration and differences in experimental technique, the values of $\sqrt{R_{k,t}}$ for transition for a given value of d/k are seen to vary only within a factor of about 2. For roughness within the linear portion of the boundary-layer velocity profile, the $\sqrt{R_{k,t}}$ is proportional to the critical projection height. This correlation, therefore, can be used to indicate, within the same accuracy, the magnitude of a submerged three-dimensional-type roughness necessary to cause premature transition.

The variety of shapes presented in this figure is seen to form some systematic variation of $\sqrt{R_{k,t}}$ with d/k, a decreasing value of the roughness parameter with increasing d/k. This correlation is reasonable, inasmuch as projections with large values of d/k are approaching protuberances of a two-dimensional nature and the laminar boundary layer has been found to be more sensitive to two-dimensional than to three-dimensional disturbances.

A similar correlation of transition induced by three-dimensional roughness has been made at supersonic speeds. Data in figure 8 (from refs. 11 to 13) show that about the same value of critical roughness Reynolds number has been found to exist up to Mach numbers of about 4. This result is particularly significant inasmuch as it indicates that for surface temperatures near adiabatic wall values, the roughness height required to cause transition is greater at supersonic speeds than at subsonic speeds because of the effect of Mach number in thickening the boundary layer in supersonic flow. At higher supersonic and hypersonic speeds, experimental information, such as this, indicates that the critical roughness Reynolds number increases to such a large extent that the whole concept of this critical roughness Reynolds number breaks down. The concept depends upon flow similarity about a particle immersed within the linear portion of the boundary-layer velocity profile. At the high supersonic and hypersonic speeds, however, available data indicate that not until the roughness is considerably greater than the total boundary-layer thickness does it begin to induce premature transition. The question may then be raised as to why this phenomenon is of any concern. After all, the attainment of laminar flow at high speeds would be desirable in order to have less skin friction and lower heating rates. It is necessary, however, to be able to predict whether or not the flow will be laminar. Until the reliability of such predictions is assured, hypersonic aircraft, spacecraft, or missiles must be designed for turbulent heat-flow conditions. This approach may be unduly conservative and may require large unnecessary penalties in payload due to unnecessary weight of heat-protection methods.

Another aspect of the reduced sensitivity of the laminar boundary layer to surface roughness at the higher speeds is involved with wind-tunnel experimentation at hypersonic Mach numbers. Figure 2 showed how the Reynolds number for natural transition increased rapidly with an increase in hypersonic Mach number. This increase in natural transition Reynolds number and a decrease in the sensitivity of the boundary layer to surface roughness at hypersonic speeds make it very difficult to fix boundary-layer transition deliberately at hypersonic speeds in wind tunnels. In many experiments it is desirable to be able to fix boundary-layer transition on wind-tunnel models. Therefore, an understanding of natural transition at hypersonic speeds and also of methods of artificially inducing transition at these speeds is very important. Such research, of course, would include both experimental and theoretical work. With reference again to figure 8 for the lower supersonic speeds, an important point can be made regarding the effects of laminar boundary-layer stability on the critical three-dimensional roughness size. It has already been mentioned that boundary-layer cooling, boundary-layer suction, and favorable pressure gradients have a stabilizing effect on the laminar layer for small disturbances and that a two-dimensional surface roughness acts as a small disturbance. Three-dimensional roughness does not have the same effects. The filled-in square symbols, which represent values of the roughness parameter $\sqrt{R_{k,t}}$ for the cool cone, agree closely with the open square symbols, which represent values for the cone at adiabatic temperature. The cool cone, of course, has a higher stability in accordance with the small-disturbance theory than the uncooled cone. These values are also in close agreement with the low-speed data of figure 8 where increased laminar stability was obtained by continuous suction and by highly favorable pressure gradients. For this large variety of conditions, an increase in laminar boundary-layer stability had very little effect on the three-dimensional roughness Reynolds number parameter for transition.

In fact, these methods of stabilizing the laminar boundary layer may actually aggravate the adverse effect of three-dimensional roughness on transition. An example of this effect is presented in figure 9 where additional hot-wire traces behind three-dimensional roughness elements on a cone are presented. The traces in the left group were made with the model surface at equilibrium temperature and those in the right group with the same surface roughness but with the surface cooled. The wall-temperature distribution for the cooled model is shown in the upper right-hand corner. The wall temperature varied from almost stagnation temperature near the cone apex to about -50° F ahead of the roughness. It is clearly demonstrated that, for the stream unit Reynolds number at which the roughness was just critical, that is, when turbulence spots began to appear with the surface at equilibrium temperature, cooling the cone surface resulted in completely turbulent boundary layer. In fact, for the cooled condition, it was necessary to decrease appreciably the stream unit Reynolds number in order to return the boundary layer to the laminar condition. Associated with the surface cooling for given values of roughness size, roughness location, stream Reynolds number, and Mach number is an increase in the actual roughness Reynolds number R_k. This increase is caused by: first, an increase in velocity at the top of the particle due to a thinning of the boundary layer and due to an increase in convexity of the velocity profile, and second, an increase in local density and a decrease in local viscosity due to the lowered boundary-layer temperature. Inasmuch as the cooling did not increase the critical value of the roughness Reynolds number but did increase the actual value of the roughness Reynolds number, it is clear that cooling will promote rather than delay boundary-layer transition. Favorable pressure gradients and boundary-layer suction will also promote rather than delay transition in the presence of three-dimensional surface roughness.

Laminar-Flow Control

Theoretical and experimental research in the 1940's and early 1950's (ref. 3) indicated that attainment of extensive regions of laminar flow was possible at subsonic speeds with boundary-layer suction. Attainment of long runs of laminar flow was possible even in the presence of adverse pressure gradients, but it was necessary that the airfoil surfaces be maintained sufficiently smooth. The continued sensitivity of the laminar flow to surface roughness, however, was and still is the main obstacle in applying laminar-flow control successfully to aircraft. Information is needed with regard to the man-hours and cost required to maintain the required aircraft surface smoothness and also with regard to the degree of roughness accumulated from such items as dust and bugs during take-off and climb to altitude. Evaluation of these maintenance and handling factors that exceed present-day practices of standard maintenance procedures can be accomplished only through actual flight experience. To provide this experience, the Northrop Corporation, under an Air Force contract, modified two B-66D airplanes in the early 1960's to incorporate laminar-flow control on both surfaces of the wings through suction slots. These airplanes were designated the X-21A and the X-21B. Figure 10 is a photograph of one of the modified airplanes and figure 11 is a closeup photograph that will give some idea of the suction-slot arrangement at the leading edge of one of the wings.

The flight tests of the X-21 airplanes have been in progress at Edwards Air Force Base since April 1963. During these tests, extensive regions of laminar flow have been obtained for the first time on an operational aircraft. In fact, full-chord laminar flow was measured at the outboard wing sections. Satisfactory operation of a complete suction system is being demonstrated and considerable advances have been made in construction techniques for laminar suction wings. The overall results, however, are not completely optimistic. The primary purpose of the program has not yet been accomplished because of two serious problems. The first is one of spanwise contamination along the stagnation line of the sweptback leading edge. It was demonstrated that any inboard turbulence due to wing-body intersections or wing-fence intersections or local disturbances such as roughness will travel along the leading edge and completely eliminate any laminar flow outboard when a Reynolds number $R_\theta = \frac{W\theta}{\nu}$ exceeds a certain value. This Reynolds number (fig. 12) is based on the tangential velocity along the leading edge W and the boundary-layer momentum thickness θ and is a function of free-stream unit Reynolds number U/ν, wing sweep Λ, and leading-edge radius r. On the X-21 airplane, this Reynolds number has been found to exceed the critical value in the inboard regions of the wing. This leading-edge contamination problem is an example of a phenomenon that was disclosed in flight. (The first indications of adverse effects of wing sweep on boundary-layer transition were obtained in flight some years ago by the British.) The phenomenon had gone unrecognized in previous wind-tunnel investigations because the leading-edge radii in the wind-tunnel tests were never high enough to get into the region for the leading-edge contamination. After the flight tests had disclosed this problem, further wind-tunnel tests were made at Northrop with a swept-wing model having a much larger leading-edge radius than that usually tested in order to acquire some quantitative data on the phenomenon. The results are illustrated in figure 13. Presented is a plan-view sketch of the leading-edge region of the sweptback wing model. The model was equipped with an inboard fence. The curved lines represent the most outboard boundaries of laminar oscillations originating either from the juncture of the leading edge with the fence or from a roughness particle at the leading edge. It is apparent that when the leading-edge Reynolds number R_θ is equal to about 84 or greater, the turbulent flow originating at the leading edge—fence juncture or originating at the particle spreads along the leading edge and prevents attainment of any laminar flow on the wing. As the value of R_θ decreases, the spanwise spread of turbulence decreases appreciably and approaches the normal angular spread of turbulence as indicated by the inboard boundary of the turbulent flow from the particle. Through these wind-tunnel tests and additional flight tests, a modified leading-edge configuration has been developed with which the leading-edge contamination problem has been eliminated. The modification includes a reduced leading-edge radius and additional leading-edge suction through slots perpendicular to the leading edge. Both the reduced radius and the increased leading-edge suction reduce the value of the leading-edge Reynolds number R_θ.

The second factor that has so far prevented accomplishment of the primary purpose of the flight program is one of a lack of repeatability in the laminar-flow results. It has not been possible in most cases to repeat the same results on the same configuration during different flights. The nonrepeatability is due primarily to the use of considerable amounts of plastic material for fairing out surface waviness in the wing. This plastic material chips during flight and results in severe and variable surface roughness. Consideration is presently being given to rebuilding part of the wings with some new construction techniques through which elimination of the surface waviness is believed possible. This type of modification is needed in order to provide the quantitative information desired on the maintenance problems for laminar-flow control in aircraft.

Another factor contributing adversely to the maintenance of laminar flow and receiving attention in the X-21 program is the effect of acoustical disturbances. Although efforts were made to provide a low noise environment on the wing through such considerations as placing the engines behind the wing on the after part of the fuselage, some indications have been observed that acoustical disturbances are influencing the extent of laminar flow. Analytical and experimental studies are necessary to investigate this problem further.

Before the application of laminar-flow control to supersonic aircraft can be considered, not only must the previous questions be answered but considerable additional aerodynamic research is required at supersonic speeds. For example, information is needed on the effectiveness of laminar-flow control in the presence of shock—boundary-layer interactions, wing-body junctures, wing sweep with the leading edge swept both ahead of and behind the Mach cone, acoustical and vibrational disturbances, and three-dimensional body flow. Extensions to the stability theory at supersonic speeds are also desired. Continuance of research programs on boundary-layer transition should, in time, result in clarification of these matters – at least, to the extent of indicating whether application of laminar boundary-layer control to supersonic aircraft offers sufficient promise to justify a major effort.

CONCLUDING REMARKS

The numerous factors contributing to transition from laminar to turbulent flow in a fluid have been reviewed in broad perspective. The intent has been to provide general background information on the various transition phenomena rather than to make a study of the problem in depth. Included are the effects on transition of such factors as pressure gradient, surface temperature, Mach number, and two- and three-dimensional types of surface roughness. The use of suction for laminar boundary-layer control is discussed briefly. Theoretical aspects of transition are compared with experimental results, and needs for further research are indicated.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 12, 1966.

APPENDIX

RELATION BETWEEN CRITICAL REYNOLDS NUMBER PROPOSED BY SCHILLER AND $R_{k,t}$

The Reynolds number proposed by Schiller (ref. 10) is defined by

$$R_{Sc} = \frac{\rho v_* k}{\mu}$$

where

ρ free-stream density

μ free-stream viscosity

v_* friction velocity

k critical roughness height

Since $v_* = \sqrt{\frac{\tau}{\rho}}$ and $\tau = \mu \frac{du}{dy}$, then

$$R_{Sc} = \sqrt{\frac{\mu \frac{du}{dy}}{\rho}}\left(\frac{\rho k}{\mu}\right) \tag{1}$$

where du/dy is the slope of the linear portion of laminar boundary-layer velocity profile. Simplifying equation (1) yields, since $u_k = k \frac{du}{dy}$,

$$R_{Sc}{}^2 = \frac{\mu \frac{du}{dy}}{\rho} \frac{\rho^2 k^2}{\mu^2} = \frac{du}{dy} \frac{\rho k^2}{\mu} = \frac{u_k}{k} \frac{\rho k^2}{\mu} = \frac{\rho u_k k}{\mu}$$

where u_k is the velocity in the linear portion of the laminar velocity profile at top of roughness. In incompressible flow, $\rho_k = \rho$ and $\mu_k = \mu$ where the subscript k denotes conditions at the top of the roughness. Then by definition

$$R_{Sc}{}^2 = \frac{\rho_k u_k k}{\mu_k} = R_{k,t}$$

or

$$R_{Sc} = \sqrt{R_{k,t}}$$

APPENDIX

Evaluation of friction velocity can be obtained from:

$$v_* = U\sqrt{\frac{c_f}{2}} \quad \text{as derived by substituting} \quad \tau = c_f q \quad \text{into} \quad v_* = \sqrt{\frac{\tau}{\rho}}$$

where

c_f	local skin-friction coefficient
q	free-stream dynamic pressure
U	free-stream velocity
τ	local shear stress

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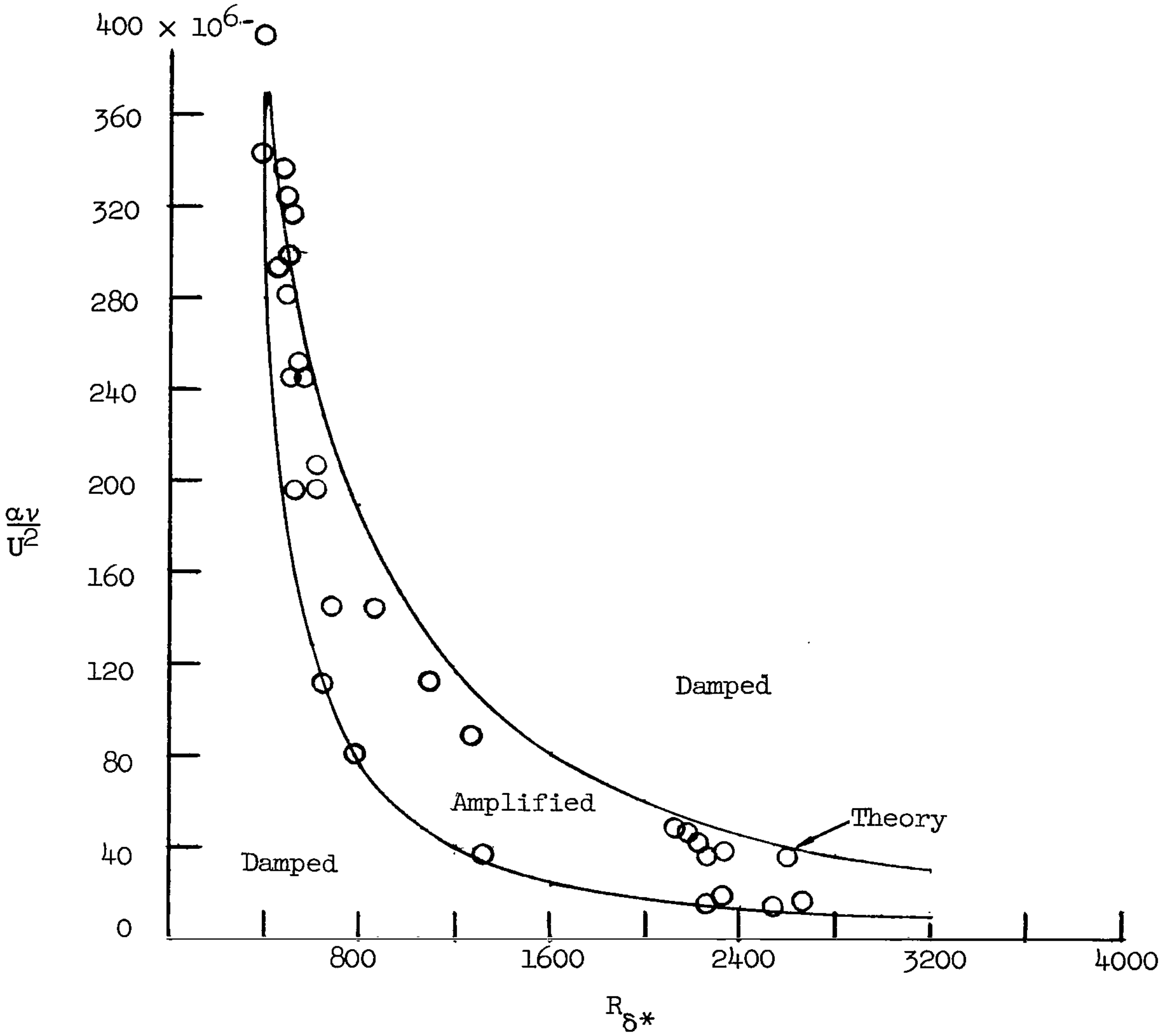


Figure 1.- Laminar stability on flat plate. (From ref. 9.)

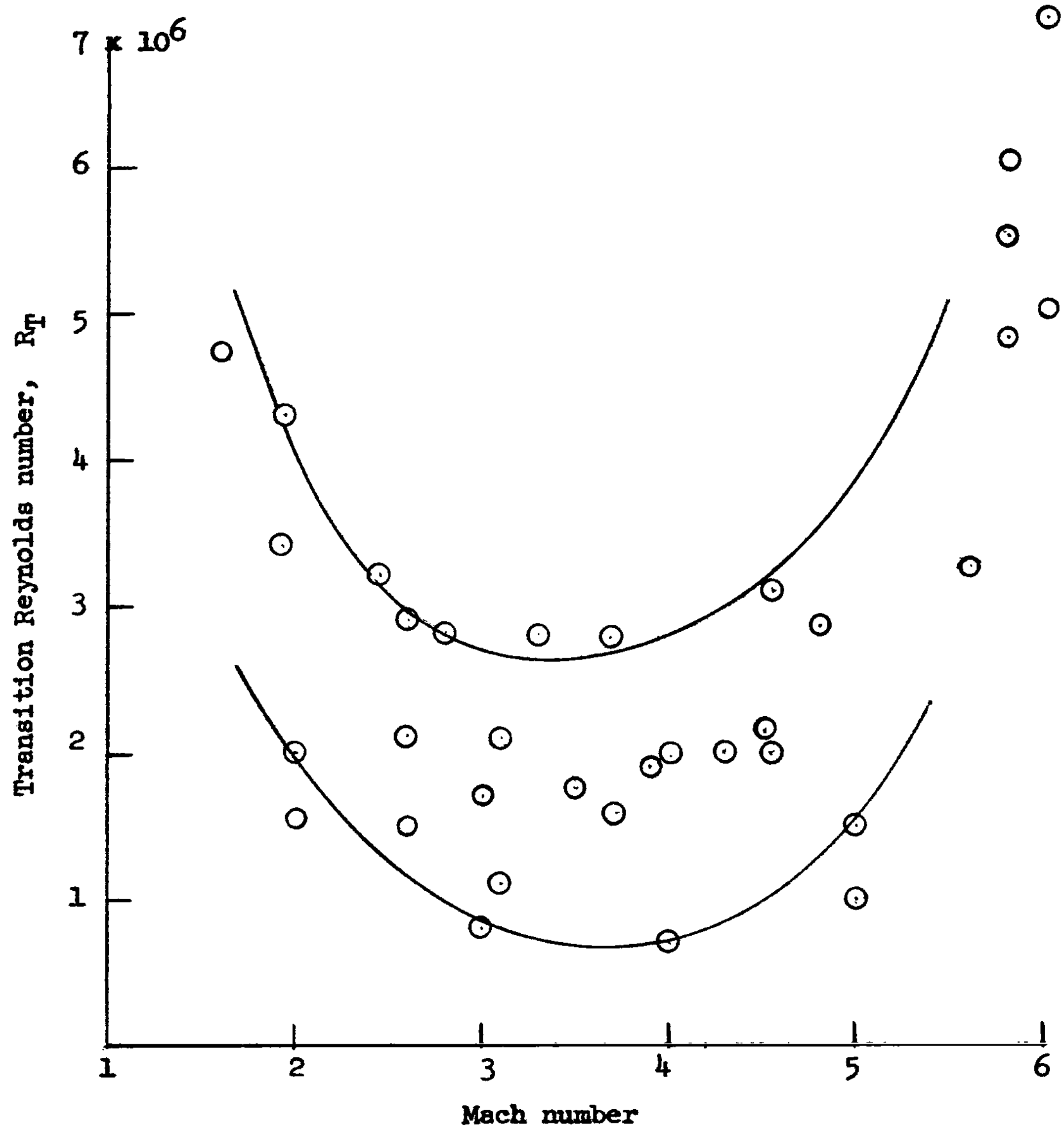


Figure 2.- Effect of Mach number on transition, insulated plates, and hollow cylinders (ref. 4).

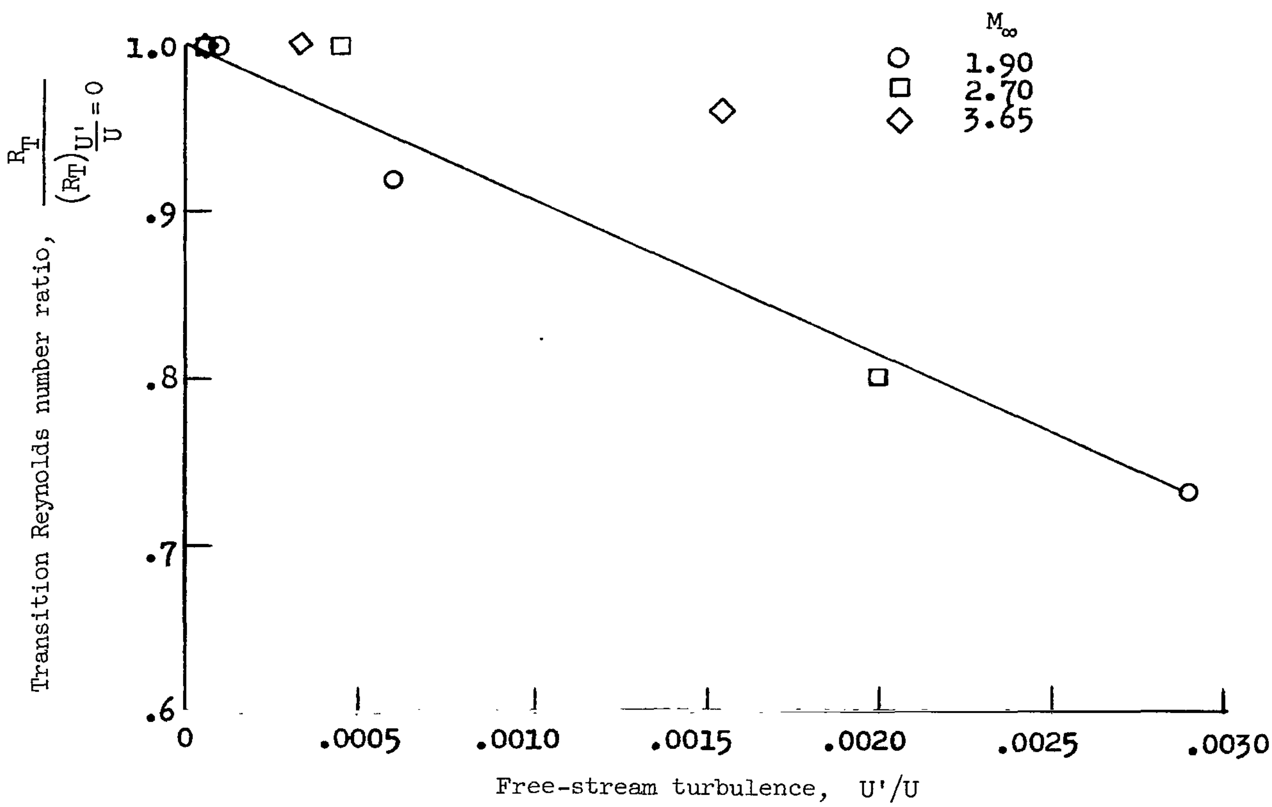


Figure 3.- Effect of turbulence level on transition (ref. 4).

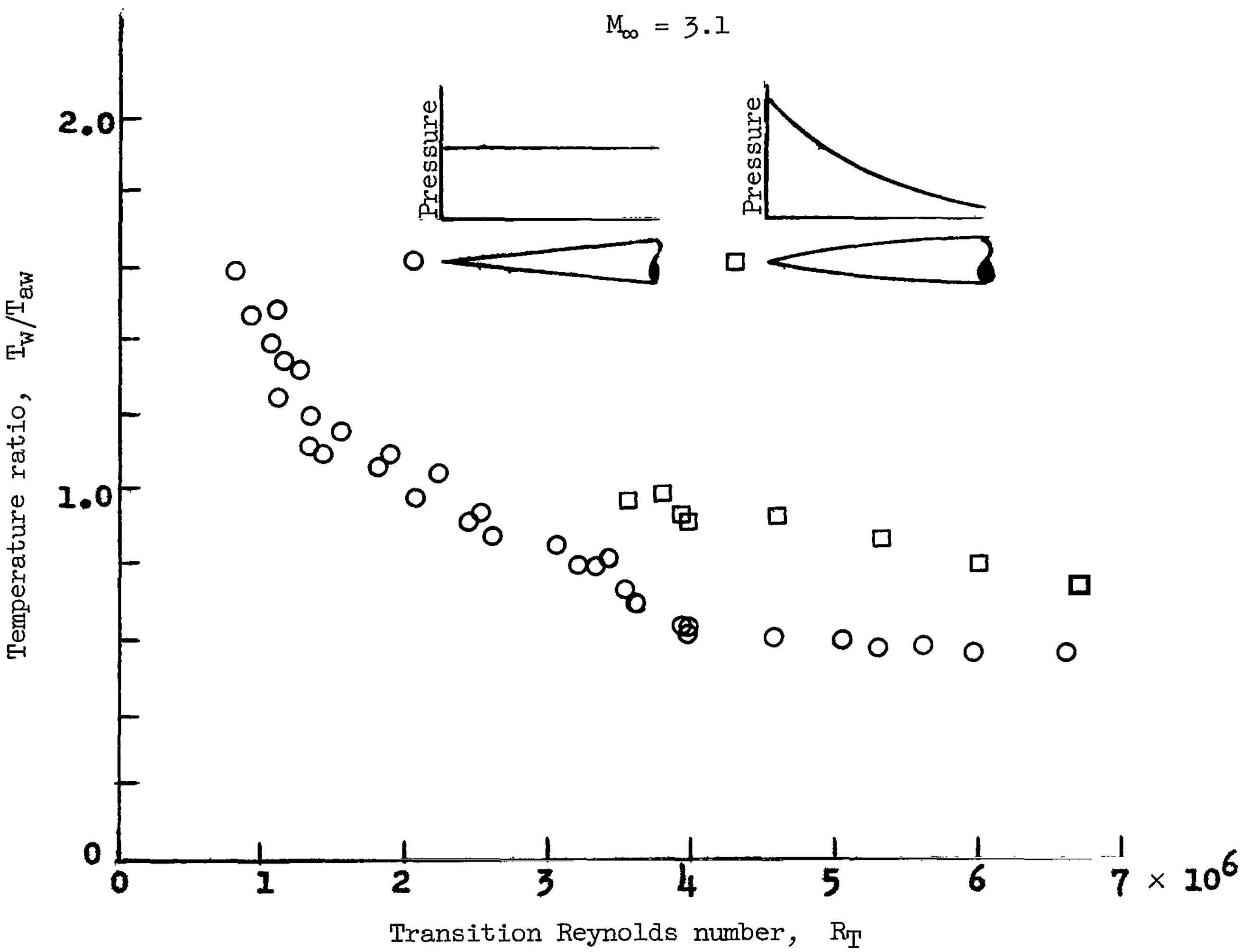


Figure 4.- Effect of surface temperature and favorable pressure gradient (ref. 4).

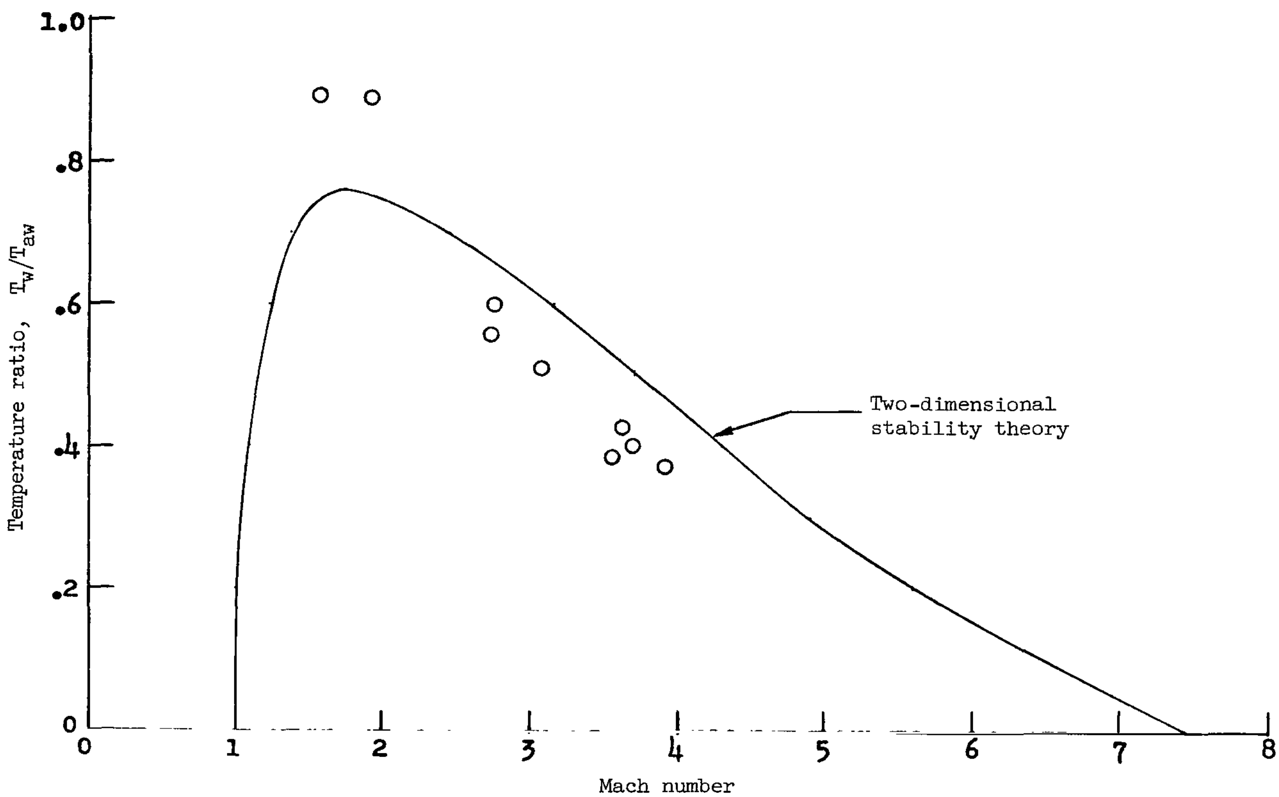


Figure 5.- Cooling effect compared with stability theory (ref. 4).

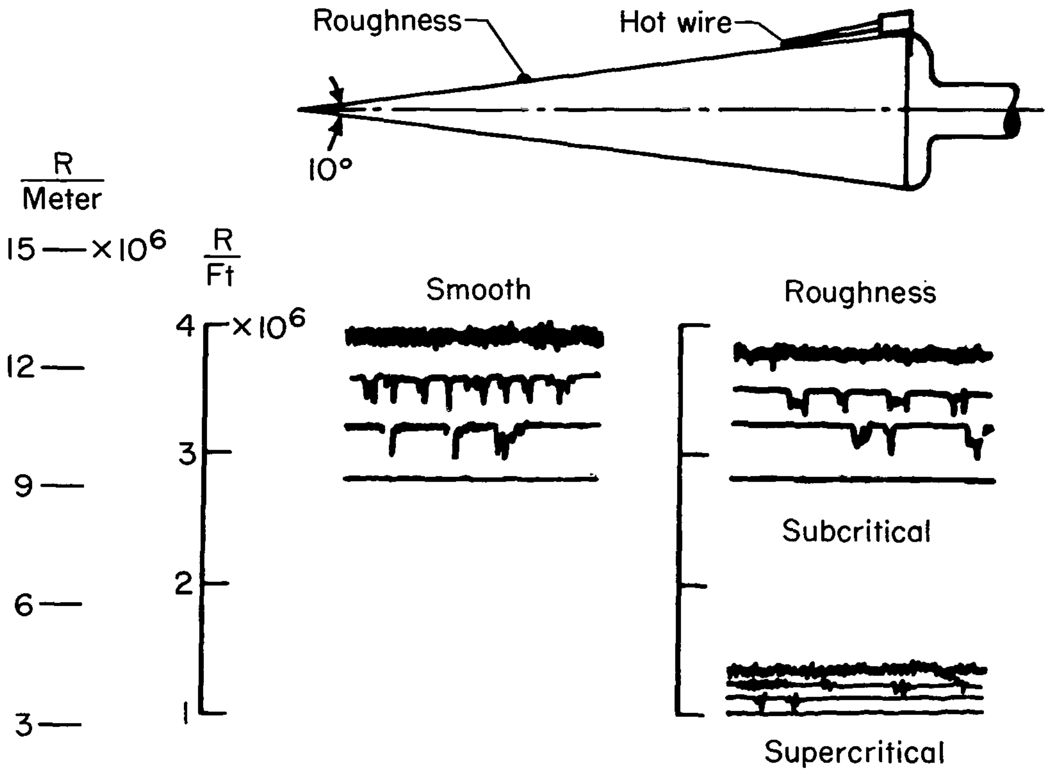


Figure 6.- Effect of granular roughness on transition Reynolds number. $M_\infty = 2.01$. (From ref. 6.)

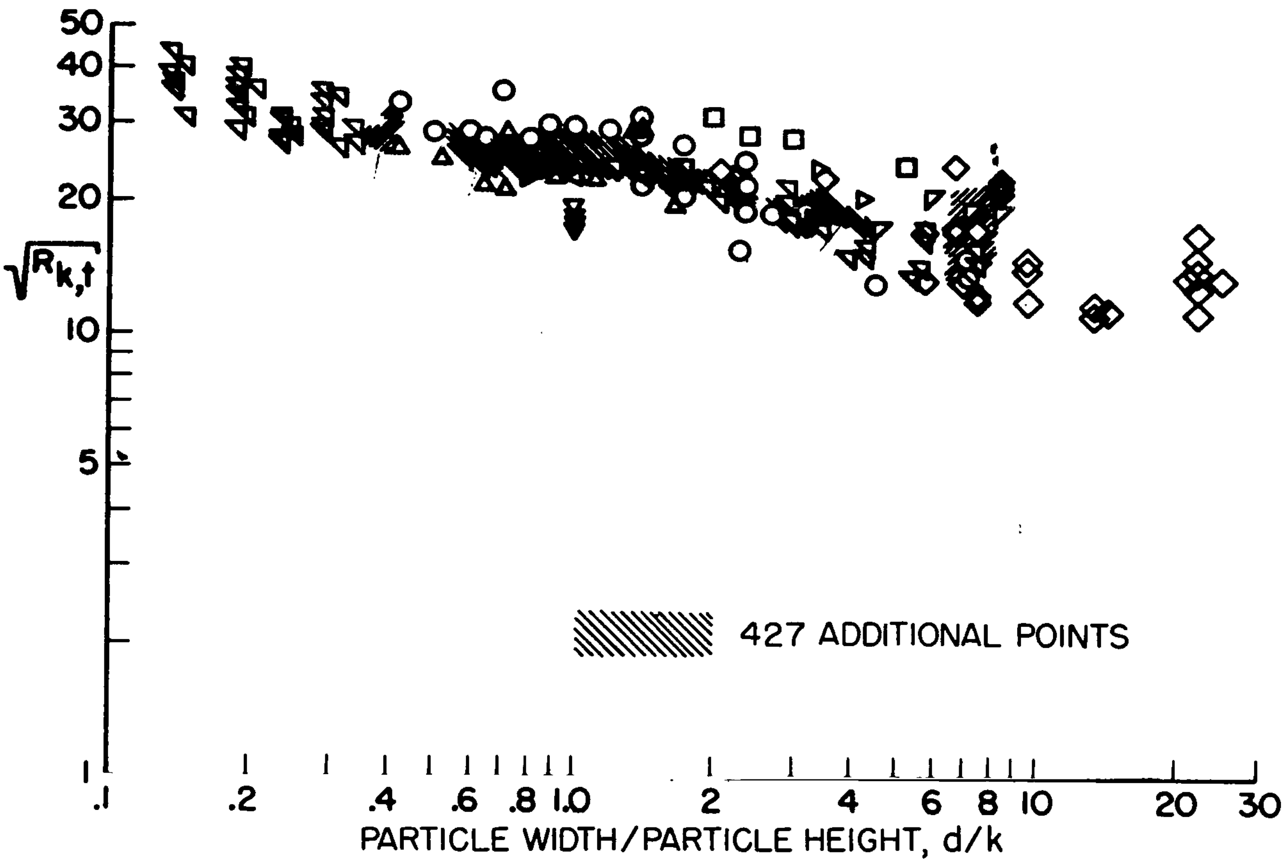


Figure 7.- Low-speed transition correlation. (From ref. 6.)

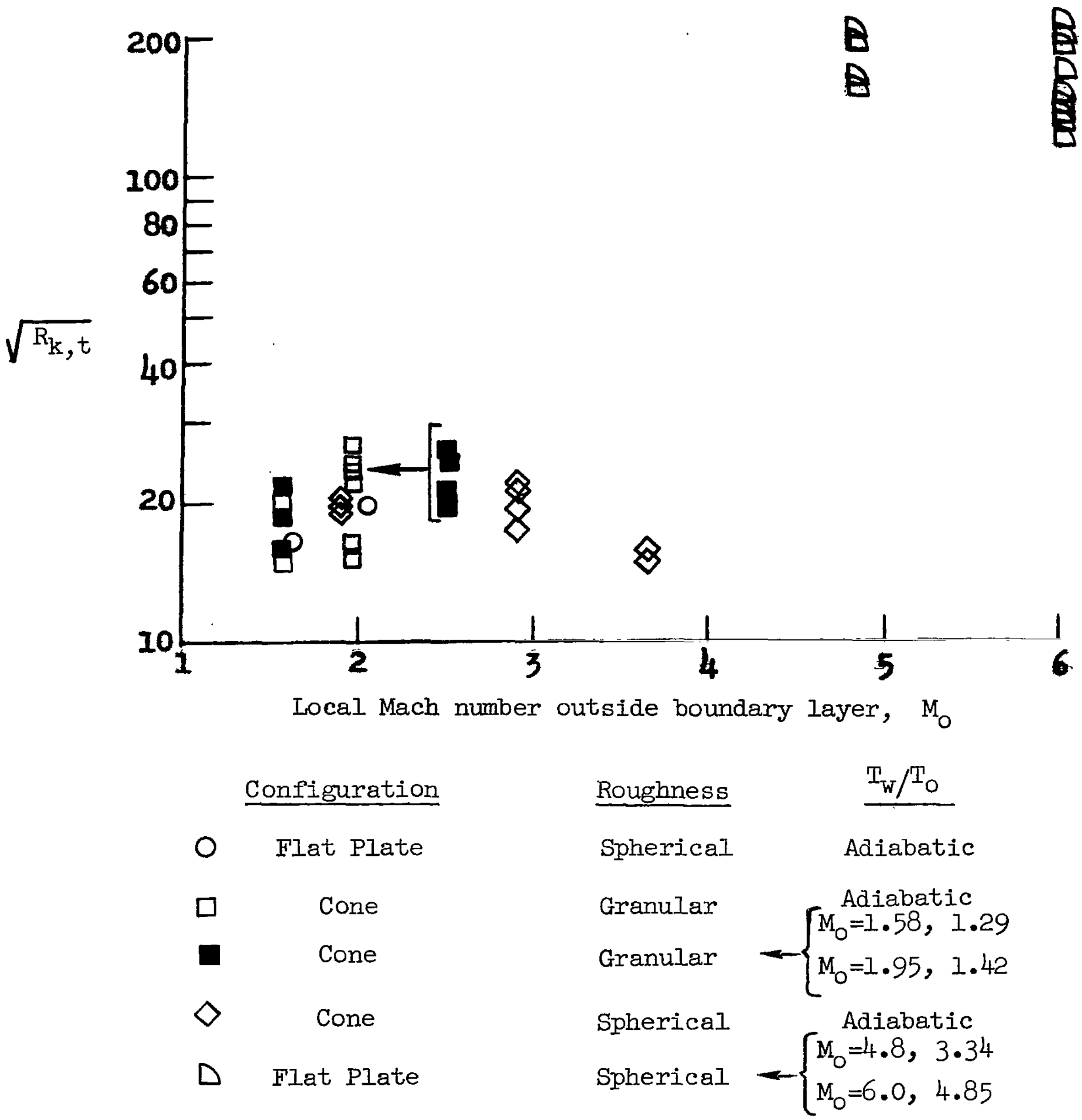


Figure 8.- Effect of Mach number on critical roughness Reynolds number. (Data from refs. 11, 12, and 13.)

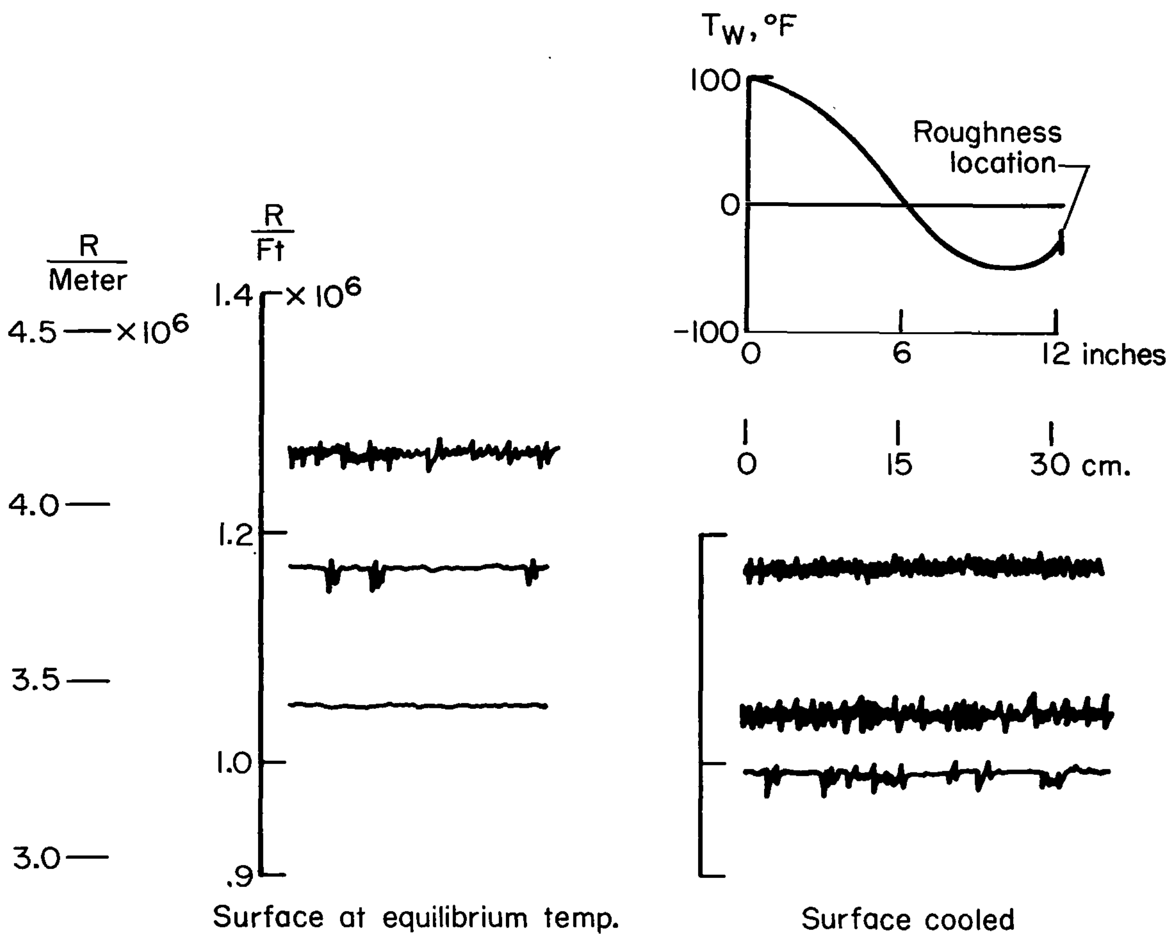


Figure 9.- Effect of surface cooling on transition Reynolds number. $M_\infty = 2.01$. (From ref. 6.)

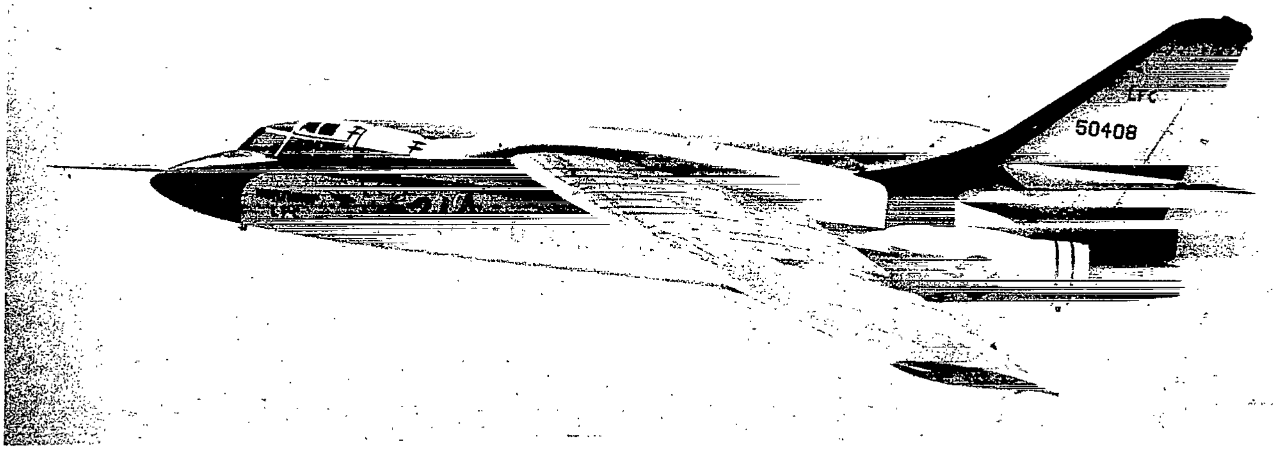


Figure 10.- Photograph of X-21A airplane. (Courtesy of U.S. Air Force.) L-66-1174

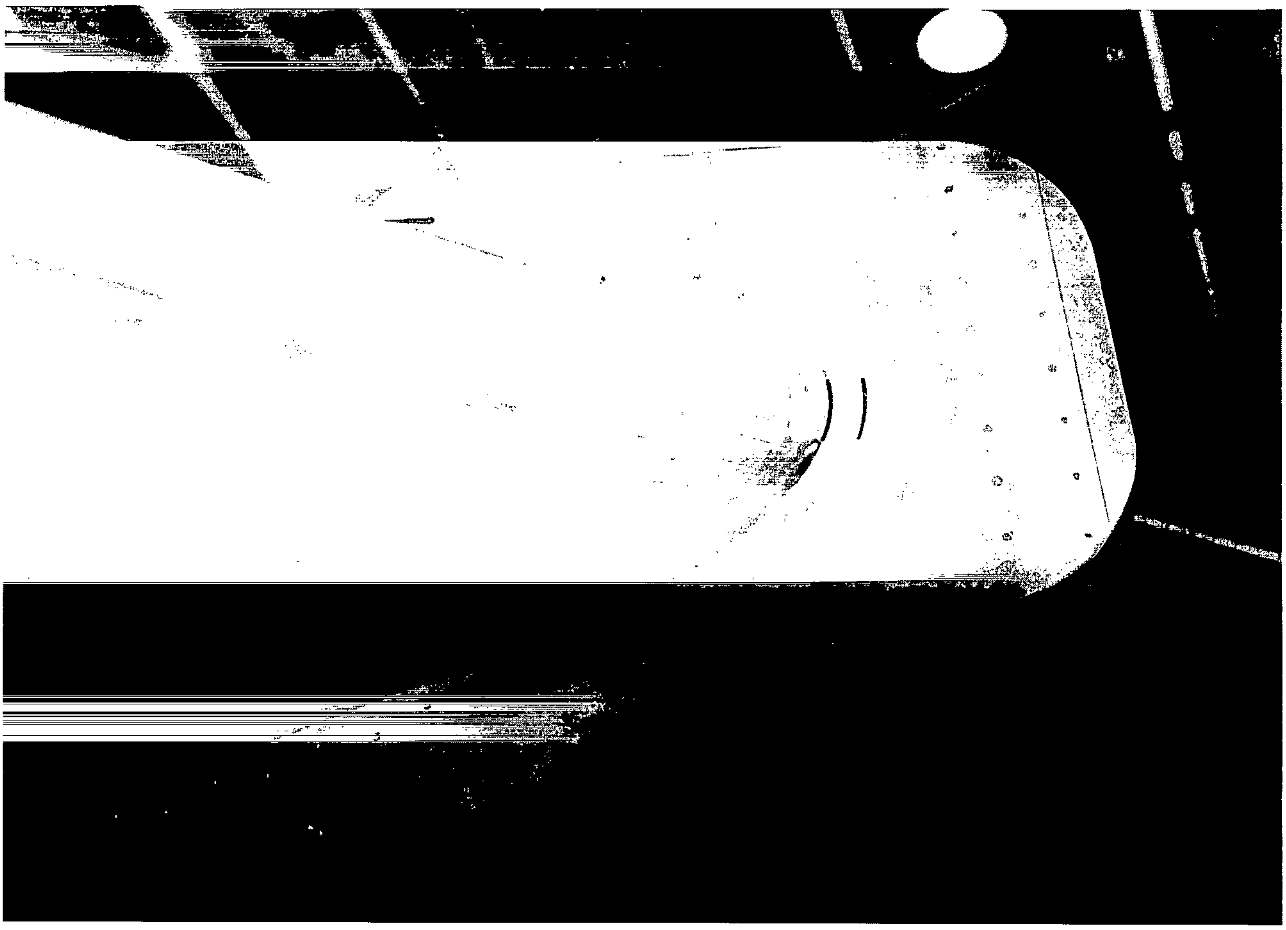
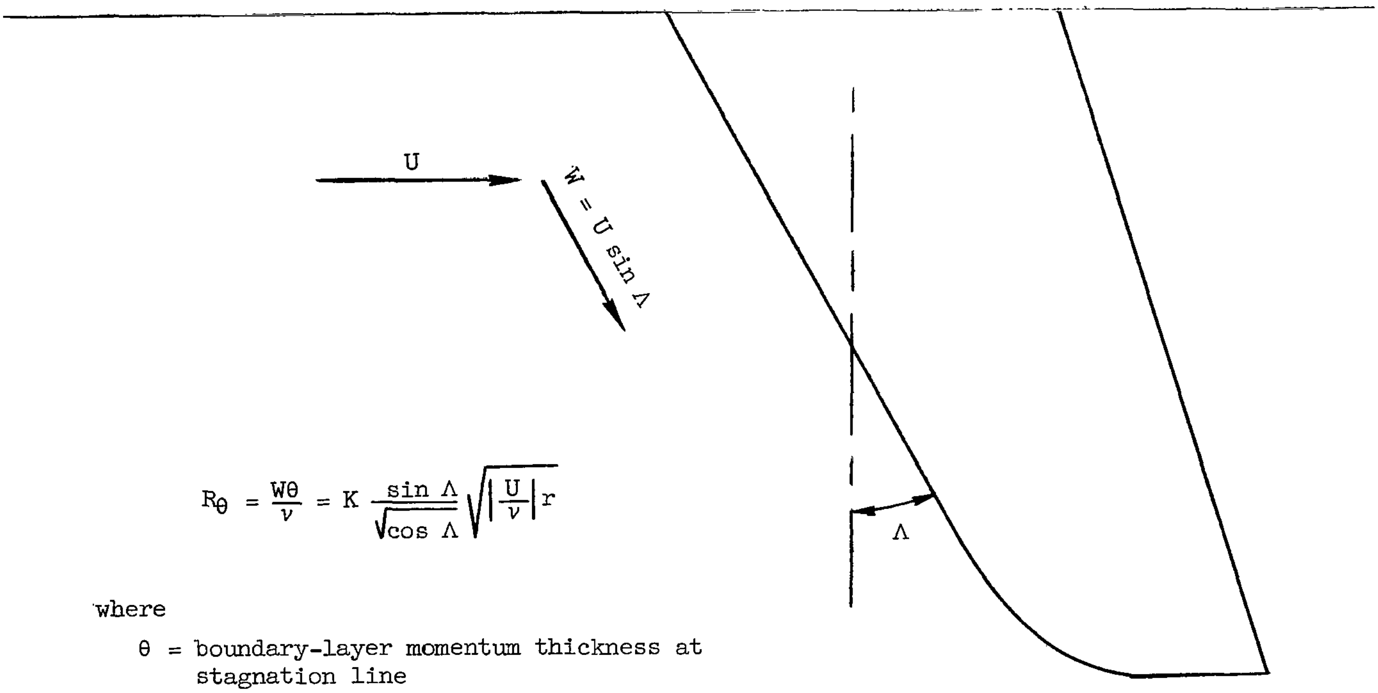


Figure 11.- Closeup photograph of leading-edge region of wing on X-21 airplane. (Courtesy of U.S. Air Force.) L-66-1175



$$R_\theta = \frac{W\theta}{\nu} = K \frac{\sin \Lambda}{\sqrt{\cos \Lambda}} \sqrt{\left|\frac{U}{\nu}\right| r}$$

where

θ = boundary-layer momentum thickness at stagnation line

r = leading-edge radius

Figure 12.- Reynolds number used for correlation of spanwise contamination.

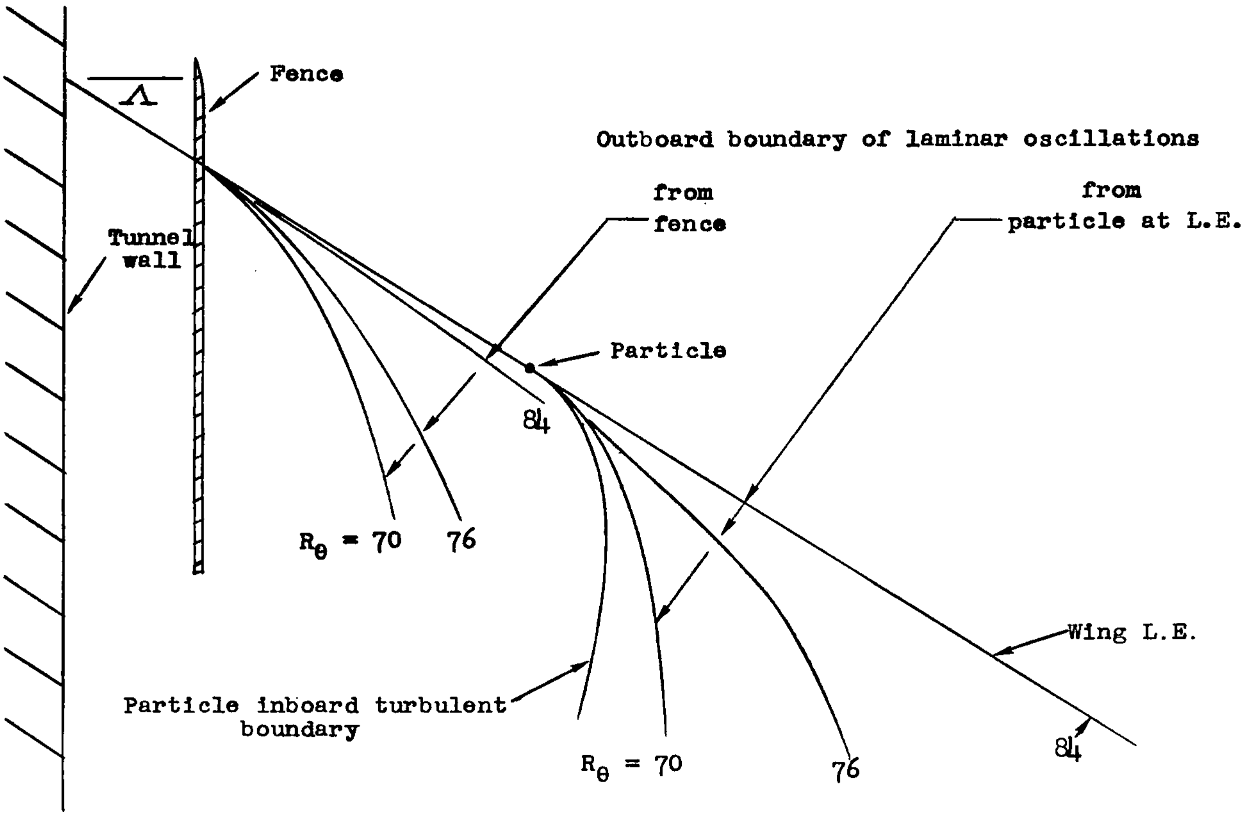


Figure 13.- Plan-view sketch of wing leading-edge region illustrating spanwise turbulent contamination. (Data by courtesy of U.S. Air Force.)

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