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COMPARISON OF PERFORMANCE OF PCM AND PFM TELEMETRY SYSTEMS

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ABSTRACT

A PFM telemetry system can be transformed into a PCM system of equivalent information capacity by a series of steps, outlined in this paper. Since each of these systems can be reduced to a number of discrete digital channels, a quantitative comparison of the systems is obtained by adding to the ratio of the relative powers required to provide equal signal strengths per channel (expressed in db) the relative powers required to provide equivalent word error rates. Since equivalent channel detectors are assumed, this method is independent of the detecting device as long as the systems are uncoded, that is, when word detection is digit by digit.

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INTRODUCTION

This paper compares the performance of PFM (Reference 1) and PCM (Reference 2) telemetry systems in terms of their basic parameters of transmission rate, bandwidth, error rate (or accuracy), and power. Difficulties arise because the systems differ in nature: the analog performance of PFM falls off gradually below some signal-to-noise threshold above which system accuracy is satisfied, whereas digital systems are usually designed to operate considerably above their relatively abrupt threshold and performance interest extends to the region where error rates are very low, since digital systems are capable of arbitrarily high precision by extending the word length. Because the percentage of digital information being sent over PFM systems is increasing, it has become more urgent that these systems be compared, and, in fact, this comparison is facilitated by the comparison of efficiency in transmitting the same data.

Since PCM is a standard type of telemetry it will not be further described here. PFM, however, is not in general use; its use has been confined almost entirely to the Goddard Space Flight Center and a few related NASA and foreign space projects. Goddard Space Flight Center use, however, is quite extensive, so the discussion in this paper is appropriate. Following the usual nomenclature PFM might better be described as PAM/FM/PM or FM. The prototype of the several variations which have been used consists of a multiplexer which successively samples 16 data lines; its output frequency-modulates an oscillator over a continuous, nominal range from 5 to 15 kc. These audio tones further frequency- or phase-modulate a radio-frequency carrier. Because of the particular circumstances of use in small, low-power, space experimentation, most projects using PFM have been designed to operate near their threshold of detectability, and, in order to capitalize on its characteristics, relatively sophisticated signal-processing equipment is utilized (Reference 3). One part of this equipment consists of a bank of contiguous filters (comb filter) which has the effect of quantizing the frequency bank into 100 discrete channels. Thus, in effect, the PFM telemetry system may be considered an ensemble of narrow-band channels, and this viewpoint is adopted in the following discussion.

PARAMETERS OF INTEREST

The relative efficiency of two telemetry systems can be expressed in terms of relative powers required to send signals at the same data rate over systems made up of comparable components, (that is, numbers of communication channels of discrete bandwidths), and also in terms of the relative error rates. The relative frequency spectra of the two systems under investigation are discussed herein, but equipment complexity of the two systems is not, although it is recognized that this may also be an important factor in the choice of a telemetry system.

The performance of analog communications systems can also be measured in terms of percentage accuracy as a function of signal-to-noise ratio. On those occasions when it is desired to transmit a number of data channels on an analog telemeter, they are frequently multiplexed in time; this process requires some kind of sampling technique, which is usually also the first step in performing an analog-to-digital conversion.

The task of the satellite designer in choosing a telemetry system is thus complicated by the fact that the performance parameters are sometimes different, and thus the choice is between apples and oranges, as it were. The purpose of the section which follows is to show how, by a series of transformations which preserve the essential constant parameter of information rate, the relative power of PFM and PCM systems can be compared.

We can compare power efficiency by the ratio of powers (expressed in db) required to deliver a given signal through the systems at a given information rate. The relative error rates can also be compared at a given input signal-to-noise ratio (also expressed in db). The relative system efficiency with respect to power and error rate will then be the algebraic sum of these power ratios, scaled from the charts provided, at the region of interest. The final choice of a telemetry system will include considerations of bandwidth required, difficulty of implementing the circuitry, the availability of ground equipment, the proportion of analog and digital data to be telemetered, and subjective factors based on the experience of the designers.

POWER EFFICIENCY COMPARISONS

To compare the power efficiency of the two telemetry systems in transmitting digital information at equal rates, we can start with a PFM system transmitting n bits per sample. This requires r channels where $r = 2^n$ and, by rule, a signal is sent over only one channel each sample time. This might be termed a parallel r-ary system.

This can be equated to a hypothetical telemeter in which there are n parallel binary channels, no restriction being placed on the number of channels which can contain signals simultaneously. With random data, an average of half of the channels would contain "ones" per sample period (assuming no power to send a zero); therefore the amount of power required of this system is $n/2$ times the power of the PFM system. The number of frequency bandwidths occupied by these signals is $n/r = n/2^n$ that occupied by the original PFM system. We will call this a parallel binary system, and the power demands of such systems with respect to those of PFM are shown in Figure 1.

The next step is to transform the parallel binary system (simultaneous signals on separate frequencies) to a serial binary system (time sequential signals on the same frequency) in which each signal is 1/n of the original sampling duration and n times its original power. This is conservative of the energy per bit, or of system power. The system we now have is serial PCM using binary digits. The frequency spectrum per channel has been multiplied by n in this transformation since the samples are now shorter by a factor of n, but since the number of channels has again been reduced (to one channel) by the factor n, the spectrum occupied by the serial system is that of a parallel system. These relationships are depicted in Figure 2 for 3- and 4-bit words (or samples) and tabulated in Table 1 for 1-, 2-, 3-, 4-, and 7-bit words. In the case of 1-bit samples, PFM degenerates into frequency shift keying (FSK). PFM was used to send 3-bit (8-level) samples in the S-3, Ariel, and IMP series, and 4-bit digital information will be sent exclusively by PFM on the IMP D and E series. Seven-bit digital or analog signals are used as a basis for comparison for sending 1 percent analog information on both systems.

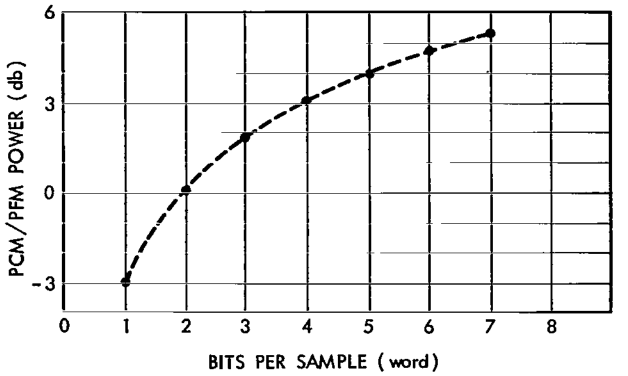


Figure 1—Relative power efficiency.

Table 1

PFM/PCM Comparisons of Power Efficiency.

Relationship	Bits/sample or bits/word, n: 1	2	3	4	7
PFM					
Number of PFM levels, r	2	4	8	16	128
Energy (or power)/ sample (or word)	1	1	1	1	1
Energy/bit	1	1/2	1/3	1/4	1/7
Transmission gain, db	0	3	4.8	6	8.45
PCM					
Parallel binary number of levels	1	2	3	4	7
Average energy/word	1	2	3	4	7
Serial binary (one level), number of bits	1	2	3	4	7
Energy/bit	1/2	1/2	1/2	1/2	1/2
Transmission gain, db	3	3	3	3	3
$\frac{\text{Noncoherent PFM}}{\text{Noncoherent PCM}}$, -3db	-3	0	1.8	3	5.45
$\frac{\text{Noncoherent PFM}}{\text{Coherent PCM}}$, -6db	-6	-3	-1.2	0	2.45

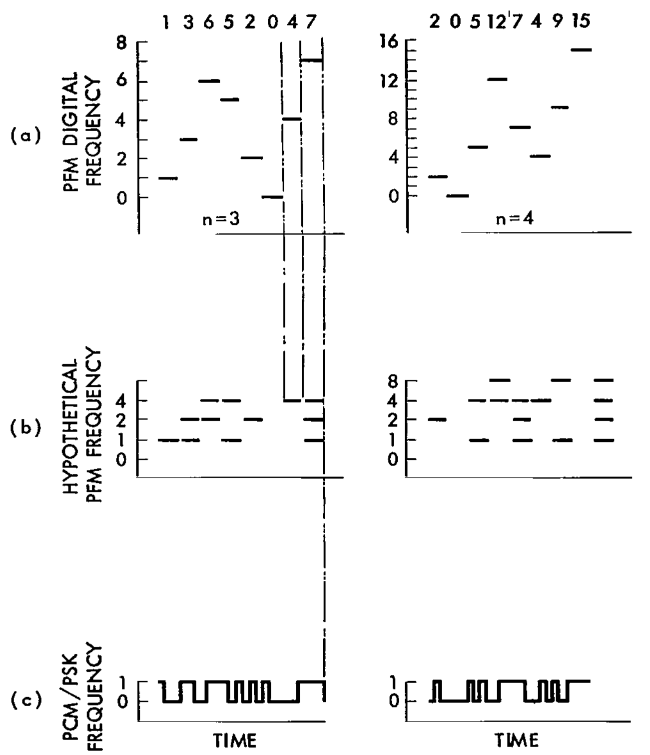
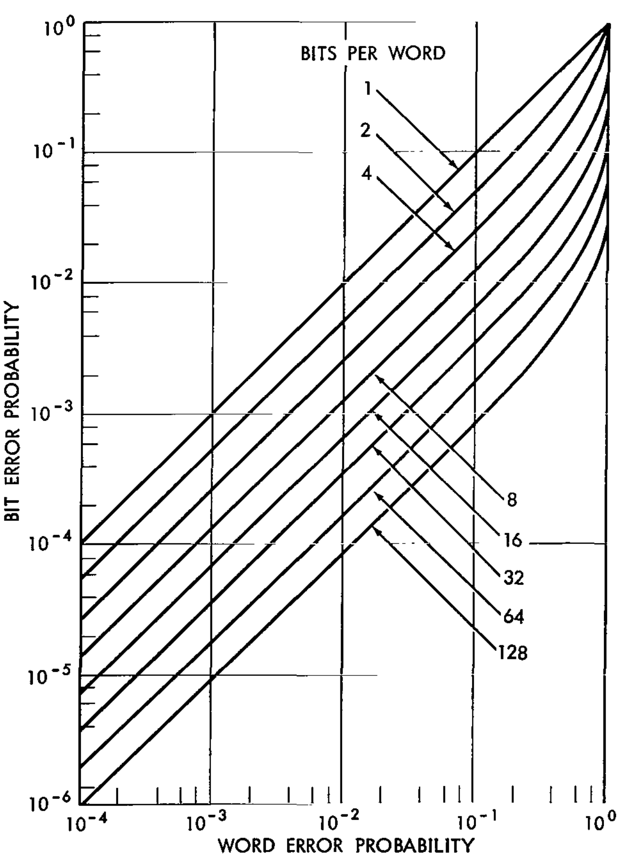


Figure 2—Conversion of PFM to PCM: (a) Parallel r-ary, (b) Parallel n, (c) Serial binary.

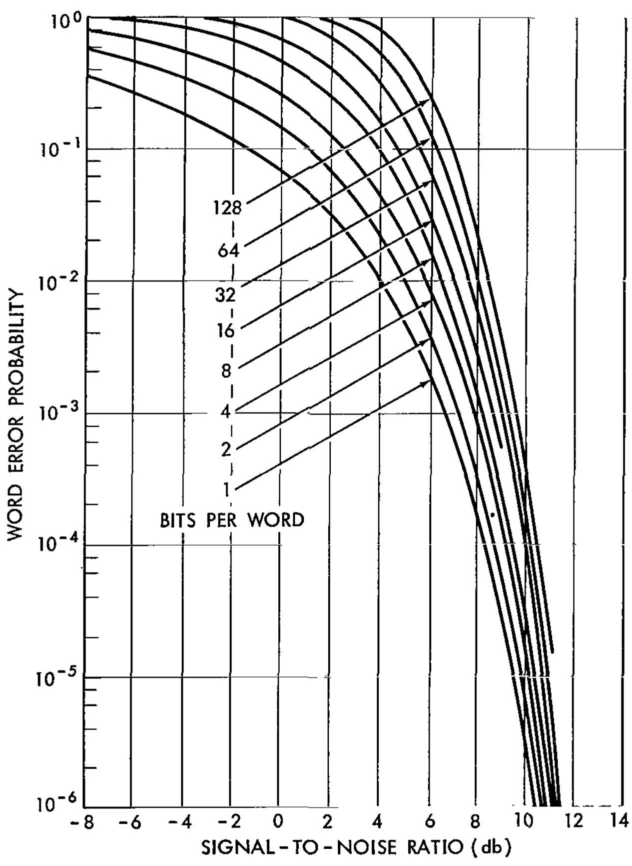
ERROR RATE COMPARISONS

PCM Performance

It has become conventional to express the performance of digital communications systems in terms of error rate versus signal-to-noise ratio, or, even more generally, bit error rate p_e versus energy per bit divided by noise power per unit bandwidth. These terms are aptly applied to a series of binary digits on a single channel, singly or grouped in words of n digits db word, coded or uncoded, the probability of detecting a digit correctly p_c and the probability of a digit error p_e being mutually exclusive and totaling 1. The probability P_c of correctly receiving a word of n bits is $P_c = p_c^n$ (the individual digit probability being assumed independent). Experimentally, digit (and word) errors are more easily measured than successes so this can be changed to $P_c = (1-p_e)^n$; then if we wish probability of word error, $P_e = 1-(1-p_e)^n$.* The probabilities of error for words of lengths up to 128 digits have been plotted against probability of bit error in Figure 3(a) and against signal-to-noise ratio in Figure 3(b).



(a) Word error probability vs. bit error probability.



(b) Word error probability vs. signal-to-noise ratio.

Figure 3—Word error probability for digital words (uncoded) of various lengths.

*This form is often preferred because it facilitates computation of conditional probabilities of long digital words at low error rates. For example, $(0.9999989)^{13}$ can easily be found by transforming it to the form $(1-0.0000011)^{13}$; in the binomial expansion of this difference we choose the first two terms, an approximation which holds as long as the error rate is small, in which case the higher order terms become negligible and the expression reduces to $1-.0000011 \times 13 = 1-0.0000143 = 0.999857$.

PFM Performance

In the case where we wish to express the performance of PFM in conveying digital information, we have a more complex situation. In pulse frequency modulation, according to present practice, digital numbers are sent 3 bits per sample by dividing the 128 channels into 8 groups representing 8 digital levels, a form of parallel octenary (octal). In any such system the number of levels required is 2^n where n represents the number of bits per sample. This, of course, is a way to increase the system capacity; by increasing the bandwidth (number of channels) per sample, the digit represented by the sample can be raised to a higher radix. The noise bandwidth of the system, however, remains substantially the same as the bandwidth of an individual channel, hence the virtue of this system.

In measuring the performance of this system, we assume that if there are 2^n discrete levels there must be 2^n channels. Each of these channels can be looked on as a separate communication channel of bandwidth b capable of transmitting $b/2^{n+1}$ bits per second. (Since the average number of bits per channel is actually $b/2 \cdot 1/2^n = b/2^{n+1}$ we see that, in terms of bandwidth utilization, this system is very inefficient, a matter of small concern when there are few satellites, but a matter which may become of greater importance as the demand for satellite telemetry increases.)

A distinguishing property of a parallel r-ary system is that, in contrast with parallel or serial binary systems, the correct number of pulses per digit is one, but the probability of a pulse and the absence of a pulse are no longer mutually exclusive except as pertain to the individual channel. For example, in an 8-level system sending 3 bits of information, where the probability of error per channel is 10^{-4}, the total probability of an error is 8×10^{-4}. Thus the probability of success in sending a five-digit word on an 8-level system is $P_c{}^8$, while the probability of error is $5 \times 8 p_e$. Since there are more ways to be incorrect than there are to be correct (more than one level can be energized) we must revise our criterion of success to exclude these ambiguous replies; therefore all pulses sent must be received, and no extra pulses shall be received. Since the probability of missing a pulse equals the probability of receiving an extra pulse on a given channel (Reference 4), this is the conditional probability that

$$P_c{}^8 \cdot P_e{}^{(40-8)} = P^8 \cdot P^{32} = P^{40}$$

for the 5-digit, 8-level system. This expression shows that PFM in transmitting digital information shares the weakness of digital systems in sending digital words; in this case an idealized PFM system sending 8 bits has the error rate of an uncoded serial binary word of 40 bits.

The approach taken in evaluating comparative error rates of PFM and PCM telemeters does not elucidate all of the features of these systems. For example, the analysis of PFM based on the comb-filter detector implies that all channels are independent and that all errors have equal weight. This is not true of the comb filter presently used, since the filters have responses to signals outside their nominal bandpass, and furthermore, the spectra of the signals contain energy over an interval greater than one channel. Thus as signal level in a channel deteriorates, its output begins to fluctuate as responses appear in adjacent channels, until finally response of all channels becomes random (since automatic gain control lowers the threshold) (see Figure 4 (a)). The discrete steps occur because the comb filter quantizes the frequency into 100 levels corresponding to inherent limitations on the accuracy of the telemetry oscillator in the spacecraft. At high signal-to-noise ratios, pulse-counting discriminators of analog or digital varieties are capable of achieving resolutions higher than this 1 percent (References 5 and 6). With these a curve resembling that in Figure 4 (b) results.

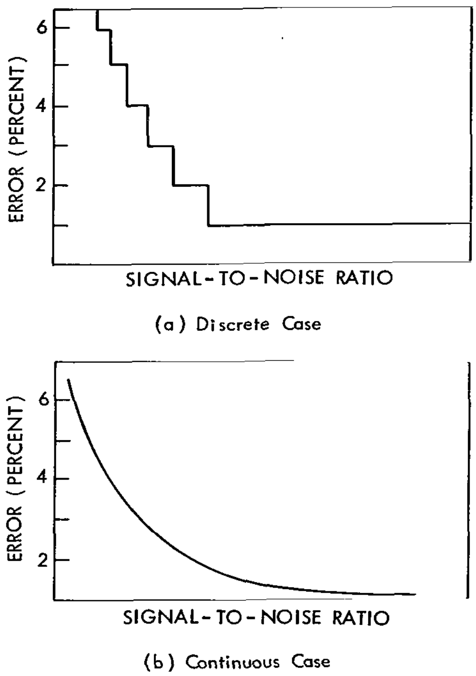


Figure 4—Performance of channel as function of signal strength.

When a digital system is used to transmit analog signals, there is quantization noise and a maximum resolution established by the length of the digital word which is substantially independent of S/N ratio. However, errors due to random sources may occur in any part of a digital word from the most to the least significant bit. Digital systems are usually operated well above their thresholds where errors are vanishingly small; however, there is a region near threshold where usefulness of data becomes sensitive to the length of the digital word. This effect has often been noticed and has led to the observation that digital systems have an abrupt threshold. The same effect has been noted on PFM telemeters when digital data is being sent* and in this analysis of PCM and PFM with comb filter detectors. It is seen that individual channels are considered identical and there are therefore no inherent qualitative differences in the systems.

A further difference remains. When PFM is used to transmit analog and digital signals near threshold, errors tend to decrease the accuracy until finally, well below threshold, the errors are random. A sample may represent several bits, and the value of the sample represents all of them, so errors affect the least significant digits first. In most binary PCM systems, probability of error is assumed to be independent of bit position (for nonimpulsive noise), and an error in any bit of a word is presumed to vitiate the value of that word. For the comparative purpose of this study, this is also presumed to be the case when digital information is being sent via PFM, since most digital words will consist of more than one sample.

A large number of curves purporting to give what is effectively the number of errors per bit versus signal-to-noise ratio have been published. Although these stem from the same theory, they vary according to the assumed spectra of the signal and the noise and choice of filter characteristics. This variation has occurred because of the difficulties arising in obtaining agreement between experimental measurements and theoretical models. The choice of a particular curve used in this discussion does not reflect a preference by the author, since, in a comparison of systems, relative

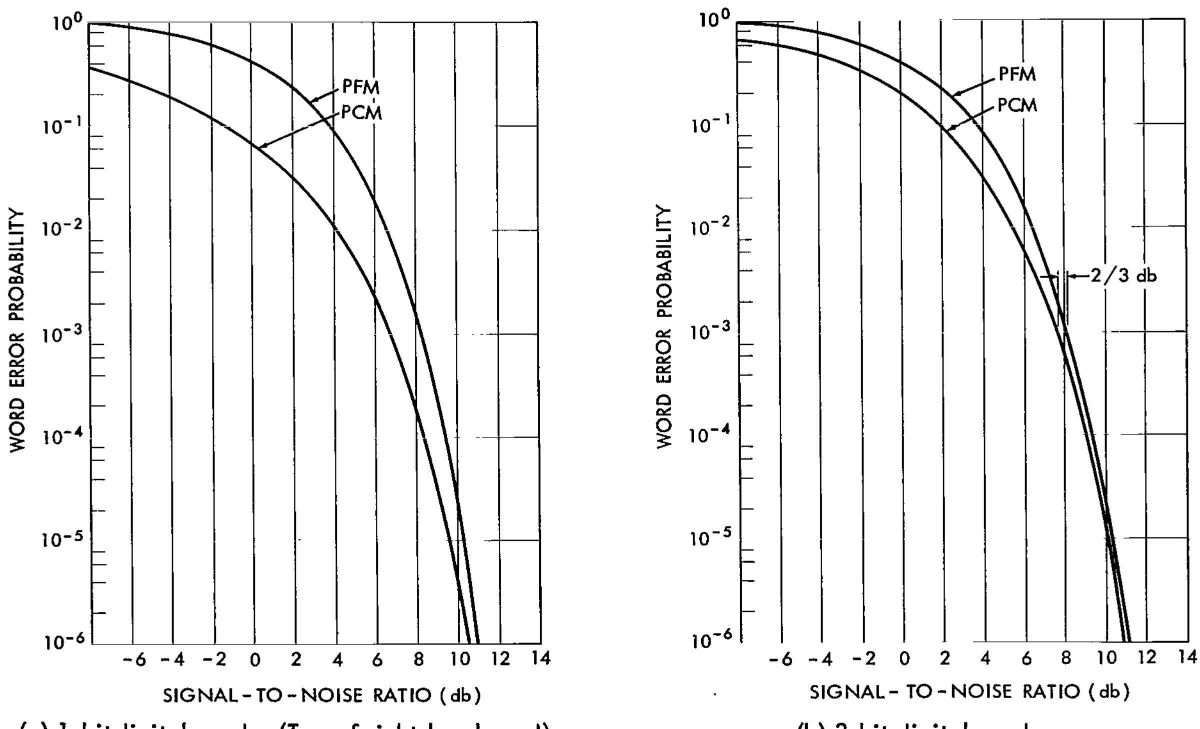
*Notably in an experiment on Explorer XII 1961 ν1 in which the contents of a 15-bit register were telemetered at 3 bits/sample and a further computer programming restriction gave the effect of a 45-bit word.

judgments are preserved irrespective of their common base. This is believed to be one of the main values of this unified approach to PFM and PCM system comparisons.

Following this precept, Figure 3 (a) has been constructed in which a family of curves relate the probabilities of word errors to bit errors for digital words of length up to 128. Values from Figure 3 (a) were then used to construct Figure 3 (b) which relates word error probability to the signal-to-noise ratio in which the base is the bit error probability which has been theoretically derived.

A series of charts have been constructed which compare error rates of 8-level PFM against uncoded PCM for 1-bit, 3-bit, 9-bit, and 15-bit digital words (Figures 5 (a), 5 (b), 5 (c), and 5 (d)). These intervals are chosen because our 8-level system of reference sends 3 bits per sample, and any other digital word length would entail a less favorable comparison to PFM since some channel capacity would be wasted. The charts use as a basis the theoretical bit error rate of uncoded PCM, the error rate of the words of various bit lengths having been determined from these basic values using the formulations of this text.

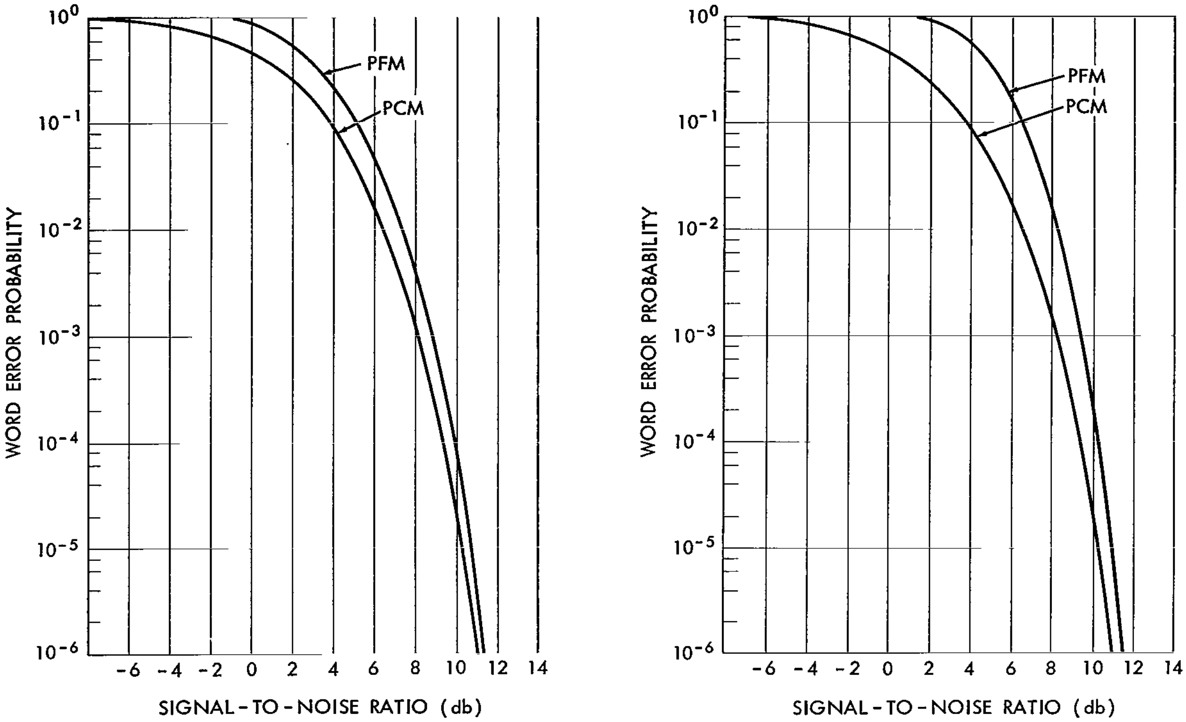
A further consideration which must be taken into account in calculating the channel capacity of existing (operational) PFM digital systems is the fact that, because of oscillator instability, the 8 digital levels are not discrete (on a 100-cycle PFM channel basis). In order to quantize them with a small chance of missing samples due to oscillator drift, the 128 individual channels of the comb filter used in the reduction are strapped together with logical "OR" circuits, with 5 channels being used for octal zero; 5 for octal 1; 7 for octal 2; 7 for octal 3; 9 for octal 4; 10 for octal 5; 14 for octal 6; and 16 for octal 7.



(a) 1-bit digital words. (Two of eight levels used).

(b) 3-bit digital words.

Figure 5—Comparisons of error rates of 8-level PFM and uncoded PCM data.



(c) 9-bit digital words.

(d) 15-bit digital words.

Figure 5 (Continued)—Comparisons of error rates of 8-level PFM and uncoded PCM data.

Thus a total of 73 channels are used to convey 8 levels of information, or approximately 9 channels per level (digital channel). This increases the probability of error in sending a digital number by a factor of 9 and reflects an increasing disparity in performance of PFM compared with a digital PCM system. It is hoped that this handicap will be eliminated in future quantized PFM telemetry systems.

USE OF FIGURES IN COMPARING SYSTEMS

We now have available the information necessary for comparing a PFM telemetry system with an equivalent PCM system. From Figure 1 or from Table 1 we can assume a PFM transmission gain, in all cases favorable to PFM, except when 1 or 2 bit samples are sent on an 8-level system. (A case of this occurred on an early satellite.) From this we can subtract a PCM transmission gain of 3 db because of the statistical properties of the signal, i.e., pulses are present in a channel approximately one-half of the time; and a further 3 db can be subtracted if coherent detection of PCM is available. These sums are tabulated in the last two rows of the table for comparison of power efficiencies of PFM to noncoherent and coherent PCM.

To the figure (in db) obtained above must be added the relative powers required (in db) to provide equal error rates at some chosen level of error rate. Some representative cases are plotted in Figures 5, 6, and 7. For example, in Figure 5 (b) we see that a PCM system having 3-bit words requires approximately 2/3 db less power to provide an error rate of 10^{-3} than an equivalent 8-level

PFM system requires. By subtracting this from the appropriate figure in Table 1, we get an overall ratio of 1.8 - 0.66 = 1.1 db for noncoherent PCM detection and -1.2 - 0.66 = 1.8 db (favorable to PCM) for coherent PCM detection

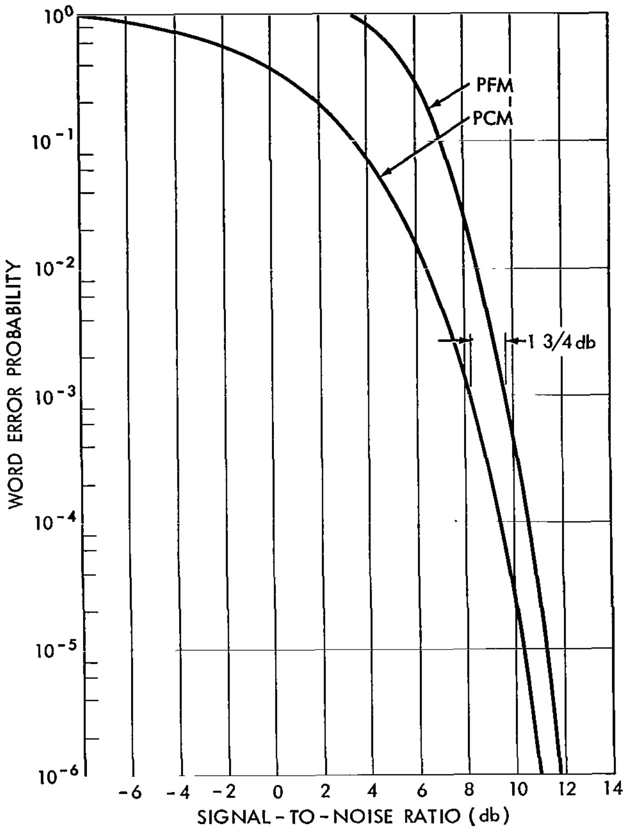


Figure 6—Comparison of error rates of 128-level PFM and uncoded PCM data, 7-bit digital words.

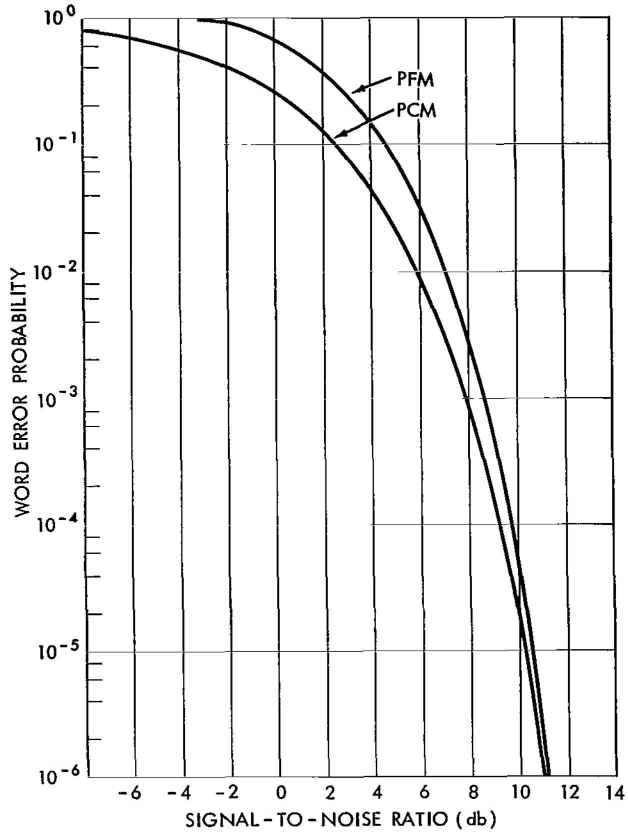


Figure 7—Comparison of error rates of 16-level PFM and uncoded PCM data, 4-bit digital words.

CONCLUSIONS

When a scientific satellite is being designed, the choice of the telemeter frequently depends to some extent on the nature and quantity of the data to be sent. Analog telemetry systems such as FM/FM and PFM are more easily adapted to analog signals, whereas digital signals are more easily handled by digital systems such as Pulse Code Modulation (PCM). As there is frequently a mixture of analog and digital signals, some analog-to-digital or digital-to-analog conversions must be made.

According to this study, the performance of past PFM systems as media for transmitting digital numbers has suffered slightly in comparison with uncoded binary PCM. This stems from the fact that a given PFM system is tantamount to a PCM of a shorter word length.

According to Table 1, PFM enjoys a 5.45-db power advantage over noncoherent PCM when sending 1 percent analog information, the type of service for which it is well adapted (this advantage falls to 2.45 db when compared with coherent PCM). When we subtract from this the $1\frac{3}{4}$ db power difference necessary to provide equal error rates of 10^{-3} obtained from Figure 6, these diminish to a PFM net advantage of $3\frac{3}{4}$ db and 0.7 db, respectively. In sending digital information with 3- and 4-bit samples PFM is closely comparable with PCM (PM), and even PCM (AM) is roughly comparable in terms of required power, with a slight advantage going to PCM.

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