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The Substitution of a Semi-infinite, Homogeneous Dielectric Medium for a Plasma Slab

S. Gotkis

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Department of ELECTRICAL ENGINEERING

THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION
Columbus, Ohio

REPORT 1691-19

REPORT
by
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
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Sponsor	National Aeronautics and Space Administration Office of Grants and Research Contracts Washington, D. C. 20546
Grant Number	NsG-448
Investigation of	Spacecraft Antenna Problems
Subject of Report	The Substitution of a Semi-infinite, Homogeneous Dielectric Medium for a Plasma Slab
Submitted by	S. Gotkis Antenna Laboratory Department of Electrical Engineering
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ABSTRACT

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The substitution of a homogeneous, semi-infinite dielectric for a finite plasma slab is studied as a means of simplifying the analyses of the impedance of an aperture antenna radiating into an inhomogeneous plasma medium. Such a substitution could be made if the reflection coefficients of the equivalent medium, as a function of angle of incidence, corresponded to the reflection coefficients of the plasma slab. The dielectric constant of the equivalent medium is found by matching reflection coefficients at some particular angle of incidence (usually 0°). The range of angular correspondence is narrow (less than 5°) for cases in which the dielectric constant dips to near or below zero. In other cases fairly wide ranges of equivalence occur.

TABLE OF CONTENTS

THE SUBSTITUTION OF A SEMI-INFINITE, HOMOGENEOUS DIELECTRIC MEDIUM FOR A PLASMA SLAB

INTRODUCTION

It is generally quite difficult to determine the impedance of an aperture antenna in the presence of an inhomogeneous layered plasma medium. One of the major difficulties is treating the inhomogeneous plasma itself. The problem would be greatly simplified if the inhomogeneous plasma could be approximated by some homogeneous, semi-infinite dielectric. The problem of an aperture antenna radiating into a semi-infinite, homogeneous medium is much easier to solve. For example, the case of an open-ended rectangular waveguide in which the interface of the semi-infinite medium is coincident with the guide aperture has been analyzed.[1]

The purpose of this report is to describe a manner in which the homogeneous, semi-infinite dielectric can be substituted for the inhomogeneous plasma. If the plane wave reflection coefficients of the homogeneous half-space are equivalent, as a function of the incidence angle, to the reflection coefficients of the original inhomogeneous plasma, the homogeneous dielectric can be substituted over the range of equivalence. The validity of this approximation may be determined by first calculating the plane wave reflection coefficients as a function of angle of incidence for the inhomogeneous plasma layer. Then at some arbitrary incidence angle (usually 0°) the reflection coefficient is matched to that of a homogeneous dielectric medium with the appropriate dielectric constant.

Once the value of the dielectric constant of the semi-infinite medium has been determined so that the reflection coefficients of the two media match for that incidence, the reflection coefficients are calculated for the homogeneous medium. For cases in which these reflection coefficients approximate well those of the inhomogeneous plasma, the homogeneous medium may be substituted for the plasma.

It is anticipated that the semi-infinite dielectric can be so chosen that reflection coefficients will coincide over a wide enough angular range to be of practical use. This would be useful for an antenna aperture somewhat removed from the face of the plasma layer. Such conditions exist during the re-entry of a space vehicle into the atmosphere. The gap between the antenna aperture and the plasma sheath is often on the order of a wavelength.

DIELECTRIC SLABS

To initially calculate the plane wave reflection coefficients of inhomogeneous media, computer programs were written for both parallel and perpendicular polarizations. The calculations were based on a technique by Richmond[2] where the fields are computed at intervals in the medium by a recursion process. Then the reflection coefficients are found from the computed fields. The programs are shown in Appendix A, Figs. 32 and 33.

Use of these programs revealed several limitations. Care must be taken to keep the distance between points of calculation as small as possible for the recursion technique to be effective. Also variation in the dielectric constant from point to point must not be too great. When the variations in the dielectric constant are large the program calculates transmission coefficients greater than unity. The programs are ineffective for cases in which the relative dielectric constants, ϵ_r, are equal to zero or varied near zero. The formulation is also inaccurate for negative dielectric constants.

As a result another program based on a new technique by Richmond[3] was written. This program is based on the exact field solutions for a layered medium. Recursion techniques are used to solve for the exact fields and the reflection coefficients. Both cases, parallel and perpendicular polarization, are handled simultaneously. Two versions of this program were written. In one version calculations are made directly from dielectric constants which are read in for each layer. In a second version electron density and collision frequency profiles are introduced and the resultant complex dielectric constant is calculated and then used. The new programs are accurate in all regions, including the regions in which the original programs were limited. In Figs. 34 and 35 (Appendix A) the statement listings are given for both versions of the program. The results presented in this report were calculated by the new programs.

The equations for plane wave reflection from a semi-infinite medium are used to determine the properties of the equivalent medium. The dielectric constant of the equivalent semi-infinite medium is determined by matching its reflection coefficient with that of the original medium for some particular angle of incidence (usually 0°). Then the reflection coefficients of the two media are compared as a function of the incidence angle. If the two coefficients agree over a sufficiently wide range of incidence, the equivalent semi-infinite medium may be substituted for the original medium. The range of agreement may not have to be more than 20°-30° because of the corresponding pattern of the antenna whose impedance is to be determined.

The equations for plane wave reflection from a semi-infinite medium are given by

$$(1) \qquad R = \frac{\cos\theta - \sqrt{\epsilon_r - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon_r - \sin^2\theta}} \qquad \text{(perpendicular polarization)}$$

and

$$(2) \qquad R = -\frac{\epsilon_r \cos\theta - \sqrt{\epsilon_r - \sin^2\theta}}{\epsilon_r \cos\theta + \sqrt{\epsilon_r - \sin^2\theta}} \qquad \text{(parallel polarization)},$$

where ϵ_r is the relative dielectric constant and θ is the angle of incidence.

To demonstrate the validity of the equivalent medium technique several dielectric constant distributions were analyzed. In Figs. 1-5 comparisons are given of the reflections from dielectric slabs with uniform dielectric constants with those of the equivalent media. In each case the polarization of the electric field is perpendicular to the plane of incidence. The angle at which the reflection coefficients of the slab and equivalent medium are matched is 0° in each case. The slabs are of constant thickness, with kd = 2.0.

The dielectric constant for Fig. 1 is $\epsilon = -0.2$, with reflection ranging from 0.77 to 1.0. The equivalent medium has a low dielectric constant of 0.01716 and hence matches only near 0° because the critical angle of total internal reflection is reached almost immediately. The critical angle of the equivalent medium occurs for $\sin\theta_c = \sqrt{\epsilon_r}$ and is equal to 7.52° in Fig. 1.

Figure 2 is similar, with the dielectric constant of the uniform slab as $\epsilon = 0.1$. The value of the dielectric constant for the equivalent medium is 0.059 for which the critical angle of the equivalent medium occurs at 14.06°. Again the equivalent medium reflection coefficients correspond to those of the slab only in the neighborhood of 0°. Figure 3 is a plot with a slab dielectric constant of $\epsilon = 0.2$. The dielectric constant of the equivalent medium is 0.088 and the critical angle occurs at 17.25°. The equivalent medium reflection coefficients correspond to those of the slab for about 10°.

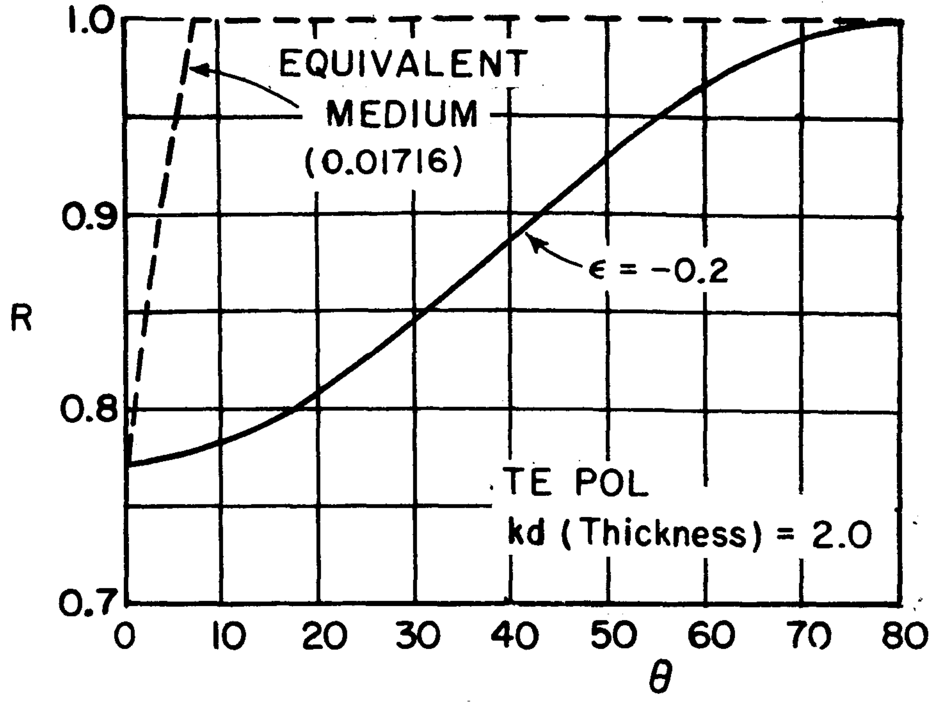

Fig. 1. Reflection vs incidence angle.

In Fig. 4 the uniform dielectric constant of the slab is $\epsilon = 0.5$. The value of the dielectric for the equivalent medium is $\epsilon = 0.265$ and the critical angle is 31°. The reflection coefficients of the slab and equivalent medium agree for about a 0°-20° range. In Fig. 5 the dielectric constant of the slab is $\epsilon = 0.8$ and that of the equivalent medium is $\epsilon = 0.64$. In this case the dielectric constant of the equivalent medium is near that of the slab and the critical angle does not occur until $\theta_c = 53.13°$. The respective reflection coefficients correspond closely over a range of 0°-40°. In Figs. 1-5 it is seen that as the value of the dielectric constant of the equivalent medium increases and the critical angle becomes greater, there is a greater range of reflection coefficient correspondence and the equivalent medium concept becomes more meaningful.

Figures 6-10 contain data for slabs of uniform dielectric constant with an incident plane wave E-field polarization parallel to the plane of incidence. The thicknesses of the slabs are again constant, with kd = 2.0. In Fig. 6 the dielectric constant of the slab is negative with $\epsilon = -0.2$. The reflection coefficients for this negative medium demonstrate no Brewster angle as might be expected for parallel polarization. The dielectric constant of the equivalent medium and the critical angle are the same as for the perpendicular case with

with $\epsilon = 0.01716$ and $\theta_c = 7.52°$. The respective reflection coefficients match only at 0°.

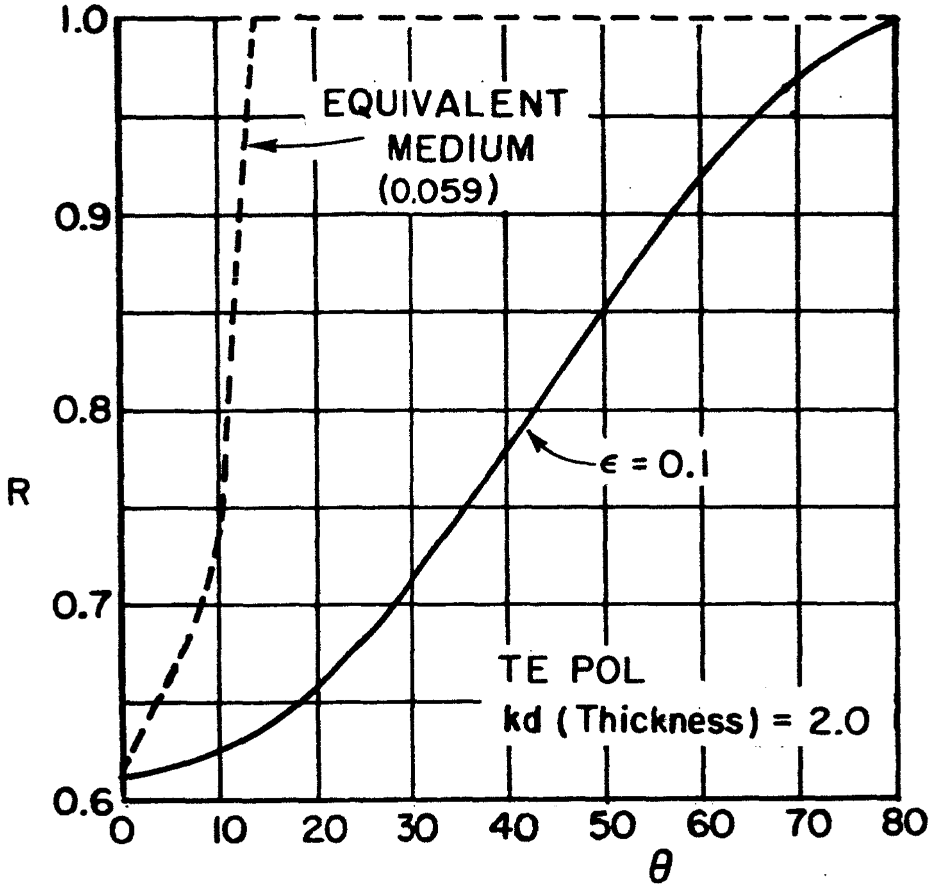

Fig. 2. Reflection vs incidence angle.

For Fig. 7 the slab dielectric constant is $\epsilon = 0.1$ and that of the equivalent medium is $\epsilon = 0.059$. The critical angle is 14.06° and reflection coefficients correspond to near that point. The reflection coefficient of the slab rapidly approaches unity after passing through the Brewster angle of zero reflection. In Fig. 8 the dielectric constants of the slab and equivalent medium are, respectively, $\epsilon = 0.2$ and $\epsilon = 0.088$. The critical angle is 17.25° and the reflection coefficients correspond over a range greater than 0°-10°. In Fig. 9 the dielectric constant of the slab is $\epsilon = 0.5$ and that of the equivalent medium is $\epsilon = 0.265$. The critical angle is 31° and there is excellent correspondence of reflection coefficients to that point. For Fig. 10 the respective dielectric constants are for the uniform slab $\epsilon = 0.8$, and for the equivalent medium $\epsilon = 0.64$. The critical angle occurs at $\theta_c = 53.13°$ and the reflection coefficients correspond over a 40° range.

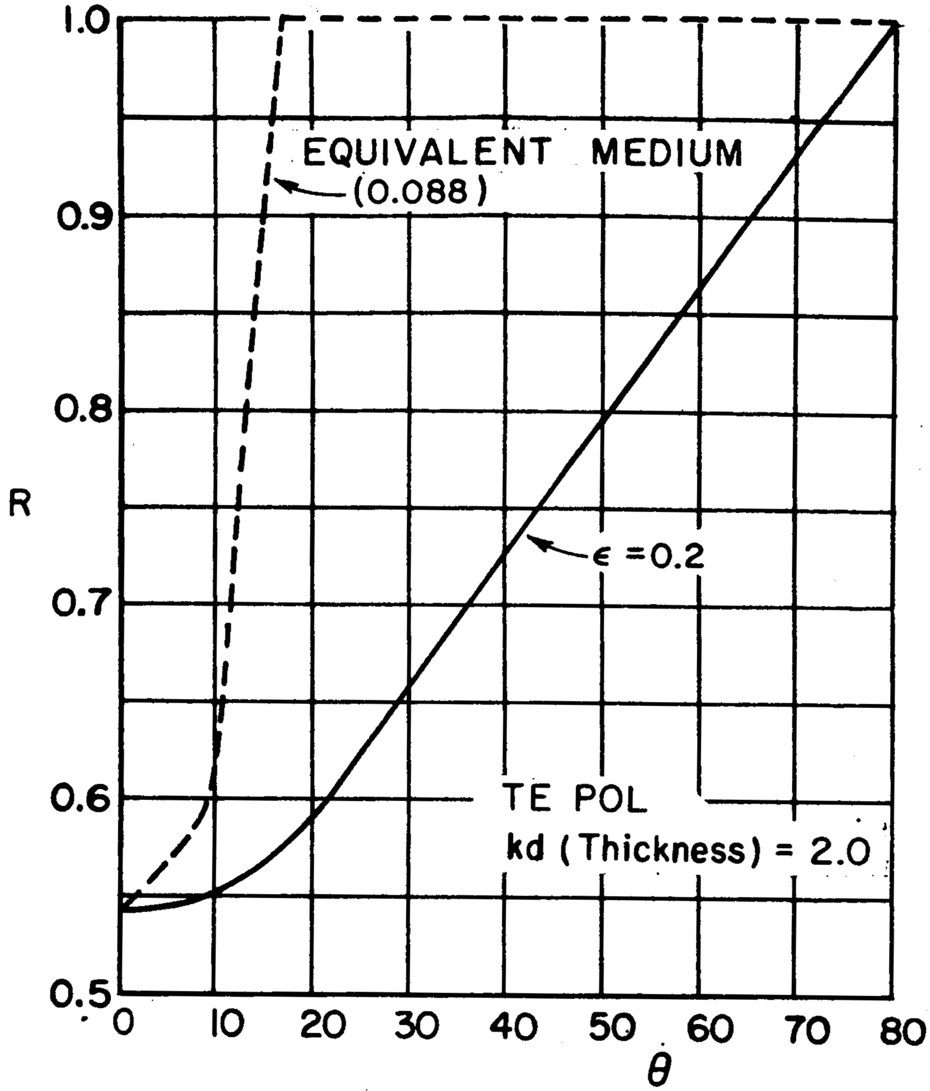

Fig. 3. Reflection vs incidence angle.

For either parallel or perpendicular polarizations, the critical angle becomes greater with increasing dielectric constant of the slab. For slabs with a low dielectric constant, such as $\epsilon = -0.2$ or $\epsilon = 0.1$, the critical angle is reached immediately as the angle of incidence varies from the normal. The equivalent medium concept is more useful where the dielectric constants are greater, and that of the equivalent medium is more nearly equal to that of the slab.

Thus far only the magnitudes of the reflection coefficients have been considered. The phase of the reflection coefficients of the uniform dielectric slab and the equivalent medium will be compared for two cases

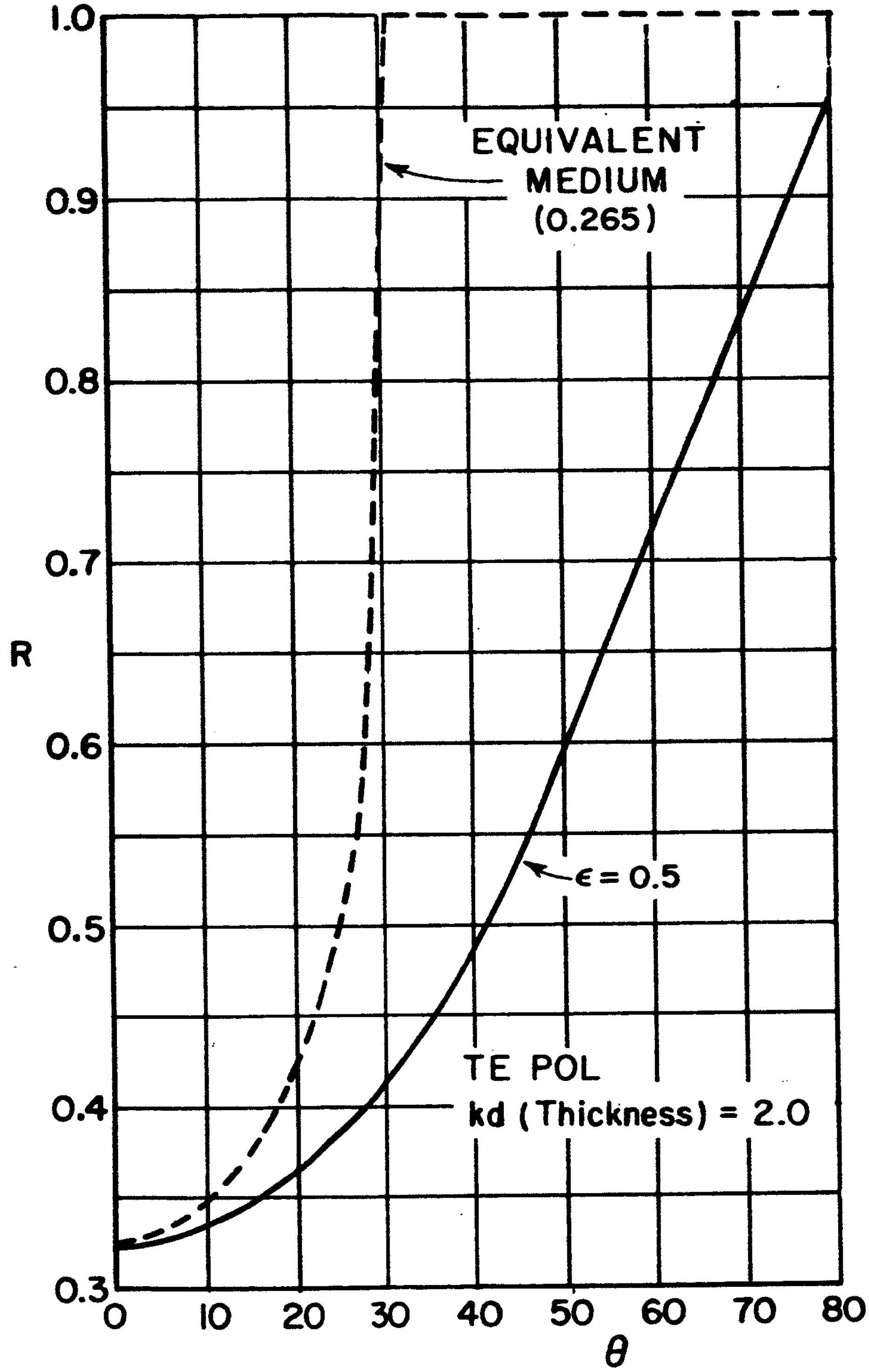

Fig. 4. Reflection vs incidence angle.

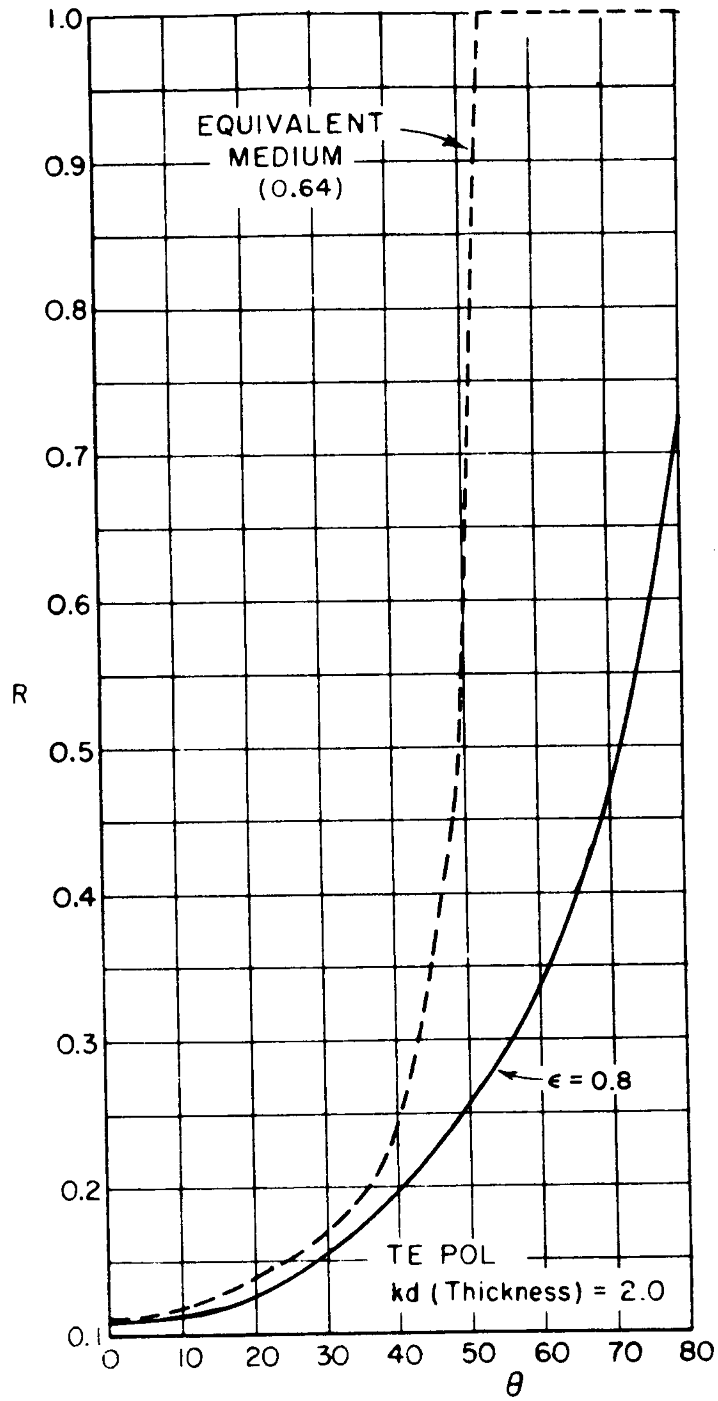

Fig. 5. Reflection vs incidence angle.

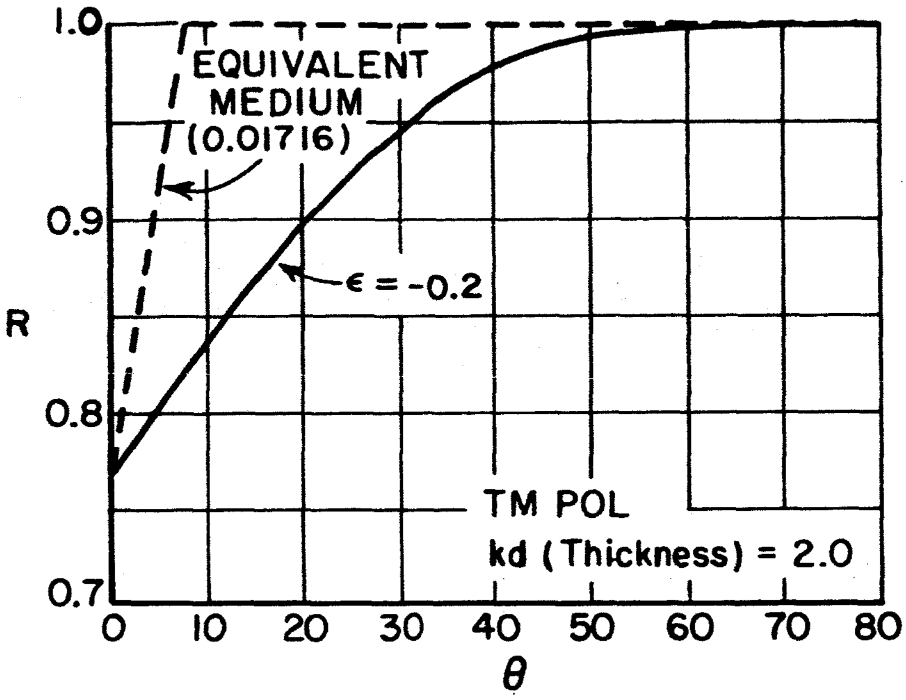

Fig. 6. Reflection vs incidence angle.

where the magnitudes correspond over a wide range. The two cases are for slabs of dielectric constant $\epsilon = 0.5$ and $\epsilon = 0.8$.

Figure 11 represents a slab dielectric constant of $\epsilon = 0.5$ and perpendicular polarization. It is seen that the relative phase angles of the reflection coefficients of the uniform slab and equivalent medium are very close for a range of approximately 30°. This correspondence of relative phase angles covers about the same range as the magnitude correspondence shown in Fig. 4. Figure 12 represents a uniform slab of $\epsilon = 0.5$ and parallel polarization. The relative phase angles of the reflection coefficients of the slab and equivalent medium are in agreement for a range of over 20°. This agrees with the reflection coefficient magnitude correspondence of Fig. 9.

Figures 13 and 14 show reflection coefficient phase angles for a uniform dielectric slab of $\epsilon = 0.8$ for perpendicular and parallel polarizations, respectively. In both cases the phase angle of the equivalent medium reflection coefficient is nearly equal to that of the slab for approximately 40°. As seen in Figs. 5 and 10 the magnitudes of the reflection coefficients agree over a similar range. Figures 11-14 demonstrate that where there is good correspondence in the reflection coefficient magnitudes of the slab and equivalent medium, the relative phase angles also agree.

Figures 15-19 give the reflection coefficient as a function of slab thickness for the dielectric slabs already discussed. The only case considered was that of parallel polarization. The reflection coefficients were analyzed for two angles of incidence, $\theta = 0°$ and $\theta = 30°$. The dielectric constants of the slabs are, respectively, $\epsilon = -0.2$, 0.1, 0.2, 0.5, and 0.8. As expected the reflection coefficients increase with increasing layer thickness. Also the reflection coefficients generally decrease with increasing dielectric constant.

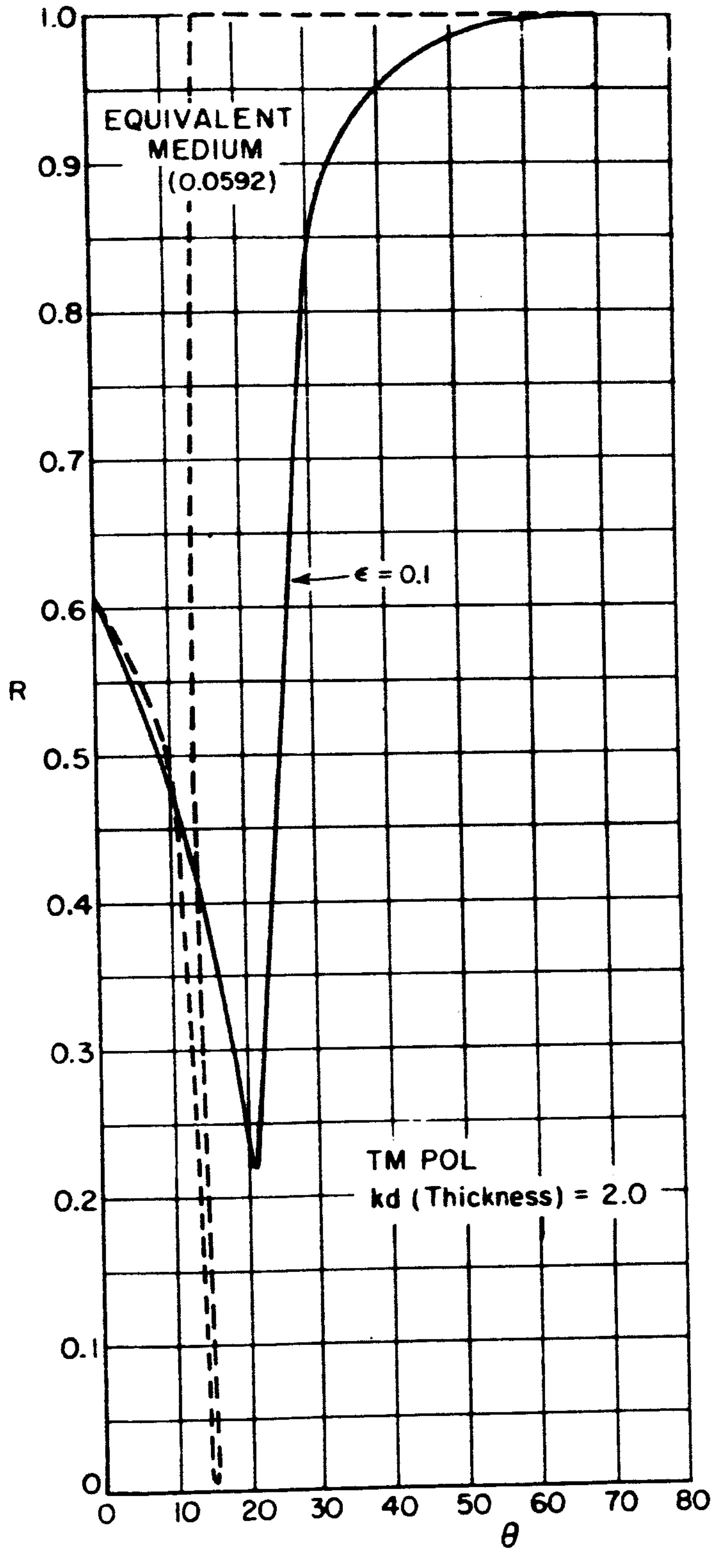

Fig. 7. Reflection vs incidence angle.

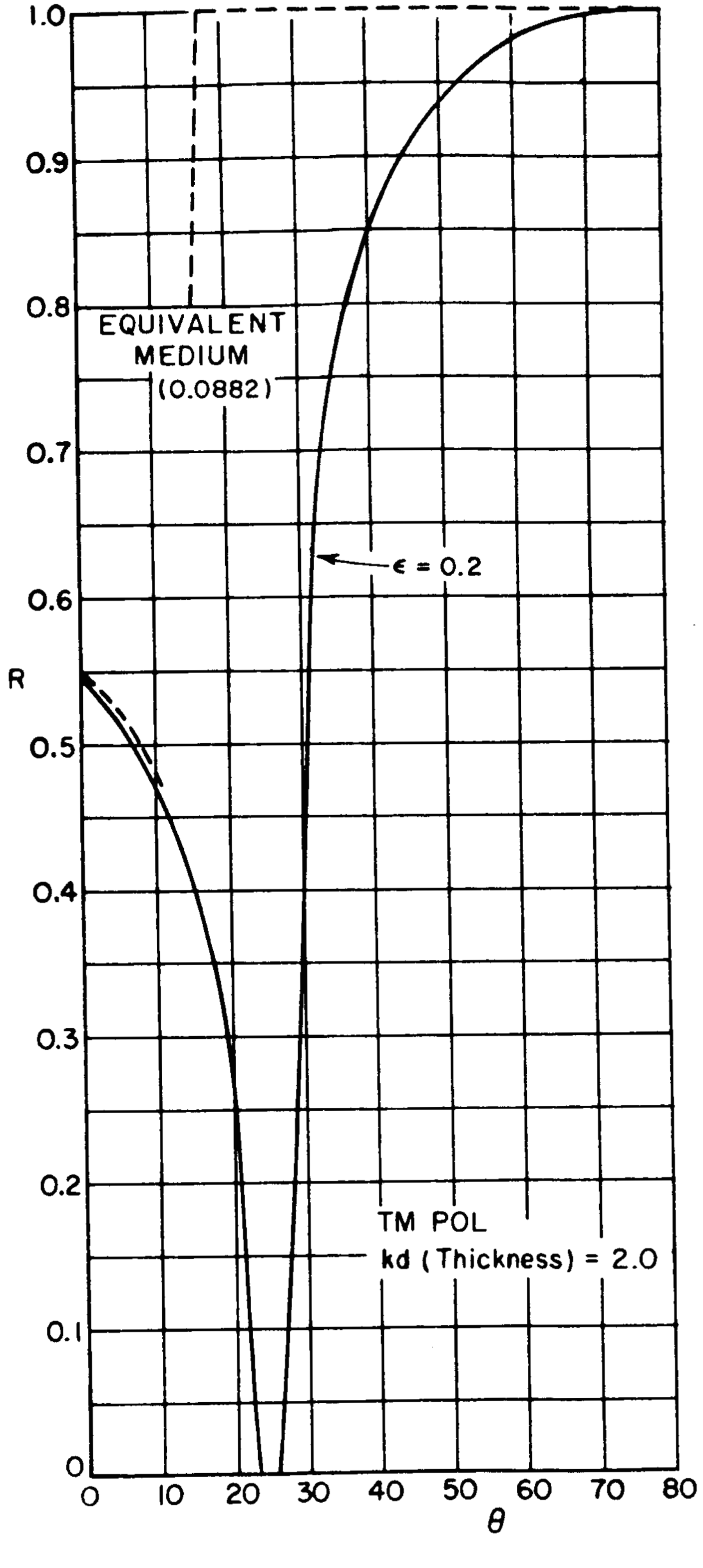

Fig. 8. Reflection vs incidence angle.

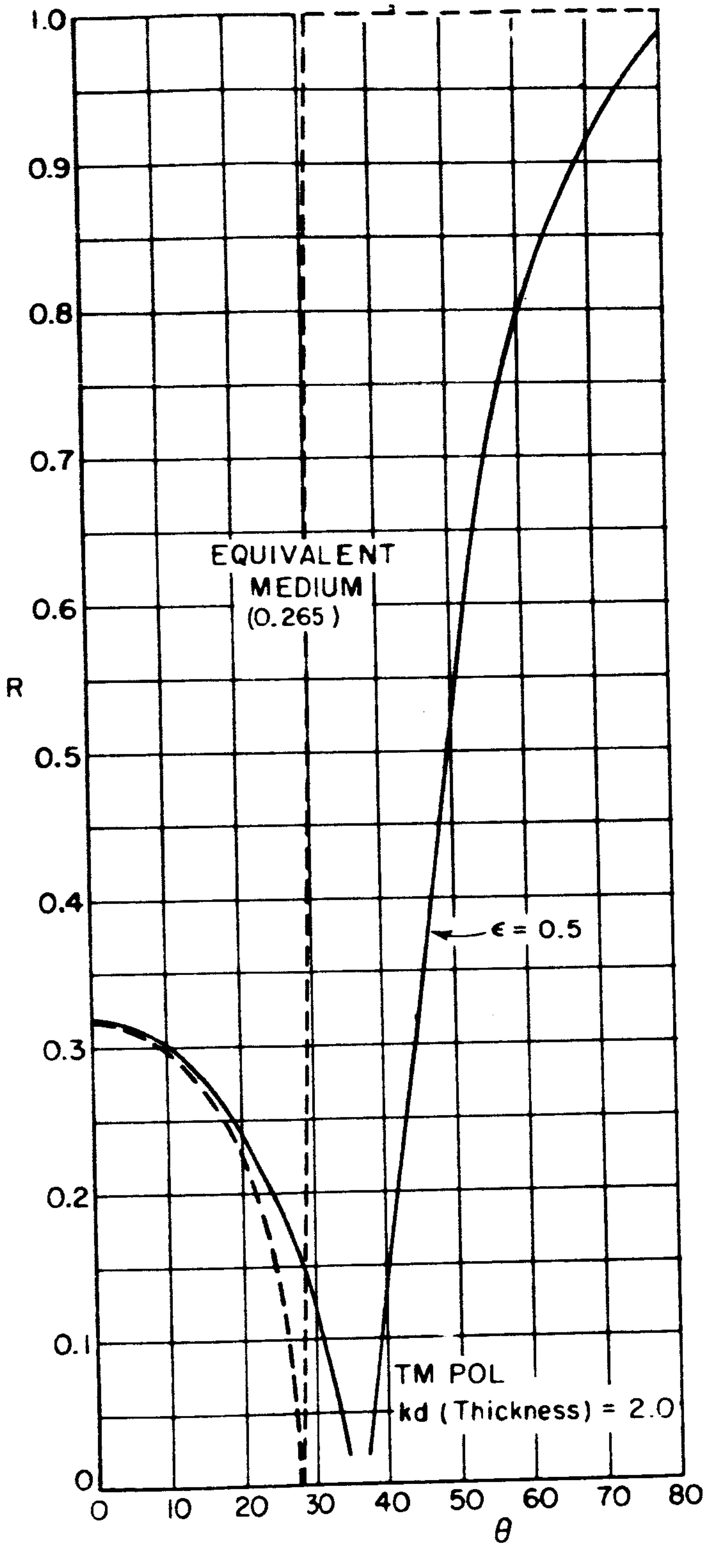

Fig. 9. Reflection vs incidence angle.

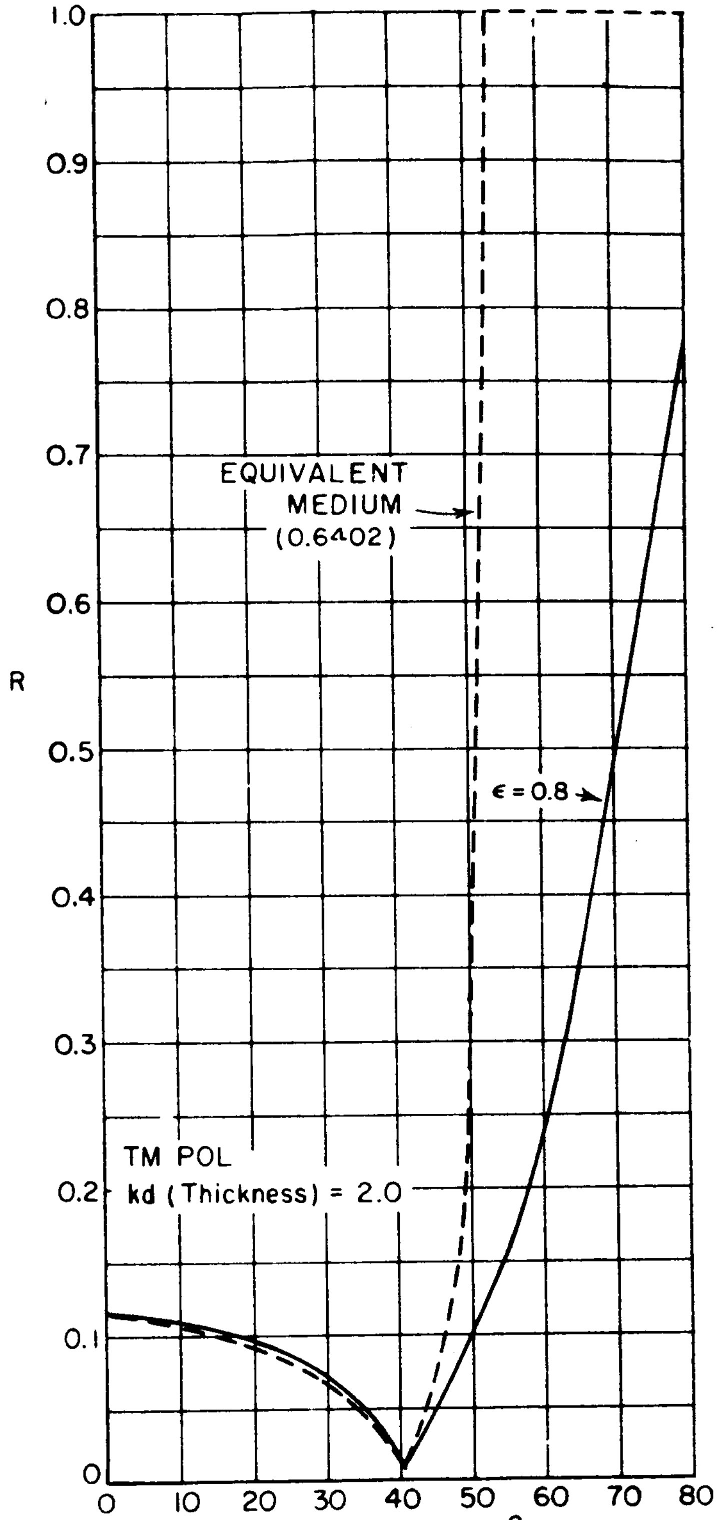

Fig. 10. Reflection vs incidence angle.

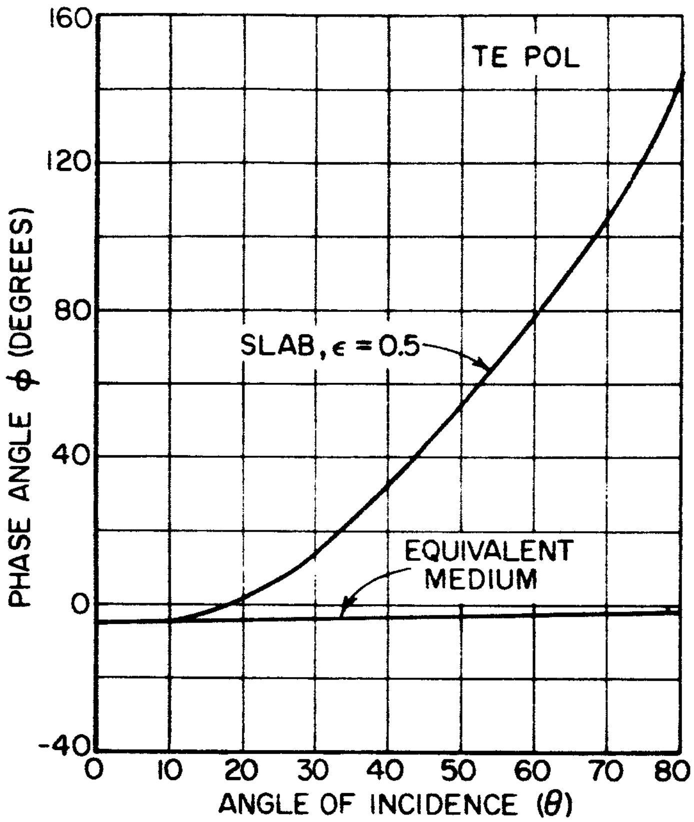

Fig. 11. Phase vs indicence angle.

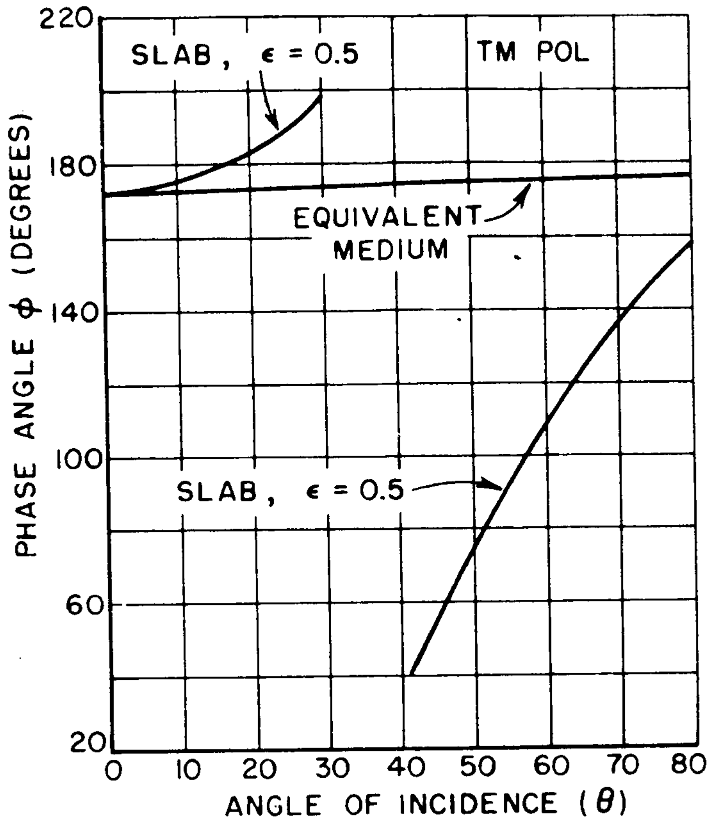

Fig. 12. Phase vs incidence angle.

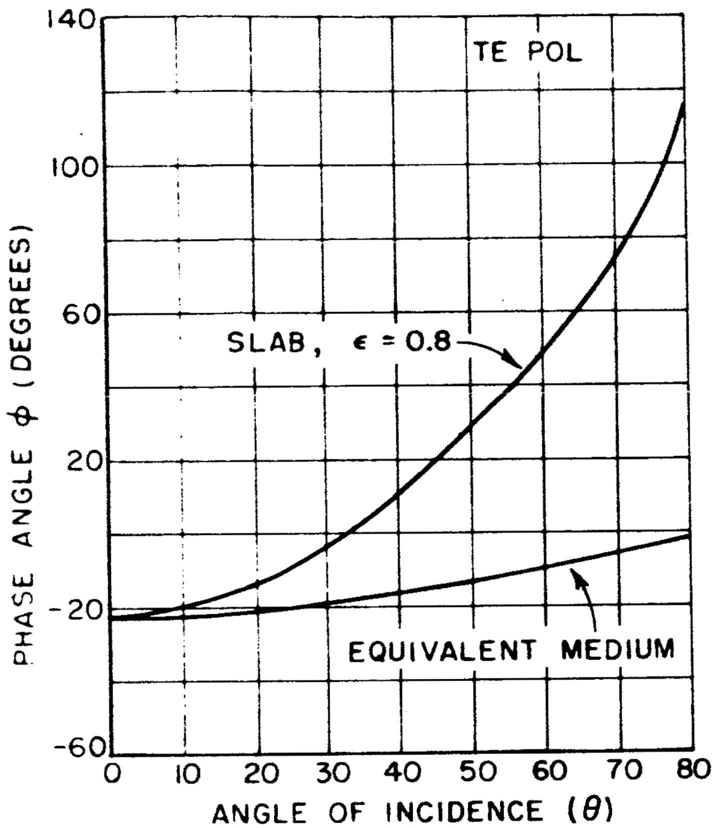

Fig. 13. Phase vs incidence angle.

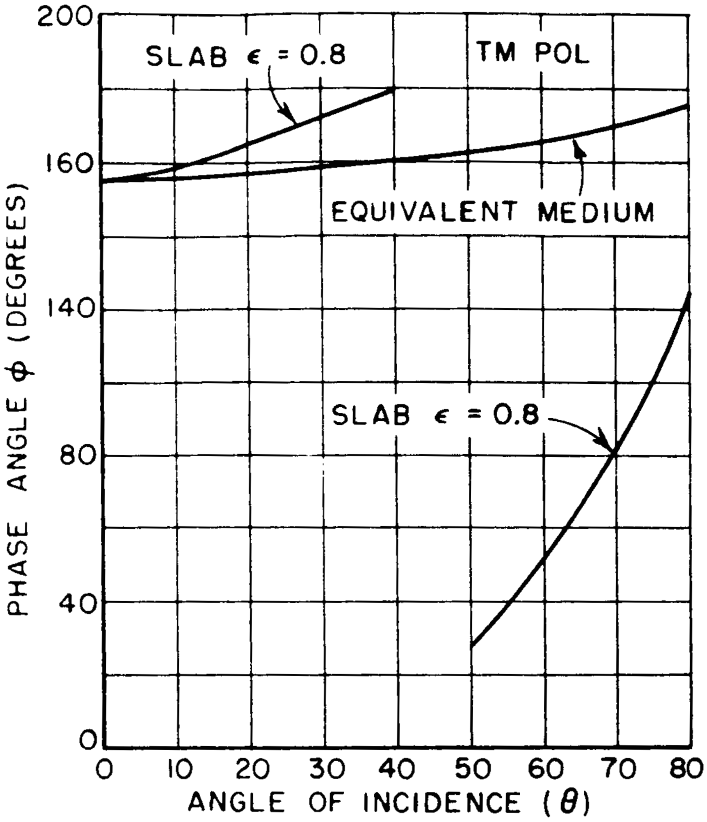

Fig. 14. Phase vs incidence angle.

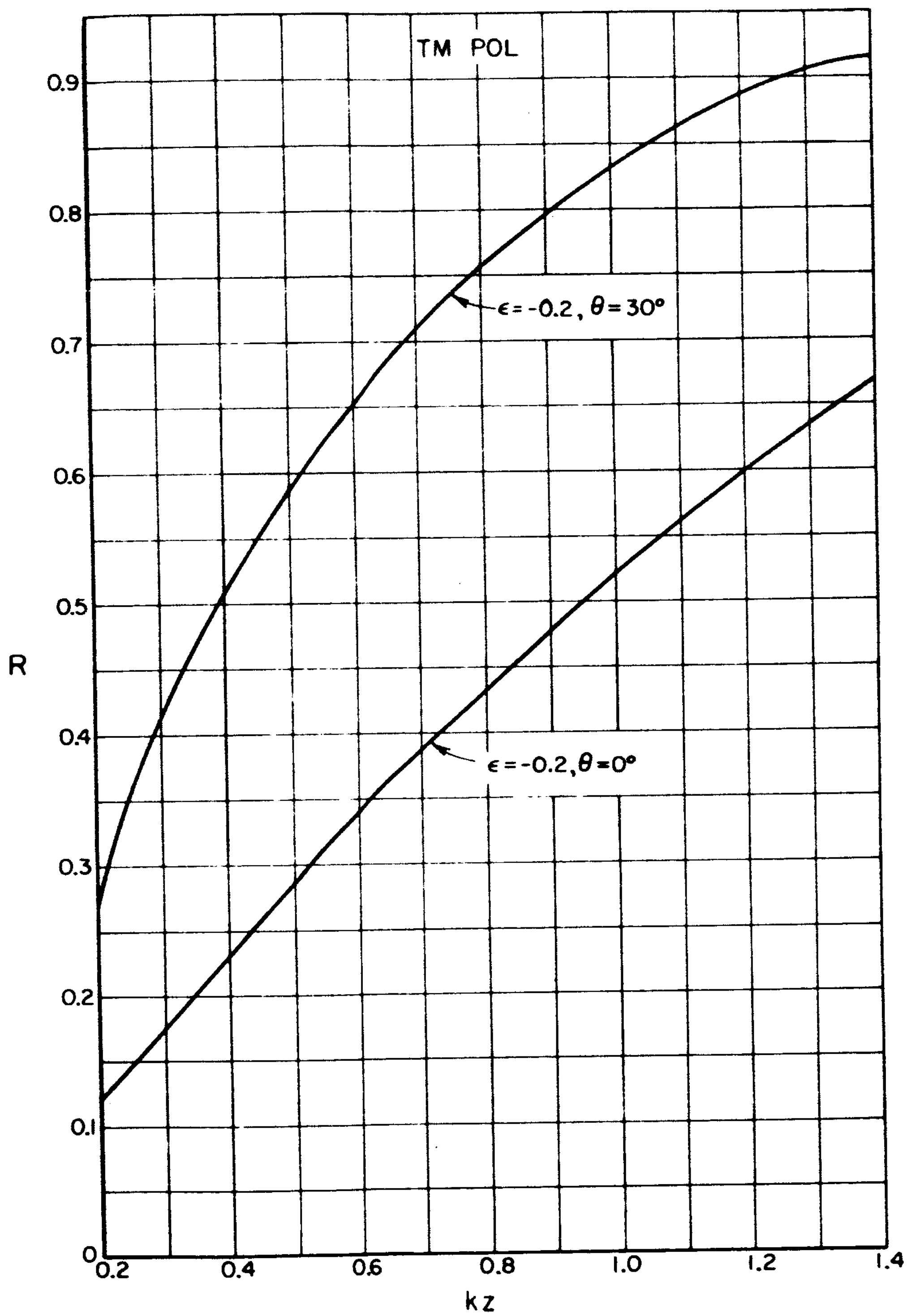

Fig. 15. Reflection coefficient vs layer thickness.

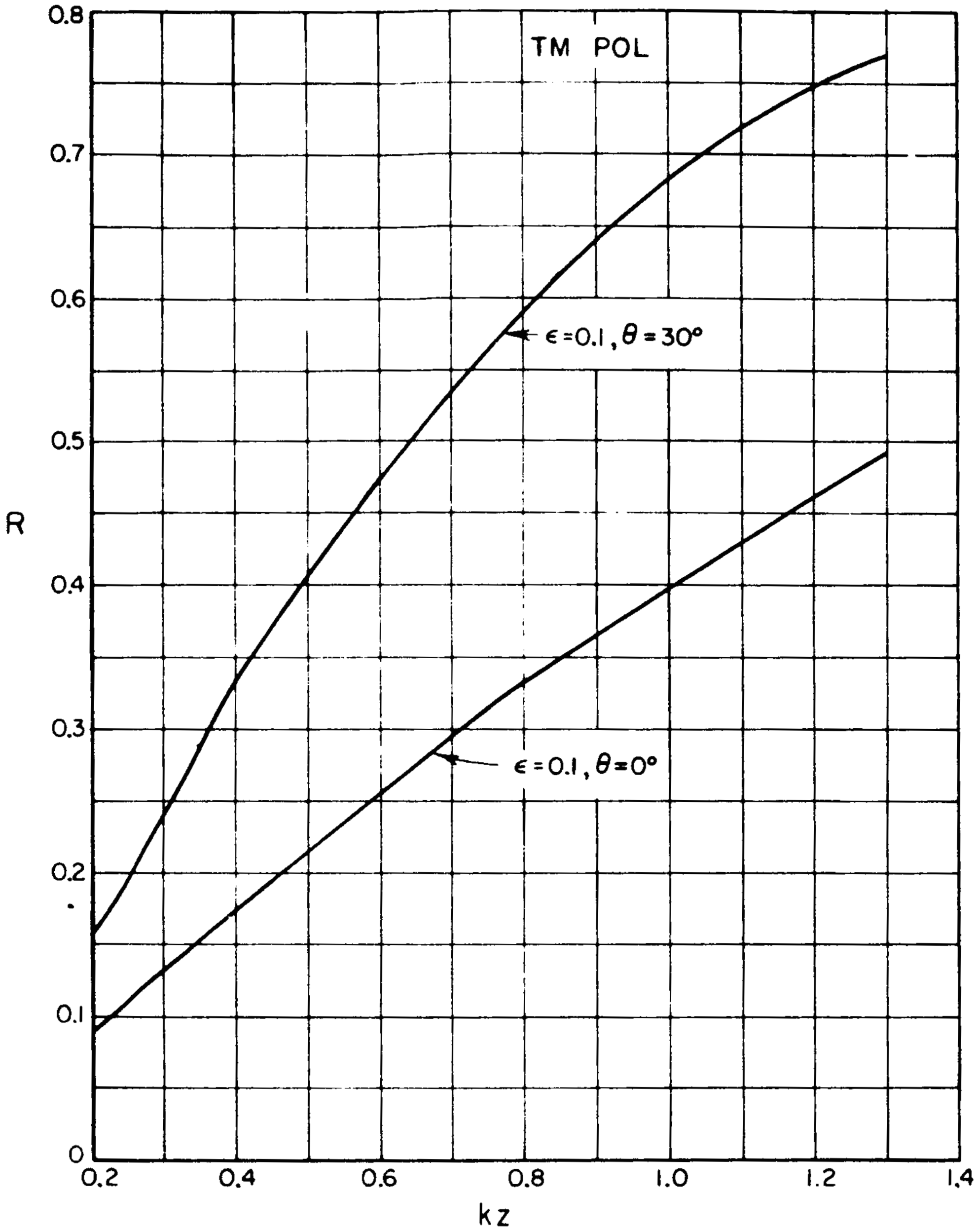

Fig. 16. Reflection coefficient vs layer thickness.

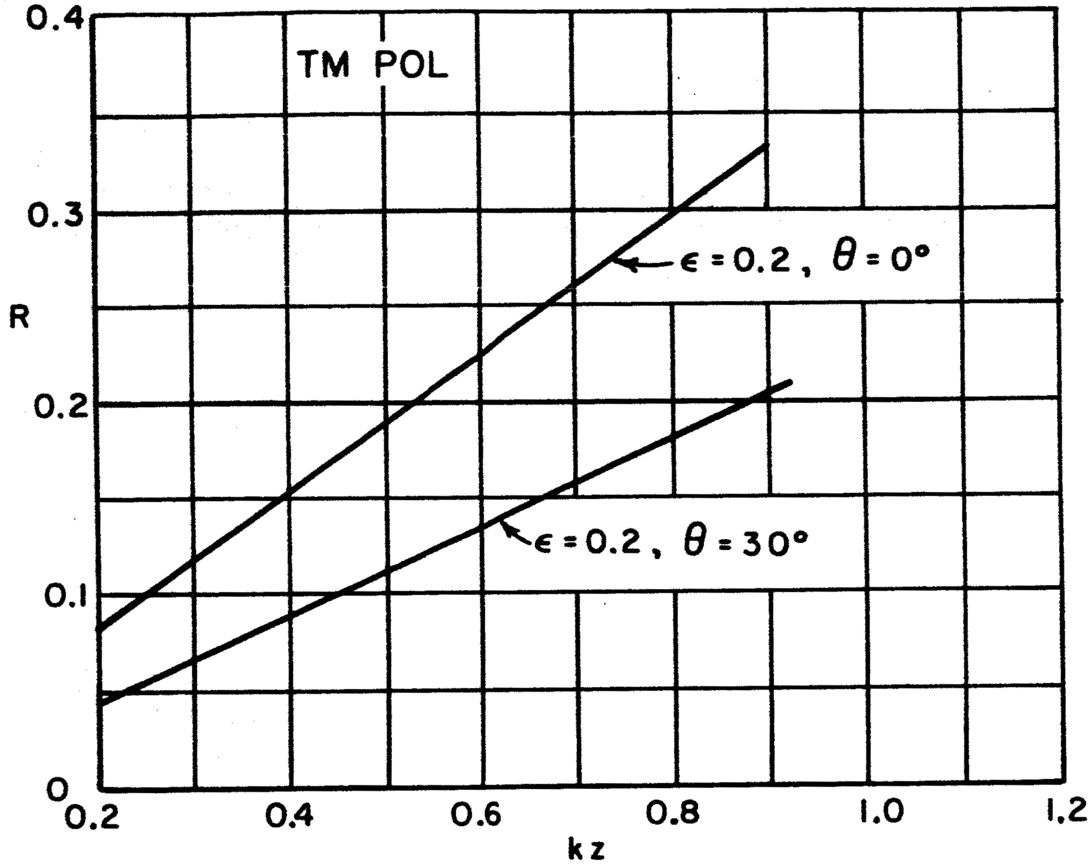

Fig. 17. Reflection coefficient vs layer thickness.

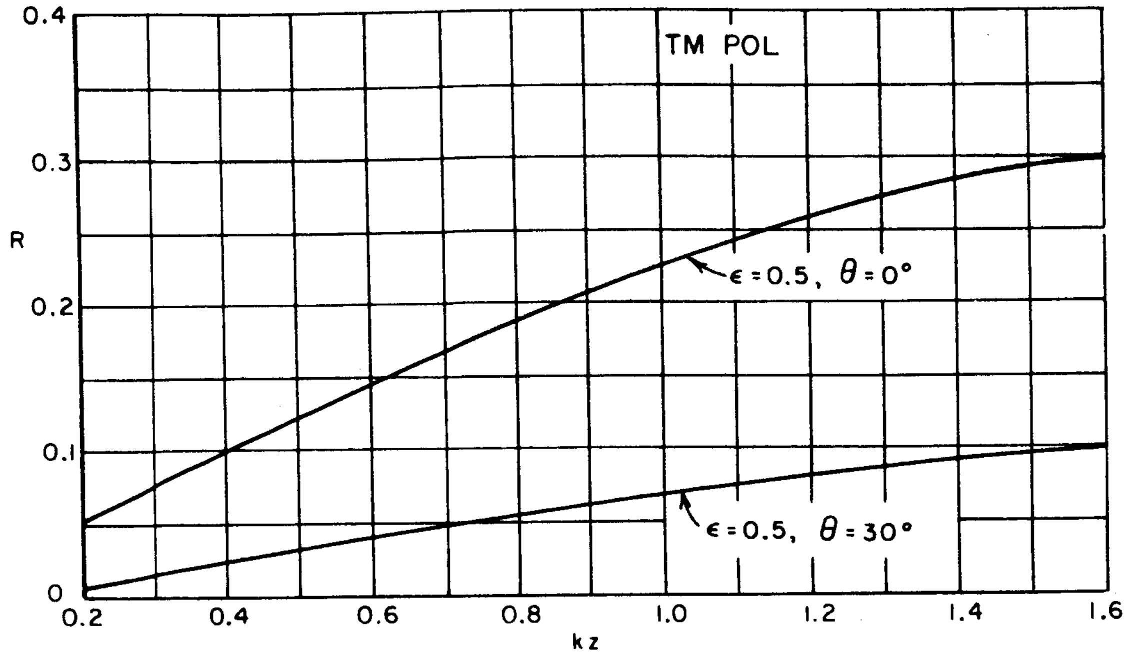

Fig. 18. Reflection coefficient vs layer thickness.

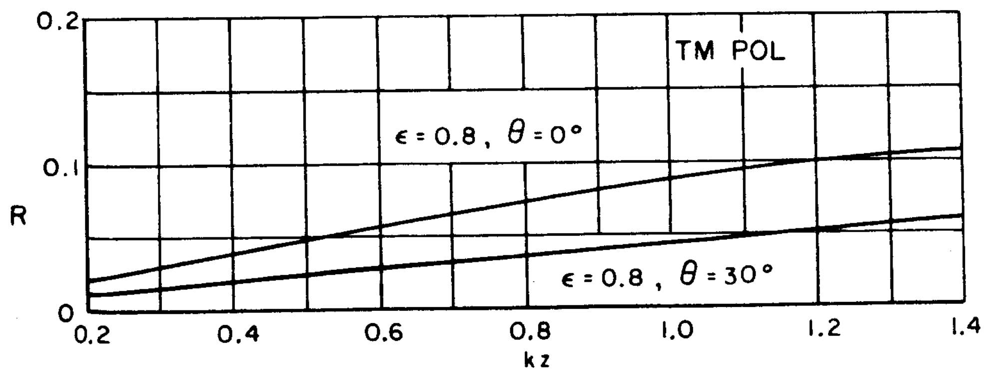

Fig. 19. Reflection coefficient vs layer thickness.

ELECTRON DENSITY PROFILES

Having examined the equivalent semi-infinite medium technique for slabs of finite thickness and uniform dielectric constant, a varying electron density profile was studied. The electron density profile is that considered by Swift[4] as shown in Fig. 20 along with the associated collision frequency profile. The complex dielectric constants for each frequency are shown in Table I. Data from the profile of Fig. 20 were sampled every 0.1 cm and the frequency was varied from 0.5 to 20.0 GHz. Figures 21 through 25 represent perpendicular or TE polarization and Figs. 26 through 30 represent parallel or TM polarization.

TABLE I
Complex Dielectric Constant
for Various Frequencies

Layer	0.5 GHz	3.0 GHz	8.0 GHz	12.0 GHz	20.0 GHz
1	-31.1-j1.53	0.106-j.0071	0.874-j.0003	0.944-j.0001	0.979-j.00002
2	-95.3-j4.59	-1.68-j.021	0.622-j.0011	0.832-j.0003	0.939-j.00007
3	-127.4-j6.13	-2.57-j.028	0.447-j.0015	0.776-j.0005	0.919-j.00009
4	-143.5-j6.89	-3.02-j.032	0.434-j.0016	0.748-j.0003	0.909-j.0001
5	-111.3-j5.36	-2.12-j.024	0.559-j.0013	0.804-j.0003	0.929-j.00008
6	-63.2-j3.06	-0.787-j.014	0.748-j.0007	0.888-j.0002	0.959-j.00004
7	-31.1-j1.53	0.106-j.007	0.874-j.0003	0.944-j.0001	0.979-j.00002
8	-15.05-j0.766	0.533-j.003	0.937-j.0001	0.972-j.00005	0.989-j.00001
9	-7.02-j0.383	0.776-j.0017	0.968-j.00009	0.986-j.00002	0.994-j.000006
10	-2.21-j0.153	0.91-j.0007	0.987-j.00003	0.994-j.00001	0.997-j.000002

Figure 21 shows the reflection as a function of incidence angle at a frequency of 0.5 GHz. The equivalent medium has a dielectric constant of $\epsilon = 0.000642$ and the critical angle is 0.48°. In this case of very high reflection by the plasma profile the calculated dielectric constants are indeed negative and rapidly varying (see Table I) and the equivalent medium concept is valid only at 0°.

For Fig. 22 the frequency of operation is 3.0 GHz and the computed dielectric profile (shown in Table I) only goes partly negative. The equivalent medium has a dielectric constant of

$\epsilon = 0.110$ and the critical angle is 19.33°. The two reflection coefficient magnitudes correspond for about 10°.

Figure 23 applies for a frequency of 8.0 GHz at which the calculated permittivity is positive and slowly varying throughout the thickness of the plasma slab. The equivalent medium has a dielectric constant of $\epsilon = 0.475$ and a critical angle of 43.62°. The magnitudes of the reflection coefficients compare favorably for approximately 30°. The equivalent medium concept becomes more useful as frequency increases.

The frequency for Fig. 24 is 12.0 GHz for which the dielectric constant of the equivalent medium is $\epsilon = 0.6708$ and the critical angle is 55.58°. The computed permittivity of the plasma slab is positive and varies much less than at lower frequencies. The reflection coefficients of the equivalent medium match those of the plasma slab for a range of 40°.

Figure 25 represents the electron density profile at a frequency of 20.0 GHz; and the calculated dielectric constants of the plasma profile vary only very slightly. The equivalent medium has a dielectric constant, $\epsilon = 0.8832$ and the critical angle is 70.07°. The reflection coefficients of the two media closely correspond for 60°.

Figures 26-30 represent the same frequencies as Figs. 21-25, with the polarization being TM or parallel. The equivalent media and critical angles are the same for frequencies of 0.5, 3.0, 8.0, 12.0, and 20.0 GHz. As in the case of TE or perpendicular polarization the reflection coefficients of the equivalent media and the plasma profile match over wider ranges with increasing frequency. The range of reflection coefficient matching varies from a single point (0°) at a frequency of 0.5 GHz to a 0°-60° coincidence for a frequency of 20.0 GHz. The variation in the calculated permittivities range from a rapidly varying negative profile at 0.5 GHz to a very slowly varying positive profile at 20.0 GHz.

Figure 31 shows the variation of reflection coefficient, R, with frequency for the case of TE or perpendicular polarization and for three different angles of incidence. The greater reflections (almost total) occur at the lower frequencies and gradually decrease to almost zero with increasing frequency.

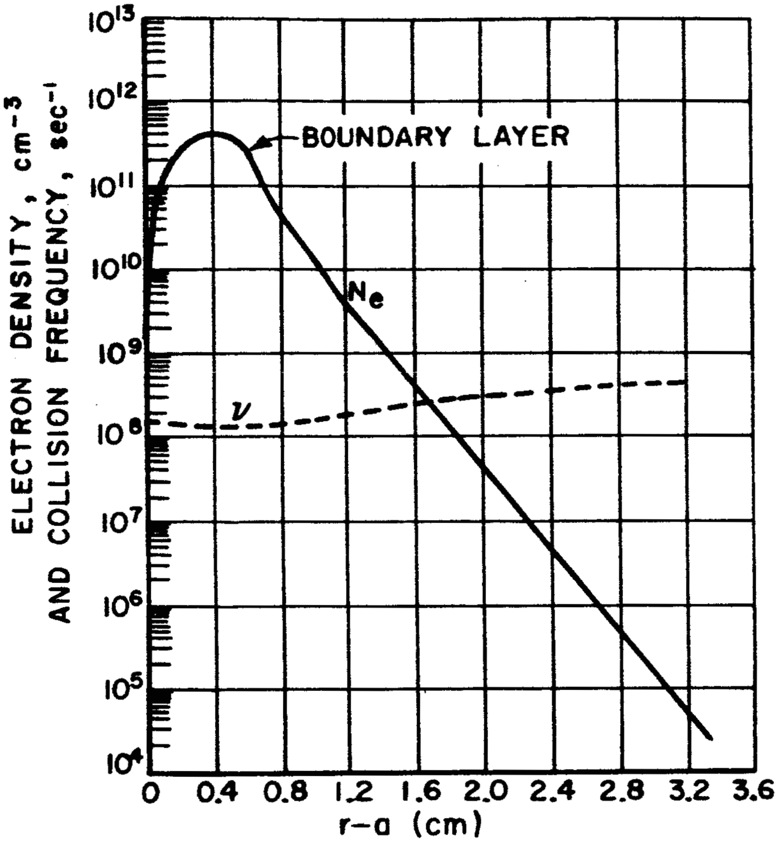

Fig. 20. Electron density and collision frequency profiles for two flow-field assumptions.

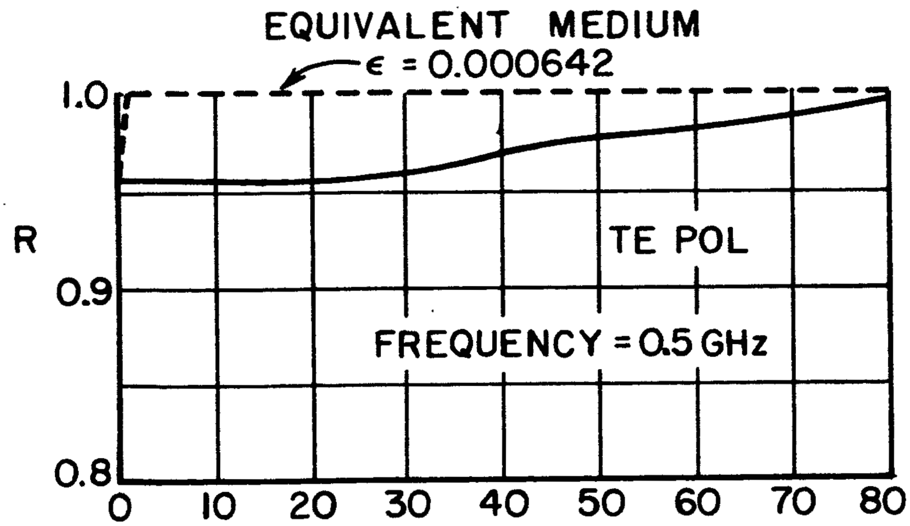

Fig. 21. R vs θ for plasma profile.

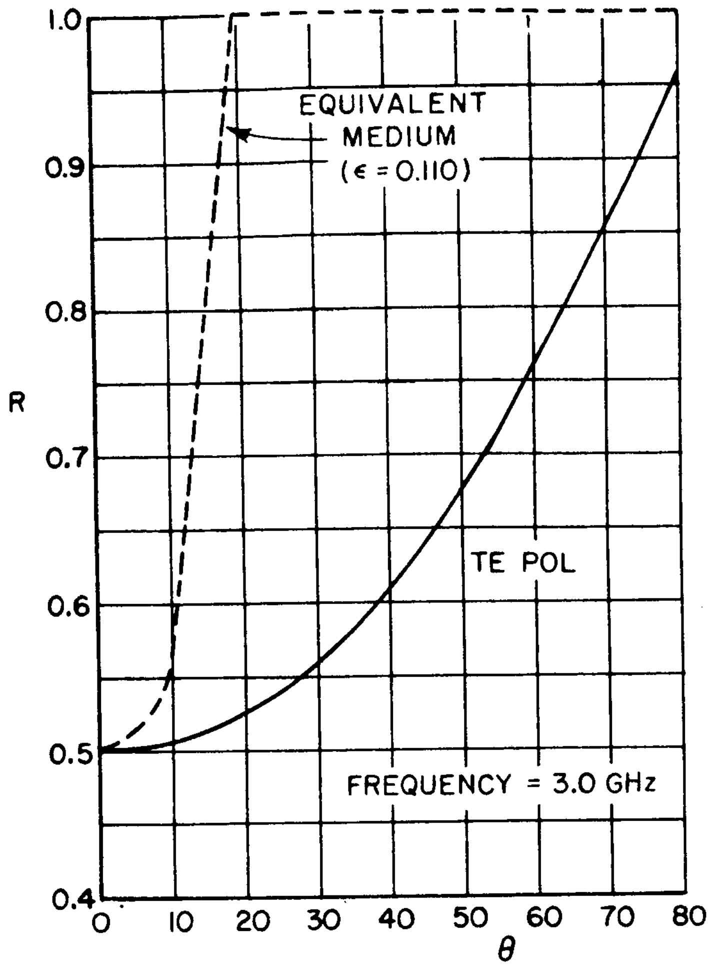

Fig. 22. R vs θ for plasma profile.

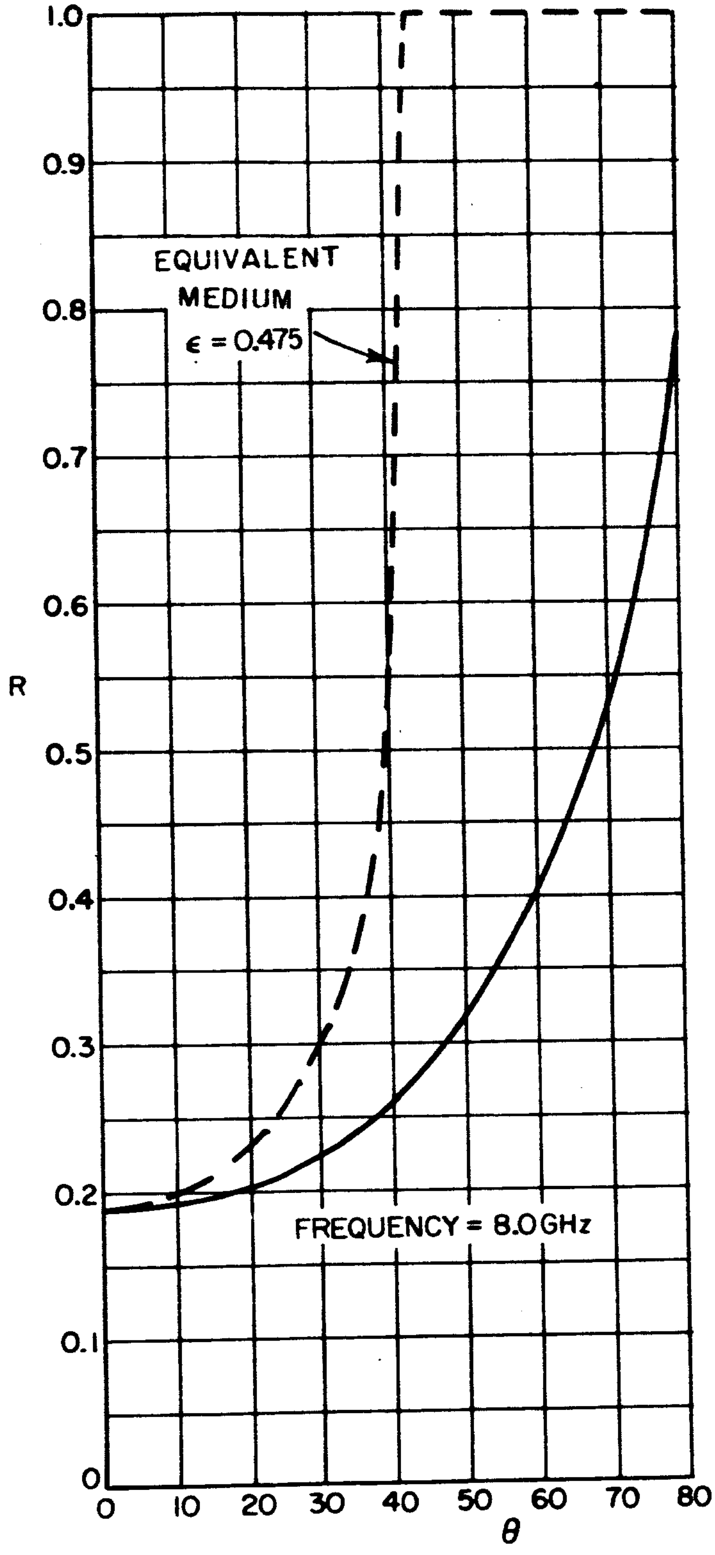

Fig. 23. R vs θ for plasma profile.

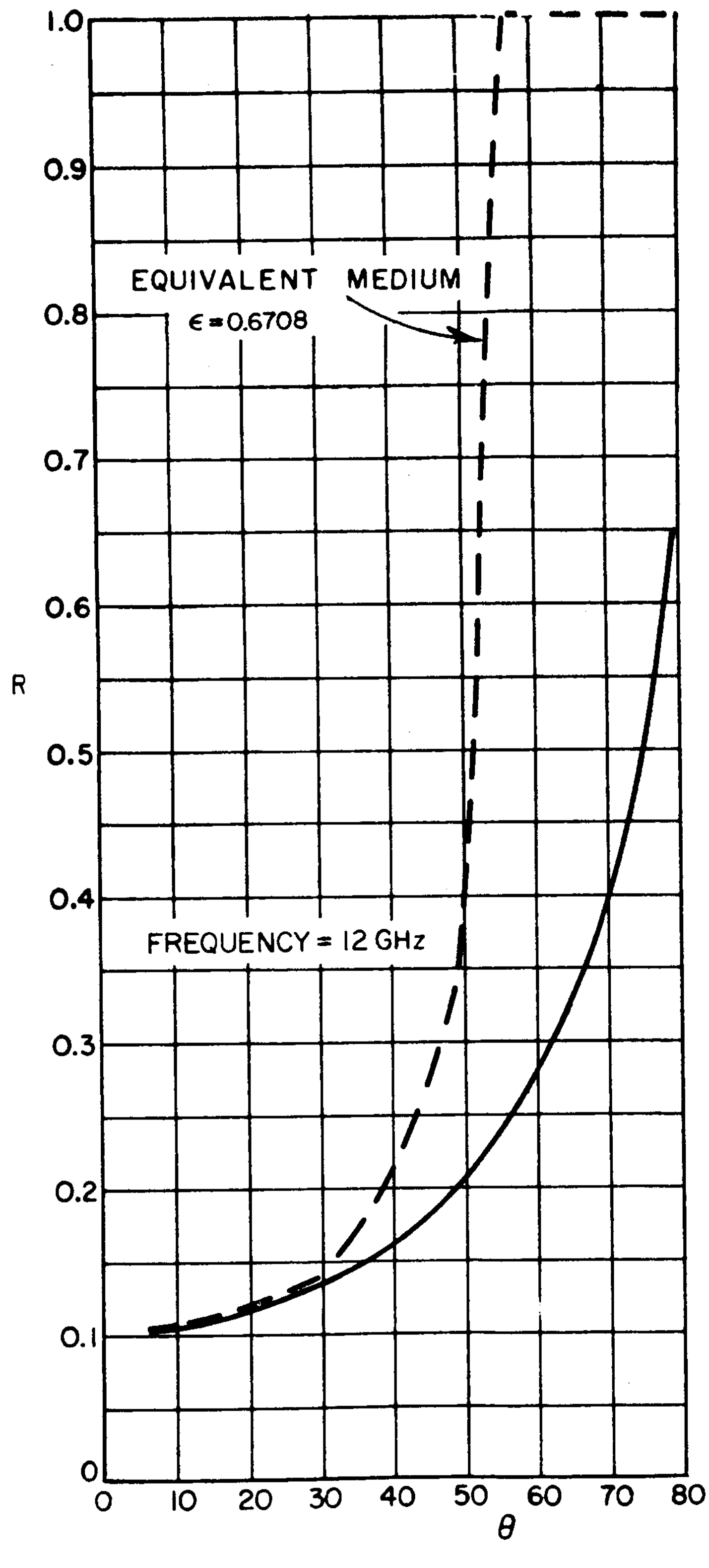

Fig. 24. R vs θ for plasma profile.

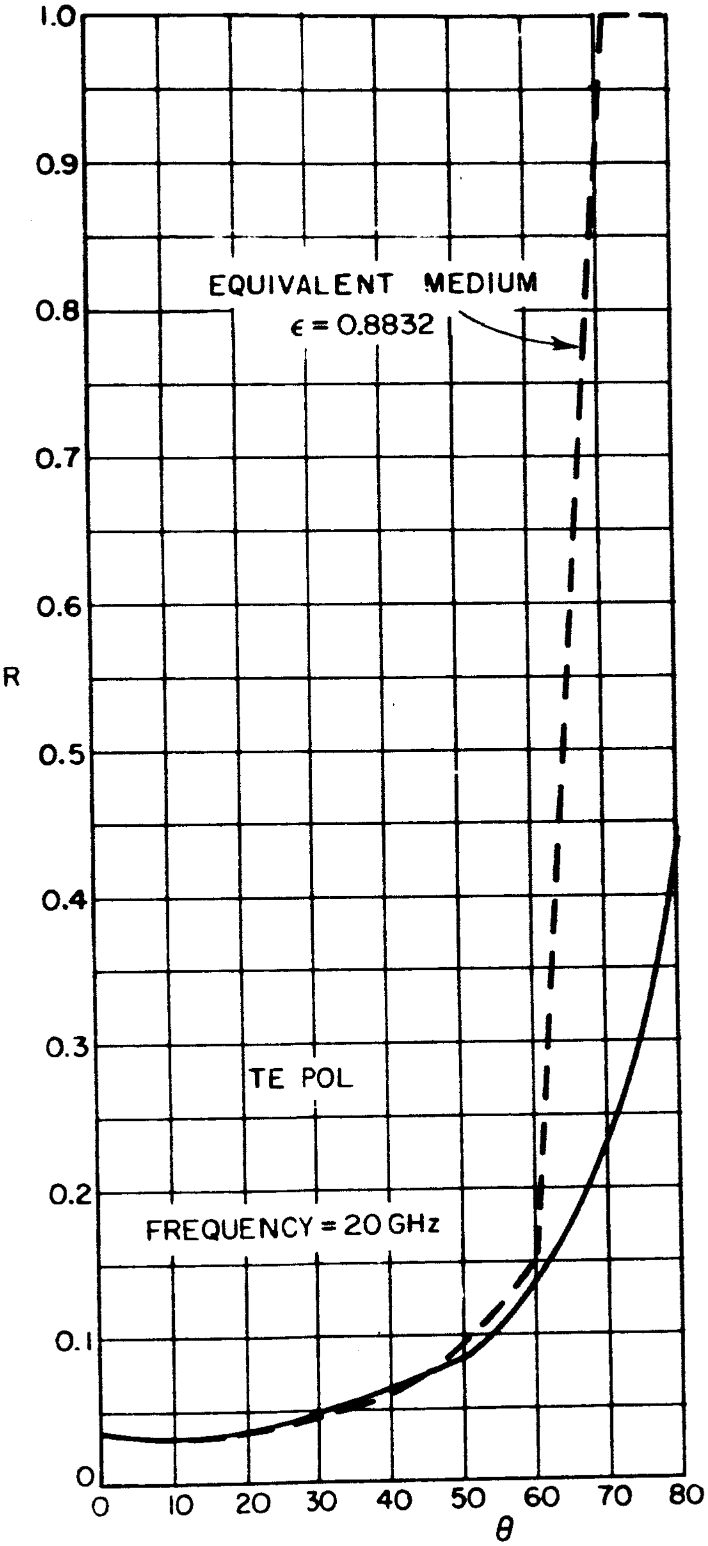

Fig. 25. R vs θ for plasma profile.

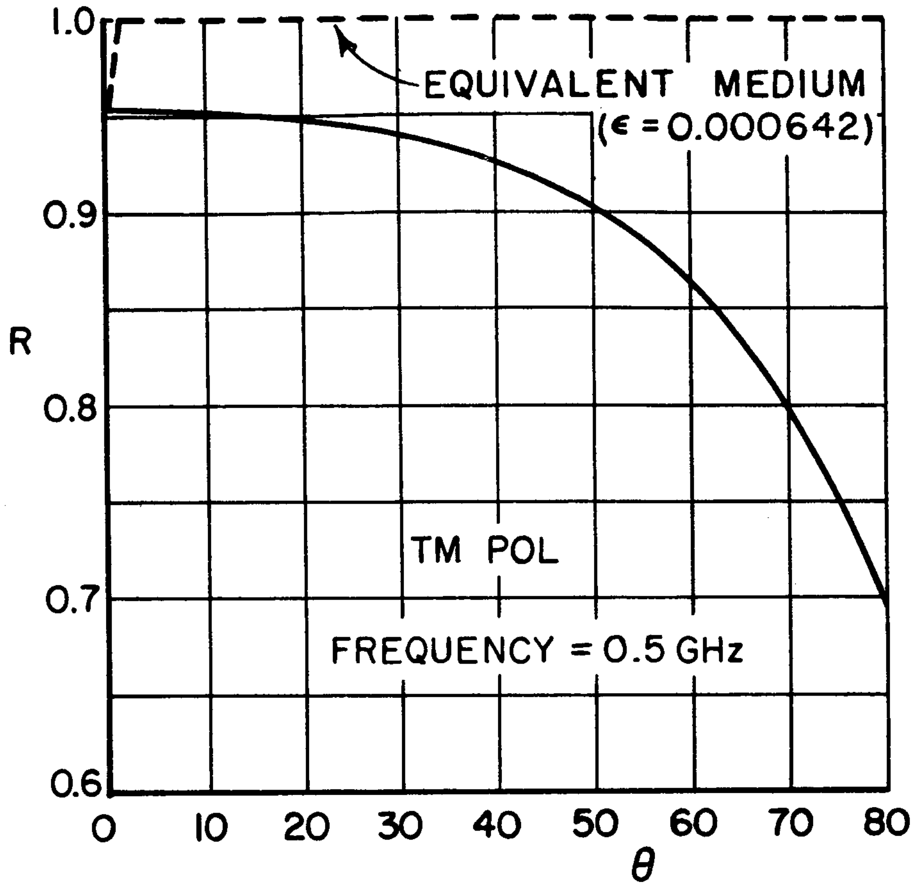

Fig. 26. R vs θ for plasma profile.

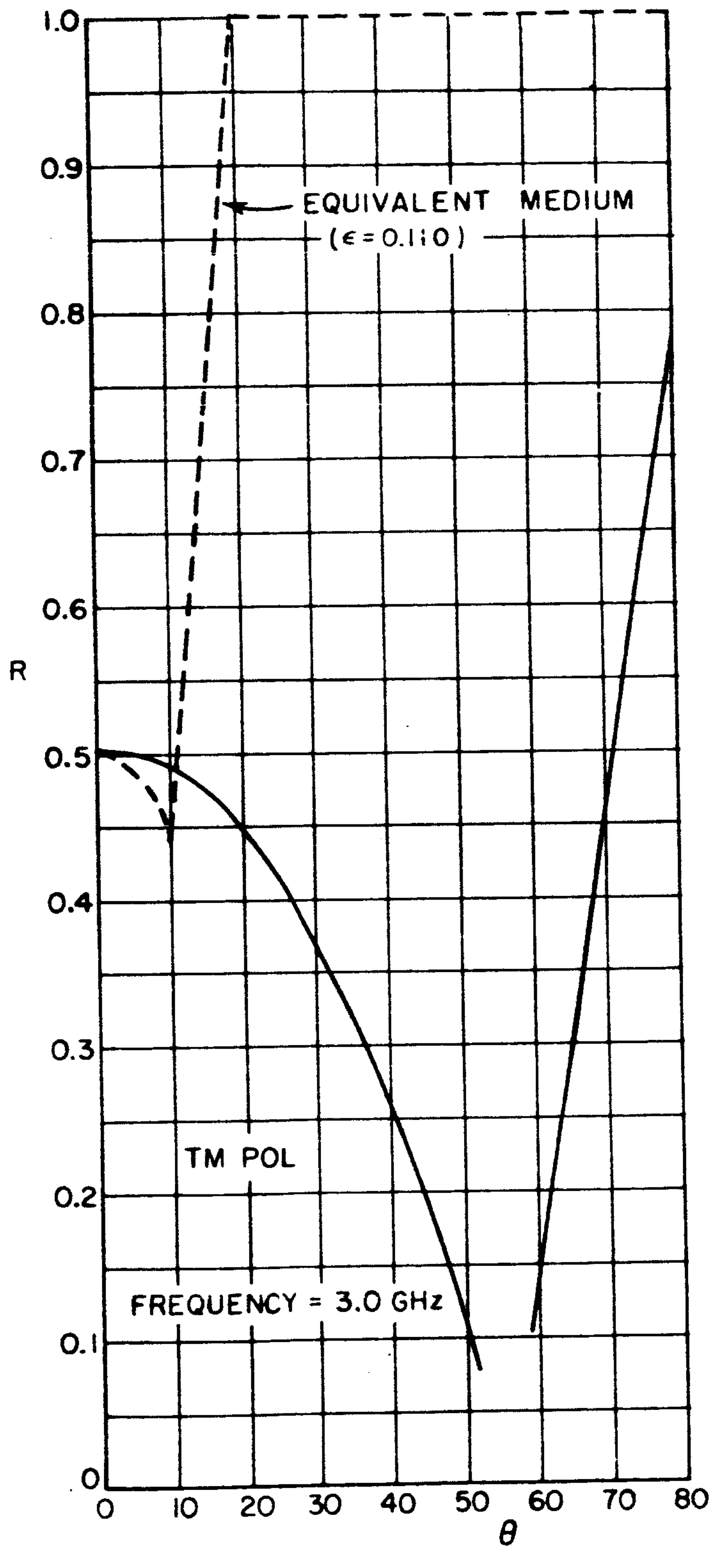

Fig. 27. R vs θ for plasma profile.

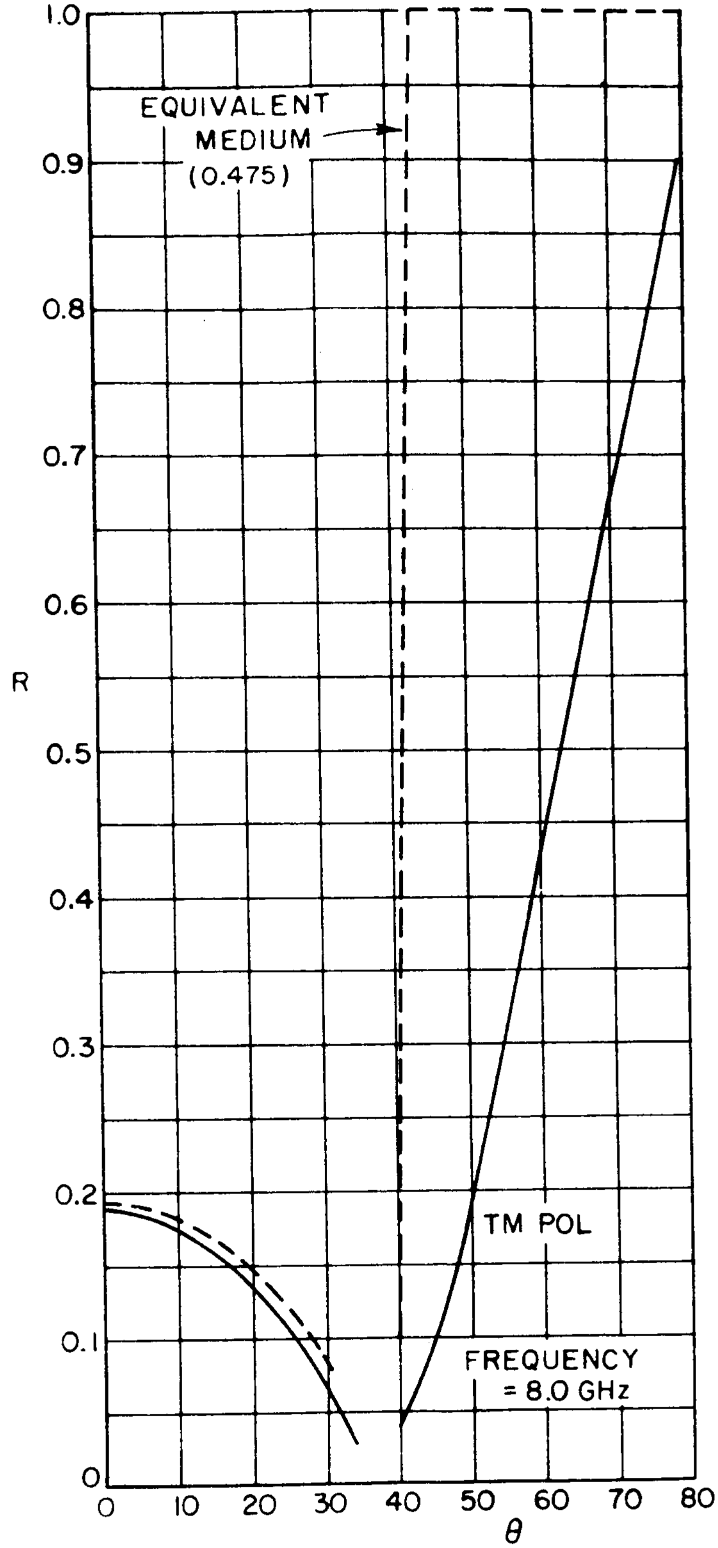

Fig. 28. R vs θ for plasma profile.

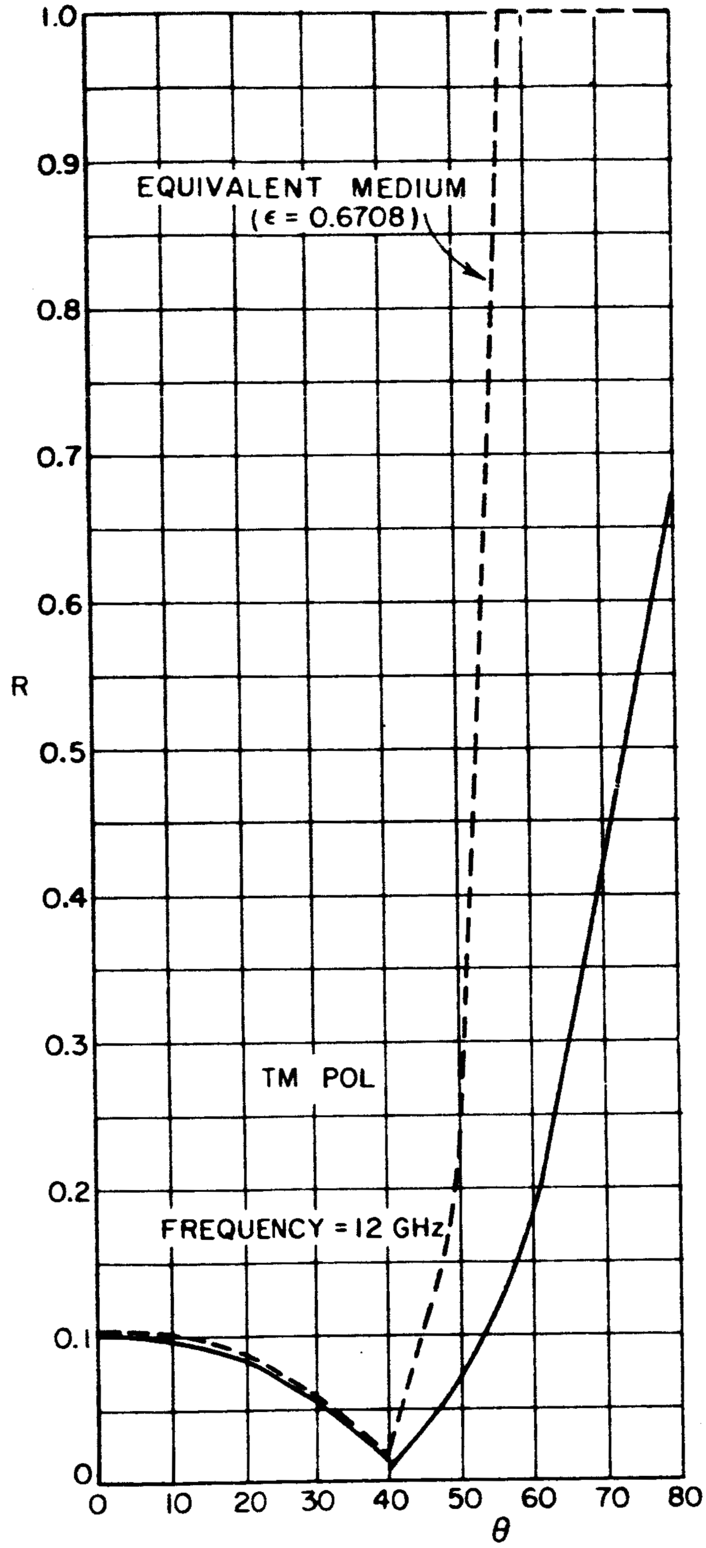

Fig. 29. R vs θ for plasma profile.

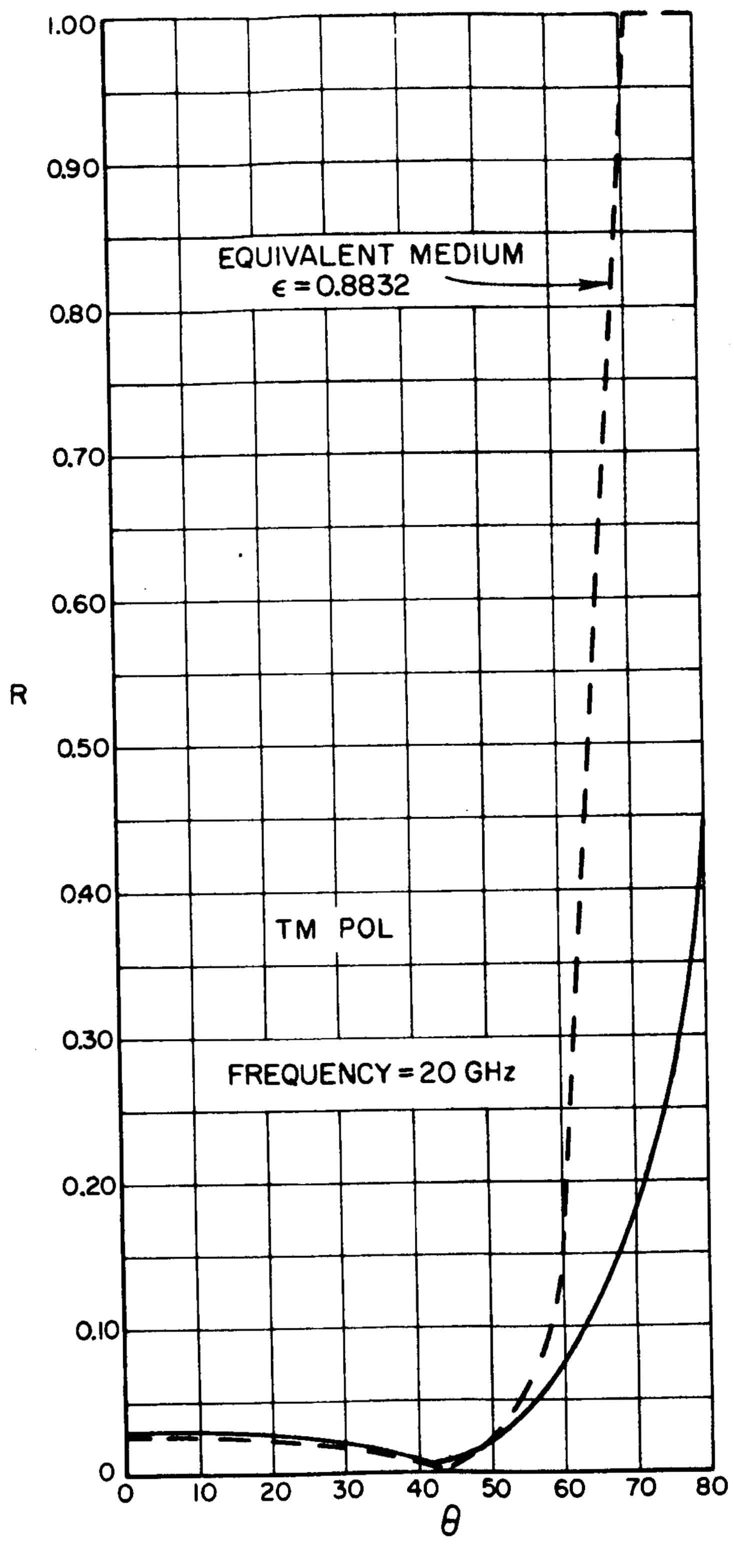

Fig. 30. R vs θ for plasma profile.

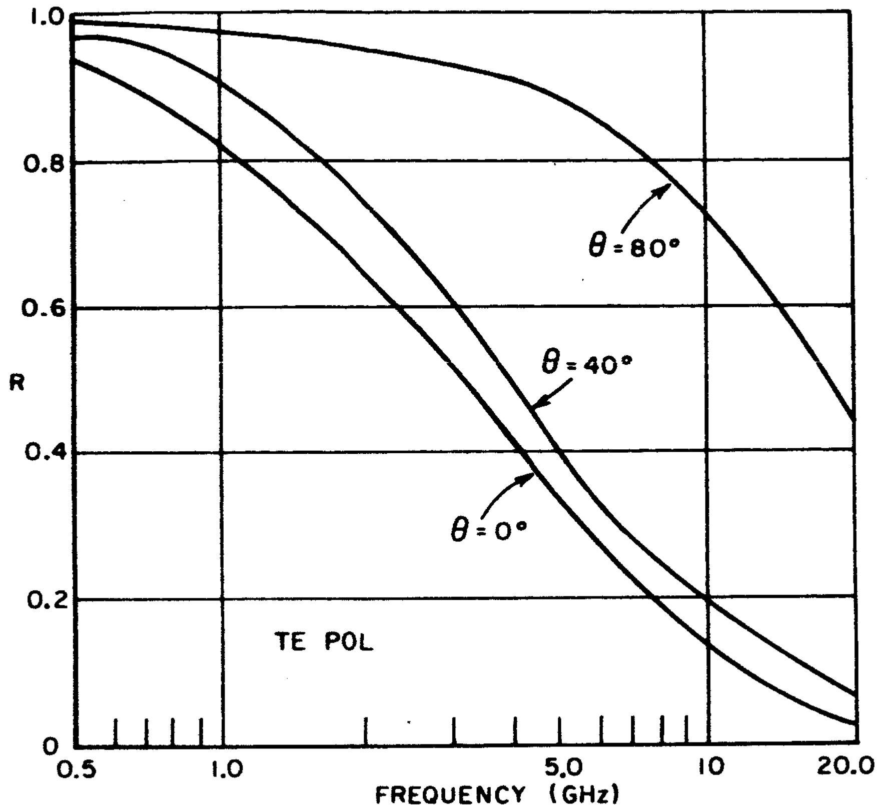

Fig. 31. R vs frequency for constant angle of incidence.

CONCLUSIONS

The substitution of an equivalent semi-infinite, homogeneous dielectric medium for an inhomogeneous, finite plasma layer is studied. Comparison of the reflection coefficients as a function of angle of incidence for various media and their equivalent media shows that in most cases there is a range in which the reflection coefficients are approximately equal. It is in this range of agreement that the substitution of the equivalent medium may be effected; the necessary range depends on the particular antenna whose properties are being studied.

The range of equivalence was determined for uniform dielectric slabs of various dielectric constant values and for an inhomogeneous plasma layer at various frequencies. The useful range of equivalence was found to be very narrow (less than 5°) for cases in which the dielectric constant dips to near or below zero. In other cases fairly wide ranges of equivalence occur.

APPENDIX A

Figures 32 through 35 give the statement listings of the programs used to calculate reflection coefficients and equivalent media. Figure 32 is the old program for TE polarization. Figure 33 is the old program for TM polarization. Figure 34 is the new program version that uses dielectric constant values as input data. Figure 35 is the new program version that uses the electron density profile as input data.

SOURCE LANGUAGE STATEMENTS

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                  COMPLEX (RR,TRM)-
                  DIMENSION (E(16))-
F T2            (7F10.5)     -
F T3            (I5,4F15.8) -
  S7              READ INPUT ,T3,(N,H,TH)-
                H2=H*H-
                  READ INPUT ,T2,((E(I),I=1,1,I.LE.N))-
                M=90./TH-
                TH=.01745329*TH-
                THETA=0-
                  DO THROUGH (S31),J=1,1,J.LE.M-
                S=SIN.(THETA)*SIN.(THETA)-
                C=COS.(THETA)-
                R1=1.-
                S1=0.-
                I=1-
                  WRITE OUTPUT ,T3,(I,R1,S1,R1)-
                A=E(1)-S-
                R2=1.-H2*A/2-H2*(E(2)-E(1))/6.-
                S2=H*C-H2*H*A*C/6-
                EMAG=SQRT.(R2*R2+S2*S2)-
                I=2-
                  WRITE OUTPUT ,T3,(I,R2,S2,EMAG)-
                  DO THROUGH (S25),I=2,1,I.LE.N-
                I1=I+1-
                A=E(I)-S-
                B=2.-H2*A-
                R3=B*R2-R1-
```

Fig. 32. TE polarizations.

```
                S3=B*S2-S1-
                EMAG=SQRT.(R3*R3+S3*S3)-
                 WRITE OUTPUT ,T3,(II,R3,S3,EMAG)-
                 TRANSFER TO (S32,S28,S20) PROVIDED (N-1)-
        S20     R1=R2-
                S1=S2-
                R2=R3-
        S25     S2=S3-
        S28     DRR=SQRT.(E(N))*H*C*2.*R2-S3+S1-
                DRI=SQRT.(E(N))*H*C*2.*S2+R3-R1-
                DDR=SQRT.(E(N))*H*C*2.*R2+S3-S1-
                DDI=SQRT.(E(N))*H*C*2.*S2-R3+R1-
                RR=(DRR+.I.DRI)/(DDR+.I.DDI)-
                RMAG=.ABS.(RR)-
                ANDN=FATAN2.(DRR,DRI)-
                ANDD=FATAN2.(DDR,DDI)-
                ANG=ANDN-ANDD-
        S30     ANG=ANG*57.29578-
                TRM=(4.*H*C)/(DDR+.I.DDI)-
                TRMAG=.ABS.(TRM)-
                THD=57.29578*THETA-
                 WRITE OUTPUT ,T2,(RMAG,ANG,THD,TRMAG)-
        S31     THETA=THETA+TH-
        S32      CALL SUBROUTINE ()=ENDJOB.()-
                 END PROGRAM (S7)-
```

Fig. 32(cont.)

SOURCE LANGUAGE STATEMENTS

```
    S1          COMPLEX (H1,H2,H3,F1,F2,F3,RR,TP,A,B,CC,E(40))-
                COMPLEX (RRT)-
                COMPLEX (CSQRTL.)-
  F T2        (7F10.5)    -
  F T3        (I5,6F10.5) -
    S7          READ INPUT ,T3,(N,Z,TH)-
    START       READ INPUT ,IN1,(NUMCAS)-
  F IN1       (I2)  -
               Z2=Z*Z-
               M=90/TH-
               TH=.01745329*TH-
    GO         NUMCAS=NUMCAS-1-
                READ INPUT ,T2,((E(I),I=1,1,I.LE.N))-
               THETA=0-
                DO THROUGH (S31),L=1,1,L.LE.M-
               S=SIN.(THETA)*SIN.(THETA)-
               C=COS.(THETA)-
               H1=1.-
               F1=C-
               I=1-
                WRITE OUTPUT ,T3,(I,H1,F1)-
               EI=.IMAG.(E(1))-
               ER=.REAL.(E(1))-
               EER=.REAL.(E(2))-
                WRITE OUTPUT ,T2,(ER,EI,EER)-
               F2=C+Z*S*EI/(ER*ER)-Z2/2*(ER-S)*C+.I.(Z*(1-S/ER+Z/2*EI*C-1./6.
                                                          *Z2*(ER-S)*(ER-S)/ER))-
               H2=1-Z*EI*C-Z2/2*(ER-S)+.I.(Z*(ER*C+Z*EI/2-1./6.*Z2*ER*(ER-S)*C))-
               H2A=.ABS.(H2)-
```

Fig. 33. TM polarizations.

```
            F2A=.ABS.(F2)-
            RR=(H2*C-F2)/(H2*C+F2)-
            RRMAG=.ABS.(RR)-
            TR=2/(H2+F2/C)-
            TRMAG=.ABS.(TR)-
            I=2-
             WRITE OUTPUT ,T3,(I,H2A,F2A,RRMAG,TRMAG)-
             WRITE OUTPUT ,T2,(RR,TR)-
             WRITE OUTPUT ,T2,(H2,F2)-
             DO THROUGH (S25),I=2,1,I.LE.N-
            II=I+1-
            CC=E(I)-E(I-1)-
            K=100-
             PROVIDED (I.E.2),K=99-
             DO THROUGH (S21),J=1,1,J.LE.K-
            EE=E(I-1)+J*CC/100.-
            A=EE-S-
            B=2.-Z2*A-
            F3=B*F2-F1+.I.(Z*CC*S*H2/(100*EE*EE))-
            H3=B*H2-H1+.I.(Z*CC*F2/100.)-
            F1=F2-
            F2=F3-
            H1=H2-
    S21     H2=H3-
            RR=(H3*C-F3)/(H3*C+F3)-
            TR=2*C/(H3*C+F3)-
            F3A=.ABS.(F3)-
            H3A=.ABS.(H3)-
            RRMAG=.ABS.(RR)-
```

Fig. 33(cont.)

```
          TRMAG=.ABS.(TR)-
          THD=57.29578*THETA-
           TRANSFER TO (S27,S28,S29) PROVIDED (.REAL.(E(I)))-
S27       RRT=(H3*C-.I.(CSQRTL.(E(I))*F3))/(H3*C+.I.(CSQRTL.(E(I))*F3))-
           TRANSFER TO (S30)-
S28       RRT=1.-
           TRANSFER TO (S30)-
S29       RRT=(H3*C-CSQRTL.(E(I))*F3)/(H3*C+CSQRTL.(E(I))*F3)-
S30       RRTM=.ABS.(RRT)-
           WRITE OUTPUT ,T3,(II,H3A,F3A,RRMAG,TRMAG,THD)-
           WRITE OUTPUT ,T2,(RR,TR,RRT,THD)-
           WRITE OUTPUT ,T2,(H3,F3)-
S25        WRITE OUTPUT ,T2,(RRTM)-
S31       THETA=THETA+TH-
S32        PROVIDED (NUMCAS.G.0), TRANSFER (GO)-
S33        CALL SUBROUTINE ()=ENDJOB.()-
           END PROGRAM (S7)-
```

Fig. 33(cont.)

```
               COMPLEX (CSQRTL.,CEXPL.,CI,EC,G,AE,AB,BE,AM,GZ,F,YI,YII,EXPI,
                              EXPII,AEP,AMP,EXP,RE,RM,FP,FM,TE,TM)-
               COMPLEX (UC(100),EC(100),G(100))-
               COMPLEX (AM)-
               COMPLEX (SQR)-
               COMPLEX (AET,AMT,BET,BMT,EXP3)-
               COMPLEX (FT)-
               COMPLEX (RQTE,RQTM)-
               DIMENSION (Z(100))-
   FIRST       READ INPUT ,8,(KKK,N)-
               READ INPUT ,7,(D,TH)-
              DR=57.295779-
               DO THROUGH (S10),I=1,1,I.LE.N-
   S10        UC(I)=1.-
              NN=N+1-
              EC(NN)=1.-
              UC(NN)=1.-
              TPI=6.2831853-
              CI=.I.1.-
              M=90/TH-
              Z(0)=.0-
               DO THROUGH (S30),I=1,1,I.LE.N-
              II=I-1-
   S30        Z(I)=Z(II)+D-
               DO THROUGH (S100),K=1,1,K.LE.KKK-
               READ INPUT ,7,((EC(I),I=1,1,I.LE.N))-
              THETA=0-
               WRITE OUTPUT ,F4,((I,Z(I),EC(I),I=1,1,I.LE.N))-
 F F4         (16X,I3,5X,F15.8,10X,2E15.7)  -
```

Fig. 34. New program. Dielectric constant version.

```
                DO THROUGH (S99),J=1,1,J.LE.M-
                THET=0.01745329*THETA-
                SS=SIN.(THET)*SIN.(THET)-
                CC=COS.(THET)-
                DO THROUGH (S40),II=1,1,II.LE.N-
     S40        G(II)=CSQRTL.(SS-EC(II))-
                GZ=CI*CC-
                G(NN)=CI*CC-
                AET=.5*(1.+UC(1)*GZ/G(1))-
                BET=.5*(1.-UC(1)*GZ/G(1))-
                AMT=.5*(1.+EC(1)*GZ/G(1))-
                BMT=.5*(1.-EC(1)*GZ/G(1))-
                WRITE OUTPUT ,F5,(THETA)-
   F F5         (///30X,6HTHETA=,F15.8//6H LAYER,9X,5HGAMMA,22X,2HAE,20X,
                                               2HBE,20X,2HAM,20X,2HBM/)-
                J1=1-
                WRITE OUTPUT ,F6,(J1,G(1),AET,BET,AMT,BMT)-
   F F6         (1X,I3,2E14.6,8E11.3)    -
                DO THROUGH (S50),II=2,1,II.LE.N-
                I=II-1-
                F=UC(II)*G(I)/(UC(I)*G(II))-
                YI=G(I)*Z(I)-
                YII=G(II)*Z(I)-
                FP=1.+F-
                FM=1.-F-
                EXPI=CEXPL.(YI)-
                EXPII=CEXPL.(YII)-
                AEP=AET-
                AMP=AMT-
                AET=.5/EXPII*(AEP*EXPI*FP+BET*FM/EXPI)-
```

Fig. 34(cont.)

```
BET=0.5*EXPII*(AEP*EXPI*FM+BET*FP/EXPI)-
EXP3=CEXPL.(CI*CC*Z(I))-
FP=1.+G(I)/(CI*CC)-
FM=1.-G(I)/(CI*CC)-
AE=.5/EXP3*(AET*EXPI*FP+BET*FM/EXPI)-
BE=0.5*EXP3*(AET*EXPI*FM+BET*FP/EXPI)-
F=EC(II)*G(I)/(EC(I)*G(II))-
FP=1.+F-
FM=1.-F-
AMT=(.5/EXPII)*(AMP*EXPI*FP+BMT*FM/EXPI)-
BMT=.5*EXPII*(AMP*EXPI*FM+BMT*FP/EXPI)-
FT=G(I)/(EC(I)*CI*CC)-
FP=1.+FT-
FM=1.-FT-
AM=.5/EXP3*(AMT*EXPI*FP+BMT*FM/EXPI)-
BM=.5*EXP3*(AMT*EXPI*FM+BMT*FP/EXPI)-
 WRITE OUTPUT ,F6,(II,G(II),AE,BE,AM,BM)-
TE=1./AE-
TM=1./AM-
ARG=2.*CC*Z(N)-
EXP=COS.(ARG)-.I.SIN.(ARG)-
RE=EXP*BE/AE-
RM=EXP*BM/AM-
TEM=.ABS.TE-
TMM=.ABS.TM-
REM=.ABS.RE-
RMM=.ABS.RM-
FIPDE=-DR*FATAN2.(.IMAG.TE,.REAL.TE)-
FIPDM=-DR*FATAN2.(.IMAG.TM,.REAL.TM)-
```

Fig. 34(cont.)

```
                PRE=DR*FATAN2.(.IMAG.RE,.REAL.RE)-
                PRM=DR*FATAN2.(.IMAG.RM,.REAL.RM)-
                 WRITE OUTPUT ,F11-
  F F11         (///22X,9HMAGNITUDE,14X,5HPHASE,15X,4HREAL,16X,4HIMAG/)
                 WRITE OUTPUT ,F7,(REM,PRE,RE)-
  F F7          (1X,13HTE REFLECTION,6X,2(F15.8,5X),2(E15.7,5X))-
                 WRITE OUTPUT ,F8,(TEM,FIPDM,TM)-
  F F8          (1X,15HTE TRANSMISSION,4X,2(F15.8,5X),2(E15.7,5X))    -
                 WRITE OUTPUT ,F9,(RMM,PRM,RM)-
  F F9          (1X,13HTM REFLECTION,6X,2(F15.8,5X),2(E15.7,5X))-
    S50          WRITE OUTPUT ,F10,(TMM,FIPDM,TM)-
  F F10         (1X,15HTM TRANSMISSION,4X,2(F15.8,5X),2(E15.7,5X))    -
                 PROVIDED (THETA.G.0.), TRANSFER (SX)-
                EQTE=((1.-REM)/(1.+REM)).P.2-
                EQTM=((1.-RMM)/(1.+RMM)).P.2-
                DQTE=-PRE/720.-
                DEL=PRM/.ABS.(PRM)-
                DQTM=(PRM-DEL*180.)/720.-
                 WRITE OUTPUT ,F12,(EQTE,DQTE,EQTM,DQTM)-
  F F12         (/50X,17HEQUIVALENT MEDIUM/5X,5HEQTE=,F15.8,5X,5HDQTE=,
                     F15.8,5X,5HEQTM=,F15.8,5X,5HDQTM=,F15.8)
    SX          SQR=CSQRTL.(EQTE-SS)-
                RQTE=(CC-SQR)/(CC+SQR)-
                PQTE=-720.*DQTE*CC-
                SQR=CSQRTL.(EQTM-SS)-
                PQTM=720.*DQTM*CC+DEL*180.-
                RQTM=(EQTM*CC-SQR)/(EQTM*CC+SQR)-
                 WRITE OUTPUT ,F13,(RQTE,PQTE,RQTM,PQTM)-
  F F13         (15X,5HRQTE=,2E15.7,5X,5HPQTE=,E15.7/5X,5HRQTM=,2E15.7,
                                               5X,5HPQTM=,E15.7) -
    S99         THETA=THETA+TH-
```

Fig. 34(cont.)

SOURCE LANGUAGE STATEMENTS

```
                  COMPLEX (CSQRTL.,CEXPI..CI.FC.G.AE,AB,BE,BM,GZ,F,YI,YII,
                              EXPI,EXPII,AEP,AMP,EXP,RE,RM,FP,FM,TE,TM)-
                  COMPLEX (UC(100),EC(100),G(100))-
                  COMPLEX (AM)-
                  COMPLEX (SQR)-
                  COMPLEX (RQTE,RQTM)-
                  COMPLEX (ARGG)-
                  DIMENSION (FNE(100),FNU(100),Z(100))-
      FIRST       READ INPUT ,8,(KKK,N)-
                  READ INPUT ,7,(D,TH)-
                  READ INPUT ,6,((FNE(I),I=1,1,I.LE.N))-
                  READ INPUT ,6,((FNU(I),I=1,1,I.LE.N))-
                 DR=57.295779-
                  WRITE OUTPUT ,F1-
                  WRITE OUTPUT ,F2,((I,FNE(I),FNU(I),I=1,1,I.LE.N))-
    F F1        (///40X,24HELECTRON DENSITY PROFILE//32X,5HLAYER,7X,
                                        7HDENSITY,10X,9HCOLLISION/)
    F F2        (33X,I3,4X,E15.7,4XE15.7)      -
                  DO THROUGH (S10),I=1,1,I.LE.N-
      S10        UC(I)=1.-
                 NN=N+1-
                 TPI=6.2831853-
                 CI=.I.1.-
                 EC(NN)=1.-
                 UC(NN)=1.-
                 M=90/TH-
                 Z(0)=.0-
                  DO THROUGH (S100),K=1,1,K.LE.KKK-
                  READ INPUT ,7,(FGC)-
                 WAVE=30./FGC-
```

Fig. 35. New program. Electron density profile version.

```
               DO THROUGH (S30),I=1,1,I.LE.N-
              II=I-1-
      S30     Z(I)=Z(II)+D*TPI/WAVE-
               DO THROUGH (S25),I=1,1,I.LE.N-
              FPL=8.97*SQRT.(FNE(I))*10.P.3-
              X=FNU(I)/(TPI*FPL)-
              Y=FGC/FPL*10.P.9-
              DEN=X*X+Y*Y-
              EC(I)=1.-1./DEN-.I.(X/(Y*DEN))-
               WRITE OUTPUT ,1,(X,Y,DEN,FPL)-
      S25      CONTINUE -
              THETA=0-
               WRITE OUTPUT ,F3,(FGC,WAVE)-
               WRITE OUTPUT ,F4,((I,Z(I),EC(I),I=1,1,I.LE.N))-
    F F3      (///30X,10HFREQUENCY=,F15.8,3HGC.,7X,11HWAVELENGTH=,
                F15.8,3HCM.//5X,5HLAYER,20X,2HKZ,26X,12HPERMITTIVITY/)
    F F4      (16X,I3,5X,F15.8,10X,2E15.7)  -
               DO THROUGH (S99),J=1,1,J.LE.M-
              THET=0.01745329*THETA-
              SS=SIN.(THET)*SIN.(THET)-
              CC=COS.(THET)-
               DO THROUGH (S40),II=1,1,II.LE.N-
              ARGG=SS-EC(II)-
              G(II)=CSQRTL.(ARGG)-
      S40      WRITE OUTPUT ,1,(THET,SS,CC,G(II),EC(II))-
              GZ=CI*CC-
              G(NN)=CI*CC-
              AE=.5*(1.+UC(1)*GZ/G(1))-
              BE=.5*(1.-UC(1)*GZ/G(1))-
              AM=.5*(1.+EC(1)*GZ/G(1))-
```

Fig. 35(cont.)

```
        BM=.5*(1.-EC(1)*GZ/G(1))-
         WRITE OUTPUT ,F5,(THETA)-
F F5    (///30X,6HTHETA=,F15.8//6HLAYER,9X,5HGAMMA.
            22X,2HAE,20X,2HBE,20X,2HAM,20X,2HBM/)
        J1=1-
         WRITE OUTPUT ,F6,(J1,G(1),AE,BE,AM,BM)-
F F6    (1X,I3,2E14.6,8E11.3)   -
         DO THROUGH (S50),II=2,1,II.LE.NN-
        I=II-1-
        F=UC(II)*G(I)/(UC(I)*G(II))-
        YI=G(I)*Z(I)-
        YII=G(II)*Z(I)-
        FP=1.+F-
        FM=1.-F-
        EXPI=CEXPL.(YI)-
        EXPII=CEXPL.(YII)-
        AEP=AE-
        AMP=AM-
        AE=(.5/EXPII)*(AEP*EXPI*FP+BE*FM/EXPI)-
        BE=.5*EXPII*(AEP*EXPI*FM+BE*FP/EXPI)-
        F=EC(II)*G(I)/(EC(I)*G(II))-
        FP=1.+F-
        FM=1.-F-
        AM=(.5/EXPII)*(AMP*EXPI*FP+BM*FM/EXPI)-
        BM=.5*EXPII*(AMP*EXPI*FM+BM*FP/EXPI)-
         WRITE OUTPUT ,F6,(II,G(II),AE,BE,AM,BM)-
  S50    CONTINUE -
        TE=1./AE-
        TM=1./AM-
        ARG=2.*CC*Z(N)-
```

Fig. 35(cont.)

```
            EXP=COS.(ARG)-.I.SIN.(ARG)-
            RE=EXP*BE/AE-
            RM=EXP*BM/AM-
            TEM=.ABS.TE-
            TMM=.ABS.TM-
            REM=.ABS.RE-
            RMM=.ABS.RM-
            FIPDE=-DR*FATAN2.(.IMAG.TE,.REAL.TE)-
            FIPDM=-DR*FATAN2.(.IMAG.TM,.REAL.TM)-
            PRE=DR*FATAN2.(.IMAG.RE,.REAL.RE)-
            PRM=DR*FATAN2.(.IMAG.RM,.REAL.RM)-
             WRITE OUTPUT ,F11-
F F11       (///22X,9HMAGNITUDE,14X,5HPHASE,15X,4HREAL,16X,4HIMAG/)      -
             WRITE OUTPUT ,F7,(REM,PRE,RE)-
F F7        (1X,13HTE REFLECTION,6X,2(F15.8,5X),2(E15.7,5X))-
             WRITE OUTPUT ,F8,(TEM,FIPDE,TE)-
F F8        (1X,15HTE TRANSMISSION,4X,2(F15.8,5X),2(E15.7,5X))    -
             WRITE OUTPUT ,F9,(RMM,PRM,RM)-
F F9        (1X,13HTM REFLECTION,6X,2(F15.8,5X),2(E15.7,5X))-
             WRITE OUTPUT ,F10,(TMM,FIPDM,TM)-
F F10       (1X,15HTM TRANSMISSION,4X,2(F15.8,5X),2(E15.7,5X))    -
             PROVIDED (THETA.G.O.), TRANSFER (SX)-
            EQTE=((1.-REM)/(1.+REM)).P.2-  .
            EQTM=((1.-RMM)/(1.+RMM)).P.2-
            DQTE=-PRE/720.-
            DEL=PRM/.ABS.(PRM)-
            DQTM=(PRM-DEL*180.)/720.-
             WRITE OUTPUT ,F12,(EQTE,DQTE,EQTM,DQTM)-
F F12       (/50X,17HEQUIVALENT MEDIUM/5X,5HEQTE=,F15.8,5X,5HDQTE=,
         -       F15.8,5X,5HEQTM=,F15.8,5X,5HDQTM=,F15.8)
```

Fig. 35(cont.)

```
    SX          SQR=CSQRTL.(EQTE-SS)-
                RQTE=(CC-SQR)/(CC+SQR)-
                PQTE=-720.*DQTE*CC-
                SQR=CSQRTL.(EQTM-SS)-
                PQTM=720.*DQTM*CC+DEL*180.-
                RQTM=(EQTM*CC-SQR)/(EQTM*CC+SQR)-
                 WRITE OUTPUT ,F13,(RQTE,PQTE,RQTM,PQTM)-
F   F13        (15X,5HRQTE=,2E15.7,5X,5HPQTE=,E15.7/5X.5HRQTM=
                                ,2E15.7,5X,   5HPQTM=,E15.7)      -
    S99         THETA=THETA+TH-
    S100         WRITE OUTPUT ,2-
                 END PROGRAM (FIRST)-
```

Fig. 35(cont.)

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