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TEST PROGRAM ON THIN-WALLED
TUBES WITH ZERO THERMAL GRADIENT

FINAL REPORT

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**NASA-Goddard Space Flight Center,
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FOR
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PREPARED BY

SPECIAL PRODUCTS & APPLIED RESEARCH DIVISION,
THE DE HAVILLAND AIRCRAFT OF CANADA, LIMITED,
MALTON ONTARIO CANADA

FOR

NASA - GODDARD SPACE FLIGHT CENTER,
GREENBELT, MARYLAND

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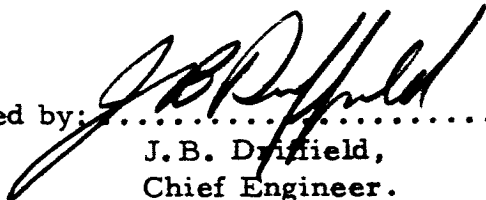
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TABLE OF CONTENTS

SECTION	TITLE	PAGE
	ABSTRACT	ii
	TABLE OF CONTENTS	iii
	LIST OF ILLUSTRATIONS	v
	LIST OF TABLES	vii
1.0	INTRODUCTION	1
1.1	Applicable Documents on Statement of Work	2
2.0	BRIEF OUTLINE OF THE THEORY OF THERMAL BENDING AND PERFORATION	4
3.0	DESCRIPTION OF PERFORATED STEMS	6
3.1	Perforation Patterns	6
3.2	Test Apparatus	7
3.3	Fabrication of STEMS	8
3.4	Storage of Perforated STEMS	9
4.0	DESCRIPTION OF THERMAL SOURCE - VACUUM CHAMBER	15
4.1	Vacuum Chamber	15
4.2	Vacuum Pumping Equipment	16
4.3	Thermal Source Unit	16
4.4	Deflection Measuring System	19
4.5	Deflection Measurement Procedure	19
5.0	RESULTS	27
5.1	Hole Pattern - Intensity Modulation	27
5.2	Deflection Measurements	27
5.3	Reflectance Measurements	32
5.4	Straightness and Diameter of Test Samples	39
5.5	STEM Strength Tests	42
5.6	Silver Film Thickness Measurements	45
6.0	DISCUSSION OF THERMAL BENDING RESULTS	66

TABLE OF CONTENTS Continued

7.0	CORRELATION TO SPACE SOLAR ENVIRONMENT	69
8.0	CONCLUSIONS AND RECOMMENDATIONS	71
8.1	Conclusions	71
8.2	Recommendations	72
9.0	NEW TECHNOLOGY	74
10.0	BIBLIOGRAPHY	75

APPENDIX I Temperature Gradients and Profile Changes
 in Long Tubular Elements Due to Incident
 Radiation.

APPENDIX II Procedure for Determining Boom Straightness

LIST OF ILLUSTRATIONS

FIGURE	TITLE	PAGE
1	Hole Pattern Transmission Apparatus: General View	11
2	Hole Pattern Transmission Apparatus: Recorder	12
3	Hole Pattern Transmission Apparatus: Receiver End	13
4	Schematic - Hole Pattern Transmission Apparatus	14
5	Schematic - Vacuum Test Facility	21
6	Vacuum Pump Assembly	22
7	Vacuum Test Chamber Showing Access Scaffold to Top Plate	23
8	General View - Top Plate and Heater Assembly	24
9	Mounting STEM In Vacuum Test Chamber	25
10	Two Co-ordinate Travelling Microscope	26
11	Typical Hole Patterns	47
12	Regular STEM Pattern with Light Transmittance Curve	48
13	Typical Spiral Pattern With Light Transmittance Curve	49
14	Hole Pattern for 5% STEM	50
15	Hole Pattern for 14% and 20% STEM	51
16a	Co-ordinate System	28
16b	Co-ordinate System for Film Records	28
17	Unperforated Sample Deflection Photographs	52
18	20% Area Removed: Deflection Photographs (Sample 16-20-2L)	53
19	5% Area Removed: Deflection Photographs (Sample 16-5-1SL)	54
20	5% Area Removed: Deflection Photographs (Sample 16-5-4SL)	55

LIST OF ILLUSTRATIONS Continued

FIGURE	TITLE	PAGE
21	Deflection Measurements Made Using Microscope	56
22	Deflection Measurements Made Using Photographic Technique	57
23	Schematic - Integrating Sphere Attachment for Bausch & Lomb Spectrophotometer	58
24	Reflectance Curves for Silver Sample Measured at SPAR and NASA-Goddard	59
25	Reflectance Curve for STEM Sample No. 16-5-1	60
26	Reflectance Curves for PT401 Paint and Oxidized BeCu Tape	61
27	Schematic STEM Bending Rig for Strength Tests	62
28	Root Failure of Perforated STEM Samples	63
29	Tip Load vs Deflection for Perforated STEM	64
30	Bending Strength for Perforated STEM Relative to Unperforated Sample	65
31	Normalized Curve of Solar and 1100°C Blackbody Radiation and Typical Silver Reflectance.	70

LIST OF TABLES

TABLE	TITLE	PAGE
3-1	Punch Diameter and Number of Punches for the 5%, 14% and 20% Dies	8
3-2	Diameter Stability of Stored, Perforated STEM	10
5-1	Vacuum Chamber Thermal Deflection Data (Microscope Measurements)	30
5-2	Vacuum Chamber Thermal Deflection Data (Photographic Data)	31
5-3	Integrated Reflectance and Absorptance (One Sample)	35
5-4	Solar Reflectance and Absorptance of Typical SPAR Electrodeposited Films	36
5-5	Reflectance Data for STEM Samples Used in Vacuum Chamber (B&L Spectronic 505 - Pressed BaSO ₄ Reference)	38
5-6	Diameter Measurement STEM 16-0-1	40
5-7	Diameter Measurement STEM 16-5-4 SL	40
5-8	Diameter Measurement STEM 16-14-1L	41
5-9	Diameter Measurement STEM 16-20-1	42
5-10	Data on Strength Test Specimens	43
5-11	Bending Strength of STEM in Weakest Position	44
6-1	Effective STEM Absorptance α_{EFF} and Thermal Deflections Compared to the Unperforated STEM	67
7-1	Percentage Area Removed for "Zero Bending" STEM In Space, Solar Environment, Outside Silver Plated (α_s), Inside PT401; $\beta_s = 0.95$.	69

ABSTRACT

This report outlines the method and results of a test program on the deflection characteristics of thin walled STEM tubes that have been perforated with a random spiral hole pattern to produce equal absorption of radiant power on the front and rear surface of the STEM, thereby reducing thermal bending due to temperature gradient.

A thermal source-vacuum chamber capable of testing the bending characteristics of 16 foot perforated STEM samples under high vacuum conditions was constructed. Studies were made on, perforation patterns, the reflectivity and absorptance characteristics of the coatings used, the straightness and diameter characteristics of perforated STEM tubes, perforated STEM strength measurements compared to unperforated samples, and film thickness measurements of the silver coatings used.

The test program demonstrated the feasibility of producing "zero bending" STEMs for the chamber environment. It also demonstrated that reproducible high reflectance (low solar absorptance) silver films can be produced by electrodeposition. Results indicated that for the 1100° C thermal source in the vacuum chamber an area removal of 5% results in a "zero bending" STEM. Under the same conditions a 12% area removed STEM would be "zero bending" under solar radiation in space. Strength tests showed that a 20% area removal reduced its strength to half that of an unperforated STEM.

It is recommended that further testing be done on this concept to further test suitable coatings, the effect of overlap and overlap spiraling to eliminate the overlap registration problem, and finally a more detailed study on the reflectivity characteristics of outer and inner coatings under a high vacuum space solar environment.

TEST PROGRAM ON THIN-WALLED TUBES

WITH ZERO THERMAL GRADIENT

1.0 INTRODUCTION

This report covers the results of a testing program on the thermal bending characteristics of thin-walled perforated STEM tubing. The object of perforation being to reduce thermal bending to zero by equalizing the amount of radiant power absorbed on the front outer surface and the rear inner surface.

During SPAR's long history of development and manufacturing of STEMs it has been well known that in space under solar illumination, temperature gradients develop across the tube which will result in the STEM bending away from the sun. Theoretical studies were made by Messrs. Kemper and Farrell in December of 1962 (Appendix I and Reference 1) to derive theoretical formulae which could be used to predict STEM bending under solar illumination. This latter study established those parameters which should be altered to reduce thermal bending, viz. small STEM diameter, small coefficient of expansion, large STEM material thickness, high thermal conductivity, and most important, an outer surface with a low solar absorptance. This resulted in SPAR developing silver electroplating techniques to produce low solar absorptance STEM coatings (see Section 5.3) and thereby reduce bending.

It was further suggested in the above study that it is possible to reduce thermal bending to zero by putting a pattern of holes (perforations) in the STEM to allow a small fraction of the incident radiant power to get to the back inner shadow surface of the STEM. By a suitable balance between the percent area removed and the outer and inner surface solar absorptivities (Reference 14), zero temperature gradient would be feasible and hence such a STEM would not bend under solar illumination.

It was the purpose of this test program to manufacture and test 0.5 inch diameter STEMs 16 feet long which, under collimated illumination would move away (positive), towards (negative) and remain stationary (zero bending) with respect to a source under high vacuum conditions (10^{-5} TORR). The test program also studied the reflectance and absorptance of the coatings used and correlated the results obtained with the theory of thermal bending.

Since the orientation of a STEM in space with respect to the sun will vary it is necessary that the perforation pattern used in a STEM be such

that the amount of radiant power received by the inner rear surface be independent of this orientation. Considerations in previous studies of STEM perforation had indicated that this would best be done by using a random pattern of holes, varying the hole sizes and number per inch of STEM to produce positive or negative bending. Test measurements of the rotational symmetry of the percentage radiant power received by the rear inner surface were made in this program using a special optical transmission apparatus, resulting in the selection of a random spiral pattern of holes as the optimum configuration for rotational symmetry.

To test thermal bending a vacuum chamber, capable of handling 16 foot samples of half inch diameter STEMs was constructed containing a line nichrome filament source running at 1,100°C at the focus of a line of cylindrical mirrors, the latter producing a collimated beam in one plane, illuminating the entire 16 foot length of the STEM. The requirement for a high vacuum in the chamber was necessary to eliminate thermal transport due to gaseous conduction and convection. Deflection measurements for various percent area removed samples were made in the test chamber by direct observation using a two-co-ordinate comparator, and by a photographic technique showing the end cross-sectional view of the STEM before and after illumination by the thermal source.

Measurements were also made on the reflectance characteristics of the outer silver plated and inner black surface of the test STEMs, the straightness and diameter of the STEM samples, and the comparative strength of perforated STEMs compared to unperforated samples. Thickness measurements on the plated samples were also performed.

1.1 Applicable Documents on Statement of Work

The complete outline of the statement of work is covered by the Contract NAS 5-9572 and the de Havilland SPAR Proposal DHC-SP-P.203, dated June 1964, as amended by de Havilland letter to NASA October 19, 1964 with attachments. Temperature gradient measurements were deleted as a requirement and were replaced by measurements of STEM tip deflection.

The work reported herein includes the results of a test program "to validate the theoretical concept whereby a positive, negative, and zero thermal gradient can be manufactured into a thin-walled STEM device. This will be accomplished by arrangement of specific sized holes in the tube, specific coatings on both the inside and the outside walls of the tube and by measuring the positive, negative and zero thermal gradient by accurate tip deflection on a 16 ft. sample. The measurements will be conducted in a vacuum of at least 2×10^{-5} Torr".

It was further stated that thickness measurements shall be made of the coatings tested and that comparative strength measurements shall be performed on perforated samples.

2.0 BRIEF OUTLINE OF THE THEORY OF THERMAL BENDING AND PERFORATION

It is shown in Appendix I that the deflection of an unperforated STEM is approximately given by:

$$x = \frac{1}{8} l^2 \frac{de}{tk} H_s \alpha_s \quad 2-1$$

where

- l = length of the STEM tube
- d = diameter of the STEM tube
- e = the coefficient of expansion
- t = thickness of the STEM wall
- k = the thermal conductivity of the STEM material
- H_s = the solar irradiance
- α_s = the solar absorptance of the outer surface of the STEM (see Section 5.3).

It is seen from equation 1, and as mentioned previously, the importance of maintaining the STEM diameter, coefficient of expansion, and solar absorptance small; while keeping the wall thickness and thermal conductivity large to reduce thermal bending to a minimum.

In order to reduce bending to "zero", that is have a zero temperature gradient, it is necessary to allow a portion of the incident radiation to pass through the outer surface of the STEM to the inner rear surface of the tube. In this case the equation for thermal bending is given by: (Reference 14)

$$x = \frac{1}{8} l^2 \frac{de}{tk} (1 - p) \cdot H_s \left[\alpha_s - \frac{p \cdot \beta_s}{1 + (1 - p)(1 - \beta_s)} \right] \quad 2-2$$

where p = fractional area removed from the outside surface allowing radiation into the interior of the tube.

β_s = the solar absorptance of the inner surface of the STEM tube.

The conditions for zero bending are then seen to be:

$$\alpha_s = \frac{p \cdot \beta_s}{1 + (1 - p)(1 - \beta_s)} \quad 2-3$$

$$\text{and } 1 - p = 0 \quad 2-4$$

Obviously condition 2-4 is academic but does indicate that thermal bending is reduced if the fractional area removed is large. For the case where the inner surface β_s value is large then to a first approximation p is related to α_s and β_s as:

$$p = \frac{\alpha_s}{\beta_s} \quad 2-5$$

That is, the fractional area removed to produce zero bending is approximately equal to the solar absorptance of the outer surface. These latter approximations of course only apply when the inner surface β_s is approximately unity ($.9 < \beta_s < 1$).

In order that equations 2-2 and 2-3 apply to a STEM in space it is necessary that p be independent of the orientation with respect to the sun, and somewhat less obvious that the radiant power (proportional to pH_s) reaching the rear inner surface be constant and independent of the orientation with respect to the sun. This problem is discussed more fully in Section 3 where it is shown that this condition is best met by using a spiral pattern of holes, the holes being positioned along the spiral in a completely random manner.

The above brief treatment of the theory of thermal bending generally outlines the parameters affecting this problem, and the parameters that one may vary to produce negligible bending. In the test program reported here the main variable parameter is the fractional area removed (p) the absorptance values α_s and β_s being kept constant by selecting silver plating for the outer surface and a matt black paint PT401, for the inner surface. The test program does however investigate the variability of α_s and β_s and the possibilities of alternate coating materials (see Section 5.3).

It should be pointed out that the above theory applies to a seamless tube, and since actual samples tested are standard STEMs with an overlap seam, some deviation from the above theory may result. This is discussed in more detail in the latter part of the report (Section 8) after the results of the test program have been presented, discussed, and correlated to the solar environment. The latter is necessary since the test chamber facility utilizes a source whose blackbody temperature (approximately $1,100^\circ\text{C}$) is much lower than that of the sun (5500°C).

3.0 DESCRIPTION OF PERFORATED STEMS

3.1 Perforation Patterns

It will be shown that only a random distribution of holes in the STEM enables the ratio,

$$\frac{\text{energy absorbed by front of STEM}}{\text{energy absorbed by rear of STEM}}$$

to remain constant and independent of the orientation of the STEM with respect to the direction of incident radiation. In particular, if the perforated STEM is placed at a known angle to a collimated beam and rotated, the intensity of the energy transmitted through the STEM will remain constant, if the hole distribution is completely random, but shows a periodicity if the hole pattern is not random.

To determine an acceptable pattern, therefore, an optical test rig, capable of taking a 0.5 inch diameter STEM up to 6 inches long, was constructed. The apparatus, described in Section 3.2, measures the intensity of the radiation transmitted through various perforated samples.

Many 6 inch samples, with holes randomly distributed, were examined on the optical test rig. Samples with a spiral distribution of holes were also constructed and altogether, over 30 different samples were investigated. Figure 11 shows some typical hole patterns investigated. Although the holes and spirals are equally pitched in a spiral distribution, the position of the first hole in any line is varied in a random manner and this ensures that holes on the front of the sample do not line up completely with the holes in the rear. It was also discovered, while fabricating spiral samples, that such samples are more rigid than ones with the holes distributed in a random manner, even though the perforated area was the same. A typical spiral hole distribution is shown in Figure 13 together with the light transmittance curve for this sample as obtained on the optical bench and recorded on the Sanborn chart recorder. Figure 12 shows a regular hole pattern together with a portion of the light transmittance curve for 2 different sample rotation speeds. The first curve is very faint since the recording pens are moving at high speed. Those curves verify that, as expected, a regular hole distribution is not acceptable as a perforated STEM pattern.

It should be noted, with the 6 inch samples, that the holes in the overlap region are deliberately lined up so as to be coincident.

Thus the overlap region gives rise to no vignetting.

3.2 Test Apparatus

The apparatus is shown schematically in Figure 4 and pictures of the rig are shown in Figures 1, 2 and 3.

The collimator consists of a spherical mirror with a focal length of 56 inches. It is seen, from Figure 4, that some coma (which reduces beam collimation) is introduced since the source is off-axis with respect to the collimator. However, this is negligibly small due to the long focal length.

The source is a 0.5 inch diameter aperture covered by a diffusing screen and illuminated by a tungsten-iodide lamp. It should be noted that the reflected beam is thus collimated to within 0.5 of an inch in 56 inches; i.e. 30' arc (the sun subtends 32' of the arc at the earth). The radiation from the source is chopped at 720 cps.

A short focal length concave spherical mirror is used to collect the radiation transmitted through the perforated sample and concentrate it on the silicon diode detector. Although the spherical mirror introduces spherical aberration, the width of the detector is greater than the transverse spherical aberration introduced. Thus all the radiant power transmitted through the 6 inch sample is collected by the detector; e.g. the transverse spherical aberration (TA^1) at the paraxial focus is given by:

$$TA^1 = \frac{h^3}{8F^2} = 0.33 \text{ in.}$$

Where F = focal length and h = distance from axis.

It can be shown that at the circle of least confusion, the transverse spherical aberration reduces to approximately 0.1 inches (i.e. the radius of the aberration circle is 0.1 inches). The detector has a diameter of 0.25 inches which thus allows all the radiant power to be collected.

The perforated sample is rotated, by means of an electric motor, with a period of approximately 3.5 seconds. The sample mount can also be turned about an axis perpendicular to the STEM so that the energy transmitted at any angle to the collimated beam can be determined.

The output from the detector is amplified and demodulated and fed into a Sanborn chart recorder. Some typical curves from the chart recorder are included in Figures 12 and 13.

3.3 Fabrication of STEMs

After selection of the hole distribution (see Section 5.1) a die was made for perforating the STEMs. To reduce edge effects as the STEM axis gets closer to the direction of incident radiation, and to have adequate strength in the punch, the hole diameter should be at least 15 times the STEM material thickness. The BeCu thickness of 0.002 inches thus necessitates hole diameters of over 0.030 inches.

Three different dies were produced, which removed 5%, 14% and 20% of the area by perforation. Two different hole patterns were used (see Figures 14 and 15). The following table summarizes the difference in dies. Each die perforates a section of STEM approximately 1.0 inches long.

TABLE 3.1

PUNCH DIAMETER AND NUMBER OF PUNCHES FOR THE 5%, 14% AND 20% DIES

Area Removed By Die	Hole Diameter	Number of Punches (excluding overlap area)	Hole Diameter in Overlap Area	Pattern
5%	0.036	69	0.046	Shown in Figure 14
14%	0.036	198	0.045	Shown in Figure 15
20%	0.043	198	0.045	Shown in Figure 15

Since perforating a STEM increases the diameter, 0.4 inch diameter unperforated, silver plated STEM material was utilized. It should be noted that when the 0.4 inch Be-Cu STEM (thickness 0.002 inches) was silver plated the edge of the STEM was rippled along its entire length. These ripples, which will be apparent on the STEMs delivered to NASA under this contract, are due to the fact that the silver plating equipment was designed to handle 0.5 inch

diameter STEM. This defect would not be present for a plating mechanical fixture modified for 0.4 inch diameter STEM.

After Ag plating, the STEMs were cut to length, inspected, and diameter and straightness measurements noted (see Section 5.4).

The STEMs were then wound on 12 inch diameter drums and sprayed with PT401, which is a catalyzed epoxy resin, (see Section 5.3) with a very high absorption coefficient, suitable for use in space (produced by Product Techniques Inc., Los Angeles, California). No bake out is required.

The STEMs were then perforated, the diameter increasing in the process (see Section 5.4). The holes in the overlap were designed to coincide with the holes underneath when the STEM diameter is 0.5 inches. Generally, the STEM diameters increased to a value less than 0.5 inches so that the overlap holes were blocked. Although many deflections were observed, and recorded, on such STEMs most results in the report refer to artificially aligned STEMs i.e. STEMs laced up every 12 inches approximately, by means of wire.

3.4 Storage of Perforated STEMs

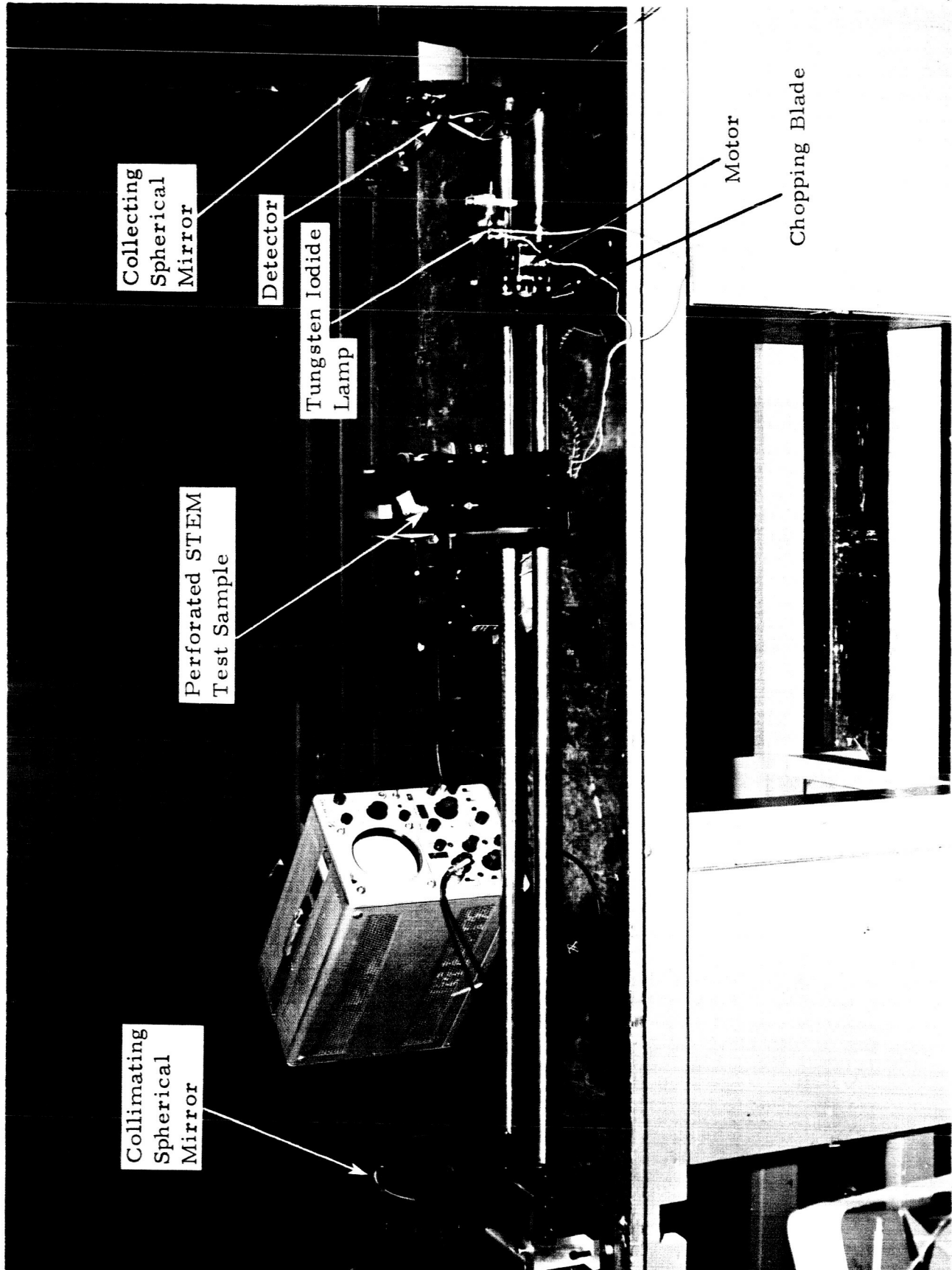
The 14% STEMs were the first perforated samples produced and it quickly became apparent that the sample diameter was increasing when the STEMs were stored as formed taped (i.e. unrolled). Eventually, they opened up so much that the STEMs were no longer closed tubes. There appears to be little doubt that the reason for this is as follows. The 14% die (the first one produced) was not hardened and after a few feet of STEM had been perforated, the die quickly deteriorated. The holes were no longer being punched cleanly but had burrs on the edges. Either the burrs on the holes or the strain placed on the STEM material, as the holes were being punched with an improper die, or a combination of both, was stress relieving the Be-Cu and causing the increase in diameter.

All other perforated STEMs (20% and 5%), which were fabricated with new dies, have shown no tendency to increase diameter when stored as formed tape on cylindrical drums, apart from the initial increase during the perforating process. To determine the stability of the diameter of a stored perforated STEM, an Ag plated, PT401 painted, 20% perforated sample, 46 feet long was stored on a 9 inch diameter drum for 10 days and the results are shown below.

DIAMETER STABILITY OF STORED PERFORATED STEMTABLE 3-2

Length from End of Element	Element Diameter (in inches)		Increase (inches)
	Initial	Final	
3 ft.	0.4745	0.4950	0.0205
15 ft.	0.4635	0.4856	0.0221
30 ft.	0.4721	0.4775	0.0054
42 ft.	0.4723	0.4837	0.0114

The above results are encouraging but the problem warrants further investigation and long term storage data should be determined.

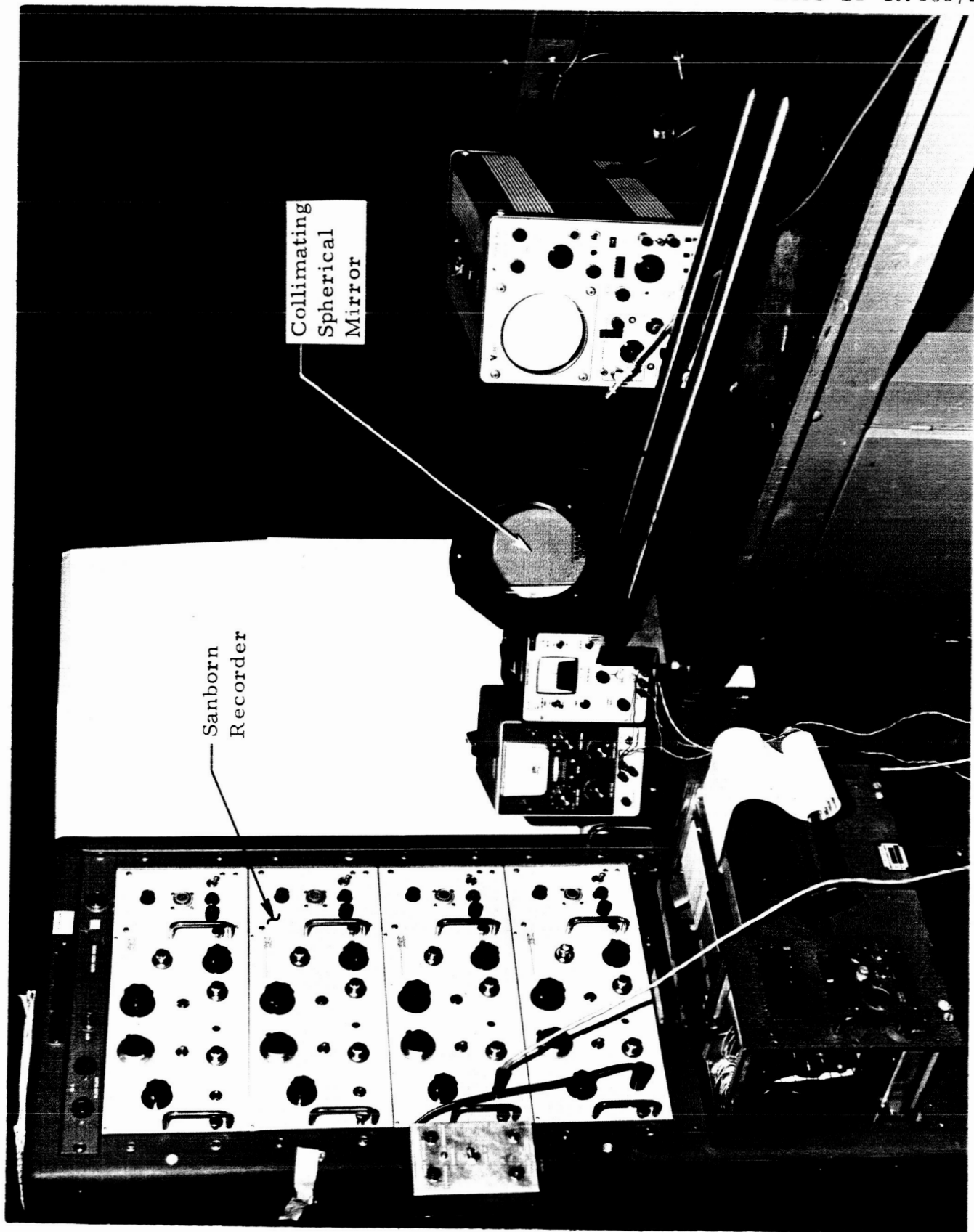


GENERAL VIEW - HOLE PATTERN TRANSMISSION MEASURING APPARATUS

Figure 1



DHC-SP-R.168/2

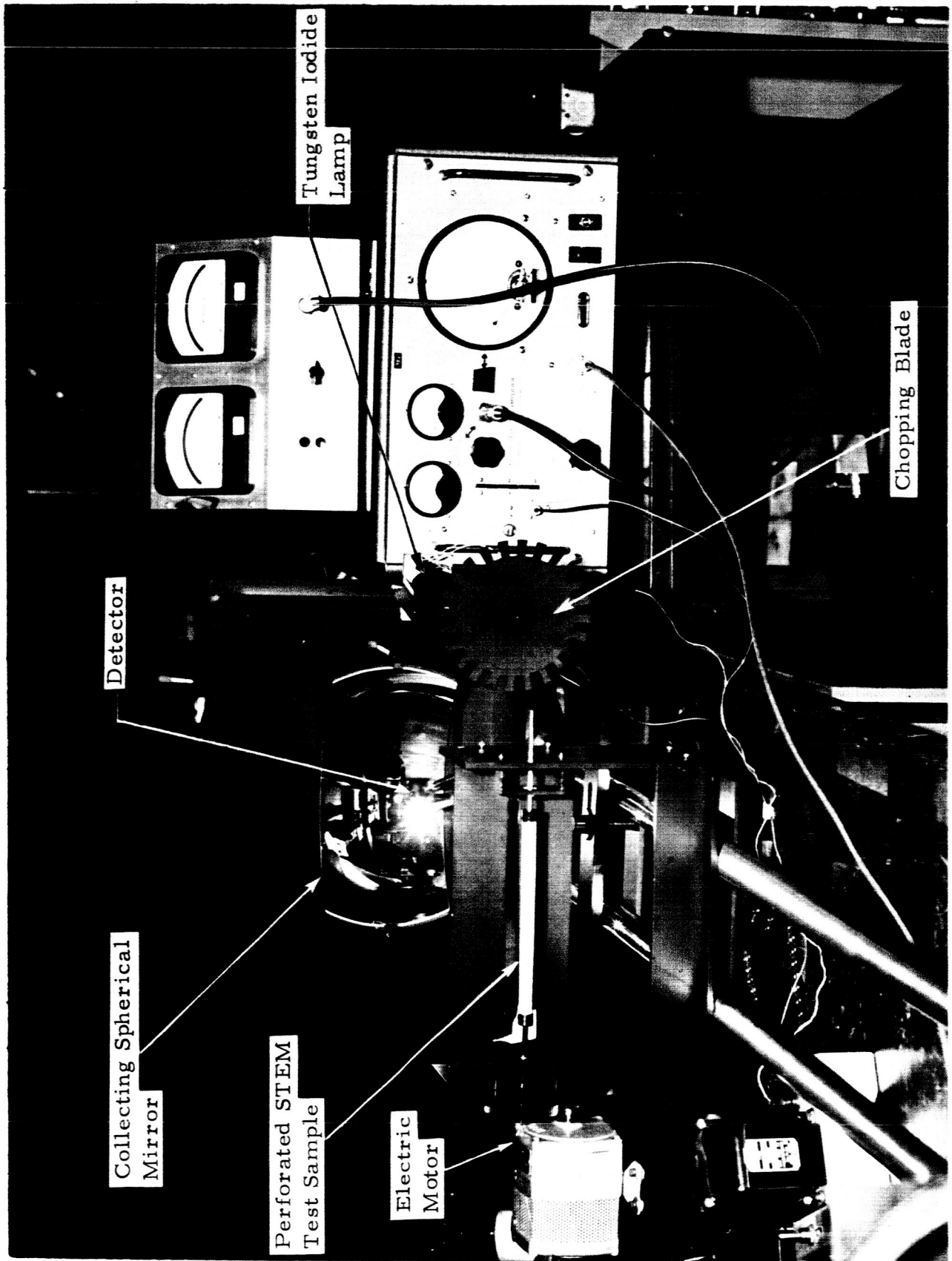


HOLE PATTERN TRANSMISSION MEASURING APPARATUS - RECORDER

Figure 2

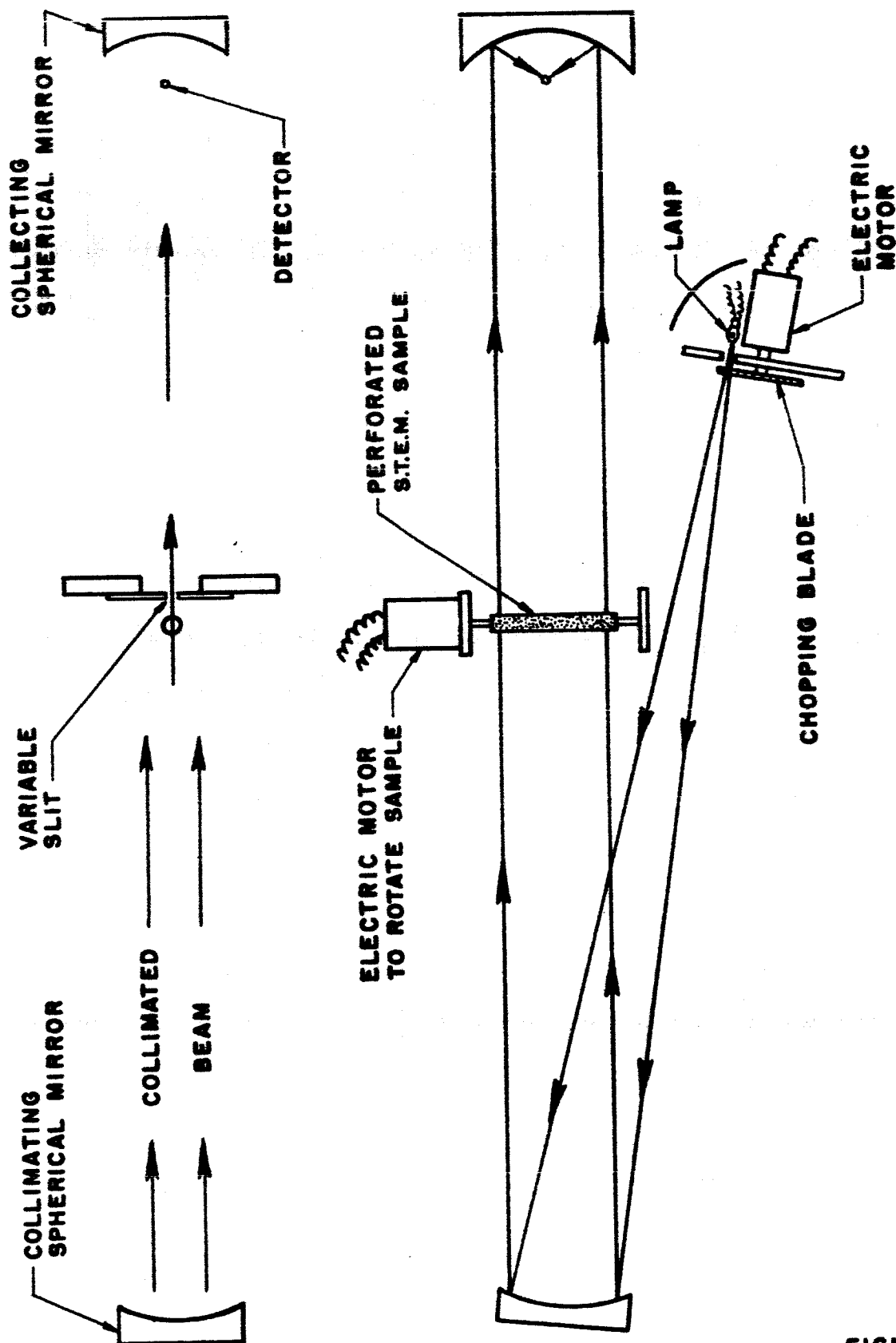


DHC-SP-R.168/2



HOLE PATTERN TRANSMISSION MEASURING APPARATUS - RECEIVER END

Figure 3



SCHEMATIC DIAGRAM OF OPTICAL APPARATUS

FIGURE 4

4.0 DESCRIPTION OF THERMAL SOURCE - VACUUM CHAMBER

The vacuum test chamber was built for perforated STEM studies and approximately simulates the total level of solar irradiance in space but not the spectral distribution. Pressures as low as 9×10^{-6} torr are obtainable.

4.1 Vacuum Chamber

The chamber shown in Figures 5, 6, 7, and 8 consists of a 16 foot high, 12.0 inch diameter, cylindrical section of steel piping. The inner walls are coated with a special matt black, absorptive, epoxy paint ("Cat-a-lac" black, part No. 463-3/8) designed for use in low vacuum systems and produced by the Finch Paint and Chemical Co., Torrance, California. The absorption coefficient in the visible is approximately 95%. A 12 inch diameter elbow, connected to the diffusion pump, enters the chamber about 2 feet from the bottom face. A 7 inch diameter viewing port (0.75 inch thick pyrex) is located in the bottom plate, and the top plate supports the heater assembly (described in Section 4.3). The top plate is bolted to the chamber and sealed with a VITON 'O' ring. An aluminum angle, containing a pair of water coolant pipes and painted matt black, runs the length of the chamber, opposite the heater, and serves as an absorptive target to absorb radiation escaping past the STEM sample and prevent reradiation into the chamber.

The STEM sample (15 ft. 9 ins. long) is mounted on a 0.5 inch diameter rotary seal, supplied by Edwards High Vacuum, which is screwed into the top plate in the chamber and is sealed by an 'O' ring. A small circular plate is mounted over the seal and is attached to the top plate by means of 4 threaded bolts. By adjusting the screws independently the compression in the 'O' ring could be varied enabling the rotary seal shaft to be tilted. Any slight bows in the STEM, which caused the tip to be outside the collimated beam could thus be compensated for by tilting the rotary seal shaft. The STEM tip could be made to swing through at least ± 2 inches.

By rotating the shaft, the STEM, could of course, be oriented with the overlap in any desired position with respect to the incident radiation.

The bottom of the chamber is supported about 2 feet above floor level to enable an observer to measure tip deflections using the travelling microscope or the photographic recording system.

4.2 Vacuum Pumping Equipment

The pump assembly is shown in position in Figures 5 and 6. A two stage gas ballasted rotary pump, supplied by Edwards High Vacuum Ltd., ("Speedivac" model ED500), with a displacement of 500 litres/minute and an ultimate vacuum of better than 10^{-4} torr, was used in conjunction with a 9 inch diameter oil vapour fractionating diffusion pump (Edwards model F903) with a capability of 5×10^{-7} torr, and a displacement of 1600 litres/sec. A 12 inch baffle valve and a 9 inch liquid nitrogen cooled vapour trap complete the pumping equipment.

Pirani gauges read the pressures in the roughing line and backing line while the chamber pressure is monitored by means of an ionization gauge head mounted in the chamber wall above the elbow.

4.2.1 Evacuating Chamber

The rotary pump evacuates the chamber to less than 0.35 torr. (the maximum pressure at which the diffusion pump can be operated) in 7 minutes. The baffle valve is then opened and a pressure of 5×10^{-5} torr is normally obtained within 30 minutes. The pressure then drops slowly and continuously while the pump is operating, another hour usually being required to reach 2×10^{-5} torr. As the system slowly outgasses, lower pressures are being obtained during deflection measurements; the lowest pressure obtained so far was 8.6×10^{-6} torr. If the vapour trap is not charged with liquid nitrogen the pressure levels off at 2×10^{-5} torr.

4.3 Thermal Source Unit

The heater assembly is illustrated in Figures 5 and 8.

The radiant power is supplied by a 15 foot length of tophet 'A' wire (80% Ni, 20% Cr.) running at approximately $1,100^{\circ}\text{C}$ and the wire diameter (B.S. gauge 20, 0.032 inch diameter) has been chosen to ensure that the irradiance at the STEM is approximately 140 milliwatts per sq. cm.

The energy radiated from the source is collected by 12 cylindrical, aluminum mirrors, each 14.4 inches long, with a radius of 0.75 inches and a width of 1.2 inches. Some of the energy from the heater wire is absorbed by the aluminum mirrors (absorption coefficient 0.20 approximately), some is collected by the cooling pipes, and

some escapes past the pipes into the chamber (where the intensity from this radiation, obeying a $1/d^2$ law, will be negligible). The required wattage to be dissipated in the filament to give an irradiance of 140 milliwatts per sq.cm. (i.e. 132 watts per sq.foot) is therefore calculated as follows:

$$\begin{aligned} \text{Mirror radius} &= 0.75 \text{ inches} \\ \text{Mirror width} &= 1.2 \text{ inches} \\ \text{Heater filament - mirror separation} &= 3/8 \text{ inch} \end{aligned}$$

The radiant power from the source is collected by the mirrors over 166° .

Ignoring errors introduced due to spherical aberration in the collimated beam, then the beam area = $\frac{15 \times 1.2 \text{ sq.ft.}}{12}$

$$= 1.5 \text{ sq. ft.}$$

Approximate reflection coefficient of mirrors = 0.8.

Required irradiance in beam = 132 watts/sq.ft. If W is the wattage in the wire then,

$$W = \frac{132 \times 360 \times 1.5 \text{ watts}}{166 \times 0.8}$$

$$= 540 \text{ watts.}$$

In practice, 800 watts was dissipated in the filament since the beam width tends to be wider than 1.2 inches due to out of focus effects, mirror aberrations, vibrations in the filament, etc.

The heater assembly is designed to permit rapid replacement of the filament by various gauges of wire thus permitting the irradiance to be varied but keeping the filament temperature constant (1100°C). Thus the STEM could be made to absorb the same total radiant power as in a solar environment, and the improvement in Ag reflectance with increasing λ compensated for (see Section 7.0). The power dissipated in the filament was read on a wattmeter.

A weight is attached to the bottom of the wire and runs in a guide tube allowing the wire to expand through approximately 4 inches when heated from ambient temperature to operating temperature.

If a heat source is to accurately simulate solar radiation in space it must: - (Reference 6)

- 1) have the same blackbody spectrum as the sun.
- 2) produce a total integrated irradiance of 450 BTU's/hours per sq.foot (or 140 milliwatts per sq.cm.) at the target and,
- 3) the incident radiation must be collimated within 32 minutes of arc.

Practical limitations made it impossible to fulfill all conditions simultaneously over as large an area as that covered in the chamber (15 ft. by 1.2 inches) and the thermal source can only satisfy condition 2. Consequently it can in no way be considered to be a "solar simulator" but a thermal source producing the same total irradiance as that which exist in space due to the sun. Figure 31 shows the blackbody spectrum of a source at 1100°C peaking at 2.1 microns compared to the solar spectrum in space. The correlation between chamber thermal deflections and those for a solar environment is considered in Section 7.0.

The reflected power from the chamber source is collimated in the horizontal plane only. The effect of non-vertical collimation on a perforated STEM is to cause a slightly more even distribution in the irradiance on the rear face (i.e. radiant power which has travelled through the holes) than the distribution which would be obtained by a near collimated source such as the sun.

Since the energy from the source is collimated in one plane only, the intensity is thus a function of $1/d$ where d is the source - STEM separation. The STEM diameter is 0.5 inches and the STEM centre is approximately 6 inches from the source; thus the irradiance at the front of the STEM is $K/5.75$ and the irradiance at a position on the rear of the STEM is $K/6.25$ where K is a constant. Since $K/5.75$ and $K/6.25$ are equal to within 8%, then the errors introduced in assuming constant irradiance can be neglected.

As shown in Figure 5, the heater wire and source are surrounded by water cooled matt black painted pipes which prevent stray radiation reaching the chamber walls and also serve to cool the mirrors and prevent expansion buckling under the heat from the source. The whole thermal source assembly is attached to the top plate. When in the chamber, a yoke is used to hold the bottom of the unit rigidly in place.

4.4 Deflection Measuring System

STEM deflections were measured using 2 different methods:

- 1) Directly using a two co-ordinate comparator, and
- 2) Photographically.

4.4.1 Two Co-ordinate Comparator Microscope System

A two co-ordinate comparator was mounted below the viewing port, with the microscope held inverted in the chassis and focussed on the STEM tip. The tip was illuminated by means of a lamp outside the chamber and deflections as small as $\pm .0005$ inch could be measured.

4.4.2 Photographic System

The microscope measuring system could be replaced by a 35 mm camera, with a 50 mm lens, mounted below the window on an aluminum frame bolted to the chamber. By using extension rings on the lens, the ratio of image size to object size is very close to unity. The camera is capable of making multi-exposures on the same frame. Typical double exposed pictures are shown in Figures 17, 18, 19 and 20.

4.5 Deflection Measurement Procedure

Deflection measurements were obtained on a typical STEM (perforated or unperforated) as follows:

- 1) The sample, of which the straightness, diameter and reflectance parameters had previously been determined, was mounted on the rotary seal and cut to a length of 15 ft. 9 ins.
- 2) The STEM was then mounted in the chamber and the shaft rotated to get the overlap in the desired position (see Section 5.2). The twist in the overlap of the STEM was noted. The rotary seal shaft was tilted slightly, as necessary, until the STEM tip was in the centre of the observation port, i.e. vertically below the rotary seal.
- 3) The chamber was then evacuated.
- 4) When the pressure reached approximately 3×10^{-5} torr the cooling water supply was turned on and the heater checked for

operation and adjusted to dissipate 800 watts.

- 5) Several tentative runs were then made to ensure consistency in STEM deflections before recording measurements were taken.

6) a) Microscope Readings

The microscope was alternately focussed on 3 points on the STEM tip and readings taken with the source off. The source was then switched on and readings again taken on the same 3 points. It is necessary to take readings on 3 points to determine any change in STEM diameter and any twisting as the STEM is deflected.

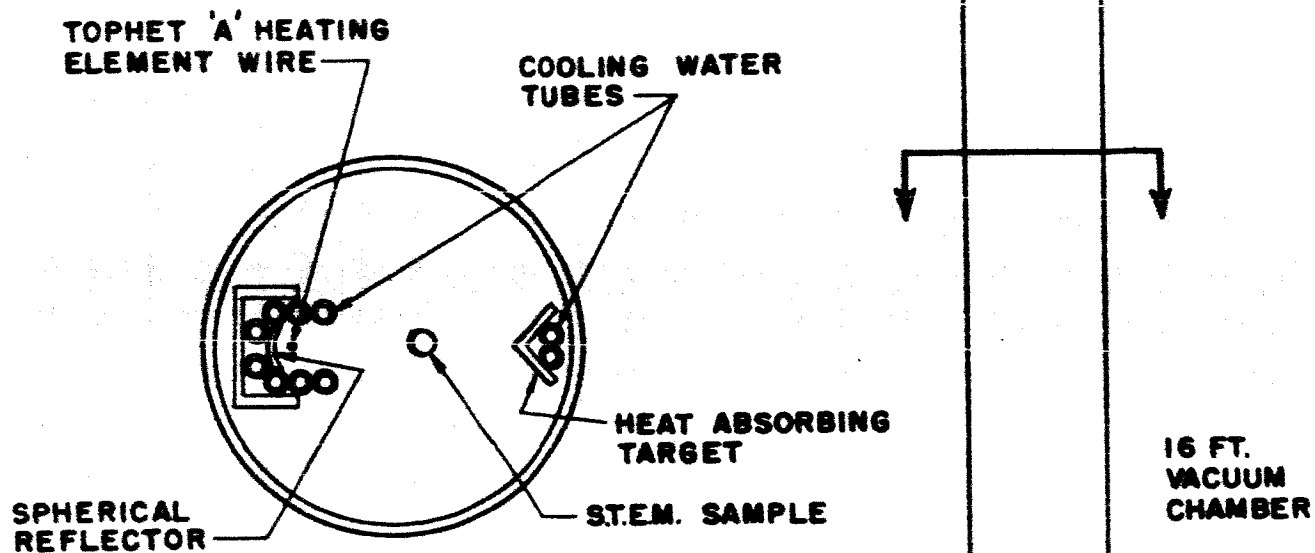
This procedure was repeated a minimum of 3 times for each STEM orientation and often results were also taken with the source running at 600 and 700 watts.

b) Photographic Procedure

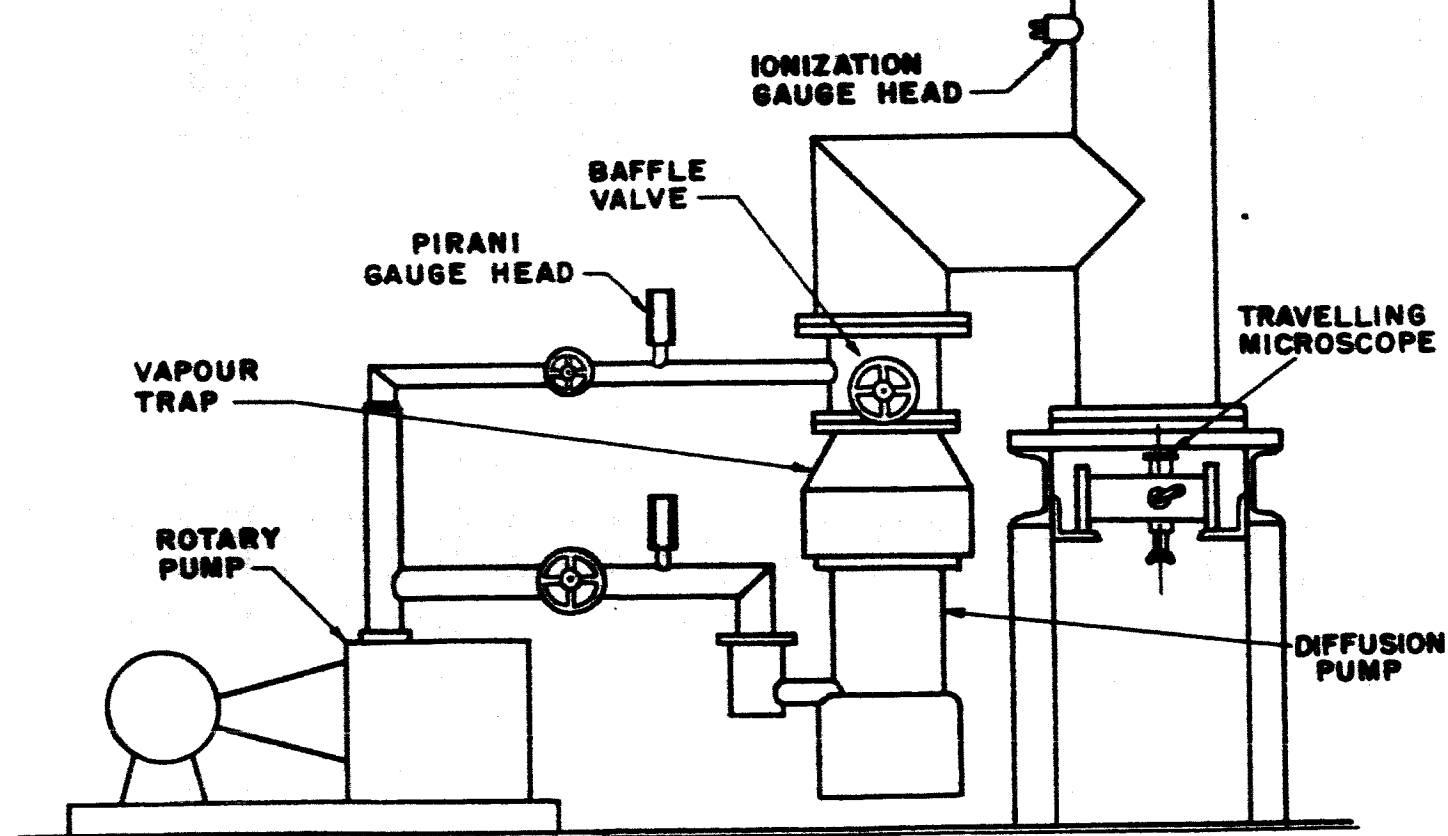
The camera was mounted in position below the window and focussed on the STEM tip by adjusting the tip - camera lens separation. An exposure of 1/2 sec. at f2.8 was then made with the source off. The source was then switched on, the shutter mechanism recocked and a double exposure made by photographing the STEM after the heater had been on precisely 1 minute. This procedure was repeated several times. The light from the source always appears on the film.

It should be noted that, after the source is switched on, the STEM swings through approximately 75% of its travel in 20 seconds. After 1 minute it has travelled through about 95% of its maximum deflection and appears to take several minutes to reach its final position.

- 7) The STEM was then rotated so that the overlap was in a new position (see Section 5.2.1 for STEM positions) and the procedure repeated.



SECTION THROUGH VACUUM CHAMBER

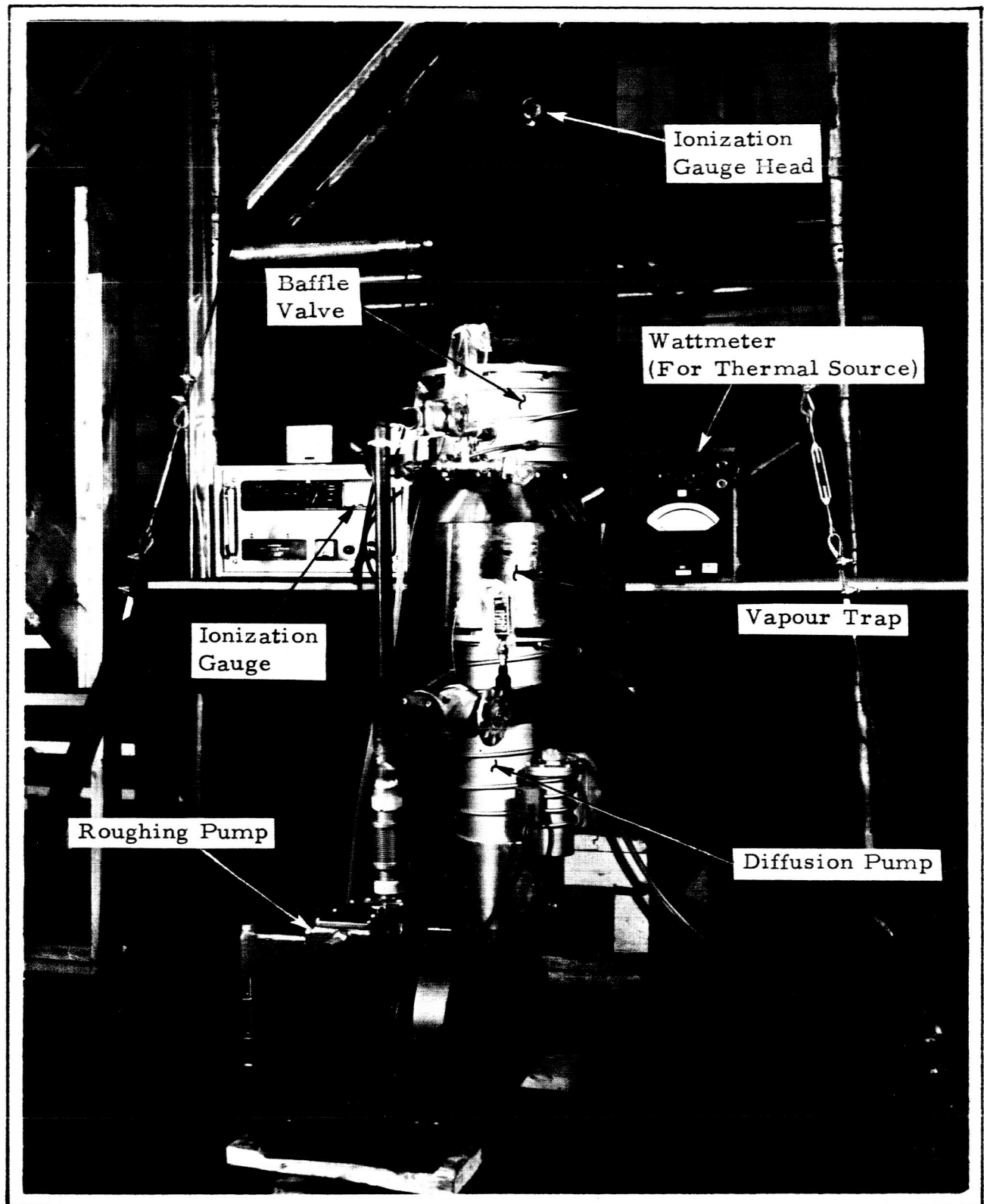


VACUUM TEST FACILITY

FIGURE 5

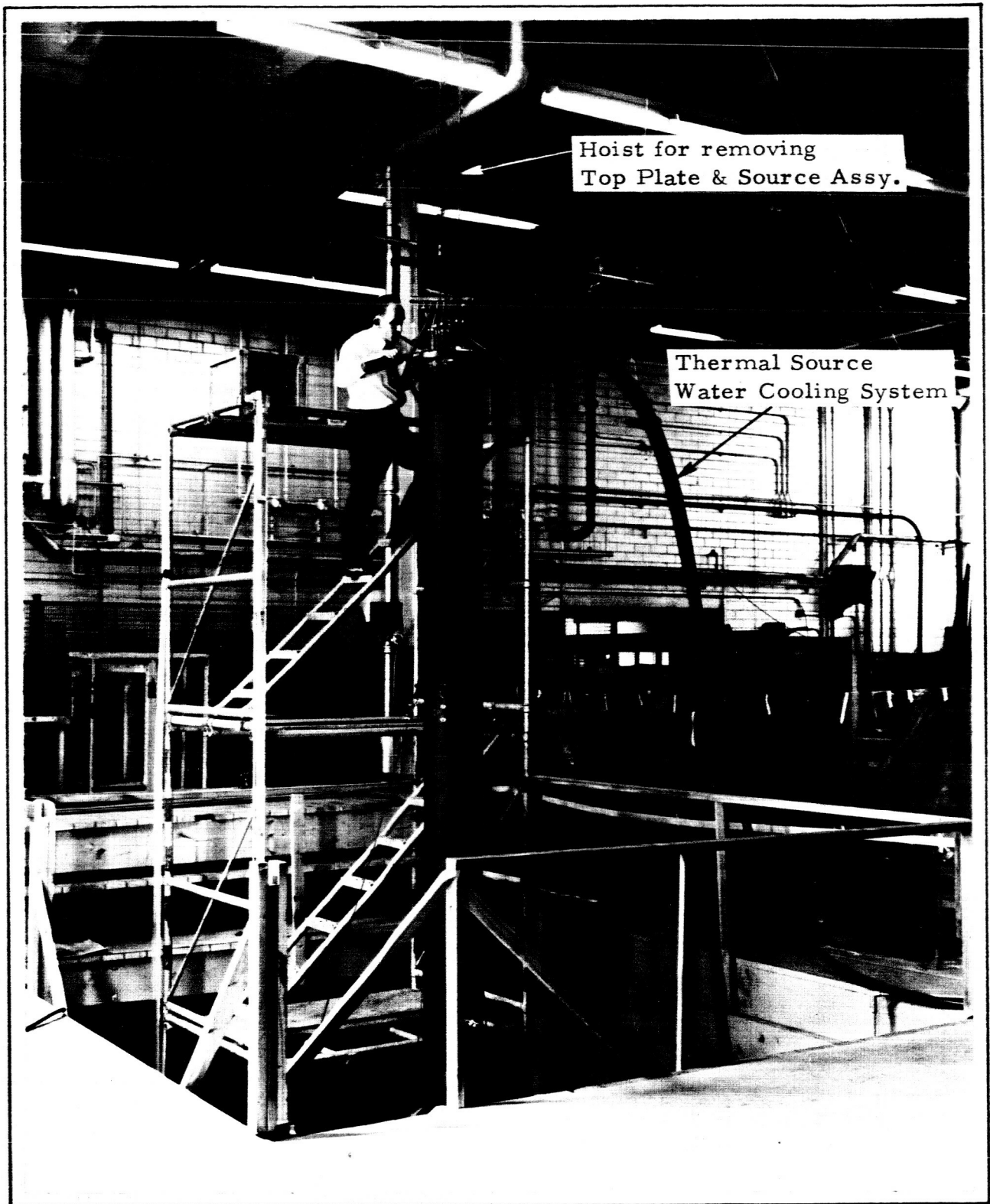


DHC-SP-R.168/2

VACUUM PUMP ASSEMBLY



DHC-SP-R.168/2

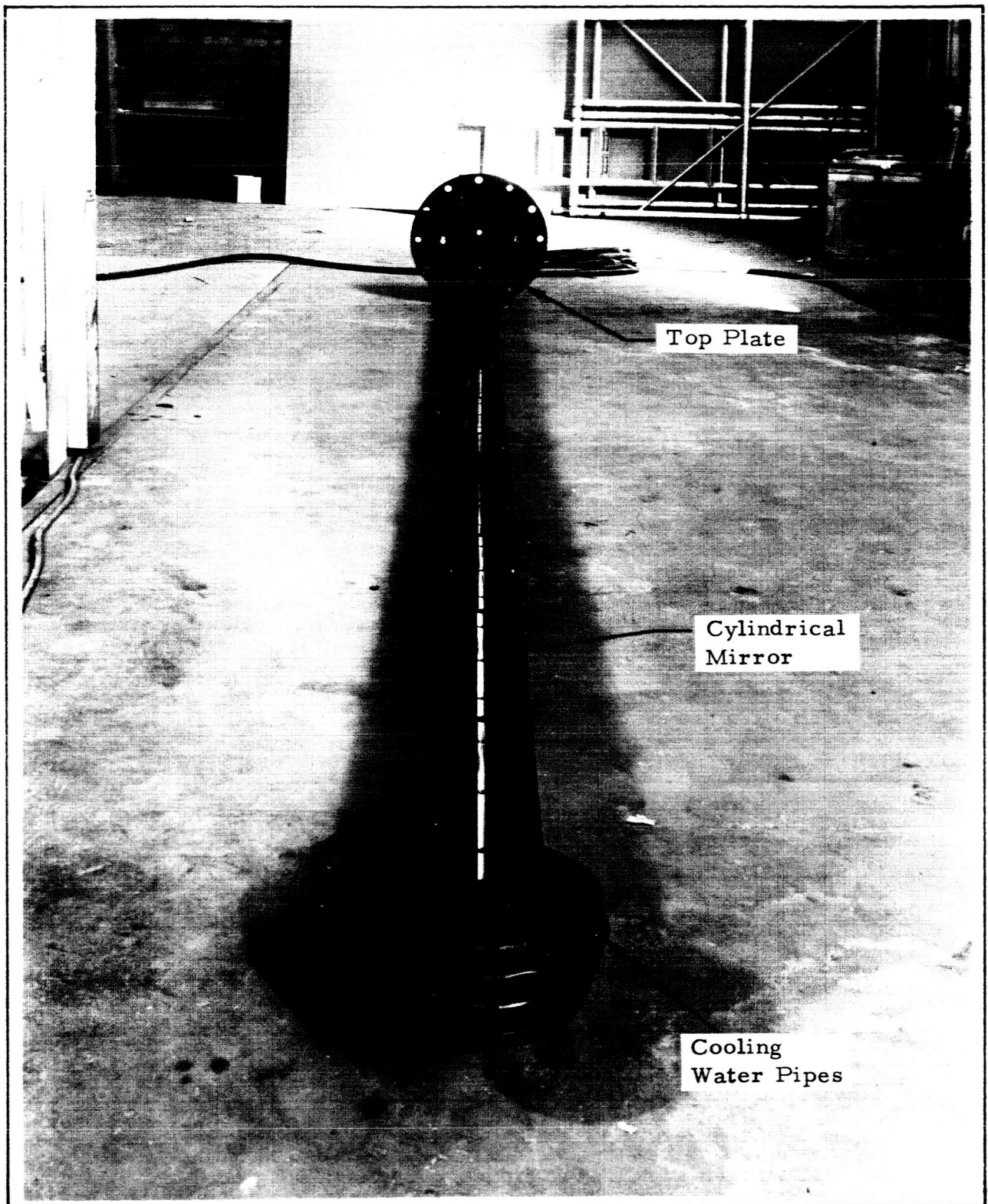


VACUUM TEST CHAMBER SHOWING
ACCESS SCAFFOLD TO TOP PLATE

Figure 7



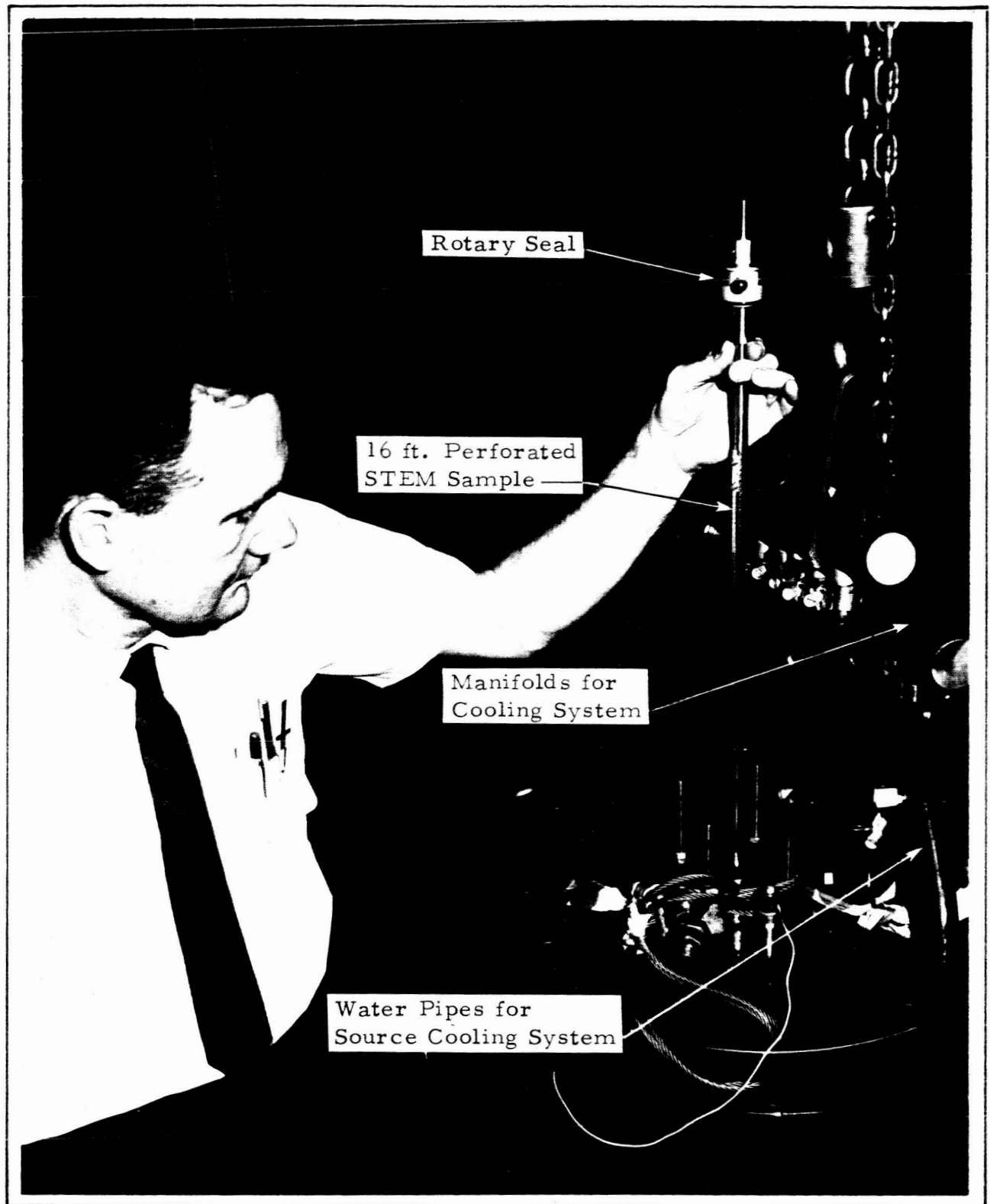
DHC-SP-R.168/2



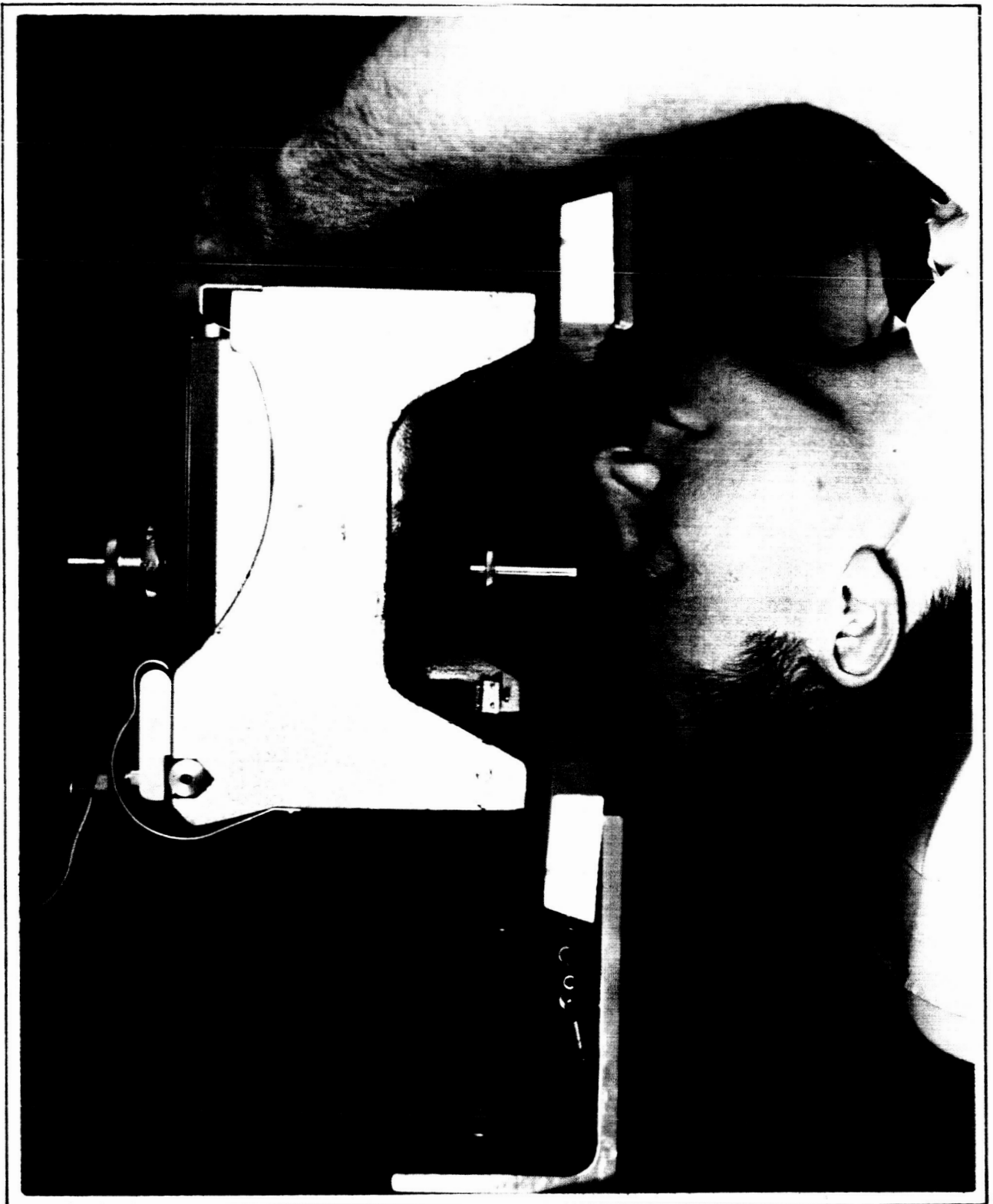
GENERAL VIEW - TOP PLATE & HEATER ASSEMBLY



DHC-SP-R.168/2



MOUNTING STEM IN VACUUM TEST CHAMBER



TWO COORDINATE TRAVELLING MICROSCOPE FOR MEASURING
STEM TIP DEFLECTION BELOW OBSERVATION PORT

Figure 10

5.0 RESULTS

5.1 Hole Pattern - Intensity Modulation

After completion of tests on many 6 inch samples on the optical test rig (see Section 3.1 and 3.2) it became obvious that those samples with a spiral distribution of holes were yielding the best results (i.e. the light transmittance curves were fairly uniform). Although many different spiral patterns were investigated, all gave better results than any of the random hole patterns without a spiral. A spiral distribution of holes was therefore selected for the die patterns. Circumstances limited the restriction of the pattern punching die to a length of approximately 1.0 inch. Obviously, a 6 inch long die would probably yield better results since there would be fewer repetitions along the STEM and would probably be used in a production process. It appears to be a general property of spiral patterns that, if they have a uniform hole - hole and line - line separation but with the position of the first hole in every line varied in a random manner, then they yield a good rotational distribution for a perforated STEM. The 2 patterns used to perforate the STEMs tested in the chamber are shown in Figures 14 and 15.

The intensity modulation curve for a typical spiral distribution is shown in Figure 13. To prove that a regular distribution of holes is unacceptable as a STEM pattern, a regular pattern was investigated on the optical test rig and the results, which are quite conclusive, are shown in Figure 12.

5.2 Deflection Measurements

5.2.1 Nomenclature and Presentation of Data

a) STEM Number

The tube number will indicate the length, perforation type, and sample number. Perforation type will be indicated by the % area removed, and sample number will be indicated by a suffix. Thus 20-1 indicates first sample, 20% area removed. Lacing will be indicated by an L suffixed to the tube number, thus we may have 20-2L to indicate second sample, 20% removed, laced.

Length will be indicated by a prefix thus 45-5-1 refers to the first 45 foot length with 5% area removed.

b) Orientation of STEM in Chamber

Four orientations at 90 degree intervals, designated by numbers 1, 2, 3, and 4, indicating the relative position of the overlap region with respect to the source. Looking at the tip of the STEM towards the mounted end we have:

Source Position	Tip of STEM	
*		orientation No. 1, overlap towards the source.
*		orientation No. 2, overlap rotated 90° clockwise.
*		orientation No. 3, overlap away from source.
*		orientation No. 4, overlap rotated 270° clockwise from orientation No. 1.

c) Axis Definition

A right handed cartesian axis system will be used, with the origin at the point of mounting the STEM (rotary seal). We shall further define that the STEM will lie along the positive Z axis, and the incident radiation is along the positive X axis. This then specifies the positive Y axis when looking at the tip of the STEM (see Figure 16). This co-ordinate axis system results in a deflection away from the source as being positive.

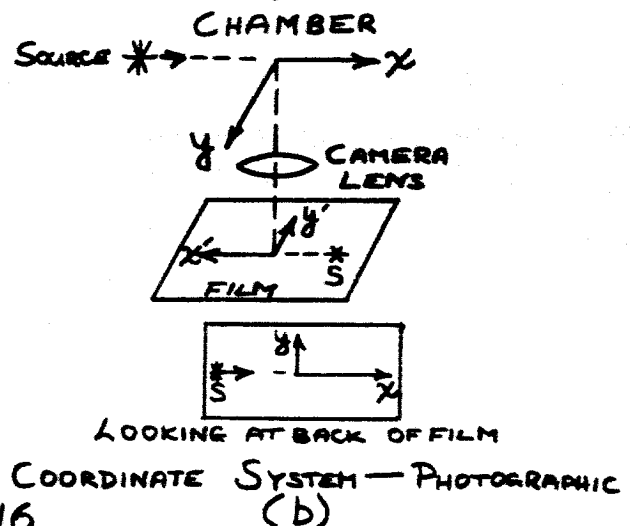
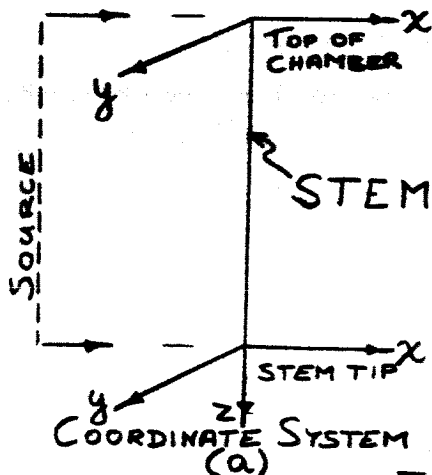


FIGURE 16

5.2.2 Chamber Deflection Data

For each STEM, measurements were repeated three times for each orientation, and in the case of reasonable agreement between the readings a mean was taken. Measurements were made of ΔX and ΔY , and their direction, where ΔX and ΔY are the small distances moved along the X and Y axis when the STEM is illuminated by the source (see Figure 17). Graphs are presented for the value of ΔX vs. % area removed for each orientation.

Only the mean values of 3 runs for each orientation for ΔX and ΔY are shown in the deflection tables. But there was little spread in ΔX and ΔY in the photographic data and results for several runs with the same orientation were quite consistent. There was, however, a larger spread in the microscope readings for reasons given in Section 4.5 paragraph 6(b).

Deflection data obtained is shown in Tables 5-1 and 5-2. Table 5-1 gives ΔX and ΔY data obtained from microscope readings only. Table 5-2 gives ΔX and ΔY data obtained from photographic measurements only. Figure 21 was drawn from the data in Table 5-1 and Figure 22 drawn from the data in Table 5-2. In Figures 21 and 22 only ΔX data is plotted (i.e. deflections along the direction of incident radiation) and ΔY values are ignored, it being assumed that ΔY deflections are mainly due to overlap effect. Every ΔX point given in Tables 5-1 and 5-2 is plotted in Figures 21 and 22 but when drawing the best fit through the points, data from unlaced STEMs have been ignored due to their large scatter. Some of the unusual cases are considered below.

a) Sample 16-5-1 SL, which was one of the samples from which data was obtained for Table 5-2, (the subscript SL denotes semi-laced i.e. although the STEM was laced, a large percentage of the holes still do not line up). When 16-5-1 is in orientation 3 (overlap away from the source), and assuming that no holes line up, let the absorption coefficient of the inside surface be β and H be the irradiance on the rear surface. As long as β is large, the following argument holds. Since no holes line up, the power absorbed by the rear surface is βH . If the holes line up perfectly the power absorbed by the rear surface is $0.95 \beta H$ since 5% is transmitted. Thus, in this orientation, the power absorbed by the rear surface is but little effected by hole alignment. We would thus expect STEM 16-5-1 SL to give good results in this orientation. This is supported by the results in Table 5-2.

TABLE 5-1

VACUUM CHAMBER THERMAL DEFLECTION DATA
 (Microscope Measurements)

Orientation ²			Sample ¹	ΔX (mean of at least 3 runs) mm
1	* S		16-14-1L	-3.4
			16-0-2	+1.8
			16-0-1	+0.75
2	* S		16-14-1L	-2.4
			16-0-2	+2.8
			16-0-1	+1.4
3	* S		16-14-1L	-2.7
			16-0-2	+4.25
			16-0-1	not available.
4	* S		16-14-1L	not available
			16-0-2	+2.94
			16-0-1	not available

- NOTES
1. CODE 16-14-1L indicates 16 foot sample, 14% area removed, sample 1, overlap laced. Sample numbers without L are unlaced.
 2. At tip of sample.

CONDITIONS: SOURCE TEMPERATURE: 1100°C
 CHAMBER PRESSURES LESS THAN
 4×10^{-5} TORR
 SOURCE WATTAGE: 800 WATTS.

TABLE 5-2

VACUUM CHAMBER THERMAL DEFLECTION DATA
 (Photographic Data)

SOURCE 800 WATTS AT 1100°C: CHAMBER PRESSURE LESS THAN
 4×10^{-5} TORR

Orientation ⁴		Sample ³	ΔX (mean of 3 runs) mm	ΔY (mean of 3 runs) mm	STEM Over- lap Twist ⁵
1	X S		16-20-2L	-7.7	80°CL ²
			16-5-1SL	+1.8	90°AC ¹
			16-20-1 (unlaced)	+3.2	180°-
			16-0-2	+3.4	90°AC
2	X S		16-20-2L	-1.45	80°CL
			16-5-1SL	+1.4	90°AC
			16-20-1 (unlaced)	-9.8	180° -
			16-0-2	+2.7	90°AC
			16-5-4SL	0.2	90°CL
3	X S		16-20-2L	-4.6	80°CL
			16-5-1SL	+0.3	90°AC
			16-20-1 (unlaced)	----	180° -
			16-0-2	+2.0	90°AC
			16-5-4SL	+0.6	90°CL
4	X S		16-20-2L	-8.9	80°CL
			16-5-1SL	+1.3	90°AC
			16-20-1 (unlaced)	-3.5	180° -
			16-0-2	+2.5	90°AC

- NOTES
1. AC - Anti-clockwise twist along entire length of STEM
 2. CL - Clock-wise twist along entire length of STEM
 3. 16-20-2L indicates 16 foot length, 20% area removed, sample 2, overlap laced. Sample numbers without L are unlaced. SL indicates partial lacing.
 4. At tip of sample.
 5. Overlap position at top mounting point determined by rotating orientation figure indicated angle.

b) However, consider STEM 16-5-1 SL in orientation 1 (overlap facing source) and again let H be the power incident on the overlap. If the holes line up completely, then the power absorbed by the rear surface is $0.05\beta \cdot H$. If only 50% of the holes line up, then the power absorbed by the rear surface is $0.025\beta \cdot H$. If no holes line up then no energy reaches the rear surface and the STEM appears like an unperforated STEM to the incident radiation. We would thus expect STEM 16-5-1SL to be deflected away from the source in this orientation, which is again supported by the data.

It is clear that hole alignment is critical in orientation 1 but is of little importance in orientation 3.

It should be noted that, when observations were made on unlaced (therefore misaligned) perforated STEMs, the deflections were always in the expected directions.

It should also be noted that the 14% STEM was originally intended to give zero deflection but the calculation of required perforated area was based on erroneous reflectance data. The determination of required perforated area for the new zero bending STEM (i.e. 5%) was based on Figure 21. If it is borne in mind that the reflectance of Ag plated STEM approaches 95% at about 2 microns, the 2 methods, deflection graphs and Equation 2-3, of determining the perforated area for a zero gradient STEM, are compatible. Further discussions of the results are given in Section 6.

Results shown in Tables 5-1 and 5-2 are not corrected for the effects of a 1g gravity field on measured STEM deflections as this effect was investigated and the reduction in deflection due to gravity was found to be less than 4% with a 16 foot sample.

5.3 Reflectance Measurements

5.3.1 General

In conjunction with this test program and other programs much work has been done on reflectance of silver plated STEM samples. Measurements have been made using an integrating sphere attachment on a Bausch & Lomb Spectronic 505 spectrophotometer. We have made minor modifications to this instrument to improve the accuracy, however we cannot extend the

range beyond 650 mμ into the infrared. We have therefore made certain assumptions as to the shape of the spectral reflectance curve over the region .650 μ to 2.5 μ, and will show how these results compare with a run made on a sample supplied by SPAR and run on a Beckman DK2A spectrophotometer at NASA-Goddard Space Flight Centre. This is an area where considerable work can be done in setting up a standardized method of measurement, using specified reference samples made under known conditions. There is much doubt as to the veracity of any method of measurement of absolute reflectance, as there is no primary standard of reflectance, and indeed we have been informed by one of the larger manufacturers of spectroscopic equipment that "we will not furnish any devices with the claim that it will make absolute reflectance measurements". Several double beam systems are available which will measure the difference in reflectivity between two samples, however this then brings us back to the problem of an absolute standard. Possible methods of tackling this problem have been suggested by Hass (Reference 5), Bennett (Reference 9) and Gier (Reference 10).

In this particular case our requirement is the more stringent as we are concerned primarily with absorptance, and this we wish to know to a high accuracy. The only convenient way of measuring absorptance at SPAR at the present is through reflectance (Reference 7), and one must remember that 1% error in reflectance at 95% reflectance represents a 20% error in absorptance.

We define absorptance for the purpose of this work as being:

$$\alpha_s = 1 - R_s \quad \text{for solar absorptance} \quad 5-1$$

$$\alpha_c = 1 - R_c \quad \text{for chamber absorptance} \quad 5-2$$

where

$$R_s = \frac{\int H_s(\lambda) R(\lambda) d\lambda}{\int H_s(\lambda) d\lambda} \quad 5-3$$

where $H_s(\lambda)$ is the solar spectral irradiance in space. Similarly

$$R_c = \frac{\int H_c(\lambda) R(\lambda) d\lambda}{\int H_c(\lambda) d\lambda} \quad 5-4$$

for the chamber (source temperature approximately 1100°C). $R(\lambda)$ is the spectral reflectance of the sample measured on the spectrophotometer.

We are also concerned with the absorptance of the black paint used on the interior of the tube, however this is of the order of 0.95 and consequently we can ignore large percent errors in reflectance compared to the errors possible in the absorptance of the silver.

5.3.2 Method of Measurement

As mentioned earlier, reflectance measurements are made using an integrating sphere attachment to a Bausch & Lomb Spectronic 505 spectrophotometer. (Figure 23). Measurements made are total reflectance, with the specularly reflected beam from the silver sample being diffused by a freshly made pressed Barium Sulphate surface. The reference surface is also Barium Sulphate, and care is taken to ensure that these two surfaces are identical. On all chart records of spectral reflectance will be found a 100% line marked "BaSO₄ reference" and this is the reflectivity of the light trap, see Figure 23, compared with a "standard" BaSO₄ surface. Both of these surfaces are freshly made at the start of a series of measurements, and precautions are taken to ensure that they are equal in reflectance before this reference run is made. The light trap is then placed in its correct position, and the reflectance of the sample measured. A special BaSO₄ holder was designed and made to fit in place of the light traps supplied by Bausch & Lomb to improve the instrument accuracy.

5.3.3 SPAR-NASA Reflectance Comparison

As a further check on the absolute accuracy of the SPAR Bausch & Lomb reflectance attachment, measurements were made on a sample by the NASA DK2A equipment and the SPAR equipment. The results of this run are shown in Figure 24. The comparison indicated that the NASA reflectance values were consistently higher than those produced with the B & L instrument at SPAR and also showed a steady increase in the reflectance of the silver sample between .65 to 2.5 microns. The lower values for the SPAR data may be due to the BaSO₄ light trap target (see Figure 23) limitation when measuring specular reflectance. Using the data of Figure 24 the solar reflectance and absorptance of the sample were computed using

Equations 5-3 and 5-4 and are shown in Table 5-3.

TABLE 5-3

INTEGRATED REFLECTANCE AND ABSORPTANCE
(one sample)

	REFLECTANCE		ABSORPTANCE	
	SOLAR R_s	CHAMBER R_c	SOLAR α_s	CHAMBER α_c
NASA ¹	.889	.975	.111	.025
SPAR ^{2,3}	.857	.925	.143	.075
RATIO NASA TO SPAR	1.04	1.06	.775	.33

1. Beckman DK2A with reflectance attachment - Aluminum reference 0.30 to 2.5 μ .
2. B & L Spectronic 505 - BaSO₄ reference, integrating sphere, 0.22 to .66 μ .
3. Assuming reflectance constant for $\lambda > .65\mu$.

It would appear from these results, assuming the NASA instrumentation to be more accurate in absolute reflectance measurements, that all SPARs values computed from the B & L reflectance measurements should be increased by a factor of 1.04.

5.3.4 Typical Reflectance Data for Electro-Deposited Silver Films

An analysis was made of 19 typical reflectance runs of silver plated STEMs. These runs are characteristic of the state-of-the-art at the beginning of 1966 and more recent runs (April 66, and the sample discussed in Section 5.3.3) indicate higher average reflectances. The results of the 19 samples are presented in Table 5-4 as the computed solar reflectance and absorptance using Equation 5-3, with the assumption that the reflectance is constant for all wavelengths greater than 0.65 microns.

TABLE 5-4

SOLAR REFLECTANCE AND ABSORPTANCE OF TYPICAL SPAR
ELECTRODEPOSITED FILMS

Sample No.	Solar Reflectance R_s	Solar Absorptance α_s
1	.843	.157
2	.839	.161
3	.799	.201
4	.783	.217
5	.835	.165
6	.812	.188
7	.861	.139
8	.820	.180
9	.859	.141
10	.871	.129
11	.854	.146
12	.844	.156
13	.885	.115
14	.812	.188
15	.868	.132
16	.838	.161
17	.849	.151
18	.878	.121
19	.848	.151

Mean Value 0.158

Std. Dev. = .027

The results indicated a mean value for the solar absorptance of 0.158 with a standard deviation of 0.027 (17%).

Correcting these results by the factor 0.775 noted in Table 5-3, we get for a corrected mean value 0.122 for the solar absorptance. As mentioned above, we would expect this mean value to be lower in more recent runs and would expect that values of 0.12 for solar absorptance be more representative of current state-of-the-art for our silver films. Future work may lower this further, as is evident in the one sample used in the NASA-SPAR comparison (Table 5-3) where $\alpha_s = .111$ was obtained. For the chamber a mean value for $\alpha_c = .045$ can be considered typical of current state-of-the-art Ag films.

5.3.5 Reflectance Data for the STEM Samples Placed in the Chamber

Earlier reflectance measurements, not shown, indicated that the reflectance of silver for wavelengths greater than 0.5μ was 0.87 on the average. It was on the basis of these early reflectance measurements that the 14% area removed STEM was selected as the "zero bending" STEM for the chamber. It has been shown in Section 5.2 that the 14% samples exhibited negative bending characteristics, indicating that too much area had been removed; this was expected at the time of the deflection measurements since more recent reflectance data indicated much higher values than the .87 value originally used. The original low value for reflectance has been attributed to inferior plating processes at that time, as discussed in detail in Section 5.3.4.

Reflectance data for the STEM samples run in the chamber are shown in Table 5-5, reflectance data is not available for all samples. As discussed in Section 5.3.3 these results are felt to be low by a factor of 1.04 around $.6\mu$, and that they will increase further as λ increases.

TABLE 5-5

**REFLECTANCE DATA FOR STEM SAMPLES USED IN VACUUM
CHAMBER**
(B & L SPECTRONIC 505 - PRESSED BaSO₄ REFERENCE)

WAVELENGTH	16-0-2	16-5-1		16-5-4
	UNPOLISHED ¹	UNPOLISHED ²	POLISHED	UNPOLISHED ¹
.30	.08	.06	.07	.09
.35	.43	.40	.42	.43
.40	.63	.51	.57	.64
.45	.72	.65	.71	.76
.50	.78	.75	.79	.84
.55	.81	.79	.83	.86
.60	.83	.82	.85	.89
.65	.86	.86	.89	.92

- NOTES
1. SAMPLE CLEAN - FREE OF FINGERPRINTS
 2. SAMPLE SHOWED EVIDENCE OF FINGERPRINTS.

5.3.6 PT 401 Paint

This paint has a very low total reflectance, and its spectral reflectance is flat in the visible region. No information is available, even from the manufacturers, for reflectivity in the infrared, and consequently the assumption was made that it was spectrally flat out to 2.5u. Since its absorptance is so high, however, the integrated solar and chamber absorptance will be relatively unaffected by a moderate change in spectral reflectance. Figure 26 shows the reflectance as measured by the usual technique on the Bausch & Lomb.

5.3.7 Oxidized Beryllium Copper

Because of the doubt as to the feasibility of using PT401 in a space application, preliminary investigations were conducted

into the feasibility of using the natural oxidation of the Be-Cu tape during the STEM forming operation. Figure 26 shows a comparison of the reflectance of this oxidized layer and the PT401, and shows the beginning of the rather rapid rise in reflectance in the near infrared. These preliminary results indicate that this is probably a feasible technique. WADC Technical Report No. 56-222 P.163 (Reference 7) indicates that a β_s of 0.5 - 0.6 for the inner surface of the STEM should be possible. More work is needed in this area, as indicated by Mr. H. S. Kerr in DHC-SP-R.168/1 Section 8.1 (Reference 11) before any definite conclusions are made.

5.4 Straightness and Diameter of Test Samples

5.4.1 Methods of Measurement

The straightness of the STEM samples was measured in a water tank using the technique laid down in DHC-SP-SP.86 (Appendix II). The diameter was measured using a micrometer, at many positions along the sample.

5.4.2 Results

Tables 5-6 to 5-9 show results of diameter measurements of samples which were measured in the chamber.

The following samples were investigated in the chamber: 16-0-1; 16-0-2; 16-5-1SL; 16-5-4SL; 16-14-1L; 16-20-1 and 16-20-2.

It will be noted that 2 of the above samples were also sent down to NASA i.e. 16-0-2 and 16-20-2 and the data for these samples is included in the straightness, diameter and reflectance data for the 6 samples sent to NASA.

The diameter measurements for the remaining samples are shown in Tables 5-6 to 5-9. Straightness of the chamber test samples were checked qualitatively by suspending the STEM by its tip and looking along the STEM to check the bow. It is simple to detect the bow in the STEM by this method and badly bowed STEMs were not put in the chamber. The diameters were normally measured in 2 perpendicular directions at each point as shown in the sketch.

TABLE 5-6

DIAMETER MEASUREMENTS
STEM 16-0-1

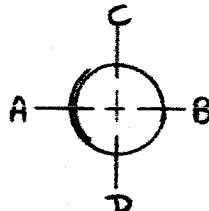
	Distance from End (ft.)	Diameter AB (inches)	Diameter CD (inches)
	0	0.496	0.497
	2	0.488	0.490
	4	0.486	0.491
	6	0.485	0.480
	8	0.493	0.479
	10	0.500	0.483
	12	0.505	0.498
	14	0.518	0.494
	15.75	0.497	0.479

TABLE 5-7

DIAMETER MEASUREMENTS
STEM 16-5-4 SL

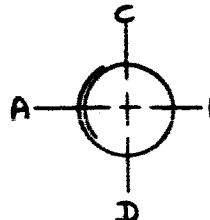
	Distance from End (ft.)	Distance AB (inches)	Distance CD (inches)
	0	0.482	0.489
	2	0.492	0.496
	4	0.499	0.500
	6	0.494	0.495



TABLE 5-7 Continued.....

Distance from End (ft.)	Distance AB (inches)	Distance CD (inches)
8	0.498	0.504
10	0.488	0.502
12	0.492	0.502
14	0.487	0.498
15.75	0.497	0.499

TABLE 5-8

DIAMETER MEASUREMENTSSTEM 16-14-1L

Distance from End (ft.)	Diameter AB (inches)	Diameter CD (inches)
0	0.514	0.516
2	0.497	0.509
4	0.496	0.508
6	0.498	0.514
8	0.504	0.508
10	0.499	0.506
12	0.499	0.509
14	0.500	0.506
15.75	0.498	0.504

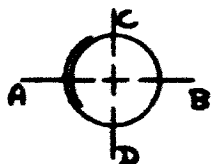
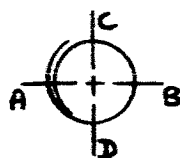


TABLE 5-9

DIAMETER MEASUREMENTS

STEM 16-20-1

	Distance from End (ft.)	Diameter AB (inches)	Diameter CD (inches)
	0	0.472	0.472
	2	0.477	0.502
	4	0.482	0.497
	6	0.503	0.498
	8	0.495	0.499
	10	0.504	0.497
	12	0.503	0.507
	14	0.499	0.490
	15.75	0.499	0.508

5.5 STEM Strength Tests

5.5.1 Introduction

The purpose of these tests was to obtain experimental values for the minimum bending stiffness and strength of silver plated STEMs of varying percentage of perforation. Tests were conducted and results analyzed as described in DHC-SP-R.158 (Reference 12).

5.5.2 Details of STEM Specimens

Details of the three 10 feet length silver plates STEM specimens were as follows:

TABLE 5-10

DATA ON STRENGTH TEST SPECIMENS

SPECIMENT	PERCENTAGE PERFORATION %	STRIP WIDTH s (IN)	UNPLATED THICKNESS t (IN)	FORMED DIAMETER d (IN)
1	0	2	0.002	0.42
2	5	2	0.002	0.45
3	20	2	0.002	0.48

MATERIAL BeCu 25

The root ends of all three specimens were encased in 2 inches of cerrobend to give fully fixed end conditions.

5.5.3 Test Rig and Testing Procedures

Each specimen was clamped vertically to the bending test rig shown in Figure 27 and rotated until its free edge was located in the weakest bending position. The free end was restrained, such that twist was prevented during bending. Horizontal tip loads were applied incrementally by means of a cord and adjustable pulley arrangement. Measurements of the corresponding horizontal tip deflection were noted and observations made on the mode of buckling and failure.

5.5.4 Results and Discussion

The results of the bending tests corrected for self weight influence are given in Figure 29. The following table is a summary of the STEM specimen bending strengths.

TABLE 5-11

BENDING STRENGTH OF STEM IN WEAKEST POSITION

SPECIMEN	P crit. TIPLOAD (LBS)	Yo crit. DEFLECTION (IN)	lr FAILURE LOCATION FROM ROOT (IN)	l crit (IN)	M crit BEND- ING MOMENT (LB IN)	RATIO TO 1
1	0.19	62	14	86	16.4	1
2	0.12	52	4	101	12.1	0.74
3	0.063	39	2	114	7.2	0.44

where l crit is the vertical distance from tip to point of failure given by

$$l \text{ crit} = (1 - 77 (Y_o/l)^2) - lr$$

and $M \text{ crit} = P \text{ crit. } l \text{ crit.}$

where l is length of specimen (unloaded).

5.5.5 Observations

All STEM specimens had a pronounced free edge rippling caused by the plating operation. This is due to the fact that the plating system has been designed for 0.5" tubing, whereas the tube used here is 0.4" diameter. It will not occur in a plating machine designed for 0.4" diameter. This effect is undesirable in that it prematurely sets off free edge buckling in the early bending stages.

The modes of failure for the two perforated STEMs are shown in Figure 28. It is interesting to note that the 0.48 inch diameter 20% perforated specimen having little overlap, fails with pronounced free edge buckling, whereas the 0.45 inch diameter, 5% perforated specimen having 30% more overlap than the latter, fails in much the same manner as a seamless tube.

5.5.6 Conclusion

It is stated on Figure 29 that the critical bending moment for the unperforated STEM is proportional to its cross-sectional circumferential length S (E = Young's Modulus, σ = poisson's ratio). It can be shown that the corresponding effective value S for the perforated STEM is given by $S(1 - 10 \cdot \sqrt{4/\pi} \times (\% \text{ perforation}))$. Using this expression the bending strength of perforated STEMs relative to that for the unperforated STEM has been plotted in Figure 30. Superimposed are the experimental results, using as a reference the unperforated test specimen result.

In the above K_B , the experimental bending constant, was assumed constant. In actual practice it is greatly influenced by the degree of STEM overlap. From experience it is known to have its maximum value for a STEM having an overlap factor between 1.4 - 1.5 (i.e. for a diameter between 0.45 - 0.42 inches in the case of the present specimens). The drop in K_B for increasing diameters is due to the increasing degree of instability of the overlap as it decreases.

5.6 Silver Film Thickness Measurements

5.6.1 General

We have made a number of attempts to measure the silver film thickness, but without any conclusive results. Three methods were used to make these measurements, and these are described below. This is an area where much work can be done, both from the point of view of measurement and a number of suggestions have come up during the course of this aspect of the program.

It should be emphasized, however, that considerable study and work has been done to optimize the plating process parameter to maximize the spectral reflectance of the silver film, as is evident from recent reflectance runs on plated samples.

5.6.2 Methods of Measurement

Two techniques of measurement were used:

- (a) The first is the only non-destructive technique

attempted. The STEM wall thickness was measured before cleaning, after cleaning, and after plating. This gave a set of figures for the nominal thickness of the STEM after forming. The cleaning process removes a thin layer from the surface, and this thickness is determined by the second measurement. The third measurement gives the final thickness of the STEM, and from the three measurements a figure for the thickness of silver is determined. This method gave the highest of the values obtained (see Section 5.6.3).

- (b) The second method involves the electrolytic removal of the silver film using a controlled current and known quantity of electrolyte. The time taken for the silver to be removed is a measure of the thickness. This method will obviously give a low value since the apparent end of the electrolytic removal is taken as a discontinuity in the current flow as the silver is removed, exposing the gold flash. However, it is probable that not all the silver has been removed at this stage, but further electrolysis would begin to remove the gold.
- (c) The third, and probably most accurate method was to use a selective chemical etch to remove the silver, and measure the base thickness before and after etching.

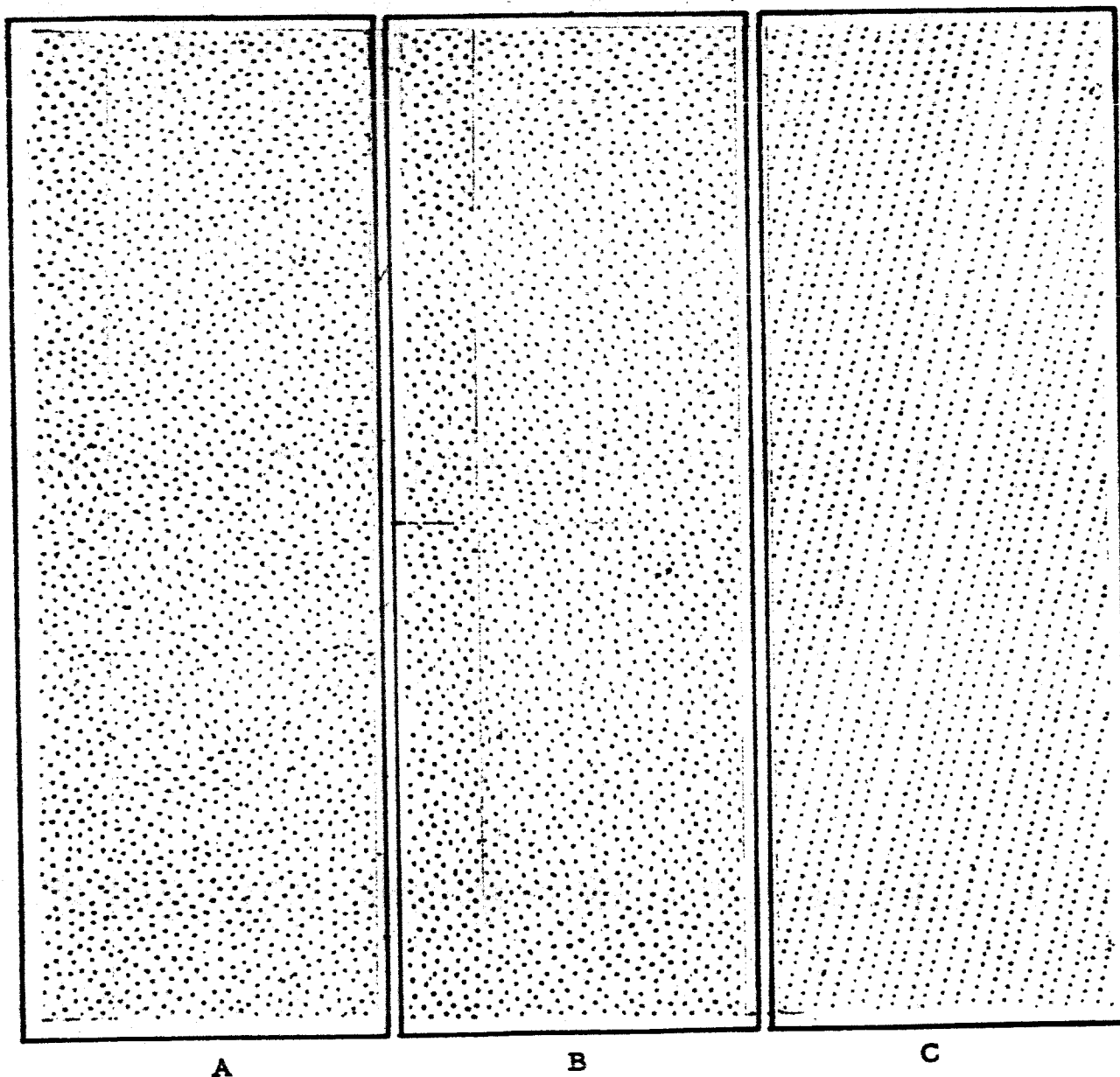
5.6.3 Results

None of the methods outlined in Section 5.6.2 are entirely satisfactory, and accuracy for each is probably only of the order of $\pm 50\%$, since 10 micro-inches is the smallest division on mechanical measurement equipment.

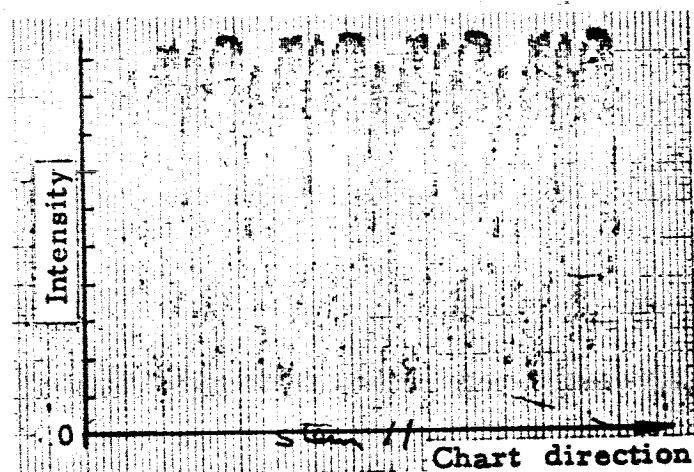
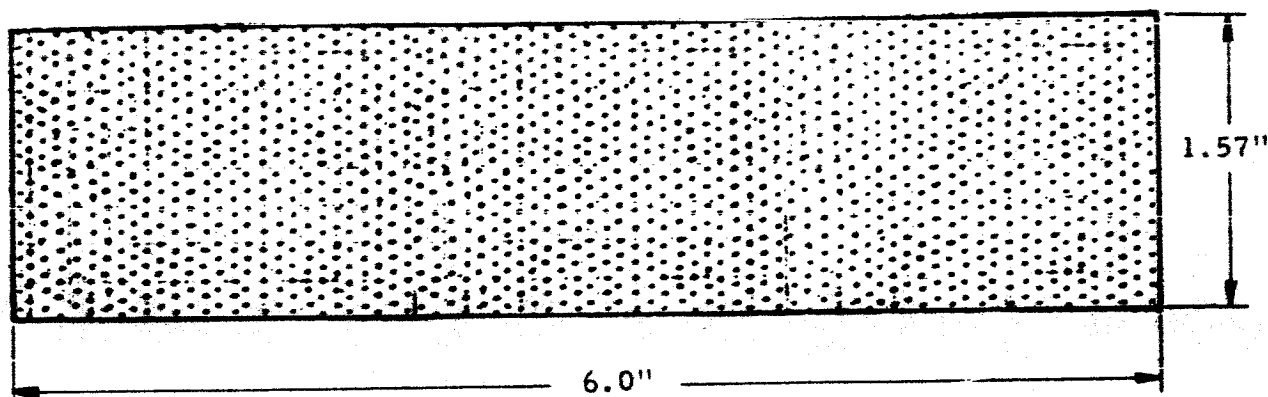
Method (a) indicates a value between 60 micro-inches and 140 micro inches.

(b) gives values of approximately 15 micro-inches. However, as mentioned previously this is probably a low figure.

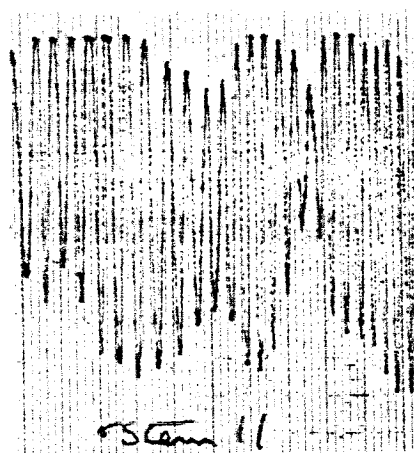
(c) this is probably the most accurate, and indicates a value between 60 and 100 micro-inches.



TYPICAL HOLE PATTERNS



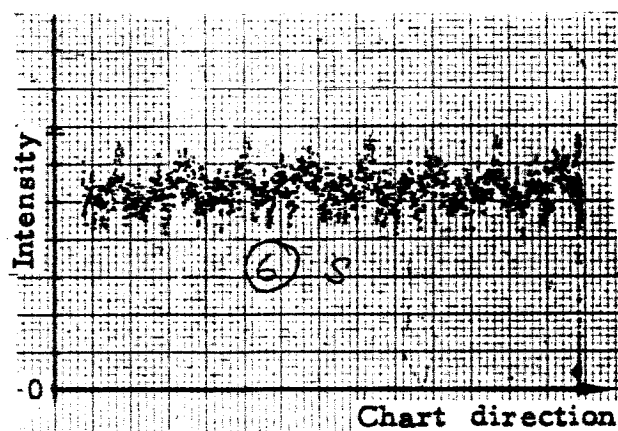
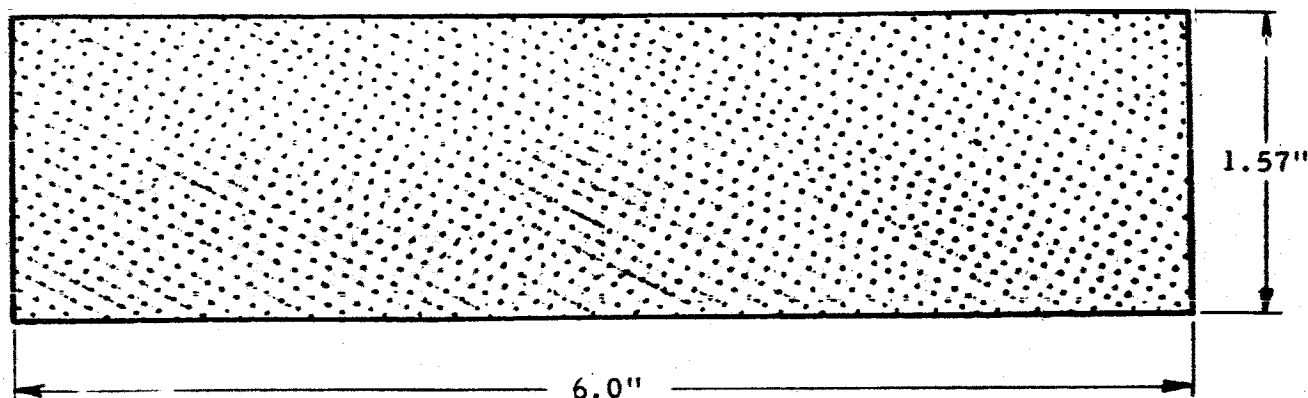
Sample Rotation Speed:- 50 Degrees Per Second
(The Poor Trace is Due to the High Speed of the Recording Pen)



Sample Rotation Speed:- 10 Degrees Per Second

REGULAR STEM PATTERN WITH LIGHT TRANSMITTANCE CURVE

Figure 12

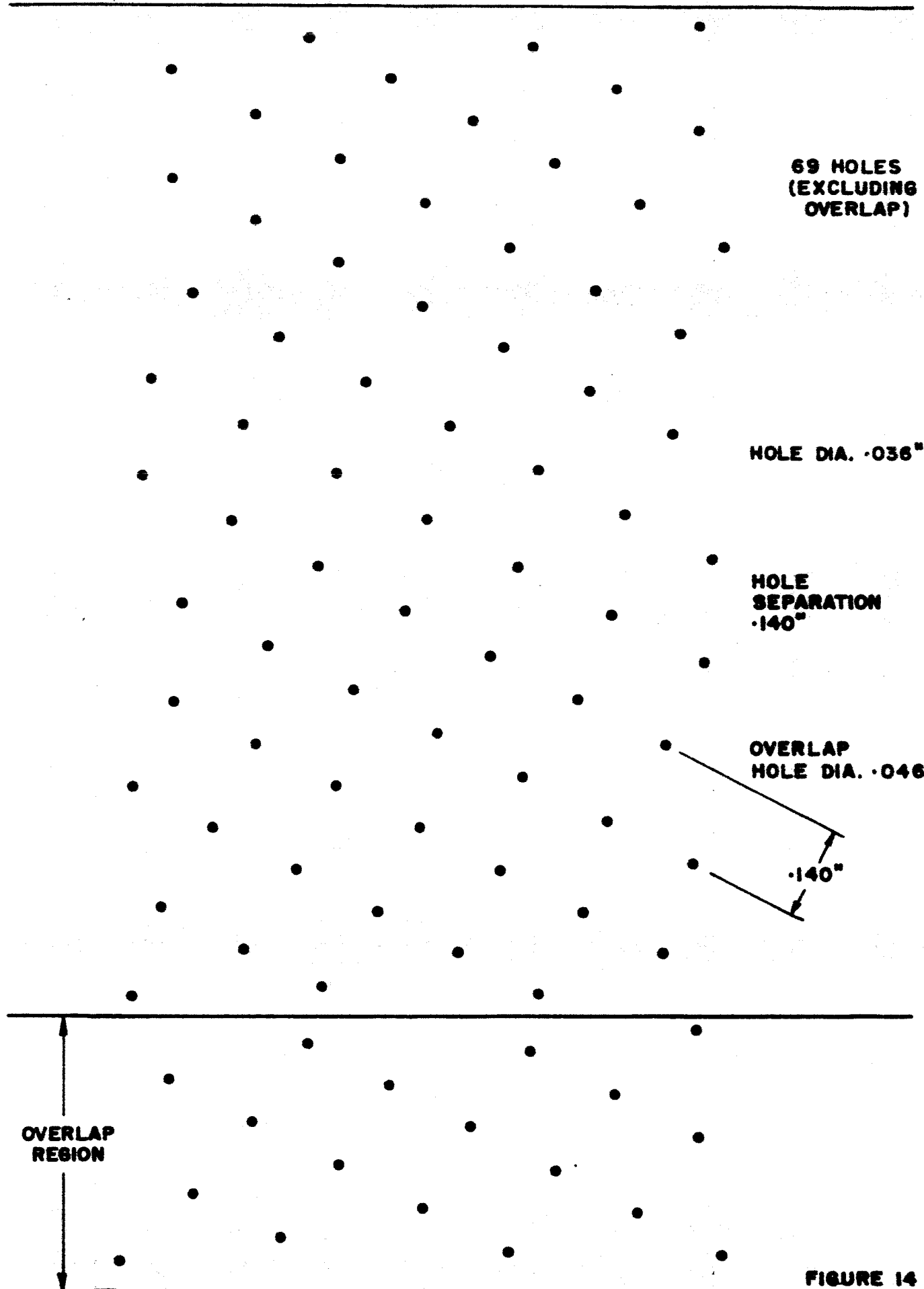


Light transmittance curve for above pattern, as measured on the optical bench with the sample rotated at right angles to the direction of the incident radiation.

The hole - hole separation and line - line separation are equal at approximately 0.087 inches but the position of the first hole in every line is varied in a random manner.

TYPICAL SPIRAL PATTERN WITH LIGHT TRANSMITTANCE CURVE

Figure 13



69 HOLES
(EXCLUDING
OVERLAP)

HOLE DIA. .036"

HOLE
SEPARATION
.140"

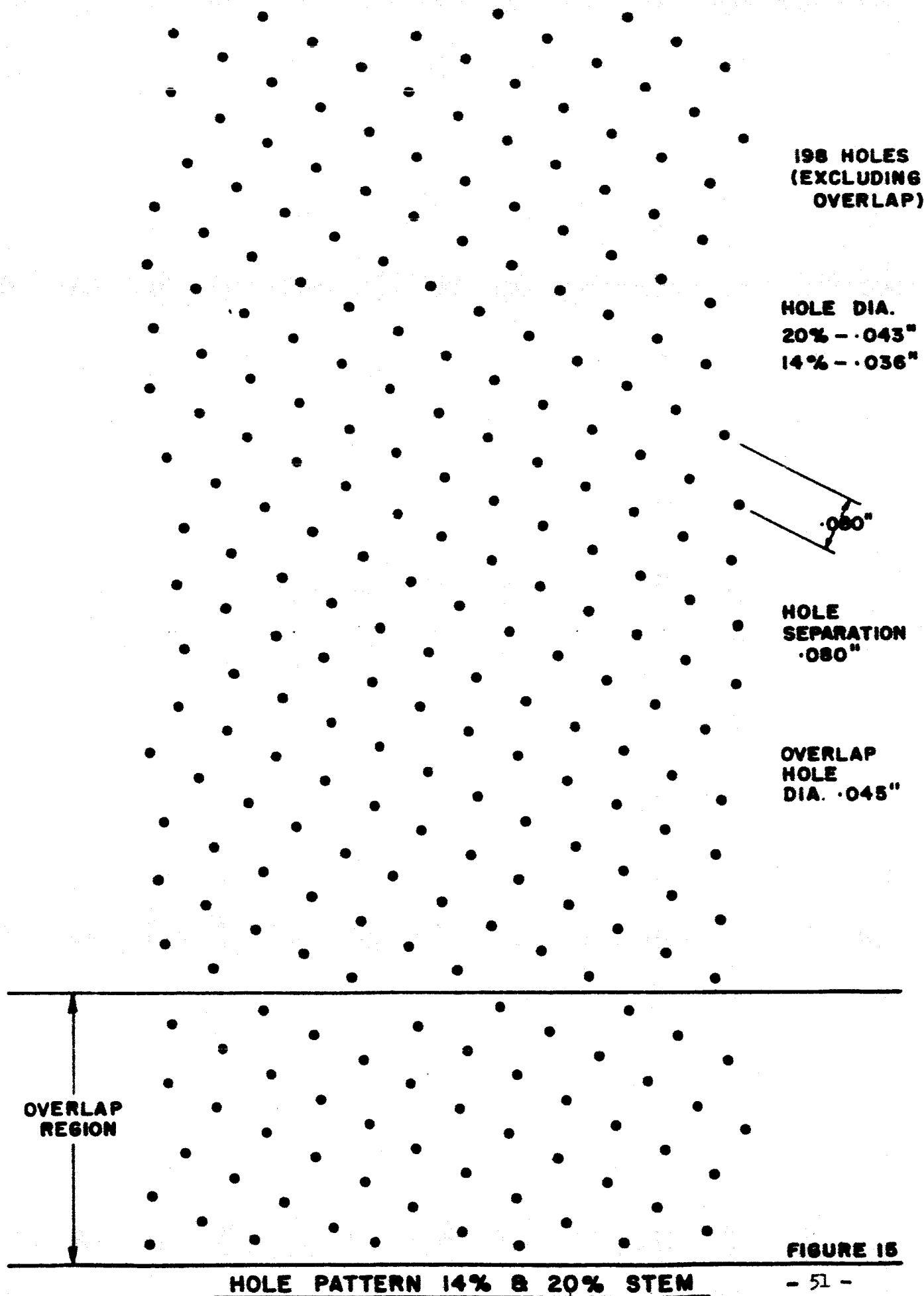
OVERLAP
HOLE DIA. .046

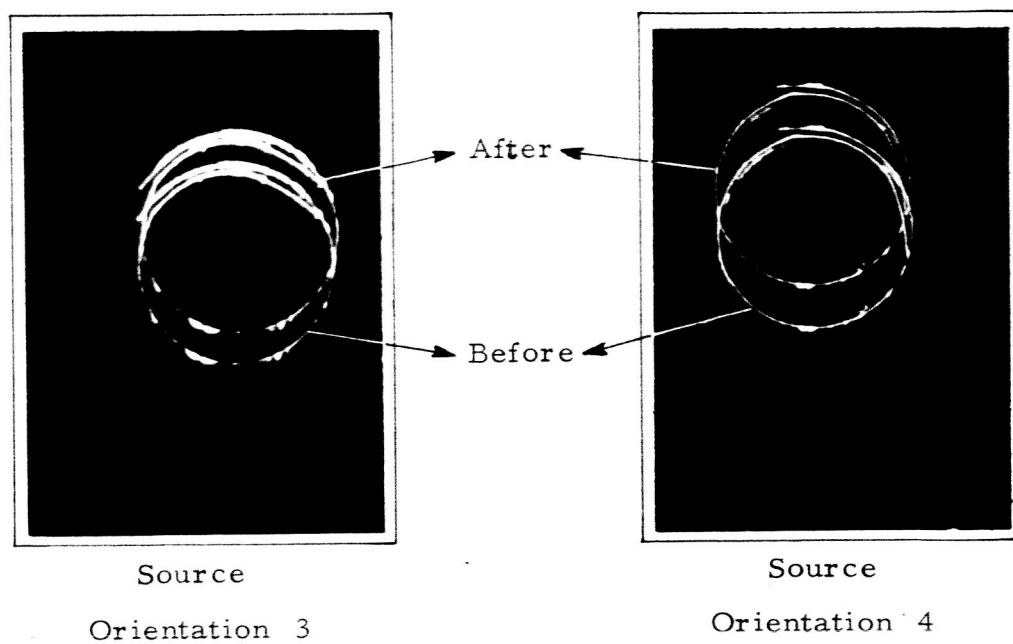
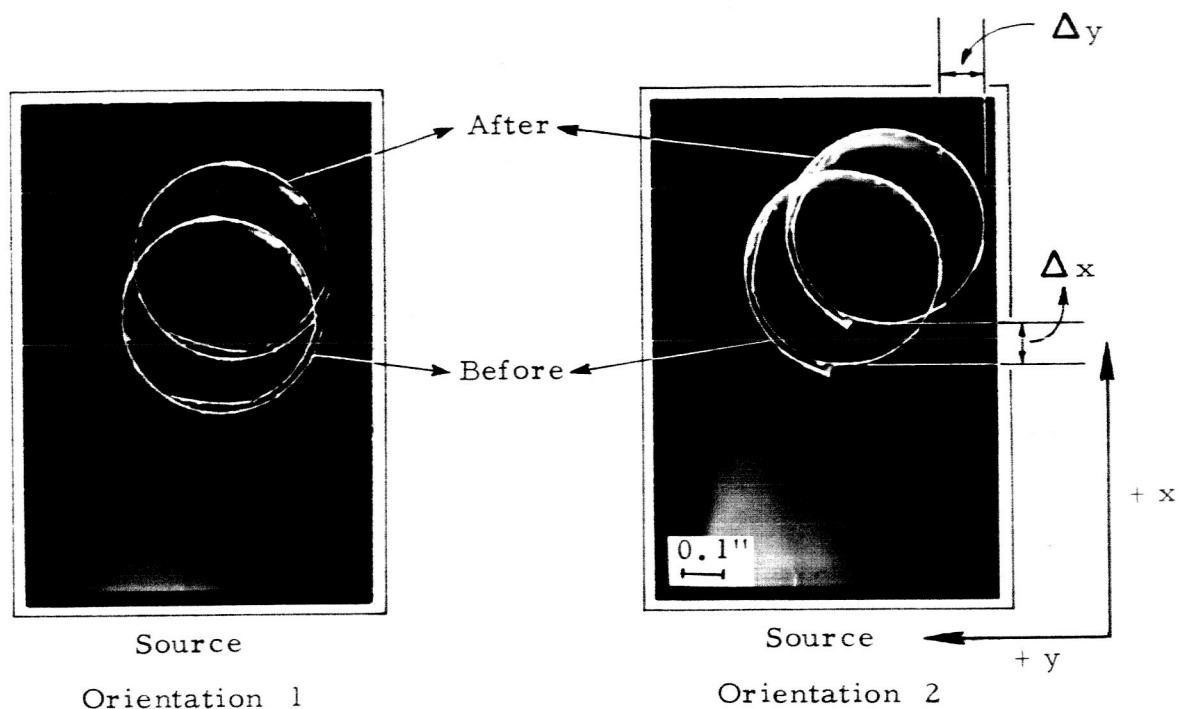
.140"

OVERLAP
REGION

HOLE PATTERN 5% STEM

FIGURE 14

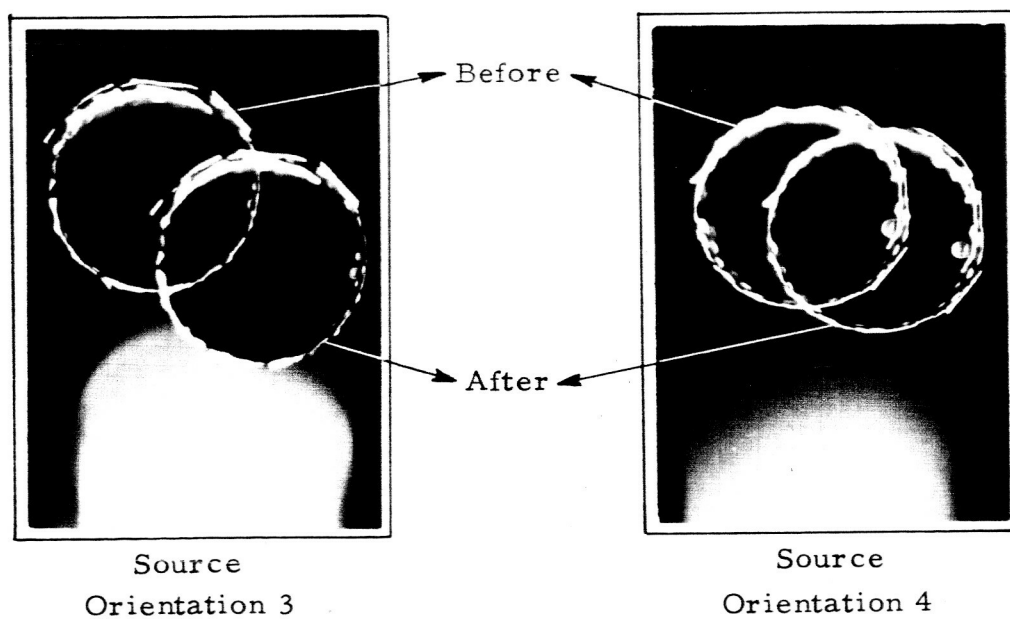
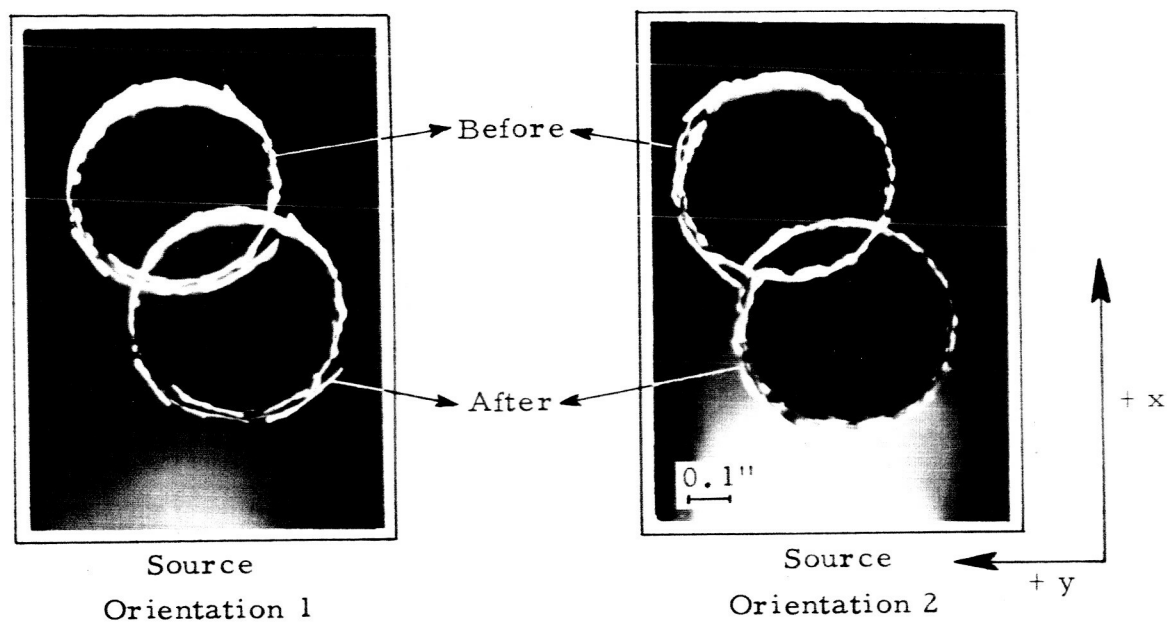




UNPERFORATED SAMPLE: DEFLECTION PHOTOGRAPHS
SAMPLE 16-0-2



DHC-SP-R.168/2



20% AREA REMOVED DEFLECTION PHOTOGRAPHS
SAMPLE 16-20-2 L



DHC-SP-R.168/2

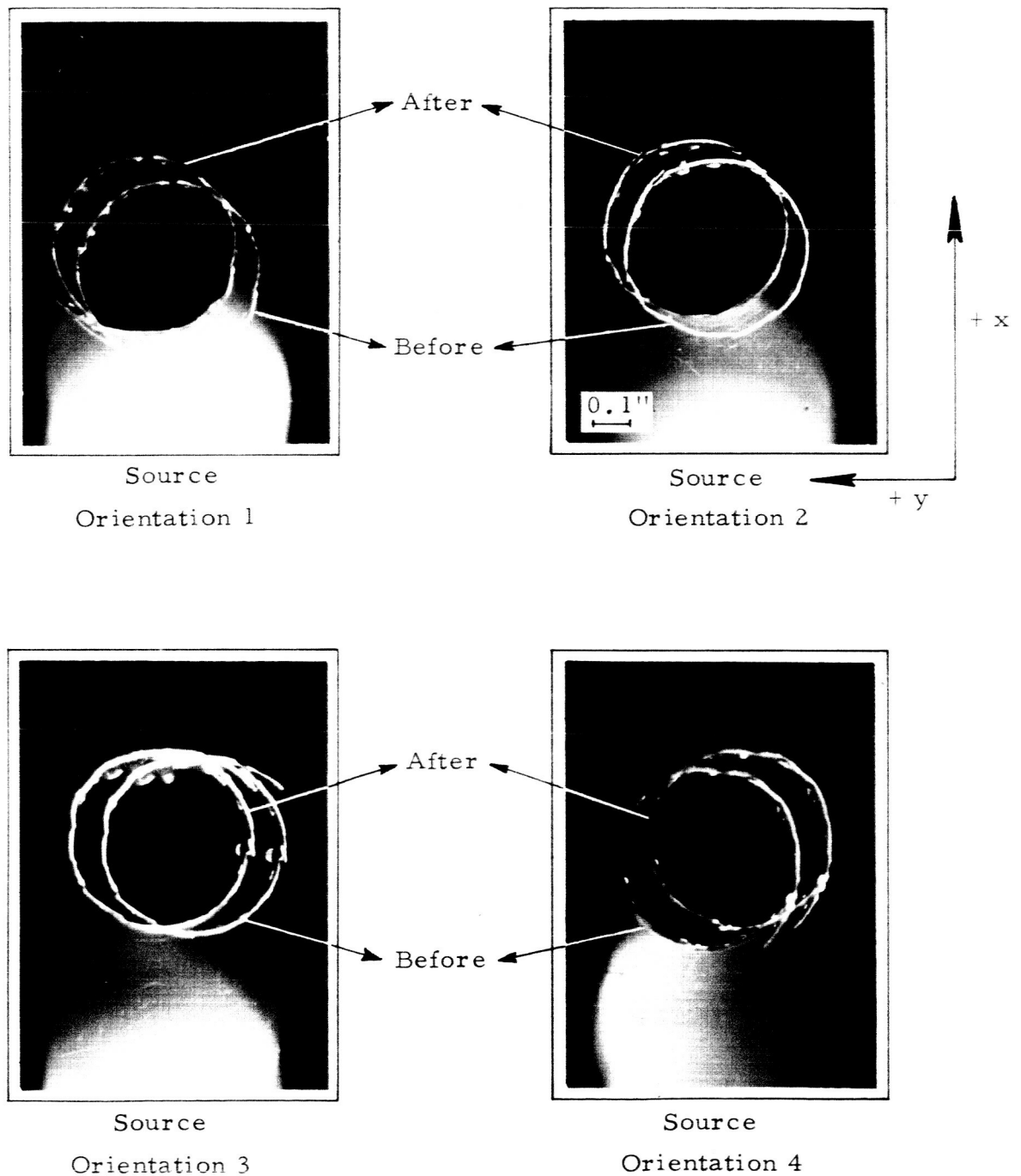
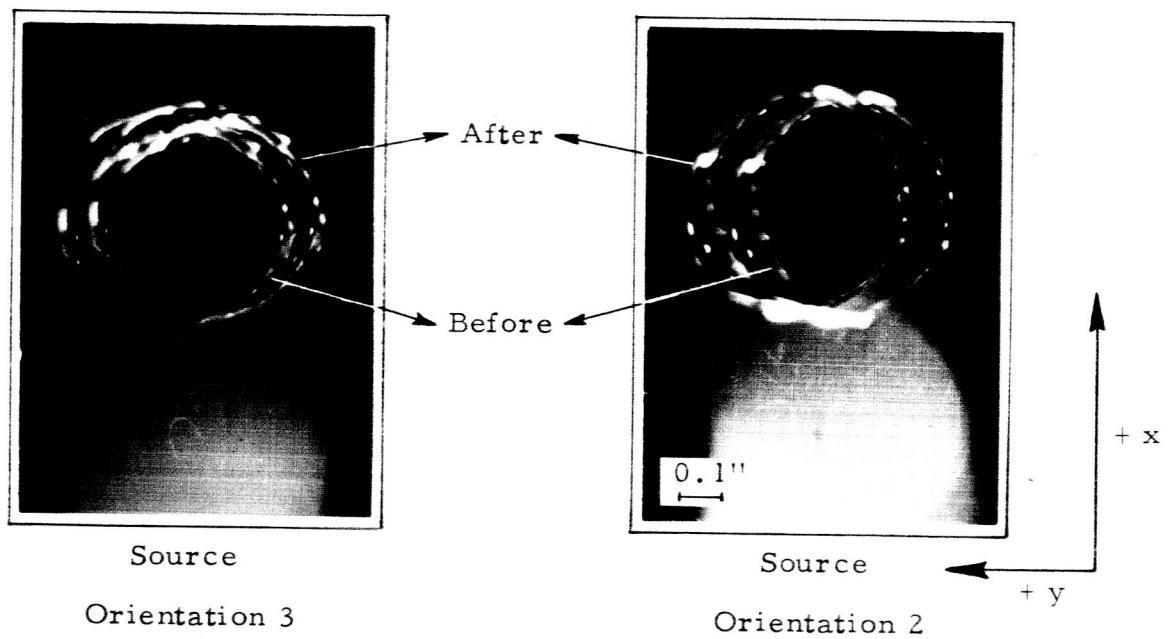
5% AREA REMOVED DEFLECTION PHOTOGRAPHSSAMPLE 16-5-1 SL

Figure 19

"ZERO BENDING" STEM

5% AREA REMOVED DEFLECTION PHOTOGRAPHS
SAMPLE 16-5-4 SL

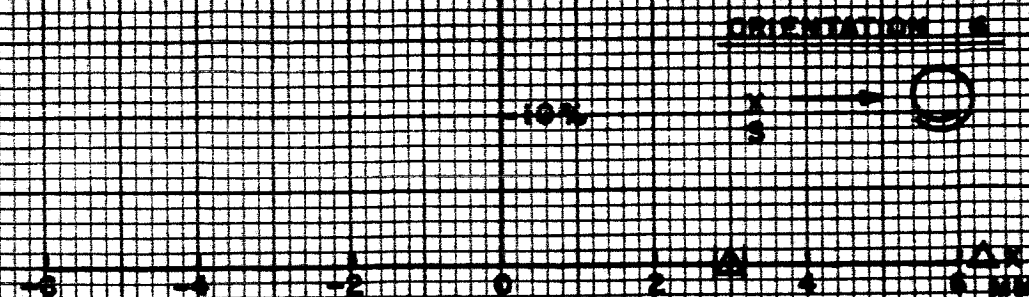
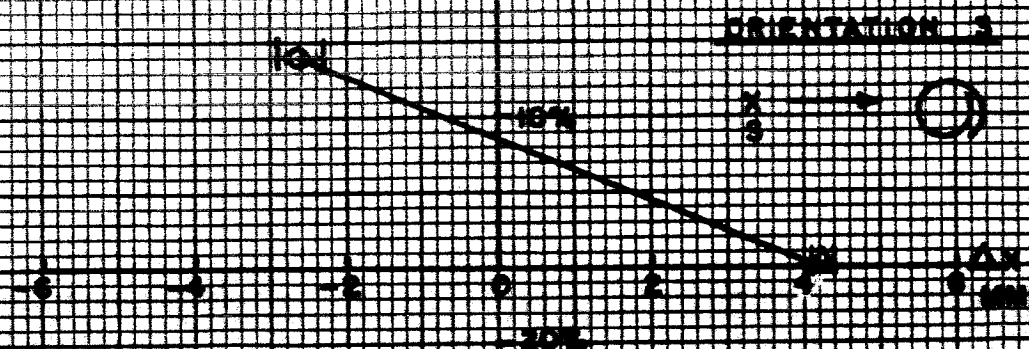
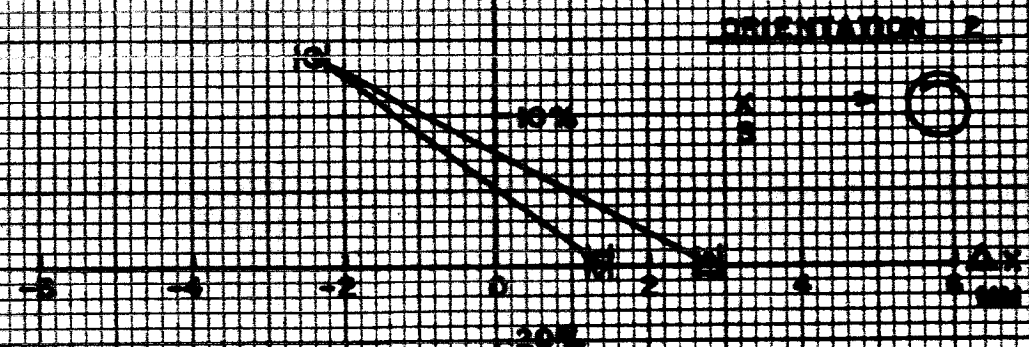
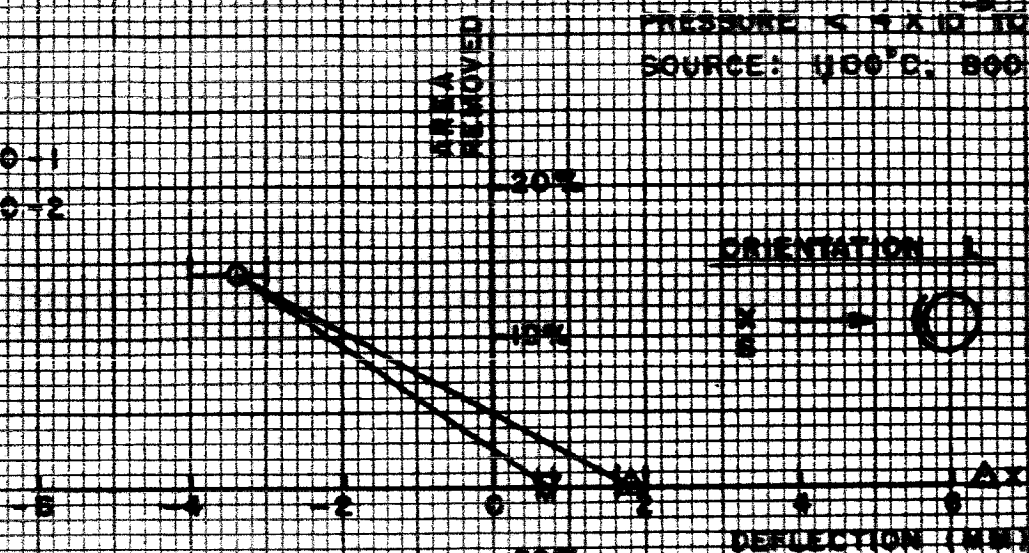
DEFLECTION MEASUREMENTS (SEE TABLE 5-1)

(OBTAINED WITH MICROSCOPE)

KEY

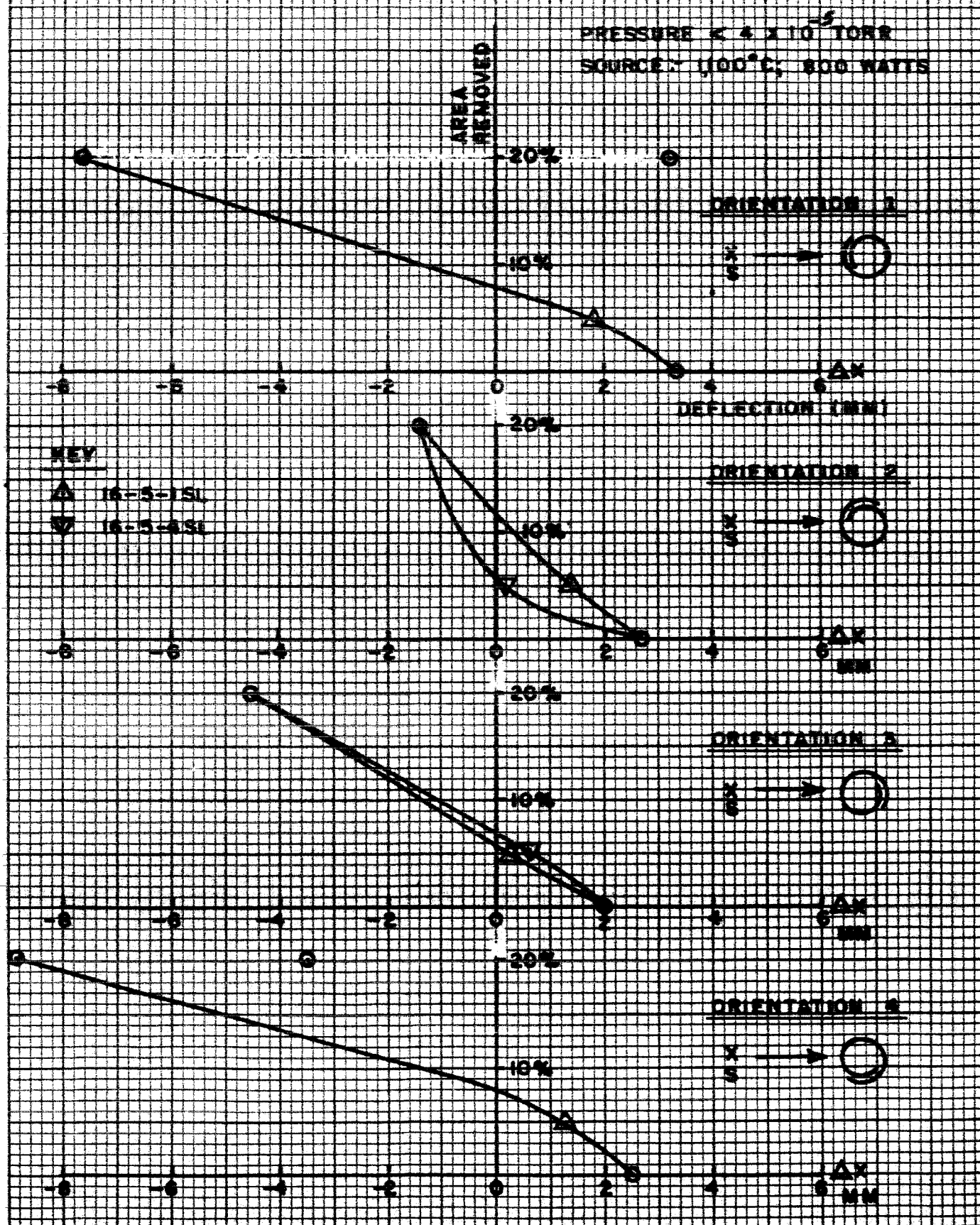
V 16-0-1

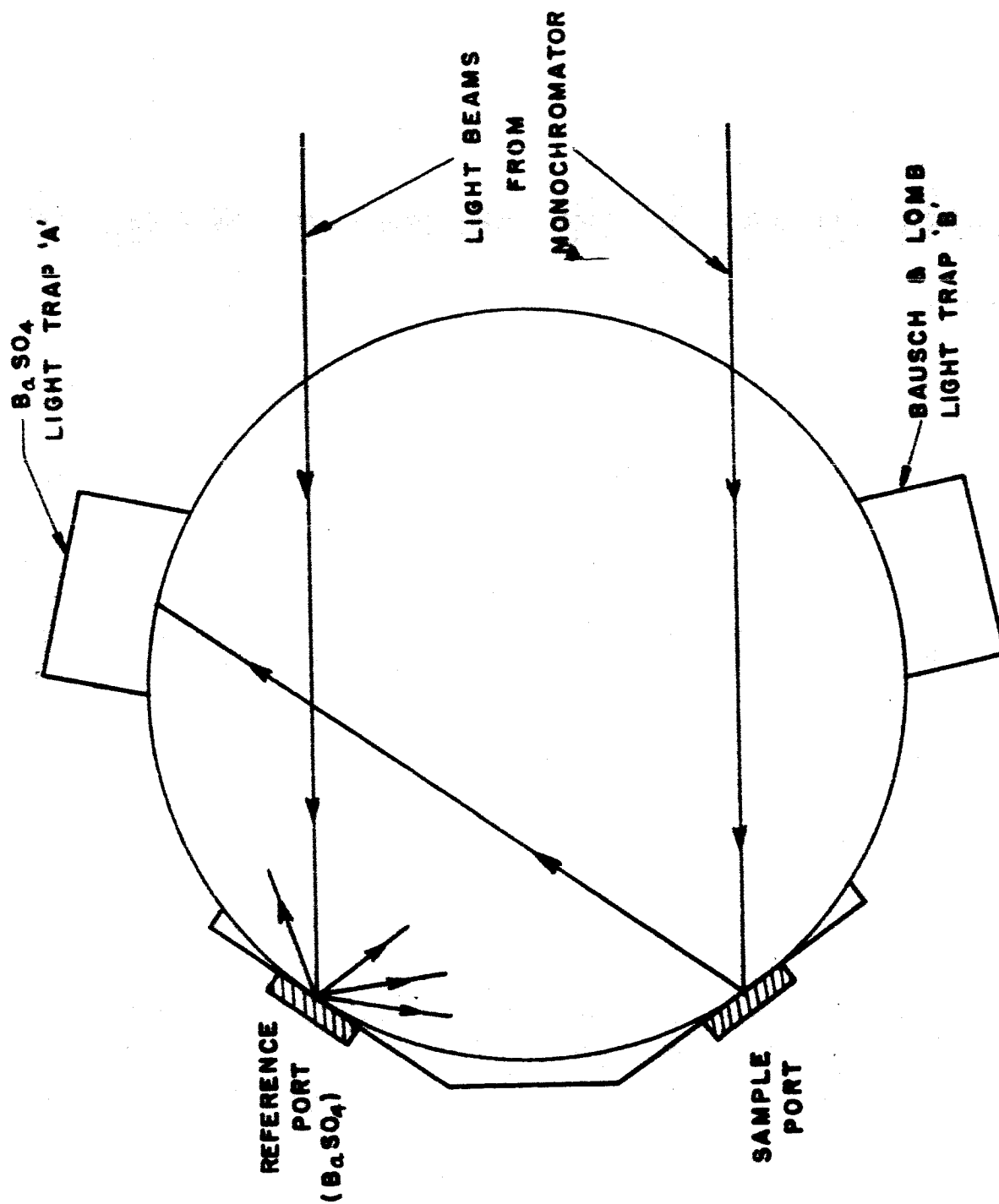
Δ 16-0-2

PRESSURE < 4 X 10 TURN
SOURCE: 100°C, 800 WATTS

DEFLECTION MEASUREMENTS (SEE TABLE 5-2)

(OBTAINED FROM PHOTOGRAPHIC DATA)





**SCHEMATIC - INTEGRATING SPHERE ATTACHMENT
FOR BAUSCH & LOMB SPECTROPHOTOMETER**

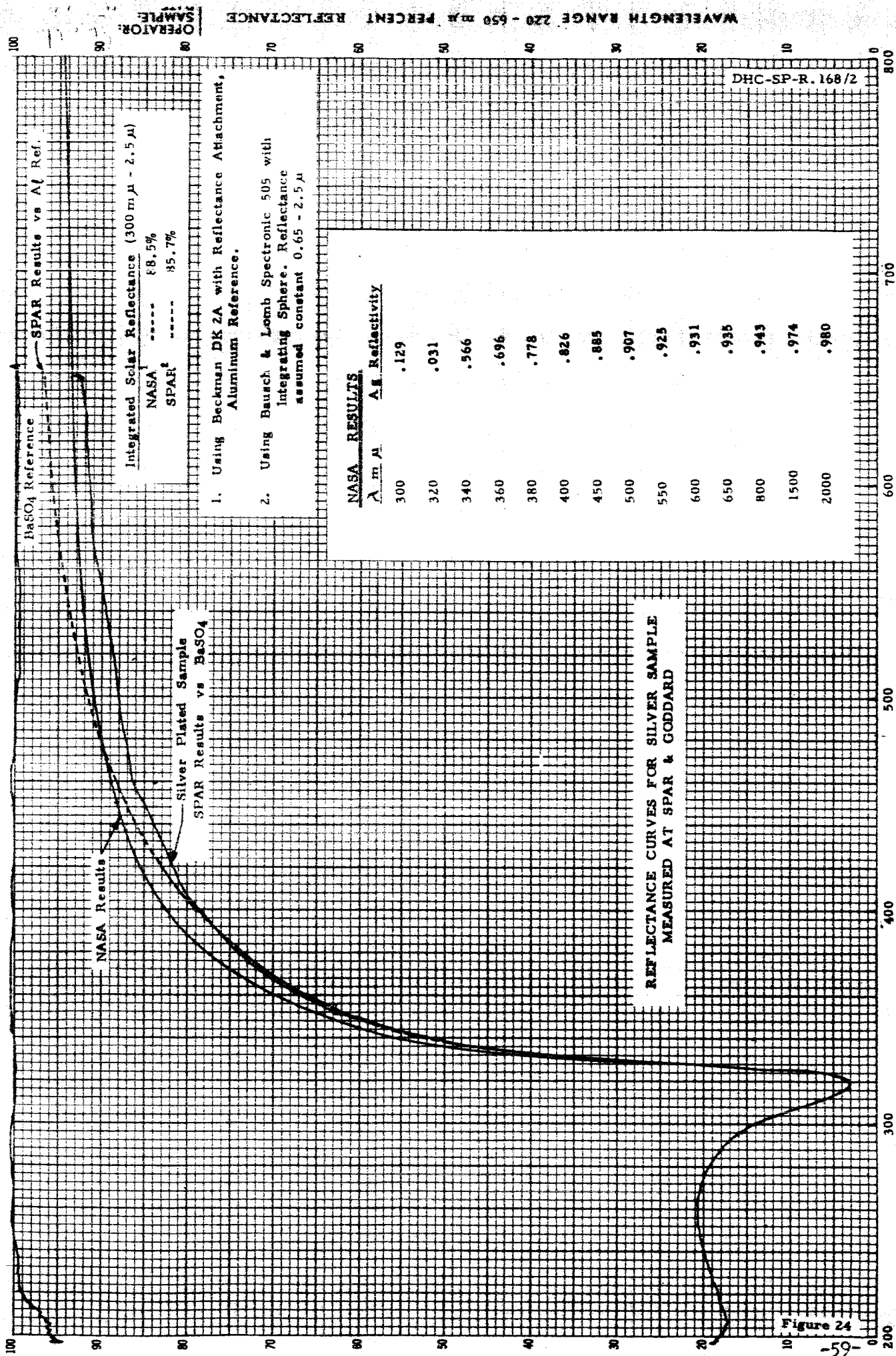
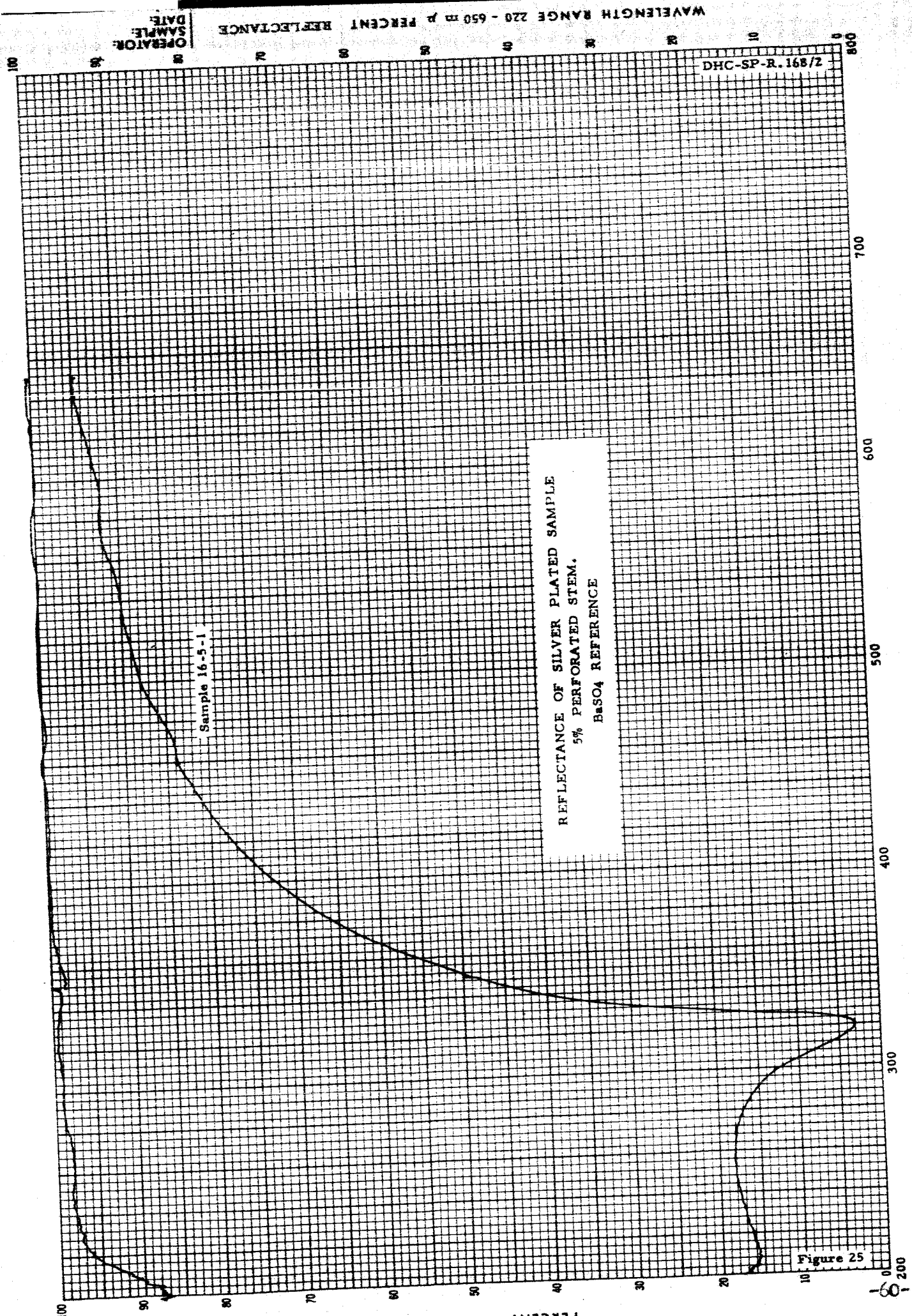
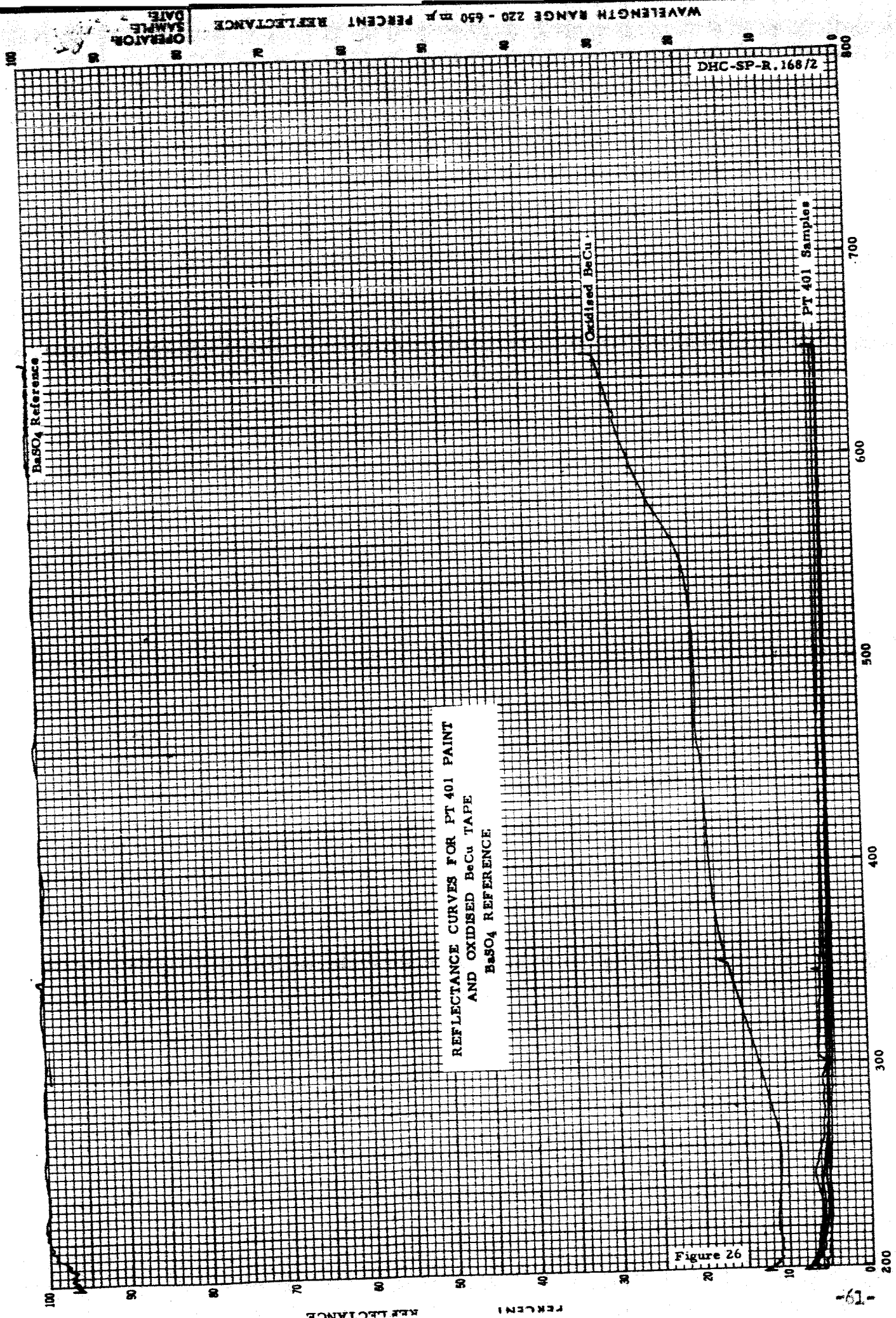
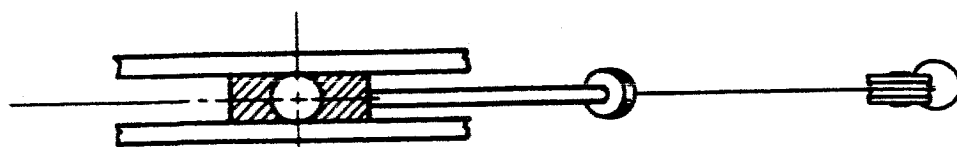
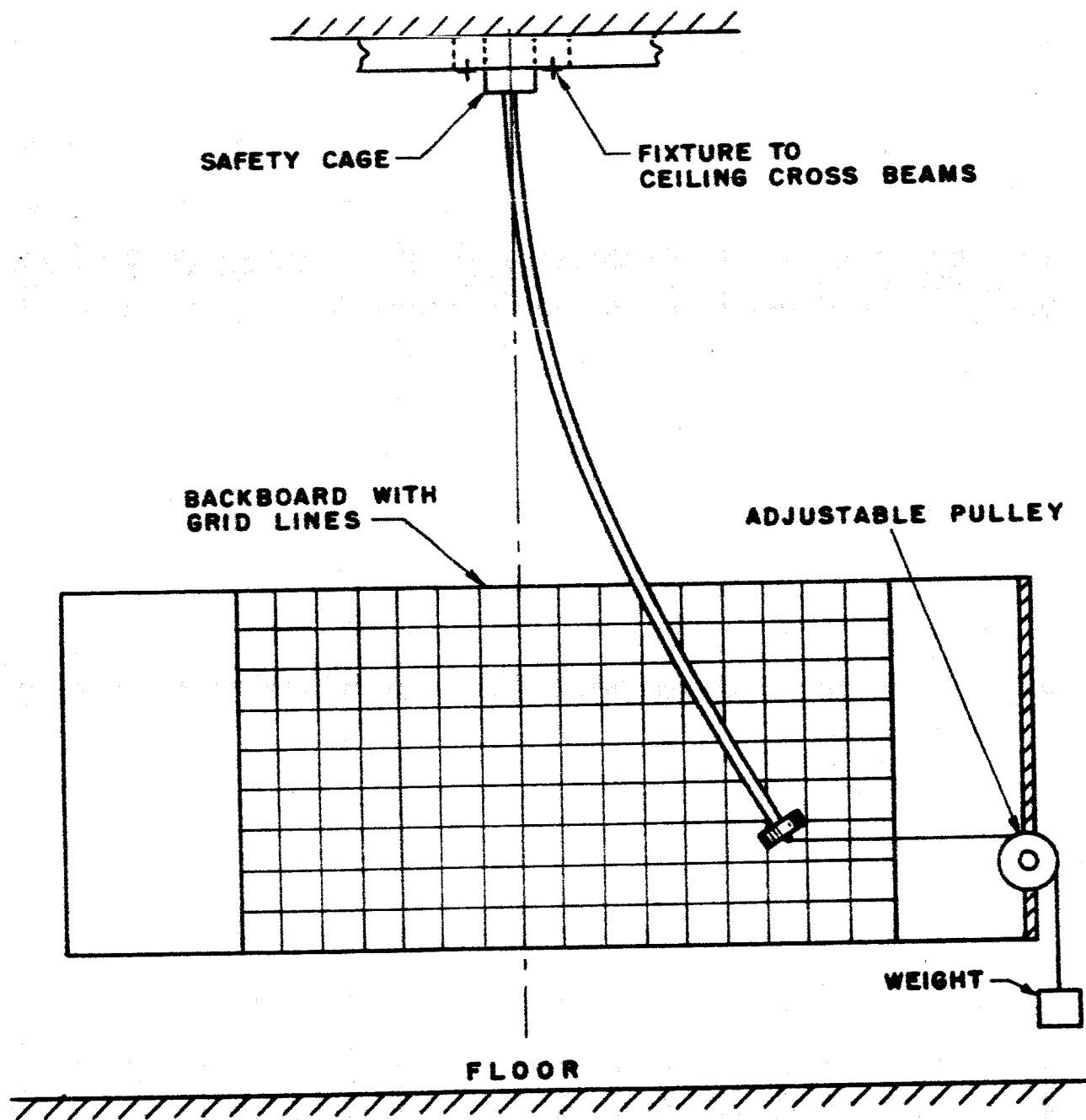


Figure 24



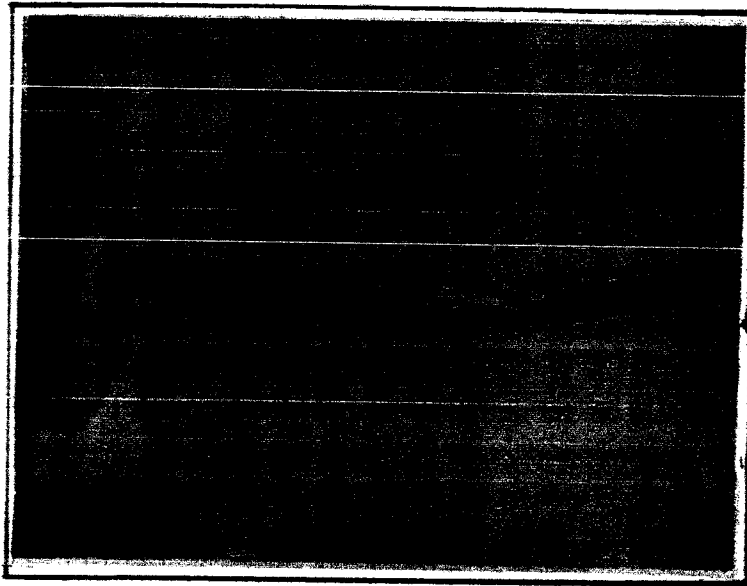




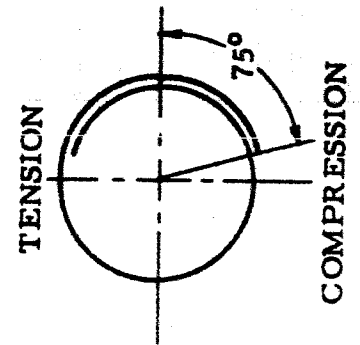
PLAN VIEW OF RIG

STEM BENDING RIG

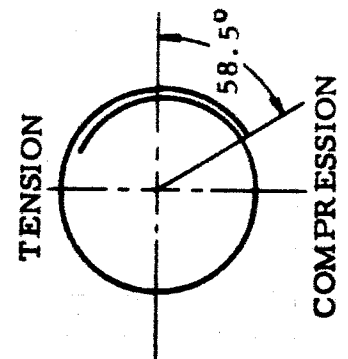
FIGURE 27



5% Sample
 Failure 4.0" From Root
 Diameter 0.45"
 Overlap Factor 1.415

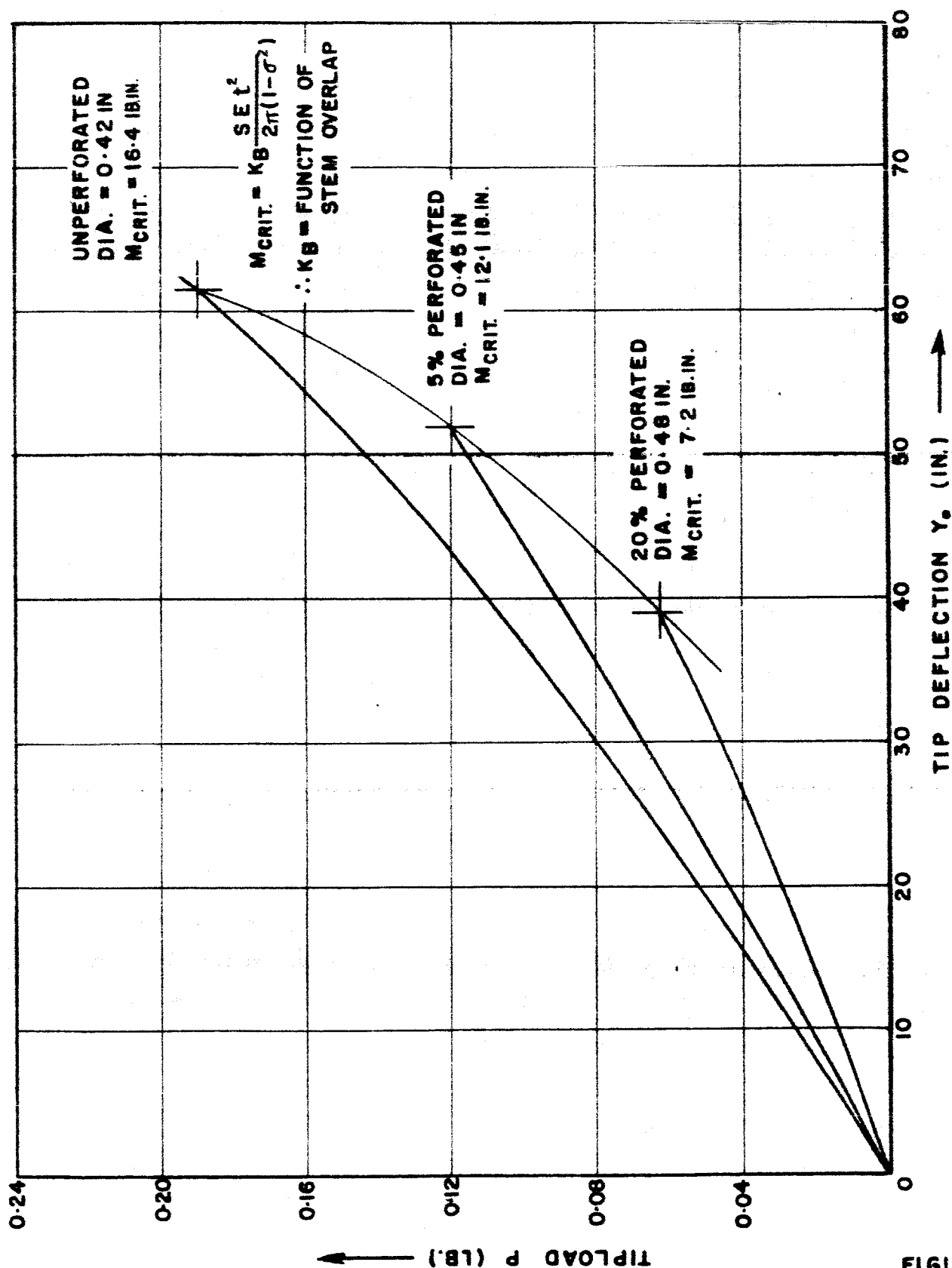


20% Sample
 Failure 2.0" From Root
 Diameter 0.48"
 Overlap Factor 1.325

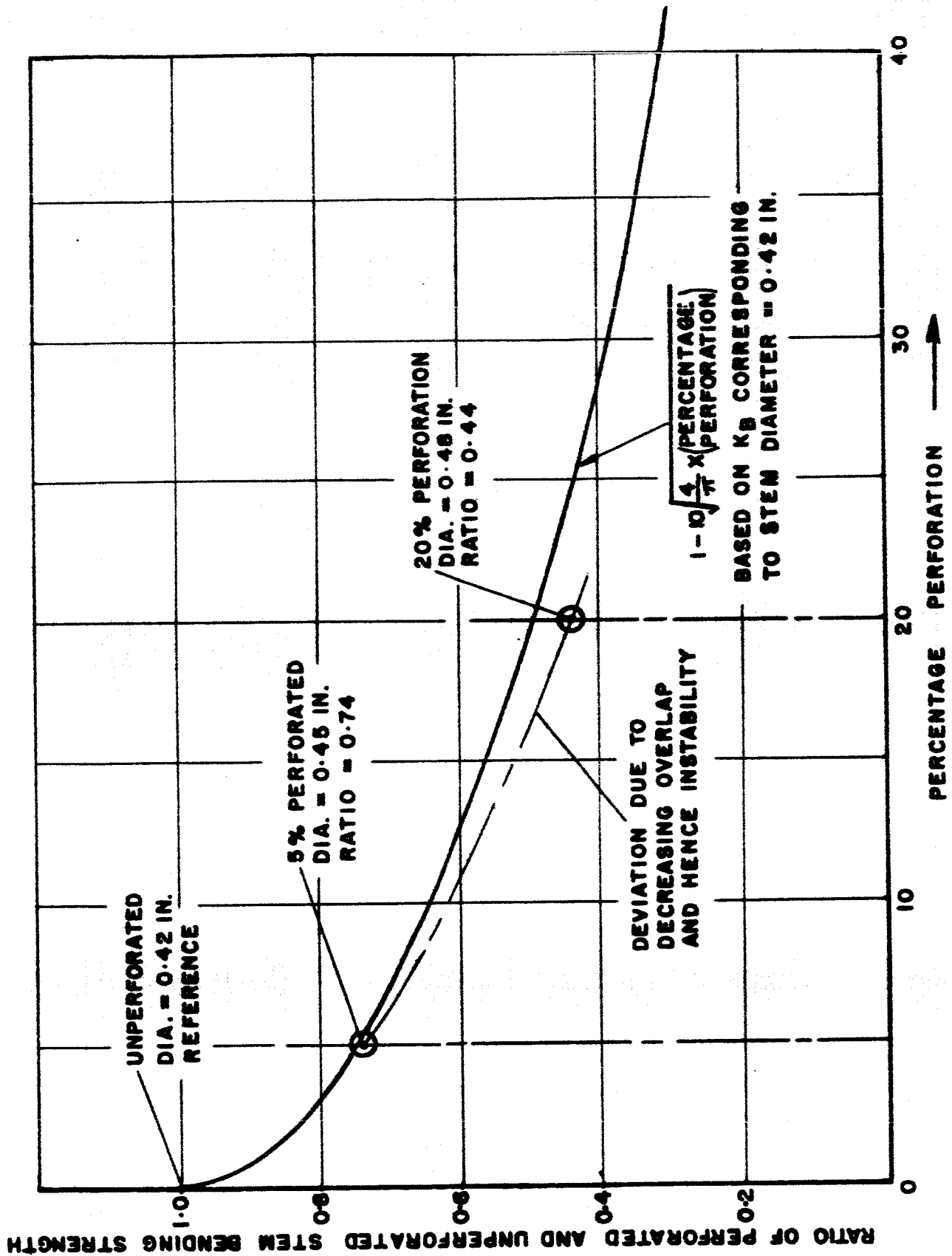


ROOT FAILURES OF PERFORATED STEMS

Figure 28



TIPLOAD VS DEFLECTION OF PERFORATED STEMS



BENDING STRENGTH OF PERFORATED STEMS RELATIVE TO
THE UNPERFORATED STEM

6.0 DISCUSSION OF THERMAL BENDING RESULTS

The results of the deflections of the various percent area removed STEMs are summarized in Figures 21 and 22. It is obvious from the curves that a zero deflection condition exists for a specific % area removed since both positive and negative bendings have been measured on the samples tested. For the chamber both the 14% and 20% STEMs yield, with minor exceptions, negative bending, while the unperforated samples have all shown a positive deflection. A correlation of these deflections on a quantitative basis using Equation 2-2 is not possible since the precise value for the irradiance level at the STEM in the chamber is not known. However, assuming that the total irradiance level for the 1100°C source is approximately 140 milliwatts per sq.cm., one would expect for an α_s of the outer surface of .045 a deflection of approximately 4 mm for an unperforated STEM. It is expected, however, that due to spherical aberration caused by the cylindrical mirrors and a possible slight out-of-focus condition for the filament, that the irradiance level could be down by a factor of 2 from the design value quoted above. This would then bring the observed zero perforation deflection in line with that predicted by Equation 2-2. Also overlap (which will be discussed at greater length below) may be affecting the results so that the equation derived for the seamless tube would not apply for the overlap configuration.

To examine the results in a relative manner one can consider the term (see Section 2 for definition of terms)

$$\alpha_{eff} = (1 - p)(\alpha_c - p\beta_c) \quad 6-1$$

as an effective absorptance value for the STEM sample, positive to give positive (unperforated) deflections, and negative when deflections are negative. Let us consider STEM samples with the following characteristics:

$$\begin{aligned} \alpha_c &= 0.045 \\ \beta_c &= 0.95 \end{aligned}$$

Then Table 6-1 shows the effective α_{eff} for the p values studied in this test program.

TABLE 6-1

EFFECTIVE STEM ABSORPTANCE α_{EFF} AND THERMAL DEFLECTIONS COMPARED TO THE UNPERFORATED STEM

STEM	α_{EFF}	DEFLECTION RATIO TO UNPERFORATED STEM	THERMAL DEFLECTION
0	+ 0.045	1.0	+ 2 mm
4.7% (Nominally 5%)	0.	---	---
14%	- 0.076	1.7	- 3.4 mm
20%	- 0.116	2.6	- 5.2 mm

Comparing the deflection values of the 14% and 20% STEM for a "zero bending" STEM deflection of +2 mm, with the actual results shown in Figures 21 and 22, indicate that they lie within the scatter of deflections obtained in the test program. Thus, within the scatter present in the technique, the results appear to agree with those predicted by the theoretical equations.

The scatter in results, which are clearly evident in the curves, are in the main due to the fact that hole registration in the overlap regions is not as good as it should be, and also because of the existence of the overlap seam and its effect on the equations derived for seamless tubes. This latter point is strongly supported by examining the more consistent results obtained when the overlap is facing away from the source, a condition most in agreement with the assumptions made in the theory of bending of perforated tubes.

It is also felt that this general overlap problem is the main factor causing the deflections observed in the y direction (transverse to the source direction).

The data (see Section 5.2) for the finally selected "zero bending" STEM (5%) strengthens further the problem currently present due to the overlap. It will be noted that for only those cases where the overlap is not facing the source at some portion of length down the chamber were the extremely small deflections noted (see Figure 20).

Additional factors that may be contributing to the scatter in the results

but ones which could not be quantitatively assessed in this test program, are as follows:

- a) Possible deflection distortion introduced at the rotary seal mount due to method of clamping and/or excessive temperature gradients in this region of the STEM.
- b) residual bow in the STEM along its length may result in irregular illumination of the STEM along its length.
- c) The twist in the overlap position along its length.

None of the above conditions could be examined quantitatively since only one viewing port is available in the chamber, this being located at the bottom or free end of the suspended STEM.

7.0 CORRELATION TO SPACE SOLAR ENVIRONMENT

The equation for zero bending (Equation 2-3) was presented in Section 2. It can be seen from this equation that the only parameters that are changed when one compares chamber results with the space solar environment are:

- (1) the total absorptance of the outer surface.
- (2) the total absorptance of the inner surface.

The latter factor being changed only very slightly, if at all. Thus a "zero bending" STEM in space will require a different percent area removed than that of the "zero bending" STEM for the chamber, due mainly to the change in the total absorptance of the silver film (see Section 5.3).

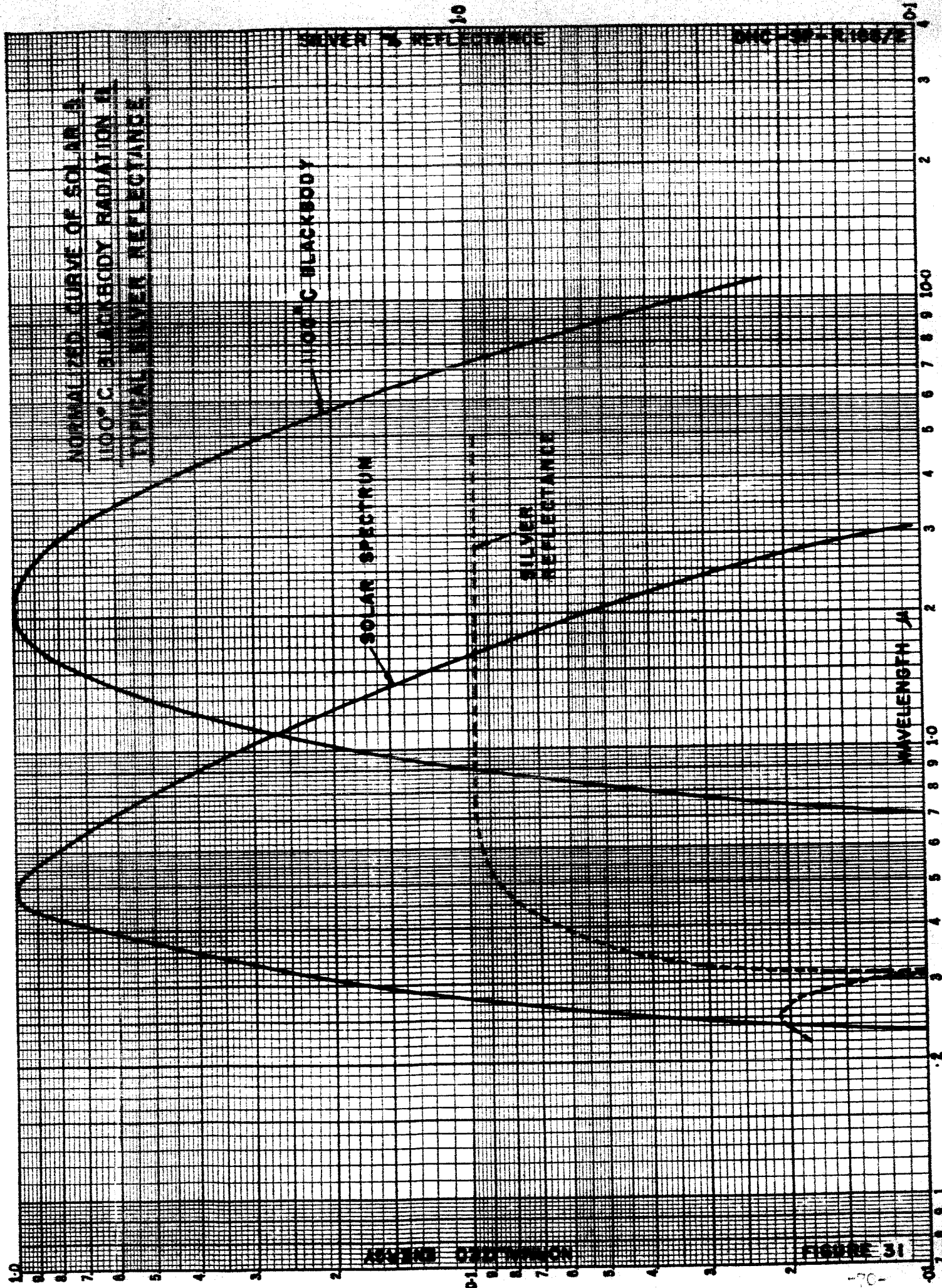
Using Equation 2-3, and taking values for α_s from 0.10 to 0.15 and $\beta_s = .95$, then the fractional area removed will be as shown in Table 7.1.

TABLE 7.1

FRACTIONAL AREA REMOVED FOR "ZERO BENDING" STEM IN SPACE
SOLAR ENVIRONMENT OUTSIDE SILVER PLATED (α_s)
INSIDE PT401 $\beta_s = 0.95$

α_s	P
.10	.11
.11	.122
.12	.132
.13	.142
.14	.154

It was shown in Section 5.5 that 20% area removal results in a bending moment strength of 7.2 inch pounds. If such an area removal was used with an excellent outside surface absorptance β_s could be as low as 0.64. This introduces the possibilities of using oxidized Be-Cu inside (see Section 5.3.7), avoiding a painted coating as the inside surface.



8.0 CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

From the presentation of deflection results (Section 5.2) as shown in Figures 17, 18, 19, 20, 21 and 22 and the discussion of results (Section 6) it is clear that the test program has clearly demonstrated the feasibility of the perforation technique to produce "zero bending" STEMs for the chamber. The data which most strongly supports the above is the results obtained for the 5% STEM (Figure 22, Orientation 3) where the overlap is away from the source, and the twist along its length at most moves the overlap to the side, but never towards the source. In this latter case X - deflections were + .3 and + .6 mm for the two samples tested. It would appear from these results that a slightly higher percentage (6-7%) removal would result in a closer approximation to the "zero bending" STEM for the chamber.

It is also evident from the deflection results that the overlap orientation with respect to the source greatly affects the results for any % area removed STEM. Varying the overlap orientation causes considerable scatter in the deflection measurements and also introduces a displacement perpendicular (Y) to the direction of the source. Two basic problems exist with the overlap:

- (1) it is difficult to get perfect hole registration between the overlap and underlap sections.
- (2) the overlap region represents a discontinuity in the thermal conduction path, which causes bending distortion, as evidenced by the random results obtained with the unperforated samples. No theory has been derived for this condition, but the problem has been considered and is discussed briefly in Appendix I.

The test program has demonstrated that reproducible high reflectance, electrodeposited, silver coatings can be made but that proper precautions in handling this material must be carried out to maintain high reflectance.

Measurements on the reflectance characteristics of oxidized Be-Cu have indicated that it is feasible to get sufficiently high solar absorptance values for this material (0.64) to use it in the space application, with 20% area removal.

8.2 Recommendations

The results and conclusion of this test program have indicated certain areas which require further study to implement the technique into a final production item having "zero bending" characteristics in space. The present study has shown that, with straight seam overlap STEM with 12% area removed, greatly reduced thermal bending would result in space. It is questionable because of the overlap problem, whether this reduction would be adequate for the more exacting applications where bending tolerances are extremely small. For the latter case further reduction of the residual bending would require studies in the areas outlined in the following sections.

8.2.1 Alternate Internal Surfaces

Problems may exist in the use of a painted inside surface, such as PT401. These arise not from the aspect of durability in space, but from the aspect that repeated handling and extension in the STEM devices may cause deterioration of the surface characteristic, thereby reducing its solar absorptance value. Tests should be conducted to see if alternate surface treatments, such as oxidized Be-Cu, oxidized Ni or oxidized Tantalum, could be used. All these surface treatments exhibit low reflectance in the visible and infrared.

8.2.2 Overlap Test Program

Further testing and experimental work should be performed to improve overlap registration and reduce its deflection distortion affect in thermal bending. A spiral technique should be tested, where a combination of spiral pitch (say 360° twist every 3 ft.) and a hole pattern along the length of the STEM are used to eliminate the variation in STEM thermal bending with its orientation with respect to the source.

8.2.3 Reflectance in Space

As shown in Equation 2-3 the equalization of the radiant power absorbed inside and outside for a given fractional area removed (1) depends on the α s and β s values. Measurements to date have been made under normal atmospheric laboratory conditions where oxides, and sulphides may be lowering the reflectance of the silver films. In space under intense irradiation, low pressure and the presence of minute quantities of ozone, these oxides, sulphides, etc., may be "cleaned" away

resulting in lower α s values compared to those measured in the laboratory. As mentioned in Section 8.2.1 various internal surface treatments can be used to obtain the desired β s, again however, careful consideration should be made on their stability and alteration in β s under the space environment.

For the above reasons work should be done to study the effect of high vacuum and solar irradiation on materials selected for this application.

9.0 NEW TECHNOLOGY

The object of this program was to test, analyze and record results of elements, the technology for which was previously produced against private company funds. The result of such records and analysis was to be used by NASA in assessing long-boom techniques, and such results have been previously documented in prior progress reports and in the body of this final report. As the documentation of testing and evaluation was the end product in this program it naturally follows that no new technology was developed under this specific contract NAS 5-9572.

10.0 BIBLIOGRAPHY

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DHC-SP-R.168/2

APPENDIX I

PART I & II

TEMPERATURE GRADIENTS AND PROFILE
CHANGES IN LONG TUBULAR ELEMENTS
DUE TO INCIDENT RADIATION

by

A. Kemper - K. Farrell

(From DHC-SP-TN.164, Reference 1)



TABLE OF CONTENTS

	Page
0.0 SUMMARY	iv

PART I

EVALUATION AND DISCUSSION OF ANALYTICAL RESULTS

1.0 INTRODUCTION	2
1.1 Specification of the Element	2
1.2 Specification of the Radiant Energy	2
2.0 THE ELEMENT IN OUTER SPACE	
2.1 The Maximum Temperature Difference Along the Perimeter	3
2.2 The Temperature Profile Along the Perimeter	4
2.3 Element Profile Change	4
2.4 Explanation of the Profile Chart	4
2.5 Several Elements on One Satellite	5
2.6 Tip Deflection	5
2.7 Minimum Radius of Curvature	5
2.8 Maximum Element Temperature	6
2.9 Element Response Time	6
3.0 THE ELEMENT EXPOSED TO RADIANT ENERGY FROM SUN AND EARTH	
3.1 Analysis and Definition of Earth Radiant Energy	7
3.2 The Maximum Temperature Difference Along the Perimeter	8
3.3 The Temperature Profile Along the Perimeter	10
3.4 Effect of Earth Radiant Energy on the Element Profile	10
3.5 Element Profiles in Earth Orbits	10
3.6 The Maximum Temperature Along the Element Axis	11
3.7 Element Response Time	12



Table of Contents (Cont'd)

	Page
4.0 DISCUSSION OF BASIC ASSUMPTIONS	
4.1 Average Temperature and Internal Radiation	12
4.2 Neglecting the Overlap	12
4.3 Suggestion of a Tolerance Band for the Evaluation Results	13
 <u>PART II</u>	
MATHEMATICAL ANALYSIS	
1.0 INTRODUCTION	15
2.0 DERIVATION OF TEMPERATURE PROFILE FOR THIN WALLED CYLINDRICAL TUBE	
2.1 Heat Balance	16
2.2 Heat Flow from Source Absorbed at Surface	17
2.3 Heat Radiated to Space	17
2.4 Heat Flow due to Conduction	18
2.5 Heat Radiated Across Interior of Tube	18
2.6 General Heat Flow Equation	20
2.7 Temperature Profile with Sun Radiation Only	21
2.8 Temperature Profile with Sun and Earth Radiation Directly Opposed	23
2.9 Temperature Profile with Sun and Earth Radiation Not Directly Opposed	26
3.0 DERIVATION OF THERMAL DISTORTION	
3.1 Thermal Distortion due to Sun, and Directly Opposed Sun and Earth Radiation	29
3.2 Thermal Distortion due to Sun and Earth Radiation Not Directly Opposed	34
4.0 DERIVATION OF ELEMENT RESPONSE TIME	37
5.0 DISCUSSION OF ASSUMPTIONS	
5.1 Interior Radiation and Surface Radiation to Space	39
5.2 Heat Flow Along Tube Length	41
5.3 Overlapped Tubes	42
APPENDICES:	
LIST OF SYMBOLS	46
REFERENCES	48



SUMMARY

The temperature difference between the bright and dark side of the specified 800 - ft. , half inch diameter, Beryllium Copper element, when exposed to sun radiation, will reach 12°F . This difference will cause the element to bend away from the radiation source in an asymptotic form. The tip deflection will be about 550 ft. , with a minimum radius of curvature of 333 ft.

The location of the point of minimum radius of curvature can be anywhere along the axis of the element from the root to the tip, depending on the element's initial direction with respect to the radiation source.

The point of minimum radius of curvature coincides with the point of maximum element temperature, which will be about 850°R with sun and earth radiation, and 785°R in outer space.

The temperature difference along the cross section, and the resulting profile change, are determined by the material and its surface absorptivity only and are independent of the emissivity; while the maximum temperature depends on the ratio of absorptivity to emissivity only.

The magnitude of the time constant of the specified element with respect to step changes in radiation is about 3 minutes. The final profile (96%) will be established in about 7 minutes after a sudden extension of the element into the radiation field.



PART I

EVALUATION AND DISCUSSION OF ANALYTICAL RESULTS



PART I

EVALUATION AND DISCUSSION
OF ANALYTICAL RESULTS

1.0 INTRODUCTION

This study of the effect of radiant energy impinging on long tubular elements in space investigates the temperature distribution along the perimeter and the axis of the element and the resulting profile changes. The results are condensed into equations and into a Profile Chart, allowing examination of the particular profile of an element in outer space, or in an orbit around the earth. The time constant of the element is also investigated analytically.

1.1 Specification of the Element

The specified element is considered as a closed tube for most of the investigations. A correction factor is discussed and proposed for an open tube with 180° overlap. Numerical values of temperatures along the perimeter and the axis, and values for the minimum radii of curvature and tip deflections are calculated for an element with the following specifications:

Element Material	Beryllium copper
Element Diameter	$d = 0.04167$ feet (0.50 inches)
Element Length	$l = 800$ feet
Wall Thickness	$t = 0.000167$ feet (0.002 inches)
Material Density	$\rho = 500$ lb/ft ³
Thermal Conductivity	$k = 44$ BTU/ft - °F - hr
Thermal Expansion Coefficient	$\epsilon = 1.04 \times 10^{-5}$
Absorptivity Ratio	$\alpha = .45$
Emissivity Ratio	$\epsilon = .10$
Specific Heat	$c = .10$ BTU/lb - °F
Overlap Factor	$\phi = 1.5$

1.2 Specification of the Radiant Energy

Sun and earth radiation, and reflection of sun radiation by the earth and the atmosphere, are taken into account. These are assumed to have the following intensities: -



Sun Radiation

$$J_S = 450 \text{ BTU/HR FT}^2$$

Earth Radiation

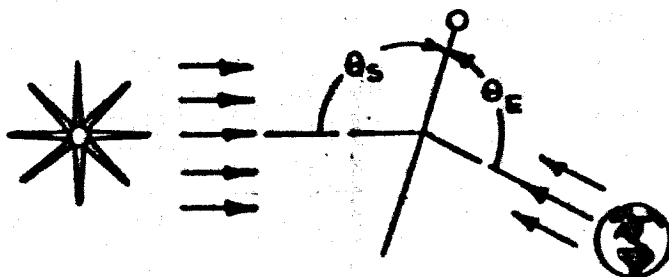
$$J_E = 70 \text{ BTU/HR FT}^2$$

Earth Reflection

$$J_R = 160 \text{ BTU/HR FT}^2$$

Sun radiation is considered to be constant in the scope of this analysis. Earth radiation and reflection are subject to correction for altitude; the values applied, referring to the upper layers of the mesosphere, which is assumed to be 500 miles above the surface of the earth.

The amount of heat entering the element is also a function of the incident angle. This angle is defined in the analysis as the angle between the radiation direction and the axis of the element as seen from the root of the element. (See diagram below.)



2.0 THE ELEMENT IN OUTER SPACE

2.1 The Maximum Temperature Difference Along the Element Perimeter

The maximum temperature difference between the bright and dark side of the element occurs when the incident angle $\theta_s = 90^\circ$. It is obtained by the equation:

$$\Delta T_s = \frac{d^2}{4tk} \alpha_s J_s \sin \theta_s \quad (1)$$

It is independent of the emissivity ratio ϵ . $T_s (\text{max})$ is 12°F for the specified element.



2.2 The Temperature Profile Along the Element Perimeter

The Temperature Profile is shown in Figure 2. It is symmetrical on both sides of the diameter from the bright side to the dark. It is obtained from the equations:

$$\Delta T_s(1) = \Delta T_{s \max} \left[\frac{1}{2\pi} \left(\frac{x}{r} \right)^2 + \cos \frac{x}{r} - .45 \right] \quad (2)$$

$$\Delta T_s(2) = \Delta T_{s \max} \left[\frac{1}{2\pi} \left(\pi - \frac{x}{r} \right)^2 - .45 \right] \quad (3)$$

Equation 2 gives the profile in the first and fourth quadrants, and equation 3 the profile in the second and third quadrants. In both cases, X is a point on the perimeter expressed in terms of $r \times \pi$

2.3 Element Profile Change

The temperature difference between the bright and dark sides of the element will cause the element to bend away from the radiation source. The profile of the bent element, the tip deflection, and the radii of curvature along the element axis, are determined by a "Profile Number" which is defined as

$$\text{Profile Number: } N_s = \frac{e}{d} \Delta T_s(\max) = \frac{1}{4} \frac{dc}{tk} J_s \alpha_s \quad (4)$$

It has a value of 3×10^{-3} for the specified element.

Using this profile number, the element shape can be derived by the equation:

$$x = \frac{1}{N_s} \ln \cos(Y \times N_s) \quad (5)$$

This profile has been plotted in Figure 2 for the specified element and a radiation incident angle $\theta_s = 90^\circ$. The X and Y axes are in feet; and the length of the element has been traced on the plotted profile. Figure 3 shows the same element for a radiation incident angle $\theta_s = 135^\circ$. Figure 4 shows the profiles of a dipole element with incident angles from 0 to 180° .

2.4 Explanation of the Profile Chart

The profile curves of Figures 2, 3 and 4 are extracted from the Profile Chart shown in Figure 5. Any 800 foot length traced on the curve



with the Profile Number 3×10^{-3} , can be a profile of the specified element. The root of the element is located at the intersection point of the incident angle line with the profile curve, and the tip of the element is found by tracing the length of the element (800 feet) clockwise on the profile curve.

The Profile Chart and equation 4 for the Profile Number, imply that elements with different design parameters can have the same profile change as long as the design parameters in equation 4 yield the same Profile Number. Hence one can examine the change of profile for a change of a particular design parameter, by comparing the profile curves with the appropriate numbers.

2.5 Several Elements on One Satellite

As long as the design ensures that the incident radiation angle is the same for all the elements, several elements on one satellite will have the same profile. A crossed dipole of elements as specified, will have a configuration as shown in Figure 6. The configuration will be maintained in the orbit around the sun as shown in Figure 7. The elements have the tendency to align themselves and the satellite radially with respect to the orbit, so that the same side of the satellite surface always faces the sun. The backward-bent elements which are exposed to the sun's pressure, will create a correcting torque as soon as the satellite deviates from the equilibrium either in the pitch or yaw axis. The dynamic properties of the elements; their damping effects, and time lags, have to be included in the orientation and stabilization system of the satellite.

2.6 Tip Deflection

The value X in Equation 5 represents the tip deflection from the initial straight profile. It can be obtained only by tracing the profile. However, for very small Profile Numbers, or for short elements, Y can be replaced by the element length: l and Equation 5 can be simplified to:

$$X = \frac{1}{2} l^2 N \quad (6)$$

Thus the deflection for a 100 foot length would be 15 ft., when exposed to sun radiation only.

2.7 Minimum Radius of Curvature

The greater the temperature difference on the cross section, the more the element will bend at that section, and the smaller will be the radius of the curvature. The location of the minimum radius of curvature



will be at the point where the incident angle is 90° . In Figure 2 the location is at the root of the element and in Figure 3 where the tangent on the element is normal to the radiation direction. The radius at any point is given by

$$R(x) = \frac{1}{N} \sin \theta(x) \quad (7)$$

where $\theta(x)$ is the angle between the tangent and the initial direction. The minimum radius of curvature for the specified element is 333 feet., when exposed to sun radiation. Angle $\theta(x)$ can be obtained from the graph or computed from the equation:

$$\theta(x) = \left(\frac{\pi}{2} - Y(x)N \right) \frac{180}{\pi} \quad (8)$$

2.8 Maximum Element Temperature

The element temperature depends on the absorptivity and emissivity ratios, and the incident radiation angle, and is otherwise independent of the element design parameters. At any point X along the axis the average element temperature is given by

$$T(x) = \left(\frac{1}{\epsilon \sigma \pi} \alpha J_s \sin \theta(x) \right)^{1/4} \quad (9)$$

where $\theta(x)$ is again the angle between the tangent on the element and the radiation direction. The location of the point of maximum temperature coincides with the location of the point of minimum radius of curvature. Figure 8 shows maximum element temperature versus the ratio α/ϵ and Figure 9 depicts the temperature along the axis of one element in two different positions with respect to the sun. The maximum temperature of the specified element is 785°R when exposed to sun radiation only.

2.9 Element Response Time

It has been assumed that the element will have found structural and thermal equilibrium when the region of highest temperature has been reached. For this condition, the time constant C_T , which is defined as the time in minutes necessary to reach 63.2% of maximum temperature rise after a small step increase in radiant energy, is: -



$$C_T = \frac{\psi}{4} \times \frac{ct\rho}{\sigma E} \times \frac{1}{T_{s\max}^3} \quad (10)$$

"C" for the specified element would be 3 minutes.

However, the actual temperature response curve for a large step increase involving sudden extension of the element into the sunlight, must be derived from the equation:

$$\Delta T(\text{step}) = \frac{\sigma E}{ct\rho\psi} (T_{s\max}^4 - T^4) \Delta \tau(\text{step}) \quad (11)$$

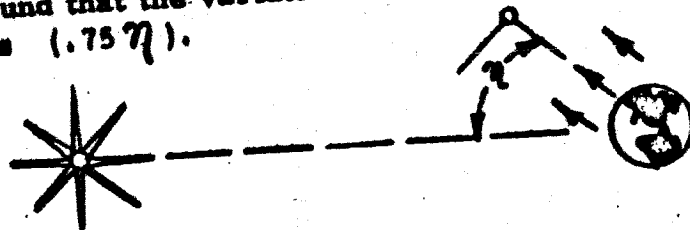
The transient temperature profile is shown for the specified element in Figure 10. The element needs 7 minutes to reach 96% of the final profile.

3.0 THE ELEMENT EXPOSED TO RADIANT ENERGY FROM SUN AND EARTH

3.1 Analysis and Definition of Earth Radiant Energy

A satellite in orbit around the Earth will be equally exposed to earth radiation due to the earth temperature at all positions of its orbit. The earth radiation constant has to be corrected for the distance of the satellite from the earth.

Reflected radiant energy can reach the satellite only as long as the sun and earth are in opposition with respect to the satellite. However, since the reflected sunlight is diffused in the atmosphere, it has been assumed that the reflection approaches zero, when the angle η between sun, Earth and Satellite reaches 120° . (See diagram below.) The variation of reflected radiation with η , between 0 and 120 degrees, can be represented by a complicated expression derived in Reference 1. However, it has been found that the variation can be reasonably approximated by the function $\cos(1.75\eta)$.



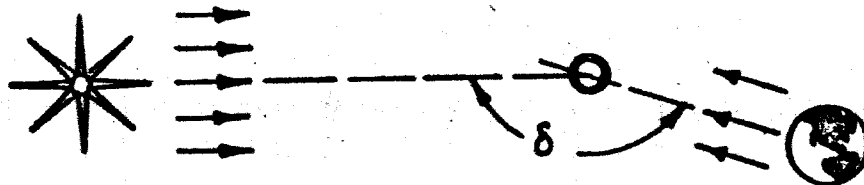


Both radiant energy sources - earth radiation and earth reflection - are subject to correction for altitude with the factor

$$F_A = \left(\frac{R+500}{R+A} \right)^2 \quad (12)$$

since radiation intensity varies inversely with the square of the distance. R is the radius of the earth, and A is the altitude of the satellite above the earth surface.

The effect of radiant energy from the earth, also depends on the magnitude of the angle between the sun and earth radiation with respect to the element cross section. This angle is illustrated in the diagram below and should not be confused with the angle η , although both may have the same value in particular positions.



3.2 The Maximum Temperature Difference Along the Perimeter

The maximum temperature difference and the profile along the perimeter is obtained by multiplying the appropriate equations 1 to 3 by the factor B to get:

$$\Delta T_{SRE} = \frac{d^2}{4tk} \alpha_s J_s \sin \theta_s \times B \quad (13)$$

wherein B represents the effect of earth reflection of sunlight and earth radiation. B is developed as follows: the total amount of impinging radiation which will cause the temperature rise consists of:

Sun radiation:	$\alpha J_s \sin \theta_s$
Earth reflection:	$\alpha J_R \cos .75\eta \times \sin \theta_E \times F_A \cos^2 \delta$
Earth radiation:	$E J_E \sin \theta_E \times F_A \cos^2 \delta$

The angles θ_s and θ_E were defined in 1.2 and the angle $.75\eta$ as well as the factor F_A in 3.1. Thus the total amount of impinging radiation is: -



$$\Sigma J = \alpha J_s \sin \theta_s \pm (\alpha J_R \cos .75\eta + \epsilon J_E) \sin \theta_E \times F_A \cos^2 \delta \quad (14)$$

$$\Sigma J = \alpha J_s \sin \theta_s \left[1 \pm \left(\frac{J_R}{J_s} \cos .75\eta + \frac{J_E}{J_s} \frac{\epsilon}{\alpha} \frac{\sin \theta_E}{\sin \theta_s} F_A \cos^2 \delta \right) \right] \quad (15)$$

where the sign + applies for $\delta < \frac{\pi}{2}$ and the sign - for $\delta > \frac{\pi}{2}$.
The term in brackets is called B thus:

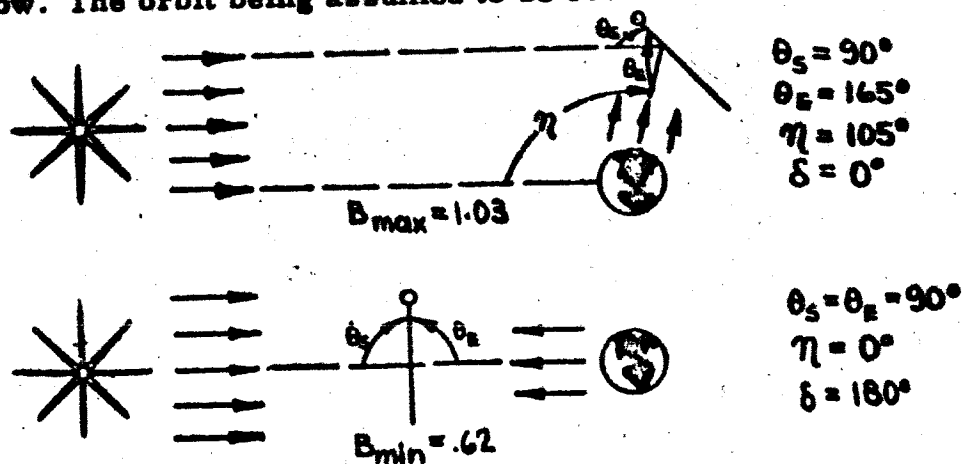
$$B_1 = 1 \pm \left(.353 \cos .75\eta + .155 \frac{\epsilon}{\alpha} \right) \frac{\sin \theta_E}{\sin \theta_s} F_A \cos^2 \delta \quad (16)$$

and when reflection approaches zero with $\eta > 120^\circ$

$$B_2 = 1 \pm .155 \frac{\epsilon}{\alpha} \frac{\sin \theta_E}{\sin \theta_s} F_A \cos^2 \delta \quad (17)$$

Inspection of the terms for B reveals that the magnitude of B depends mainly on the magnitude of the four angles which define the position of the element with respect to the sun and earth. This is because the ratio $\frac{\epsilon}{\alpha}$ is very small, and is constant for the specified element.

The extreme values for B occur in the positions in the diagram below. The orbit being assumed to be 500 miles above the earth's surface;





3.3 The Temperature Profile Along the Perimeter

The temperature profiles for the above defined positions are shown in Figure 11. They represent the borderlines for all possible conditions. They are computed from the equation:

$$\Delta T_{ser} = \Delta T_s \times B \quad (18)$$

Since B_{min} will increase, and B_{max} will decrease with orbits at higher altitudes, both will approach Unit 1 at an altitude of about 20,000 miles. This means that the influence of earth radiation and reflection will become virtually negligible at this altitude and above. The temperature profile then will be that indicated by the dotted line. This is the same as shown in Fig. 1.

3.4 Effect on Earth Radiant Energy on the Element Profile

The Profile Number of the element will be changed in the same ratio as the temperature gradient along the perimeter is changed. Thus equation 4 becomes:

$$N_{ser} = N_s \times B \quad (19)$$

Because B_{min} was found to be .62 for the specified element with $\epsilon_a = .22$ the minimum Profile Number for the specified element when exposed to sun and earth radiant energy will be:

$$N_{ser(min)} = 1.86 \times 10^{-3}$$

The appropriate profile for this Profile Number is marked in Figure 12 on the Profile Chart, for comparison with the profile for the number 3×10^{-3} . It is the lowest Profile Number which can occur in the most favourable position with sun and earth in opposition, with respect to the satellite. For all other positions particularly for all higher altitudes the Profile Number will increase until it approaches the value 3×10^{-3} at an altitude of about 20,000 ft.

3.5 The Elements in Orbits Around the Earth

Unless the satellite travels across the shadow of the earth, where the elements will have almost a straight profile - the elements have the tendency to align themselves to the direction of the sun radiation. This



applies for all orbits, polar orbits as well as for equatorial or semi-equatorial orbits.

In a polar orbit (Figure 13), with the satellite orientated to the earth so that always the same surface will face the earth, the elements will keep their configuration with respect to the earth and to the sun. They will not influence the orientation and stabilizing system of the satellite.

In a semi-equatorial orbit however as shown in Figure 14 the elements will try to align themselves and the satellite to the sun radiation, so the satellite will perform one revolution around its axis while performing one orbit cycle. Thus when the satellite would have an orientation control to face the earth always with the same side of the surface the orientation control has to cater for the disturbance caused by the elements.

In such cases the elements would take profiles as shown in Figure 15 and 16. In Figure 15 only one element is assumed, pointing to the earth in its initial position. For Figure 16 a dipole of two elements is assumed in the plane of the orbit, as tangents on the orbit path in their initial position. This configuration assumes that the orientation control keeps the satellite always in the position facing the earth with the same side of the surface.

3.6 Maximum Element Temperature Along the Element Axis

The temperature of the element on the point of Minimum Radius of Curvature is a function of the total radiant energy impinging on the element. Thus

$J_s \alpha \sin \theta_s$ in equation 9 is replaced by:

$$\sum J = J_s \alpha \sin \theta_s + (J_R \alpha \cos .75 \eta + J_E E) F_A \sin \theta_E \cos^2 \delta$$

which yields

$$T_{ser} = T_s \times B^{1/4} \quad (20)$$

but with always the sign + after 1'. (Equation 16 in 3.2.)



Thus the maximum temperature occurs for

$$F_A = 1 \text{ and } \theta_s = \theta_E = 90^\circ \text{ and } \eta = 0^\circ \text{ and } \delta = 180^\circ,$$

when sun and earth are in opposition with respect to the satellite.

In this case the maximum temperature for the specified element is:

$$T_{ser(max)} = 850^\circ R$$

3.7 Element Response Time

Replacing $T_{s \max}$ in equation 10 by $T_{ser} = T_{s \max} B^{-3/4}$ yields the response time of the element when exposed to sun and earth radiant energy simultaneously. It would be about 2.35 minutes instead of about 3 minutes so that the overall pattern of the transient curve in this case will be similar to the curve as shown in Figure 10.

4.0 DISCUSSIONS OF BASIC ASSUMPTIONS

4.1 Average Temperature and Internal Radiation

The derivation of equation 1 to 3 assume an average element temperature along the perimeter of the tube for element surface radiation into space. More rigorous analysis has shown that for the specified element, the computed numerical values can be regarded as 2% higher than the true values. It was also found that inclusion of the internal radiation between the inside surface of the element would decrease the computed values in the same order. Both assumptions suggest, therefore, the introduction of a correction factor of 0.96 for average temperature.

4.2 Neglecting the Overlap

A closed tube of equal wall thickness has been assumed for this analysis. (i.e. - equal conductivity along the cross section.) The overlapped tube however has cross sections of varying thickness and undetermined thermal content. Difficulties arise when the overlap is included in the equations, due to the fact that the overlapped sections can have multiple positions with respect to the radiation source. They will change their position continuously, due to twisting effect of very long elements.

The evaluation of four distinct configurations suggest that a 180° overlap will decrease the temperature gradient in the order of 25%, calling for a further correction factor of 0.75.



4.3 Suggestion of a Tolerance Band for the Results

If the correction factors are included, all the computed numerical values of this analysis, would be about 30% to high. However, a number of other assumptions may call for corrections in the other direction. For instance, all the assumed numerical values of the absorptivity and emissivity ratios, as well as the values of the thermal conductivity, and the thermal expansion coefficient, may be larger or smaller than assumed. In view of this it is felt that the actual profiles will be within a $\pm 30\%$ band about the computed values. This tolerance can be significantly reduced when α and ϵ values for the material are more accurately determined.

TEMPERATURE PROFILE ALONG THE PERIMETER
OF A TUBULAR ELEMENT IN OUTER SPACE
ELEMENT PROFILE: $N=3 \times 10^{-3}$

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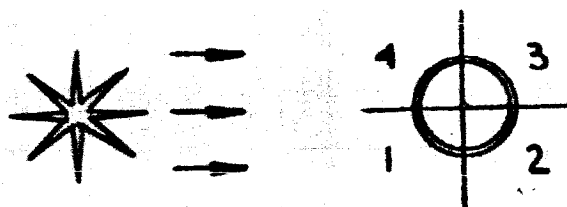
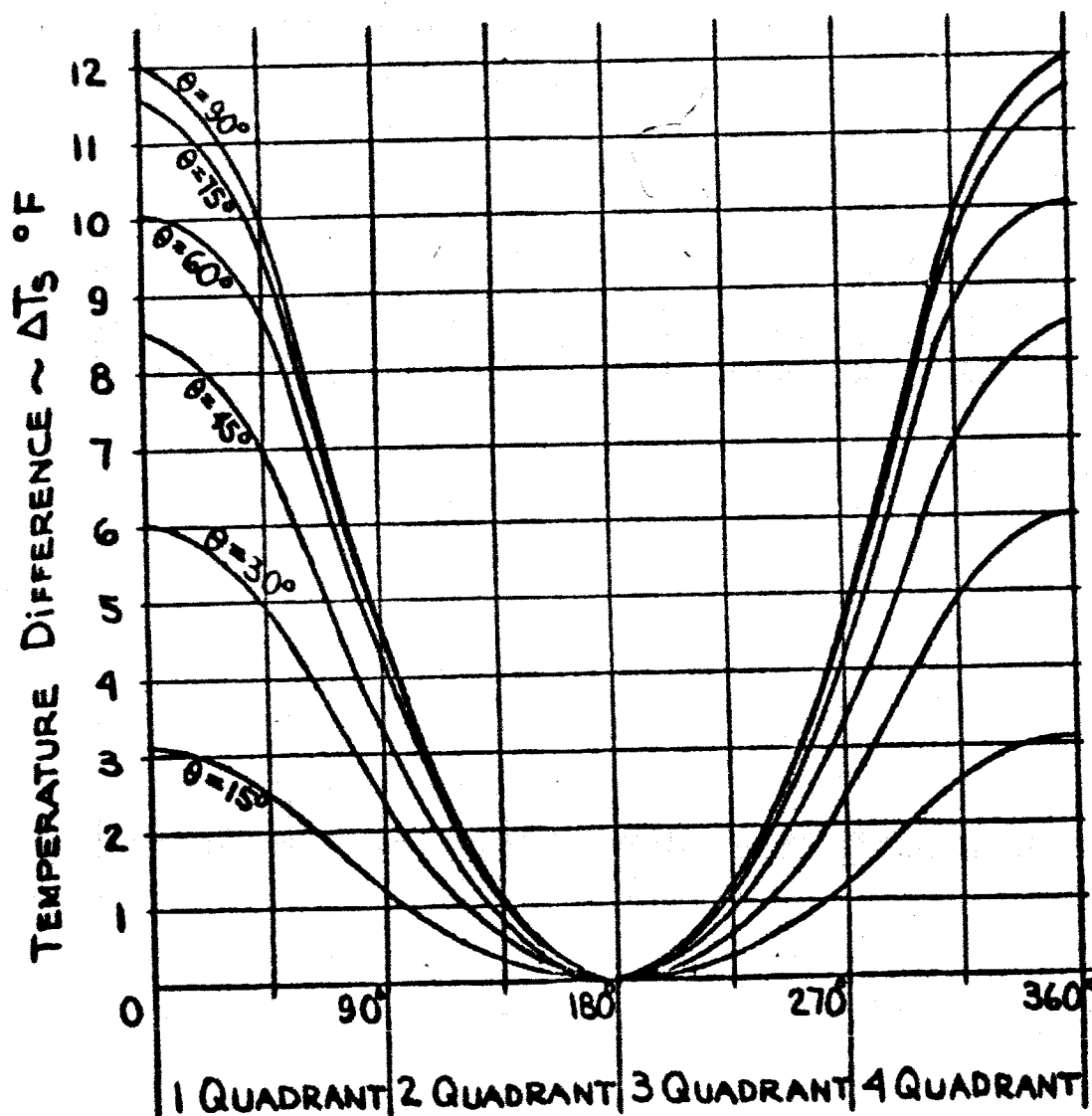


Fig. 1



$$\Delta T_s = \frac{1}{4} \frac{d^2}{t k} J_s \alpha_s \sin \theta_s$$



DEFLECTION PROFILES OF A TUBULAR ELEMENT
IN OUTER SPACE
ELEMENT PROFILE: $N = 3 \times 10^{-3}$ 800 FT. LONG

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Fig. 2

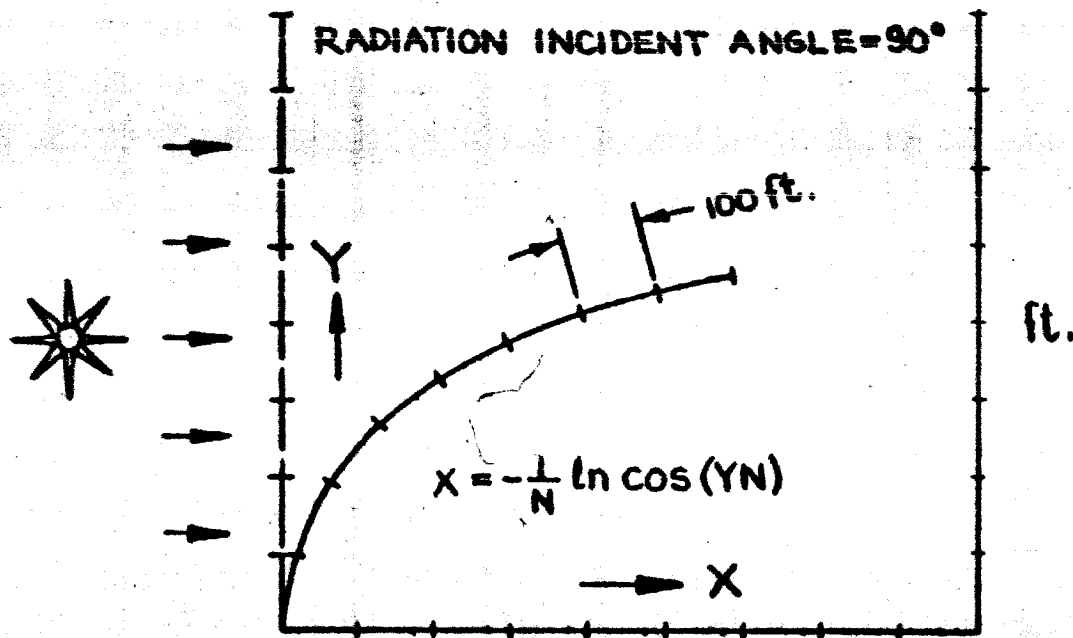
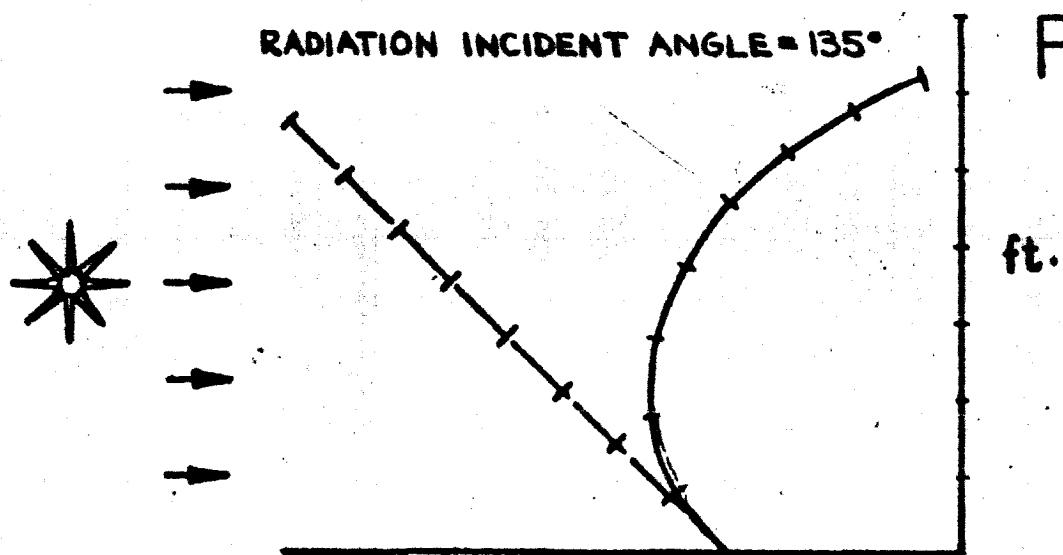
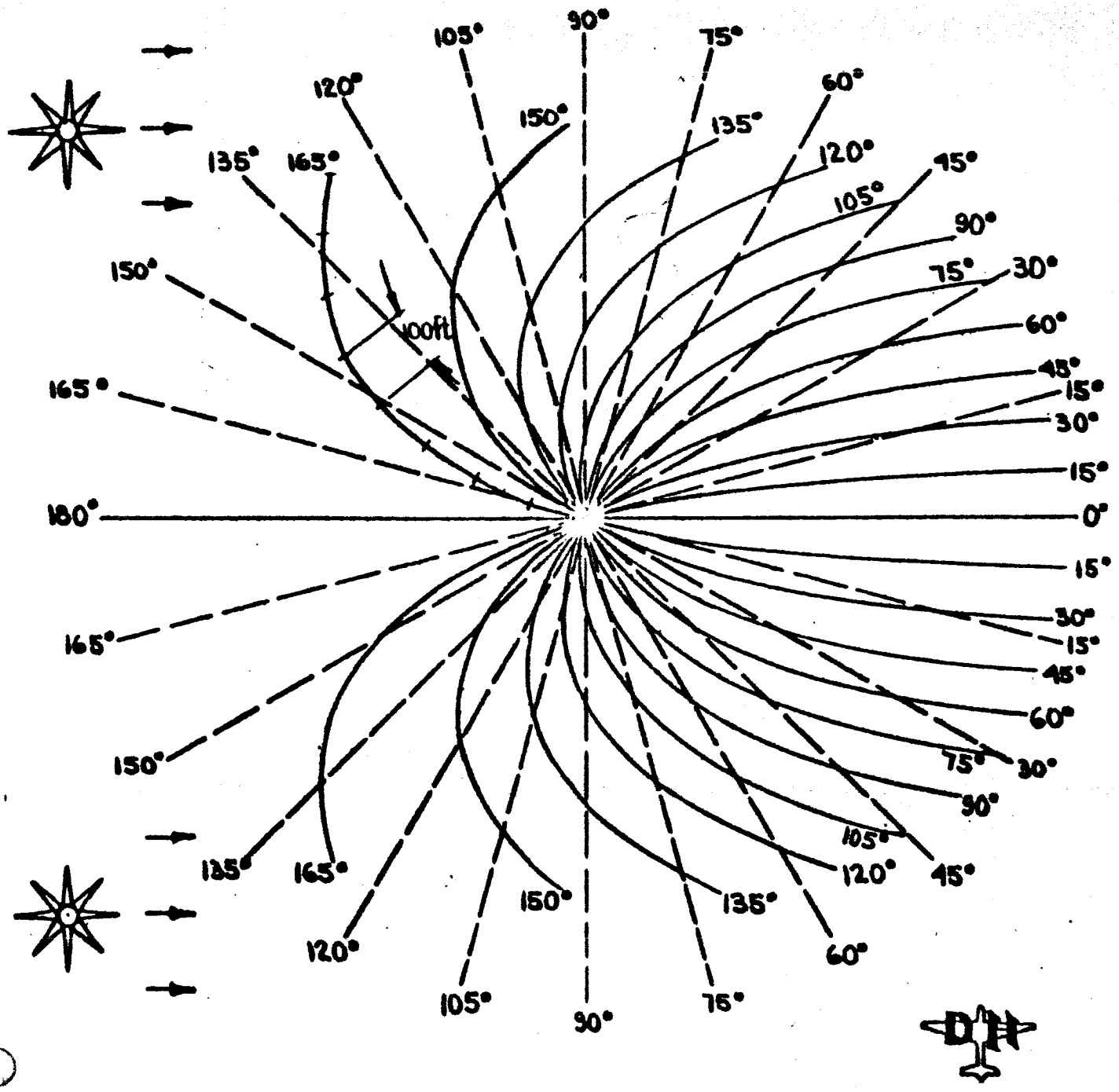


Fig. 3



DEFLECTION PROFILES OF A TUBULAR ELEMENT IN OUTER SPACE FOR VARIOUS RADIATION INCIDENT ANGLES ELEMENT PROFILE: $N = 3 \times 10^{-3}$; 800 FT. LONG	GRAPH NO.: 37	
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Fig. 4



DEFLECTION PROFILE CHART

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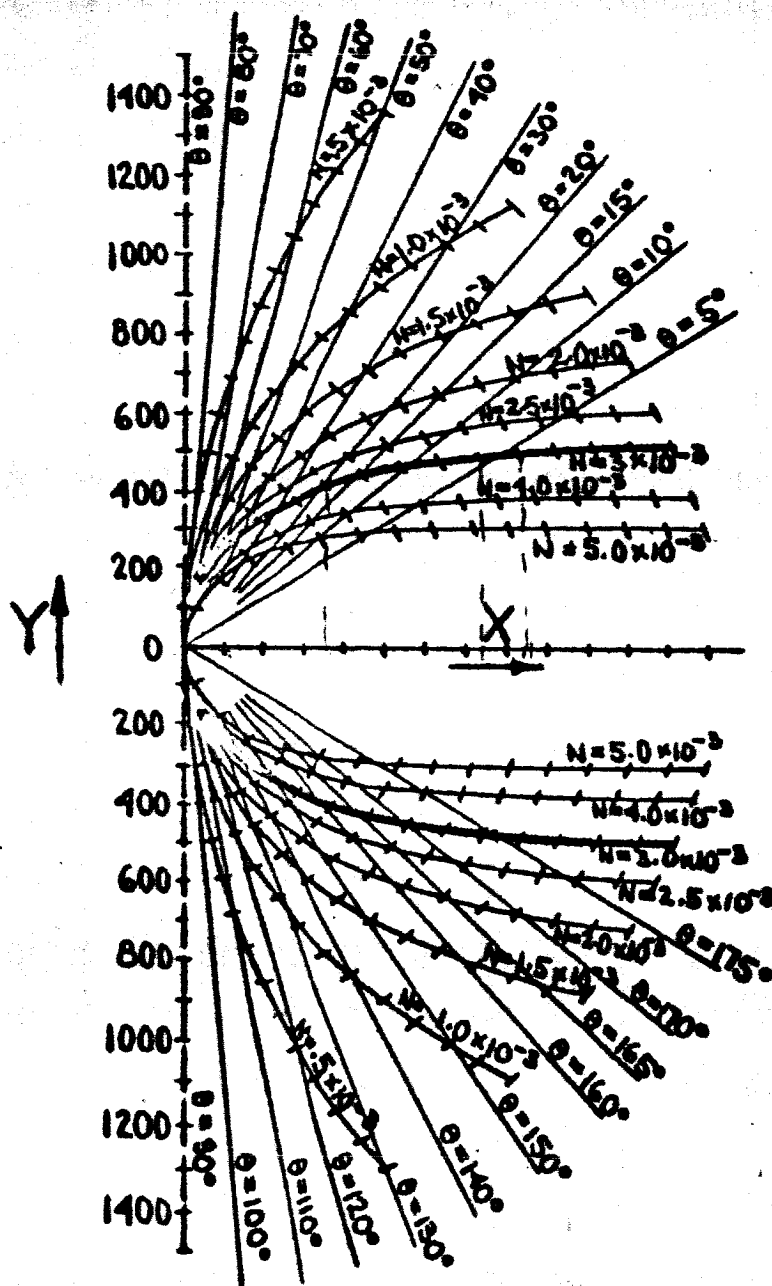
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$$X = -\frac{1}{N} \ln \cos(\gamma N)$$

Fig. 5



DEFLECTION PROFILES OF TUBULAR ELEMENTS
IN OUTER SPACE
ELEMENT PROFILE: $N = 3 \times 10^{-3}$; 800 FT. LONG

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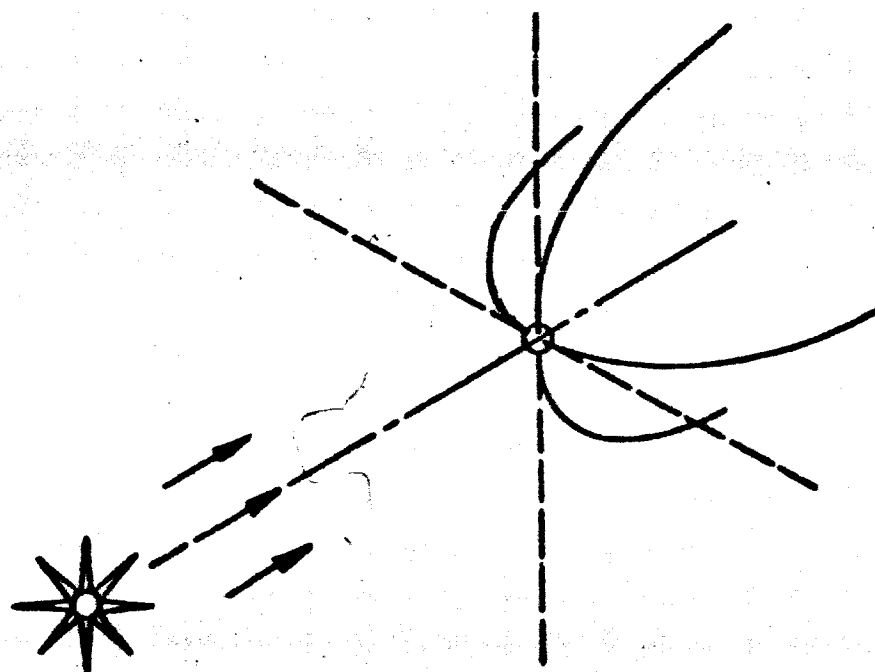


FIG. 6

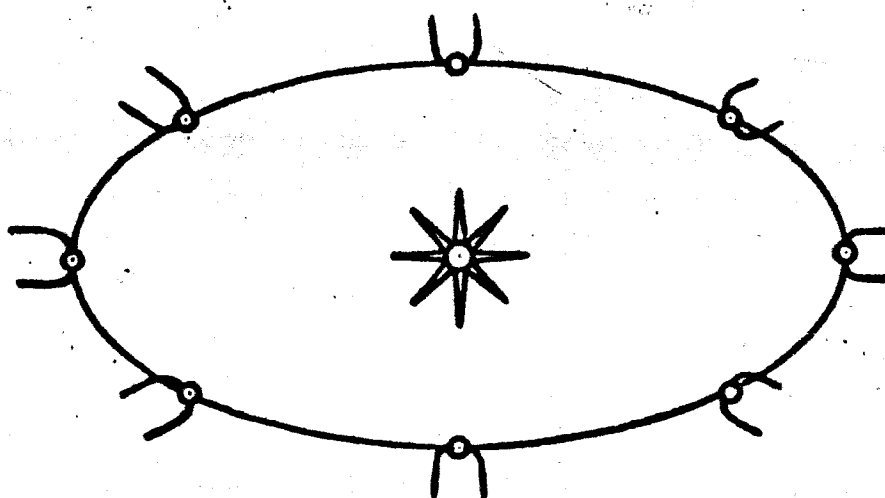


FIG. 7



DHC-SP-R.168/2

MAXIMUM TEMPERATURE OF A TUBULAR ELEMENT
IN OUTER SPACE

GRAPH NO.: 40

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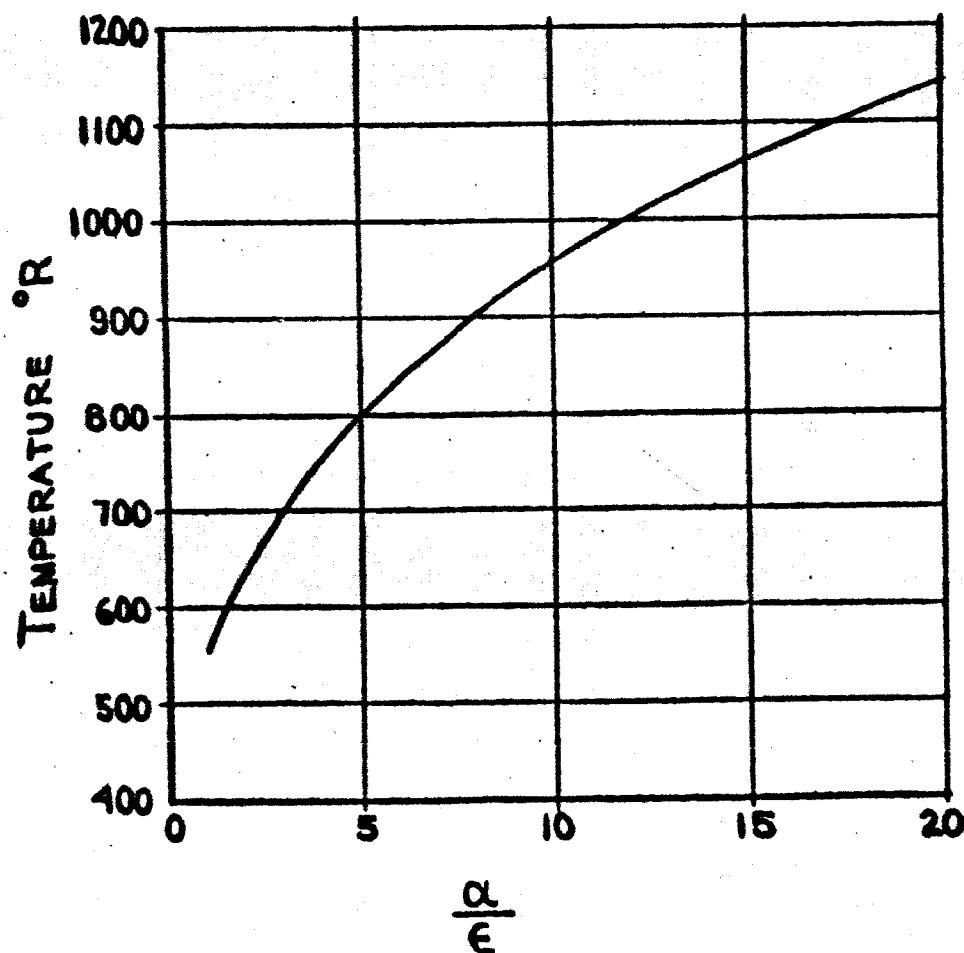
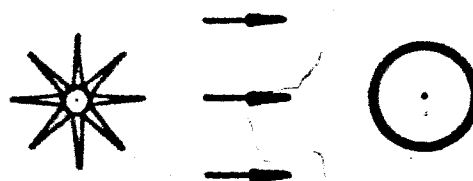
DATE:

10-1-63

APPR.:

$$T_m = \left(\frac{\alpha_s J_s}{\sigma \epsilon_s \pi} \right)^{1/4}$$

Fig. 8



DHC-SP-R. 168/2

TEMPERATURE ALONG THE AXIS OF A TUBULAR
ELEMENT IN OUTER SPACE
ELEMENT PROFILE: $N = 3 \times 10^{-3}$

GRAPH NO.: 41A

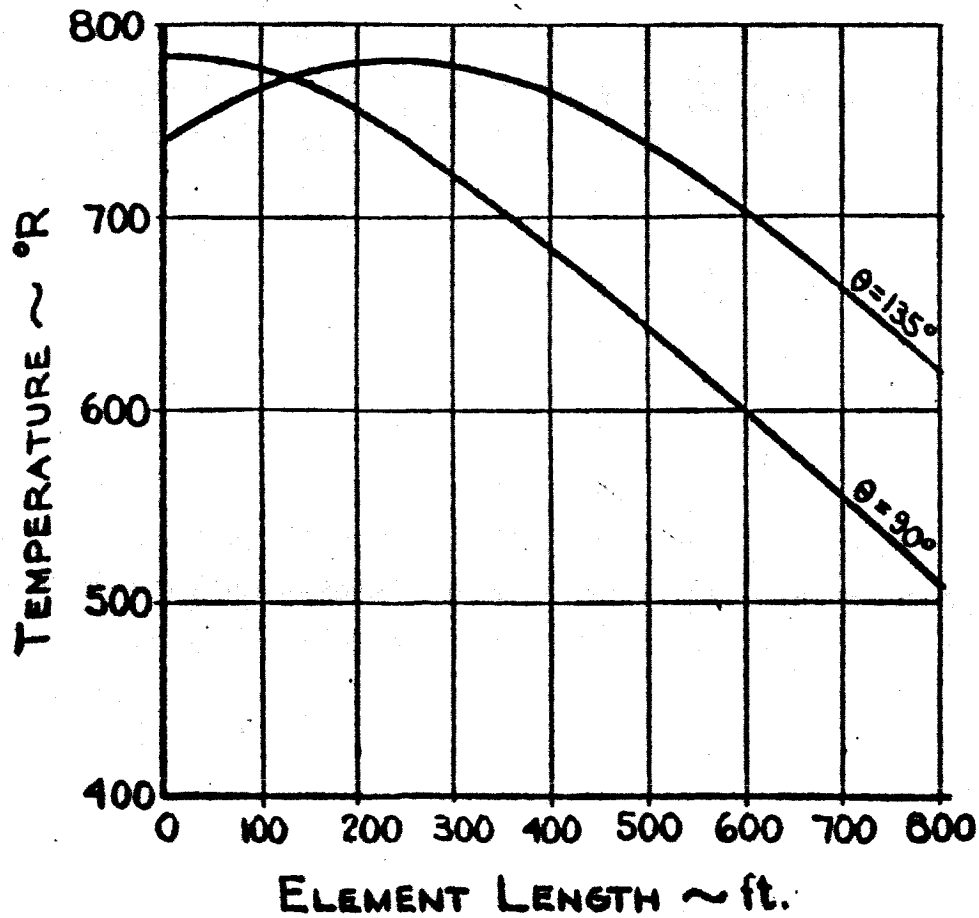
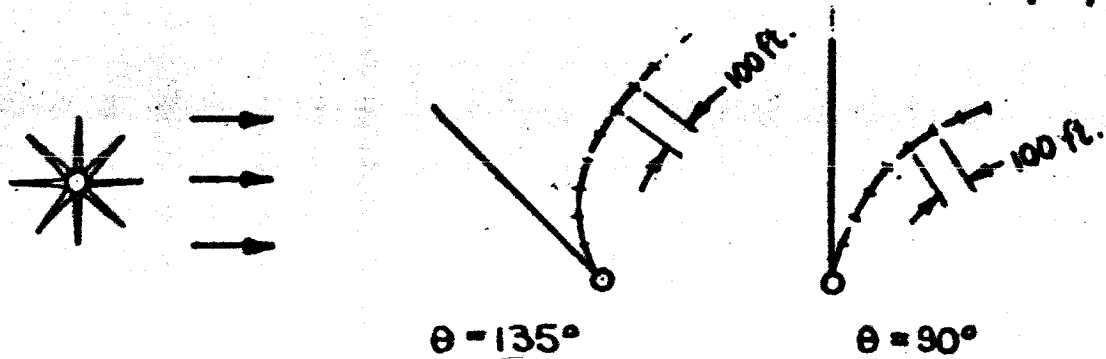
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APPR.: *A/K*

Fig. 9



TRANSIENT TEMPERATURE OF A TUBULAR
ELEMENT IN OUTER SPACE DUE TO A STEP
CHANGE IN VARIATION FROM 0 TO 100%

GRAPH NO.: 42

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DATE:

11-1-63

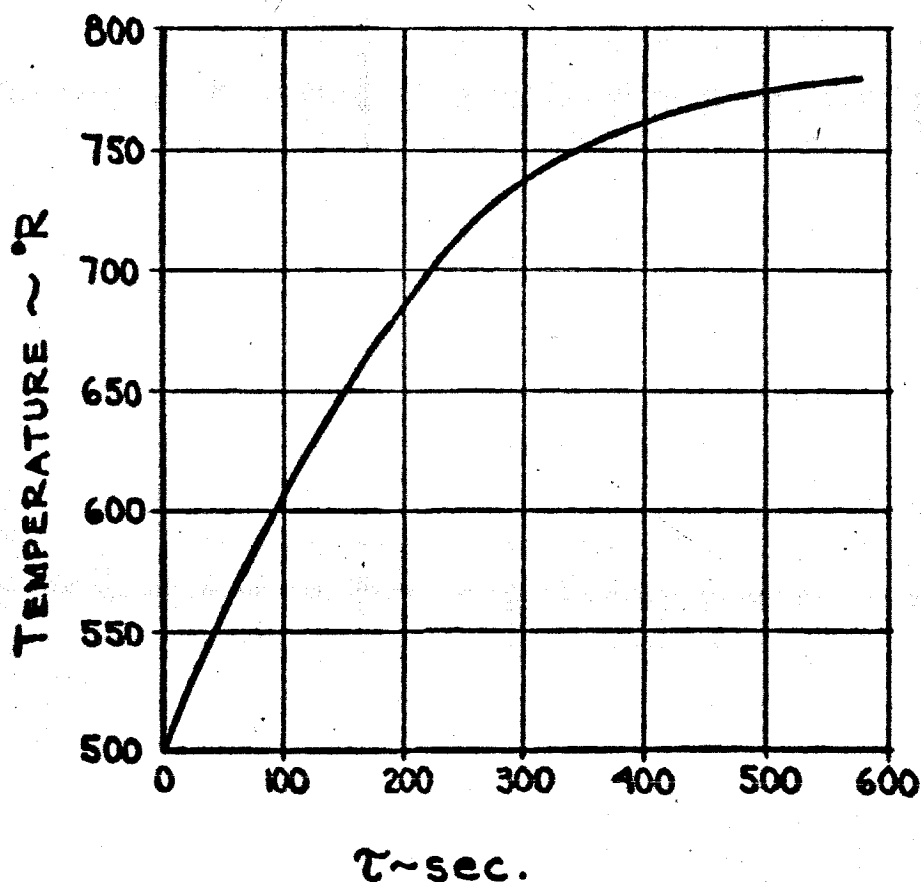
APPR.:

OVERLAP = 180°
 $C_p = .1 \text{ Btu/lb.-°F}$
 $\rho = 495 \text{ lbs/ft}^3$

$t = .002 \text{ in.}$
 $\epsilon = .1$
 $\psi = 1.5$

Fig. 10

$$\Delta T = \frac{\sigma \epsilon}{\psi \rho C_p t} (T_{\max}^1 - T^1) \Delta \tau$$

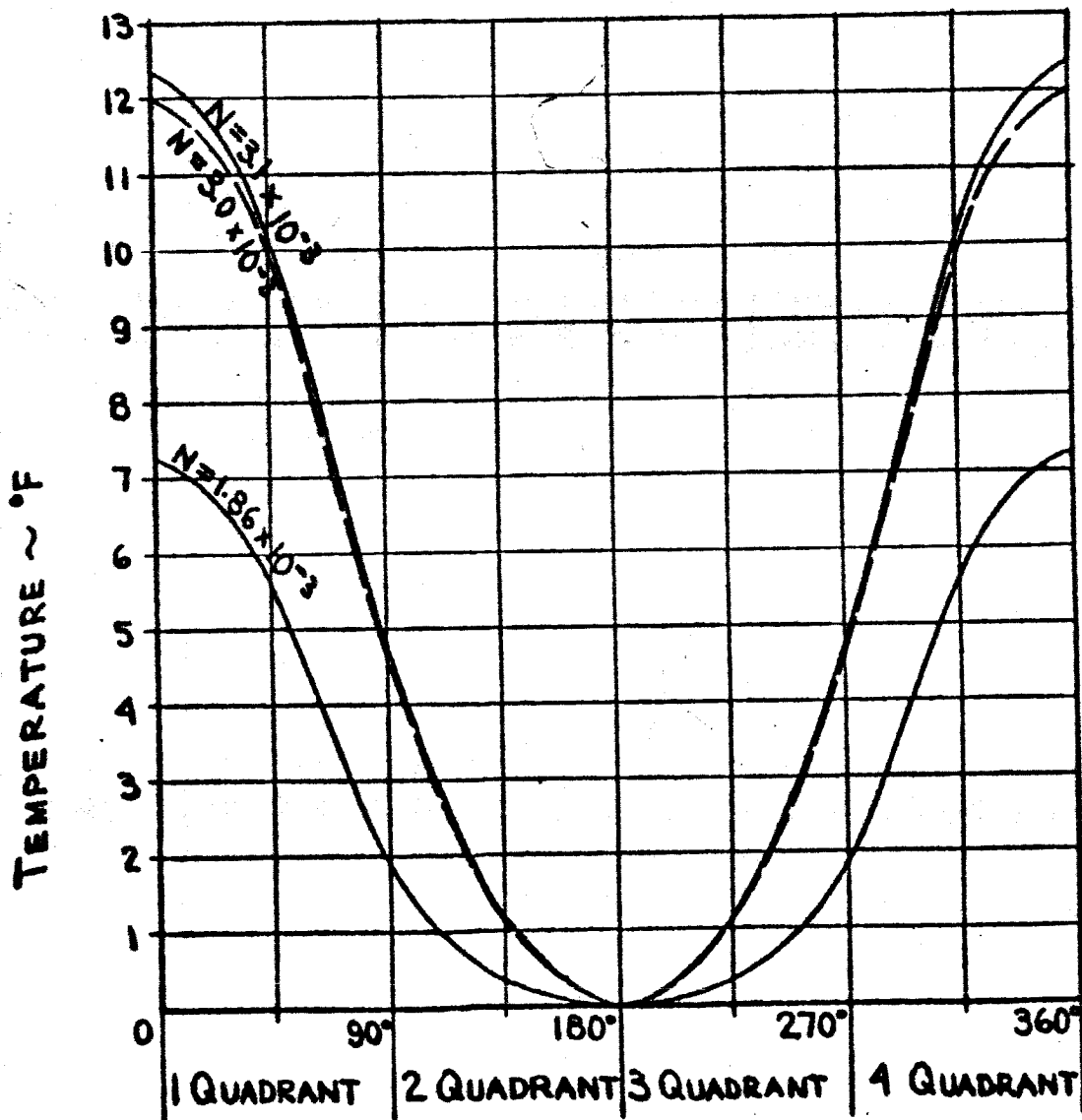
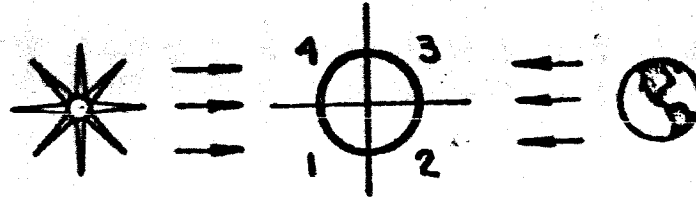


DHC-SP-R. 168/2

TEMPERATURE PROFILE ALONG THE PERIMETER OF
A TUBULAR ELEMENT IN ORBITS AROUND THE EARTH
ELEMENT PROFILE : $N = 1.86 \times 10^{-3}$ TO 3.1×10^{-3}

GRAPH NO. : 43	
DRAWN: <i>DR</i>	DATE
APPR.:	11-1-63

Fig. 11



DEFLECTION PROFILE CHART

GRAPH NO. : 38

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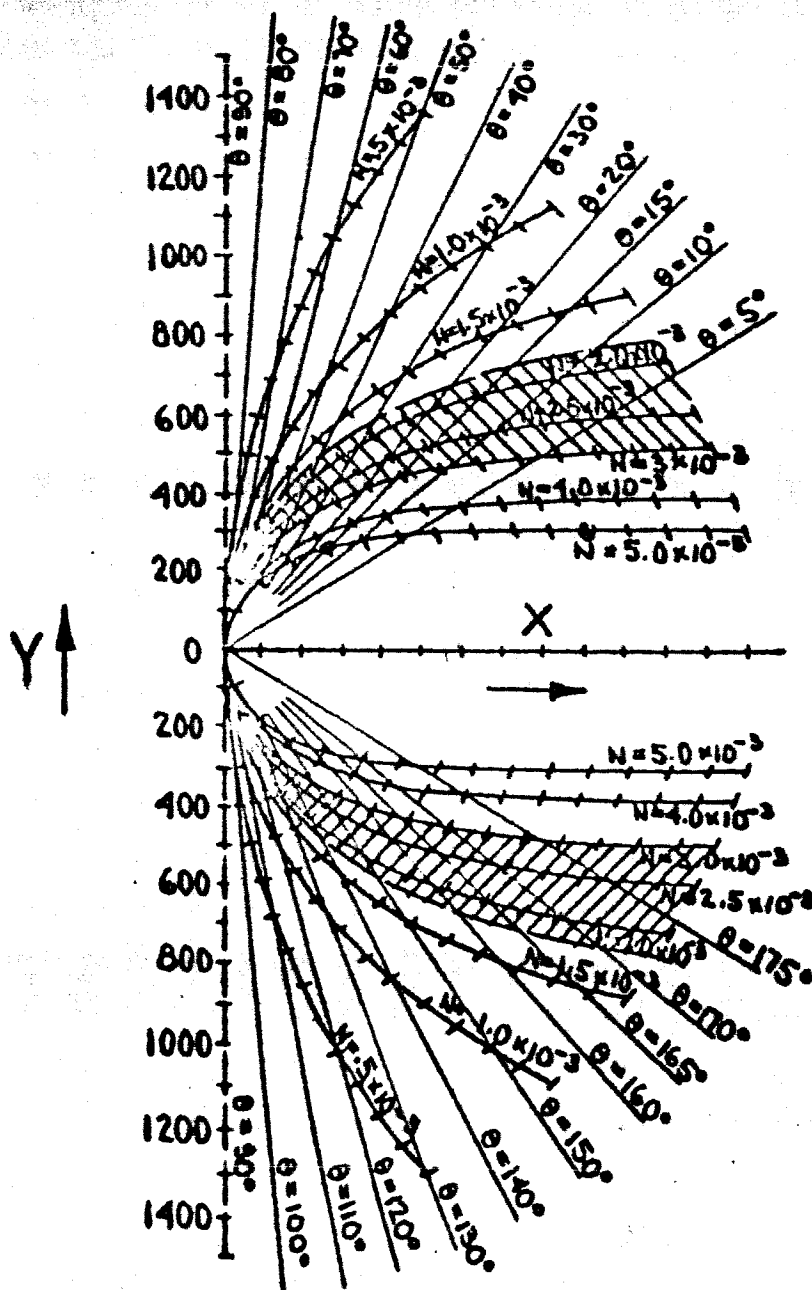
DATE:

10-1-63

APPR.:

$$X = -\frac{1}{N} \ln \cos(\theta N)$$

FIG. 12



DEFLECTION PROFILES OF TUBULAR ELEMENTS
IN ORBIT AROUND THE EARTH
ELEMENT PROFILE: $N = 1.86 \times 10^{-3}$ TO 3.1×10^{-3}

GRAPH NO.: 44

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11-1-63

APPR.:

POLAR ORBIT

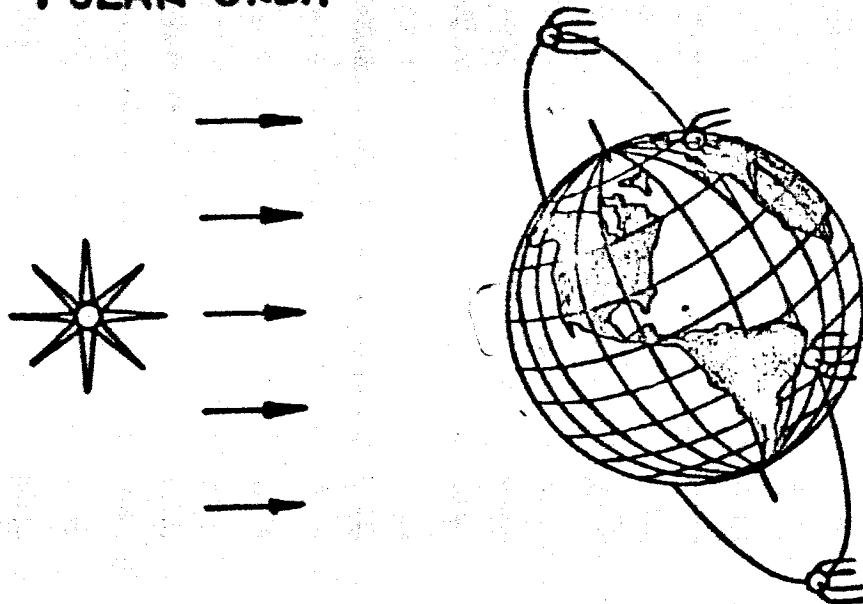


Fig. 13

SEMI-EQUATORIAL ORBIT

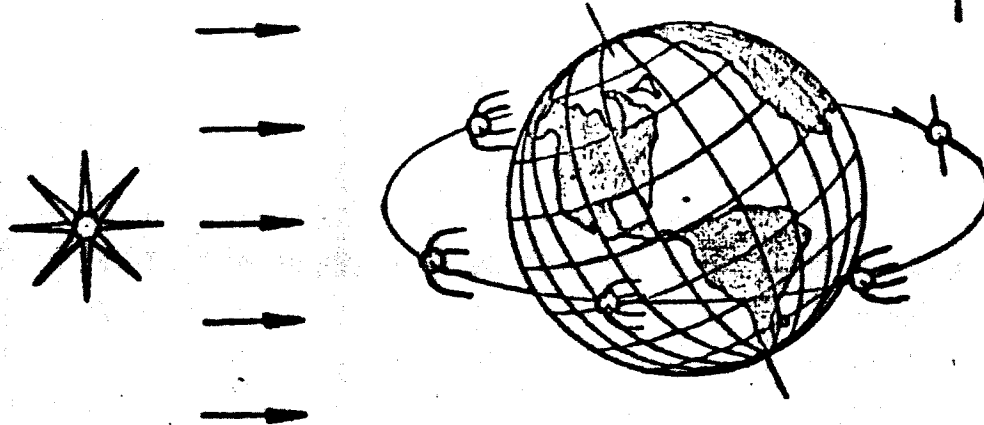


Fig. 14



DEFLECTION PROFILES OF TUBULAR ELEMENTS
IN ORBITS AROUND THE EARTH
ELEMENT PROFILE: $N=1.86 \times 10^{-3}$ TO 3.1×10^{-3}

GRAPH NO.: 45

DRAWN: *QW*

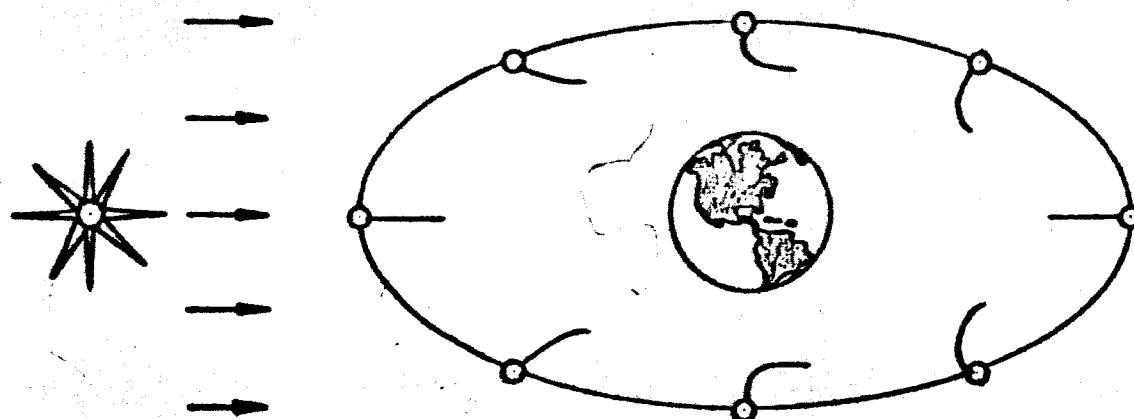
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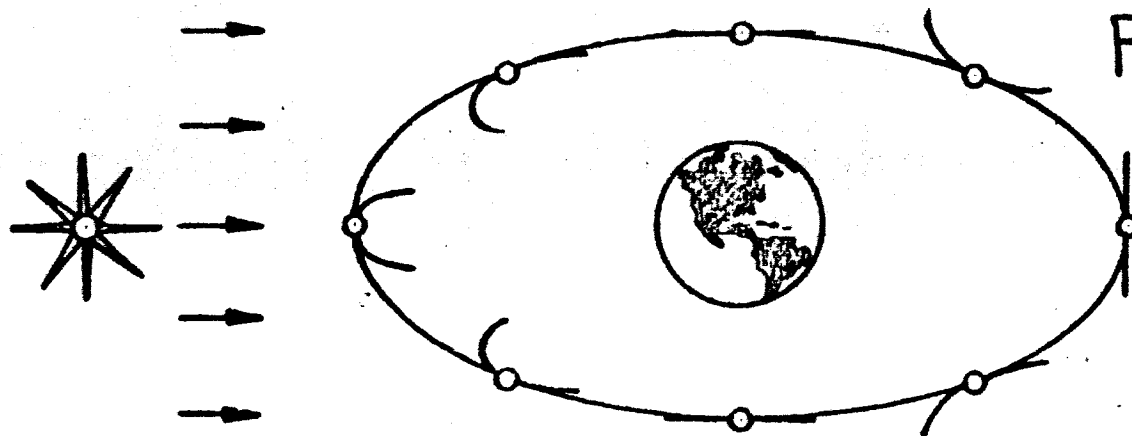
ONE ELEMENT POINTING TO THE EARTH

Fig. 15



DIPOLE ELEMENTS AS TANGENT TO THE ORBIT

Fig. 16



DHC-SP-R. 168/2



DHC-SP-R.168/2

PART II

MATHEMATICAL ANALYSIS

PART II

MATHEMATICAL ANALYSIS

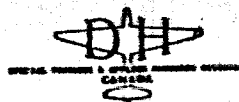
1.0 INTRODUCTION

The purpose of this part of the report is to show the mathematical development of the formulas and equations needed to calculate distorted shapes of long thin-walled tubes subjected to heat radiation in space. For the most part, the equations are developed in general form, but in certain cases where complexities warrant it, numerical values are used to arrive at working equations. In these cases the equations are made to apply to a 0.5 inch diameter beryllium copper tube, 0.002 inches thick, subjected to radiation in space just outside the earth's atmosphere. Thus the principle sources of radiation considered are the sun, which delivers heat at an intensity of 450 BTU/HR-FT^2 , and the earth, which delivers its own heat at an intensity of 70 BTU/HR-FT^2 and reflected sun heat at an intensity of 160 BTU/HR-FT^2 .

The method of obtaining the required equations is to develop temperature profiles on the tube cross-section by consideration of the various radiation and conduction heat flows. The temperature profiles are then used to determine the thermal distortion of the tube.

Since the elements are made of flat metal strips formed into circular cross-section tubes with approximately 180 degrees of overlap, it is assumed that there is always some area of contact in the overlapped section. Because of the forming method used in producing the tubes, the most probable place for contact to occur is at the outer free edge of the formed strip. This type of contact makes the overlapped tube similar to a seamless tube from a heat transfer point of view, and for this reason the temperature profile development in section 2.0 deals only with seamless tubes. The effect of increased area of contact in the overlapped section on temperature profiles is discussed in section 5.0.

Equations for the distorted shape of tubular elements are developed in section 3.0. These equations are of interest mainly because they show clearly what factors will increase or decrease element deflections. Actual magnitudes of deflections obtained from these equations are only approximate because of the assumptions used in the mathematical development.



2.0 DERIVATION OF TEMPERATURE PROFILES FOR THIN-WALLED CYLINDRICAL TUBES

2.1 Heat Balance

The temperature profile of a thin-walled cylindrical tube can be determined by equating the heat absorbed to the heat rejected for a differential element of the tube. Since the thickness t is much smaller than the radius r , (less than $0.01 r$) temperature gradients in the radial direction will be negligible compared to temperature gradients around the perimeter, and the differential element shown in Figure 1 (a) can be changed to a rectangular element of length $dx = r d\theta$, as shown in Figure 1 (b). The heat flows indicated in Figure 1 (b) are defined as follows:

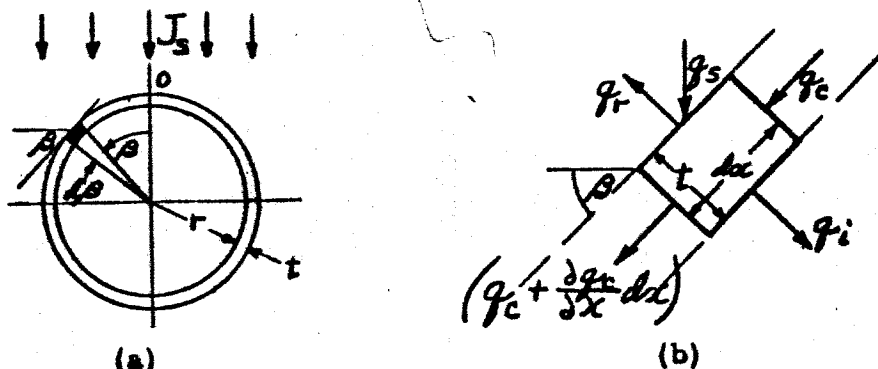


Figure 1

q_s = Radiant heat from sun or other source absorbed at the surface of the tube.

q_r = Heat radiated to space.

q_c = Heat flow around tube due to conduction.

q_i = Heat radiation across interior of tube.

Heat flow along the length of the tube is assumed to be zero, thus allowing all further calculations to be based on a unit length of tube. This assumption is valid as long as the heat source remains constant over long lengths of the tube.



Equating heat into heat out yields

$$q_s + q_c = q_r + q_i + \left(q_c + \frac{\partial q_c}{\partial x} dx \right)$$
$$-\frac{\partial q_c}{\partial x} dx = q_r + q_i - q_s \quad (1)$$

The temperature profile can be obtained from equation (1) by substituting for the indicated heat flows.

2.2 Heat Flow From Source Absorbed at Surface

If the source of radiant heat is the sun, the heat flow is:

$$q_s = J_s \alpha_s dx \cos\left(\frac{x}{r}\right) \quad (2)$$

where J_s is the radiation intensity and α_s is the solar absorptivity of the material.

If the source of radiant heat is the earth, the heat flow is:

$$q_s = (J_R \alpha_s + J_E \epsilon) dx \cos\left(\frac{x}{r}\right) \quad (3)$$

where J_R is the intensity of sunlight reflected by the earth, J_E is the intensity of infra-red earth radiation and ϵ is the emissivity of the tube material. It is assumed the absorptivity of any material for earth reflected sunlight is the same as that for direct sunlight.

2.3 Heat Radiated to Space

If a surface radiates to black body (space) surroundings, the heat flow is:

$$q_r = \sigma \epsilon T^4 dx \quad (4)$$



2.4 Heat Flow Due to Conduction

$$q_c = -k t \frac{dT}{dx}$$

from which:

$$\frac{\partial q_c}{\partial x} = -k t \frac{d^2 T}{dx^2} \quad (5)$$

2.5 Heat Radiated Across Interior Of Tube

Consider the interior surface of a tube divided into n elements of length Δx , as shown in Figure 2. The amount of radiation leaving element 1 and arriving at any element p is:

$$q_{1 \rightarrow p} = J_1 \frac{\cos \beta_1 \cos \beta_p (\Delta x_1)(\Delta x_p)}{\pi y^2}$$

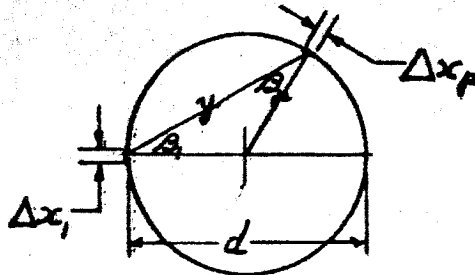


Figure 2

where J_1 is the surface radiosity of element 1. Since the elements lie on a circle, $\beta_1 = \beta_p$, and $y = d \cos \beta_1$. Therefore,

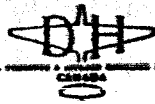
$$q_{1 \rightarrow p} = J_1 \frac{\cos^2 \beta_1 (\Delta x)^2}{\pi d^2 \cos^2 \beta_1} = J_1 \frac{(\Delta x)^2}{\pi d^2}$$

Similarly:

$$q_{p \rightarrow 1} = J_p \frac{(\Delta x)^2}{\pi d^2}$$

Thus the net heat flow from 1 to p is:

$$q_{1p} = (J_1 - J_p) \frac{(\Delta x)^2}{\pi d^2}$$



And the total heat flow from 1 to all the other elements is:

$$q_1 = \left[(n-1)J_1 - \frac{n}{2}J_1 \right] \frac{(\Delta x)^2}{\pi d^2} \quad (7)$$

Another expression for the heat leaving element 1 can be derived from the fact that it must be equal to the negative sum of the heat flows leaving the other elements. I.E.:

$$\sum_{r=1}^n q_r = 0$$

$$q_1 = - \sum_{r=2}^n q_r \quad (8)$$

Using the general expression for heat leaving the surface of element p

$$q_r = \frac{\epsilon(\Delta x)}{\gamma} (E_{b,r} - J_r) \quad (9)$$

where $E_{b,p}$ is the black body emissive power of the element and γ is the reflectivity, equation (8) becomes

$$q_1 = - \sum_{r=2}^n (E_{b,r} - J_r) \frac{\epsilon(\Delta x)}{\gamma} \quad (10)$$

Combining equations (7), (9), (10), and realizing that $E_{b,p} = \sigma T_p^4$, gives the following result:

$$q_1 = \left[\frac{(n-1)(\Delta x)\sigma T_1^4 - \sigma \epsilon \sum_{r=2}^n T_r^4 (\Delta x)}{\pi d^2 \epsilon + \gamma n (\Delta x)} \right] (\Delta x) \quad (11)$$



If the number of elements is large such that Δx approaches zero, then

$$(n-1)\Delta x \cong n\Delta x = \pi d$$

and

$$\sigma \epsilon \sum_{i=1}^n T_p^4(\Delta x) \cong \sigma \epsilon \int_0^{\pi d} T_p^4 dx = \pi d \sigma \epsilon T_m^4$$

where T_m is the mean heat transfer temperature of the tube. Thus the required expression for radiation across the interior of the tube becomes:

$$q_i = \frac{\sigma \epsilon}{\epsilon d + \gamma} (T^4 - T_m^4) dx \quad (12)$$

2.6 General Heat Flow Equation

Substitution for the various heat flows in equation (1) gives the general heat balance in terms of temperatures, from which the temperature profile may be obtained. Considering only sun radiation for the moment, equation (1) becomes:

$$k t \frac{d^2 T}{dx^2} dx = \sigma \epsilon T^4 dx + \frac{\sigma \epsilon}{\epsilon d + \gamma} (T^4 - T_m^4) dx - I \alpha_s \cos\left(\frac{x}{r}\right) dx$$

from which

$$\frac{d^2 T}{dx^2} = \frac{\sigma \epsilon T_m^4}{k t} \left[\left(\frac{T}{T_m} \right)^4 + \left(\frac{1}{\epsilon d + \gamma} \right) \left(\left(\frac{T}{T_m} \right)^4 - 1 \right) \right] - \frac{I \alpha_s}{k t} \cos\left(\frac{x}{r}\right) \quad (13)$$

Since it is expected that the mean tube temperature will be around 800° R, and that the actual temperature will have a maximum range of $T = T_m \pm 10^\circ$, it can be seen that the function of $(T/T_m)^4$ will range from 1.05 to 1 while $\cos(x/r)$ ranges from 1 to 0. Therefore to simplify equation (13) so that an analytic solution may be obtained, it will be assumed that $(T/T_m)^4$ is a constant numerically equal to 1. The heat balance therefore becomes: -

$$\frac{d^2T}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} - \frac{J_s \alpha_s}{kt} \cos\left(\frac{x}{r}\right) \quad (14)$$

Stated in another way, equation (14) neglects the heat flow across the interior of the tube, and assumes that the heat radiated to space from the tube surface does not vary from point to point around the perimeter.

2.7 Temperature Profile with Sun Radiation Only

If the sun is the only source of radiation impinging on the tube surface, the profile may be obtained from two differential equations. One equation gives the profile in the first quadrant, the other gives the profile in the second quadrant. Figure 3 shows that by symmetry the profiles in the third and fourth quadrants are identical to those in the second and first quadrants respectively.

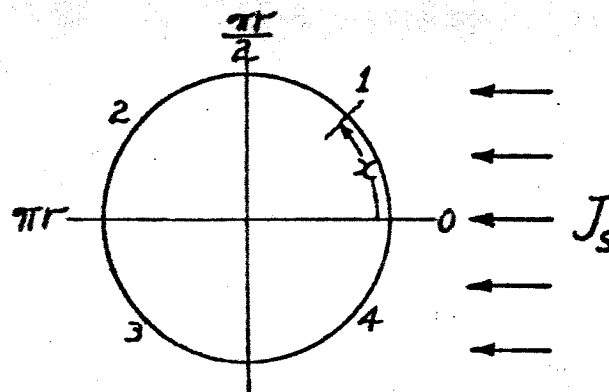


Figure 3

The two equations may be written directly from equation (14) realizing that the source is zero in the second quadrant.

$$\frac{d^2T_1}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} - \frac{J_s \alpha_s}{kt} \cos\left(\frac{x}{r}\right) \quad \left[0 < x < \frac{\pi r}{2}\right] \quad (15)$$

$$\frac{d^2T_2}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} \quad \left[\frac{\pi r}{2} < x < \pi r\right]$$

Equations (15) can be integrated directly to give:

$$\begin{aligned}\frac{dT_1}{dx} &= \frac{r}{kt} \left[\sigma \epsilon T_m^4 \left(\frac{x}{r} \right) - J_3 \alpha_3 \sin \left(\frac{x}{r} \right) + C_1 \right] \\ \frac{dT_2}{dx} &= \frac{r}{kt} \left[\sigma \epsilon T_m^4 \left(\frac{x}{r} \right) + C_2 \right] \\ T_1 &= \frac{r^2}{kt} \left[\frac{\sigma \epsilon T_m^4}{2} \left(\frac{x}{r} \right)^2 + J_3 \alpha_3 \cos \left(\frac{x}{r} \right) + C_1 \left(\frac{x}{r} \right) + C_3 \right] \\ T_2 &= \frac{r^2}{kt} \left[\frac{\sigma \epsilon T_m^4}{2} \left(\frac{x}{r} \right)^2 + C_2 \left(\frac{x}{r} \right) + C_4 \right]\end{aligned}\tag{16}$$

The constants of integration in equations (16) can be solved for by applying the boundary conditions of continuous heat flow and temperature at $x = \pi r/2$, and zero heat flow at $x = 0$ and $x = \pi r$. Stated mathematically:

$$\begin{aligned}\frac{dT_1}{dx}(x=0) &= \frac{dT_2}{dx}(x=\pi r) = 0 \\ \frac{dT_1}{dx}(x=\frac{\pi r}{2}) &= \frac{dT_2}{dx}(x=\frac{\pi r}{2}) \\ T_1(x=\frac{\pi r}{2}) &= T_2(x=\frac{\pi r}{2})\end{aligned}$$

from which:

$$\begin{aligned}T_1 &= \frac{r^2}{kt} \left[\frac{\sigma \epsilon T_m^4}{2} \left(\frac{x}{r} \right)^2 + J_3 \alpha_3 \cos \left(\frac{x}{r} \right) + C \right] \\ T_2 &= \frac{r^2}{kt} \left[\frac{\sigma \epsilon T_m^4}{2} \left(\pi - \frac{x}{r} \right)^2 + C \right]\end{aligned}\tag{17}$$

The unknown constant C can be seen by inspection to be the lowest temperature on the cross section. It can be evaluated in terms of the mean heat transfer temperature by means of the following expression:

$$\pi r T_m^4 = \int_0^{\frac{\pi r}{2}} T_1^4 dx + \int_{\frac{\pi r}{2}}^{\pi r} T_2^4 dx$$



Because the variation of T_1 and T_2 is small compared to the value of T_m , this can be approximated by:

$$T_m = \frac{1}{\pi r} \left[\int_0^{\frac{\pi}{2}} T_1 dx + \int_{\frac{\pi}{2}}^{\pi} T_2 dx \right] \quad (18)$$

Using equation (18) to evaluate C, and substituting C back into equations (17) gives the desired temperature profile. A further substitution can be made from an overall heat balance of heat absorbed to heat radiated:

$$J_s \alpha_s = \pi r \epsilon T_m^4$$

The equations for temperature therefore become:

$$\begin{aligned} T_1 &= T_m + \frac{r^2}{kL} \left[\frac{1}{2\pi} \left(\frac{x}{r} \right)^2 + \cos \left(\frac{x}{r} \right) - 0.45 \right] \\ T_2 &= T_m + \frac{r^2}{kL} \left[\frac{1}{2\pi} \left(\pi - \frac{x}{r} \right)^2 - 0.45 \right] \end{aligned} \quad (19)$$

Inspection shows that the maximum temperature always occurs at $x = 0$, and the minimum at $x = \pi r$. The equation for maximum temperature difference is therefore:

$$T_{1max} - T_{2min} = \Delta T_{max} = \frac{r^2}{kL} J_s \alpha_s \quad (20)$$

2.8 Temperature Profile with Sun and Earth Radiation Directly Opposed

If sun and earth radiation are directly opposed, the problem may be solved by two differential equations, since the profile will still be symmetrical as shown by Figure 4.

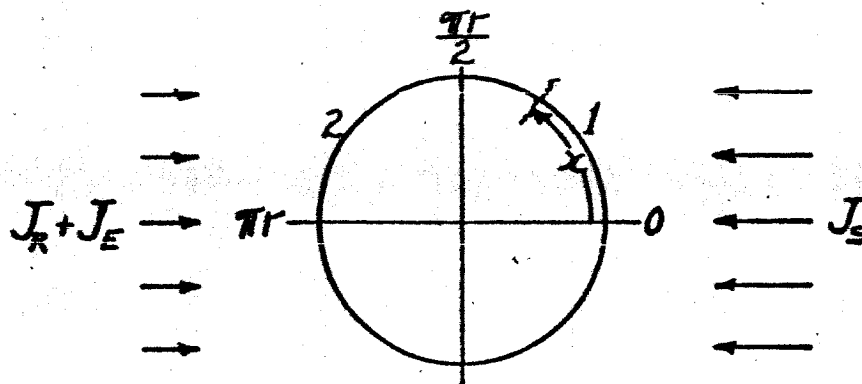


Figure 4

The two equations may be written directly from equation (14) as follows:

$$\frac{d^2 T_1}{dx^2} = \frac{\epsilon \epsilon T_m^4}{k t} - \frac{J_S \alpha_s}{k t} \cos\left(\frac{x}{r}\right) \quad \left[0 < x < \frac{\pi r}{2}\right] \quad (21)$$

$$\frac{d^2 T_2}{dx^2} = \frac{\epsilon \epsilon T_m^4}{k t} + \frac{1}{k t} (J_R \alpha_s + J_E \epsilon) \cos\left(\frac{x}{r}\right) \quad \left[\frac{\pi r}{2} < x < \pi r\right]$$

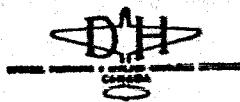
Replacing $J_S \propto S$ by S_1 , and $(J_R \propto S + J_E \epsilon)$ by S_2 ,

the temperature profile may be obtained by direct integration of equations (15), the boundary conditions being exactly the same as with sun radiation only. The desired profile is therefore:

$$T_1 = T_m + \frac{r^2}{k t} (S_1 + S_2) \left[\frac{1}{2\pi} \left(\frac{x}{r}\right)^2 + \frac{S_1}{S_1 + S_2} \cos\left(\frac{x}{r}\right) - 0.45 \right] \quad (22)$$

$$T_2 = T_m + \frac{r^2}{k t} (S_1 + S_2) \left[\frac{1}{2\pi} \left(\pi - \frac{x}{r}\right)^2 + \frac{S_2}{S_1 + S_2} \cos\left(\frac{x}{r}\right) - 0.45 \right]$$

It is not obvious from equations (16) where the maximum and minimum temperatures occur in the profile, the positions depending to a large extent on the relative values of S_1 and S_2 . To determine the maxima and minima, equations (16) must be differentiated and set equal to zero;



$$\frac{dT_1}{dx} = 0 = \frac{1}{\pi r \left(\frac{x}{r}\right)} - \frac{1}{r \left(\frac{s_1}{s_1 + s_2}\right)} \sin\left(\frac{x}{r}\right) \quad a)$$

$$\frac{dT_2}{dx} = 0 = -\frac{1}{\pi r \left(\pi - \frac{x}{r}\right)} + \frac{1}{r \left(\frac{s_2}{s_1 + s_2}\right)} \sin\left(\frac{x}{r}\right) \quad b)$$

from which:

$$\frac{s_1}{s_2} = \frac{1}{\left[\pi \sin\left(\frac{x}{r}\right) / \left(\frac{x}{r}\right)\right] - 1} = \frac{J_s \alpha_s}{J_R \alpha_s + J_E \epsilon} \quad a)$$

$$\frac{s_1}{s_2} = \left[\pi \sin\left(\frac{x}{r}\right) / \left(\frac{x}{r}\right)\right] - 1 = \frac{J_s \alpha_s}{J_R \alpha_s + J_E \epsilon} \quad b)$$

Using the values of J_s , J_R and J_E which occur at the outer edges of the earth's atmosphere, values of α_s / ϵ can be obtained from equations a) and b) for various values of (x/r) .

$$\left(\frac{x}{r}\right) = 0; \quad \frac{s_1}{s_2} = \frac{1}{\pi - 1} \quad \frac{\alpha_s}{\epsilon} = 0.087 \quad a)$$

$$\left(\frac{x}{r}\right) = \frac{\pi}{2}; \quad \frac{s_1}{s_2} = 1 \quad \frac{\alpha_s}{\epsilon} = 0.242 \quad a) \quad b)$$

$$\left(\frac{x}{r}\right) = \pi; \quad \frac{s_1}{s_2} = \pi - 1 \quad \frac{\alpha_s}{\epsilon} = 1.39 \quad b)$$

The significance of these results is that they give the range of values of α_s / ϵ for which the minimum temperature lies in a certain quadrant. From the boundary conditions of the problem, it is known that the maximum temperature must be at either $x = 0$ or $x = \pi r$. The result of equation b) at $x = \pi r$ shows that the minimum temperature will be at $x = \pi r$ for values of $\alpha_s / \epsilon > 1.39$, which is the range that covers most polished metal surfaces. It can be seen, therefore, that the maximum temperature difference across a tube for directly opposed sun and earth radiation will be: -

$$\Delta T_{max} = T_1(x=0) - T_2(x=\pi r)$$

$$\Delta T_{max} = \frac{r^2}{kL} [J_S \alpha_S - (J_R \alpha_S + J_E \epsilon)] \quad (23)$$

2.9 Temperature Profile With Sun and Earth Radiation Not Directly Opposed

If the tube is oriented such that the sun and earth radiation directions have an angle δ between, the problem of deriving the temperature profile becomes more complicated. Four differential equations are required, corresponding to the four segments shown in Figure 5.

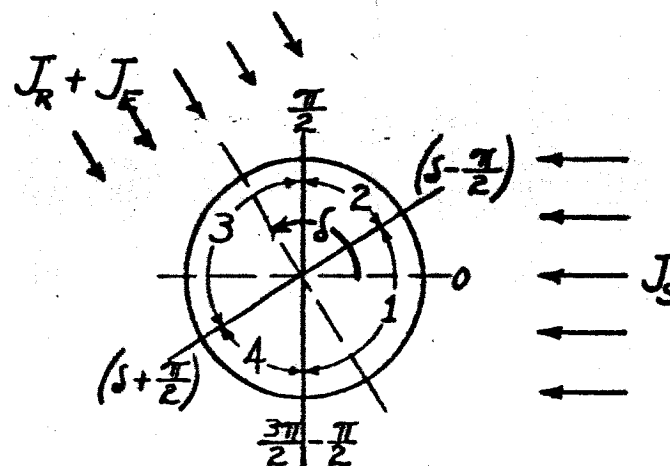


Figure 5

The four equations may be written directly from equation (14) as follows:

$$\frac{d^2 T_1}{dx^2} = \frac{\epsilon E T_m^4}{kL} - \frac{S_1}{kL} \cos\left(\frac{x}{r}\right) \quad \left[-\frac{\pi}{2} < \frac{x}{r} < s - \frac{\pi}{2}\right] \quad (24)$$

$$\frac{d^2 T_2}{dx^2} = \frac{\epsilon E T_m^4}{kL} - \frac{S_1}{kL} \cos\left(\frac{x}{r}\right) - \frac{S_2}{kL} \cos\left(s - \frac{x}{r}\right) \quad \left[s - \frac{\pi}{2} < \frac{x}{r} < \frac{\pi}{2}\right]$$



$$\frac{d^2 T_2}{dx^2} = \frac{\epsilon E T_m^4}{k t} - \frac{S_2}{k t} \cos\left(\delta - \frac{x}{r}\right) \quad \left[\frac{\pi}{2} < \frac{x}{r} < \delta + \frac{\pi}{2}\right]$$

$$\frac{d^2 T_4}{dx^2} = \frac{\epsilon E T_m^4}{k t} \quad \left[\delta + \frac{\pi}{2} < \frac{x}{r} < \frac{3\pi}{2}\right]$$

Equations (18) may be integrated directly to give the temperature profile, the boundary conditions being continuity of temperature and heat flow between the segments.

$$T_1 = \frac{r^2}{k t} \left[\frac{S_1 + S_2}{2\pi} \left(\frac{x}{r}\right)^2 + S_1 \cos\left(\frac{x}{r}\right) + S_2 \left(1 - \frac{x}{r}\right) - 2S_2 \delta - S_1 \frac{\pi}{2} + C \right] \quad (25)$$

$$T_2 = \frac{r^2}{k t} \left[\frac{S_1 + S_2}{2\pi} \left(\frac{x}{r}\right)^2 + S_1 \cos\left(\frac{x}{r}\right) + S_2 \cos\left(\delta - \frac{x}{r}\right) - S_2 \left(\frac{\delta}{\pi}\right) \left(\frac{x}{r}\right) - S_2 \left(\delta + \frac{\pi}{2}\right) - S_1 \left(\frac{\pi}{2}\right) + C \right]$$

$$T_3 = \frac{r^2}{k t} \left[\frac{S_1 + S_2}{2\pi} \left(\frac{x}{r}\right)^2 + S_2 \cos\left(\delta - \frac{x}{r}\right) + \left(S_1 + S_2 \frac{\delta}{\pi}\right) \left(\frac{x}{r}\right) - S_2 \left(\delta + \frac{\pi}{2}\right) + C \right]$$

$$T_4 = \frac{r^2}{k t} \left[\frac{S_1 + S_2}{2\pi} \left(\frac{x}{r}\right)^2 - S_1 \left(\frac{x}{r}\right) - S_2 \left(1 + \frac{\delta}{\pi}\right) \left(\frac{x}{r}\right) + C \right]$$

To obtain the maximum temperature difference across the cross section of the tube, the positions of maximum and minimum temperature must first be established by differentiating equations (19) and setting the derivatives equal to zero.

$$\frac{dT_1}{dx} = 0 = \frac{S_1 + S_2}{\pi} \left(\frac{x}{r}\right) - S_1 \sin\left(\frac{x}{r}\right) + S_2 \left(1 - \frac{\delta}{\pi}\right) \quad (26)$$

$$\frac{dT_2}{dx} = 0 = \frac{S_1 + S_2}{\pi} \left(\frac{x}{r}\right) - S_1 \sin\left(\frac{x}{r}\right) + S_2 \sin\left(\delta - \frac{x}{r}\right) - S_2 \left(\frac{\delta}{\pi}\right)$$



$$\frac{dT_2}{dx} = 0 = \frac{S_1 + S_2}{\pi} \left(\frac{x}{r} \right) + S_2 \sin \left(\delta - \frac{x}{r} \right) - \left(S_1 + S_2 \frac{f}{\pi} \right)$$

$$\frac{dT_4}{dx} = 0 = \frac{S_1 + S_2}{\pi} \left(\frac{x}{r} \right) - S_1 - S_2 \left(1 + \frac{f}{\pi} \right)$$

It can be seen from equations (20) that the values of x/r to give maxima or minima depend on the angle δ and the ratios $S_1/S_1 + S_2$ and $S_2/S_1 + S_2$. These two ratios may be considered constant if the ratio α_3/ϵ is greater than 3, which is the case that will be considered here. It can also be seen from Figure 5 and the results of Sections 2.8 that the maximum temperature must occur in segment 1 or 2, and the minimum in segment 3 or 4. Figure 6 shows the functions $(x/r)_{\max}$ and $(x/r)_{\min}$ plotted against δ , and also shows the segments in which they occur.

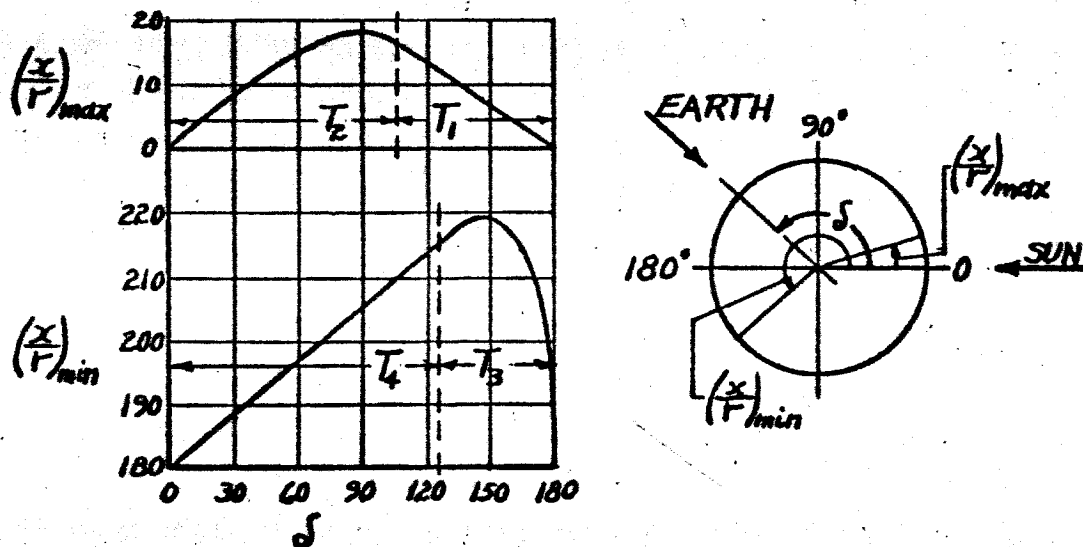


Figure 6

By substituting values of $(x/r)_{\max}$ and $(x/r)_{\min}$ back into equations (20), the maximum temperature difference across the tube can be obtained for any value of δ . In general, an expression for this temperature difference would be of the form:

$$\Delta T_{\max} = \frac{r^2}{kL} f(S_1, S_2, \delta) \quad (27)$$



where f indicates an unknown functional relationship. Taking S_1 and S_2 to be constant in this analysis, ΔT_{max} can be plotted as a function of δ . Figure 7 shows that the functional relationship approaches a cosine squared curve, such that temperature difference across the tube can be approximated by the formula:

$$\Delta T_{max} = \frac{r^2}{kL} (S_1 + S_2 \cos^2 \delta) \quad [0 < \delta < \frac{\pi}{2}] \quad (28)$$

$$\Delta T_{max} = \frac{r^2}{kL} (S_1 - S_2 \cos^2 \delta) \quad [\frac{\pi}{2} < \delta < \pi]$$

For an element on a satellite orbiting the earth, the assumption that S_2 is a constant is not correct, since it will change with distance from the earth's surface and with position relative to the earth's bright side. However, numerical computations for differing values of S_2 show that equation (22) is reasonably accurate regardless of the value of S_2 , in the region $\delta > 90^\circ$.

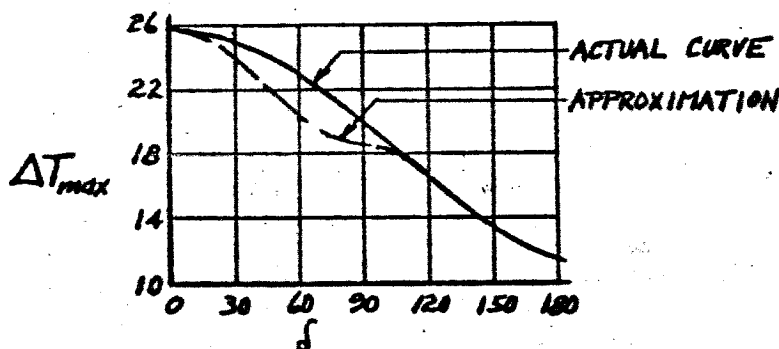


Figure 7

3.0 DERIVATION OF THERMAL DISTORTION

3.1 Thermal Distortion Due to Sun and Directly Opposed Sun and Earth Radiation

It has been shown that a thin-walled tube in space will develop a temperature difference across its cross-section due to radiation incident on its surface. Because of this temperature difference, longitudinal strips

of the tube will expand to different lengths, causing distortion of the tube. Consider a short length of such a tube, as shown in Figure 8. If the temperature gradient across the tube is linear with respect to a diameter passing through the points of maximum and minimum temperature, the distortion will be uniform bending about a radius of curvature R_c , and there will be no distortion of the cross-section.

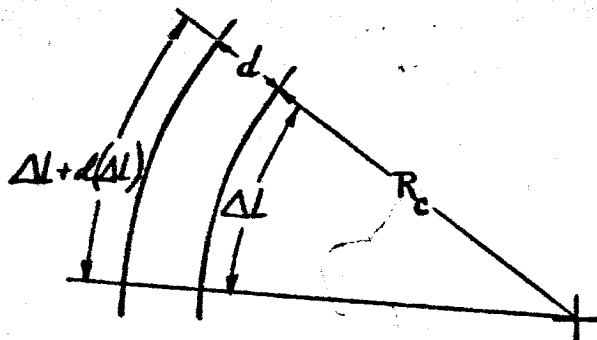


Figure 8

From the geometry of Figure 8

$$\frac{\Delta L}{R_c} = \frac{\Delta L + d(\Delta L)}{R_c + d}$$

From which

$$\frac{1}{R_c} = \frac{1}{d} \times \frac{d(\Delta L)}{\Delta L} = \frac{1}{d} e \Delta T \quad (29)$$

where e is the coefficient of thermal expansion of the tube material and ΔT is the temperature difference across the diameter.

at an angle θ to the axis and, as shown in Figure 9.

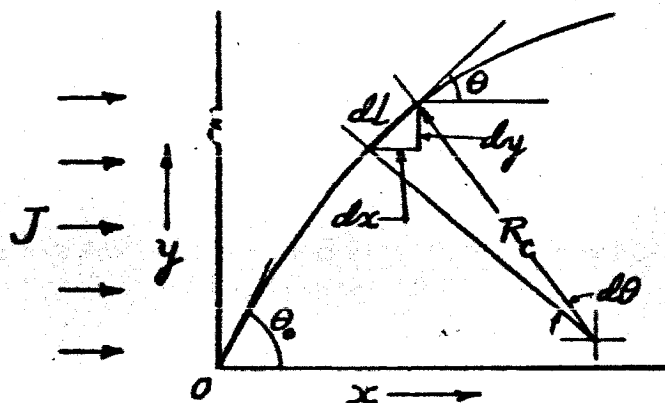


Figure 9

If heat flow along the length of the tube is neglected, the temperature difference across the tube at any angle θ is:

$$\Delta T = \Delta T_{\max} \sin \theta \quad (30)$$

where $\Delta T_{\max}$ is the temperature difference at the fixed end.

Considering an element of length dl at any angle θ :

$$\begin{aligned} -R_c d\theta &= dl \\ \frac{1}{R_c} &= -\frac{d\theta}{dl} = -\frac{d\theta}{dy} \sin \theta \end{aligned} \quad (31)$$

Substituting equation (24) into (23) and equating to (25) gives the following:

$$\begin{aligned} -\frac{d\theta}{dy} \sin \theta &= \frac{e \Delta T_{\max}}{d} \sin \theta \\ d\theta &= -N dy \end{aligned} \quad (32)$$

where $N = e \Delta T_{\max} / d$. Integrating equation (26) and applying the condition that at $y = 0$, $\theta = \theta_0$;

$$\theta = \theta_0 - Ny \quad (33)$$

Taking the cotangent of equation (27) gives:

$$\begin{aligned} \cot \theta &= \frac{dx}{dy} = \cot(\theta_0 - Ny) \\ dx &= \cot(\theta_0 - Ny) dy \end{aligned} \quad (34)$$

Integrating equation (28) and applying the condition that at $y = 0$, $x = 0$;

$$x = -\frac{1}{N} \ln \left[\frac{\sin(\theta_0 - Ny)}{\sin \theta_0} \right] \quad (35)$$

Equation (29) is the equation for the shape a tube will take when radiation causes a linear temperature gradient along a diameter. Since θ_0 can have any value from 0 to 180 degrees, it is useful to take $\theta_0 = \pi/2$ in which case equation (29) becomes

$$x = -\frac{1}{N} \ln \cos(Ny) \quad (36)$$

Equation (30) can then be plotted as shown in Figure 5, Part I of this report, with the y axis positive in either direction from zero, and the angles θ indicated at the appropriate points on the curve. In this way, the factor N is calculated from the value of $\Delta T_{\max}$, obtained by considering the radiation to be at right angles to the tube.



Distance along the curve in unit lengths of tube is also plotted in Figure 5, Part I. This is obtained by combining equations (24) and (25) in a slightly different manner to give

$$\frac{1}{R_c} = -\frac{d\theta}{dL} = N \sin \theta$$
$$dL = -\frac{1}{N} \csc \theta d\theta \quad (37)$$

Integrating equation (31) and applying the condition (from Figure 5, Part I) that at $l = 0$, $\theta = \pi/2$

$$L = -\frac{1}{N} \ln \tan \frac{\theta}{2} \quad (38)$$

For short tubes, Figure 5, Part I, shows that deflections are small compared to the tube length. Thus the ordinate y in equation (30) may be replaced by distance l along the tube, giving the deflection x as

$$x = -\frac{1}{N} \ln \cos(NL) \quad (39)$$

In particular if $Nl \leq 0.3$, equation (33) simplifies to

$$x = \frac{N}{2} l^2 \quad (40)$$

with a deviation of less than 2%.

Equation (34) or Figure 5, Part I, may be used to give the distortion shape of a tube with a temperature difference across its cross-section induced either by sun radiation or by directly opposed earth and sun radiation. However, it must be remembered that the shape is only approximate,

since actual temperature gradients with respect to a diameter are not linear as was first stipulated.

Figure 11 shows some typical temperature profiles plotted against tube diameter. It is expected that the small deviations from a linear gradient will cause distortion of the cross-section, thus changing somewhat the bending shape of the tube. To calculate the final bending shape would require the

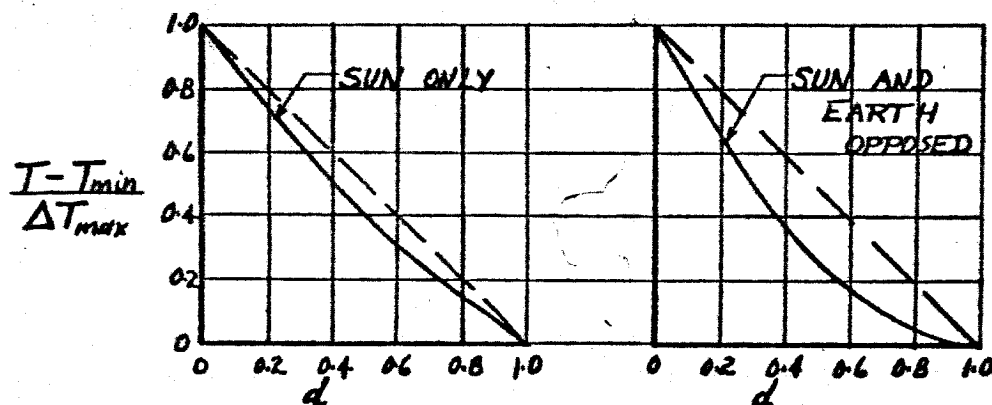


Figure 11

minimizing of lengthwise and cross-sectional strain energies to determine the shape of least energy. However, this final shape would not be markedly different from that already derived. Therefore the task of deriving it will not be undertaken in this report.

3.2 Thermal Distortion Due to Sun and Earth Radiation Not Directly Opposed

If the sun and earth radiations are not directly opposed, the problem of determining the distortion shape becomes more complicated. Not only is the temperature gradient non-linear along a diameter, the points of maximum and minimum temperature do not lie on a diameter. However, the problem can be simplified somewhat by assuming that the temperature gradient due to a single source is linear along a diameter. Since temperature profiles are solutions of linear differential equations, the profile due to two sources with an angle δ between is the sum of two profiles due to the sources acting separately. Also, since the separate profiles are assumed linear, the temperature difference across the tube in the direction of the sun radiation is proportional to $S_1 + S_2 \cos \delta$ (see figure 12) and the temperature difference at right angles to the direction of sun radiation is proportional to $S_2 \sin \delta$.

In this manner any problem involving two radiation sources can be transformed into a problem of two different sources acting at right angles to each other.

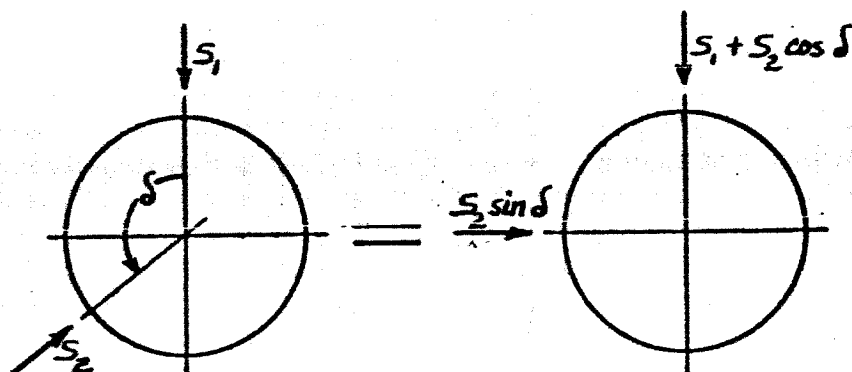


Figure 12

Consider now a differential length dl of the tube in space, as shown in Figure 13. The effective source S_3 will cause bending of the tube about a

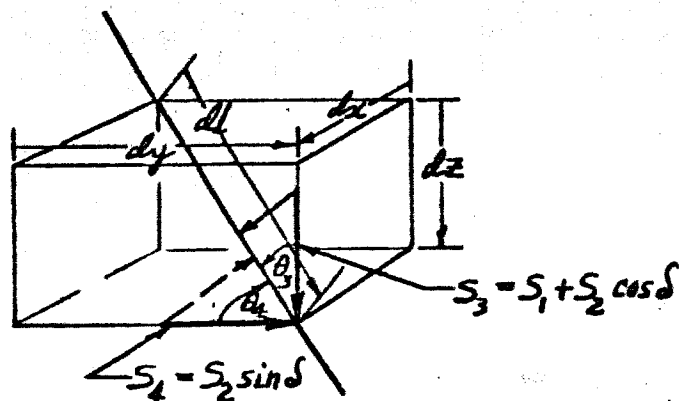


Figure 13

radius of curvature R_{c3} in the direction of $S_3 \sin \theta_3$. Similarly S_4 will cause bending about a radius R_{c4} in the direction of $S_4 \sin \theta_4$. Using equations previously developed:

$$\frac{1}{R_{c3}} = -\frac{d\theta_3}{dl} = -\frac{d\theta_3}{dz} \cos \theta_3 = \frac{c\Delta T_{max}}{d} \sin \theta_3 \quad (41)$$

$$\frac{1}{R_{c4}} = -\frac{d\theta_4}{dl} = -\frac{d\theta_4}{dy} \cos \theta_4 = \frac{e\Delta T_{max}}{d} \sin \theta_4$$

where

$$\Delta T_{3max} = \frac{F^2}{kL} (S_1 + S_2 \cos S)$$

$$\Delta T_{4max} = \frac{F^2}{kL} (S_2 \sin S)$$

Integrating equations (35) and applying the conditions that at $y = z = 0$, $\theta_3 = \theta_4 = \pi/2$;

$$\begin{aligned} z &= -\frac{1}{N_3} \ln \sin \theta_3 \\ y &= -\frac{1}{N_4} \ln \sin \theta_4 \end{aligned} \quad (42)$$

where

$$N_3 = \frac{e \Delta T_{3max}}{d}$$

$$N_4 = \frac{e \Delta T_{4max}}{d}$$

Equations (35) may be integrated in terms of the length l , and then solved for in terms of θ_3 and θ_4 to give:

$$\begin{aligned} \theta_3 &= 2 \arctan [e^{-N_3 l}] \\ \theta_4 &= 2 \arctan [e^{-N_4 l}] \end{aligned} \quad (43)$$

Equations (36) and (37) may be used to solve for the components of deflection (y and z) at any distance l along a tube which is being deformed by two sources of radiation with an angle S between, each initially at right angles to the tube. The shape of the deformed tube is shown in Figure 14.

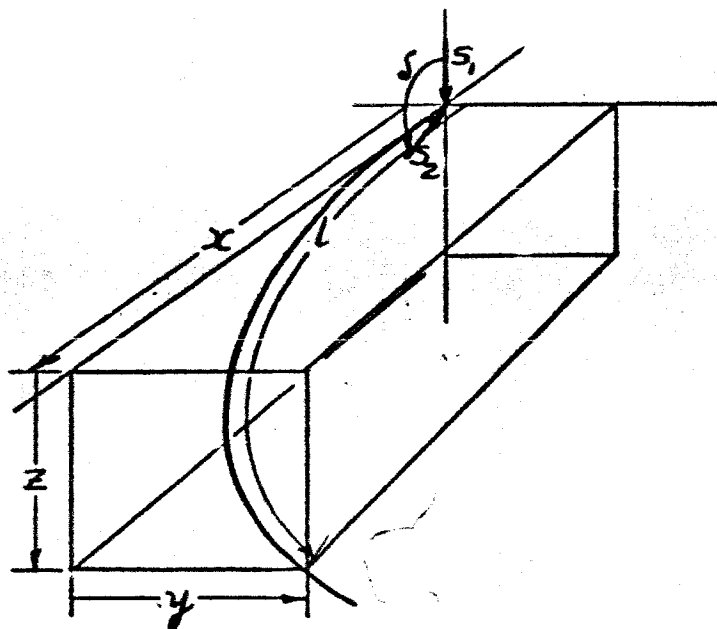


Figure 14

Again it must be pointed out that the equations for y and z give approximate deflections because of the assumptions stipulated in the derivation.

4.0 DERIVATION OF ELEMENT RESPONSE TIME

An estimate of the length of time required for an element to build up a temperature gradient across its cross-section can be obtained by investigating the response of the mean tube temperature to sudden exposure of the tube to sun radiation. This assumes that the temperature gradients calculated in Section 2.0 will be realized when the element is in thermal equilibrium.

Consider an element initially at a uniform temperature T_{m_0} . If it is suddenly placed in sunlight, a balance of the heat flows can be made as follows:

$$\text{Heat Radiation In} - \text{Heat radiated out} = \text{Heat absorbed.}$$

Substituting for the heat flows gives the following:

$$dI_s \alpha_s - \pi d r \epsilon T_m^4 = c \rho \phi \pi d t \frac{dT}{d\tau} \quad (44)$$

where c is the specific heat of the material, ϕ is the overlap factor and τ is time.

Equation (38) can be re-arranged to allow numerical integration. Substituting for the sun radiation in terms of the final equilibrium mean temperature T_{mf} gives the required expression:

$$\Delta T_m = \frac{r \epsilon}{c \rho \phi L} (T_{mf}^4 - T_m^4) \Delta \tau \quad (45)$$

The temperature T_m plotted as a function of time τ is shown in Figure 10, Part I, of this report, for a temperature increase ($T_{mf} - T_{m0}$) of 285°F.

If the step increase in temperature is small such that $T_m = T_{m0} + \Delta T_m$, where ΔT_m is very small compared to T_{m0} , then equation (39) can be simplified to:

$$\frac{dT_m}{d\tau} = \frac{d(\Delta T_m)}{d\tau} = \frac{r \epsilon}{c \rho \phi L} [(T_{m0} + \Delta T_{max})^4 - (T_{m0} + \Delta T_m)^4] \quad (46)$$

Since ΔT_m is small,

$$(T_{m0} + \Delta T_{max})^4 - (T_{m0} + \Delta T_m)^4 = 4 T_{m0}^3 (\Delta T_{max} - \Delta T_m)$$

and equation (40) reduces to:

$$\frac{d(\Delta T_m)}{d\tau} + \frac{4 r \epsilon T_{m0}^3}{c \rho \phi L} (\Delta T_m) = \frac{4 r \epsilon T_{m0}^3}{c \rho \phi L} (\Delta T_{max}) \quad (47)$$



The solution to equation (41) is:

$$\Delta T_m = \Delta T_{max} [1 - e^{-(t/C_t)}]$$

where

$$C_t = \frac{c \rho \phi t}{4 \pi \epsilon T_m^3} \quad (48)$$

and is referred to as the time constant of the element.

The value of the time constant C_t is of interest since it represents the time required for the element to reach 63.2% of its maximum temperature rise, and can be used in a stability analysis of an antenna which undergoes a small change in orientation with respect to the radiation heat source.

5.0 DISCUSSION OF ASSUMPTIONS

5.1 Interior Radiation and Surface Radiation to Space

Two assumptions made in the derivations of the temperature profiles were that the interior radiation could be neglected and that the tube would radiate to space at its mean temperature. To determine the validity of these assumptions, the heat flow equation will be solved again for the simple case of sun radiation only, taking all heat flows into account. Equation (13) may be rewritten in the following way:

$$\frac{d^2 T}{dx^2} - \frac{\pi \epsilon}{k t} \left(1 + \frac{1}{\epsilon_d + \gamma} \right) (T - T_m) = - \frac{\pi \epsilon T_m^4}{k t} - \frac{I_s \alpha_s \cos(\chi)}{k t} \quad (49)$$

Since the difference between T and T_m is very small, the following simplification may be used, with an error of less than 2%.

$$T^4 - T_m^4 = [T_m + (T - T_m)]^4 - T_m^4 = 4T_m^3(T - T_m)$$

Therefore equation (43) becomes:

$$\frac{d^2T}{dx^2} - \left[\frac{\epsilon \epsilon T_m^3}{kL} \left(4 + \frac{4}{\epsilon d + r} \right) \right] T = - \left[\frac{\epsilon \epsilon T_m^3}{kL} \left(3 + \frac{4}{\epsilon d + r} \right) \right] - \frac{J_s \alpha_s}{kL} \cos\left(\frac{x}{r}\right) \quad (50)$$

Replacing the constants in equation (44) by a^2 , b and d , the two equations of heat flow for sun radiation only acting on the tube may be written as follows:

$$\frac{d^2T_1}{dx^2} - a^2T_1 = -b - d \cos\left(\frac{x}{r}\right) \quad \left[0 < x < \frac{\pi r}{2} \right] \quad (51)$$

$$\frac{d^2T_2}{dx^2} - a^2T_2 = -b \quad \left[\frac{\pi r}{2} < x < \pi r \right]$$

The solution to equations (45) is:

$$T_1 = \frac{rd}{2a(1+a^2r^2)} \frac{\cosh(ax)}{\sinh\left(\frac{a\pi r}{2}\right)} + \frac{b}{a^2} + \frac{r^2d}{1+a^2r^2} \cos\left(\frac{x}{r}\right) \quad (52)$$

$$T_2 = \frac{rd}{2a(1+a^2r^2)} \left[\frac{\cosh(ax)}{\sinh\left(\frac{a\pi r}{2}\right)} - 2 \sinh a\left(x - \frac{\pi r}{2}\right) \right] + \frac{b}{a^2}$$

The maximum temperature difference across the tube is therefore:

$$T_1(x=0) - T_2(x=\pi r) = \Delta T_{max} = \frac{r^2d}{1+a^2r^2}$$

$$\Delta T_{max} = f \frac{r^2}{kL} J_s \alpha_s \quad (53)$$

where

$$f = \frac{1}{1 + \alpha^2 r^2} = \frac{1}{1 + \left(4 + \frac{4}{\epsilon_d + \gamma}\right) \frac{F^2}{k t} \sigma \epsilon T_m^3}$$

It can be seen that the factor f is always slightly smaller than unity. For the particular case of a one-half inch beryllium copper tube, 0.002 inches thick, f has a value of 0.96.

It is therefore to be expected that neglecting interior radiation in the tube and assuming radiation to space at the mean temperature of the tube will give temperature differences across the tube which are approximately 4% too high.

5.2 Heat Flow Along the Tube Length

Another assumption made in the derivation of the temperature profile was that the heat flow along the length of the tube is zero. However, it can be seen that because of the distorted tube shape, the mean temperature varies along the tube length according to the formula:

$$T_m^4 = T_{m_0}^4 \sin \theta \quad (54)$$

where T_{m_0} is the mean temperature at the fixed end, and θ is the angle at any point on the tube, as shown in Figure 9.

To obtain the average heat flux, equation (48) must be differentiated with respect to length l and multiplied by the conductivity k :

$$-k \frac{dT_m}{dl} = -\frac{k}{4} T_{m_0} (\sin \theta)^{\frac{3}{4}} \cos \theta \frac{d\theta}{dl} \quad (55)$$

Substituting for $d\theta/dl$ from equation (31) gives:

$$-k \frac{dT_m}{dl} = \frac{kNT_{m0}}{4} (\sin\theta)^{\frac{1}{4}} \cos\theta \quad (56)$$

Equation (50) can be differentiated with respect to θ and set equal to zero to find the point of maximum heat flux along the length. If this is done, it is found that $\theta_{\max}$ equals 26.6 degrees, and the expression for maximum heat flux becomes:

$$\left(-k \frac{dT_m}{dl}\right)_{\max} = 0.183 kNT_{m0} \quad (57)$$

The average heat flux around the perimeter of the tube at θ equals 26.6 degrees is:

$$-k \frac{dT}{dx} = \frac{k\Delta T_{\max} \sin(26.6^\circ)}{0.5 \pi d} \quad (58)$$

Taking a ratio of equation (51) to (52), and substituting for N , gives, for the beryllium copper tube;

$$\frac{\text{flux along tube length}}{\text{flux around tube perimeter}} = 0.64 e T_{m0} \approx .005$$

This result indicates that the heat flux along the length of the tube is less than one-half percent of the average heat flux around the perimeter of the tube, and will therefore have a negligible effect on the temperature distributions calculated in section 2.

5.3 Overlapped Tubes

In deriving temperature profiles in Section 2, it was assumed that the tube was seamless and of constant thickness. If the tube is overlapped 180 degrees, this assumption implies that there is enough contact in the overlapped section to give a conduction heat flow area equivalent to the constant area of the seamless tube. The amount of contact required in this

case would be small, as illustrated in Figure 15 (a). If the contact area were larger, as shown in Figure 15 (b), the problem would be similar to a seamless tube having one side twice the thickness of the other. For this case, the temperature difference

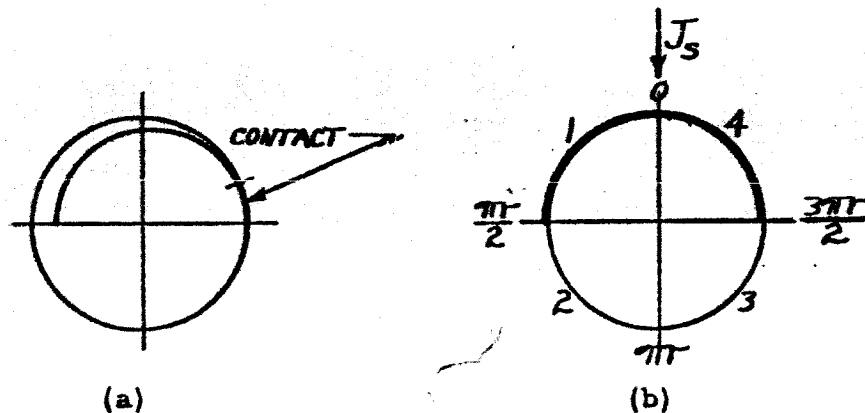


Figure 15

across the tube will be derived with sun radiation at $x = 0$, $\pi r/2$ and πr .

When the sun radiation is at $x = 0$, the problem is symmetrical and the two equations of heat flow can be written as follows:

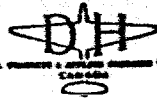
$$\frac{d^2 T_1}{dx^2} = \frac{\epsilon \epsilon T_m^4}{2kt} - \frac{J_s \alpha_s}{2kt} \cos\left(\frac{x}{r}\right) \quad \left[0 < x < \frac{\pi r}{2}\right] \quad (59)$$

$$\frac{d^2 T_2}{dx^2} = \frac{\epsilon \epsilon T_m^4}{kt} \quad \left[\frac{\pi r}{2} < x < \pi r\right]$$

Equations (53) can be integrated as in Section 2, with the same boundary conditions, to give:

$$T_1 = \frac{r^2}{kt} J_s \alpha_s \left[\frac{1}{4\pi} \left(\frac{x}{r}\right)^2 + \frac{1}{2} \cos\left(\frac{x}{r}\right) + \frac{1}{4\pi} \left(\frac{\pi}{2}\right)^2 + C \right] \quad (60)$$

$$T_2 = \frac{r^2}{kt} J_s \alpha_s \left[\frac{1}{2\pi} \left(\pi - \frac{x}{r}\right)^2 + C \right]$$



The maximum temperature difference across the tube is therefore:

$$T_1(x=0) - T_2(x=\pi r) = \Delta T_{max} = \frac{r^2}{kt} J_s \alpha_s \left(\frac{1}{2} + \frac{\pi}{16} \right) \quad (61)$$

$$\Delta T_{max} = 0.696 \frac{r^2}{kt} J_s \alpha_s$$

Similarly it can be shown that when the sun radiation is at $x = \pi r/2$, the maximum temperature difference is:

$$\Delta T_{max} = \left(1 - \frac{\pi}{16} \right) \frac{r^2}{kt} J_s \alpha_s = 0.804 \frac{r^2}{kt} J_s \alpha_s \quad (62)$$

When the sun radiation is at $x = \pi r/2$, the problem is no longer symmetrical, and four equations of heat flow are required for the four quadrants of the tube. The heat flow equations are:

$$\frac{d^2 T_1}{dx^2} = \frac{\sigma \epsilon T_m^4}{2kt} - \frac{J_s \alpha_s}{2kt} \sin\left(\frac{x}{r}\right) \quad \left[0 < x < \frac{\pi r}{2} \right] \quad (63)$$

$$\frac{d^2 T_2}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} - \frac{J_s \alpha_s}{kt} \sin\left(\frac{x}{r}\right) \quad \left[\frac{\pi r}{2} < x < \pi r \right]$$

$$\frac{d^2 T_3}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} \quad \left[\pi r < x < \frac{3\pi r}{2} \right]$$

$$\frac{d^2 T_4}{dx^2} = \frac{\sigma \epsilon T_m^4}{2kt} \quad \left[\frac{3\pi r}{2} < x < 2\pi r \right]$$

The solutions to equations (57) are:

$$\begin{aligned}
 T_1 &= \frac{r^2}{kt} J_5 \alpha_s \left[\frac{1}{4\pi} \left(\frac{x}{r} \right)^2 + \frac{1}{2} \sin \left(\frac{x}{r} \right) - 0.197 \left(\frac{x}{r} \right) + 0.387 + C \right] \\
 T_2 &= \frac{r^2}{kt} J_5 \alpha_s \left[\frac{1}{2\pi} \left(\frac{x}{r} \right)^2 + \sin \left(\frac{x}{r} \right) - 0.394 \left(\frac{x}{r} \right) + C \right] \\
 T_3 &= \frac{r^2}{kt} J_5 \alpha_s \left[\frac{1}{2\pi} \left(\frac{x}{r} - \pi \right)^2 - 0.394 \left(\frac{x}{r} \right) + 1.57 + C \right] \\
 T_4 &= \frac{r^2}{kt} J_5 \alpha_s \left[\frac{1}{4\pi} \left(\frac{x}{r} - \pi \right)^2 - 0.197 \left(\frac{x}{r} \right) + 0.84 + C \right]
 \end{aligned} \tag{64}$$

By observation, it can be seen that the maximum temperature must occur in T_2 , and the minimum in T_3 . By differentiation, the maximum temperature is found to occur at $x/r = 0.55\pi$ and the minimum at $x/r = 1.394\pi$. Using these values of x/r , the maximum temperature difference is:

$$T_2(x=0.55\pi) - T_3(x=1.394\pi) = \Delta T_{max} = 0.7 \frac{r^2}{kt} J_5 \alpha_s \tag{65}$$

Thus it can be seen that complete contact between layers in an overlapped tube decreases the maximum temperature difference across the tube by approximately 25%, for sun radiation only.

Further cases have not been considered because of the complexity of the equations involved, but it is felt that complete contact in the overlap will reduce temperature differences by the same order of magnitude for all cases.

LIST OF SYMBOLS

		Units
A	Satellite altitude above earth surface	miles
B	Earth radiant energy factor (see 3.2, Part I)	
F _A	Altitude correction factor (see 3.1, Part I)	
J	Radiant energy	BTU-hr ⁻¹ -ft ⁻²
N	Element profile number (see 2.3, Part I)	ft ⁻¹
R	Earth radius	miles
R _c	Radius of curvature	feet
T	Temperature	°R
Index S, s	Refers to sun radiation	
Index E, e	Refers to earth radiation	
Index R, r	Refers to earth reflection	
c	Specific heat	BTU-lb ⁻¹ -°F ⁻¹
d	Element diameter	feet
e	Thermal expansion coefficient	in/in
k	Thermal conductivity	BTU-ft ⁻¹ -°F ⁻¹ -hr ⁻¹
l	Element length	feet
t	Element thickness	feet
r	Element radius	feet
x, y, z	Axes of co-ordinate system	
α	Absorptivity - outer surface	
β	Absorptivity - inner surface	



List of Symbols (cont'd)

γ	Reflectivity ratio	
δ	Angle between sun and earth radiation direction with respect to the cross section of the element	degrees
ϵ	Emissivity ratio	
χ	Angle between perpendiculars from earth to satellite and earth to sun	degrees
θ	Radiation incident angle - angle between the element axis and the radiation direction seen from the root of the element.	degrees
ϕ	Overlap factor	
ρ	Material density	lb/ft ³
σ	Stefan-Boltzmann constant	BTU/ft ² hr.(°F) ⁴
τ	Time	minutes
C_T	Element time constant	minutes
q	Heat flow	BTU/hr.
E_b	Black body emissive power	BTU/hr-ft ²
C	Constant of integration	



REFERENCES

1. LEVIN, E.: Reflected Radiation Received by an Earth Satellite; ARS Journal, Vol, 32, No. 9, Sept., 1962.
2. KREITH, FRANK: Principles of Heat Transfer; International Textbook Company, Aug., 1960.
3. BETZ, OLSON, SCHURIN, MORRIS: Determination of Emissivity and Reflectivity Data on Aircraft Structural Materials, Part II; WADC Technical Report 56-222, Armor Research Foundation, October, 1958.

APPENDIX II

PROCEDURE FOR DETERMINING
BOOM STRAIGHTNESS

(From DHC-SP-SP.86)

February 7, 1966

by

W.J. Craib

Quality Control

PRODUCTION PROCESS STANDARD

PAGE ...1.. OF ..7....

ISSUE ..2.....

DATE ..Feb. 7/66.....

E. PROCEDURE FOR DETERMINING BOOM STRAIGHTNESS

COMPILED BY: *M. J. Craig*CHECKED BY: *H. J. R.*

APPROVED

J. B. ...
.....
PRODUCTION ENGINEERING*E. R. ...*
.....
ENGINEERING*H. J. ...*
.....
QUALITY CONTROL & INSPECTION*N/A*
.....
LABORATORY

AMENDMENT NO.

DATE INCORPORATED

FEBRUARY 7, 1966

PAGE 2 OF 7

TITLE: Procedure for Determining Boom Straightness1.0 Applicability

- 1.1 All boom elements formed at SPAR that are governed by a customer or SPAR straightness requirement.

2.0 Scope

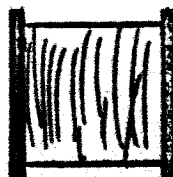
- 2.1 This Standard lays down the procedure to be used in determining the straightness of formed Beryllium Copper (plated or unplated) boom elements.

3.0 Test Equipment3.1 Water Tank (150 Ft)

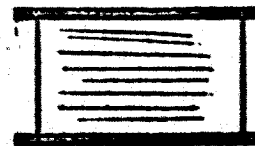
- 3.1.1 Support Floats
- 3.1.2 Measuring Trolley
- 3.1.3 End Fitting
- 3.1.4 Datum Line

3.2 Boom Element Handling Fixture4.0 Procedure

- 4.1 Remove the covers from the water tank.
- 4.2 Mount handling fixture, on which the formed element has been rolled, to the water tank end fitting so that the element will extend down the length of the water tank.
- 4.3 Adjust the end fitting so that the under-surface of the element is at the same level as the support surface of the floats, and the element will extend down the length of the water tank on a plane parallel to the horizontal.
- 4.4 Readings shall be taken in two planes. The spool attitude for the two planes shall be as shown below.
- NOTE: - Use plastic gloves when handling the element.



0°



90°

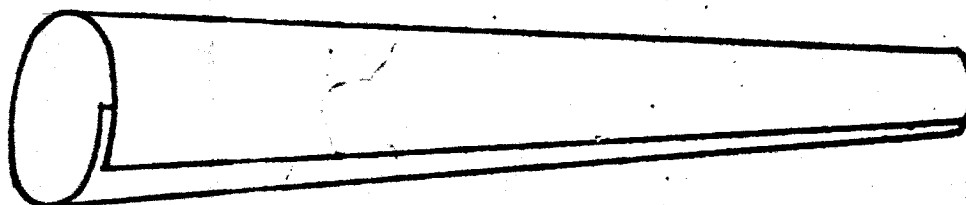
SPOOL ATTITUDE FOR TWO PLANES

FEBRUARY 7, 1966

PAGE 3 OF 7

4.0 Procedure (Cont'd)

- 4.5 As the element extends over the water, place the support floats under the element at 36" intervals.
- 4.6 When the element is fully extended and supported, adjust the end fitting so that the element is correctly aligned with the floats and is not touching the sides of the water tank.
- 4.7 Using plastic gloves align the overlap along the tube so that it forms a straight line, as shown below, and repeat adjustment per 4.6.



- 4.8 Place the covers on the water tank and allow sufficient time for the element to take-up its natural position.
- 4.9 Place the measuring trolley on the rails on top of the water tank.
- 4.10 The first station (0) shall be at the extended tip of the element.
- 4.11 Move the measuring trolley to the first (0) station (stations at 6 ft. intervals) and take a reading on the datum line and the element as illustrated in Figure 1. Record readings and footage.
- 4.12 Obtain the position of the element with respect to datum by subtracting the datum reading from the element reading. Record and plot on graph, as illustrated in Figure 2.
- 4.13 Repeat 4.11 and 4.12 for each station to the end of the element.
- 4.14 Draw a curve through the plotted "element positions with respect to datum" and join the ends of the curve with a straight line (see figure 2).

NOTE: - Use the same graph sheet for 0° and for 90° positions.

- 4.15 Remove the covers from the water tank and rotate the element 90° clockwise by adjusting the end fitting.
- 4.16 Repeat adjustments per. 4.3, 4.6 and 4.7
- 4.17 Repeat 4.8, 4.9 and 4.10.

FEBRUARY 7, 1966

PAGE 4 OF 7

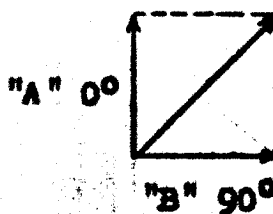
4.0 Procedure (Cont'd)

- 4.18 Repeat 4.11 through 4.14 except that datum readings first taken shall be used.
- 4.19 Take the "element position" reading from each station* at 0° and 90° and calculate the vector sum.
- 4.20 Plot the vector sums on a graph and join with a curve.
- 4.21 The curve per 4.20 shall be within the limits specified in the applicable specification or drawing.
- 4.22 Rewind the element.

5.0 Records

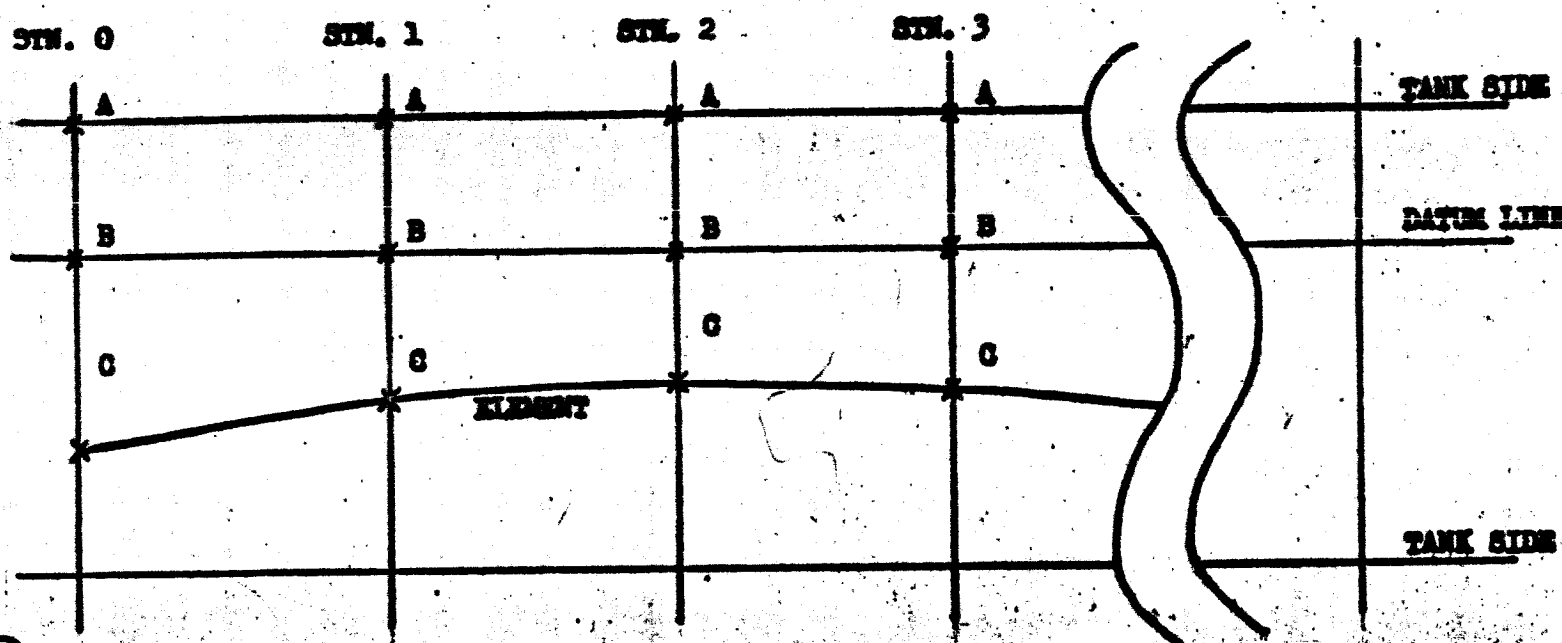
- 5.1 The data sheet (Page 7) and the graph shall be stamped and dated by the witnessing inspector and shall be filed with the applicable inspection records.
- 5.2 The data sheet and graph shall also be identified with the tube forming batch number.

* DISTANCE "A" and "B" at each station as shown on Figure 2.



FEBRUARY 7, 1966

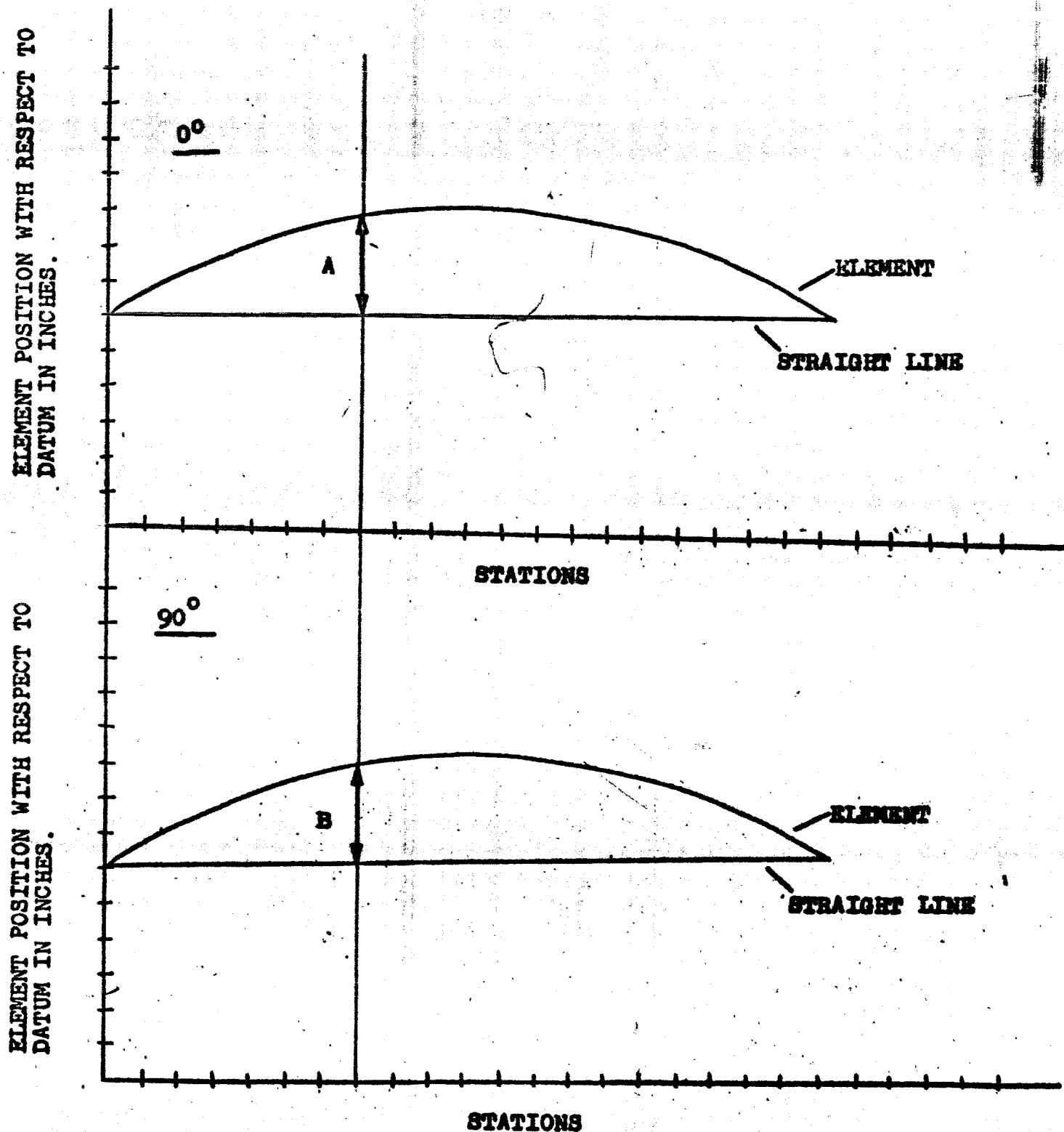
PAGE 5 OF 7



DISTANCE "AB" AT EACH STATION = DATUM READING
 DISTANCE "AC" AT EACH STATION = ELEMENT READING
 ELEMENT POSITION WITH RESPECT TO DATUM = ELEMENT
 READING (AO) - DATUM READING (AB).

APPENDIX II FIGURE I

FEBRUARY 7, 1963
PAGE 6 OF 7



FEBRUARY 7, 1956.
PAGE 7 of 7.DATA SHEET

Station	Footage at Station	Datum Reading	Element Reading		Element Position with Respect to Datum	
			0°	90°	0°	90°
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						
26						
27						
28						

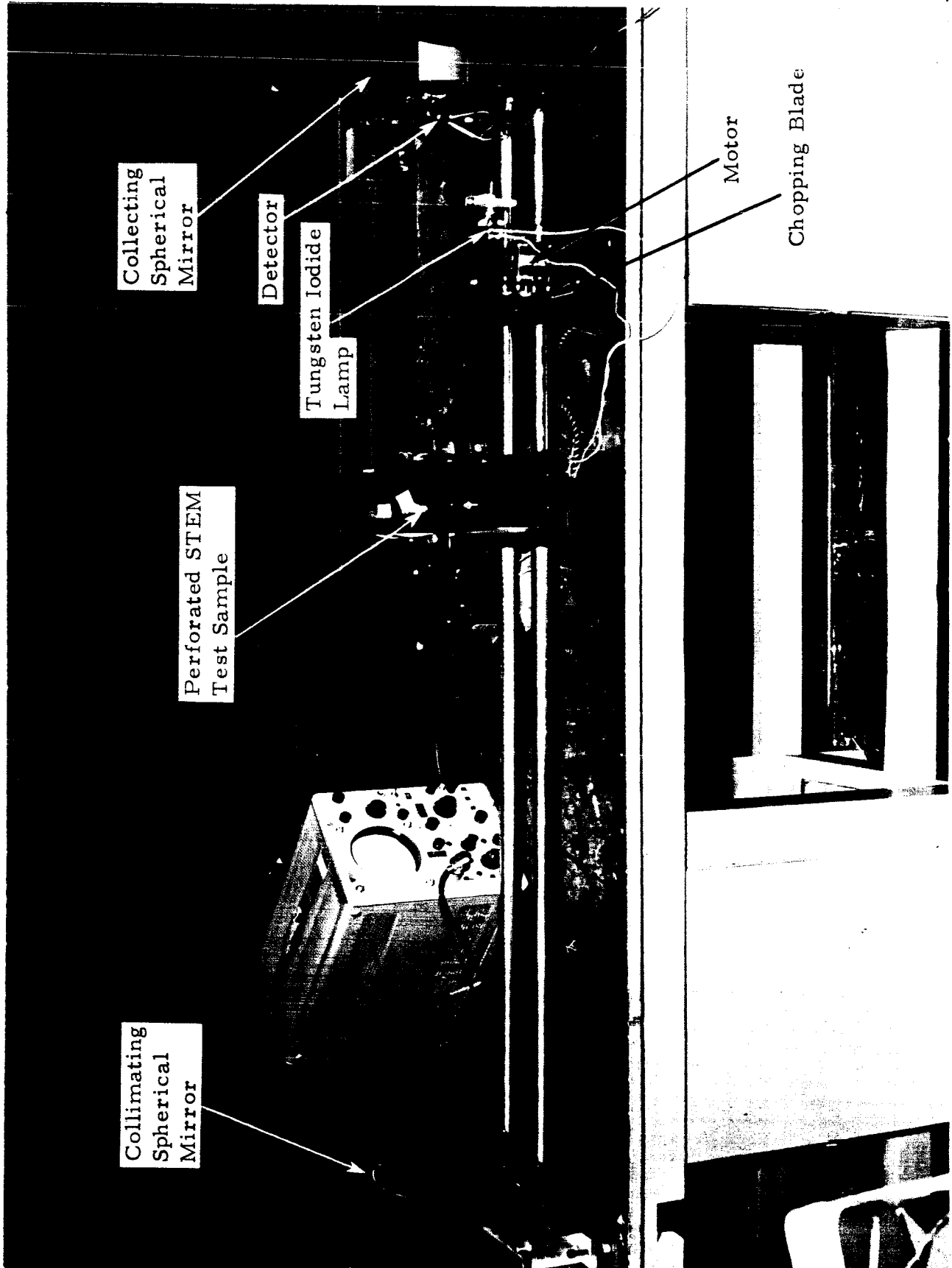
TUBE FORMING NO: _____

INSPECTOR: _____

DATE: _____



DHC-SP-R.168/2



GENERAL VIEW - HOLE PATTERN TRANSMISSION MEASURING APPARATUS

Figure 1



DHC-SP-R.168/2

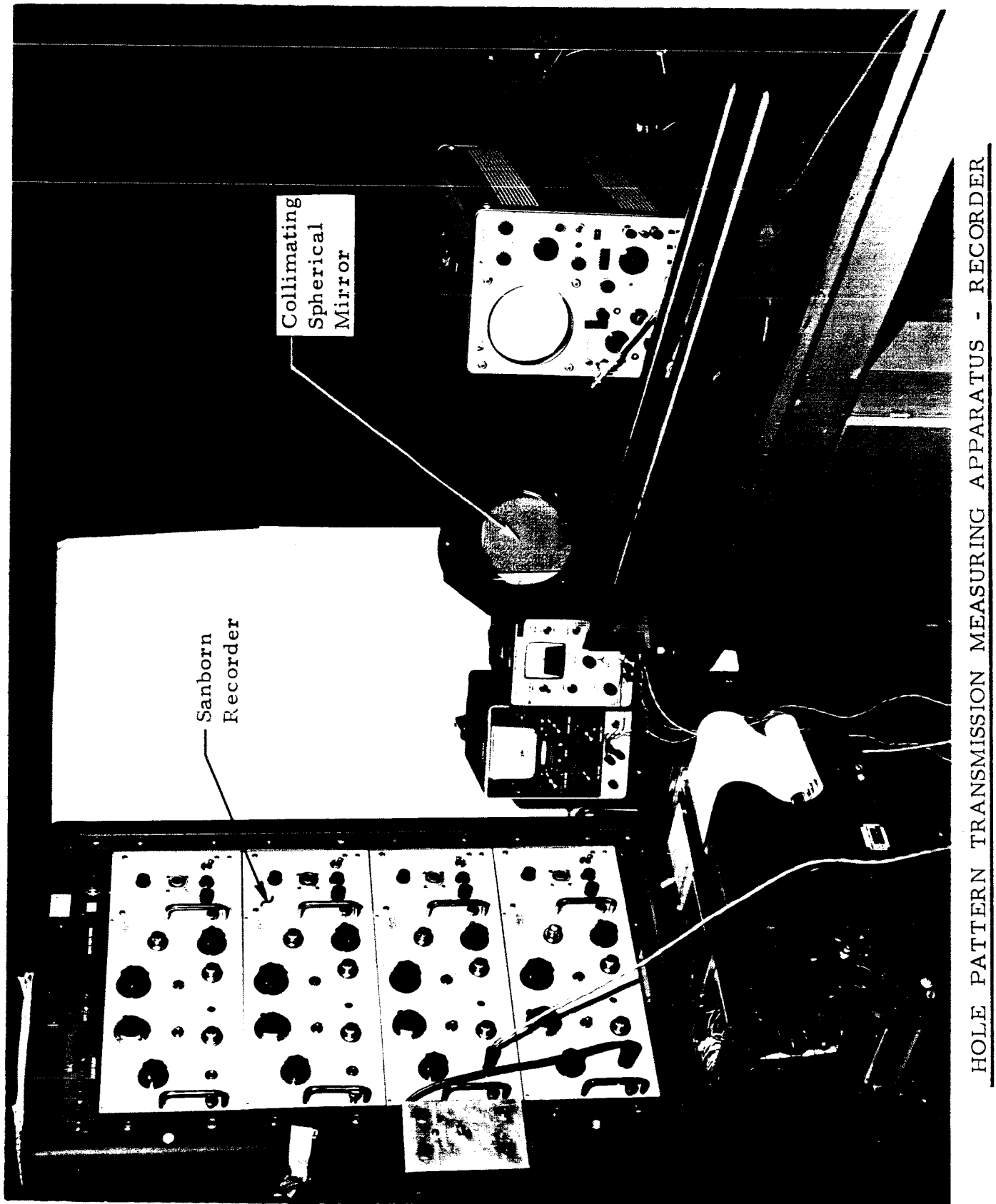
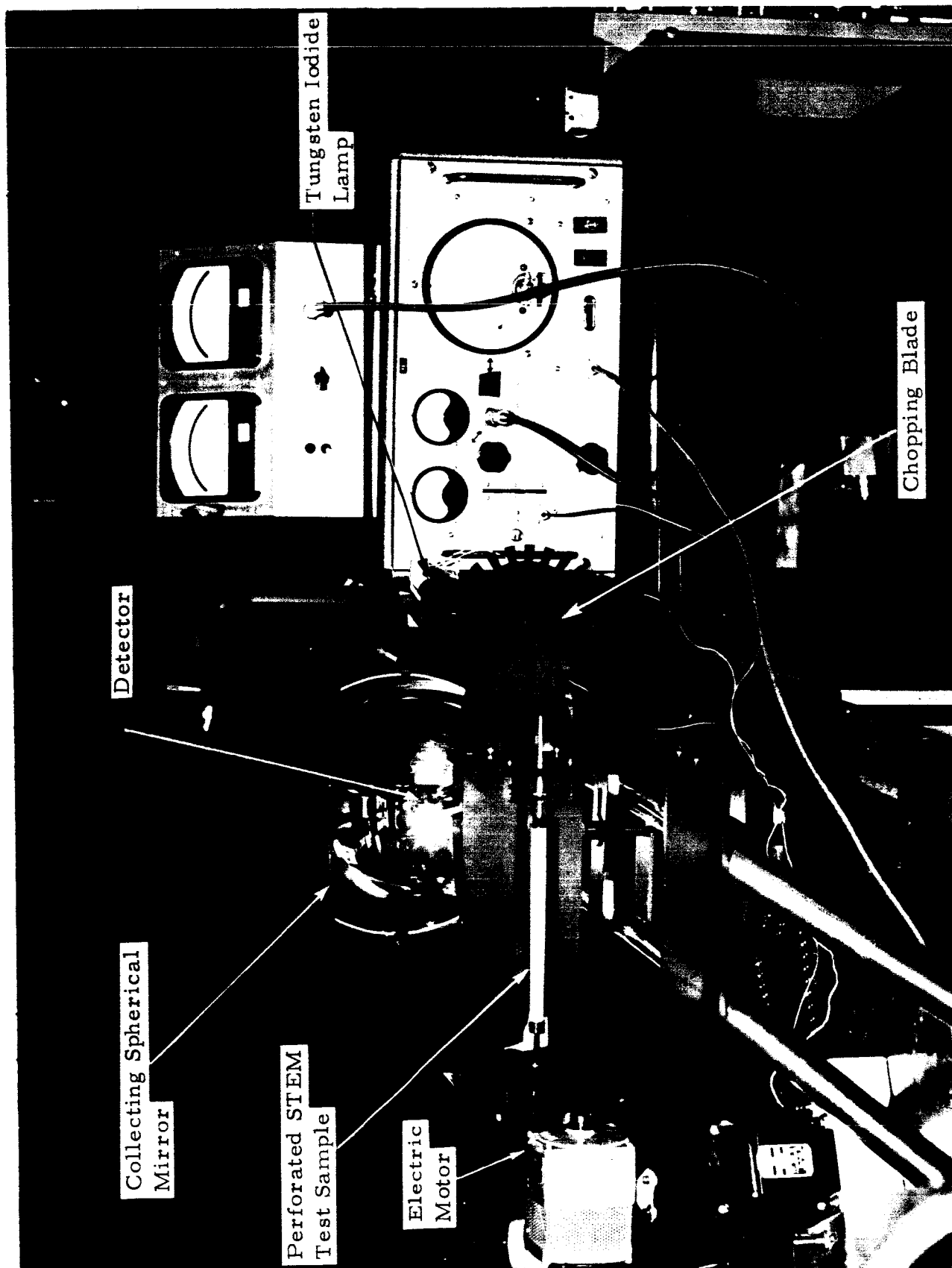


Figure 2



DHC-SP-R.168/2

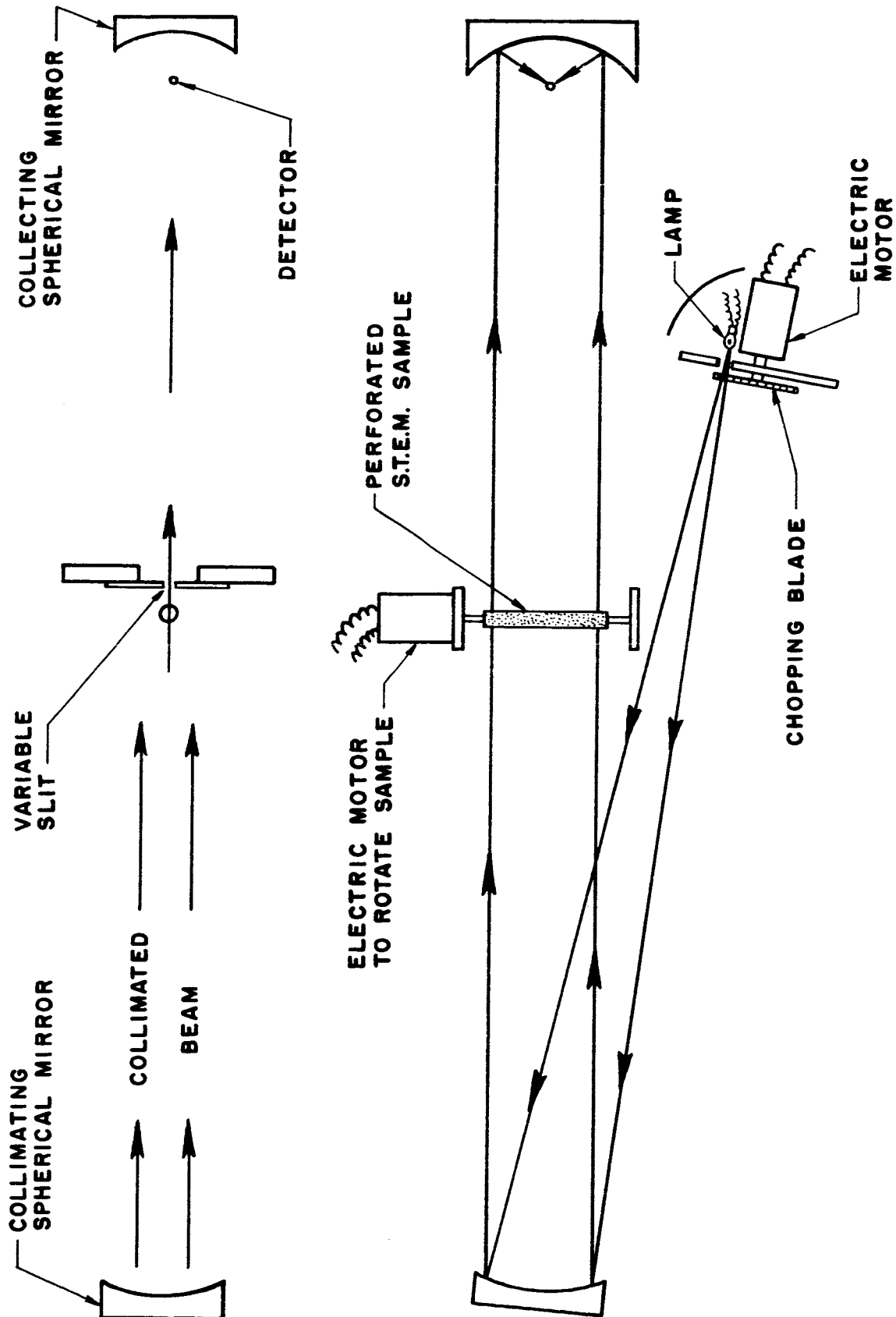


HOLE PATTERN TRANSMISSION MEASURING APPARATUS - RECEIVER END

Figure 3



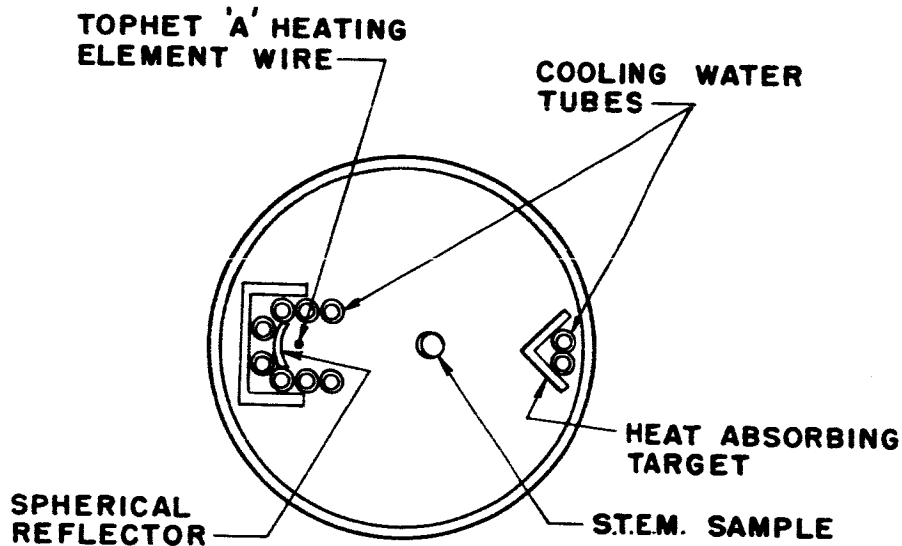
DHC-SP-R.168/2



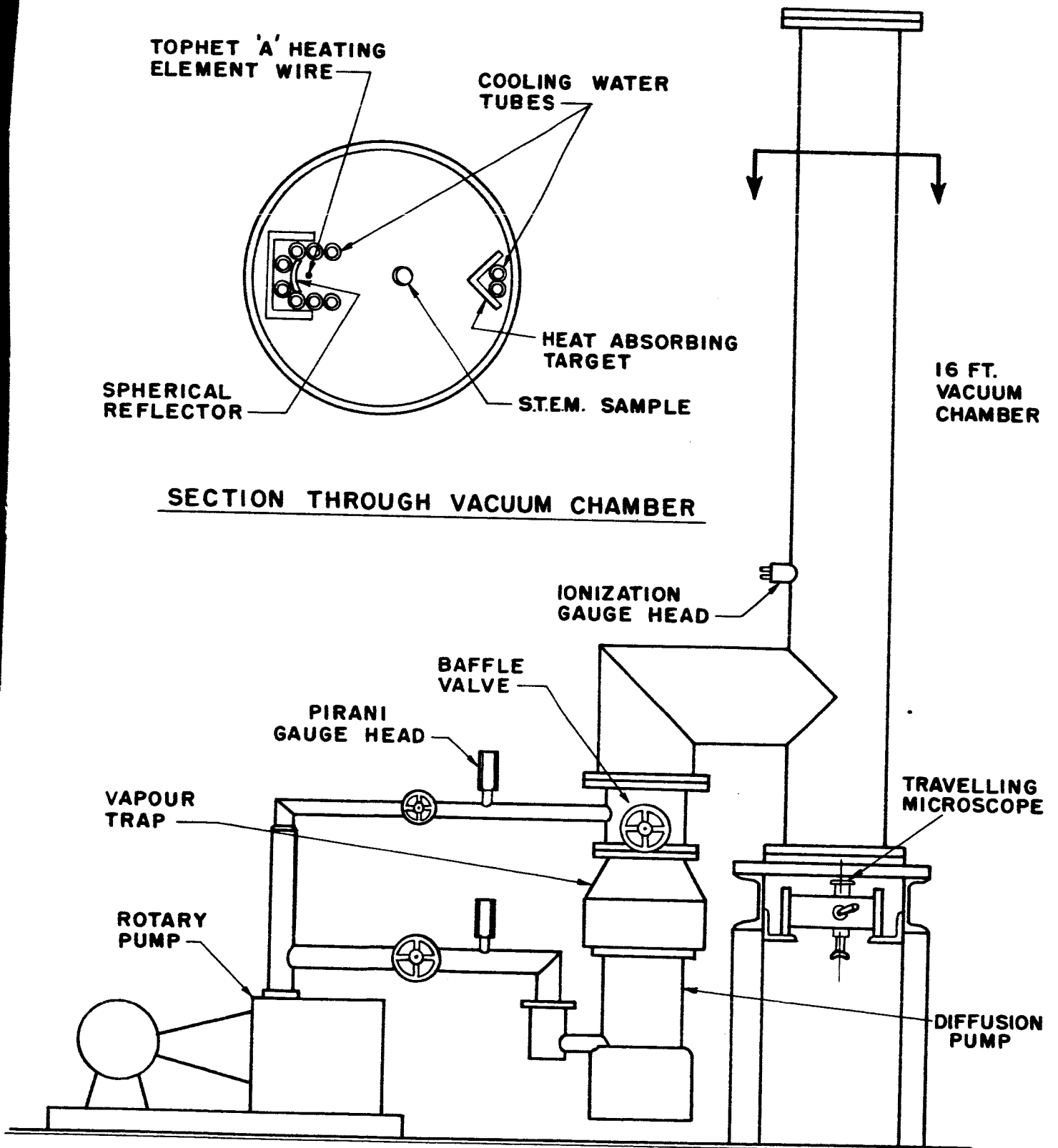
SCHEMATIC DIAGRAM OF OPTICAL APPARATUS



DHC - SP - R.168/2



SECTION THROUGH VACUUM CHAMBER

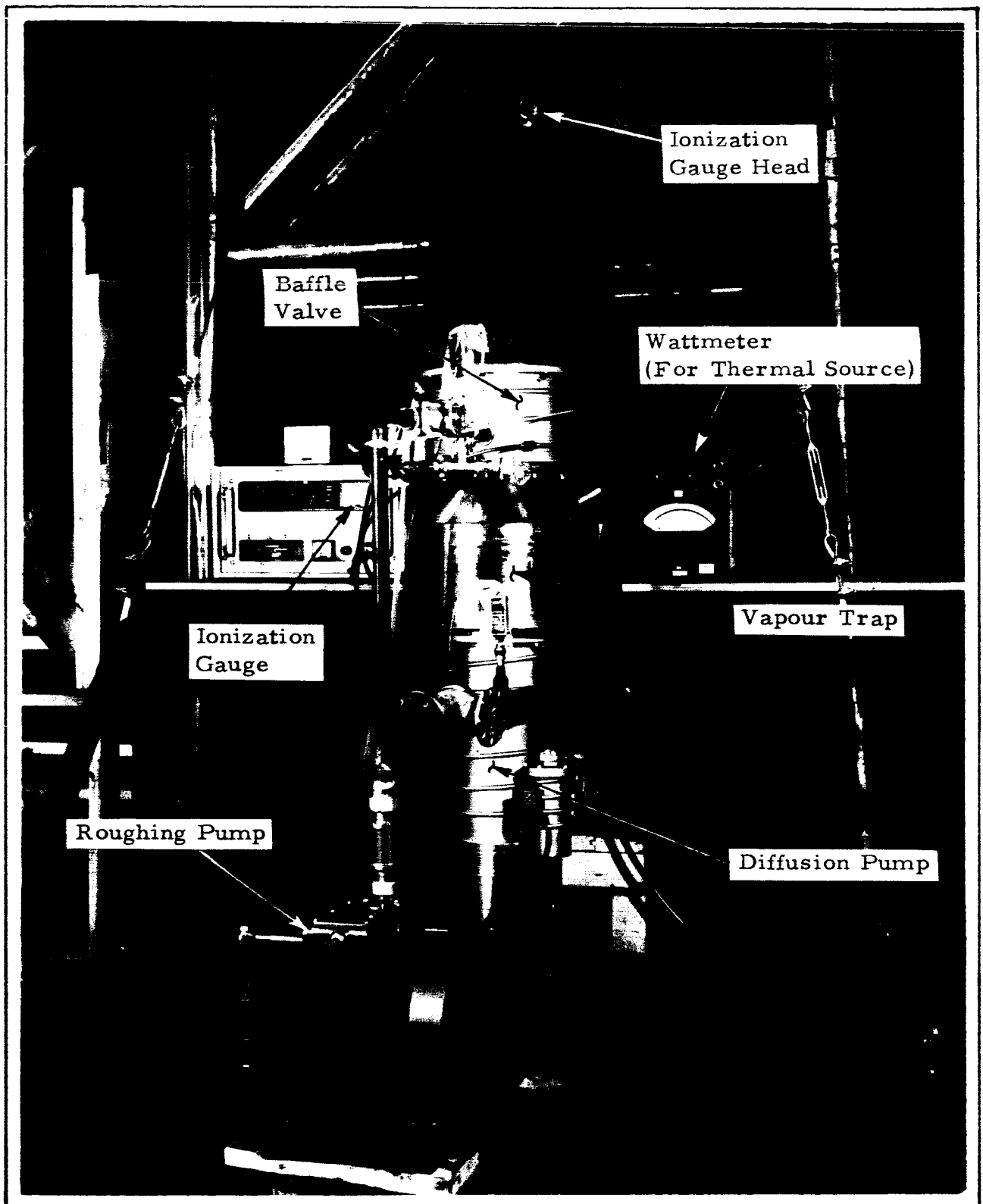


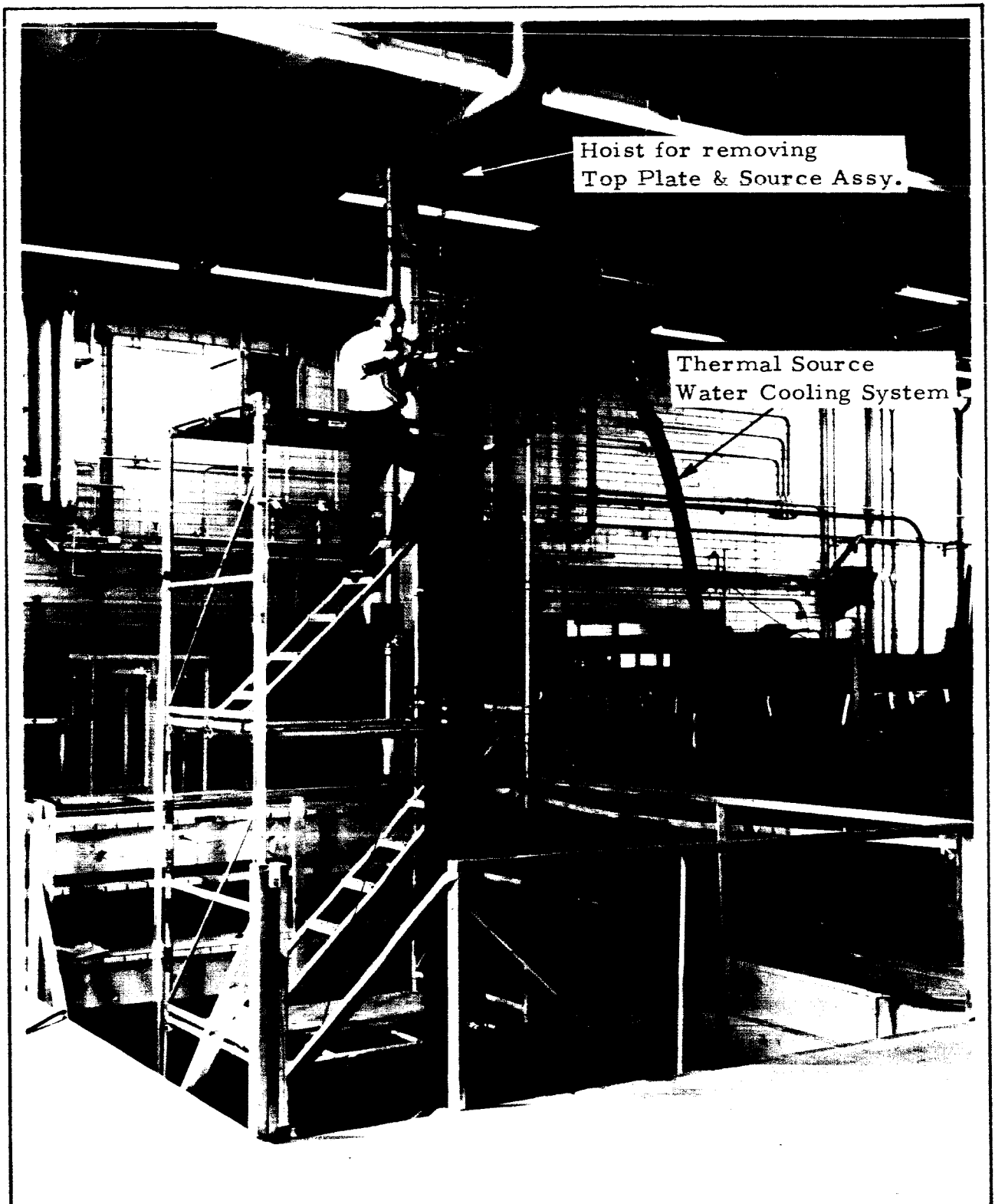
VACUUM TEST FACILITY

FIGURE 5



DHC-SP-R.168/2

VACUUM PUMP ASSEMBLY



VACUUM TEST CHAMBER SHOWING
ACCESS SCAFFOLD TO TOP PLATE

Figure 7

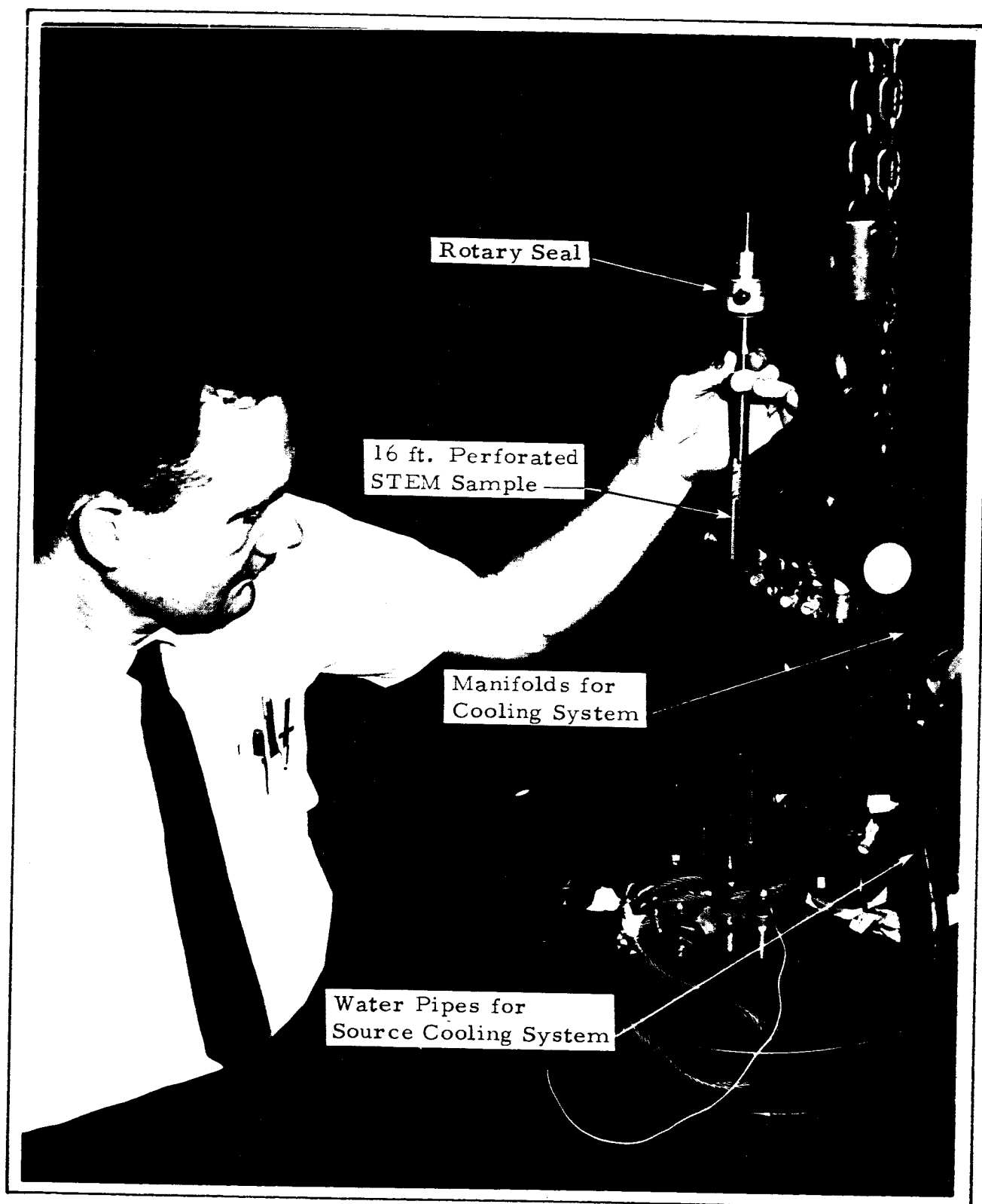


GENERAL VIEW - TOP PLATE & HEATER ASSEMBLY

Figure 8



DHC-SP-R.168/2

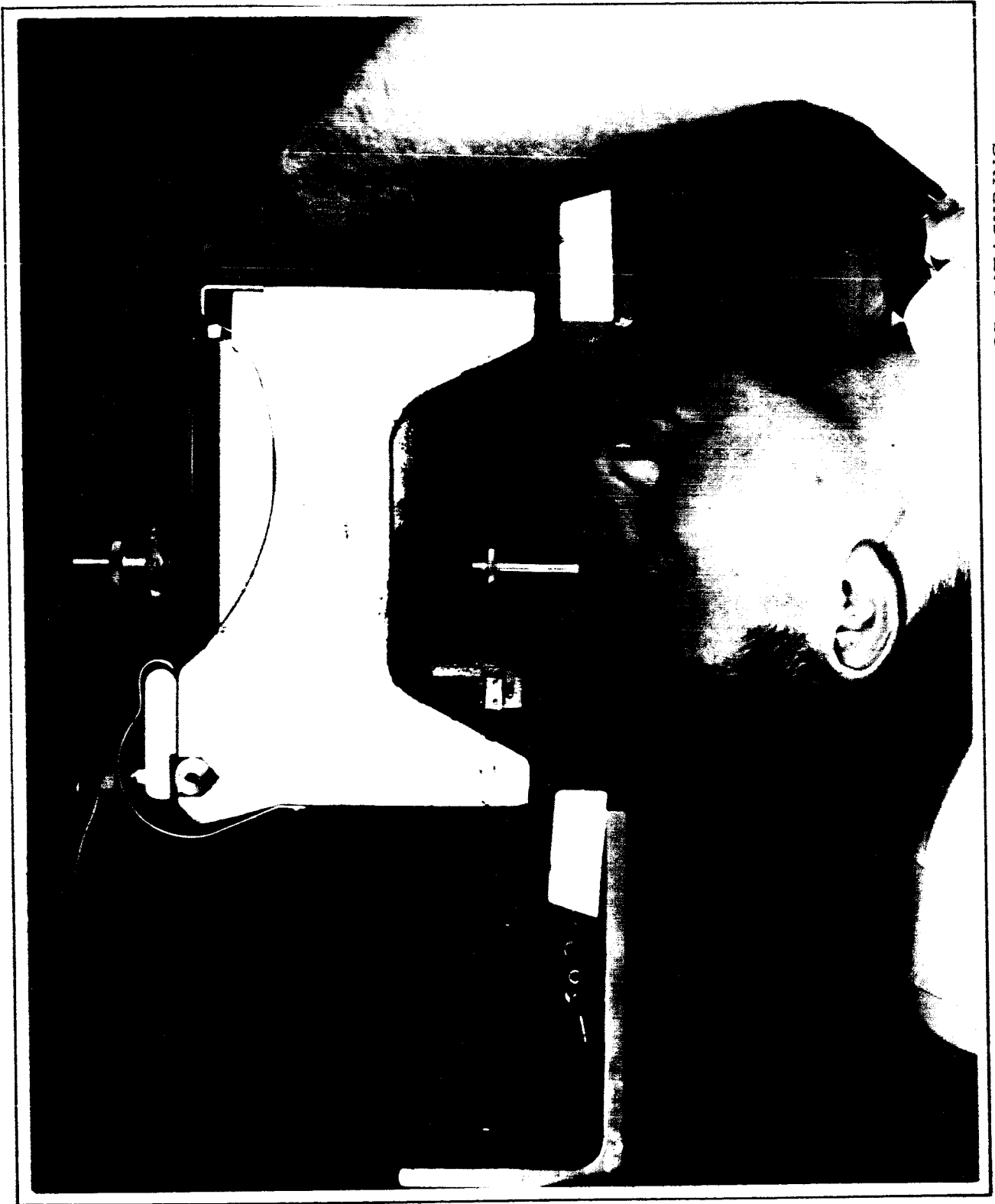


MOUNTING STEM IN VACUUM TEST CHAMBER

Figure 9



DHC-SP-R.168/2

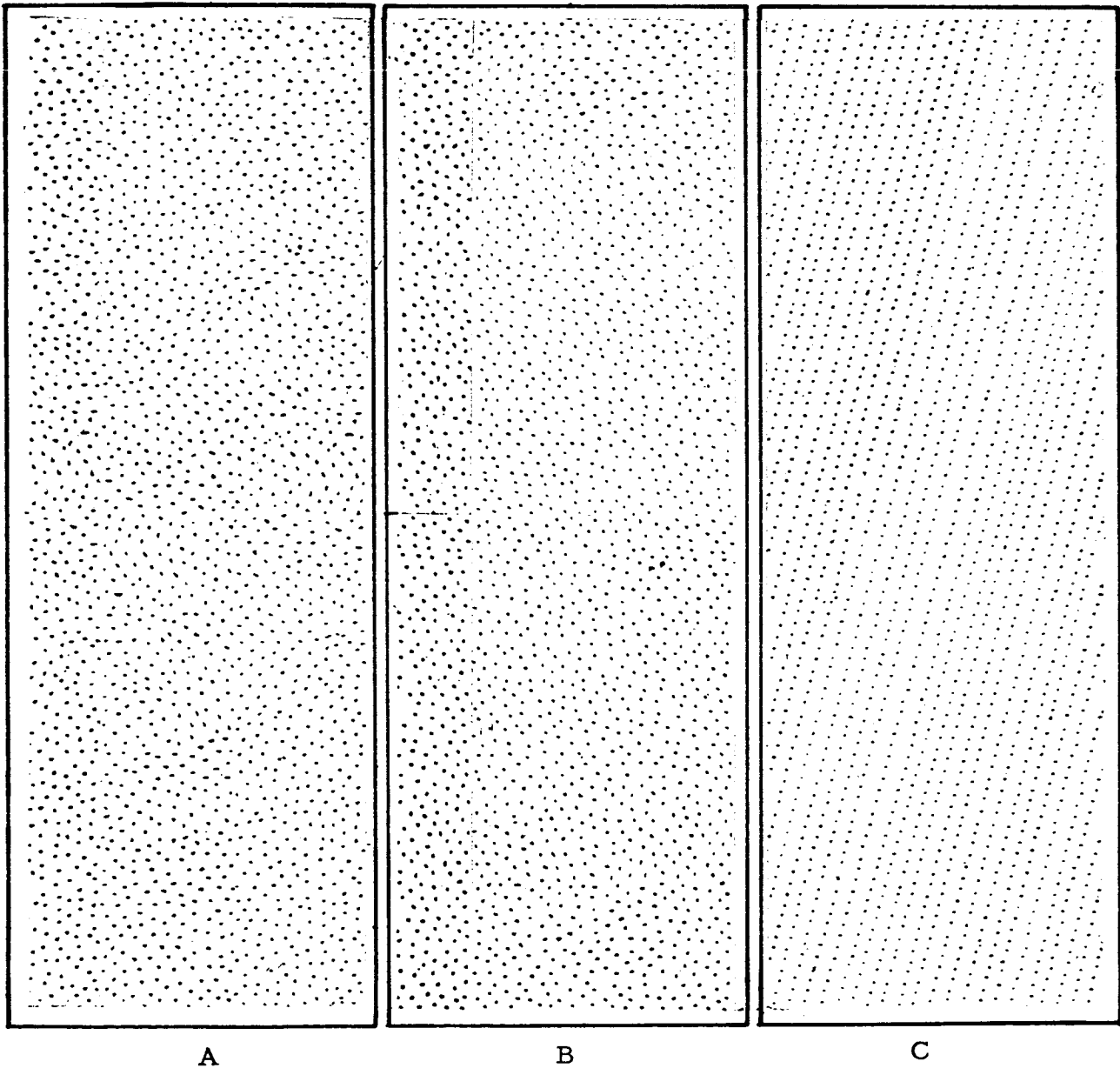


TWO COORDINATE TRAVELLING MICROSCOPE FOR MEASURING
STEM TIP DEFLECTION BELOW OBSERVATION PORT

Figure 10



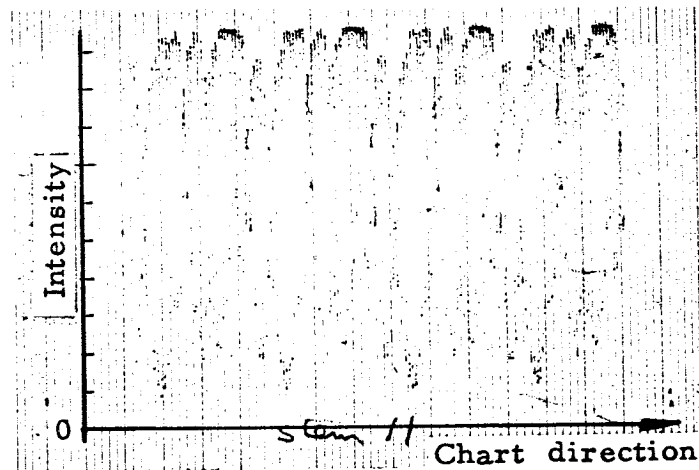
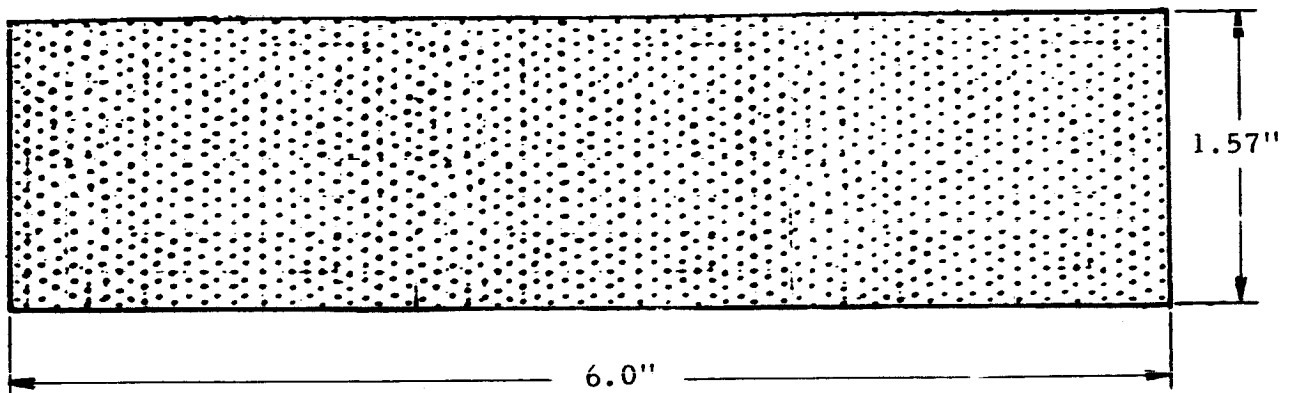
DHC-SP-R.168/2



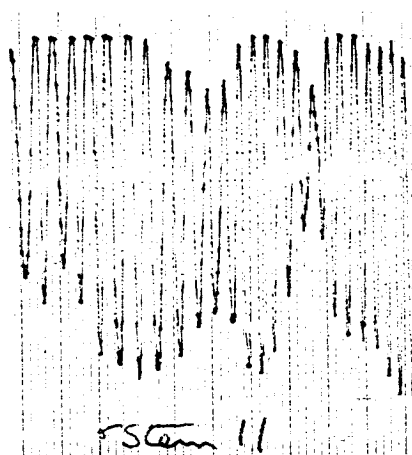
TYPICAL HOLE PATTERNS



DHC-SP-R.168/2



Sample Rotation Speed:- 50 Degrees Per Second
 (The Poor Trace is Due to the High Speed of the Recording Pen)



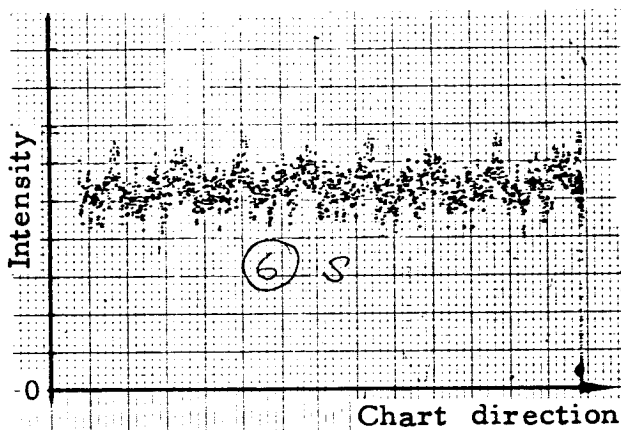
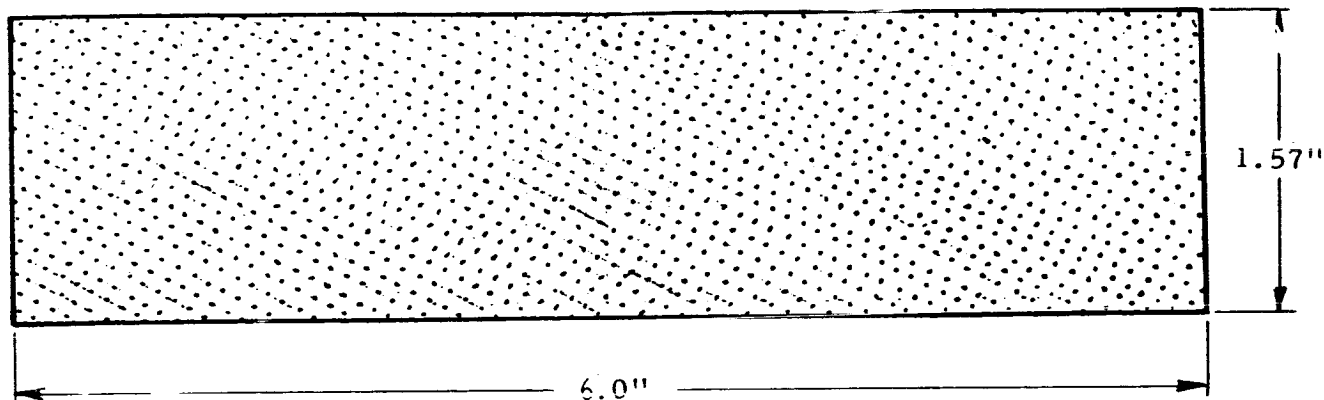
Sample Rotation Speed:- 10 Degrees Per Second

REGULAR STEM PATTERN WITH LIGHT TRANSMITTANCE CURVE

Figure 12



DHC-SP-R.168/2

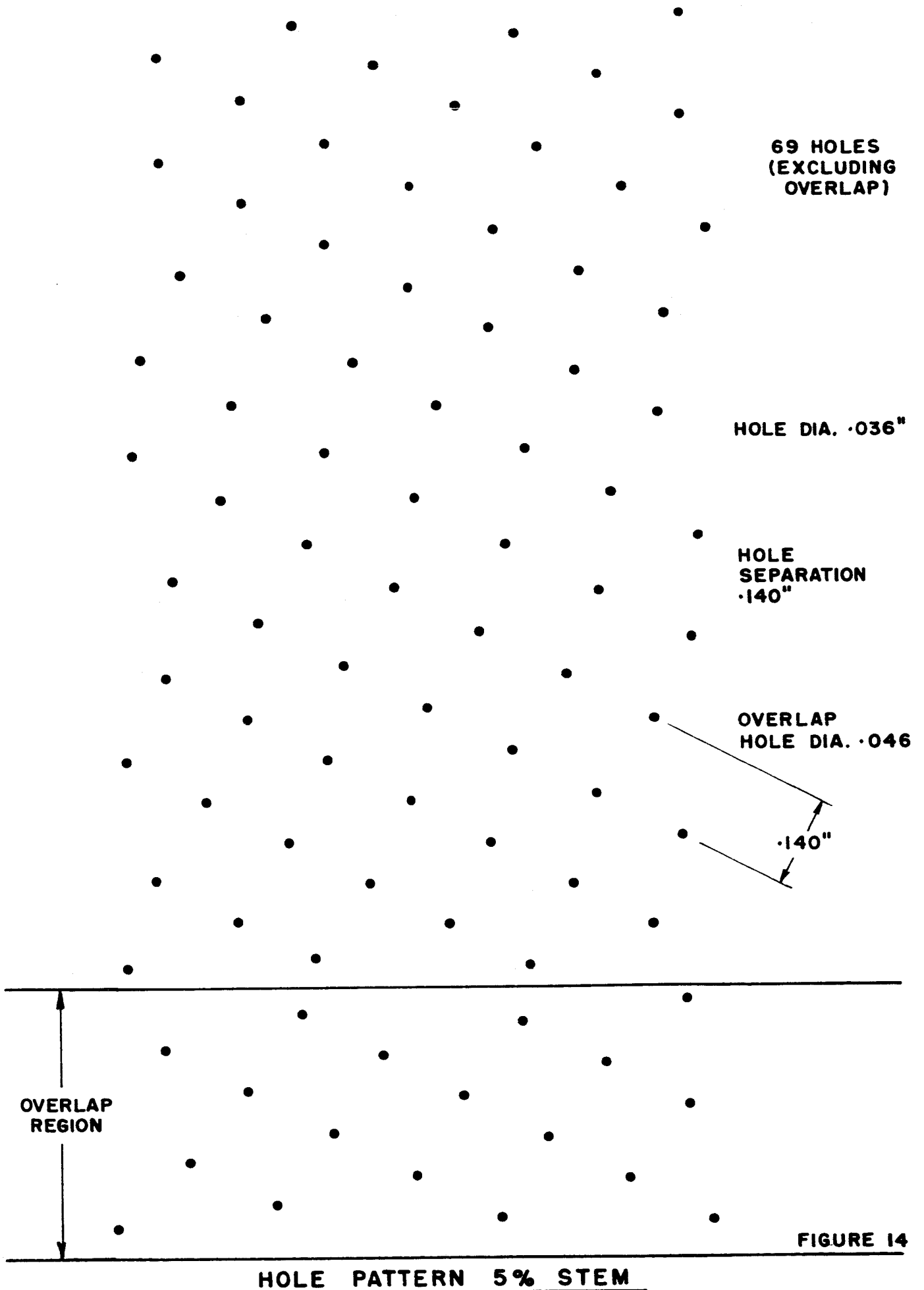


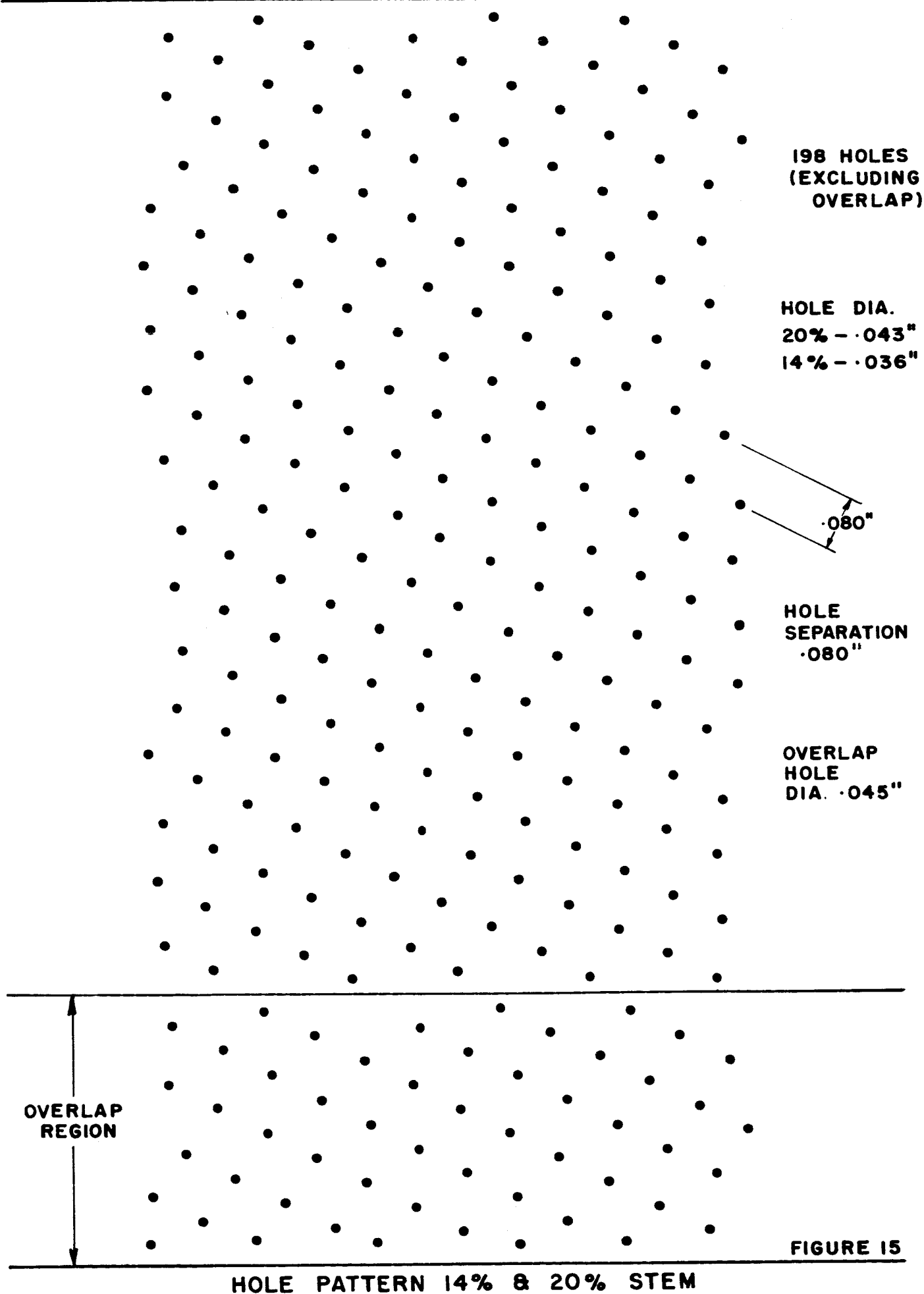
Light transmittance curve for above pattern, as measured on the optical bench with the sample rotated at right angles to the direction of the incident radiation.

The hole - hole separation and line - line separation are equal at approximately 0.087 inches but the position of the first hole in every line is varied in a random manner.

TYPICAL SPIRAL PATTERN WITH LIGHT TRANSMITTANCE CURVE

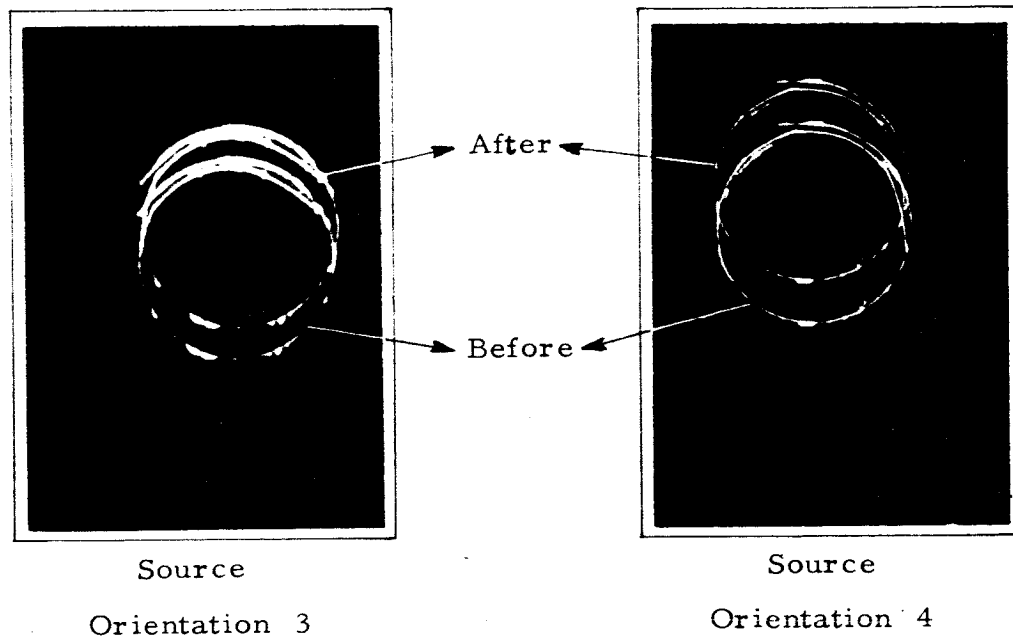
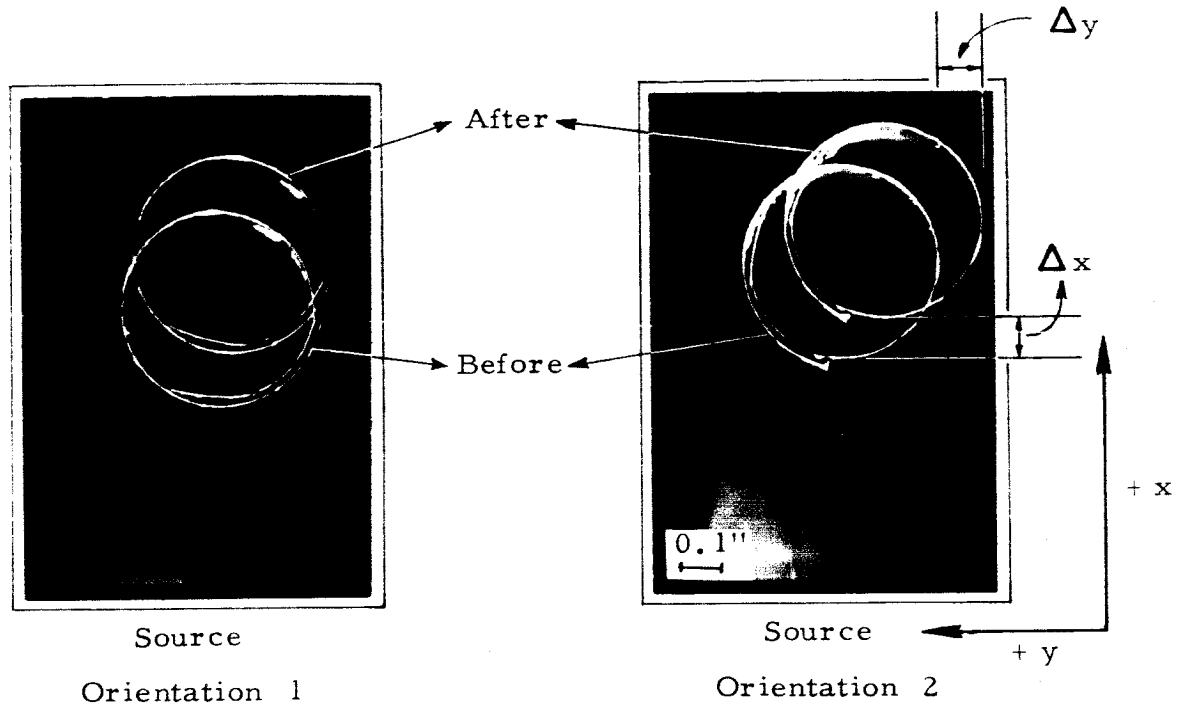
Figure 13







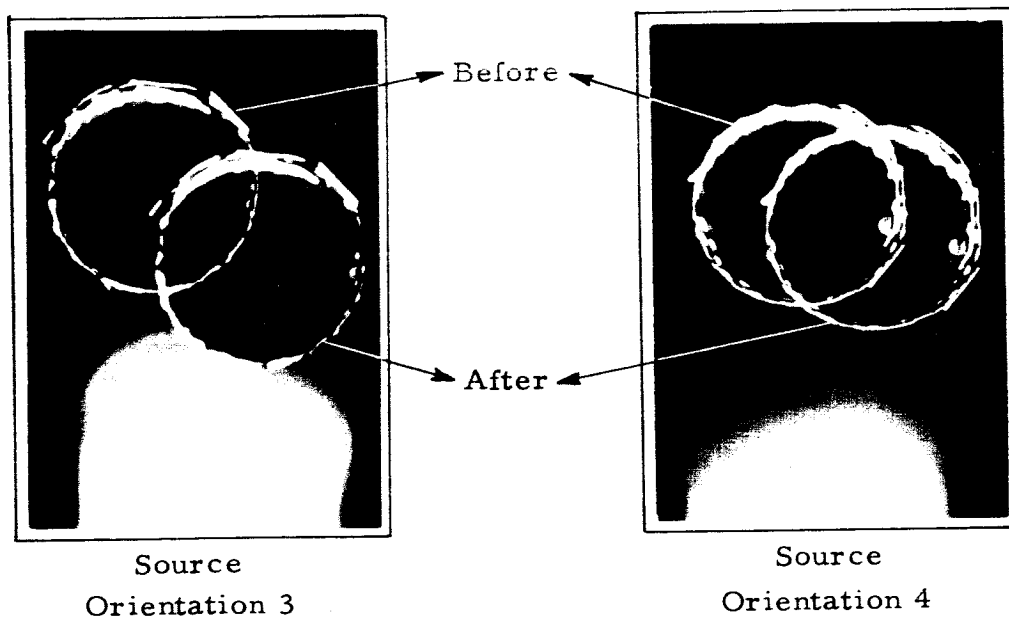
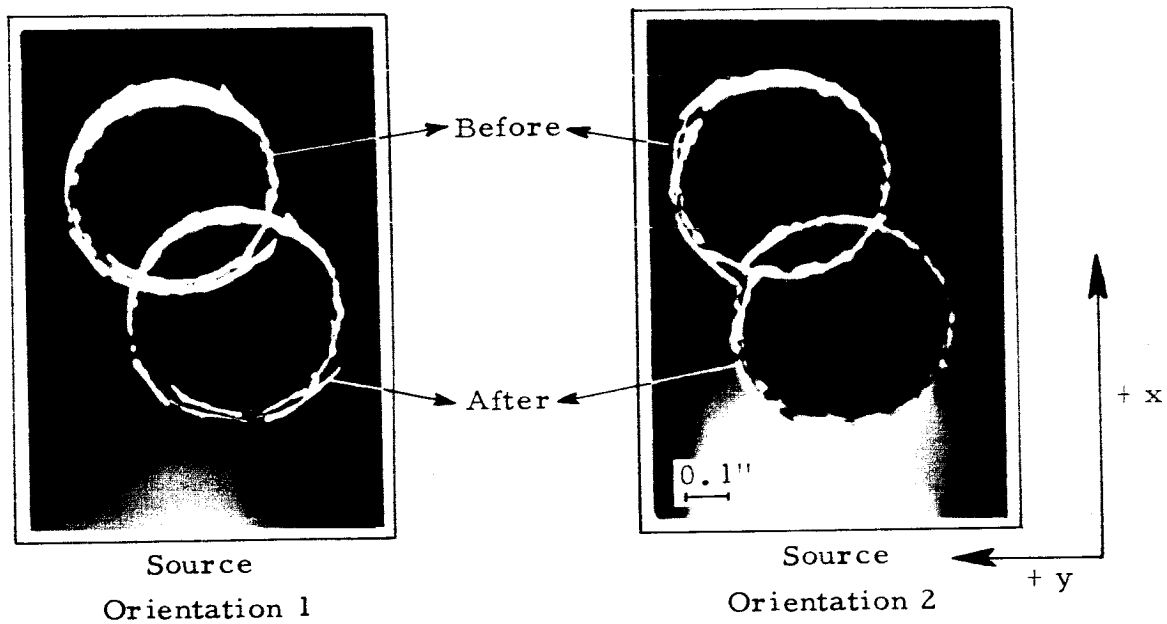
DHC-SP-R.168/2



UNPERFORATED SAMPLE: DEFLECTION PHOTOGRAPHS
SAMPLE 16-0-2



DHC-SP-R.168/2



20% AREA REMOVED DEFLECTION PHOTOGRAPHS
SAMPLE 16-20-2 L



DHC-SP-R.168/2

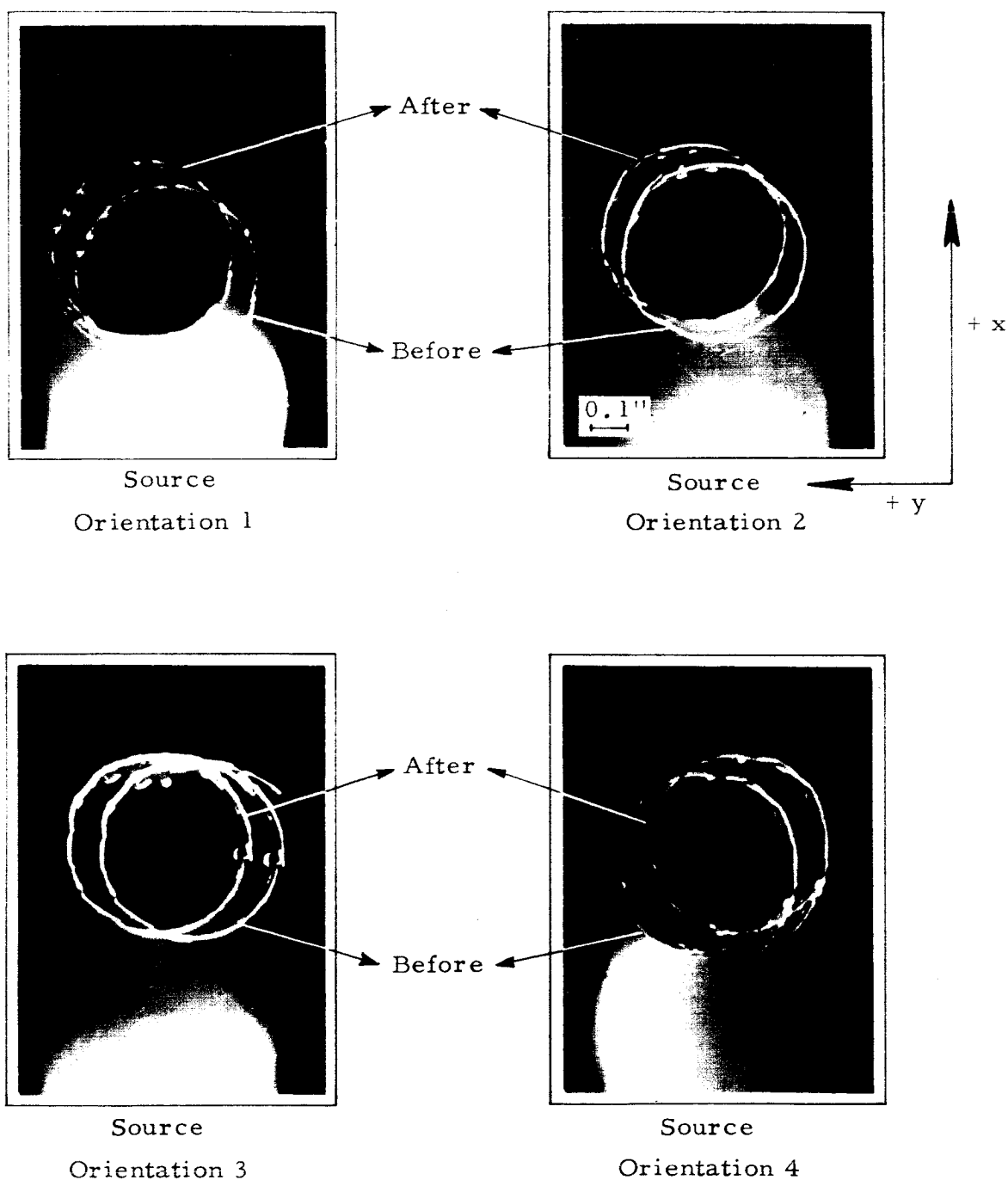
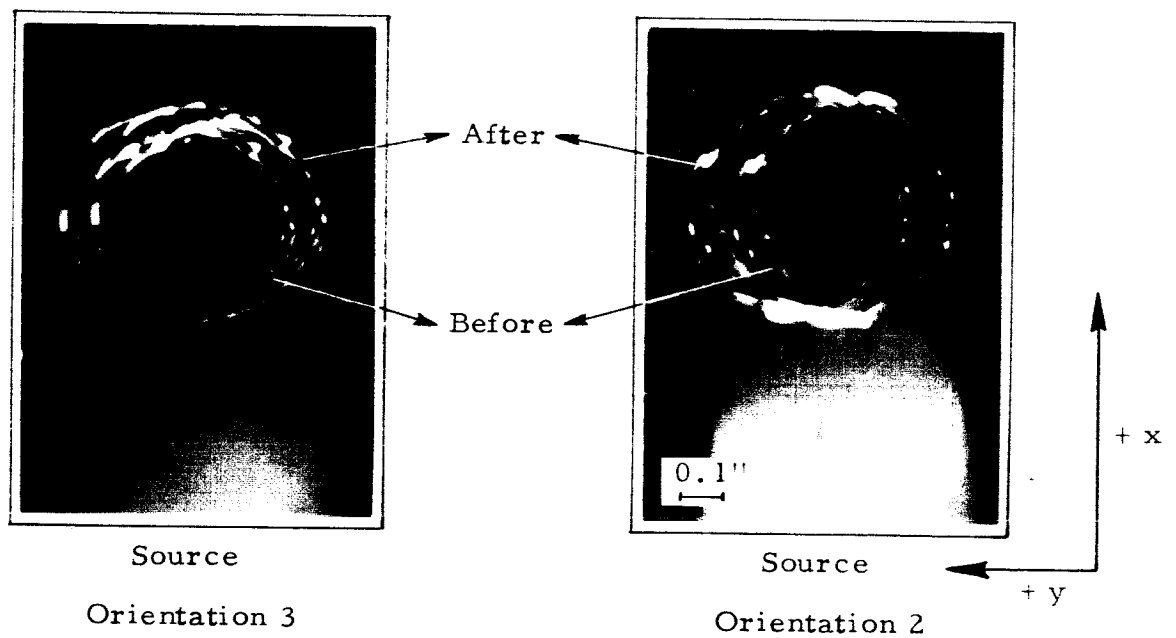
5% AREA REMOVED DEFLECTION PHOTOGRAPHSSAMPLE 16-5-1 SL

Figure 19



DHC-SP-R.168/2

"ZERO BENDING" STEM

5% AREA REMOVED DEFLECTION PHOTOGRAPHS
SAMPLE 16-5-4 SL

DEFLECTION MEASUREMENTS (SEE TABLE 5-1)

(OBTAINED WITH MICROSCOPE)

PRESSURE $< 4 \times 10^{-5}$ TORR

SOURCE: 1100°C; 800 WATTS

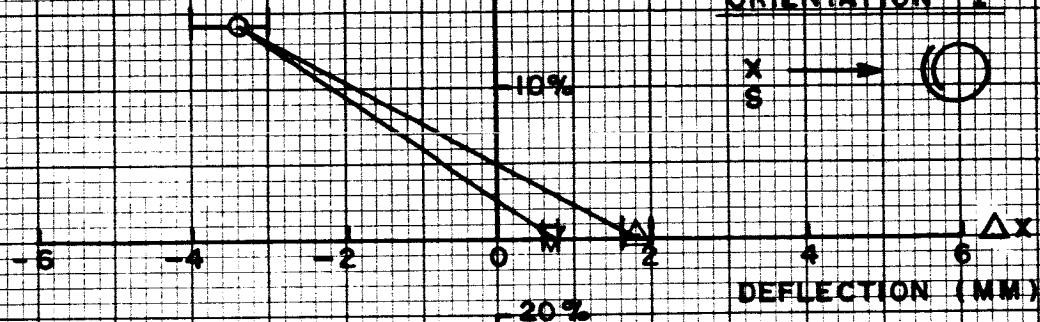
KEY

▽ 16-0-1

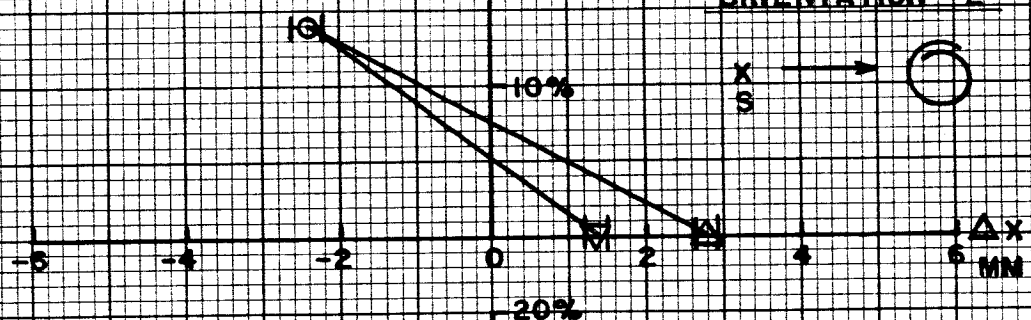
△ 16-0-2

AREA
REMOVED

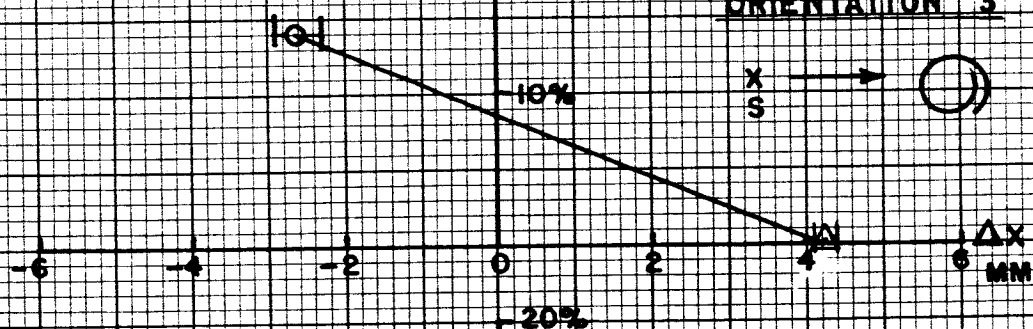
ORIENTATION 1



ORIENTATION 2



ORIENTATION 3



ORIENTATION 4

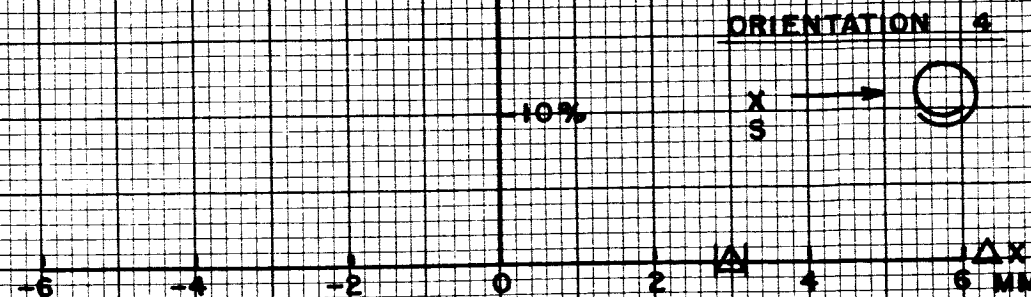
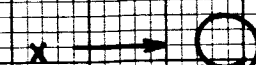


FIGURE 21

DEFLECTION MEASUREMENTS (SEE TABLE 5-2) **(OBTAINED FROM PHOTOGRAPHIC DATA)**

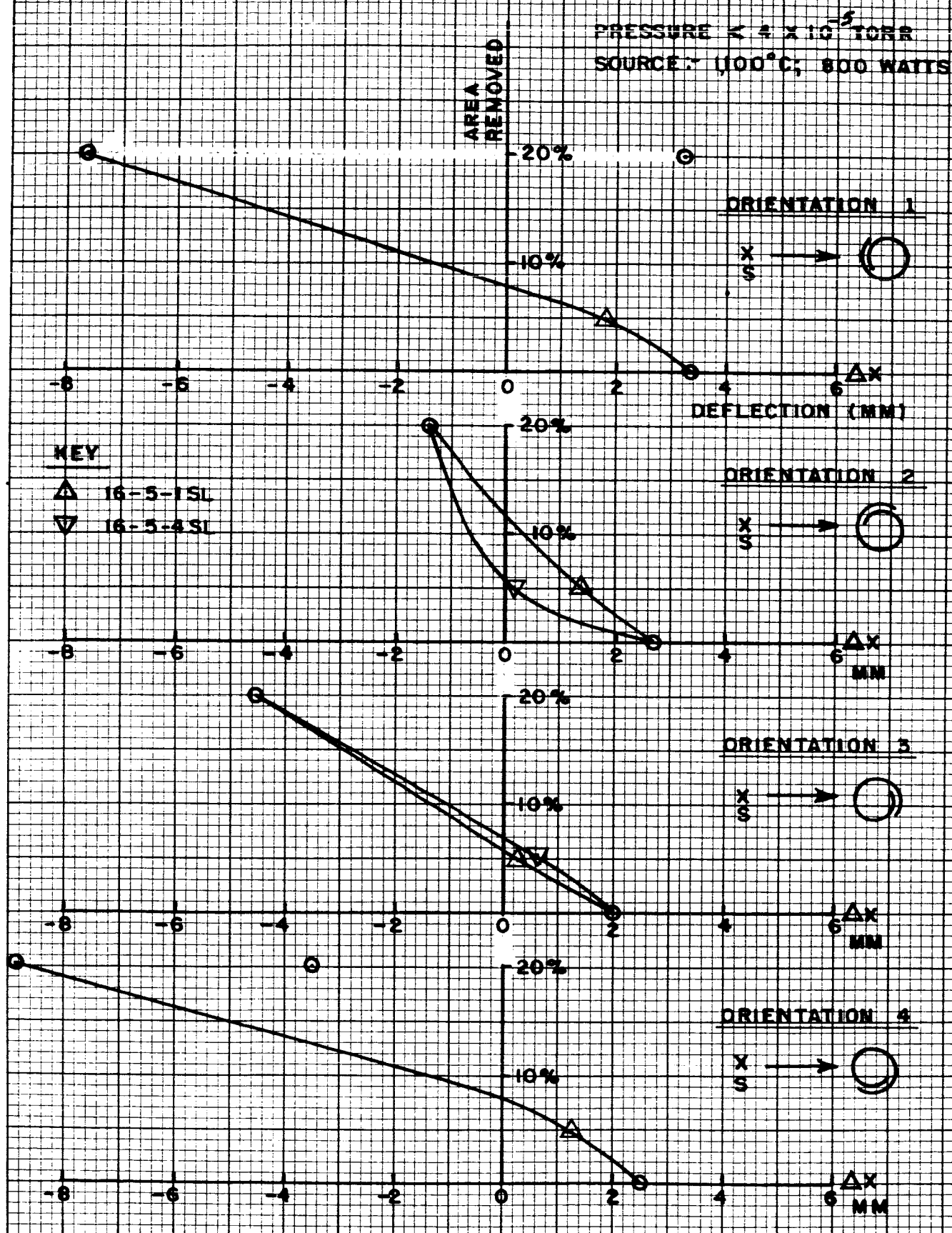
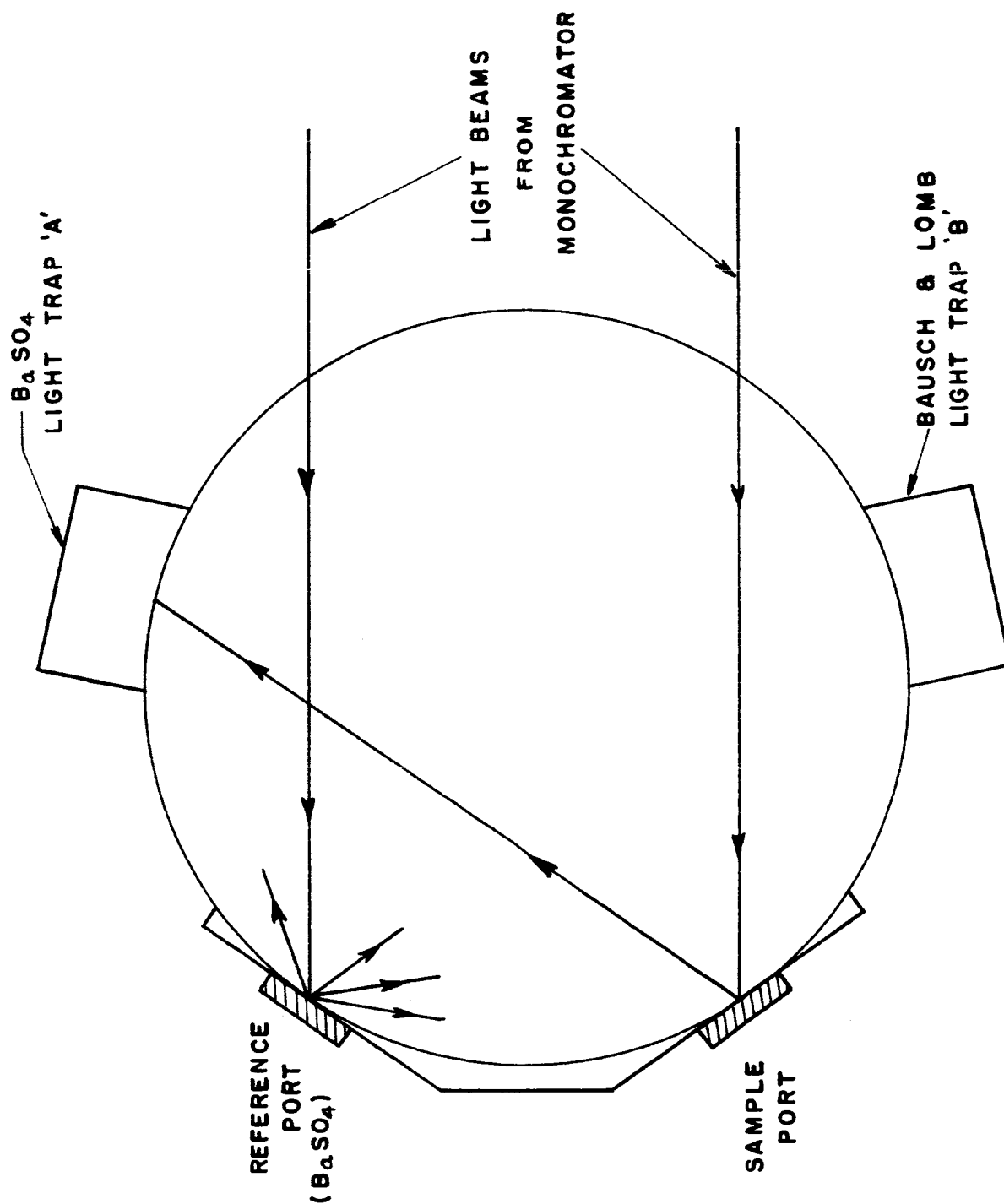


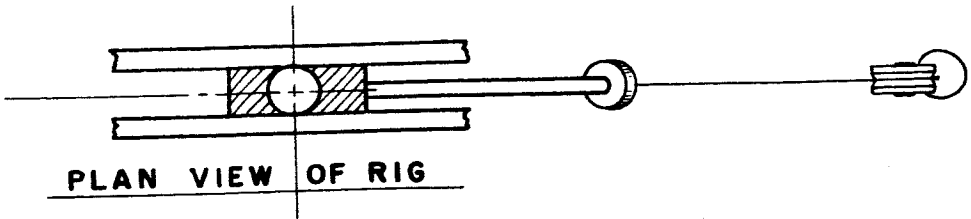
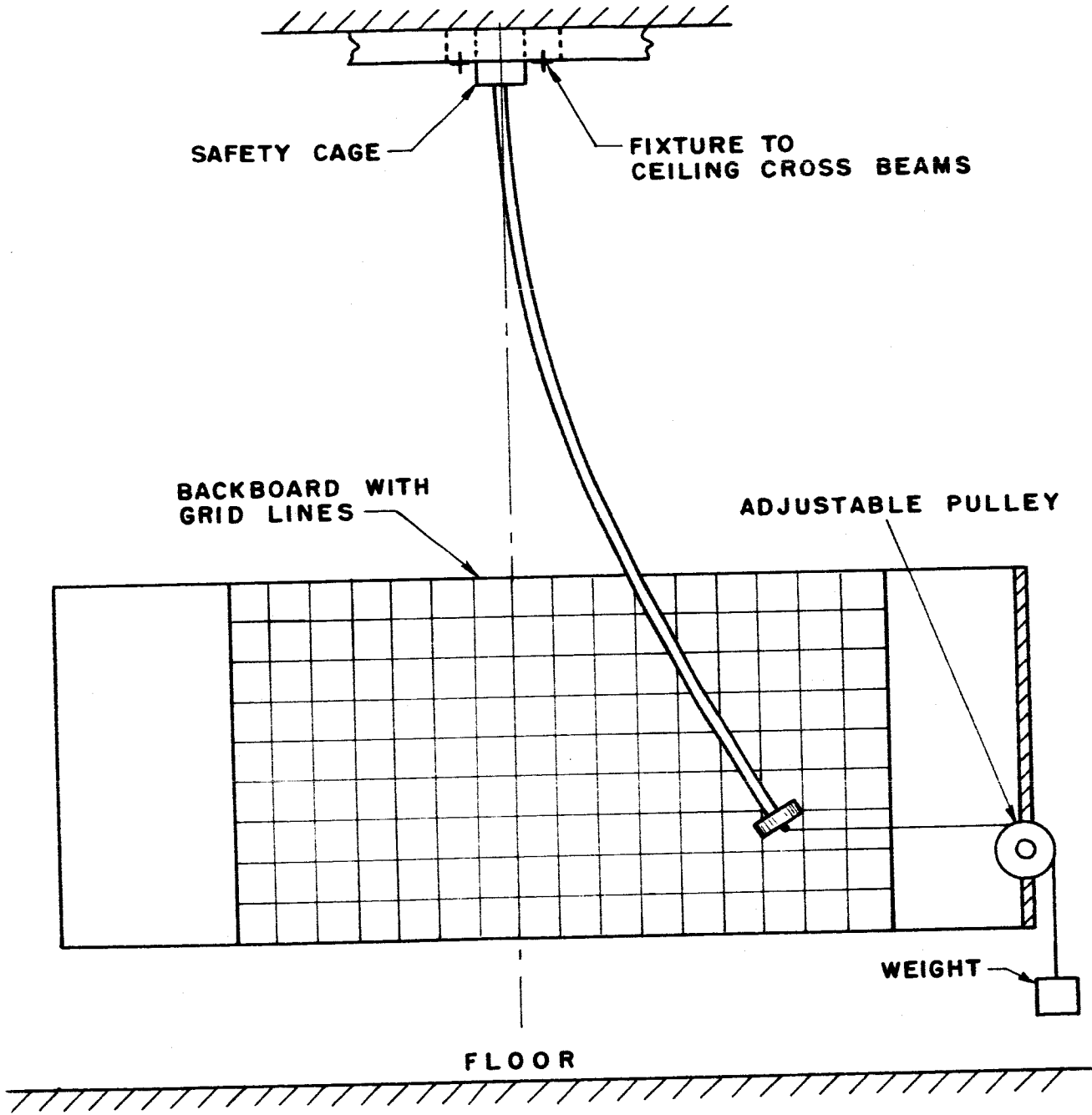
FIGURE 22



SCHEMATIC - INTEGRATING SPHERE ATTACHMENT
 FOR BAUSCH & LOMB SPECTROPHOTOMETER

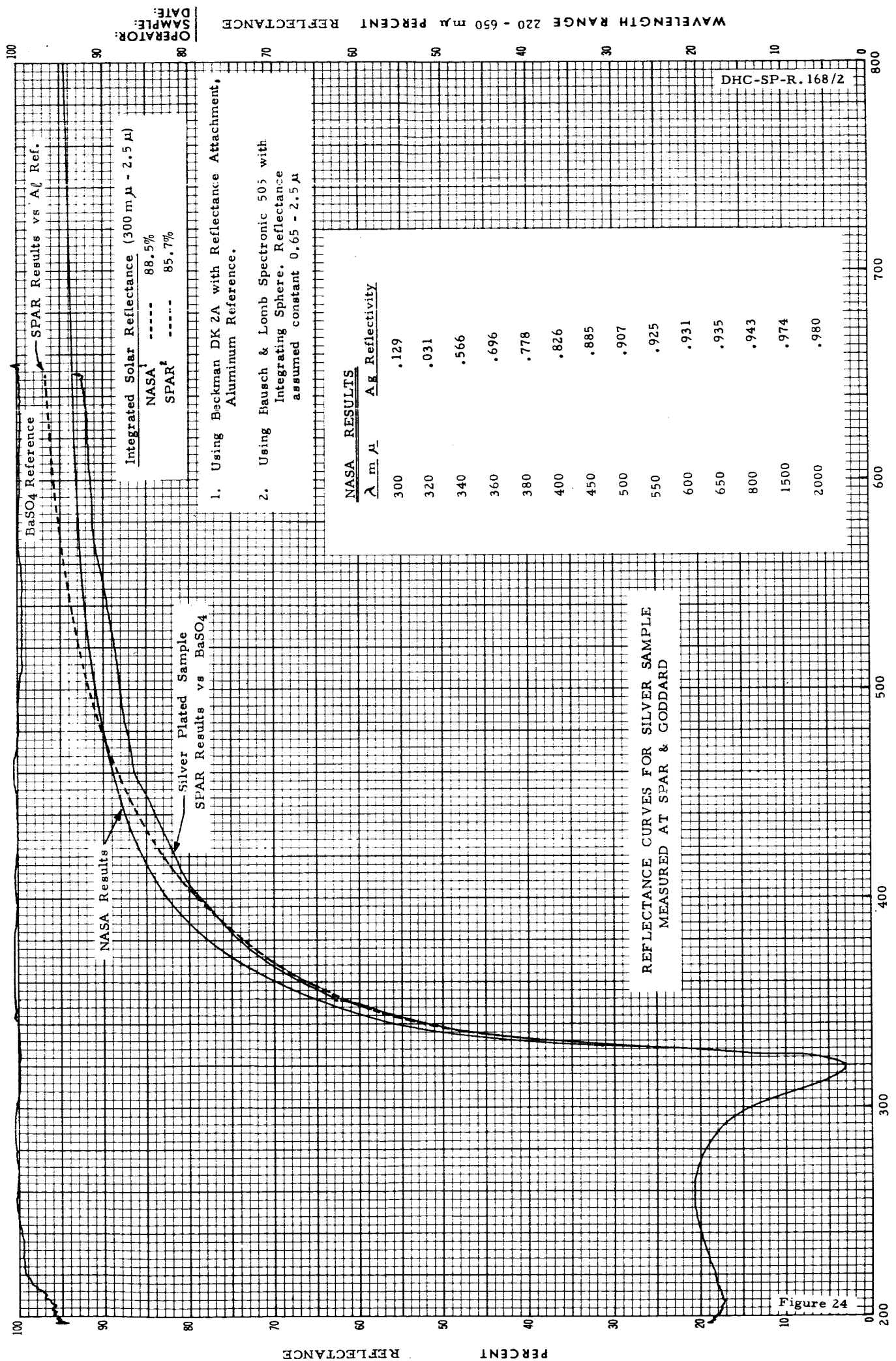


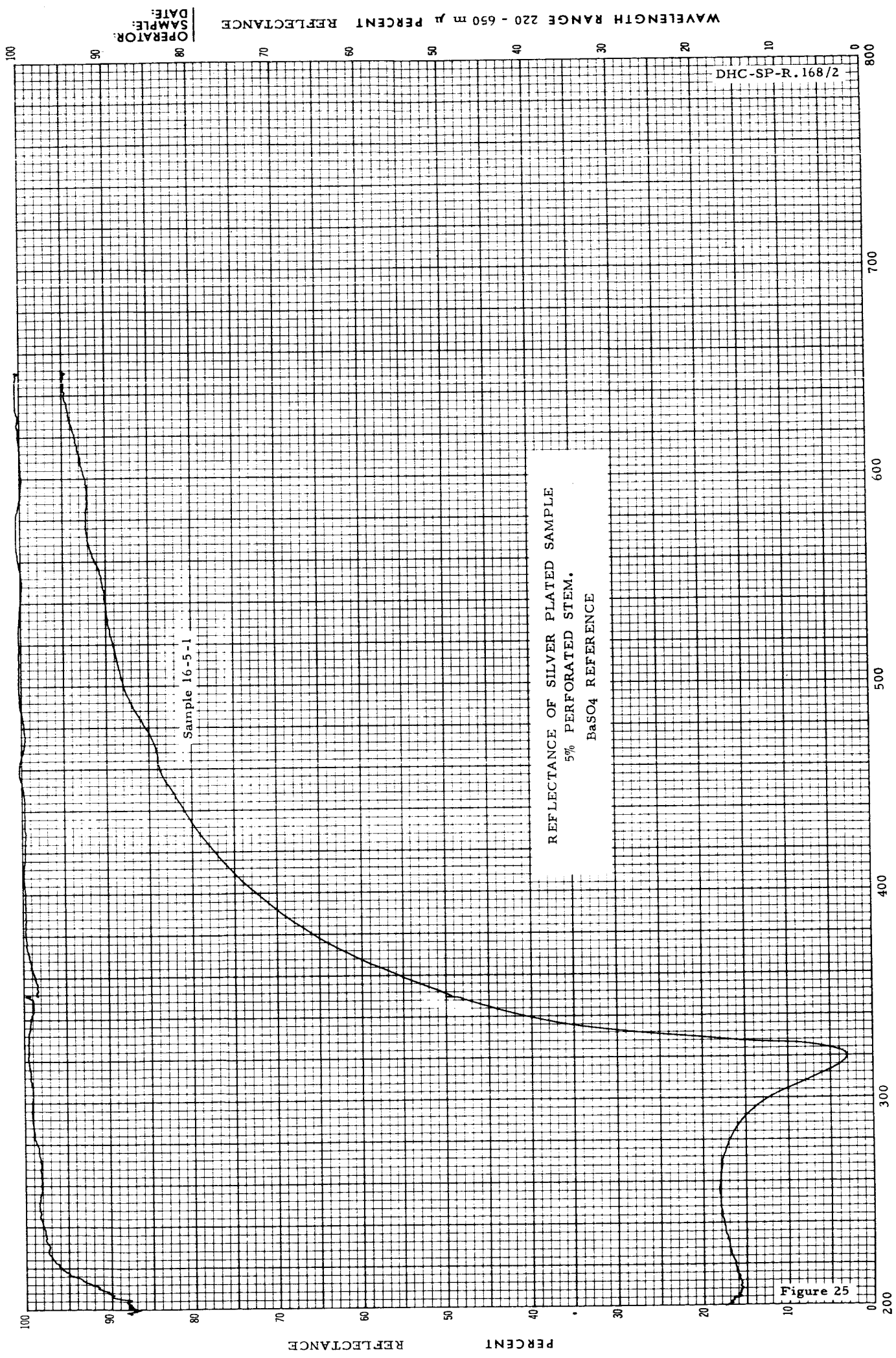
DHC-SP-R.168/2

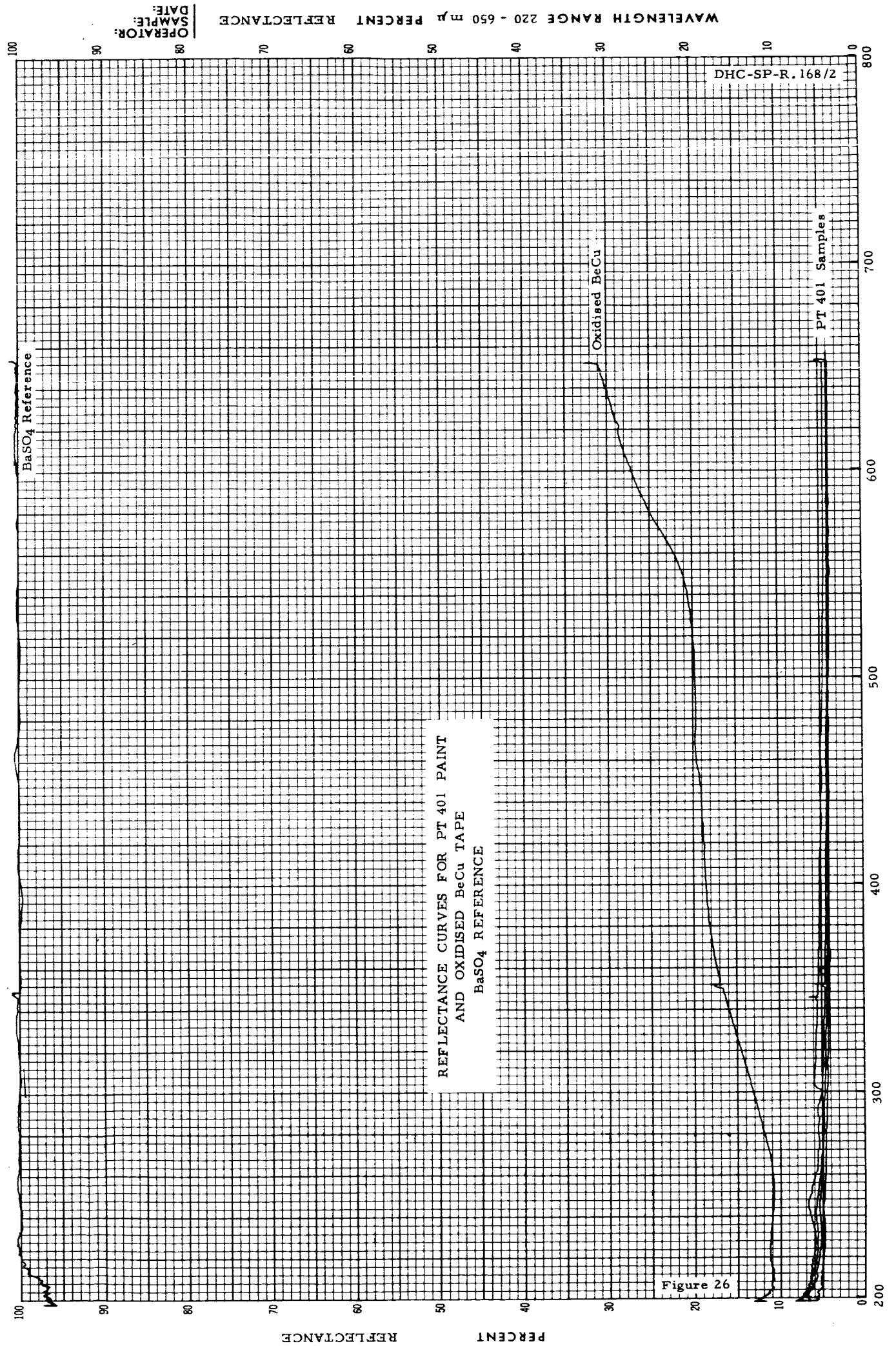


STEM BENDING RIG

FIGURE 27

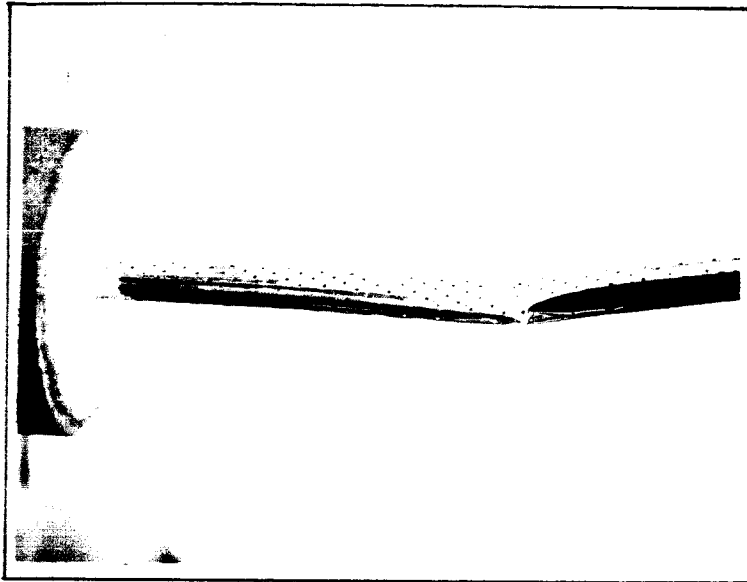




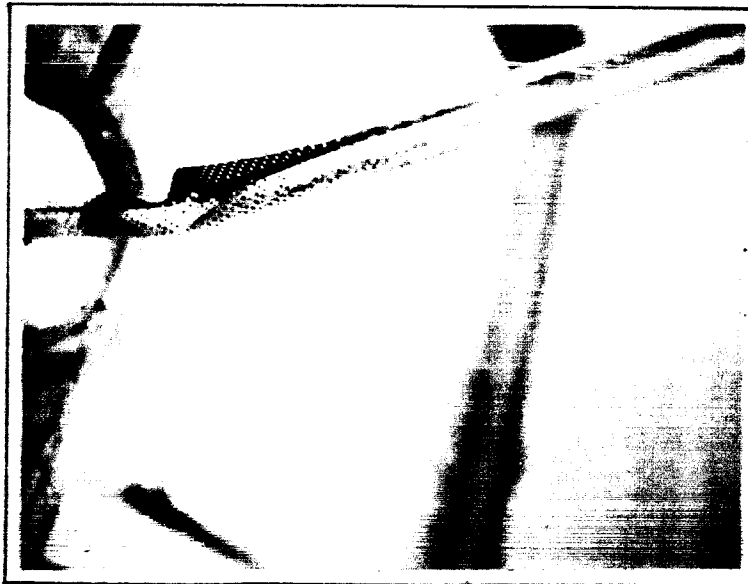
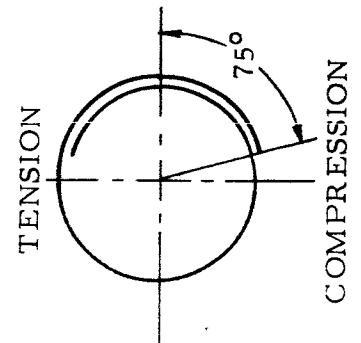




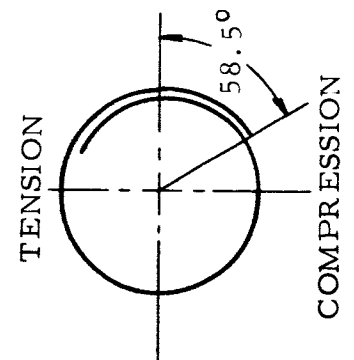
DHC-SP-R.168/2



5% Sample
Failure 4.0" From Root
Diameter 0.45"
Overlap Factor 1.415



20% Sample
Failure 2.0" From Root
Diameter 0.48"
Overlap Factor 1.325

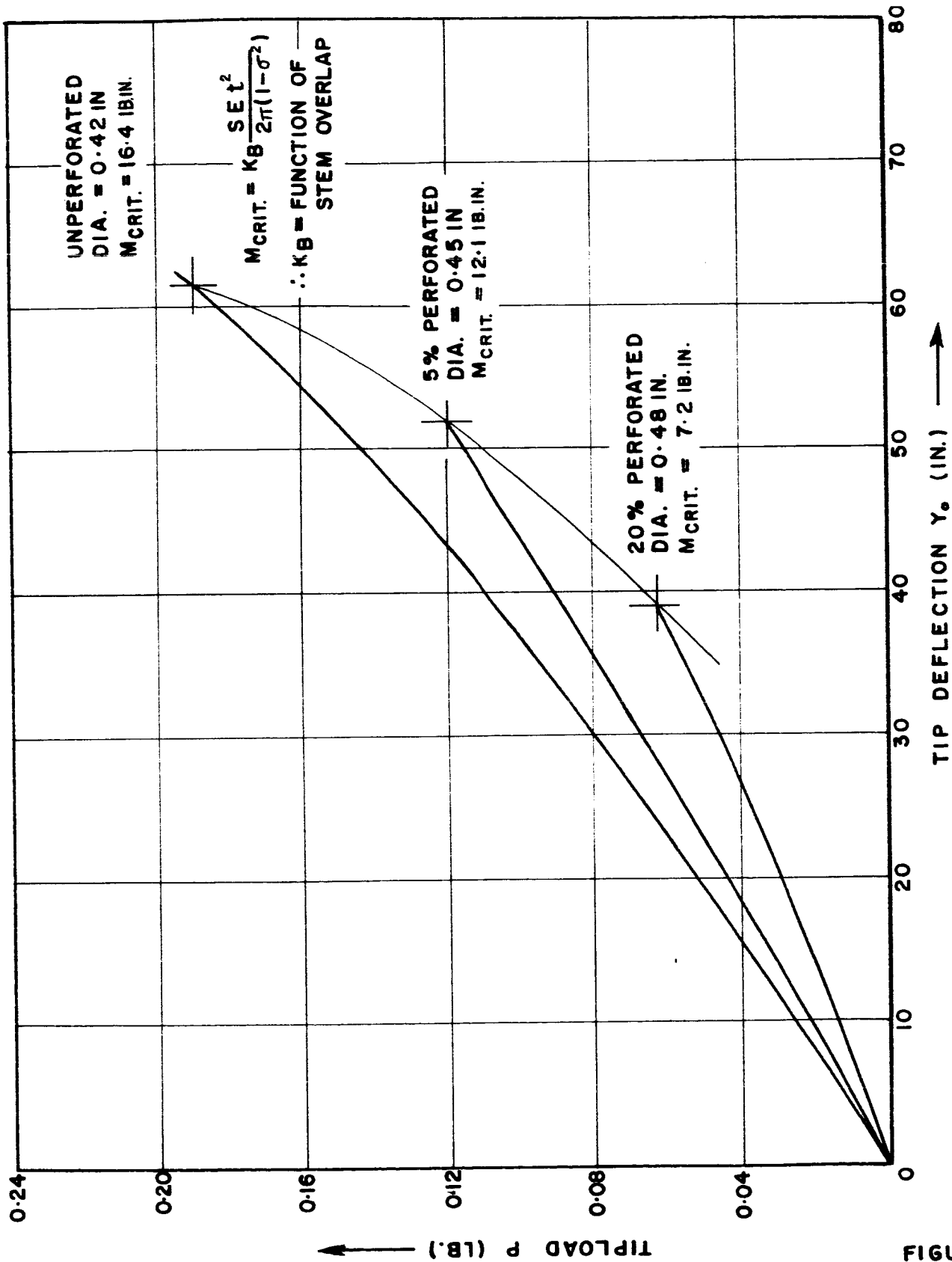


ROOT FAILURES OF PERFORATED STEMS

Figure 28



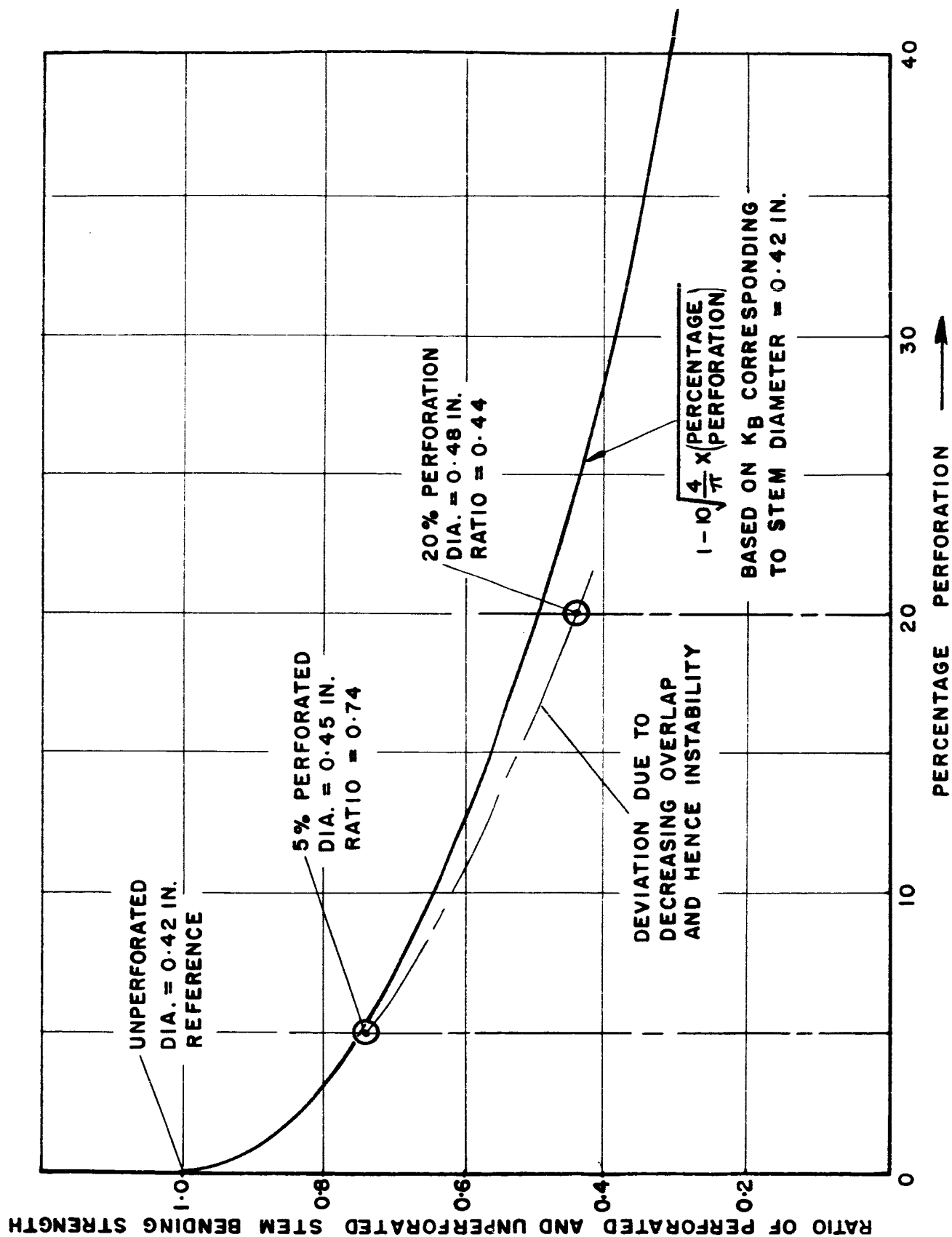
DHC - SP - R.168/2



TIPLOAD VS DEFLECTION OF PERFORATED STEMS



DHC-SP-R.168/2



**BENDING STRENGTH OF PERFORATED STEMS RELATIVE TO
THE UNPERFORATED STEM**

FIGURE 30

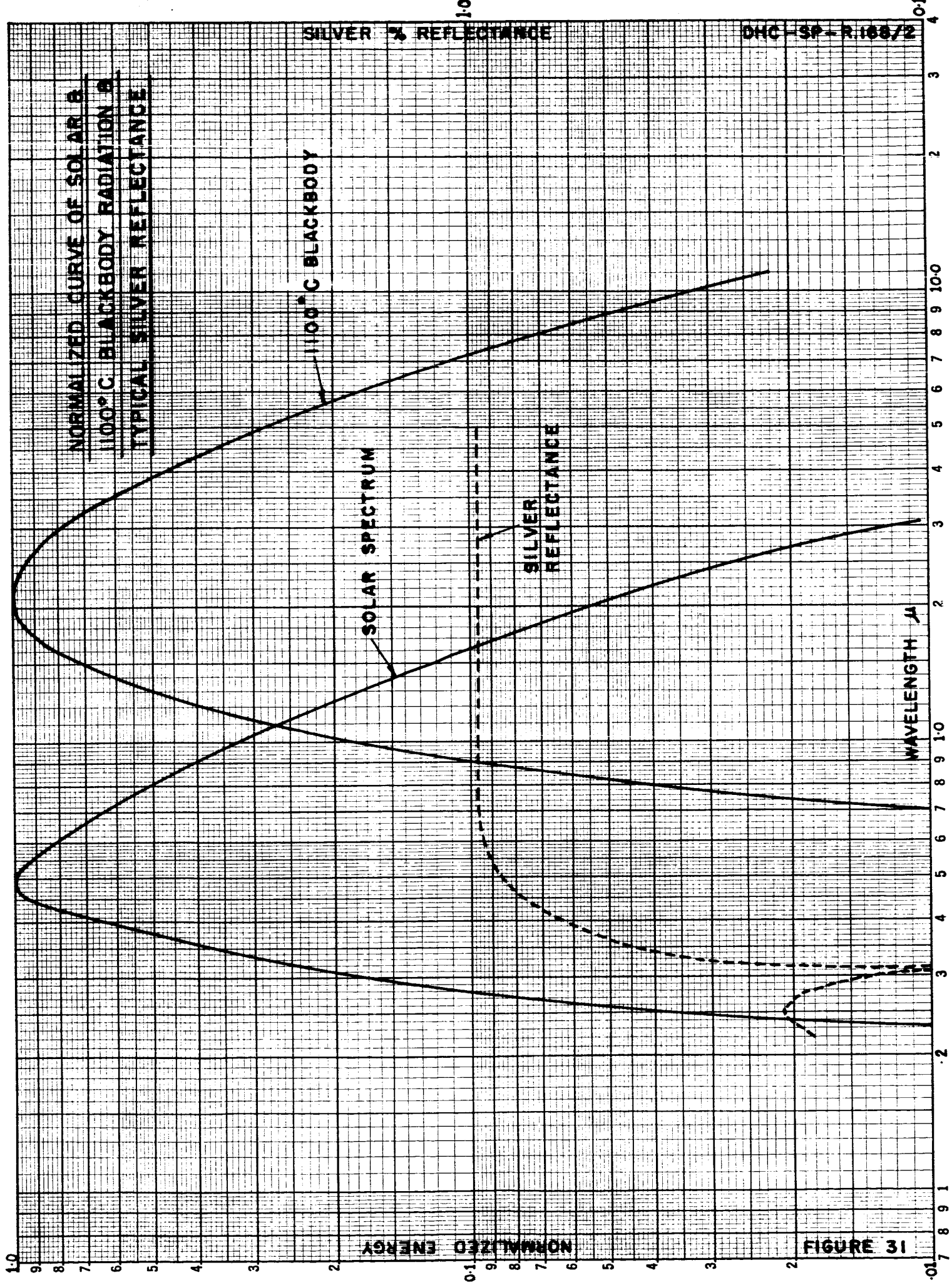


FIGURE 31