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SUMMARY

An analytical study was conducted to determine the boundary curve of combinations of altitude and descent rate for which abort during the touchdown phase of a lunar landing is possible.

The study determined the altitude required for a safe abort from the touchdown phase as a function of various parameters and operational considerations of the landing vehicle. These parameters included reaction attitude control power, abort (ascent) engine thrust, use of the attitude rockets for additional lift, and values at the start of abort of such quantities as pitch angle, pitch rate, and descent rate. The study established the minimum altitude threshold for aborting the landing based on these parameters, with consideration given to lags due to staging of the descent engine, pilot response, and engine firing delays. Only the first phase of the abort, that of arresting descent speed prior to touchdown, was considered.

The results of the study indicate that a wide variation in the boundary curves can be obtained with variations in the assumed parameters. Also, if proper consideration is given to pitch angle at abort and to reaction-control and abort engine thrust levels, an abort capability can be provided throughout the entire touchdown phase.

INTRODUCTION

The lunar-orbit-rendezvous method of accomplishing the lunar landing missions as conceived at the present time consists of a lunar excursion module (LEM) being separated from a command module in a circular lunar orbit and inserted into a coast trajectory which brings the LEM to an altitude at which a powered descent trajectory is initiated. The descent trajectory brings the LEM to an altitude, on the order of 1000 feet (305 meters), where its rate of descent is near zero in the vicinity of the landing site. After a short period of hover and translation to survey the landing area, the touchdown is accomplished. The various phases of the lunar landing mission are discussed in references 1 to 5.

Of major importance throughout the landing mission is the capability of safely aborting the mission and returning to the command module. The most critical region of the lunar landing mission from abort considerations appears to be that of the final descent from hover to touchdown. This criticality is due to the short times available for accomplishing an abort at such low altitudes.

Because of the critical nature of this region an analytical study was conducted to determine the boundary curves of rate of descent as a function of altitude for which safe aborts in this region are possible.

The study provides boundary curves which define the combinations of rate of descent and altitude for which safe aborts can be made and the regions where safe aborts are impossible and a landing must be attempted. The study covered a wide range of landing-vehicle parameters and conditions at the time of abort as well as operational considerations.

SYMBOLS

The units used for the physical quantities defined in this paper are given both in the U.S. Customary Units and in the International System of Units (SI). Factors relating these two systems of units are presented in reference 6.

g_e acceleration due to gravity on earth, 32.17 feet per second^2 (9.80 meters per second^2)

g_m acceleration due to gravity on moon, 5.32 feet per second^2 (1.62 meters per second^2)

h altitude, feet (meters)

h_1 altitude at end of pitch maneuver, feet (meters)

$\dot{h}_1$ rate of descent at end of pitch maneuver, $\dot{h}_0 + \dot{h}_R$, feet per second (meters per second)

I_Y pitch-axis moment of inertia, slug-foot^2, (kilogram-meter^2)

l moment arm of reaction control system (RCS) engines, feet (meters)

m	mass of landing vehicle, slugs (kilograms)
T_A	abort engine thrust, pounds (newtons)
T_{RCS}	RCS thrust, pounds (newtons)
t	time, seconds
θ	pitch angle, degrees
τ	total resultant thrust during pitch maneuver, pounds (newtons)

Subscripts:

A	relative to abort engine use
f	terminal conditions at end of abort
o	initiation of abort procedure
R	relative to pilot response time and lags in landing-vehicle staging and abort engine firing
θ_o	at time when $\theta = \theta_o$
$\theta_o/2$	at time when $\theta = \theta_o/2$
0	at time when $\theta = 0$

A dot over a quantity denotes first derivative with respect to time; two dots denote a second derivative with respect to time.

METHOD OF ANALYSIS

The following assumptions were made for the study:

1. An abort was considered completed when the rate of descent was nulled.

2. The descent engine of the landing vehicle was staged at the start of the abort.

3. The thrust axis of the abort (ascent) engine was required to be vertical (with respect to local horizontal) before the engine would be started. This requirement was

met by rotating the vehicle about its center of gravity to a vertical position, a procedure hereinafter referred to as rotating.

4. The mass of the landing vehicle throughout the abort procedure remained constant after staging.

5. The reaction attitude control system engines were fixed parallel to the fixed abort engine and provided some lift.

6. The reaction control system (RCS) engines were of the on-off type, and attitude was controlled through pure couples.

7. A constant reaction time of 3 seconds was necessary. (The reaction time consists of time required to stage the descent engine, pilot response time, and lag times in firing the abort engine.) Pilot response times are discussed in reference 7.

An abort from the touchdown phase of a lunar landing mission as assumed for this study is shown schematically in figure 1. During the time from $t = 0$ to $t = t_R$ the pilot of the landing vehicle reacts to the abort situation, stages the descent engine, and fires the RCS engines so that two of the engines provide a pure couple to rotate the vehicle to a vertical position while the other RCS engines provide some lifting acceleration. From t_R to $t_R + t_0$ the pilot is changing the vehicle pitch attitude to zero. The exact equations of motion for this region, t_R to $t_R + t_0$, are derived in equations (1) to (7) of the appendix. A digital computer was used to solve these equations for the results presented. However, in the absence of a computer, an approximate solution presented in the appendix (eqs. (8) to (15)) could be used for this region. At $t_0 + t_R$ the pilot starts the main abort engine, and at $t_A + t_0 + t_R$ the rate of descent of the vehicle has become zero. At this time, when the rate of descent has been nulled, the abort is considered complete. Equation (16) in the appendix is applicable to this region. Throughout the study both horizontal travel and velocity effects were neglected.

The landing vehicle characteristics used in the study are illustrated in figure 2. This figure indicates a two-stage vehicle with four pairs of RCS engines (designated as T_1, T_2, T_3, and T_4) on the ascent stage. These engines are assumed to provide both the pitch-angle correction moment and some lift during an abort and were of the on-off type. The pitch maneuver was performed by creating a pure couple by using the two RCS engines in the desired pitch plane to obtain an angular acceleration $\ddot{\theta}$ (given in eq. (1)). This acceleration was maintained until an angle $\theta_0/2$ was reached. At this time the thrust in the two engines was switched to counter the initial $\ddot{\theta}$ and thus bring the pitch angle to zero, at which time all RCS engines are assumed to be cut off and the main engine started. During this pitch maneuver four of the RCS engines are fired with T_1 and T_3 (fig. 2) producing the pitch acceleration $\ddot{\theta}$ while T_2 and T_4 provide some small amount of acceleration, depending on the pitch angle. The time required to complete the total pitch maneuver is given in equation (4).

Some analysis was made for comparison purposes of aborts which allowed firing the main engine without rotating and while rotating.

RESULTS AND DISCUSSION

For the present study the abort was considered in three distinct parts: the recognition of an abort situation and pilot reaction to it including staging, the pitch maneuver, and the firing of the abort engine until the rate of descent was nulled. The results are presented in this order. Figure 3 shows the rate of descent acquired and the altitude lost during the total time assumed for both pilot and vehicle to respond to the abort situation plotted as a function of this combined response time. These curves are given for various initial rates of descent. Figure 4 indicates the rate of descent that the landing vehicle acquires during the pitch maneuver as a function of the pitch angle at the time of the abort initiation for various levels of control capability. These curves could be obtained approximately by using equation (14) with $\dot{h}_o = 0$ and considering only the time required to rotate the vehicle to $\theta = 0$.

The RCS engines provide both lift and pitch acceleration. In figure 4 the total resultant thrust τ is equal to $2T_{RCS}$ where T_{RCS} is the thrust of each of the RCS engines. As is shown by the curves, the rate of descent acquired increases rapidly as the pitch acceleration decreases. This would indicate the need for a relatively large pitch acceleration. However, trade-offs must be made for various considerations, such as effect of $\ddot{\theta}$ on control of accelerations caused by cross-coupling effects.

Figure 5 gives the relation between the rate of descent at the end of the pitch maneuver and the distance fallen during the pitch maneuver, taking into account the velocity accumulated and the distance fallen during the combined pilot response time and landing-vehicle lag time (assumed to be 3 seconds) taken from figure 3. These values are shown in figure 5 for various rates of descent at the initiation of the abort procedures and RCS thrust. Approximations of these curves could be found by using equations (14) and (15). The rate of descent $\dot{h}_1$ consists of that velocity acquired during the 3 seconds of combined pilot response time and vehicle firing and staging lag times (which gives an initial rate of descent of 16 ft/sec (4.88 m/s) at the beginning of the pitch maneuver) plus the rate of descent $\dot{h}_{\theta_o \to 0}$ acquired during the pitch maneuver. These curves again emphasize the need for relatively large thrust engines for the RCS from an abort standpoint alone.

Figure 6 indicates the relation between $\dot{h}_A$ and the distance fallen while correcting $\dot{h}_A$ to zero for various accelerations of the main abort engine T_A/m. The quantity $\dot{h}_A$ is the total rate of descent acquired during the combined reaction time, the vehicle rotation-to-vertical maneuver, plus the initial rate of descent $\dot{h}_o$ at the initiation of the abort procedures. Figure 6, as would be expected, indicates that the larger the thrust capability of the abort engines the shorter the distance fallen. However, the size of the abort engine is governed by various considerations and thus a wide range of accelerations was covered.

By using figures 4 to 6 and the initial pitch angle and the characteristics of the landing vehicle, a boundary curve of $\dot{h}_o$ as a function of h can be established. Figures 4 to 6 can also be combined into a nomograph to obtain results for various combinations of the parameters and conditions covered. An example of the nomograph is shown in figure 7 for a particular vehicle configuration which was assumed to be the nominal landing vehicle for the present study and which had the following characteristics:

Angular acceleration, $\ddot{\theta}$	6 deg/sec^2
Thrust of RCS engines, T_{RCS}	100 lb (444.82 N)
Total resultant thrust of RCS engines during pitch maneuver, τ_{RCS}	200 lb (889.64 N)
Moment arm of RCS engines, l	5.5 ft (1.68 m)
Acceleration of main abort engine, T_A/m	$0.35g_e$

The method used to determine the total altitude that would be lost in the abort maneuver is also shown in figure 7. For this example the pure couple of two of the attitude rockets in the pitch plane produces an instantaneous pitch acceleration of 6 deg/sec^2 whereas the other two rockets, each having 100 lb (444.82 N) of thrust, produce a varying lift force depending on the pitch angle. The pitch angle assumed for this example was 40^o at the start of the abort and, as seen in figure 7(a), a vertical velocity of -24.6 ft/sec (-7.50 m/s) is accumulated by the landing vehicle while the pitch angle is corrected to point the main abort engine down. In addition to this rate of descent, the pilot and system lags (totaling 3 sec) assumed to accrue during the abort initiation increase the rate of descent to 40.6 ft/sec (12.37 m/s). Figure 7(b) reflects this 3-sec delay in both descent rate and distance fallen. If the line from the 40^o data point is extended from figure 7(a) to figure 7(b), it crosses curves for $\dot{h}_1$ as a function of h_1 at values of altitude loss that differ according to the rate of descent of the landing vehicle when the abort was initiated. The additional altitude drop occurring while the assumed $0.35g_e$ main abort engine completes the abort phase by reducing the vertical velocity to zero is shown in figure 7(c). A summary plot of the altitude values in figures 7(b) and 7(c) provides the boundary curve in figure 8 and indicates the general shape of the resultant abort boundary curves. Each point on the curve indicates a minimum altitude and an associated rate of descent for which abort can safely be made. Combinations of rate of descent and altitude which lie above this curve do not allow a safe abort whereas those below the curve do allow a safe abort. For example, for a rate of descent of 40 ft/sec (12.19 m/s) and a pitch angle of 40° the vehicle must be at or above 1050 ft (320.0 m) for a safe abort. Also shown in figure 8 is a curve representing the combinations of $\dot{h}_o$ and altitude for a free-fall trajectory, with no staging or thrusting assumed, which is limited by an impact velocity of -20 ft/sec (-6.10 m/s); that is, combinations of rate of descent and altitude for which the vehicle will hit the moon with rate of descent of 20 ft/sec. The impact velocity of -20 ft/sec is considered an upper limit of impact velocity for various safety and structural reasons. Thus the region between the two curves in figure 8 gives the combinations of $\dot{h}_o$ and altitude for which abort is prohibited and a safe or survivable landing is not probable. This region requires study for means of making the two curves overlap.

Figure 9 shows the effects of variations in parameters from those for the nominal vehicle. Curve 1 in figure 9 represents the nominal-vehicle boundary curve. Curves 2 and 3 show the effect on the nominal boundary curve of either decreasing the lag time from 3 seconds to 1.5 seconds or increasing the angular acceleration of the vehicle from 6 deg/sec^2 to 25 deg/sec^2. These curves indicate that the changes in these parameters provide essentially the same increase in abort capability. Curve 4 indicates the effect of changing the acceleration of the abort engine from $0.35g_e$ to $0.75g_e$. This increase in thrust decreases the minimum abort altitude at $\dot{h} = 0$ to 200 ft (60.9 m). Curve 5 represents the case where the pitch angle is 0° at the time of abort, the descent engine is staged immediately, and the abort engine fired with no need for rotating. This, of course, is the minimum boundary curve for the ascent-engine thrust levels and lags chosen. To have $\theta = 0$ throughout the entire letdown phase would require translation rockets. The free-fall curve shown in figure 8 is repeated in figure 9. Figure 9 shows that large variations in the curves can be caused by suitable parameter variations. The maximum possible variation caused by restricting the pitch angle to 0° is given by curve 5. This restriction permits aborts or survivable free falls throughout the entire touchdown phase.

By using the assumed nominal landing-vehicle configuration, an example of a typical landing of this vehicle can be defined. The landing vehicle would remain below the nominal boundary curve shown in figure 9 with an angle equal to or less than 40° down to an altitude of approximately 310 ft (94.49 m), but at altitudes below 310 ft, the pitch angle must be 5° or less. This allows an abort capability down to the altitude at which free fall to the surface is possible under the assumed impact velocity of -20 ft/sec (-6.10 m/s) for the unstaged vehicle.

For comparison purposes a limited study of some operational procedures other than staging, rotating, and then firing the abort engines was conducted. These procedures were (1) staging with rotation while firing main abort engines and (2) staging and firing main abort engines at the pitch angle existing at abort initiation with no vehicle rotation until rate of descent is nulled. The results obtained for these procedures are shown in figure 10 along with the nominal-landing-vehicle curve and the free-fall curve. For the procedure where the pitch angle at abort initiation is held and the main engine is fired without rotation, figure 10 indicates an improvement in abort capability, providing an abort or safe landing capability throughout the entire touchdown phase. The maneuver which includes staging immediately and firing the abort engines while rotating the vehicle to a vertical position provides an even greater improvement in abort capability than that provided by holding the angle constant. This procedure would appear to be operationally the most desirable of those studied. The equations derived in the appendix apply equally well to these particular exceptions to the procedure assumed for the study. The restriction, requiring the engines to be vertical before firing, that is generally used in the present report provides conservative answers relative to the two exceptions studied.

Although the main study included herein dealt with aborts in the final letdown from hover to touchdown, it is logical to extend the rate-of-descent and altitude parameters to include those encountered during the powered-descent phase of a lunar landing. An example of such an extension is shown in figure 11. The landing assumed was a gravity turn from 50 000 ft (15 240 m) to approximately 2500 ft (762 m) with a constant descent-engine thrust of 10 500 lb (46 706.1 N), again using the nominal landing vehicle. Since the values of pitch angle and rate of descent are continuously changing, the results could not be presented in the same way as in figures 8, 9, and 10. Figure 11 shows altitude as a function of time in the powered-descent trajectory for the assumed gravity turn and shows the results of substituting the angles and rates of descents along the trajectory into the equations presented in the appendix. Thus the solid curve represents the actual altitudes along the landing trajectory and the dashed curve represents the altitude required to abort safely. The hatched area in the figure indicates a critical region from abort considerations, since in this region the required altitude for abort is greater than the altitude for the assumed gravity-turn landing trajectory. Considerations such as illustrated in figure 11 need to be included in the decision concerning the type of landing required.

CONCLUDING REMARKS

An analysis of descent-rate and altitude boundaries for abort from the touchdown approach of a lunar landing has been made for combinations of abort conditions and vehicle capability.

Various trade-offs of the parameters affecting control are necessary to give abort capability throughout the entire region from hover to touchdown. For a fixed main abort engine with constant thrust, the primary trade-off is the capability of the landing vehicle to point the abort engine downward rapidly enough to satisfy existing attitude errors, descent rate, and altitude margins without the loss of overall stability. To accomplish this, the range of attitude control power is important, since high attitude control power is required at abort whereas very low attitude control power is required for precise control during rendezvous docking.

An additional trade-off is the orientation of the reaction control system (RCS) engines. If the RCS rockets all point in the same direction as the main abort engine, as was assumed in this study, the additional lift enhances to some small degree the abort capability of the vehicle. A pure couple of two RCS engines provides attitude control while the remaining rockets provide thrust and thus expand the range of altitudes and velocities for which abort is possible.

Results also indicate that decreasing the combined vehicle-pilot lag time from 3 to 1.5 seconds or increasing the vehicle angular acceleration increases to some extent the abort capability of the vehicle. It can also be concluded that although doubling the main engine thrust gives substantial increases in abort capability it still does not provide abort capability during the entire landing mission. The study also indicated that, in order to provide abort capability through the entire touchdown phase, the vehicle must be capable of firing the abort engines while tilted and during rotation to a vertical position; if this is not possible or practical the landing vehicle must be kept erect, dictating the need for translation rockets. Therefore, changes in combinations of the landing-vehicle parameters will give the abort capability necessary.

Extension of the investigation to the powered-descent phase indicates that in the latter part of the trajectory the altitude-velocity relation may become critical from an abort standpoint and should be investigated more completely.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., March 14, 1966.

APPENDIX

DERIVATION OF EQUATIONS OF MOTION

The angular acceleration about the pitch axis is

$$\ddot{\theta} = \frac{2T_{RCS}l}{I_Y} \qquad \left(\ddot{\theta} \text{ positive clockwise}\right)\rightarrow \tag{1}$$

where T_{RCS} is the thrust of each RCS engine (see fig. 2). By using basic constant-acceleration equations, the time required to change the initial pitch angle θ_o to $\theta_o/2$ is given by the equation

$$t_{\theta_o/2} = \sqrt{-\frac{\theta_o}{\ddot{\theta}}} \tag{2}$$

(Eq. (2) and the rest of this appendix are based on the assumption of no pitch rate at the start of abort. The calculations are planar in nature with only the pitch plane considered.) Similar consideration of time required to change $\theta_o/2$ to 0, where $\dot{\theta}_o \neq 0$, shows that

$$t_{\frac{\theta_o}{2}\rightarrow 0} = \sqrt{\frac{\theta_o}{\ddot{\theta}}} \tag{3}$$

and, therefore,

$$t_{\theta_o\rightarrow 0} = 2\sqrt{\left|\frac{\theta_o}{\ddot{\theta}}\right|} \tag{4}$$

While the landing vehicle is correcting any pitch angle it is falling toward the moon, and the thrust of the RCS engines is providing, in addition to a pitch moment, some acceleration in the local vertical direction. This acceleration gives a resultant acceleration which may be expressed

$$\ddot{h} = \frac{\tau}{m}\cos\theta - g_m \qquad \left(\ddot{h} \text{ positive upward}\right)\rightarrow \tag{5}$$

where τ is the total resultant thrust during vehicle rotation. (In the study no main-abort-engine thrust was assumed during the rotation; however, τ in eq. (5) could include any main-engine thrust used during the pitch maneuver.)

APPENDIX

In equation (5), θ is continuously varying as the vehicle is being rotated to a vertical position and for constant acceleration can be represented by

$$\theta = \frac{\ddot{\theta}t^2}{2} + \dot{\theta}t + \theta_o \tag{6}$$

Thus, equation (5) can be written

$$\ddot{h} = \frac{\tau}{m}\cos\left(\frac{\ddot{\theta}t^2}{2} + \dot{\theta}t + \theta_o\right) - g_m \tag{7}$$

Equations (1) to (7) are exact and are used in a digital-computer program to obtain the results presented in the present report. However, very good approximate solutions for the first and second integrals of equation (7) can be developed. These approximate solutions provide simplicity and reasonable accuracy. In order to obtain an approximate solution a series expansion for

$$\cos\left(\frac{\ddot{\theta}t^2}{2} + \dot{\theta}t + \theta_o\right)$$

was used. The expansion as used is

$$\cos\left(\frac{\ddot{\theta}t^2}{2} + \dot{\theta}t + \theta_o\right) = 1 - \frac{\left(\frac{\ddot{\theta}t^2}{2} + \dot{\theta}t + \theta_o\right)^2}{2!} + \frac{\left(\frac{\ddot{\theta}t^2}{2} + \dot{\theta}t + \theta_o\right)^4}{4!} - \dots \tag{8}$$

It was found that for the range of conditions studied only the first three terms of the expansion are required for a sufficiently accurate approximation. Based on this requirement, the terms of equation (7) were expanded and grouped and the equation for vertical acceleration produced by thrusting during the rotation process becomes

$$\ddot{h} = \frac{\tau}{m}\left[1 - \frac{\theta_o^2}{2} + \frac{\theta_o^4}{4!} - \theta_o\dot{\theta}t\left(1 - \frac{\theta_o^2}{3!}\right) + \frac{t^2}{2}\left(-\ddot{\theta}\theta_o - \dot{\theta}^2 + \frac{\ddot{\theta}\theta_o^3}{3!} + \frac{\dot{\theta}^2\theta_o^2}{2}\right)\right.$$

$$+ \frac{\dot{\theta}t^3}{2}\left(\frac{\ddot{\theta}\theta_o^2}{2} - \ddot{\theta} + \frac{\dot{\theta}^2\theta_o}{3}\right) + \frac{t^4}{2}\left(\frac{\ddot{\theta}^2\theta_o^2}{8} - \frac{\ddot{\theta}^2}{4} + \frac{\ddot{\theta}\theta_o\dot{\theta}^2}{2} + \frac{\dot{\theta}^4}{12}\right) + \frac{\ddot{\theta}\dot{\theta}t^5}{2}\left(\frac{\dot{\theta}^2}{3!} + \frac{\theta_o\ddot{\theta}}{4}\right)$$

$$\left. + \frac{\ddot{\theta}^2t^6}{16}\left(\dot{\theta}^2 + \frac{\ddot{\theta}\theta_o}{3}\right) + \frac{\ddot{\theta}^3\dot{\theta}t^7}{(2)(4!)} + \frac{\ddot{\theta}^4t^8}{(16)(4!)}\right] - g_m \tag{9}$$

APPENDIX

Integration of this equation gives the following expressions for the altitude rate

$$\begin{aligned}\dot{h} = \int_0^t \ddot{h}\, dt = t\Bigg\{\frac{\tau}{m}\Bigg[&\left(1 - \frac{\theta_o^2}{2} + \frac{\theta_o^4}{4!}\right) - \frac{\theta_o \dot{\theta} t}{2}\left(1 - \frac{\theta_o^2}{3!}\right) + \frac{t^2}{6}\left(\frac{\ddot{\theta}\theta_o^3}{3!} + \frac{\dot{\theta}^2\theta^2}{2} - \ddot{\theta}\theta_o - \dot{\theta}^2\right) \\ &+ \frac{\dot{\theta}t^3}{8}\left(\frac{\ddot{\theta}\theta_o^2}{2} - \ddot{\theta} + \frac{\dot{\theta}^2\theta_o}{3}\right) + \frac{t^4}{10}\left(\frac{\ddot{\theta}^2\theta_o^2}{8} - \frac{\ddot{\theta}^2}{4} + \frac{\ddot{\theta}\theta_o\dot{\theta}^2}{2} + \frac{\dot{\theta}^4}{12}\right) + \frac{\ddot{\theta}\dot{\theta}t^5}{12}\left(\frac{\dot{\theta}^2}{3!} + \frac{\theta_o\ddot{\theta}}{4}\right) \\ &+ \frac{\ddot{\theta}^2 t^6}{112}\left(\dot{\theta}^2 + \frac{\ddot{\theta}\theta_o}{3}\right) + \frac{\ddot{\theta}^3\dot{\theta}t^7}{(16)(4!)} + \frac{\ddot{\theta}^4 t^8}{(144)(4!)}\Bigg] - g_m\Bigg\} + \left(\dot{h}_o + \dot{h}_R\right)\end{aligned} \tag{10}$$

and for the altitude

$$\begin{aligned}h = \int_0^t \dot{h}\, dt = \frac{t^2}{2}\Bigg\{\frac{\tau}{m}\Bigg[&\left(1 - \frac{\theta_o^2}{2} + \frac{\theta_o^4}{4!}\right) - \frac{\theta_o \dot{\theta} t}{3}\left(1 - \frac{\theta_o^2}{3!}\right) + \frac{t^2}{12}\left(\frac{\ddot{\theta}\theta_o^3}{3!} + \frac{\dot{\theta}^2\theta^2}{2} - \ddot{\theta}\theta_o - \dot{\theta}^2\right) \\ &+ \frac{\dot{\theta}t^3}{20}\left(\frac{\ddot{\theta}\theta_o^2}{2} - \ddot{\theta} + \frac{\dot{\theta}^2\theta_o}{3}\right) + \frac{t^4}{30}\left(\frac{\ddot{\theta}^2\theta_o^2}{8} - \frac{\ddot{\theta}^2}{4} + \frac{\ddot{\theta}\theta_o\dot{\theta}^2}{2} + \frac{\dot{\theta}^4}{12}\right) + \frac{\ddot{\theta}\dot{\theta}t^5}{42}\left(\frac{\dot{\theta}^2}{3!} + \frac{\theta_o\ddot{\theta}}{4}\right) \\ &+ \frac{\ddot{\theta}^2 t^6}{448}\left(\dot{\theta}^2 + \frac{\ddot{\theta}\theta_o}{3}\right) + \frac{\ddot{\theta}^3\dot{\theta}t^7}{(72)(4!)} + \frac{\ddot{\theta}^4 t^8}{(720)(4!)}\Bigg] - g_m\Bigg\} + \left(\dot{h}_o + \dot{h}_R\right)t + \dot{h}_o t_R + h_o\end{aligned} \tag{11}$$

To determine the altitude rate developed and the distance traveled by the landing vehicle during the pitch maneuver, equations (10) and (11) must be used in two steps. The first step is the period between the start of the pitch maneuver ($\theta = \theta_o$) and the point where $\theta = \theta_o/2$. During this step $\ddot{\theta}$ is directed so that θ decreases, and $\dot{\theta}_o$ is assumed to be zero. The time required to accomplish this step is given by equation (2). The second step is the period between $\theta = \theta_o/2$ and $\theta = 0$ where the angular acceleration is equal to, but opposite in direction from, the original acceleration. This shows the angular rate generated during the first step to be zero where θ becomes zero. At this point the RCS engines were assumed to be shut off. The time for this period is given by equation (3).

With these considerations taken into account and with $\dot{\theta}_o = 0$ for the time from t_R to $t_R + t_{\theta_o/2}$ and $\dot{\theta}_o = \ddot{\theta}\sqrt{-\frac{\theta_o}{\ddot{\theta}}}$ for the time from $t_R + t_{\theta_o/2}$ to $t_R + t_0$, equations (10) and (11), respectively, can be written

$$\dot{h}_{\theta_o/2} = \sqrt{-\frac{\theta_o}{\ddot{\theta}}}\left[\frac{\tau}{m}\left(1 - 0.358\theta_o^2 + 0.0237\theta_o^4\right) - g_m\right] + \dot{h}_o + \dot{h}_R \tag{12}$$

$$h_{\theta_o/2} = -\frac{\theta_o}{2\ddot{\theta}}\left[\frac{\tau}{m}\left(1 - 0.425\theta_o^2 + 0.0318\theta_o^4\right) - g_m\right] + \left(\dot{h}_o + \dot{h}_R\right)\sqrt{-\frac{\theta_o}{\ddot{\theta}}}$$

$$+ \dot{h}_o t_R - \frac{g_m}{2} t_R^2 + h_o \tag{13}$$

for the time from 0 to $t_{\theta_o/2}$ and can be written

$$\dot{h}_0 = \sqrt{-\frac{\theta_o}{\ddot{\theta}}}\left[\frac{\tau}{m}\left(1 - 0.0583\theta_o^2 + 0.0007\theta_o^4\right) - g_m\right] + \dot{h}_{\theta/2} \tag{14}$$

$$h_0 = -\frac{\theta_o}{2\ddot{\theta}}\left[\frac{\tau}{m}\left(1 - 0.0729\theta_o^2 + 0.0010\theta_o^4\right) - g_m\right] + \dot{h}_{\theta_o/2}\sqrt{-\frac{\theta_o}{\ddot{\theta}}} + h_{\theta_o/2} \tag{15}$$

for the time from $t_{\theta_o/2}$ to t_0. Combining the respective altitude rates and altitude equations from equations (12) to (15) gives the final form of the equations for the total pitch-maneuver period. These equations are

$$\dot{h}_0 = \sqrt{\left|\frac{\theta_o}{\ddot{\theta}}\right|}\left[\frac{\tau}{m}\left(2 - 0.416\theta_o^2 + 0.0244\theta_o^4\right) - 2g_m\right] + \dot{h}_o + \dot{h}_R \tag{16}$$

and

$$h_0 = \left|\frac{\theta_o}{2\ddot{\theta}}\right|\left[\frac{\tau}{m}\left(4 - 1.214\theta_o^2 + 0.0802\theta_o^4\right) - 4g_m\right]$$

$$+ 2\left(\dot{h}_o + \dot{h}_R\right)\sqrt{-\frac{\theta_o}{\ddot{\theta}}} + \dot{h}_o t_R - \frac{g_m t_R^2}{2} + h_o \tag{17}$$

APPENDIX

Thus equations (16) and (17) give the rate of descent and altitude at the end of the rotation of the landing vehicle $\left(t = t_R + t_0\right.$ in fig. $\left.1\right)$. They include the altitude and rate of descent which were developed during the response time.

The region from $t = t_R + t_0$ to $t = t_R + t_0 + t_A$ consists of the time required to correct any rate of descent at the end of the pitch maneuver, given by equation (16), to zero or some acceptable value by using the abort engine. In order to find the distance traveled during this correction, the following simple equation is used.

$$\dot{h}_f^2 = h_0^2 + 2\left(\frac{T_A}{m} - g_m\right)h_A \qquad (18)$$

where T_A/m is the abort-engine acceleration capability. Thus, by combining the values of altitude h_0 and h_A obtained from equations (17) and (18) and using the initial rates of descent $\dot{h}_o$, the boundary curves presented herein are obtained.

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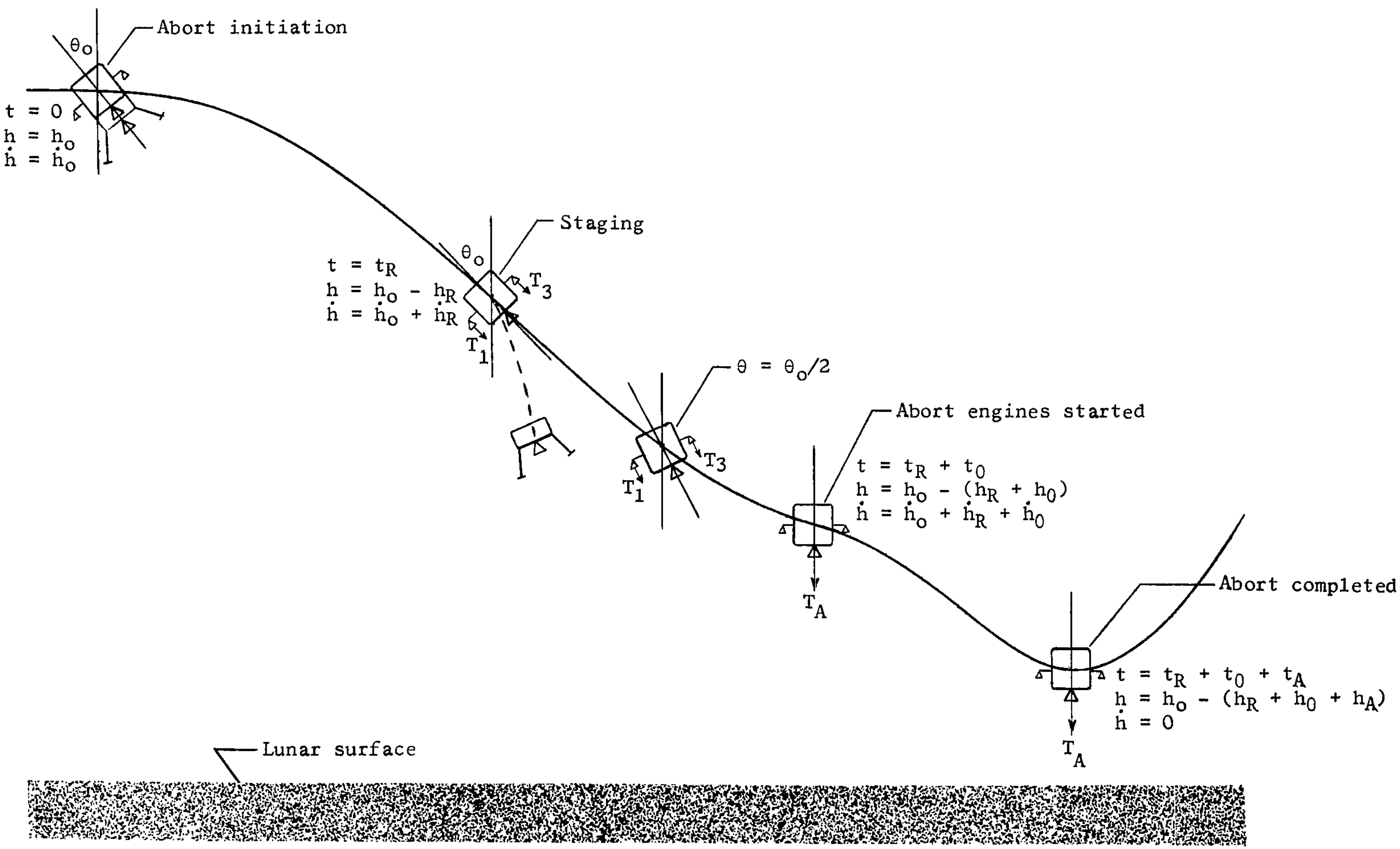


Figure 1.- Abort profile for landing vehicle with time, altitude, and rate of descent indicated.

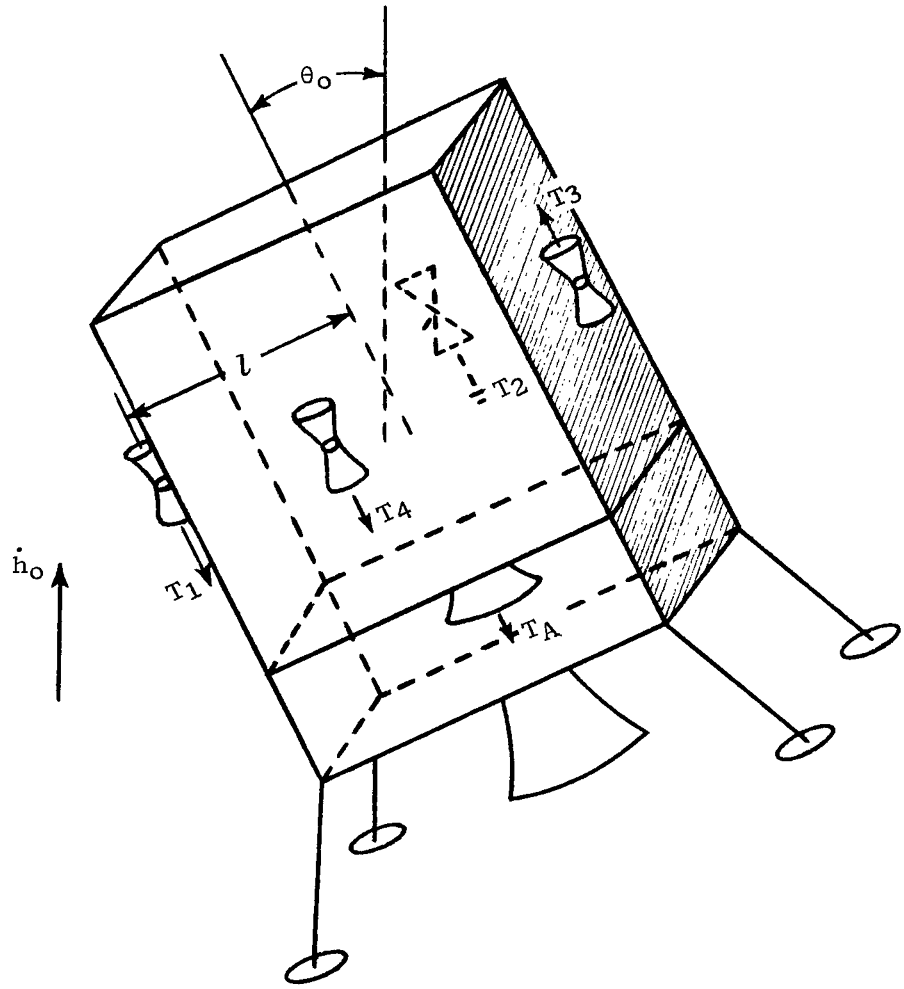


Figure 2.- Landing-vehicle configuration.

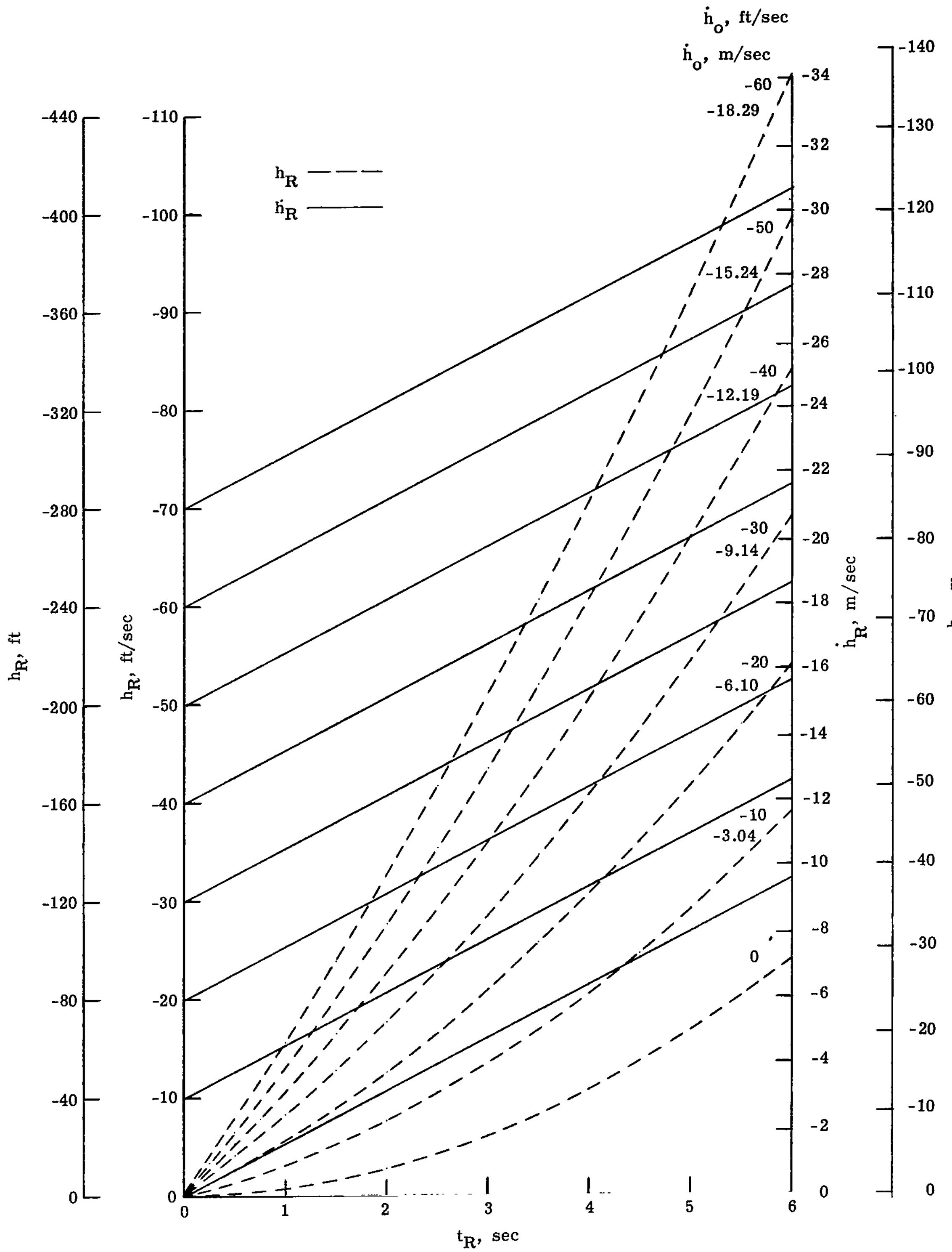


Figure 3.- Distance fallen and rate of descent acquired as a function of pilot response time plus system lag time for various initial rates of descent.

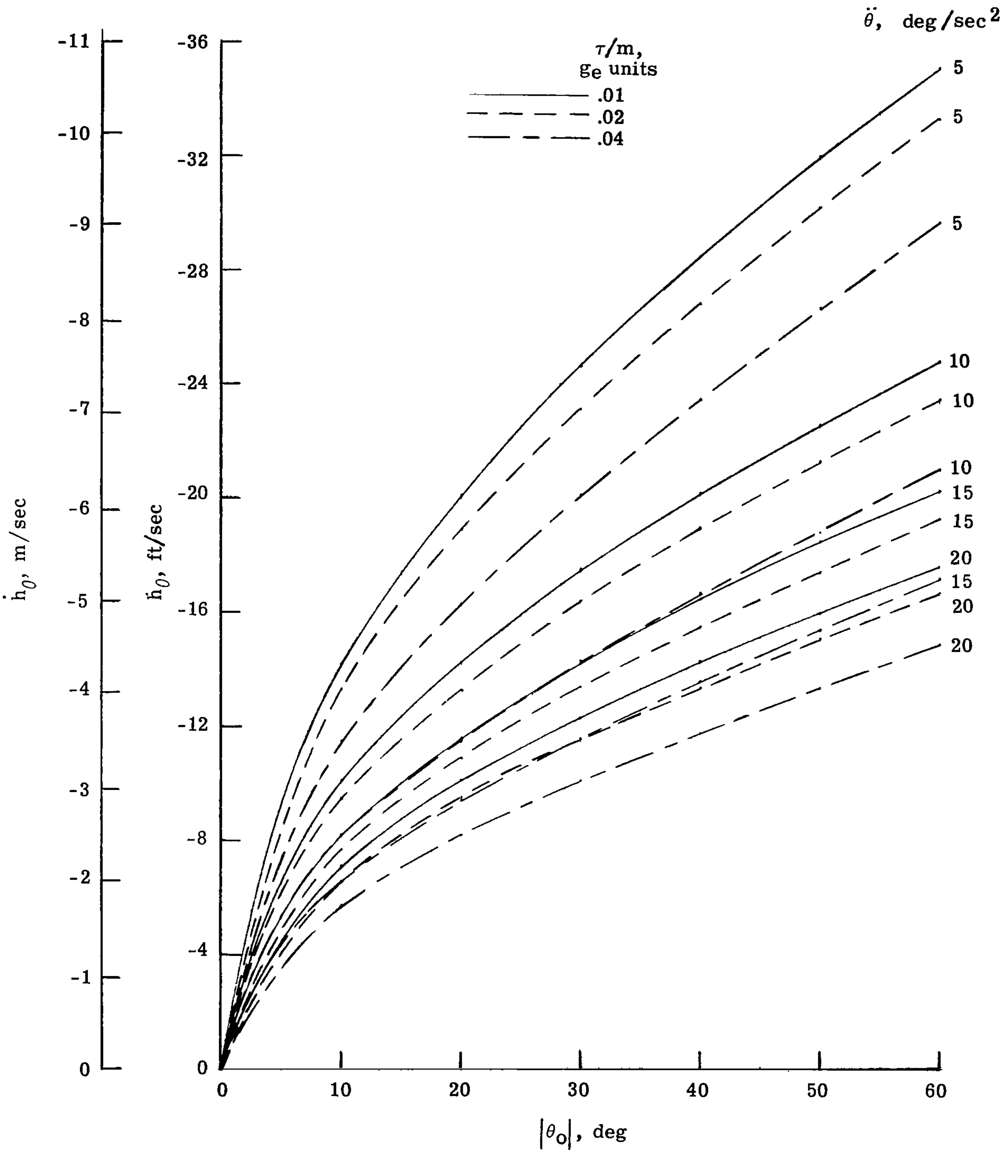


Figure 4.- Rate of descent acquired during correction for θ_0 (not including $\dot{h}_0$ or $\dot{h}_R$) for total RCS angular acceleration levels.

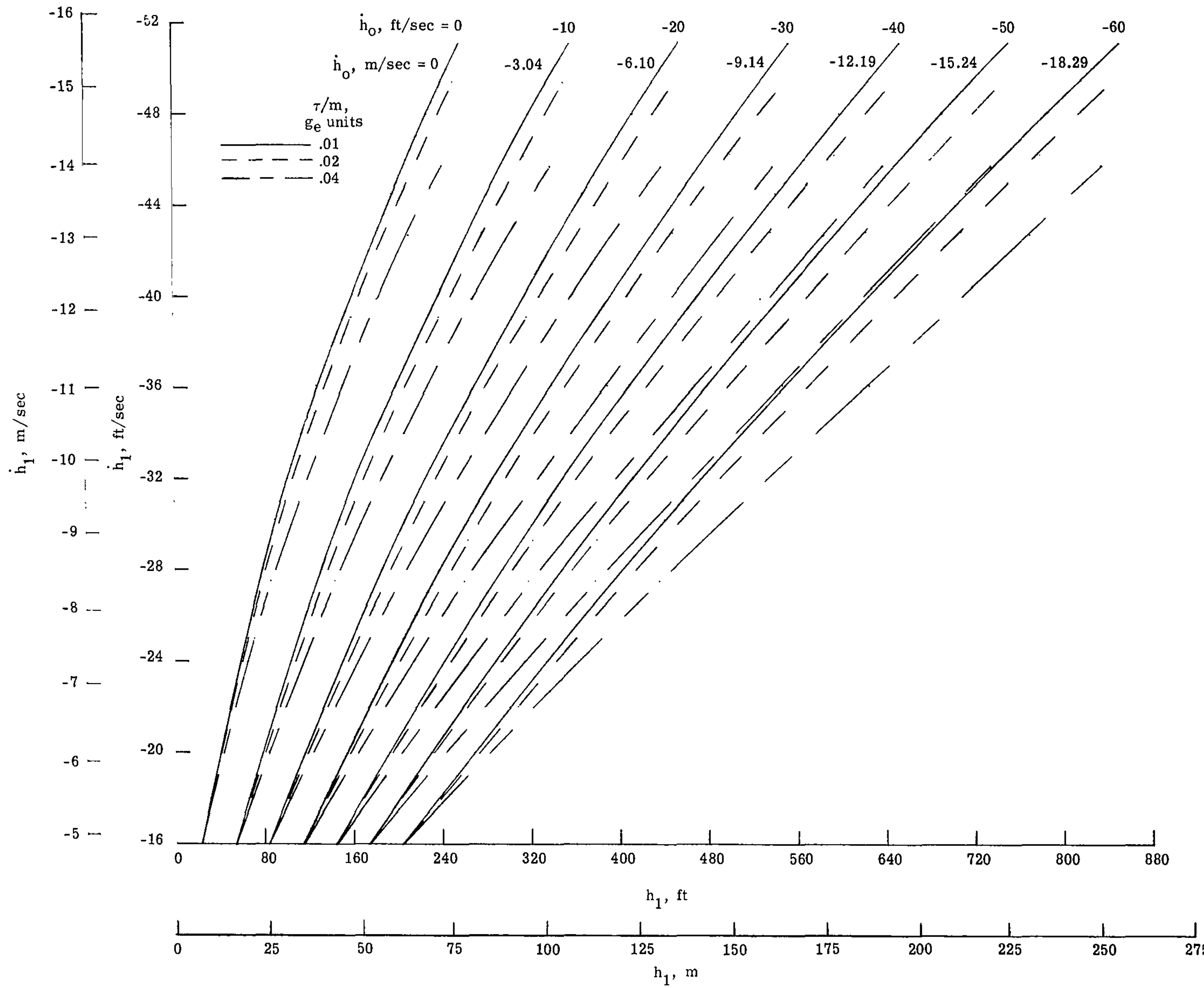


Figure 5.- Rate of descent at end of pitch maneuver $\left(\dot{h}_1 = \dot{h}_0 + \dot{h}_R\right)$ as a function of distance fallen while correcting for θ_0 plus distance fallen during system and human lag time for various initial rates of descent.

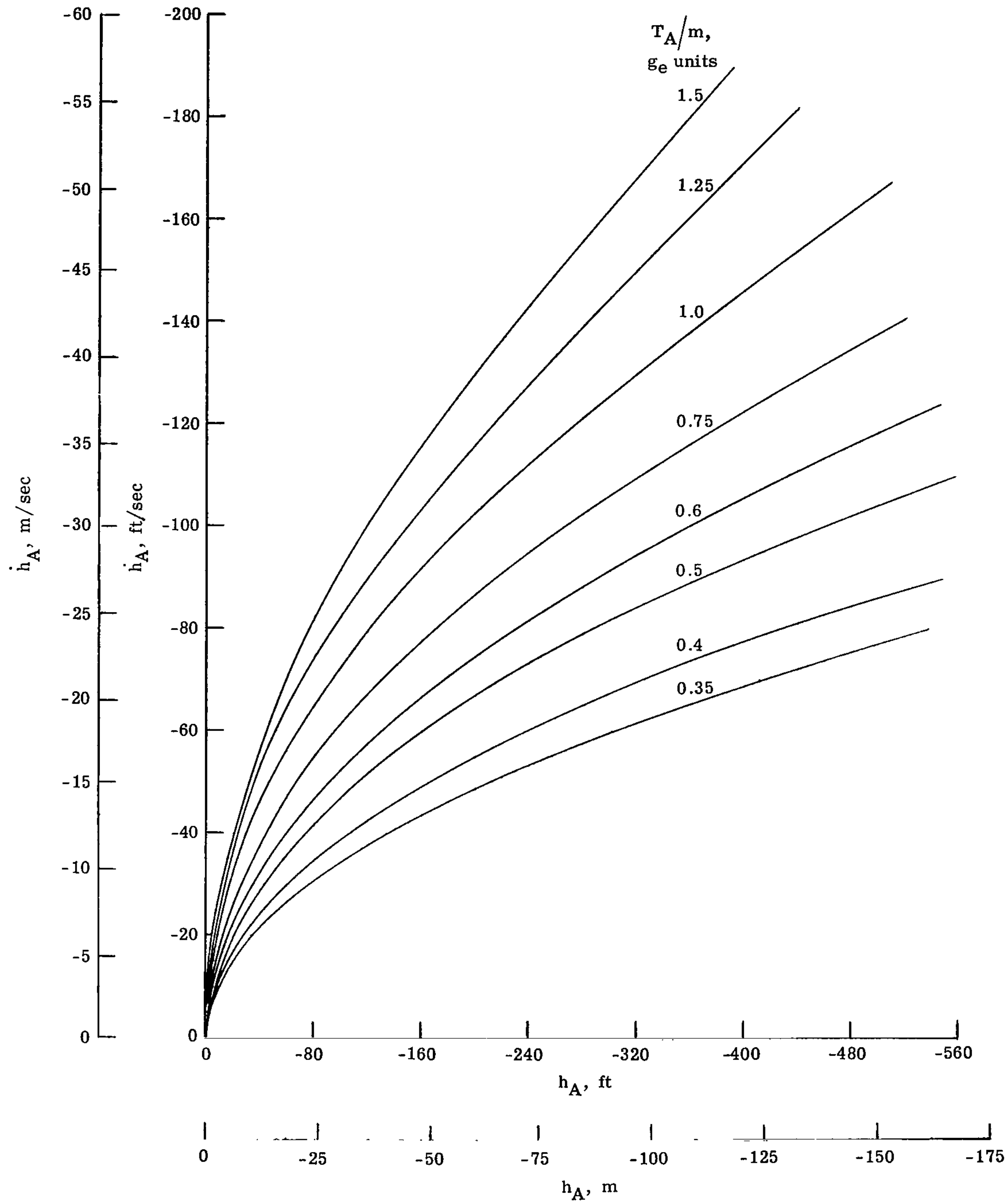
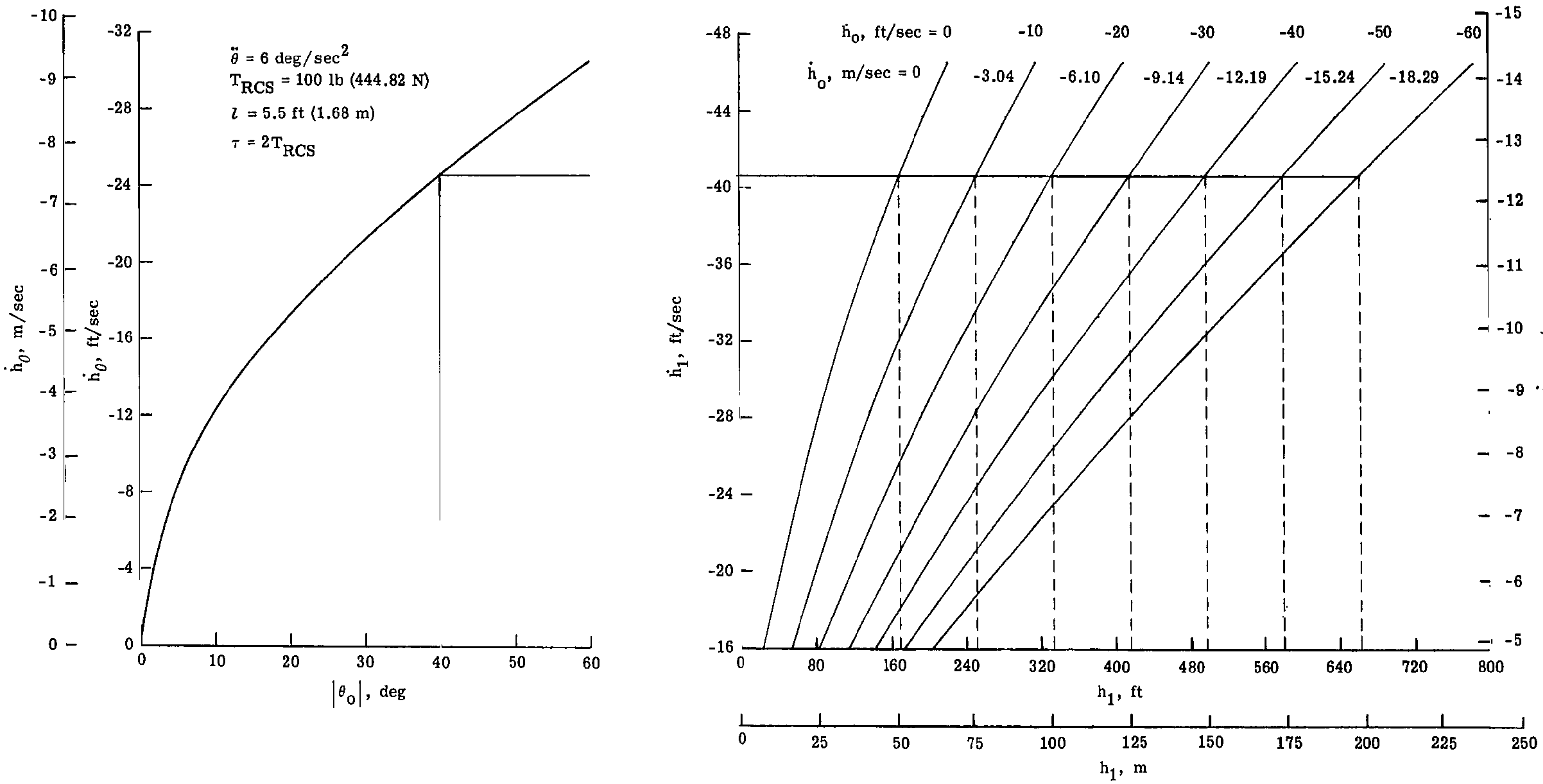


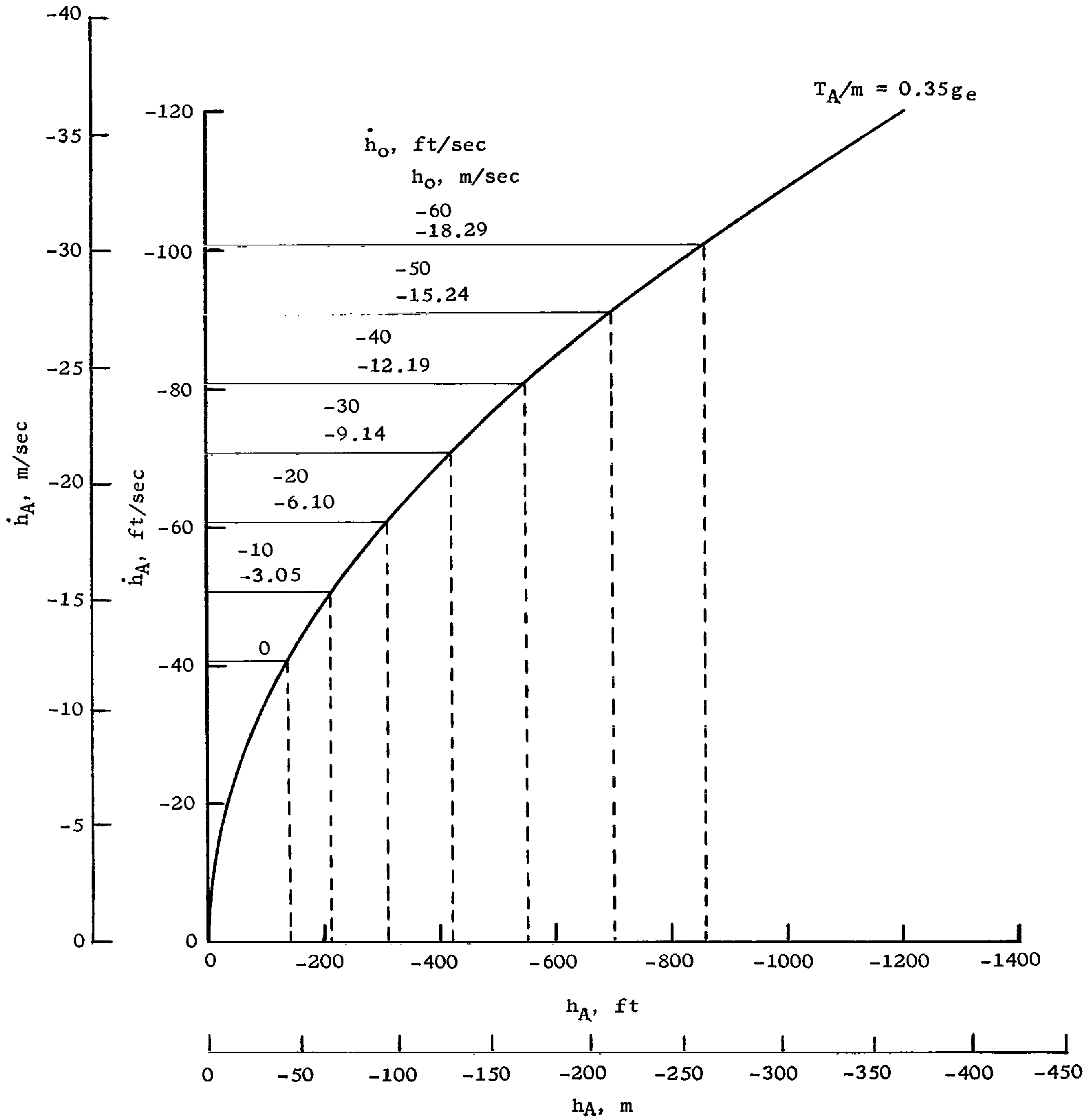
Figure 6.- Altitude lost while nulling $\dot{h}_A$ with abort engines for various T_A/m.



(a) $\dot{h}_0$ obtained while correcting for θ_0.

(b) Distance fallen during correction for θ_0 and during reaction time. $(\dot{h}_1 = \dot{h}_R + \dot{h}_0)$

Figure 7.- Curves used to determine the abort boundary conditions, assuming a nominal landing-vehicle configuration.



(c) Distance fallen while bringing $\dot{h}_A$ to zero. $\left(\dot{h}_A = \dot{h}_o + \dot{h}_1\right)$

Figure 7.- Concluded.

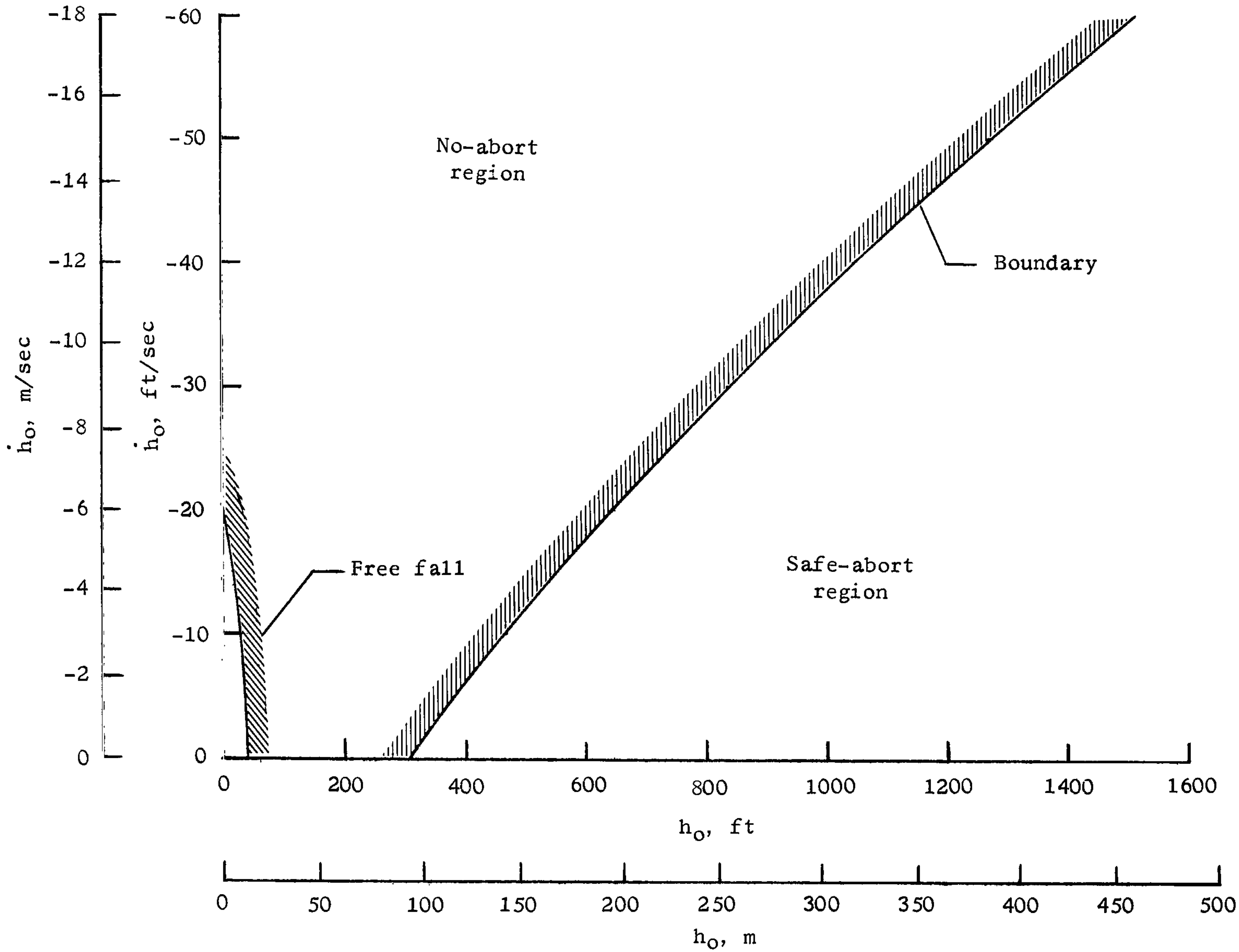


Figure 8.- Nominal-landing-vehicle configuration abort boundary conditions.

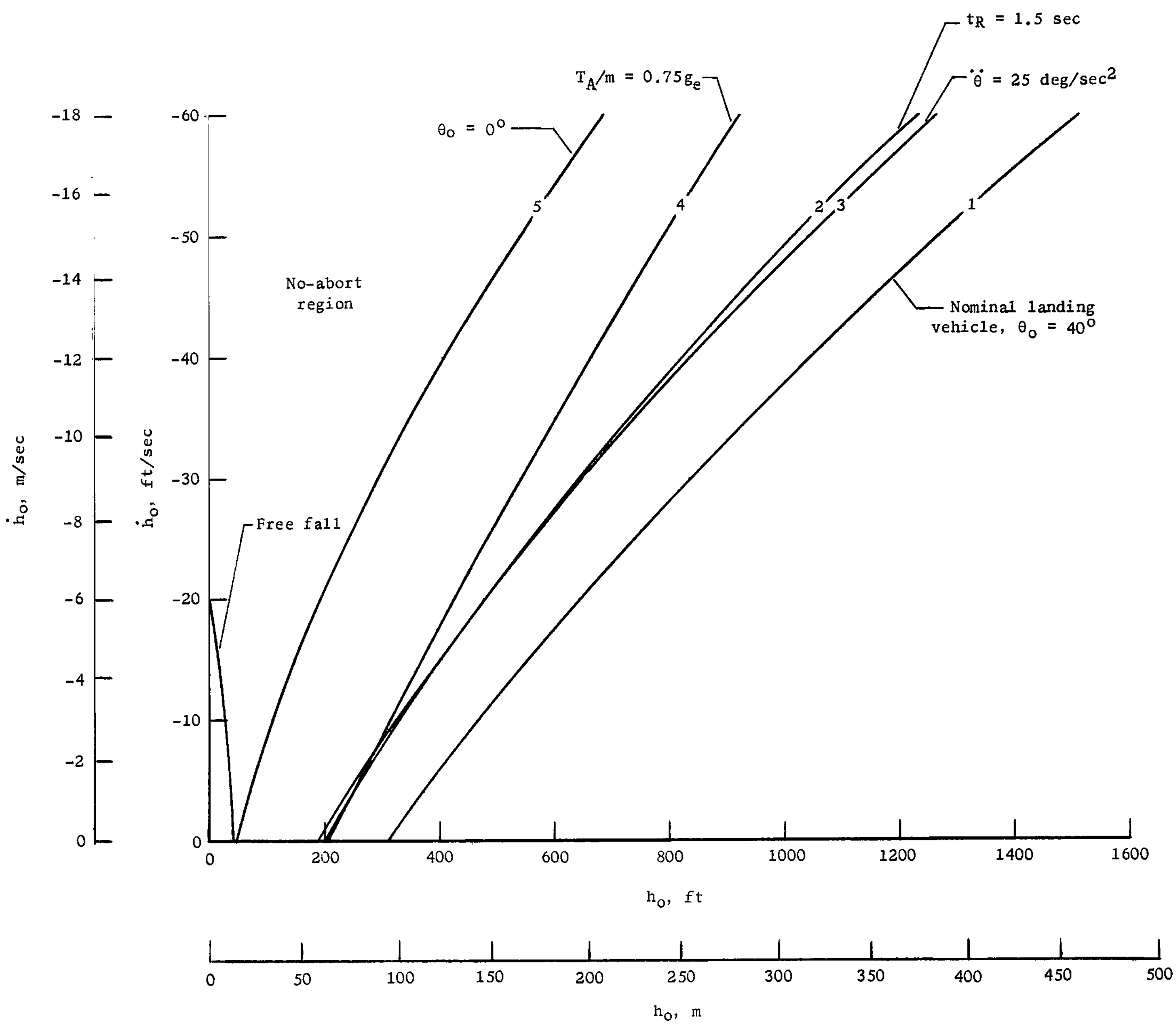


Figure 9.- Effects on nominal boundary curve of changes in landing-vehicle parameters.

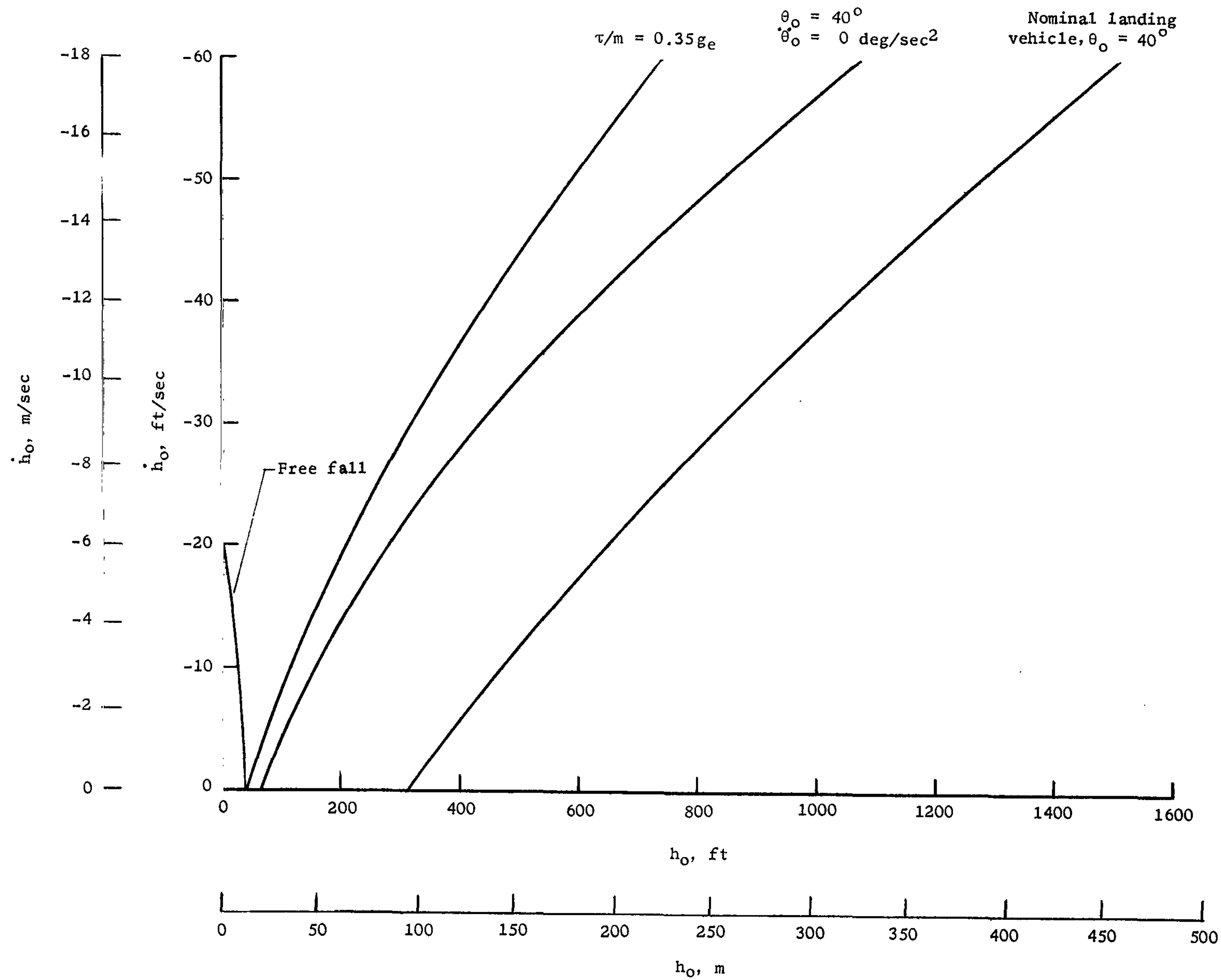


Figure 10.- Variation in abort boundary curves for changes in operational procedure.

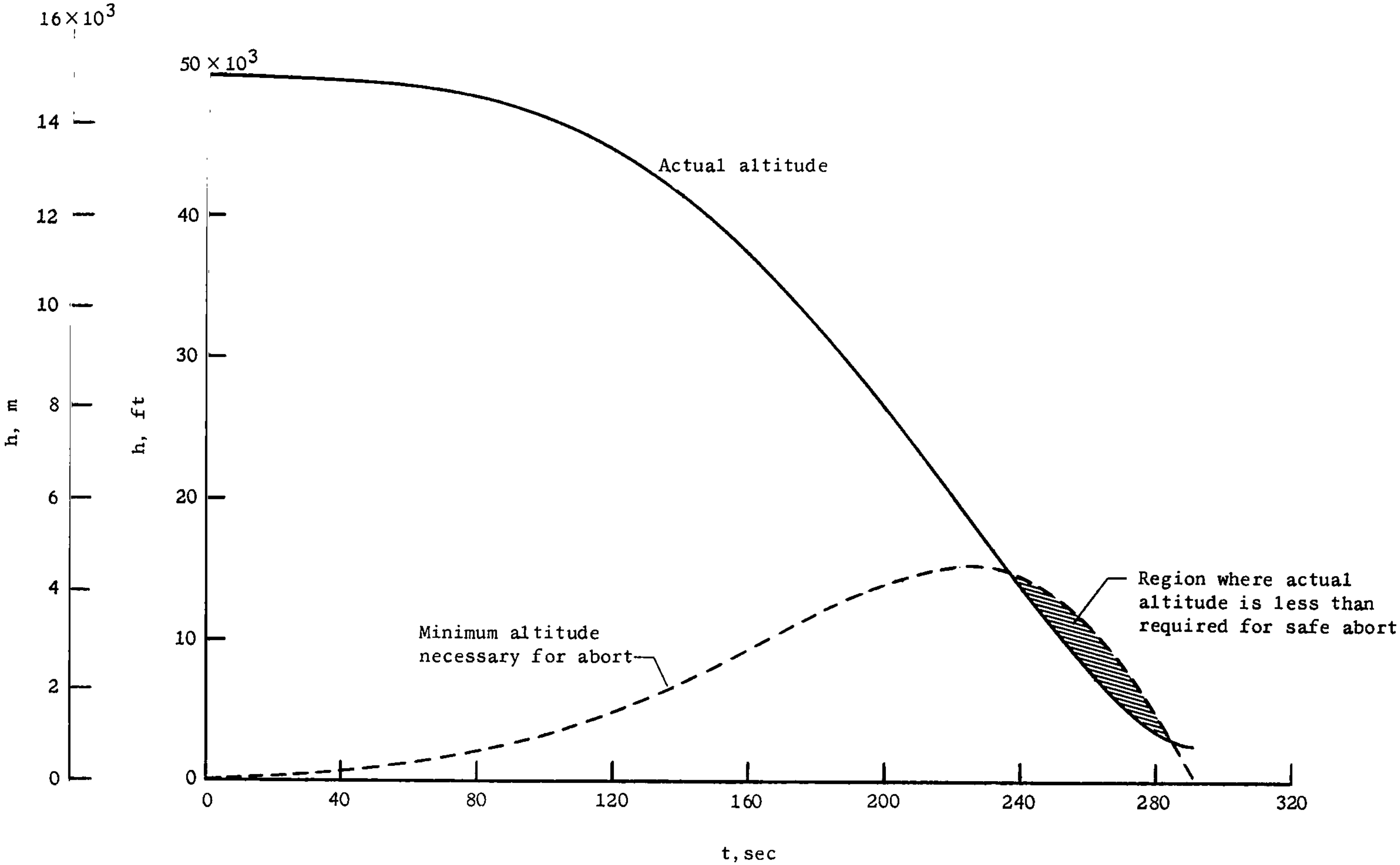


Figure 11.- Comparison of altitude for typical gravity-turn landing trajectory and necessary altitude for abort.

NASA-Langley, 1966 L-4327

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