RADIATION OF AN ELEMENTARY CYLINDER ANTENNA THROUGH A SLOTTED ENCLOSURE

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ABSTRACT

The shrouding of an elementary cylinder antenna by a concentric perfectly conducting axially slotted shell is considered. The problem is reduced to a Fredholm integral equation of the first kind which is solved for the case of a narrow shell shot. The solution indicates that the slotted shell inhibits the antenna radiation only for certain parameter combinations; for most others, the radiation remains the same and for some it is enhanced.

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Introduction

In this paper we treat the radiation problem of a cylinder antenna shrouded by a concentric axially slotted shell. The source of the cylinder antenna is an infinite axial slot uniformly excited, with the electric field in the circumferential direction. In the following five sections we present the analyses of the problem. For further details one may also refer to a Radiation Laboratory Report by A. Olte and Y. Hayashi[1].

In the first section we reduce the boundary value problem of the antenna by employing a conventional series representation of the fields to a Fredholm integral equation of the first kind. The integral equation uniquely determines the tangential electric field of the slotted shell for the given source on the cylinder surface. The kernel of the integral equation is complex, non-hermitian, and has a logarithmic singularity.

In the second section we briefly discuss the physical aspects of a singular integral equation which follows by differentiation of the Fredholm integral equation of the first kind. Hayashi[2] was the first to derive the singular equation in a similar problem.

In the third section we report a solution of the Fredholm integral equation for the case of a narrow shell slot. Recognizing that in a narrow slot the field distribution is dominated by the edge singularity we are led to a slot field representation

[1] Olte, A. and Y. Hayashi, "On the Antenna Radiation Through a Plasma Sheath," University of Michigan Radiation Laboratory Report No. 5825-1-F, June 1964.

[2] Hayashi, Y., "Electromagnetic Field for Circular Boundaries with Slots," Proc. Jap. Academy of Sci., submitted for publication, 1964.

by a Fourier series of a kind where the first term is the dominant one. This idea has already been applied by Morse and Feshbach[3] in discussing the scattering of an electromagnetic wave normally incident on an axially slotted, perfectly conducting cylindrical shell.

In the fourth section of this chapter we report the numerical calculations which are based on the above solution of the Fredholm integral equation. The purpose of the calculations is to exhibit the influence of the slotted shell on the radiation of the cylinder antenna. The cylinder diameters considered are, in wavelength, from $0.2/\pi$ to $1.8/\pi$. The radial spacings between the cylinder and the shell are $0.1/\pi$, $0.05/\pi$, and $0.025/\pi$. It will be obvious that for the parameter values considered the slotted shell does not significantly modify the form of the cylinder antenna radiation pattern. However, the pattern is rotated by the angle between the source on the cylinder surface and the shell slot, although for the cylinder diameters considered the radiation is nearly omnidirectional anyway. Even for the largest diameter antenna considered the radiation field is omnidirectional to within ± 25 per cent[4]. Therefore, we have chosen to report the ratio of the radiated power with the slotted shell and without it, as a function of the source and the shell slot separation angle.

In the fifth section of this chapter we discuss the accuracy of the approximate solution of the integral equation, and the error reflected in the power radiated. We also discuss some of the physical implications of the solution.

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[3] Morse, P.M. and H. Feshbach, Methods of Theoretical Physics, Part II, McGraw-Hill, New York, pp. 1387-1388, 1953.

[4] Wait, J.R., Electromagnetic Radiation from Cylindrical Structures, Pergamon Press, New York, p. 30, 1959.

In the last section we briefly discuss and summarize the main features of the results. The slotted shell prevents radiation only for certain parameter combinations; for most others, the radiation remains the same and for some it is even enhanced. This section concludes with a discussion of some extensions of the work.

Reduction of the Boundary Value Problem to a Fredholm Integral Equation of the First Kind

We consider a wedge waveguide of width $2\theta_o$ feeding in the lowest order transverse electric mode a perfectly conducting circular cylinder of radius a, as shown in Figure 1. The cylinder is concentrically shrouded by a vanishingly thin perfectly conducting shell of radius b. The shell has an axial slot of width $2\phi_o$. The center-to-center circumferential displacement of the shell slot and the cylinder slot is indicated by the angle ϑ. We employ a right-hand circular cylindrical coordinate system (r, ϕ, z) for which ϕ is measured counter-clockwise from the center of the shell slot and z is along the axis of the cylinder. The constitutive parameters ϵ and μ are assumed to be real. The rational MKS system of units is used and the time dependence of $e^{j\omega t}$ is implied for all field quantities.

We have a two-dimensional problem, since both the antenna structure and the source is independent of z. Furthermore, it is a three region problem: $0 \leq r \geq a$; $a \leq r \leq b$; $r \geq b$. For this particular study we limit it to a two region problem by considering the tangential electric field of the cylinder slot as given. We set out to find the fields in the coaxial region and in the free space. The fields we represent in a series form[5].

[5] Stratton, J.A., Electromagnetic Theory, McGraw-Hill, New York, p. 361, 1941.

For $a \leq r \geq b$

$$H_z = k^2 \sum_{n=-\infty}^{\infty} \left\{ B_n J_n(kr) + C_n N_n(kr) \right\} e^{-jn\phi} \tag{1}$$

$$E_\phi = j\omega\mu k \sum_{n=-\infty}^{\infty} \left\{ B_n J_n'(kr) + C_n N_n'(kr) \right\} e^{-jn\phi} \tag{2}$$

$$E_r = -\frac{\omega\mu}{r} \sum_{n=-\infty}^{\infty} n \left\{ B_n J_n(kr) + C_n N_n(kr) \right\} e^{-jn\phi} \tag{3}$$

and for $r \geq b$

$$H_z = \sum_{n=-\infty}^{\infty} k^2 A_n H_n^{(2)}(kr) e^{-jn\phi} \tag{4}$$

$$E_\phi = \sum_{n=-\infty}^{\infty} kj\omega\mu\, A_n H_n^{(2)\prime}(kr) e^{-jn\phi} \tag{5}$$

$$E_r = -\sum_{n=-\infty}^{\infty} \frac{n\omega\mu}{r} A_n H_n^{(2)}(kr) e^{-jn\phi}, \tag{6}$$

where $k = \omega\sqrt{\mu\epsilon}$.

We regard

$$E_\phi(a, \phi) \equiv f(\phi), \qquad \theta - \theta_o \leq \phi \leq \theta + \theta_o$$

as given and we seek to find

$$E_\phi(b, \phi) \equiv E(\phi), \qquad -\phi_o \leq \phi \leq \phi_o$$

We require that

$$j\omega\mu k \sum_{n=-\infty}^{\infty} \left\{ B_n J_n'(kb) + C_n N_n'(kb) \right\} e^{-jn\phi} \begin{cases} = E(\phi), & -\phi_o \leq \phi \leq \phi_o \\ = 0, & \phi_o \leq \phi \leq 2\pi - \phi_o \end{cases} \tag{7}$$

$$j\omega\mu k \sum_{n=-\infty}^{\infty} \left\{ B_n J_n'(ka) + C_n N_n'(ka) \right\} e^{-jn\phi} = f(\phi), \quad \theta-\theta_o \le \phi \le \theta+\theta_o \tag{8}$$

$$= 0, \theta+\theta_o \le \phi \le 2\pi - (\theta-\theta_o)$$

$$\sum_{n=-\infty}^{\infty} j\omega\mu k A_n H_n^{(2)'}(kb) e^{-jn\phi} = E(\phi), \quad -\phi_o \le \phi \le \phi_o$$

$$= 0, \quad \phi_o \le \phi \le 2\pi - \phi_o. \tag{9}$$

Because of the orthogonality of the circular functions we obtain from (7), (8) and (9) respectively

$$j\omega\mu k \left\{ B_n J_n'(kb) + C_n N_n'(kb) \right\} 2\pi = \int_{-\phi_o}^{\phi_o} E(\phi') e^{jn\phi'} d\phi' \tag{10}$$

$$j\omega\mu k \left\{ B_n J_n'(ka) + C_n N_n'(ka) \right\} 2\pi = \int_{\theta-\theta_o}^{\theta+\theta_o} f(\eta) e^{jn\eta} d\eta \tag{11}$$

$$j\omega\mu k A_n H_n^{(2)'}(kb) 2\pi = \int_{-\phi_o}^{\phi_o} E(\phi') e^{jn\phi'} d\phi' , \tag{12}$$

where ϕ' and η are dummy variables of integration. From (10) and (11) we obtain

$$j2\pi\omega\mu k D_n(ka, kb) B_n \quad N_n' \int_{\theta-\theta_o}^{\theta+\theta_o} f(\eta) d^{jn\eta} d\eta - N_n(ka) \int_{-\phi_o}^{\phi_o} E(\phi') e^{jn\phi'} d\phi' \tag{13}$$

$$j2\pi\omega\mu k D_n(ka, kb) C_n = -J_n'(kb) \int_{\theta-\theta_o}^{\theta+\theta_o} f(\eta) e^{jn\eta} d\eta + J_n'(ka) \int_{-\phi_o}^{\phi_o} E(\phi') e^{jn\phi'} d\phi' , \tag{14}$$

where

$$D_n(ka, kb) = J_n'(ka) N_n'(kb) - J_n'(kb) N_n'(ka) . \tag{15}$$

We observe that (12), (13) and (14) give us the Fourier coefficients of the fields for both regions once the electric slot fields are known. However, we do not know

the shell slot field. We seek to find it by enforcing the remaining boundary condition: the continuity of the tangential magnetic field through the shell slot, i. e.

$$\sum_{n=-\infty}^{\infty} A_n H_n^{(2)}(kb) e^{-jn\phi} = \sum_{n=-\infty}^{\infty} \left[B_n J_n(kb) + C_n N_n(kb)\right] e^{-jn\phi}, \quad -\phi_o \leq \phi \leq \phi_o. \tag{16}$$

We eliminate the coefficients in (16), and after factoring and transposing of terms obtain

$$\sum_{n=-\infty}^{\infty} \left[\frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \quad \frac{1}{D_n(ka, kb)} \int_{-\phi_o}^{\phi_o} E(\phi') e^{jn\phi'} d\phi' \right] e^{-jn\phi}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{D_n(ka, kb)} \int_{\theta-\theta_o}^{\theta+\theta_o} f(\eta) e^{jn\eta} d\eta \right] e^{-jn\phi}, \quad -\phi_o \leq \phi \leq \phi_o \tag{17}$$

Interchanging integration with summation, we obtain an integral equation of the form

$$\int_{-\phi_o}^{\phi_o} E(\phi') K(\phi', \phi) d\phi' = g(\theta, \phi); \; -\phi_o \leq \phi \leq \phi_o, \tag{18}$$

where

$$K(\phi', \phi) = \sum_{n=0}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \quad \frac{\epsilon_n}{D_n(ka, kb)} \cos n(\phi' - \phi)$$

$$g(\theta, \phi) = \int_{\theta-\theta_o}^{\theta+\theta_o} f(\eta) \, K^{(a)}(\eta, \phi) d\eta \tag{19}$$

$$K^{(a)}(\eta, \phi) = \sum_{n=0}^{\infty} \frac{\epsilon_n}{D_n(ka, kb)} \cos n(\eta - \phi)$$

with $\epsilon_n = 1$ for $n = 0$, and $\epsilon_n = 2$ for $n = 1, 2, 3, \ldots$

The kernel $K(\phi', \phi)$ has a logarithmic singularity at $\phi = \phi'$. This can be easily shown by considering those terms of the series for which $n >> kb$. The kernel is non-hermitian since $K(\phi', \phi) \neq K(\phi, \phi')$. The kernel $K^{(a)}(\eta, \phi)$ is a continuous function of η and ϕ. We see that the known function of the integral equation is obtained by transforming the electric field of the cylinder slot according to (19).

We have reduced the boundary value problem to a Fredholm integral equation of the first kind with a non-hermitian kernel. This is a unique statement of the original problem and no additional conditions need be imposed.

The Appearance of the Singular Integral Equation

It is well known that the normal component of the electric field must be continuous through an aperture. It is also well known that the continuity of the tangential magnetic field through the aperture automatically insures the continuity of the normal electrical field component as well. The reverse, however, is not true and therefore the continuity of the normal component of the electric field through the aperture is not sufficient for a boundary condition.

The continuity condition of the normal component of the electric field through the shell aperture in our case corresponds to an integral equation that is obtained by differentiating the Fredholm integral equation of the first kind with respect to ϕ. We have then

$$P \int_{-\phi_o}^{\phi_o} E(\phi') K'(\phi', \phi) d\phi' = \int_{\theta-\theta_o}^{\theta+\theta_o} f(\eta) K^{(a)'}(\eta, \phi) d\eta, \qquad -\phi_o < \phi < \phi_o, \tag{20}$$

where the primes indicate that the kernels are to be differentiated with respect to

ϕ. The integral of this type in a similar problem was first obtained by Hayashi as noted in the Introduction. The singularity of the kernel $K(\phi', \phi)$ by differentiation has been increased to a Cauchy type singularity and therefore we have to take the integral in the prinicipal value sense which we explicitly indicate by P in front of the integration sign. Whereas the Fredholm integral equation exists as $\phi \to \pm\phi_o$, (20) does not and we restrict ϕ to the open interval $-\phi_o < \phi < \phi_o$. The kernel $K^{(a)'}(\eta, \phi)$ is continuous and the right hand side of (20) exists even as $\phi \to \pm\phi_o$. Evidently the Cauchy integral equation for this particular problem admits a set of solutions. Somehow one has to choose a solution that satisfies the Fredholm integral equation. One may possibly choose from the set a solution that satisfies the well known condition of the edge singularity at $\phi = \pm\phi_o$. This solution is a sum of a certain particular integral and a certain general solution of the homogeneous equation of (20), both parts satisfying the edge singularity condition independent of each other. The amplitude of the general solution of the homogeneous equation clearly remains arbitrary. If finding the proper amplitude of this part of the solution gives us the unique slot field, then it may be determined by substituting the total solution in the Fredholm integral equation. These appear to by the physical aspects of the singular integral integral equation method used by Hayashi in constructing a formal solution to a similar boundary value problem.

Solution of the Fredholm Integral Equation for the Case of a Narrow Slot

The Fredholm integral equation of the first kind (18) was derived for the cylinder slot of arbitrary width $2\theta_o$. We simplify the problem, but retain its essential features by letting $\theta_o \to 0$, because then we can let the slot field assume a Diras δ-function distribution, i.e. $f(\phi) = V_a \delta(\theta - \phi)/a$ where V_a is the slot voltage.

In this case then

$$g(\theta, \phi) = V_a K^{(a)}(\theta, \phi)/a \tag{21}$$

We observe that $K(\phi', \phi)$ and $K^{(a)}(\theta, \phi)$ may be divided into even and odd parts with respect to both variables, i.e.

$$K(\phi', \phi) = K_e(\phi', \phi) + K_o(\phi', \phi) \tag{22}$$

$$K^{(a)}(\theta, \phi) = K_e^{(a)}(\theta, \phi) + K_o^{(a)}(\theta, \phi), \tag{23}$$

where

$$K_e(\phi', \phi) = \sum_{n=0}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{\epsilon_n}{D_n(ka, kb)} \cos(n\phi') \cos(n\phi) \tag{24a}$$

$$K_o(\phi', \phi) = \sum_{n=1}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{2}{D_n(ka, kb)} \sin(n\phi') \sin(n\phi) \tag{24b}$$

$$K_e^{(a)}(\theta, \phi) = \sum_{n=0}^{\infty} \frac{\epsilon_n}{D_n(ka, kb)} \cos(n\theta) \cos(n\phi) \tag{24c}$$

$$K_o^{(a)}(\theta, \phi) = \sum_{n=1}^{\infty} \frac{2}{D_n(ka, kb)} \sin(n\theta) \sin(n\phi) \tag{24d}$$

Since the unknown slot field may also be represented by an even and an odd part, $E(\phi) = E_e(\phi) + E_o(\phi)$, the Fredholm integral equation becomes

$$\int_{-\phi_o}^{\phi_o} E_e(\phi')K_e(\phi', \phi)d\phi' + \int_{-\phi_o}^{\phi_o} E_o(\phi')K_o(\phi', \phi)d\phi' = \frac{1}{a} V_a \left[K_e^{(a)}(\theta, \phi) + K_o^{(a)}(\theta, \phi)\right],$$

$$-\phi_o \le \phi \le \phi_o. \tag{25}$$

We observe that the first integral is an even function in ϕ, while the second integral is an odd function, therefore we have that

$$\int_{-\phi_o}^{\phi_o} E_e(\phi')K_e(\phi',\phi)d\phi' = \frac{1}{a} V_a K_e^{(a)}(\theta,\phi) \tag{26}$$

$$\int_{-\phi_o}^{\phi_o} E_o(\phi')K_o(\phi',\phi)d\phi' = \frac{1}{a} V_a K_o^{(a)}(\theta,\phi) \ . \tag{27}$$

We have succeeded in breaking the Fredholm integral equation into two integral equations of the same kind. The first one determines the even part of the shell slot field and the second one, the odd part. Stating the problem in this form means that we invert two small matrices instead of one large one in order to obtain the same accuracy in the solution.

Electromagnetic fields cannot have large spatial variations over distances that are small compared to the wavelength, except in the vicinity of the sources, at the discontinuities in the medium constitutive parameters, and at sharp conducting edges. The slot field of a narrow slot is therefore dominated by the edge singularity. We separate this out in the first term of a Fourier representation of the even and odd parts of the slot field, i.e.

$$E_e(\phi) = \frac{a_o}{\pi\sqrt{\phi_o^2-\phi^2}} + \sum_{q=1}^{\infty} a_q \cos\frac{q\pi\phi}{\phi_o} \tag{28}$$

and

$$E_o(\phi) = \frac{\phi b_1}{\pi\sqrt{\phi_o^2-\phi^2}} + \sum_{q=2}^{\infty} b_q \sin\frac{q\pi\phi}{\phi_o} \ . \tag{29}$$

We are guided in selecting these particular forms of the field singularities by the work of Sommerfeld[6] and Millar[7] on the diffraction by an infinite slit in a vanishingly thin perfectly conducting plane screen. Using these fields representations we convert each integral equation into an infinite set of algebraic equations by straightfoward integrations. We summarize the results for (26) in matrix notation as

$$\left[h_{mq}^{(e)}\right] \left. a_q \right] = \frac{1}{a} V_a \left. \ell_m^{(e)} \right] \tag{30}$$

where

$$h_{oo}^{(e)} = \sum_{n=0}^{\infty} \epsilon_n \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{J_o(n\phi_o)}{D_n(ka,kb)} \frac{\sin(n\phi_o)}{n\phi_o}$$

$$h_{oq}^{(e)} = 4\phi_o(-1)^{q+1} \sum_{n=1}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{1}{D_n(ka,kb)} \frac{\sin^2(n\phi_o)}{(q\pi)^2-(n\phi_o)^2}, \quad q \leq 1$$

$$h_{mo}^{(e)} = \sum_{n=1}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{J_o(n\phi_o)}{D_n(ka,kb)} \frac{n\sin(n\phi_o)}{(m\pi)^2-(n\phi_o)^2}, \quad m \geq 1$$

$$h_{mq}^{(e)} = 2\phi_o^2(-1)^{q+1} \sum_{n=1}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{1}{D_n(ka,kb)} \frac{n^2\sin^2(n\phi_o)}{\left[(q\pi)^2-(n\phi_o)^2\right]\left[(m\pi)^2-(n\phi_o)^2\right]}$$

$$m \geq 1, \quad q \geq 1,$$

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[6] Sommerfeld, A. Optics, Lectures on Theoretical Physics, Vol. IV, Academic Press, New York and London, pp 273-289, 1964.

[7] Millar, R. F., "A Note on Diffraction by an Infinite Slit," Can. J. Phys., 38, No. 1, pp 38-47, 1960.

$$\ell_o^{(e)} = \sum_{n=0}^{\infty} \left[\frac{\epsilon_n}{D_n(ka, kb)} \frac{\sin(n\phi_o)}{n\phi_o} \right] \cos(n\theta)$$

$$\ell_m^{(e)} = \sum_{n=1}^{\infty} \left[\frac{n}{D_n(ka, kb)} \frac{\sin(n\phi_o)}{(m\pi)^2 - (n\phi_o)^2} \right] \cos(n\theta), \quad m \geq 1 .$$

and for (27) as

$$\left[h_{mq}^{(o)} \quad b_q \right] = \frac{1}{a} V_a \, \ell_m^{(o)} \Big] , \tag{31}$$

where

$$h_{m1}^{(o)} = \frac{4\phi_o}{\pi} \sum_{n=1}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{1}{D_n(ka, kb)} \frac{\sin(n\phi_o)}{(m\pi)^2 - (n\phi_o)^2} \int_0^{n\phi_o} \sqrt{1 - \left(\frac{x}{n\phi_o}\right)^2} \cos x \, dx,$$

$$h_{mq} = 4\phi_o (-1)^{q+1} \sum_{n=1}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{1}{D_n(ka, kb)} \frac{q\pi \sin^2(n\phi_o)}{\left[(q\pi)^2 - (n\phi_o)^2\right] \left[(m\pi)^2 - (n\phi_o)^2\right]}$$

$$\ell_m^{(o)} = \sum_{n=1}^{\infty} \left[\frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \frac{2}{D_n(ka, kb)} \frac{\sin(n\phi_o)}{(m\pi)^2 - (n\phi_o)^2} \right] \sin(n\theta), \quad m \geq 1 .$$

We have reduced the problem of finding the shell slot electric field to an inversion of two complex matrices. By the inclusion of the edge singularity in the first term for each part of the slot field we have enhanced the first column at the expense of the rest of the matrix. The enhancement is particularly large for a narrow slot. Thus the problem has been set up in a way so as to lead to the narrow slot approximation. However, before we delve into the numerical details, we should indicate the physical quantities we want to find.

We are interested in the narrow slot antennas. Further we restrict the radial separation of the shell from the cylinder to a small fraction of the wavelength

λ, i. e. $kb-ka << 1$. Under these conditions the form of the radiation patter of the cylinder with the shell will be essentially the same as for the cylinder alone. However, the power radiated into the free space will depend most probably on the orientation of the shell. Therefore, we want to show how the radiated power depends on the slot field in the next few paragraphs.

The power radiated per unit length by the shell slot is given by

$$P = \frac{1}{2} R_e \int_0^{2\pi} \bar{E} \times \bar{H}^* \cdot \bar{a}_r \, r d\phi = \sum_{n=-\infty}^{\infty} 2\omega\mu \, k^2 A_n A_n^* \tag{32}$$

Substituting (12) in the preceding equation and (28) and (29) in the resulting equation, we have, after the interchange of integration with summation

$$P = \frac{1}{2\pi^2 \omega\mu} \sum_{n=0}^{\infty} \frac{\epsilon_n}{\left|H_n^{(2)'}(kb)\right|^2} \left\{ \left| J_o(n\phi_o) a_o - 2\phi_o^2 n \sin(n\phi_o) \sum_{q=1}^{\infty} \frac{(-1)^q a_q}{(q\pi)^2 - (n\phi_o)^2} \right|^2 + \left| 2\phi_o \sin(n\phi_o) \sum_{q=1}^{\infty} \frac{(-1)^q q\pi b_q}{(q\pi)^2 - (n\phi_o)^2} \right|^2 \right\} . \tag{33}$$

From the behavior of the denominator in the general term of the series we conclude that only $2kb$ terms need be considered independent of the slot width. A computation will show that for a narrow slot the second part of the numerator is negligible compared to a_o term. Only when b_1 becomes substantially larger than a_o do we have to take, in some special cases, the second part into consideration. Also in the first part of the numerator the a_q term is negligible compared to the a_o of the narrow slot. Thus we have that for a narrow slot,

$$P(ka, kb, \theta) \simeq \frac{|a_o|^2}{2\pi^2 \omega\mu} \sum_{n=0}^{2kb} \frac{\epsilon_n J_o^2(n\phi_o)}{\left|H_n^{(2)'}(kb)\right|^2} . \tag{34}$$

The power radiated by the cylinder without the shell is given by

$$P(ka) \simeq \frac{1}{2\omega\mu(\pi a)^2} \left|V_a\right|^2 \sum_{n=0}^{2ka} \frac{\epsilon_n}{\left|H_n^{(2)'}(ka)\right|^2} \tag{35}$$

We define the voltage of the slotted shell as

$$V_b \equiv b \int_{-\phi_o}^{\phi_o} E(\phi)d\phi \ , \tag{36}$$

and it follows that

$$V_b = ba_o \ . \tag{37}$$

The ratio of equations (34) and (35) we may write as

$$\frac{P(ka, kb, \theta)}{P(ka)} \simeq \left|\frac{V_b}{V_a}\right|^2 \left(\frac{a}{b}\right)^2 F(ka, kb, \phi_o) \ , \tag{38}$$

where

$$F(ka, kb, \phi_o) = \left[\sum_{n=0}^{2kb} \frac{\epsilon_n J_o^2(n\phi_o)}{\left|H_n^{(2)'}(kb)\right|^2}\right] \left[\sum_{n=0}^{2ka} \frac{\epsilon_n}{\left|H_n^{(2)'}(ka)\right|^2}\right]^{-1} . \tag{39}$$

For further discussion we elect to keep $V_a = a$ volts, whether or not the cylinder is enclosed by the shell. Then (38) takes the form

$$\frac{P(ka, kb, \theta)}{P(ka)} \simeq \left|a_o\right|^2 F(ka, kb, \phi_o) \ , \tag{40}$$

and a_o we regard as a dimensionless quantity. The last formula gives the enhancement (or depression) of radiation when the cylinder is enclosed by the shell. Aside from the essentially geometric factor F, all depends on the amplitude of the a_o term in the shell slot field expression.

Since Maxwell's equations are linear the result (38) is independent of the amplitude of the source voltage V_a; thus the right hand sides of (38) and (40) must be equal, and we obtain

$$\left|\frac{V_b}{V_a}\right| = \frac{kb}{ka}\left|a_o\right| \tag{41}$$

The last formula expresses the transformer properties of the shell. We may regard $(kb/ka)\left|a_o\right|$ as the transformer turns ratio.

We are fortunate that for a narrow slot the a_o term is also dominant in the slot field representation. This assertion is expected to hold when

$$\phi_o \ll \pi; \quad 2b\phi_o \ll \lambda; \quad b-a \geq 2b\phi_o . \tag{42}$$

From (30) with the understanding that $V_a = a$, we have

$$a_o \simeq \ell_o^{(e)}/h_{oo}^{(e)} \tag{43}$$

When $D_n(ka, kb) \rightarrow 0$, then (43) takes on the particularly simple form

$$a_o \simeq \frac{H_n^{(2)'}(kb)}{H_n^{(2)'}(ka)} \; \frac{\cos(n\theta)}{J_o(n\phi_o)} . \tag{44}$$

The lowest order root of $D_1(ka, kb)$ occurs[8] when $kb \simeq 2-ka$, $ka \leq 1$, that is, when the mean circumference is approximately equal to the wavelength. The lowest order root of $D_2(ka, kb)$ occurs when $kb \simeq 4-ka$, $ka \leq 2$. Of particular interest are also the roots of $D_0(ka, kb)$, because then a_o does not depend on the angle θ. We list the lowest order root of $D_0(ka, kb)$[9] for some parameter values of possible interest in Table 1.

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[8] Truell, R., "Concerning the Roots of $J_n'(x)N_n'(kx)-J_n'(kx)N_n'(x)=0$," J. Appl. Phys., 14, pp 350-352, 1943.

[9] Jahnke-Emde-Lösch, Tables of Higher Functions, McGraw-Hill, New York, p. 198, 1960.

TABLE I: PARAMETERS FOR LOWEST ORDER ROOT OF $D_0(ka, kb)$

ka	kb/ka
31.427	1.1
15.7275	1.2
10.4993	1.3
7.8875	1.4
6.2702	1.5

The Results

We are left with the task of computing a_o. The series expansion of the numerator and denominator of a_o involve Bessel functions and Neumann functions. In view of the intended applications of the results of this study, we restrict the radius of the shell to $kb \leq 2$.

We use the exact series for computing the cylindrical functions of order 0, 1, 2, 3, 4, and 5. Recursion relations are used to compute orders 6 and 7. For orders greater than 7 we use the small argument[10] approximation to obtain

$$\frac{1}{D_n(ka, kb)} \simeq - \frac{\pi(kb)^2}{n\left[1 - (\frac{ka}{kb})^{2n}\right]} (\frac{ka}{kb})^{n+1} , \tag{45}$$

$$\frac{H_n^{(2)\prime}(ka)}{H_n^{(2)\prime}(kb)} \simeq (\frac{kb}{ka})^{n+1} . \tag{46}$$

We sum 90 terms in the series determining $h_{oo}^{(e)}$ and $\ell_o^{(e)}$. We retain only five terms in the $F(ka, kb, \phi_o)$ series. The computations were performed on a digital computer for the slot width of 0.06 radians, and ka in steps of 0.2 from 0.2 to 1.8, for each step, kb-ka = 0.05, 0.10, 0.20.

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[10] Ibid., pp. 135-139.

We elect to present the results in the following forms. We choose $10 \log \left[P(ka, kb, \theta)/P(ka)\right]$ as the ordinate and θ as the abscissa in Figures 2 through 10. A zero on the ordinate axis means that the source radiates the same amount of power into the free space with and without the slotted shell. A negative number, say -20, means that introduction of the slotted shell decreases the radiated power to one-hundredth of the previous value, while +20 means that the radiated power is increased by a factor on one hundred. The origin on the ordinate axis $(\theta = 0^{o})$ means that the cylinder slot is under the shell slot, and $\pm\theta = 180^{o}$ means that the slots are on the opposite sides of the cylinder. The radiation curves are even functions in θ.

All along we assume that the source amplitude on the cylinder remains unchanged, i.e. the voltage of the cylinder slot remains at 'a' volts. Thus these calculations do not include the de-tuning of the antenna that must arise in most cases when it is surrounded by a perfectly conducting slotted shell. However, these calculations do show the effect the slotted shell has on the coupling between the line source of the cylindrical antenna and the radiation field in the free space.

In Figure 2, the antenna diameter is $\frac{0.1}{\pi} \lambda$ $(ka = 0.2)$, where λ is the free space wavelength. Enclosing this antenna by the slotted shell increases the radiated power, the bigger the separation between the antenna and the shell, the more radiation we get, which is only weakly dependent on the source and the slot separation angle θ. For $kb-ka = 0.05, 0.10, 0.20$, the radiated power is increased by 3, 7.5 and 13 db, respectively. The maximum increase is when $\theta = 180^o$ and the minimum when $\theta = 0^o$. The difference between maximum and minimum for a given kb is only about 2db.

In Figure 3, we have increased the antenna diameter to ka = 0.40 and the closer the shell is to the antenna the more radiation we get, which is just the opposite situation we had when ka = 0.2 in the preceding figure. When kb-ka = 0.05, the radiation is increased by 11.5 db when $\theta = 0^o$, and increased to 23.5 db when $\theta \rightarrow 180^o$. When we increase kb-ka to 0.10, the radiation very significantly decreases: when $\theta = 0^o$ we have only -1db, however the radiation increases to 13 db as θ increases to 180^o. Increasing kb-ka to 0.20 further reduces the radiation: at $\theta = 0^o$ we have -18 db and the radiation increases only to +2 db as $\theta \rightarrow 180^o$.

In Figure 4, we have ka = 0.6 and the farther the shell is from the antenna, the more it depresses the radiation. When kb-ka = 0.05, the radiation is -3 db at $\theta = 0$, but decreases to a deep minimum (the approximation gives zero radiated power, but this is not expected to be true) at $\theta = 36^o$, and from then rapidly rises to +6 db as $\theta \rightarrow 180^o$. Increasing kb-ka to 0.10 depresses radiation at $\theta = 0^o$ to -5 db and the deep minima moves to $\theta = 41^o$; as $\theta \rightarrow 180^o$ the radiation recovers to +2.5 db. As kb-ka $\rightarrow$ 0.2 the curve is depressed all along below zero, and the deep minima moves further to the right.

In Figure 5 the increase of ka to 0.80 has brought, maintaining the same order, all three curves closer together, and minima have moved further to the right. When $\theta = 0^o$ the radiation is a few db below zero; at 180^o either just above or below zero.

In Figure 6 we have ka = 1.0. The minima have moved farther to the right and for the three curves occur between 90° and 100°. The order of the curves has been reversed: the larger the kb-ka value, the more radiation we get into the free space. At $\theta = 0^{\circ}$, and 180°, kb-ka = 0.05 gives 0 db, kb-ka = 0.10 gives +0.5 db and kb-ka = 0.20 gives 2 db.

In Figure 7 we have ka = 1.2 and the minima are occurring between 105° and 115°. The curve ordering remains the same as in the preceding figure, but the radiation is enhanced for most θ angles, especially so for kb - ka = 0.20. For this curve the radiation at $\theta = 0^{\circ}$ and 180° is +13 db and +16db, respectively.

In Figure 8 we have increased the antenna radius to ka = 1.4, and the minimas now occur between 115° and 120°. The curves have started to reverse the order. The kb-ka = 0.20 curve has dropped about 15 db below the other curves and is approximately where the other two curves were in the preceding figure. For the high curves the maximum radiation is 18 db. All three curves show a second minimum starting to form at $\theta = 0^{\circ}$.

In Figure 9 we have increased ka to 1.6, and the reversal of the curve order has been completed, i.e. increased kb-ka decreases the radiation. Also two sets of minima have formed: the new set is between 10° to 20° and the old set has moved to 125° to 130°. The radiation is increased only by 6.5 db at the maximum.

In Figure 10 we have ka = 1.8, and all three curves have moved very closely together. The first minima occur at about 33° and the second at about 130°. At 0° the radiation is -1db; at 80° and 180°, it is ± 1 db.

Discussion

In the figures presented, we have increased the radius of the cylinder, a, in nine equal steps from $0.1/\pi\,\lambda$ to $0.9/\pi\,\lambda$. In all nine cases we have shown the effect of the slotted shell on the radiation when the radial separation between the cylinder and the shell is $0.025/\pi\,\lambda$, $0.05/\pi\,\lambda$, and $0.10/\pi\,\lambda$. The shell increases radiation independently of θ when $a < 0.1/\pi\,\lambda$. A very substantial increase in radiation is maintained as the radius of the cylinder is increased to $0.2/\pi\,\lambda$. As a is increased to $0.3/\pi\,\lambda$ and beyond, a deep minimum appears in the curves which indicates that for those angles the slotted shell decouples the cylindrical antenna from the free space. When a is $0.4/\pi\,\lambda$, or $0.5/\pi\,\lambda$ the slotted shell leaves the antenna radiation largely unaffected for extensive ranges of θ, except when the slot is in the 90° range from the source where then deep minima occur. As a is increased to $0.6/\pi\,\lambda$ the antenna radiation is enhanced, and also becomes sensitive to the cylinder and the slotted shell separation distance. The same thing remains true as a is further increased to $0.7/\pi\,\lambda$, except that a new minimum appears to form at $\theta = 0^{\circ}$. As a is further increased to $0.8/\pi\,\lambda$, both the radiation enhancement and the sensitivity of the radiation enhancement on the cylindrical antenna and the slotted shell separation markedly decrease for all θ values. Two deep minima have formed as well. Increasing a further to $0.9/\pi\,\lambda$ reduces the radiation enhancement practically to zero, and the radiation becomes independent of the antenna and the slotted shell radial separation. This also occurred at $a = 0.5/\pi\,\lambda$.

This phenomenon appears to be associated with the 'resonances' in the coaxial cavity formed by the cylindrical antenna and the coaxial shell. The 'resonances' occur when $D_n(ka, kb) = 0$. For our range of variables the first 'resonance' appears when $ka \sim 1$, the second when $ka \sim 2$, i.e. $a \sim 0.5/\pi\ \lambda$, and $1/\pi\ \lambda$, respectively.

In equation (40) essentially a geometrical factor relates the radiated power per unit length to the square of the amplitude of the a_o coefficient. This factor we denoted by F, and in particular cases consider here, the infinite series was approximated by five terms, i.e.

$$F(ka, kb, \phi_o) = \left[\sum_{n=0}^{5} \frac{\epsilon_n J_o^2(n\phi_o)}{\left| H_n^{(2)'}(kb) \right|^2} \right] \left[\sum_{n=0}^{5} \frac{\epsilon_n}{\left| H_n^{(2)'}(ka) \right|^2} \right]^{-1} \tag{47}$$

We plot $F(ka, kb, \phi_o)$ in Figure 11 for the same ranges of variables as appeared in the preceding figures. Since the shell is close to the cylinder in the three cases considered we have that the factor F is close to unity when $ka > 1$. Only for $ka < 1$ do we have a substantial increase of the factor above unity. Using this factor we can very simply obtain the ratio of the slot voltages from the preceding figures. From (38) we have

$$10 \log \left| \frac{V_b}{V_a} \right|^2 = 10 \log \frac{P(ka, kb, \theta)}{P(ka)} - 10 \log \left[\left(\frac{ka}{kb} \right)^2 F(ka, kb, \phi_o) \right] . \tag{48}$$

We notice that in the second term on the right hand side the argument of the log is close to unity. Thus for most parameter configurations the Figures 2 through 10 give also directly the ratio of the slot voltages as a function of θ. We may add that in this approximation the phase of V_a is constant between the minima, and it suffers a 180^o change as one goes through a minimum. The slot voltage phase is given by the phase of $1/h_{\infty}^{(e)}$ and the 180° phase change comes from the sign reversal of $\ell_{o}^{(e)}$ on going through the minimum.

We have presented the approximate solution of the integral equation (26), and discussed to some extent the physical consequences of this solution. The method of approximation suggested itself from the results of the narrow slot in a plane screen. The use of the same leading terms in the slot field representation is justified largely on the physical grounds and it leads to the geometrical restrictions (42). Although we feel the approximation is a good one for the range of parameters discussed, a quantitative statement of the slot voltage approximation would be very desirable. We may truncate the matrix in (30) and invert it. This procedure is laborious, the results may not be conclusive as to the error in any case, and particularly so when the original approximation is a good one. We choose to go back to the integral equation itself. We rewrite it in the form

$$\left[K_e^{(a)}(\theta,\phi)\right]^{-1}\int_{-\phi_o}^{\phi_o} E_e(\phi')K_e(\phi',\phi)d\phi' - 1 = 0 \quad . \tag{49}$$

When we use the approximate solution, the left hand side of the above equation will not be quite zero. This difference we denote by $\Delta^{(e)}(\phi)$, i.e.

$$\Delta^{(e)}(\phi) = \left[K_e^{(a)}(\theta,\phi)\right]^{-1}\int_{-\phi_o}^{\phi_o} E_e'(\phi')K_e(\phi',\phi)d\phi' - 1, \quad -\phi_o \leq \phi \leq \phi_o \tag{50}$$

where the prime on the field indicates that it is the approximate solution we have obtained, i.e.

$$E'_e(\phi') = \frac{\ell_o^{(e)}(\theta)}{h_{oo}^{(e)}} \; \frac{1}{\pi\sqrt{\phi_o^2 - (\phi')^2}} \; . \qquad (51)$$

We find that

$$\Delta^{(e)}(\phi) = \frac{\ell_o^{(e)}(\theta)}{h_{oo}^{(e)}} \left[K_e^{(a)}(\theta,\phi)\right]^{-1} \sum_{n=0}^{\infty} \frac{H_n^{(2)'}(ka)}{H_n^{(2)'}(kb)} \; \frac{\epsilon_n J_o(n\phi_o)}{D_n(ka,kb)} \cos(n\phi) - 1 \; , \qquad (52)$$

In Table II we present some of the calculations from (52) for the parameters of Figures 2, 5, 6 and 9. The $\ell_o^{(e)}(\theta)$ and the $K_e^{(a)}(\theta,\phi)$ series have been summed to 90 terms, the other two series to 200 terms. The calculations showed that the approximate solution (51) 'satisfies' the integral equation essentially independent of the angle θ. In the table we have shown how $\Delta^{(e)}(\phi)$ depends on the field point coordinate ϕ. We have carried out the computations for $\pm\phi = 0$, 0.015, and 0.030, i.e. at the slot center, half-way to either end, and at either slot edge.

TABLE II: VALUES OF $\Delta^{(e)}(\phi)$

		$\Delta^{(e)}$ for $\phi_o = 0.030$		
ka	kb-ka	$\phi=0$	$\phi=\pm 0.015$	$\phi=\pm 0.030$
0.40	0.05	0.030-j0.013	-0.063+j0.032	0.20+j0.10
0.40	0.10	-0.015-j0.0045	0.037+j0.0097	-0.12-j0.030
0.40	0.20	-0.010-j0.0017	0.022+j0.0035	-0.072-j0.011
1.0	0.05	0.0004-j0.0000002	-0.0004+j0.000001	0.0014-j0.000004
1.0	0.10	0.0010-j0.000008	-0.0016+j0.00002	0.005-j0.00006
1.0	0.20	0.0040-j0.00023	-0.0077+j0.0005	0.024-j0.0016
1.2	0.05	0.0052-j0.00001	-0.004+j0.0002	0.017-j0.00067
1.2	0.10	0.0082-j0.00055	-0.013+j0.0015	0.043-j0.0048
1.2	0.20	0.024-j0.041	-0.047+j0.090	0.14-j0.27
1.8	0.05	-0.0060+j0.000027	0.0033+j0.000085	0.013-j0.00033
1.8	0.10	-0.0050-j0.000037	0.0046+j0.00018	-0.015-j0.00062
1.8	0.20	-0.0036-j0.0001	0.0053+j0.00026	0.017-j0.00083

The approximate solution 'satisfies' the integral equation better at the slot center and the worst at the slot edges. Also it appears that the integral equation is 'satisfied' better when the radiation is insensitive to the cylinder and the shell radial separation, i.e. when we are closer to some particular co-axial 'resonance' than when in between them.

In order to be able to make a quantitative estimate of the error we rewrite (49) in the form

$$\int_{-\phi_o}^{\phi_o} \left\{E_e(\phi') - E'_e(\phi')\right\} K_e(\phi',\phi)d\phi' = K_e^{(a)}(\theta,\phi)\left[-\Delta^{(e)}(\phi)\right] , \qquad (53)$$

Taking the absolute values we have

$$\left|\int_{-\phi_o}^{\phi_o} \left\{E_e(\phi')-E'_e(\phi')\right\} K_e(\phi',\phi)d\phi'\right| \leq \left|K_e^{(a)}(\theta,\phi)\right| \left|\Delta^{(e)}(\phi_o)\right| . \qquad (54)$$

Since $K_e^{(a)}(\theta,\phi)$ is a real function we may argue that at the maximum

$$E_e(\phi)-E'_e(\phi) = \left|\Delta^{(e)}(\phi_o)\right| E_e(\phi) , \qquad (55)$$

and hence

$$E'_e(\phi) = \left[1 - \left|\Delta^{(e)}(\phi_o)\right|\right] E_e(\phi) . \qquad (56)$$

From Table II and the last formula we compute that the maximum possible error in the slot field is 22 per cent for ka = 0.40, 2.4 per cent for ka = 0.0, 32 per cent for ka = 1.2 and 1.7 per cent for ka = 1.8. For the radiation this corresponds to maximum errors of 1.7, 0.2, 2.4 and 0.16 db, respectively. Thus, we feel that the maximum possible error in the data plotted with the exception of the radiation minima, should be a few tenths of a db when the curves are close together, and a few db when they are separated, i.e. when we are far from a particular co-axial resonance. The nature of (33) indicates the minima in the radiation coupling curves for the slot width considered should be at least 25 db deep. Some further work is necessary to establish the depth of the minima.

We have computed the radiation through the shell for three different shell spacings from the cylinder. For the parameters we have selected it would appear that bringing the shell closer to the cylinder in most cases gives stronger radiation than letting the shell be farther away from the cylinder. However, the type of the problems we have does not allow us to extrapolate these results in either direction. Further calculations are necessary to establish this behavior. However, we may offer some comments based on simple physical arguments. It would appear that the radiation goes to zero as the slotted shell coalesces with the cylinder for $\phi_o < \theta < 2\pi - \phi_o$, i.e. the source is not under the shell slot, because then the source is enclosed by a perfectly conducting medium which precludes radiation. When the source is under the shell slot $(-\phi_o < \theta < \phi_o)$ as $b \rightarrow a$, then of course we have the familiar situation of a source on the perfectly conducting cylinder. These comments are also supported by some additional calculations for the case of Figure 3 (ka = 0.40). These were done for kb-ka = 0.025, 0.010, 0.005 and the radiation was successively reduced as kb-ka was decreased from 0.050. The other limit, that of increasing kb-ka to infinity for a given ka has no practical significance. Increasing kb-ka beyond 0.20 we may expect the radiation alternatively to increase and decrease.

We may also ask the physical question: How does the radiation get out when the source is not directly visible through the slot? In an attempt to read some physics into the mathematics we may compute the Fourier coefficient of the E_ϕ component of the electric field (3) in the co-axial space. Taking the cylinder slot voltage at 'a' volts, we have that

$$j\omega\mu k\left[B_n J_n'(kr)+C_n N_n'(kr)\right] \simeq \frac{1}{2\pi D_n(ka,kb)}\left\{\left[J_n'(kr)N_n'(kb)-N_n'(kr)J_n'(kb)\right]e^{jn\theta}\right.$$
$$\left.-\left[J_n'(kr)N_n'(ka)-N_n'(kr)J_n'(ka)\right]\ a_o J_o(n\phi_o)\right\}, \qquad (57)$$

where the approximate sign enters because we use the narrow slot approximation for the shell slot field. The first few coefficients we cannot discuss without some numerical computations. However, when $kb < 2$ and $n \gg kb$, we may use the small argument approximation of the cylindrical functions, and obtain

$$j\omega\mu k\left[B_n J_n'(kr)+C_n N_n'(kr)\right] \simeq \frac{1}{2\pi}\left\{\left[\left(\frac{ka}{kr}\right)^{n-1}-\left(\frac{ka}{kb}\right)^{n+1}\left(\frac{kr}{kb}\right)^{n-1}\right]e^{jn\theta}\right.$$
$$\left.+\left[\left(\frac{kr}{kb}\right)^{n-1}-\left(\frac{ka}{kr}\right)^{n+1}\left(\frac{ka}{kb}\right)^{n-1}\right]\ a_o J_o(n\phi_o)\right\}, \quad kb < 2, \quad n \gg kb . \qquad (58)$$

From (58) we have that on the cylinder surface

$$j\omega\mu k\left[B_n J_n'(ka)+C_n N_n'(ka)\right] \simeq \frac{1}{2\pi}\ e^{jn\theta}, , \qquad (59)$$

and on the shell surface

$$j\omega\mu k\left[B_n J_n'(kb)+C_n N_n'(kb)\right] \simeq \frac{1}{2\pi}\ a_o J_o(n\phi_o) . \qquad (60)$$

It is clear that many coefficients are necessary to approximate reasonably well the E_ϕ component in the co-axial space. The same statement applies to the other two field components. No Fourier coefficient alone under any conditions dominates the field components in the co-axial region, even in the case when for some particular coefficient $D_n(ka, kb) \to 0$. From this behavior of the Fourier coefficients we may conclude that no simple model may be devised to explain the transfer of power through the co-axial region. The power transfer results from the interference of very many Fourier coefficients.

Conclusions

We briefly summarize the salient features of the numerical results of this study. With a fixed magnetic line source on the cylinder the addition of a relatively close fitting slotted shell:

1) enhances radiation for all source and shell slot separation angles θ when the diameter $< 0.5/\pi \ \lambda$,

2) leaves radiation roughly unchanged when $0.5/\pi \ \lambda <$ the cylinder diameter $< 1.1/\pi \ \lambda$, except for a deep minima in the vicinity of $\theta \sim \pm 90^o$,

3) enhances radiation when $1.1/\pi \ \lambda <$ the cylinder diameter $< 1.5/\pi \ \lambda$ except for a deep minima in the vicinity of $\theta \sim \pm 110^o$,

4) leaves radiation roughly unchanged when $1.5/\pi \ \lambda <$ the cylinder diameter $2/\pi \ \lambda$, except for a deep minima in the vicinity of $\theta \sim \pm 45^o$, $\pm 135^o$.

The deep minima in case 2) is associated with the lowest root of $D_1(ka, kb) = 0$, and in case 4) with the lowest root of $D_2(ka, kb) = 0$. It appears that radiation will have no deep minima when $D_C(ka, kb) = 0$, or very close to zero. Some of the antenna parameters for which this will occur are shown in Table I .

We also note that the theoretical work on the problem has been carried to the point where the numerical calculations can be carried out for wider slots, if we so desire. In this particular theoretical work the solution of the Fredholm integral equation of the first kind is based on the truncation of two infinite matrices. It would appear that this particular method of solving the fundamental integral equation of the problem has been put in the most favorable form for carrying out further computations.

Some obvious extensions of the theoretical work accomplished so far include consideration of a dielectric under the shell that is different from that on the outside and the application of the method presented to the case of the narrow slot. On the basis of this study one would then be in a position to choose a dielectric constant ratio such that for a narrow frequency band of interest the cylinder antenna would radiate through the slotted shell for all angular positions of the shell slot.

Another extension is to consider a finite thickness, partially transparent plasma sheath with an infinite axial slot. The radiation then would depend not only on the energy that leaks out through the slot, but also on the leakage through the plasma sheath itself. This is a new boundary value problem and it is expected to be somewhat more difficult than the one discussed in this paper.

Another problem of some potential interest is to consider that the cylinder slot is excited by a wedge waveguide which has a magnetic line source at the origin. This in fact is a complete antenna problem. In order to find the slot fields of the cylinder slot and the sheath slot we have to solve two simultaneous integral equations. The solution of this problem would also show the extent to which the shell depresses the cylinder slot field.

The further problem of relating these solutions to practical antenna configurations is also of continuing importance. For this, physical intuition and understanding of the canonical problem may not be sufficient, and some judicious experiments may have to be undertaken.

LIST OF FIGURES

Figure 1. Cylinder Antenna Shrouded by a Slotted Shell

Figure 2. Radiation Through the Slotted Shell for ka = 0.20

Figure 3. Radiation Through the Slotted Shell for ka = 0.40

Figure 4. Radiation Through the Slotted Shell for ka = 0.60

Figure 5. Radiation Through the Slotted Shell for ka = 0.80

Figure 6. Radiation Through the Slotted Shell for ka = 1.0

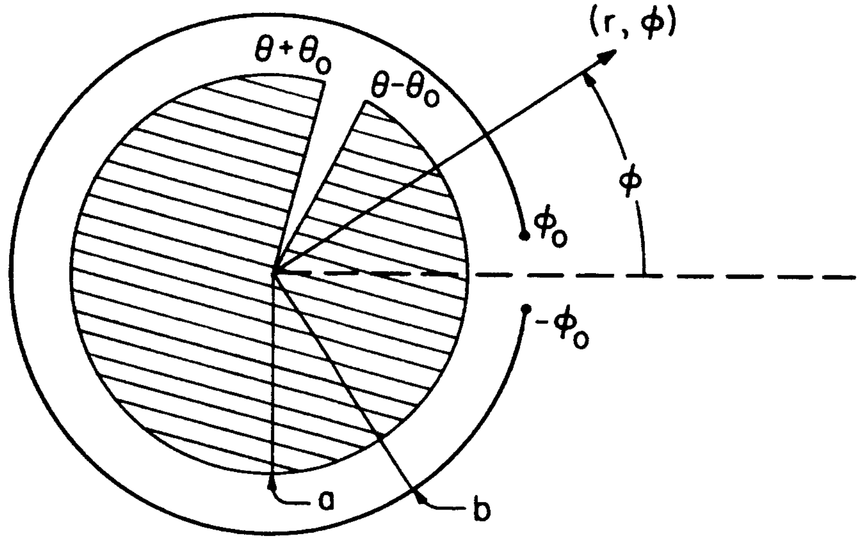
Figure 7. Radiation Through the Slotted Shell for ka = 1.2

Figure 8. Radiation Through the Slotted Shell for ka = 1.4

Figure 9. Radiation Through the Slotted Shell for ka = 1.6

Figure 10. Radiation Through the Slotted Shell for ka = 1.8

Figure 11. Comparative Radiation Efficiency of Cylinders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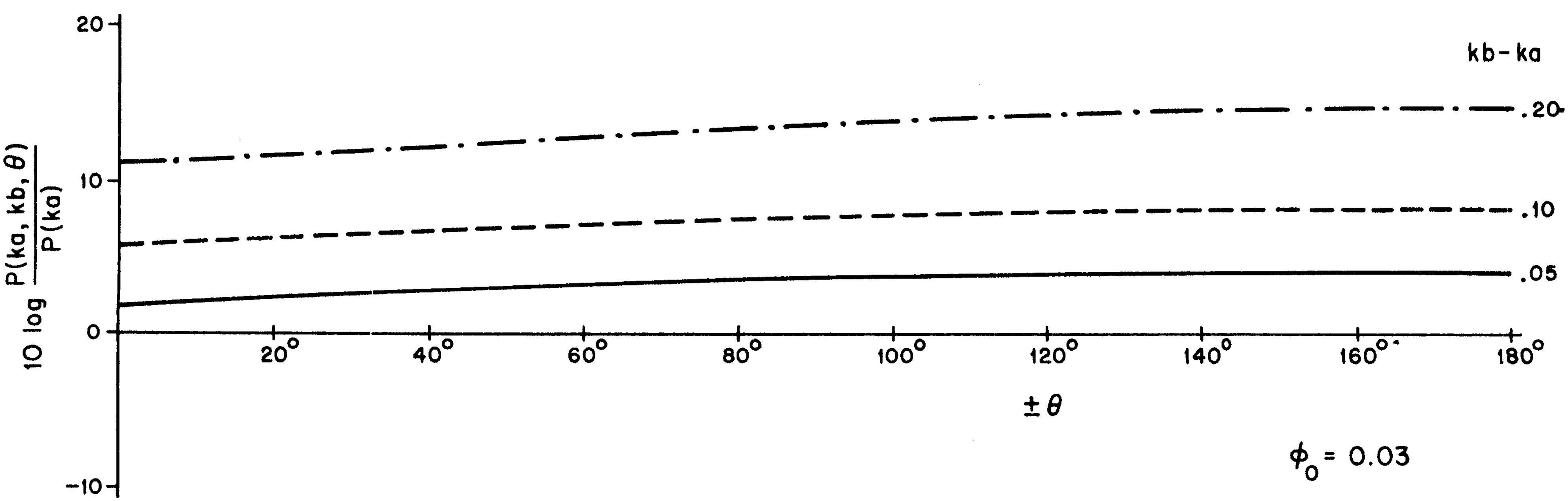
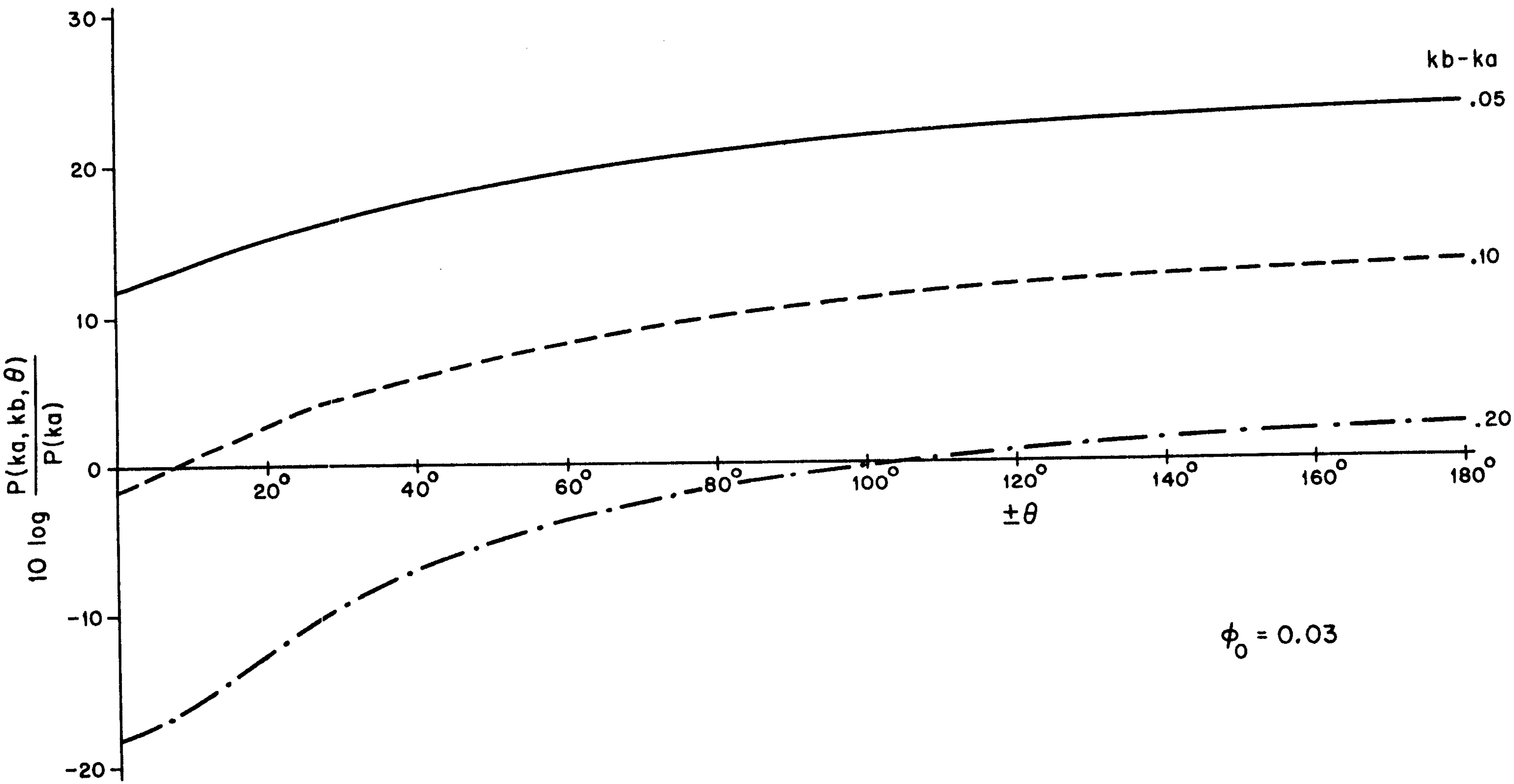
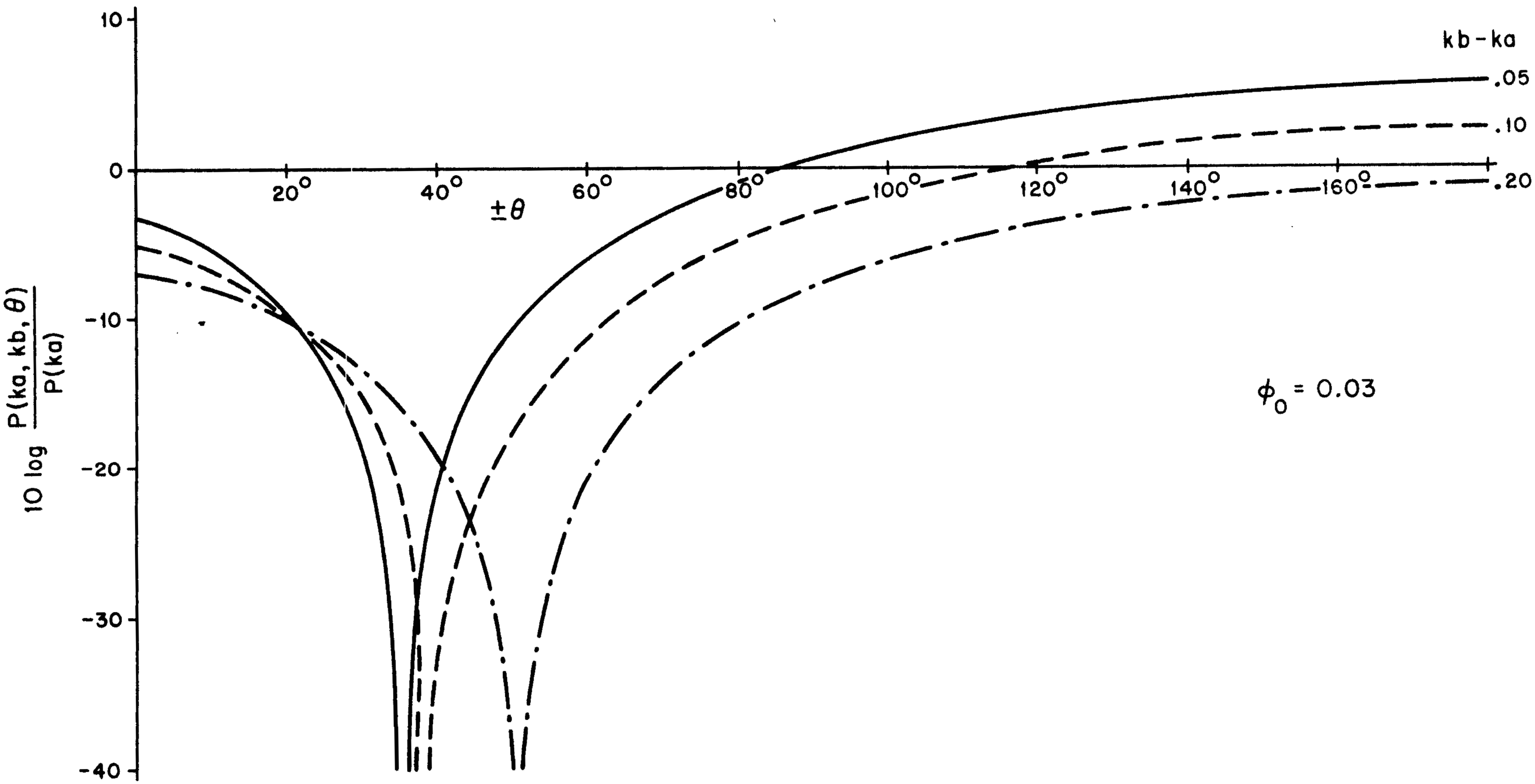
θ+θo
θ-θo
ϕ
ϕo
-ϕo
a
b

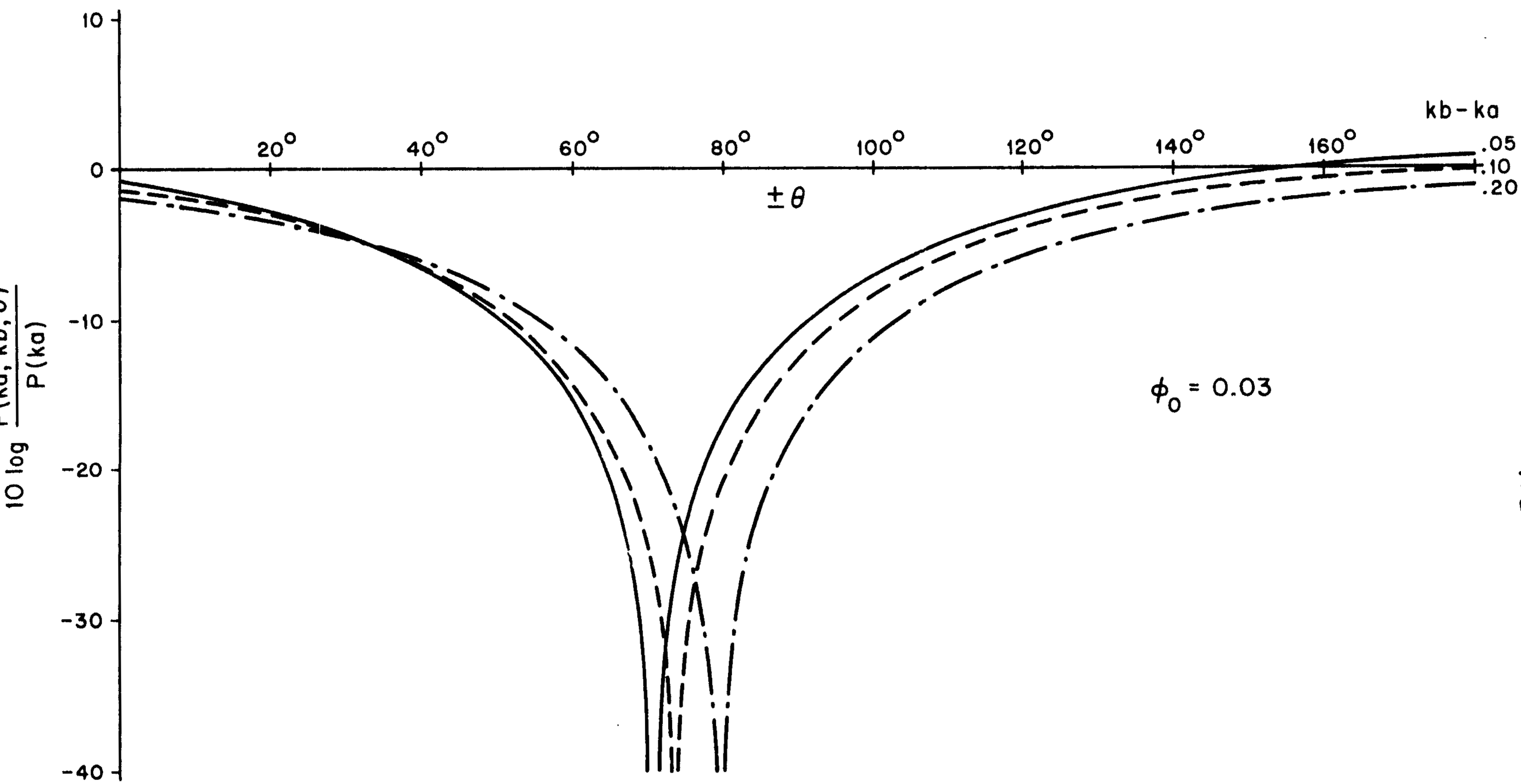


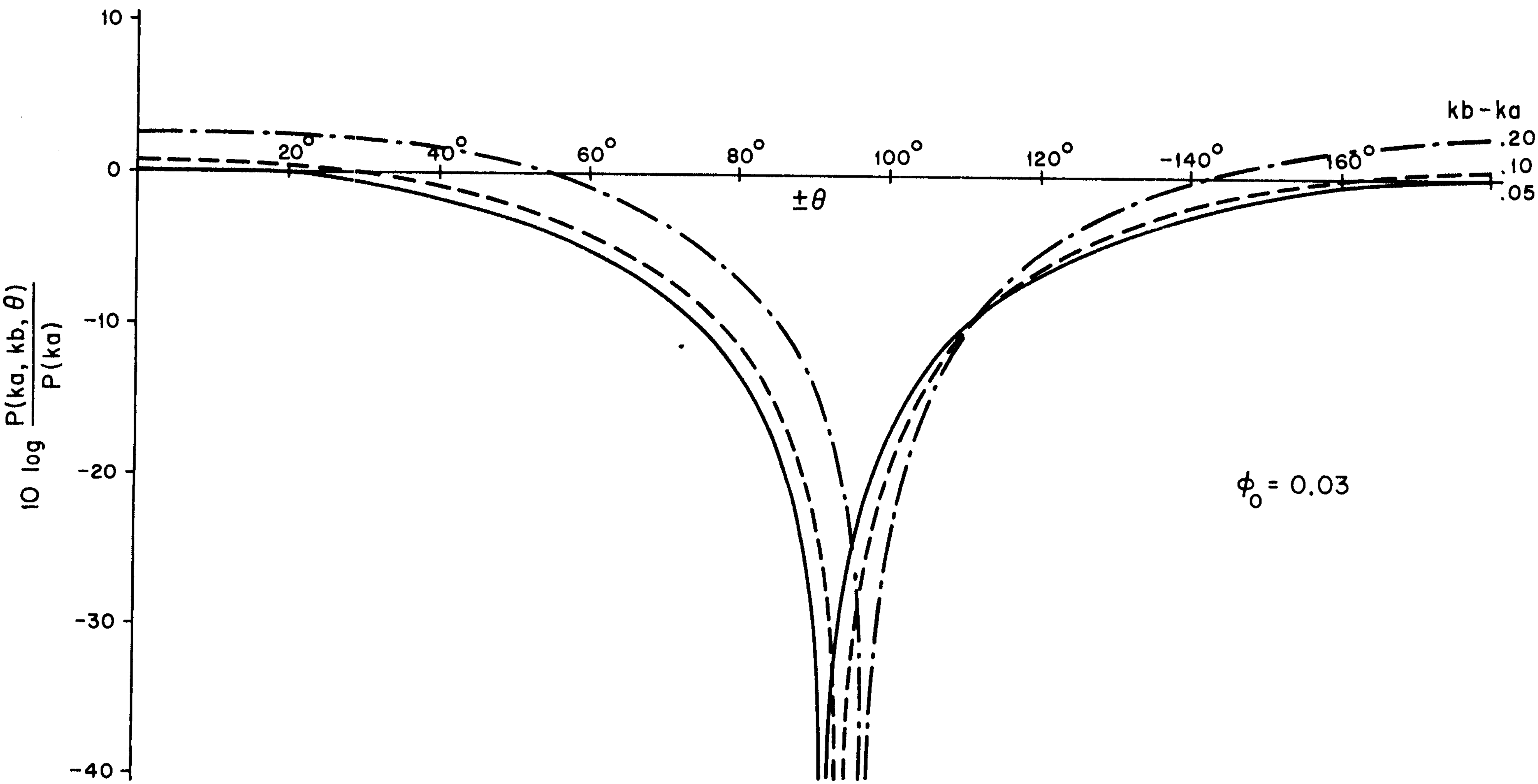
10 log P(ka, kb, θ) / P(ka)
20
10
0
-10
kb-ka
.20
.10
.05
20°
40°
60°
80°
100°
120°
140°
160°
180°
±θ
φ0 = 0.03



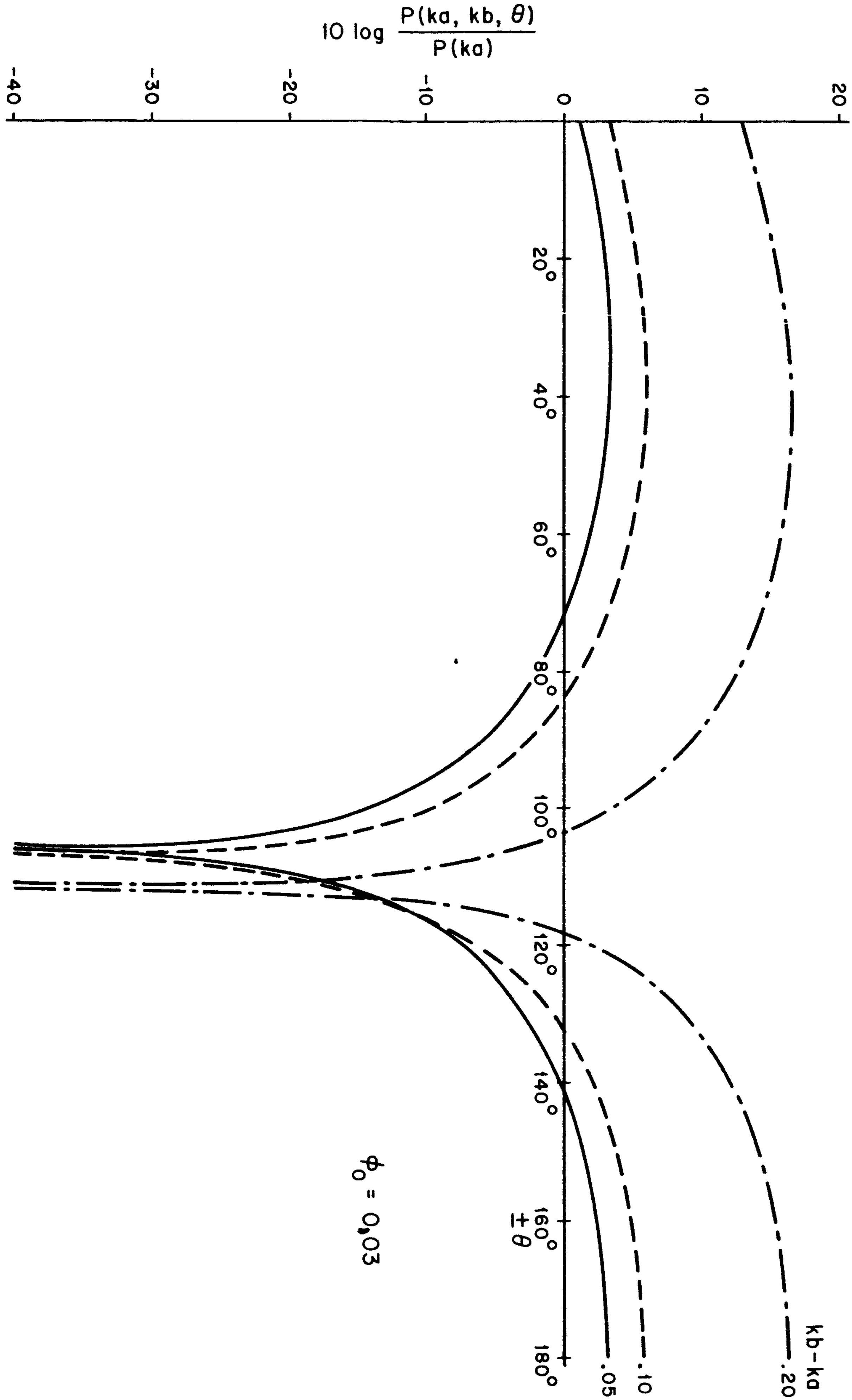
kb-ka
.05
.10
.20
30
20
10
0
-10
-20
$10 \log \frac{P(ka, kb, \theta)}{P(ka)}$
20°
40°
60°
80°
100°
120°
140°
160°
180°
$\pm\theta$
$\phi_0 = 0.03$


10 log $\frac{P(ka, kb, \theta)}{P(ka)}$
10
0
-10
-20
-30
-40
20°
40°
60°
80°
100°
120°
140°
160°
$\pm\theta$
kb - ka
.05
.10
.20
ϕ_0 = 0.03

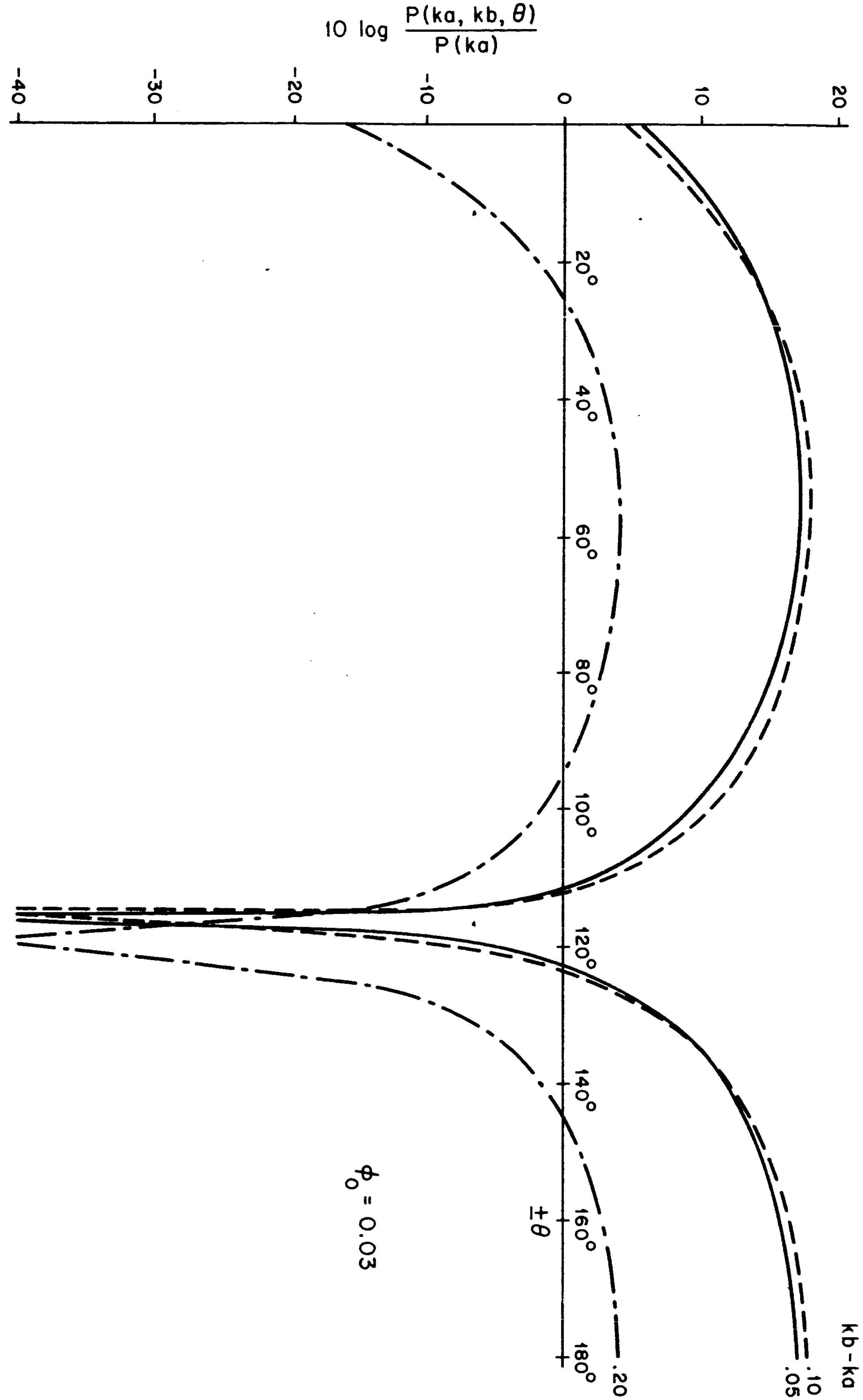

10 log $\frac{P(ka, kb, \theta)}{P(ka)}$
10
0
-10
-20
-30
-40
20°
40°
60°
80°
100°
120°
140°
160°
$\pm\theta$
kb - ka
.05
.10
.20
ϕ_0 = 0.03



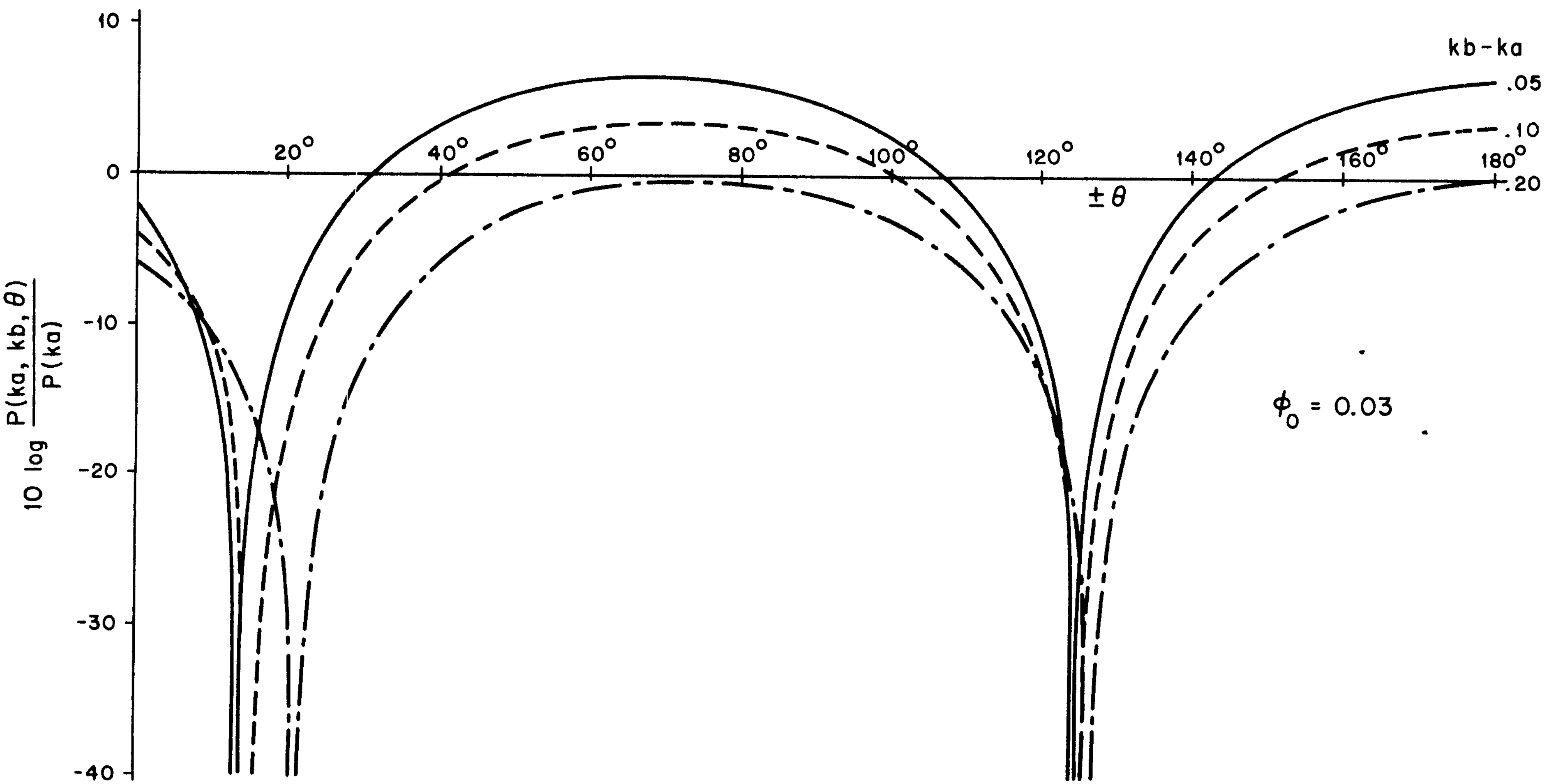
10
0
-10
-20
-30
-40
10 log P(ka, kb, θ) / P(ka)
20°
40°
60°
80°
100°
120°
-140°
160°
±θ
kb - ka
.20
.10
.05
φ0 = 0.03

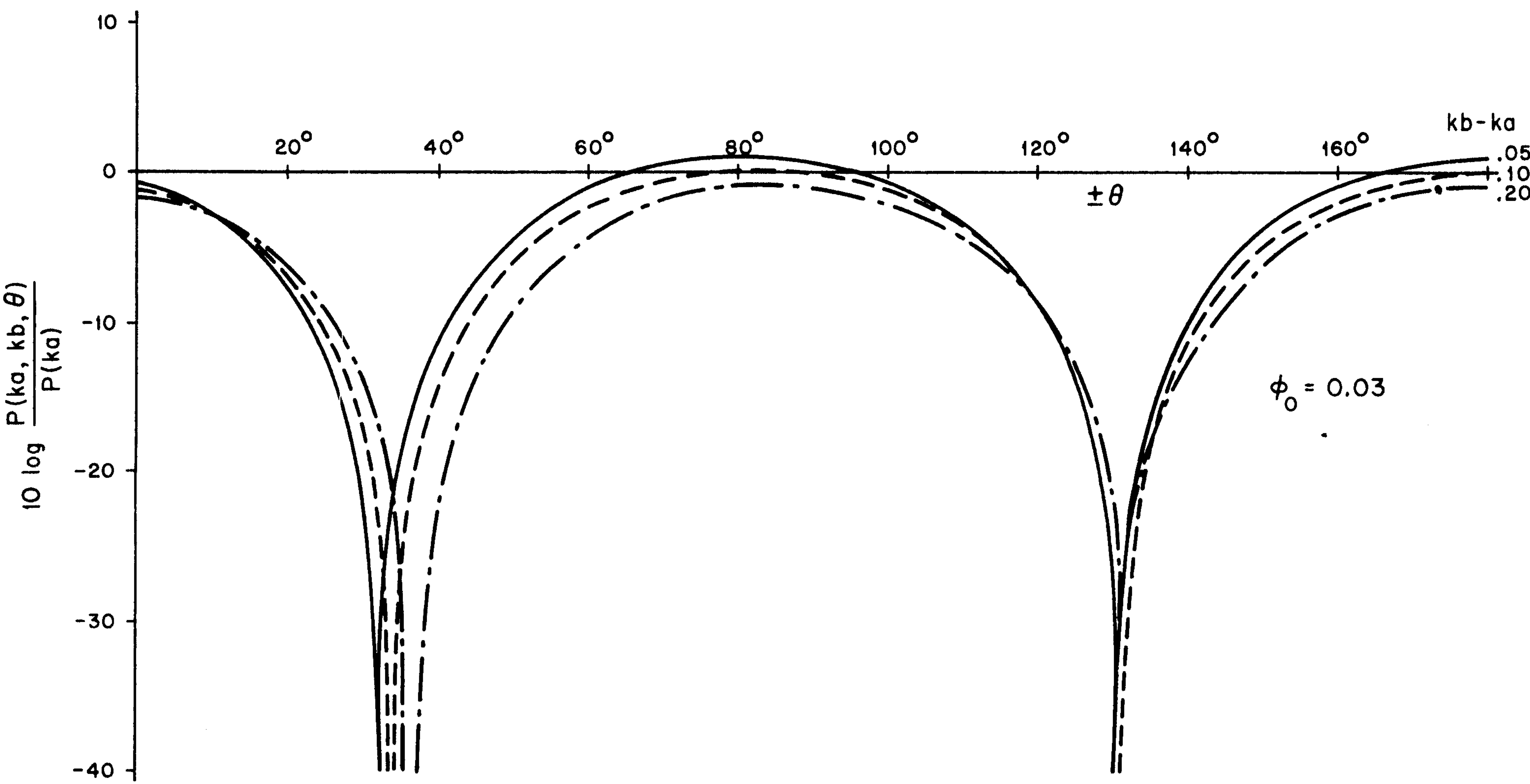


10 log P(ka, kb, θ) / P(ka)
20
10
0
-10
-20
-30
-40
20°
40°
60°
80°
100°
120°
140°
160°
180°
±θ
ϕ_0 = 0,03
kb-ka
.20
.10
.05

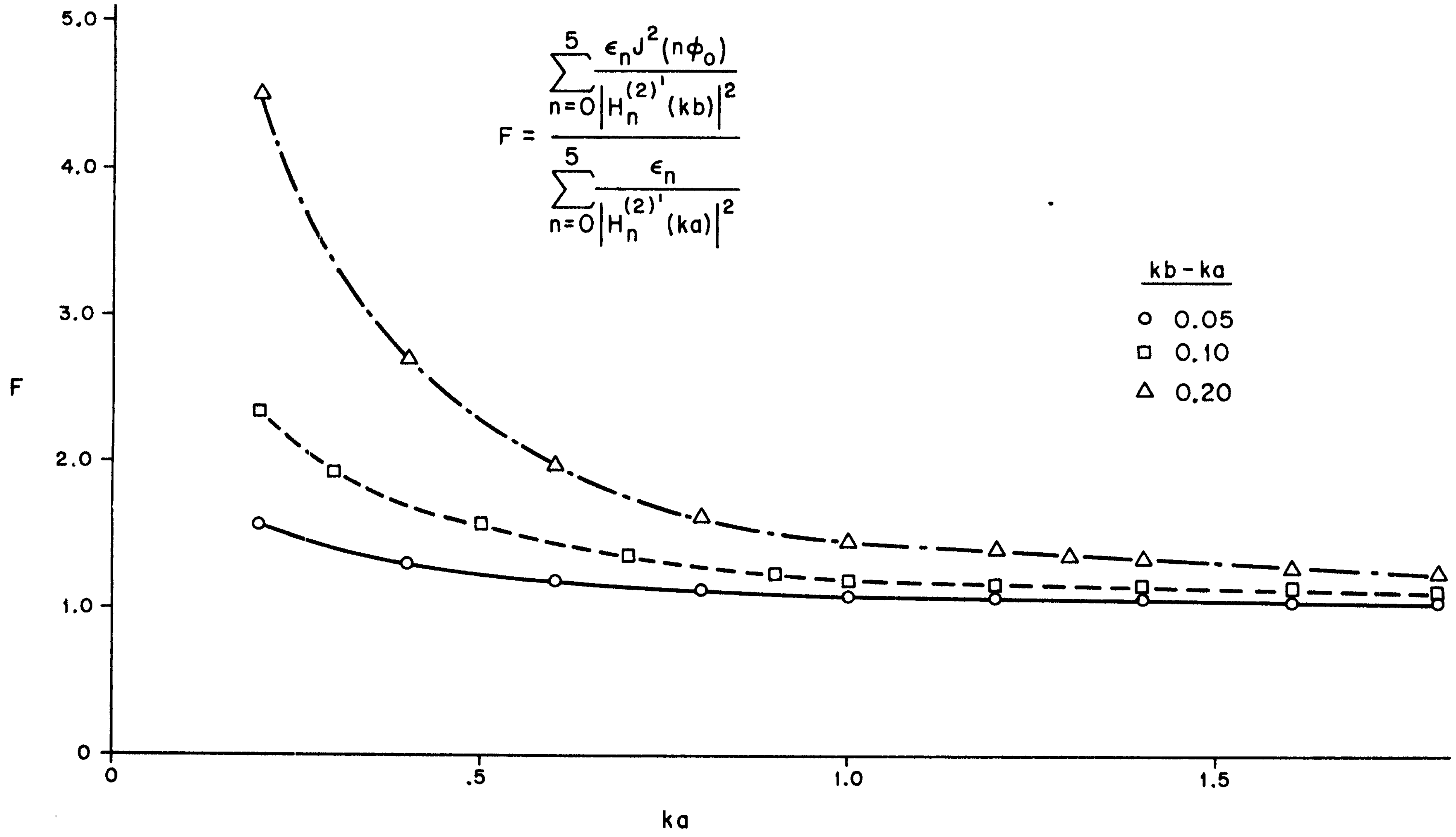


10 log $\frac{P(ka, kb, \theta)}{P(ka)}$
20
10
0
-10
-20
-30
-40
20°
40°
60°
80°
100°
120°
140°
160°
180°
$\pm\theta$
$\phi_0 = 0.03$
kb - ka
.10
.05
.20


10 log P(ka, kb, θ) / P(ka)
10
0
-10
-20
-30
-40
20°
40°
60°
80°
100°
120°
140°
160°
180°
±θ
kb-ka
.05
.10
.20
φ0 = 0.03



10 log P(ka, kb, θ) / P(ka)
10
0
-10
-20
-30
-40
20°
40°
60°
80°
100°
120°
140°
160°
±θ
kb-ka
.05
.10
.20
φ0 = 0.03


$$F = \frac{\sum_{n=0}^{5} \frac{\epsilon_n J^2(n\phi_0)}{\left|H_n^{(2)'}(kb)\right|^2}}{\sum_{n=0}^{5} \frac{\epsilon_n}{\left|H_n^{(2)'}(ka)\right|^2}}$$
kb - ka
0.05
0.10
0.20
F
5.0
4.0
3.0
2.0
1.0
0
.5
1.0
1.5
ka