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FLUTTER OF MULTIPLE-STREAMWISE BAY RECTANGULAR PANELS AT LOW SUPERSONIC MACH NUMBER

by D. R. Kobett

Prepared by
MIDWEST RESEARCH INSTITUTE
Kansas City, Mo.
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1966

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PREFACE

This report covers research sponsored by the National Aeronautics and Space Administration, Langley Research Center, under Contract No. NAS1-4900. Mr. D. R. Kobett, project leader, was responsible for overall program direction. Mr. D. I. Sommerville carried out the requisite modifications and improvements of the computer program.

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NOTATION

a	chord of one panel
b	span of one panel
B	half wavelength in Fourier expansion of spanwise deflection shape (Eq. (3))
$B_{n,u}$	coefficient in Fourier expansion of spanwise deflection shape (Eq. (3))
c_∞	velocity of sound in undisturbed stream
E	modulus of elasticity of panel material
g	structural damping coefficient
i	$\sqrt{-1}$
I	identity matrix
k	$\omega a/U$; nondimensional flutter frequency
L	number of streamwise panels
ℓ	streamwise mode number
M	Mach number
N	number of spanwise panels
n	spanwise mode number
p	aerodynamic pressure
Q_m	flutter vector
$q_{\ell,n}$	elements of flutter vector
q	dynamic pressure in undisturbed stream
q_j	modulus of j^{th} component of flutter vector (Eq. (8); Table II)
t	time

U	velocity in undisturbed stream
w	panel deflection
x	nondimensional streamwise coordinate (dimensional coordinate = xa)
y	nondimensional spanwise coordinate (dimensional coordinate = yb)
Z	$\tau^3 E/q(1-\nu^2)$; bending stiffness - dynamic pressure parameter
θ_j	phase angle of j^{th} component of flutter vector (Eq. (8); Table II)
μ	$\tau\rho_s/\rho$; mass ratio parameter
ν	Poisson's ratio
ρ	mass density in undisturbed stream
ρ_s	mass density of panel material
τ	ratio of panel thickness to chord
ϕ	spanwise coordinate in sine series expansion of spanwise deflection shape (Eq. (3))
Φ_m, Φ_j	streamwise deflection function
Ψ_n	spanwise deflection function
ω	dimensional flutter frequency

SUMMARY

A flutter analysis is conducted of simply supported flat panel arrays made up of one spanwise bay and one, two and three streamwise bays. The arrays are of finite span in contrast with previous analyses which treat either the two-dimensional case or the case of infinite span divided into equal bays. Use is made of linear plate theory and complete, linearized, three-dimensional, inviscid aerodynamic theory. The effects of structural damping and aerodynamic and structural coupling are included. Solutions to the flutter equations are obtained using the Galerkin technique.

Critical flutter boundaries are computed for panels of aspect ratio 2 and 4, Mach number 1.35, and structural damping coefficient equal to 0.01. The thickness required to prevent flutter is shown to increase with increasing number of streamwise bays. The effect of increasing the number of bays from one to two is considerably larger than the effect of adding a third panel to a two bay array. The effect is also larger for the larger aspect ratio.

The flutter mode shapes of the multiple bay arrays are dominated by structurally uncoupled modes. It is concluded that aerodynamic coupling is the primary mechanism of destabilization.

I. INTRODUCTION

Panel flutter is defined as the oscillatory instability induced in a thin panel in an airstream by interaction between the airstream and panel motion. The phenomenon is of interest to missile and aircraft designers where the vehicle skin is made up of thin panel arrays whose sustained oscillation would be objectionable. Extensive theoretical and experimental effort has therefore been invested in attempts to clarify the flutter phenomenon and, in particular, to develop a method(s) for predicting its occurrence. Although these efforts have met with some success, many relevant questions remain to be answered. The present report is concerned with one of these questions, namely, the effect of multiple streamwise bays on the flutter characteristics of an array of flat panels in the low supersonic Mach number regime.

Early investigators concentrated on the analysis of panel arrays with one bay in the streamwise direction. However, panels ordinarily occur in arrays with multiple streamwise bays; so the later configuration soon came under study. Panel arrays with multiple streamwise bays have been investigated by Rodden[2]*, Dowell[3], Zeydel [4,5] and Lock[6]. References 2,3,4 and 6 are analyses of two-dimensional arrays (i.e., arrays composed of infinite aspect ratio panels), while the case of an array of infinite span separated into uniform finite bays is treated in [5]. All of the analyses use linear plate theory to describe the structure and all but [3] use exact linearized aerodynamic theory for the pressure formulation (Ref. 3 uses the piston theory approximation).

Rodden[2] and Dowell[3] both showed that the number of streamwise bays has little effect on the flutter of pinned edge panels for Mach number greater than 1.56. In [4] Zeydel used the Galerkin technique to compute flutter boundaries (conditions at the onset of flutter) for two-dimensional pinned edge panel arrays at lower Mach numbers. He found increasing the number of streamwise bays to have a pronounced destabilizing effect (i.e., causes flutter to occur for a wider range of conditions). The analysis excluded the effects of structural coupling and structural damping. Lock[6] included structural coupling and structural damping in his analysis and computed flutter boundaries for one and two bay clamped edge arrays and two bay pinned edge arrays throughout the Mach number range 1.2 to 2.0. He found that the flutter of clamped edge arrays was insensitive to the number of streamwise bays while his results for the pinned edge arrays agreed with Zeydel's in indicating a pronounced effect at the lower Mach numbers.

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* Numbers in brackets refer to the references.

In [5] Zeydel extended his previous analysis to include panel arrays with:

(a) Infinite span divided into finite bays,

(b) Edge conditions varying between pinned and clamped, and

(c) One and two streamwise bays.

Structural coupling and structural damping were included and flutter boundaries were computed for Mach number 1.35 and aspect ratios of 4 and infinity. His infinite aspect ratio results, interpreted in terms of aluminum panels at sea level, show the flutter of clamped edge panels to be insensitive to the number of streamwise bays, in agreement with Lock; the pinned edge panels show a pronounced sensitivity in agreement with his (Zeydel's) previous results. The panels of aspect ratio 4 were found to have flutter characteristics very similar to those of the infinite aspect ratio panels.

The purpose of the present analysis is to gain further insight into the practical aspects of panel flutter by investigating arrays of finite span and moderate aspect ratio where three-dimensionality is emphasized. Finite panel arrays with one spanwise bay and one, two and three streamwise bays are analyzed. Exact linearized aerodynamic theory and a Galerkin approach are used and the effects of structural coupling and structural damping are included. Flutter boundaries are computed for simply supported arrays at Mach number 1.35 for panel aspect ratios of 2 and 4.

The analytical technique is described in detail in [1] and therefore only briefly discussed here in part II. The numerical procedure is described in part III and the results are presented and discussed in part IV.

II. ANALYSIS

The physical systems to be analyzed consist of finite arrays of similar flat panels exposed on one side to a uniform supersonic stream of Mach number M (see Fig. 1).* The panel edges are free to rotate but restrained against transverse deflection. To make the aerodynamics tractable it is assumed that the array is bordered by an infinite impermeable surface extending upstream and that acoustic pressures on the underside of the array are neglected. Analysis details are completely described in [1] and are therefore only briefly outlined here.

The dynamic equation of motion for the transverse deflection, w, is obtained using linear plate theory and exact, linearized aerodynamic theory.

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* All tables and figures appear in the Appendix.

The deflection is then approximated by

$$w = e^{i\omega t} \sum_{\ell=m_1}^{m_2} q_{\ell,n} \Phi_\ell(x) \psi_n(y) \tag{1}$$

where ω = oscillation frequency

t = time

$\Phi_\ell(x)$ = natural vibration mode shape of a multi-bay beam satisfying the boundary conditions on the spanwise edges of the panel array (ℓ refers to mode number)

$\psi_n(y)$ = natural vibration mode shape of a multi-bay beam satisfying the boundary conditions on the streamwise edges of the panel array (n refers to mode number)

$q_{\ell,n}$ = weighting coefficients

Observe that the streamwise (x) deflection shape is approximated by a summation of functions $\Phi_\ell(x)$ and the spanwise (y) deflection shape by the single function $\psi_n(y)$. Previous experience has shown that the single function representation for the spanwise shape is satisfactory for the determination of stability boundaries.

An approximate solution is sought using the Galerkin technique. The procedure leads to a set of equations

$$\sum_{\ell=m_1}^{m_2} q_{\ell,n} \left\{ ZA_{\bar{m},\ell} - \mu k^2 B_{\bar{m},\ell} \right\} + \int_0^L \int_0^N p(x,y) \Phi_{\bar{m}}(x) \psi_n(y) dy dx = 0 \tag{2}$$

$$\bar{m} = m_1 \ldots m_2$$

where $A_{\bar{m},\ell}$ and $B_{\bar{m},\ell}$ are arrays which depend only on the number of streamwise and chordwise panels and the boundary conditions on the panel edges.

k = reduced frequency* = $\omega a/U$

μ = mass ratio parameter = $\tau\rho_s/\rho$

Z = dynamic pressure - stiffness parameter = $\tau^3 E/q(1-\nu^2)$

The term $p(x,y)$ is the pressure on the deflected panels which is to be obtained from exact, linearized, three-dimensional, inviscid aerodynamic theory. A completely general solution is not available for $p(x,y)$ and use is therefore made of a result from [7] which gives the perturbation pressure $p_{u,m}$ on a surface in harmonic motion with arbitrary chordwise deflection $\Phi_m(x)$, and sinusoidal spanwise deflection $\sin u\pi\phi/B$. The total perturbation pressure on the surface is obtained from a superposition of $p_{u,m}$ terms by expanding the spanwise deflection shape in a sine series,

$$\psi_n = \sum_u B_{m,u} \sin u\pi\phi/B \tag{3}$$

The expansion (3) gives ψ_n flanked by periodic reflections. By properly choosing the wavelength B the reflections can be uncoupled aerodynamically, thus achieving the effect of an isolated finite panel array.**

The approach described above ultimately yields a formulation for the transverse deflection w expressible in matrix form as

$$\left\{ ZI + \left[1/(1+ig)\right] \left[\mu k^2 C_{\overline{m},m} + D_{\overline{m},m}\right] \right\} \left\{Q_m\right\} = \left\{0\right\} \tag{4}$$

In (4), I is the identity matrix and Q_m is the "flutter" vector with elements equal to the $q_{\ell,n}$ of equation (1). The matrices $C_{\overline{m},m}$ and $D_{\overline{m},m}$ are functions of:

Number of chordwise and spanwise panels

Panel boundary conditions

Mach number

Aspect ratio

Reduced frequency, k .

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* See nomenclature list for detailed definition of notation.

** For a detailed discussion see [1] pages 10 and 60.

Solutions to the eigenvalue equation (4) represent points of neutral stability, i.e., conditions under which the panels oscillate harmonically under the influence of the air stream. Loci of these points of neutral stability define the flutter boundaries which are the objective of the present analysis.

The techniques of the numerical computation of the flutter boundaries are described in the next section.

III. NUMERICAL PROCEDURE

Flutter boundaries are obtained by the following procedure. The general physical situation is defined first by specifying the following:

(1) Number of spanwise panels*

(2) Number of chordwise panels

(3) Panel boundary conditions*

(4) Mach number*

(5) Aspect ratio

(6) Magnitude of structural damping*

Equation (4) can then be expressed functionally as

$$\left\{ ZI + \left[\mu k^2 P_{\overline{m},m} + R_{\overline{m},m}(k) \right] \right\} \left\{ Q_m \right\} = \left\{ 0 \right\} \tag{5}$$

where $P_{\overline{m},m}$ is a known complex array and $R_{\overline{m},m}$ is a complex array depending only on reduced frequency k. The criterion for a nontrivial solution of (5) is

$$\mathrm{Det} \left\{ ZI + \left[\mu k^2 P_{\overline{m},m} + R_{\overline{m},m}(k) \right] \right\} = 0 \tag{6}$$

---

* A fixed value is assigned to this item in the present analysis. The computer program can accommodate a range of values within the restrictions defined in [1].

A special algorithm [1] is used to compute the pairs of real μ and Z and the corresponding vectors Q_m which satisfy (6) for successively selected values of reduced frequency k . These pairs of values are then plotted in the $1/\mu$ - $Z^{1/3}$ plane.* Distinct continuous stability boundaries are constructed by connecting the plotted points on the basis of continuity of reduced frequency and modal content of the vector Q_m .** Multiple stability boundaries are thereby obtained for each physical situation analyzed (Figs. 3-8).

Each boundary determined in the above manner corresponds to a locus of points of neutral stability, tacitly assumed to divide regions of flutter (unstable) from regions of no flutter (stable). For the closed boundaries such as the second mode loop in Fig. 3, the unstable region is inside the loop. For the open boundaries the unstable region is to the left.

A given combination of material and altitude defines a hyperbola in the generalized parameter plane because

$$Z^{1/3} = \frac{\rho}{\rho_s}\left\{\frac{2E}{\rho M^2 c_\infty^2(1-\nu^2)}\right\}^{1/3}\left(\frac{1}{1/\mu}\right) \tag{7}$$

The intersection of the hyperbola with a stability boundary identifies the minimum panel thickness*** required to prevent flutter in the mode characterized by the associated Q_m . The right-most intersection is critical since it determines the minimum thickness required to completely suppress flutter.

In carrying out the above procedure for determining stability boundaries it is also necessary to specify the following:

Spanwise mode ψ_n

Range of the parameter $1/\mu$

Number of terms in the expansion of ψ_n (Eq. (3))

Chordwise modes Φ_m

Reduced frequency k

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* The conventional $1/\mu$ - $Z^{1/3}$ plane is selected for displaying the boundaries in preference to the optional μ - Z plane.

** The computer program provides Q_m in polar form normalized on the maximum element to facilitate analysis of the modal content.

*** The term "thickness" is consistently used throughout this report to denote the nondimensional ratio of panel thickness to chord.

Permanent values were assigned to the first three items as follows. The fundamental beam vibration mode (π frequency) was selected for the spanwise mode, ψ_n, on the basis of previous experience. Solutions of (6) were obtained throughout the range $0.002 \leq 1/\mu \leq 0.5$ to insure complete definition of boundaries in the range of physical interest $0.01 \leq 1/\mu \leq 0.2$. Twenty non-zero terms were used in the expansion of ψ_n (Eq. (3)) after initial comparative computations showed a variation of less than 0.1 per cent in computed results using 15 and 20 terms.* (The number of terms used has negligible effect on computation time.)

Convergence of the vector Q_m is the criterion used for selection of the chordwise modes, Φ_m, to be used in the analysis. The modes were selected to insure infallible identification of the critical envelope of boundaries and definition of the envelope location to a precision commensurate with the precision of the input parameters.

Reduced frequencies were selected by observing the development of the stability boundaries as the computations progressed. Typically, initial computations were made for a set of frequencies ranging from about 0.2 to 2.0 in steps of 0.4. These computations yielded points on the boundaries which guided the selection of frequencies for subsequent calculations. By repeating the above loop the boundaries were gradually defined throughout the range of interest of the parameter $1/\mu$.

IV. NUMERICAL RESULTS

Stability boundaries were computed for arrays of simply supported panels for Mach number 1.35 and a structural damping coefficient of 0.01. Six configurations were analyzed, namely, arrays with one, two and three chordwise bays, each with alternate panel aspect ratios of 2 and 4. The stability boundaries are shown in Figs. 3 to 8** and the critical (right-most) envelopes for the two aspect ratios are superimposed in Figs. 9 and 10 to illustrate the effect of the number of chordwise bays. The broken line curve in the latter two figures is the hyperbola defined by aluminum panels at sea level flight conditions.

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* Recall that the spanwise deflection shape, ψ_n, is expanded in a sine series (Eq. (3)). The expansion in effect yields an odd periodic function with ψ_n as one-half cycle aerodynamically isolated from its antisymmetric reflections. For a detailed discussion see [1] pp. 10 and 60.

** Where some of the boundaries appear to be extrapolated in the increasing $1/\mu$ direction, computed points are available outside the plot scale to guide construction of the boundary. Boundaries shown by broken lines were not computed in detail because they are obviously subcritical.

Before discussing the individual boundaries it is useful to point out some common features not conveniently shown in the figures. Each boundary is identifiable with a particular mode corresponding to the maximum element in the flutter vector Q_m *. Coupling between modes varies along any given boundary but the dominant mode is the same throughout. Weakest coupling between modes occurs to the right in the figures, this being particularly noticeable in the closed loops typified by the second mode contours in Figs. 3 and 4. The latter feature is beneficial because it means that definition of the critical (right-most) envelope can be accomplished using a moderate number of chordwise modes.

Computational details are discussed next followed by examination of the effects of aspect ratio and number of chordwise modes.

A. Computational Details

Figures 3 and 4 illustrate the stability boundaries for the panel arrays with one chordwise bay. These boundaries were obtained using the first four natural beam modes for Φ_m. Satisfactory convergence of the boundaries was verified by obtaining at least one point on each boundary in a six mode computation. The adequacy of four mode analyses is also implied by the fact that the critical envelope is made up of those portions of the first and second mode boundaries where coupling with the fourth mode is negligible. (See Table II where the modulus of the fourth mode element is shown to be less than 0.5 per cent of the maximum.) Four boundaries were obtained, associated with the four chordwise modes Φ_m.

Stability boundaries for the arrays with two chordwise panels are shown in Figs. 5 and 6. The first and third mode boundaries were obtained initially using the first four chordwise modes; the boundaries shown were then obtained using modes one to six. Addition of the two higher modes had imperceptible effect on the location of the boundaries (although reduced frequencies increased slightly). The fifth mode boundary was obtained using modes three through eight. Its location was verified in computations using modes one through six and three through six wherein points were obtained that are indistinguishable from the plotted boundary when superimposed on the figures. The sixth mode boundary, obtained using modes one to six and three to eight, is not defined in as much detail as the others because it is obviously subcritical. It is noteworthy that second and fourth mode boundaries were not identified even though a wide range of frequencies was investigated.

* A few boundaries are denoted as coupled mode boundaries indicating that two or more elements are much larger than the rest. The largest is named first.

These boundaries may either exist outside the $1/\mu$ range of interest or be confined to a region in the flutter parameter plane too small to be detected by the numerical procedure. It is reasonable to conclude, however, that the second and fourth mode boundaries are unimportant on the basis that they are structurally coupled modes and Figs. 5 and 6 indicate the uncoupled modes to be most critical. The relative importance of the structurally uncoupled modes is more clearly illustrated in the following case of panel arrays with three chordwise bays.

Stability boundaries for panel arrays with three chordwise bays are shown in Figs. 7 and 8 where it can be seen that the structurally uncoupled modes one, four and seven are most critical. The first and fourth mode boundaries were obtained using natural modes one to six and the seventh mode boundary using modes four to nine. Analyses using six consecutive modes are sufficient because strongest coupling occurs with the adjacent modes and the coupling falls off rapidly away from the dominant mode. There is, for example, no appreciable coupling between modes one, four and seven on the critical sides of any of the three boundaries. The coupling is stronger on the portions to the left, as noted earlier, but this is not important to identification of the critical envelope. The remaining boundaries shown in Figs. 7 and 8 were obtained using the first six chordwise modes. Convergence was not investigated in detail because these boundaries are clearly subcritical. In the course of computation some points were obtained which indicate the presence of higher mode boundaries. These points (not plotted) all fall to the left of the boundaries shown.

The critical envelopes of the stability boundaries are repeated in superposition in Figs. 9 and 10 to illustrate the effect of the number of streamwise bays. The broken line curve in the figures is the hyperbola corresponding to aluminum panels at sea level flight conditions (Eq. (7)). The following material and altitude properties were used to define the hyperbola.

$$E = 10.5 \times 10^6 \text{ psi} = 7.239 \times 10^{10} \text{ Newtons/meter}^2$$

$$\nu = 0.318$$

$$c_\infty = 13440 \text{ in/sec} = 341 \text{ meters/sec}$$

$$\rho = 1.1468 \times 10^{-7} \text{ lb-sec}^2/\text{in}^4 = 1.2256 \text{ kilograms/meter}^3$$

$$\rho_s = 2.59 \times 10^{-4} \text{ lb-sec}^2/\text{in}^4 = 2.7679 \times 10^3 \text{ kilograms/meter}^3$$

Minimum thicknesses required to prevent flutter of aluminum panels at sea level, obtained from the intersections of the hyperbola with the critical envelopes, are given in Table I. The corresponding flutter vectors are given in Table II; a precise definition of the tabulated quantities is given in the following paragraph.

The deflection shape at flutter of a panel array with L streamwise bays, each of length a and width b, is given by

$$F(x,y,t) = e^{i\omega t}\sin(\pi y/b)\left\{\sum_j q_j e^{i\theta_j}\Phi_j(x)\right\} \tag{8}$$

where x is the streamwise coordinate, y the spanwise coordinate, and $\Phi_j(x)$ the j^{th} natural vibration mode shape of a continuous, simply supported beam with L equal bays.* The quantities given in Table II are the q_j and θ_j of the above formulation. For the present analysis the $\Phi_j(x)$ are normalized on the basis of unit rms amplitude, i.e., each $\Phi_j(x)$ is multiplied by a suitable constant to satisfy the condition

$$\left\{\frac{1}{L}\int_0^L \Phi_j^2(x)dx\right\}^{1/2} = 1$$

The above choice for normalization yields almost the same results for q_j in Table II as normalization on the basis of unit maximum amplitudes.**

The numerical results will now be discussed and compared with the results obtained by other investigators.

B. Effect of Number of Streamwise Bays

The numerical results show that increasing the number of streamwise bays in an array of simply supported flat panels has a pronounced destabilizing effect, i.e., the array becomes much more susceptible to flutter. The critical envelopes in Figs. 9 and 10 show this effect to extend throughout the entire practical range of the mass ratio parameter $1/\mu$. The figures further show that the destabilizing effect is

(a) Larger for the larger aspect ratio, and

(b) Proportionately greater when the number of streamwise bays is increased from one to two than when a third panel is added to a two bay array.

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* For formulation of the $\Phi_j(x)$ see for example Ref. 1. Normalized mode shapes are shown in Fig. 2.

** If normalization on the basis of unit maximum amplitude is employed, the q_j will be identical for the one bay arrays; the structurally coupled mode components will increase by a factor of 1.04 for the two bay arrays and 1.2 - 1.4 for the three bay arrays. The phase angle θ_j is unaffected.

Table I illustrates the above effect quantitatively for the case of aluminum panels at sea level flight conditions. It is seen in the table that for panels of aspect ratio 4, adding a second streamwise bay almost doubles the thickness required to prevent flutter. Adding a third panel increases the thickness requirement to 2.2 times that necessary to prevent flutter of the single panel. For panels of aspect ratio 2 the corresponding thickness ratios are 1.44 and 1.51.

A second general result is that the critical envelope of boundaries for the multi-bay arrays is dominated by the structurally uncoupled natural streamwise modes.* Since this implies relatively weak structural coupling between the panels it must be concluded that aerodynamic coupling is the primary cause of the observed destabilization which accompanies the increase in the number of streamwise bays.

Another interesting feature is illustrated by Table I. For the case of aspect ratio 4 the single panel flutters in the second mode whereas the arrays with two and three bays flutter in the first mode. The transfer of flutter mode correlates with the pronounced increase in thickness required to prevent flutter. However, in the case of panels with aspect ratio 2, the thickness required does no increase so drastically. This can probably be partly accounted for by the fact that, in one sense, the flutter mode does not change as the number of bays is increased. For instance, Table I shows that the one, two and three bay arrays flutter in the second, third and fourth modes, respectively. However, in each case the deflection shape in any one panel is a complete sine wave. Therefore, from the individual panel point of view there is no transfer of flutter mode as the number of bays increases.

The preceding observations on the effects of increasing the number of streamwise bays agree qualitatively with previous analyses of two-dimensional panel arrays (Refs. 2-6). Quantitative comparison can be made with Zeydel's analysis [4] of aluminum panels at sea level which omits the effects of structural coupling and damping. At Mach number 1.35 Zeydel found the thickness required to prevent flutter of two and three bay arrays to increase by factors of 1.56 and 1.68, respectively, over the thickness required for a single panel. These ratios are smaller than the corresponding values of 1.92 and 2.21 obtained in the present analysis for panels of aspect ratio 4. However, at this particular Mach number, Zeydel's results show that flutter occurs in the first mode for all cases. Moving to Mach number 1.4 where the flutter mode transfers from two to one, as observed in the present analysis, the thickness ratios are 1.98 and 2.31. These latter ratios are in close agreement with present results. The above comparison implies that the results obtained here for Mach number 1.35 must not be generalized to other Mach numbers in the low supersonic regime.

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* The odd numbered modes for the two bay arrays and modes 1, 4, 7, 10, etc. for the three bay arrays. These modes are characterized by vanishing bending moments at the panel boundaries.

C. Effect of Aspect Ratio

The preceding discussion deals primarily with the effect of the number of streamwise bays on the flutter of finite flat panel arrays composed of one spanwise bay and one, two and three streamwise bays. It is now desirable to consider the influence of aspect ratio. The most apparent influence is the stabilization obtained by decreasing the aspect ratio. This effect is illustrated in Figs. 3 through 8 by the overall contraction of the unstable regions which accompanies reduction of the aspect ratio from 4 to 2. The figures also show that the stabilizing effect becomes more pronounced as the number of streamwise bays increases. Quantitative evaluation of the above influence can be obtained from Table I for aluminum panels at sea level. For the single panels the thickness to prevent flutter reduces by 19 per cent in going from aspect ratio 4 to 2. The corresponding thickness reductions for the two and three bay arrays are 39 per cent and 44 per cent, respectively. It is also interesting to observe that the thickness to prevent flutter of a three bay array of panels of aspect ratio 2 is only 1.23 times the thickness required for a single panel of aspect ratio 4. These results suggest that reduction of aspect ratio may be preferable to increasing panel thickness as a means for preventing the flutter of multi-bay arrays.

A second effect of aspect ratio (not conveniently shown in the figures) is an increase in coupling between modes with decreasing aspect ratio. This implies increased structural coupling between streamwise panels. The latter probably contributes to increased stability, because Figs. 3 through 8 show that the structurally coupled modes are less easily excited. It also means that as aspect ratio decreases it becomes more important to include the effect of structural coupling in a flutter analysis.

D. Effect of Finite Span

The present analytical technique is similar to that of [5,8] with the exception that finite span arrays are investigated here whereas in [5,8] the span is assumed to extend to infinity in discrete, uniform bays. Although only limited comparison can be made between the two techniques because of the present restriction to one Mach number, it appears that the finite span approach yields slightly less conservative results, i.e., computed thickness to prevent flutter is slightly smaller. The effect also appears to be somewhat greater for the smaller aspect ratio panels. For example, critical thicknesses for single panels at sea level are shown below as obtained with the two techniques.

Aspect Ratio	Thickness to Prevent Flutter Reference 8	 Present Analysis
2	0.00790	0.00758
4	0.00928	0.00925

Although the above results show little variation it would be of interest to compare the two techniques in application to multiple streamwise bay arrays of smaller aspect ratio.

V. CONCLUSIONS

On the basis of results obtained in the present flutter analysis of simply supported, multiple streamwise bay flat panel arrays for Mach number 1.35 and structural damping coefficient equal to 0.01 it is concluded that:

(1) The addition of streamwise panels is destabilizing.

(2) The destabilizing effect is proportionately greater when the number of streamwise bays is increased from one to two, than when a third panel is added to a two bay array.

(3) The destabilizing effect is greater for panels of large aspect ratio.

(4) The primary mechanism of destabilization is aerodynamic coupling between the streamwise panels.

(5) Decreasing panel aspect ratio has a stabilizing effect which increases with number of streamwise panels.

(6) The qualitative features of the results obtained in this analysis can be extrapolated to other Mach numbers in the low supersonic regime. However, the quantitative effect of increasing the number of streamwise bays is Mach number dependent, and cannot be obtained by extrapolation.

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APPENDIX

TABLES I AND II AND FIGURES 1 THROUGH 10

TABLE I

MINIMUM THICKNESS TO PREVENT FLUTTER OF ALUMINUM PANELS AT SEA LEVEL

Aspect Ratio	Number of Bays	Reduced Frequency	Minimum Thickness to Prevent Flutter	Dominant Mode in Flutter Vector
4	1	1.215	0.00925	2
2	1	1.0385	.00758	2
4	2	0.595	.01783	1
2	2	1.505	.01074	3
4	3	0.715	.02040	1
2	3	1.555	.01143	4

TABLE II

TABULATION OF FLUTTER VECTOR

Aspect Ratio - 2
Number of Streamwise Bays

1		2		3	
q_j	θ_j	q_j	θ_j	q_j	θ_j
0.18037	2.9309	0.01937	-0.5421	0.01766	2.7602
1.0	0	0.08126	-0.4662	0.02007	2.6405
0.08102	3.0772	1.0	0	0.08409	-0.4930
0.00436	-0.2846	0.09730	3.1156	1.0	0
-	-	0.00575	-0.7410	0.15014	3.0535
-	-	0.01754	-0.3126	0.00979	-1.1932

Aspect Ratio - 4
Number of Streamwise Bays

1		2		3	
q_j	θ_j	q_j	θ_j	q_j	θ_j
0.10596	2.8718	1.0	0	1.0	0
1.0	0	0.04929	0.1589	0.06885	0.2431
0.04753	3.0701	0.00133	4.7564	0.00097	-3.7153
0.00182	-0.3906	0.00901	2.9923	0.00360	-3.3447
-	-	0.00018	4.6434	0.00124	-0.6262
-	-	-	-	0.00429	-3.4258

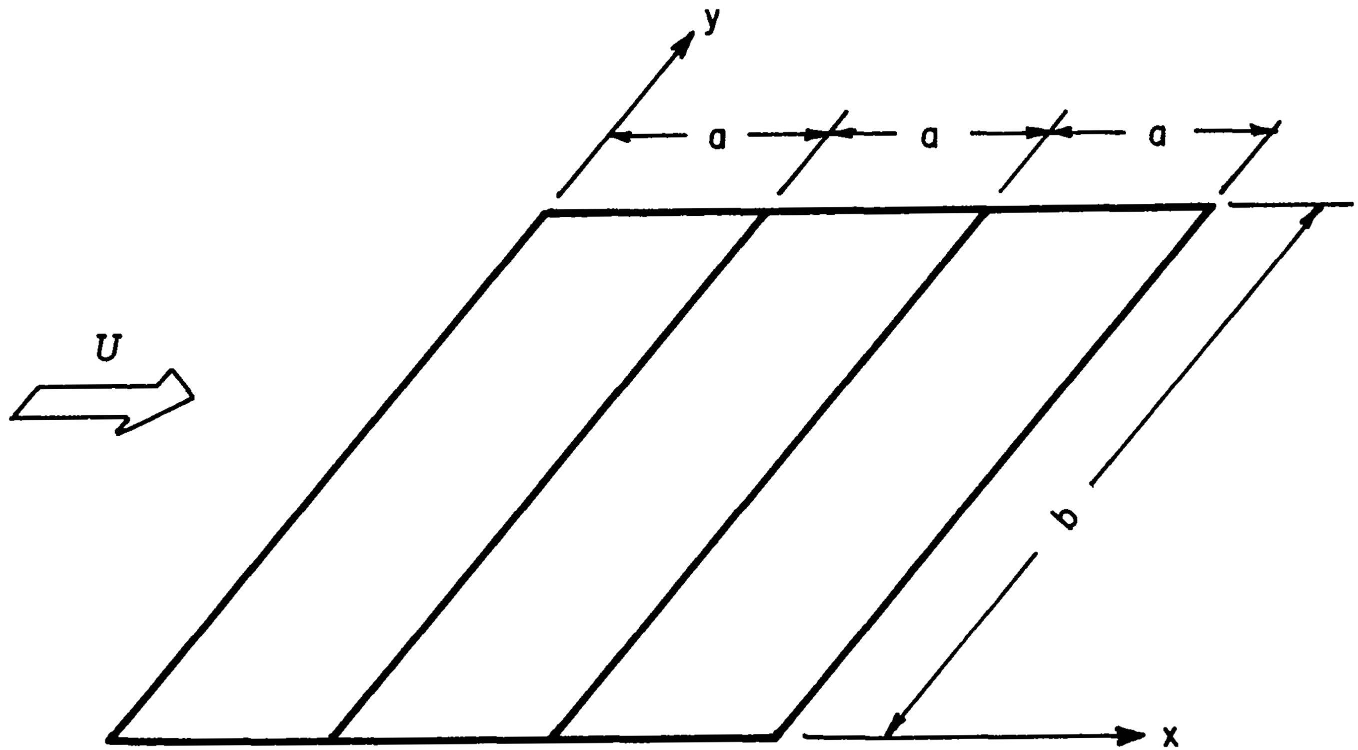

Fig. 1 - Typical Panel Array

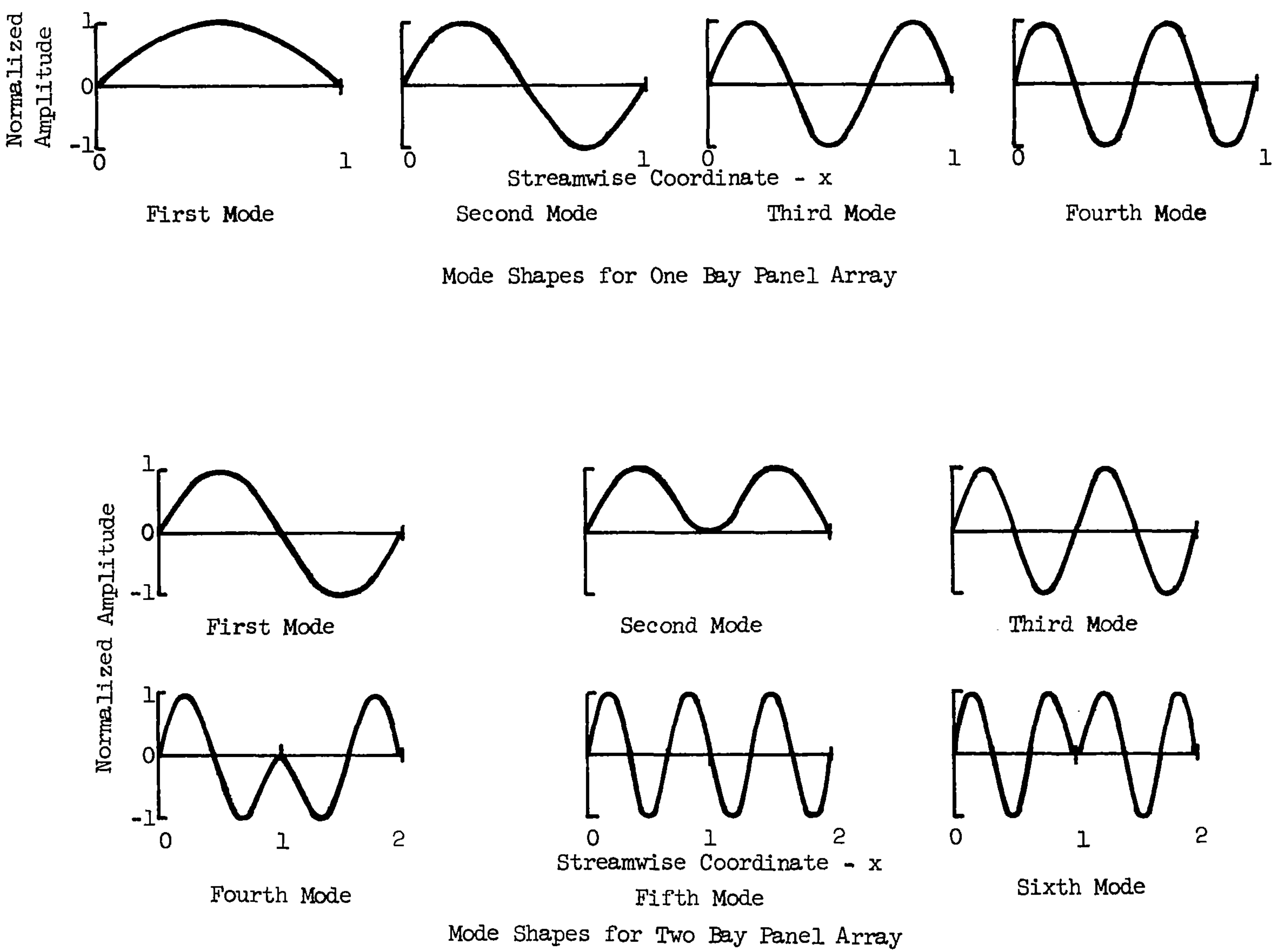

Fig. 2a - Mode Shapes for Panel Arrays with One and Two Streamwise Bays

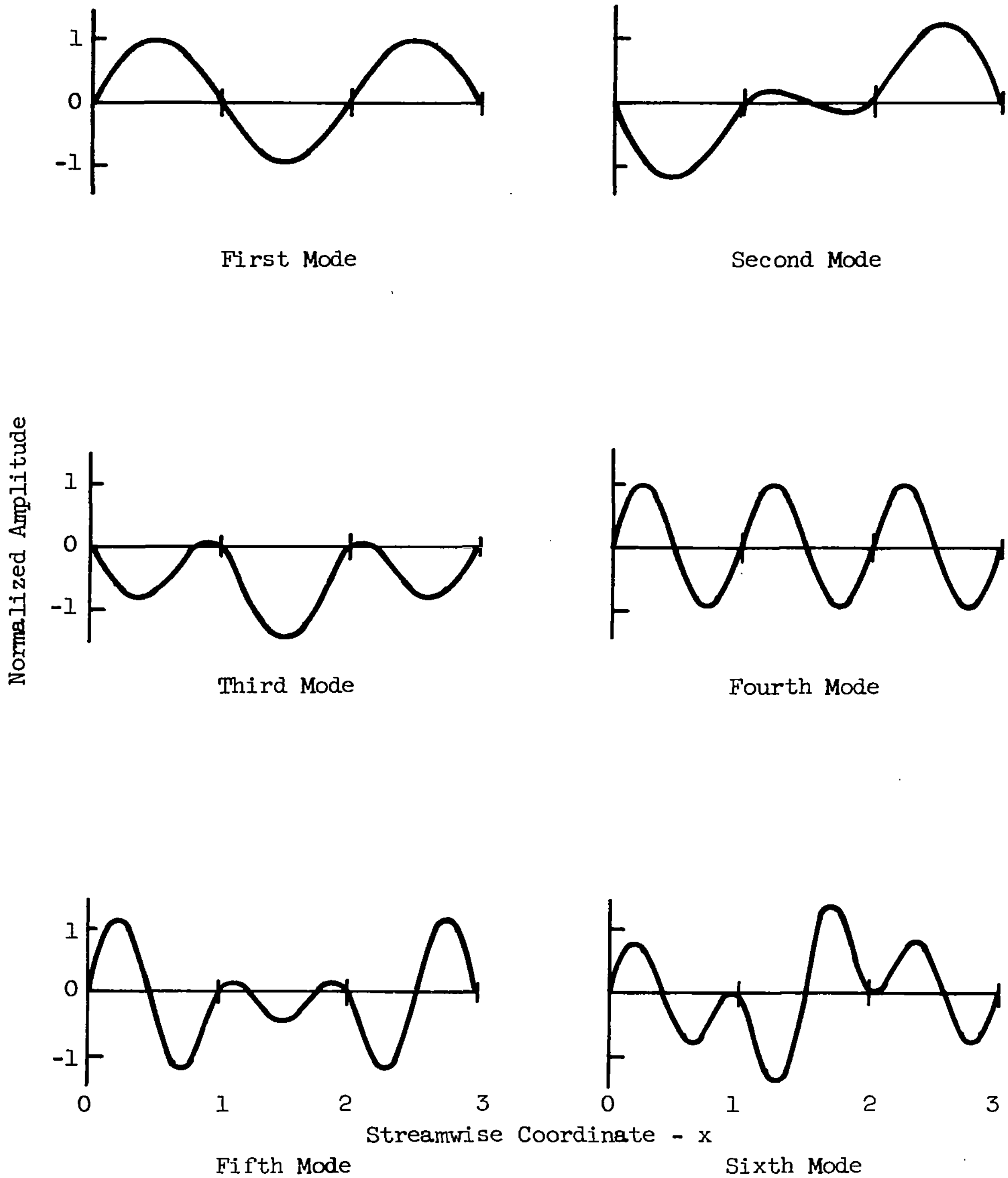

Fig. 2b - Mode Shapes for Panel Arrays with Three Streamwise Bays

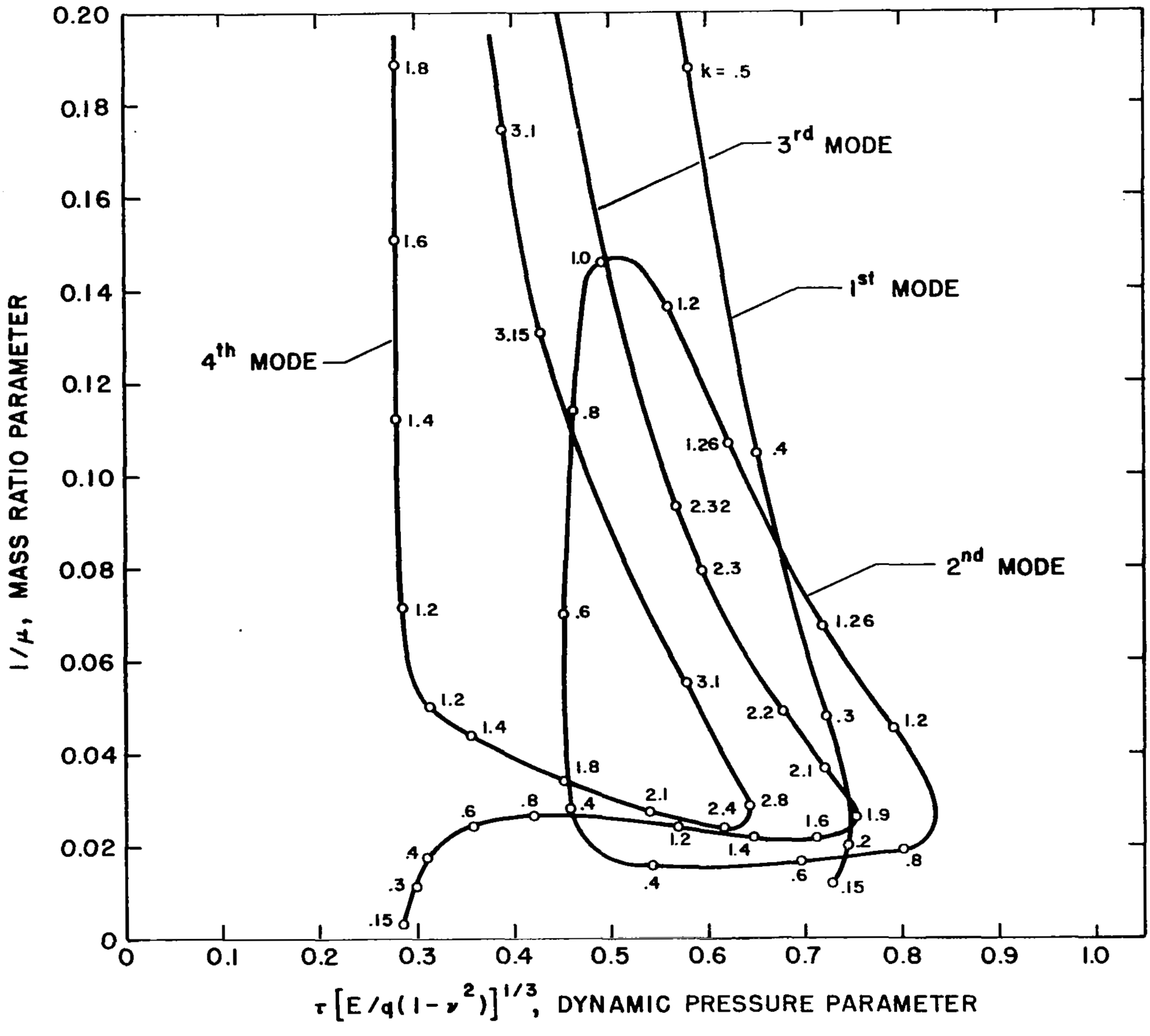

Fig. 3 - Stability Boundaries for a Single Panel of Aspect Ratio 4, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

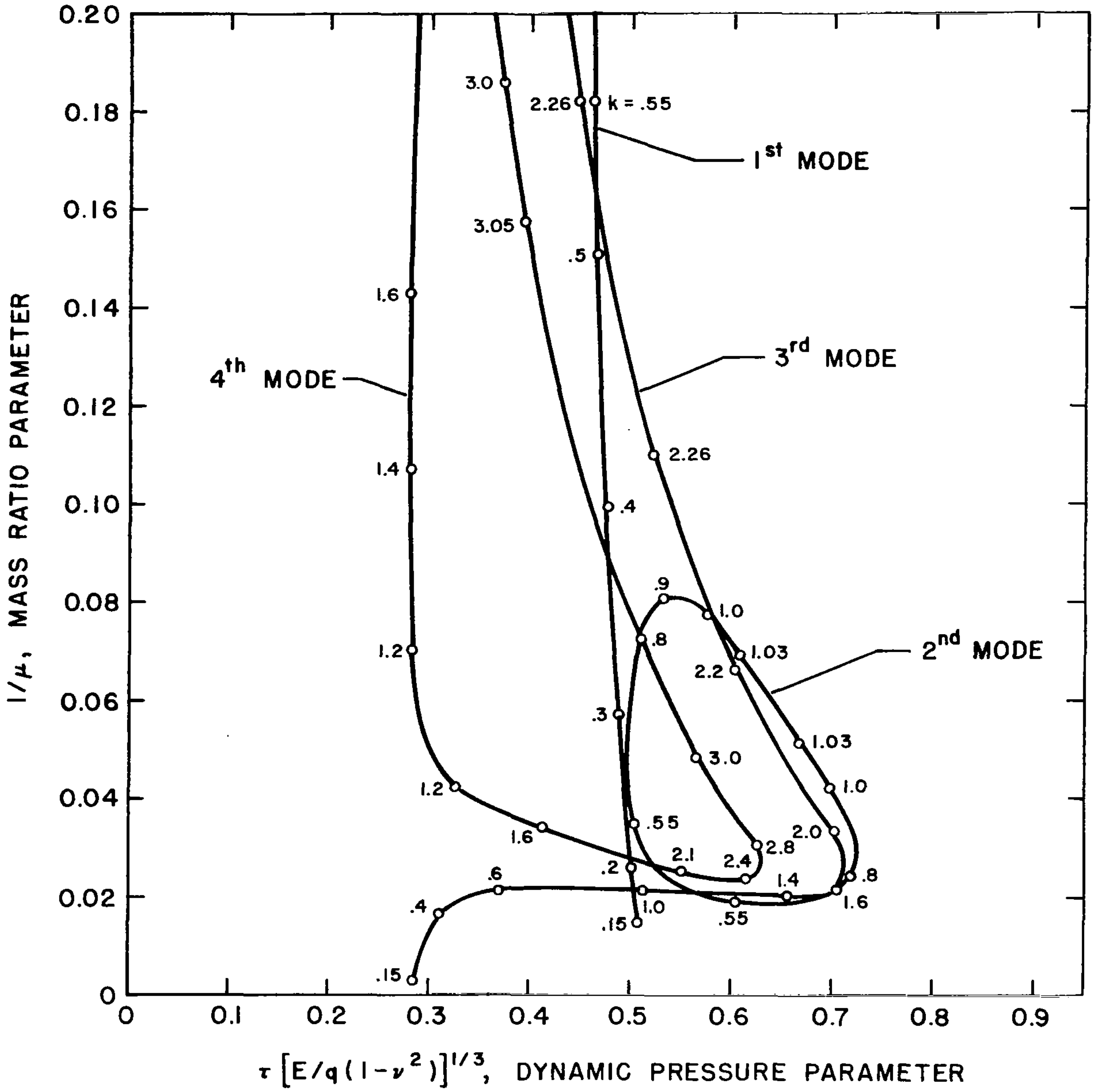

Fig. 4 - Stability Boundaries for a Single Panel of Aspect Ratio 2, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

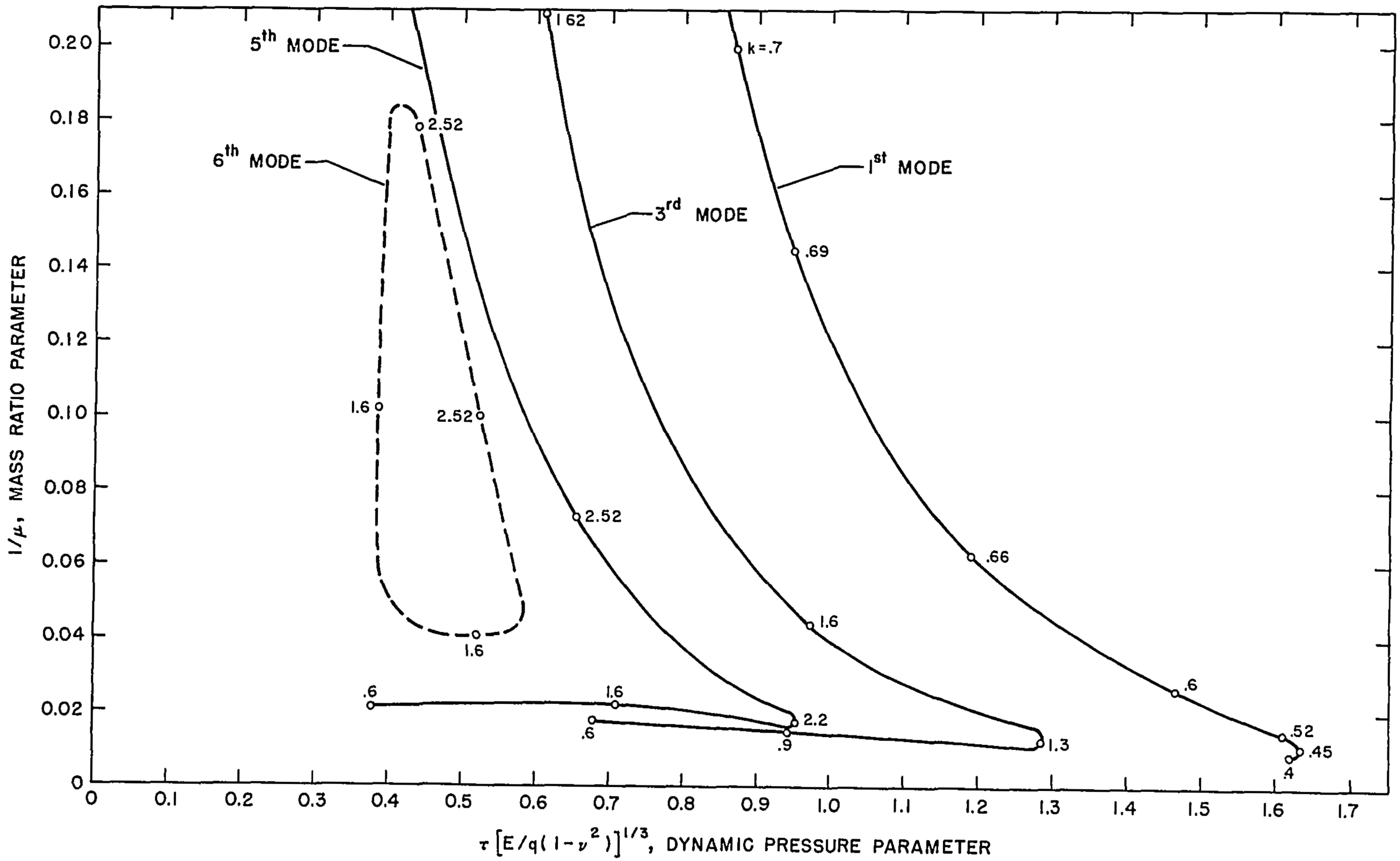

Fig. 5 - Stability Boundaries for an Array of Two Streamwise Panels of Aspect Ratio 4, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

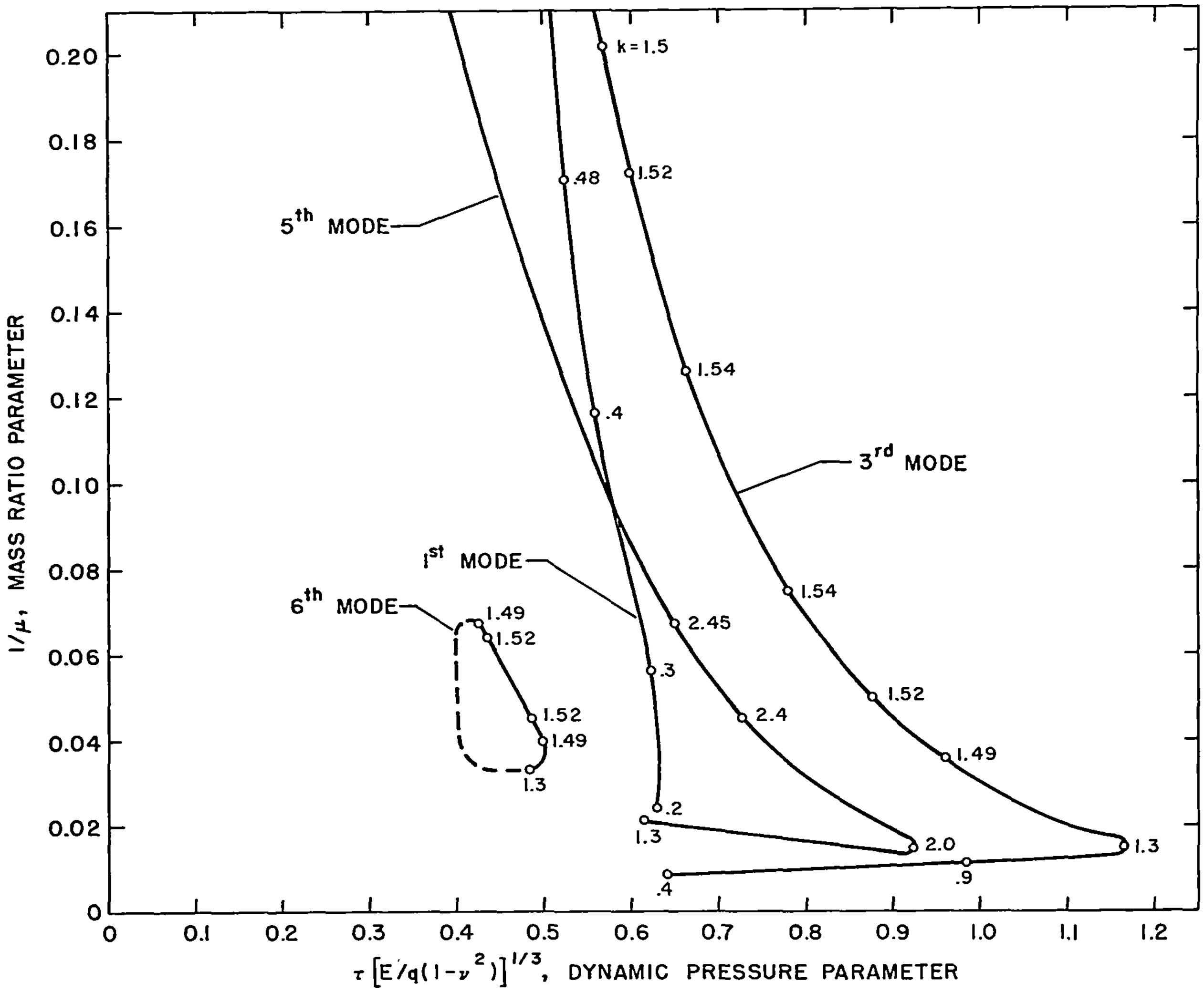

Fig. 6 - Stability Boundaries for an Array of Two Streamwise Panels of Aspect Ratio 2, Mach Number = 1.35 and Structural Damping Coefficient $g = 0.01$

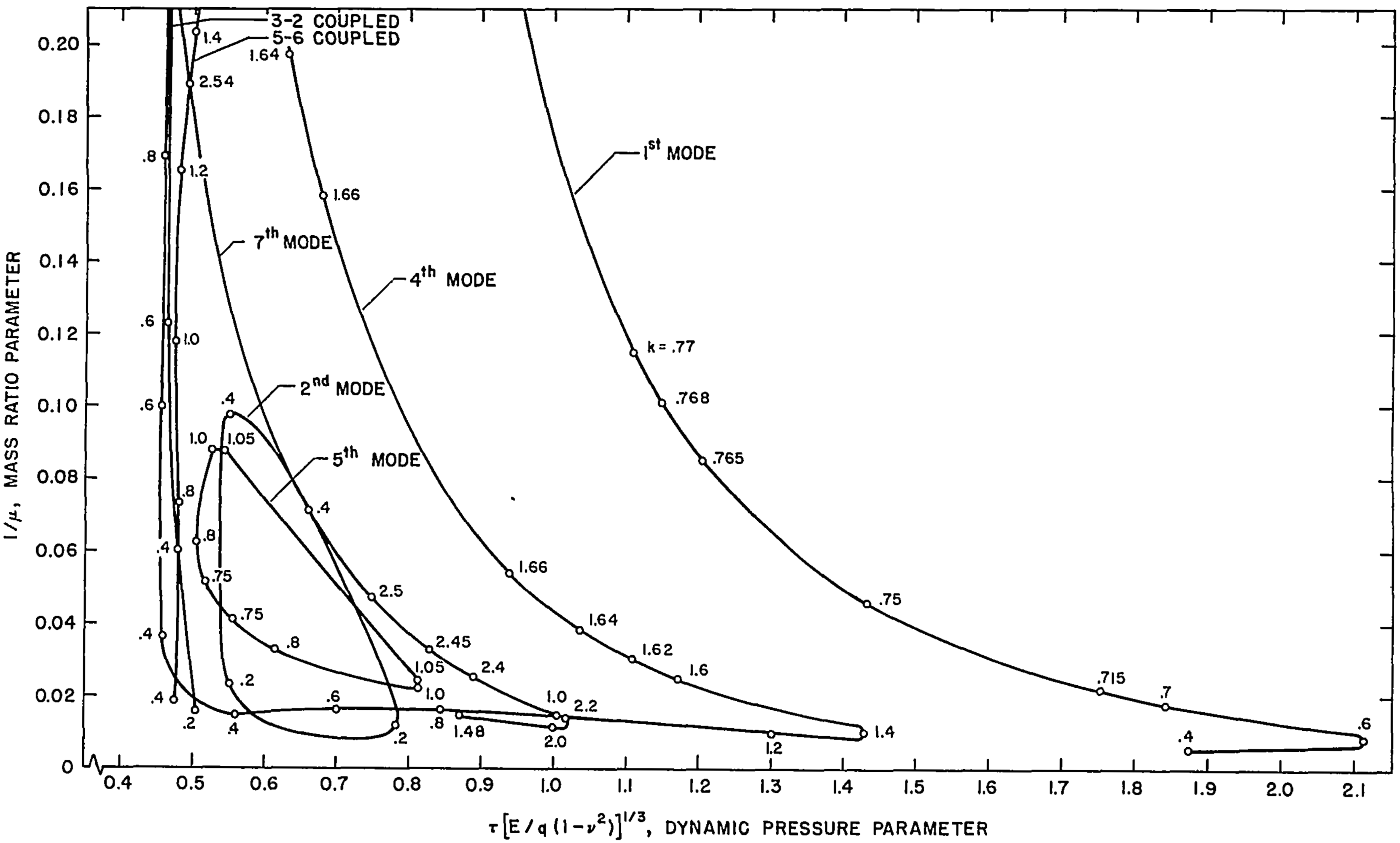

Fig. 7 - Stability Boundaries for an Array of Three Streamwise Panels of Aspect Ratio 4, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

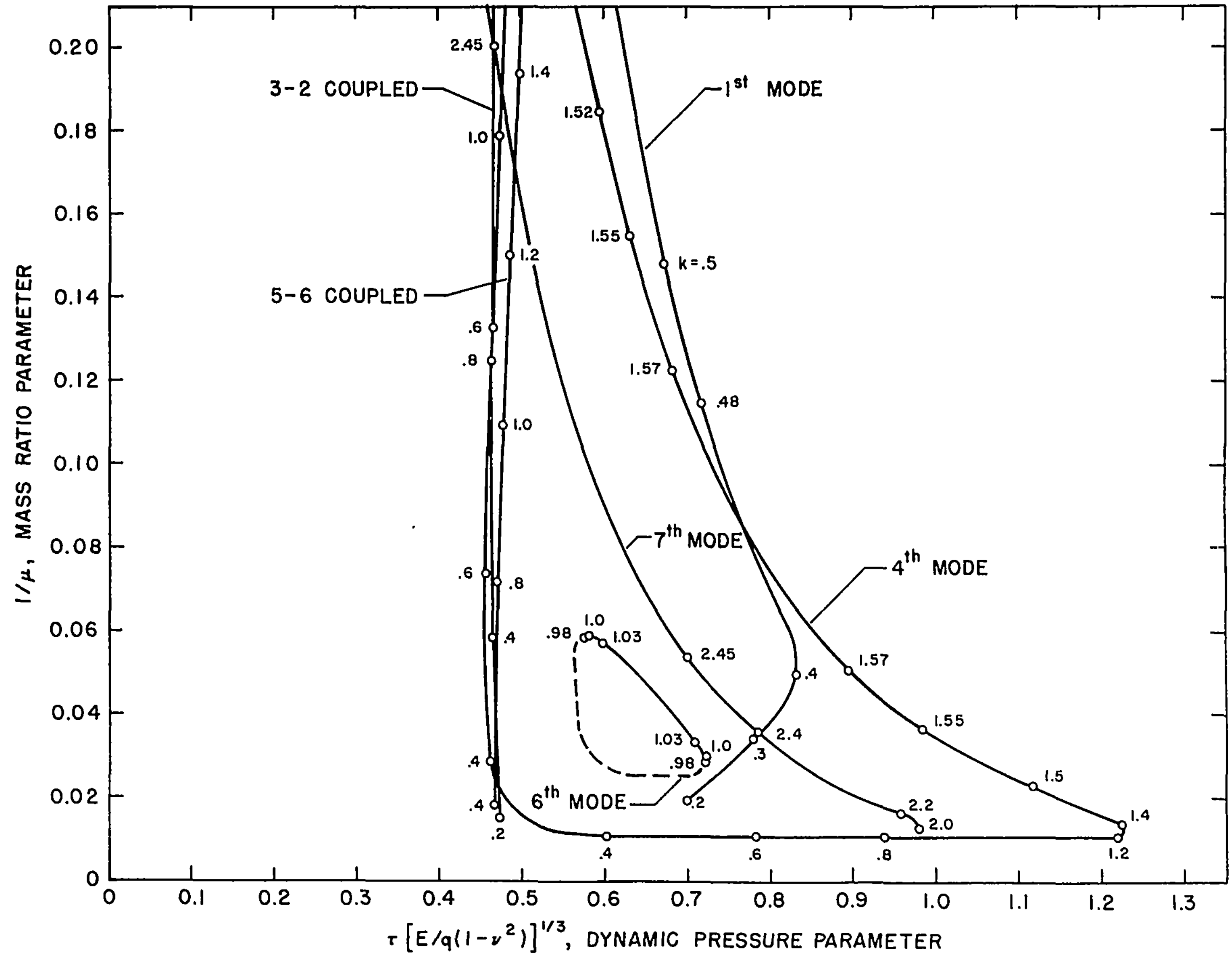

Fig. 8 - Stability Boundaries for an Array of Three Streamwise Panels of Aspect Ratio 2, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

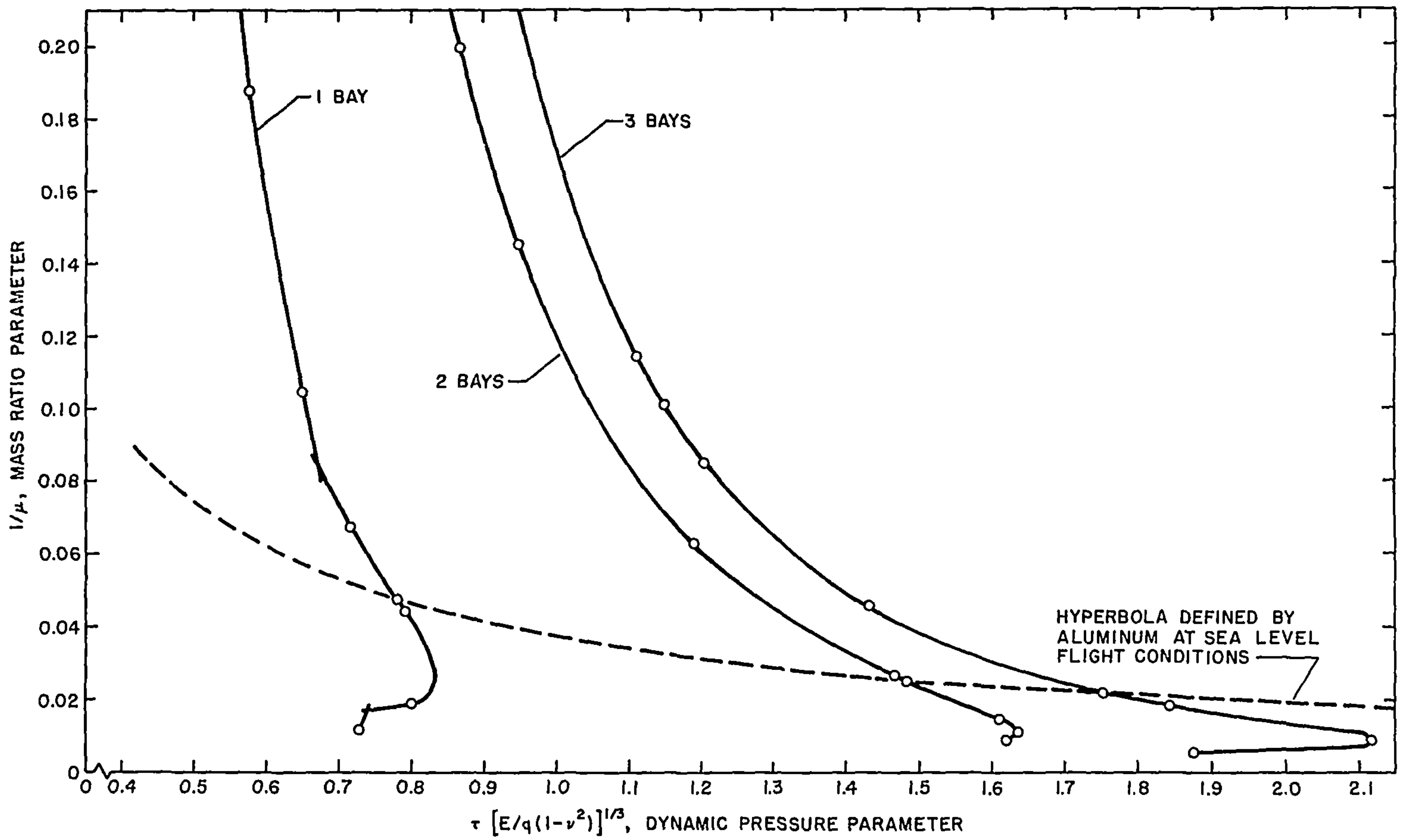

Fig. 9 - Critical Flutter Boundaries for Panel Arrays with Multiple Streamwise Bays - Panels of Aspect Ratio 4, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

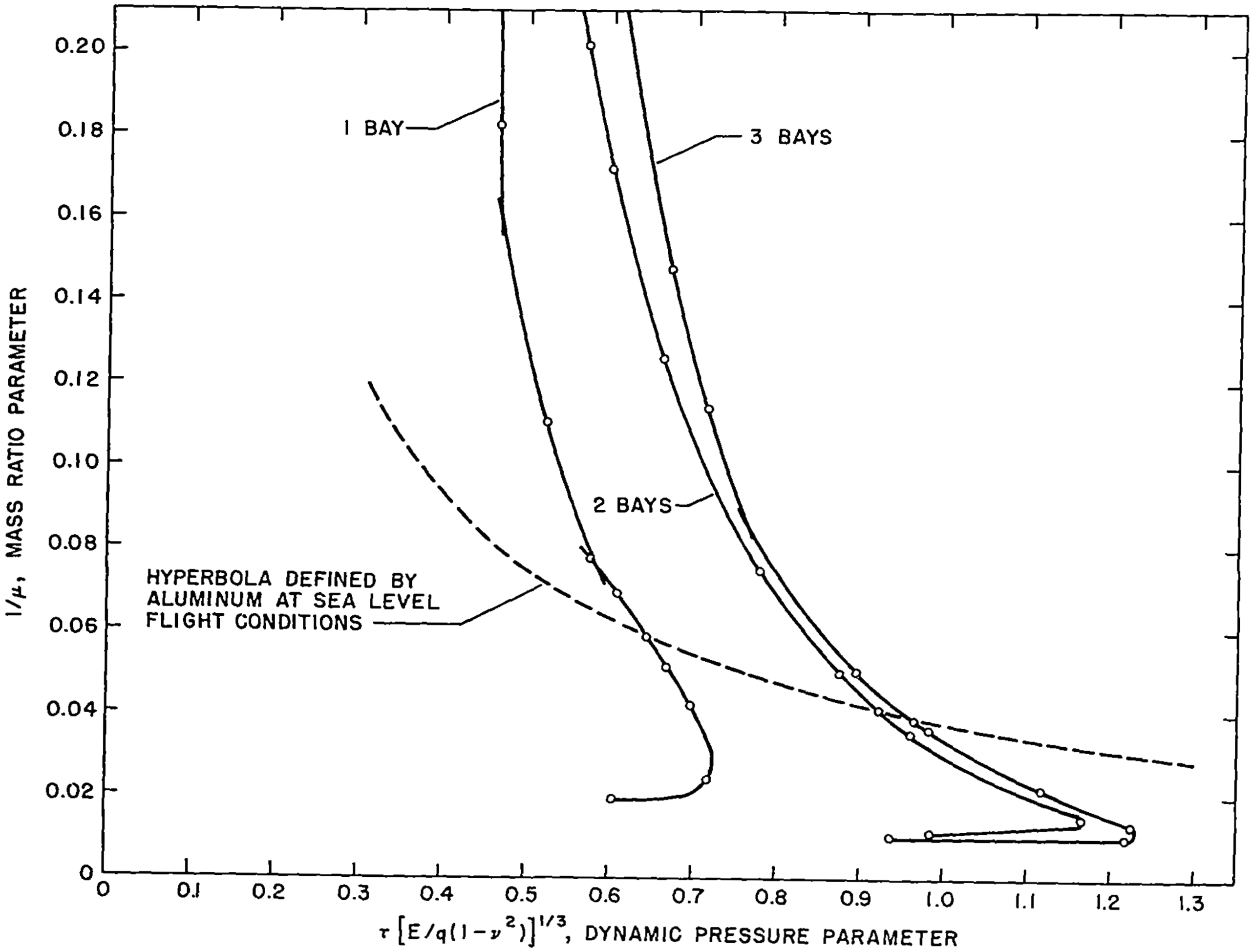

Fig. 10 - Critical Flutter Boundaries for Panel Arrays with Multiple Streamwise Bays - Panels of Aspect Ratio 2, Mach Number = 1.35 and Structural Damping Coefficient g = 0.01

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