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ERROR RATE REDUCTION OF PARITY CHECKED TELEMETRY DATA BY A LIKELIHOOD DELETION STRATEGY

by Dale R. Lumb and Frank Neuman

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SUMMARY

Scientists with experiments on interplanetary space probes receive telemetered data which contain undetected errors. The significance of the error rate depends upon the experiment. This paper presents a technique for reducing the error rate by deleting words that are likely to have errors which are not detected by a parity check code. Consequently trade-offs can be made between the word error rate and the word deletion rate for binary data words that are received from a deep space probe at a constant transmission rate. These trade-offs are based on a consideration of the matched filter (data demodulator) level output of each received bit, in addition to the binary decision data usually used from the data demodulator.

In the report, the theoretical foundations for the proposed error detection scheme are laid. It is shown that the theoretical model agrees reasonably well with results obtained from existing space communications hardware. Also, in the interest of a simple application of the described scheme, it was determined that a four level quantization of the data demodulator output as well as a four category specification of the word quality was sufficient to approach the performance of the method with the actual levels used in the elaborate calculations.

INTRODUCTION

Coding studies applicable to space communications are being carried on by several investigating groups (e.g., refs. 1, 2, and 3). The more powerful techniques under development involve fairly sophisticated encoding schemes. However, in the interest of spacecraft simplicity and reliability, telemetry schemes usually have been kept as simple as possible. For instance, Pioneer VI and Mariner II and IV have provision only for discrete changes of telemetry bit rates; also a fixed telemetry word size is used with a simple check for errors in the word based on one parity bit added per word. This parity check permits detection of an odd number of errors per word. Words tagged as having a parity error are usually deleted while words with an even number of errors go undetected. This paper describes a method for reducing these undetected errors.

The method is based on information contained in the data demodulation process. Matched filter detection of biphase modulated signals makes possible bit-by-bit decisions of binary "zeros" or "ones" with no regard for the quality of the individual received bits. The scheme described uses matched filter output values (at the bit decision times) to compute the likelihood that a word is in error even though parity has not detected the error. Based on a predetermined error probability threshold, words suspected of containing an error may be deleted.

The deletion scheme proposed here is especially applicable to deep space probe telemetry data. A deep space scientific probe typically carries several experiments for measuring physical phenomena. The data from these experiments are cast into words of fixed size and are then combined into data frames by time division multiplexing. The spacecraft-to-earth communications link sets the error rate which is the same for all experiments. However, for meaningful data, some experiments are affected more than others by the error rate. For example, in experiments measuring transient phenomena, data words in error cannot be readily recognized by values of adjacent data points. Thus, this type of experiment requires a very low error rate. On the other hand for experiments measuring slowly varying phenomena, a wrong data word might be easily recognizable. Thus, the experimenter could tolerate a relatively high error rate. Such different requirements create a problem when a decision must be made to decrease the bit rate as the space probe gets farther away from the earth. This problem is alleviated when there is a measure of the quality to be assigned to each word which has not been tagged in error by the parity check. The experimenters requiring the lowest undetected error rate can then reduce the error rate by deleting questionable words.

The word deletion scheme was developed and tested for a data channel with independent additive Gaussian noise. Specific attention was given to simplifying hardware requirements by searching for the smallest number of bits with which the quality of the signal at the matched filter output could be characterized without significant loss of information. The search was conducted by writing a computer program for the model and testing it with the simulated data. Also, a theoretical formulation was derived for hardware design. Finally, the scheme was proven by using data obtained from a test with Pioneer VI equipment in conjunction with the Goldstone Deep Space Instrumentation Facility.

SYMBOLS

n — number of bits in a word

P_B — probability of removing a zero error word by strategy S1 (measure of "bad" removal)

P_{c_j} — area under the normal probability distribution curve corresponding to a correct bit in the quantization interval j

$P(C_j)$ — probability that a received bit r_j is correct given its quantized I and D level

P_{e_j} — area under the normal probability distribution curve corresponding to an error bit in the quantization interval j

$P(E_j)$ — probability that a received bit r_j is in error given its quantized I and D level

P_e — channel bit error probability

P_{ev_k} — event probability

P_G — probability of removing an untagged single error word by strategy S1 (measure of "good" removal)

$P(R/xE)$ — probability of removal of an error word by strategy S2 given that there are "x" errors in a word

P_{WE} — word error probability

$P(xE)$ — probability that "x" errors occur in a word

$P(\geq 2E/\neq 1E)$ — probability of two or more errors in a word given that there is not one error in the word

R_D — deletion rate

W — word

X_T — deletion threshold

WORD ERROR PROBABILITY AND DELETION RATE BASED ON PARITY ERROR CHECK

The following sections serve to introduce the concepts of word error probability and word deletion rate. It will be shown that with a parity error check a certain number of words known to be in error will be deleted, thus reducing both the word error probability as well as the number of data words accepted.

In data transmission from space probes two types of data formats have been predominantly used, namely, NRZ-L and NRZ-M. The abbreviation NRZ-L stands for nonreturn-to-zero level where a binary 1 is represented by one phase and a 0 by another phase of the transmitted signal; and NRZ-M stands for nonreturn-to-zero mark where a phase change indicates that a 1 was sent. For coherent detection of binary antipodal (phase shift keying) signals by means of self-synchronization techniques, both data formats possess certain advantages and disadvantages, which will be discussed in the following sections.

Parity Error Check Code for NRZ-L Data

The bit detector demodulates the data with phase ambiguity, after bit acquisition. This ambiguity means the demodulated data bits may be the true phases representing the binary 0 and 1 data as transmitted, or they may be the complements of the original data. For NRZ-L data this ambiguity can be resolved after the bit acquisition. The true phase of the data as transmitted can be determined by inserting known words periodically into the data; however, these additional words reduce the available data rate. The advantage of NRZ-L will become apparent in later sections.

The word error rate detected by a parity check can be calculated from the bit error probability. For NRZ-L transmission which uses $n - 1$ data bits per word, a parity bit is added for each word, such that the modulo two sum of the n bits is 1 (for odd parity) or 0 (for even parity). If an odd number of errors occurs within a word, the parity computation of the received data word will disclose such errors. If the errors are statistically independent, they can be represented by a binomial model where the probability of a detected word error is

$$\sum_{j=1,3,5,\ldots}^{n} \binom{n}{j} P_e^j (1 - P_e)^{n-j} \tag{1}$$

where P_e is the bit error probability and n is the number of bits in a word. A measure of the relative importance of the first term of equation (1) compared to the remaining terms is given by the conditional probability that one error occurs given an odd number of errors occurs:

$$P(1E/1E \text{ or } 3E \text{ or } \ldots) = \frac{P(1E)}{P(1E \text{ or } 3E \text{ or } \ldots)}$$

For a seven bit word this conditional probability is plotted in figure 1 as a function of P_e. This curve shows that for P_e less than 5 percent, the detected word error rate is within 1.5 percent of the probability that one error occurs in a word:

$$P(1E) = \binom{7}{1} P_e (1 - P_e)^6 \tag{2}$$

Since an error condition is known to exist from the parity check, the word is usually discarded, and thus $P(1E)$ also represents the deletion rate, R_D, in the data received.

The undetected word error rate is important since it determines whether the data can be used by experimenters. An even number of errors in a word causes an undetected word error condition whose probability can be expressed as

$$\sum_{j=2,4,6,\ldots}^{n} \binom{n}{j} P_e^j (1 - P_e)^{n-j} \tag{3}$$

The relative predominance of the first term in this expression is again represented by the conditional probability that two errors occur given that an even number of errors occurs (see fig. 1). This figure shows that for P_e less than 5 percent, the undetected word error rate is dominated by the probability that two errors occur in a word

$$P(2E) = \binom{n}{2}P_e^2(1 - P_e)^{n-2} \tag{4}$$

The word error probability based on the words remaining after single error words tagged by parity have been removed is then

$$P_{WE} = \frac{P(2E)}{1 - R_D} \tag{5}$$

where R_D is the deletion rate

$$R_D = P(1E) \tag{6}$$

In figure 2 the word error probability has been plotted versus deletion rate, with bit error probability as a parameter. This curve is shown for both NRZ-L and NRZ-M data. The derivation of P_{WE} and R_D for NRZ-M data is given in the next section. It is noted that NRZ-M data have a consistently higher word error probability and deletion rate for the same bit error probability.

Parity Error Check for NRZ-M Data

The adverse effects of the phase ambiguity in the data demodulator, which were discussed in the previous section, can be avoided if the value of the digit is indicated by the change of phase, or lack of change, rather than by the phase itself. The NRZ-M format is one such method which can be viewed as a transformation from the NRZ-L format. This transformation is illustrated in figures 3(a) and (b). The NRZ-M format is used for Pioneer VI telemetry data.

Several problems result from the choice of a transition sensitive code such as NRZ-M, particularly when it is used in connection with a parity check. The bit detector usually makes errors singly. This is the case when the additive white Gaussian noise assumption is valid. A single bit error from the bit detector, called a channel error, is followed by an adjacent error, when the data are converted from NRZ-M to the original form.

The double error conjecture is shown to be true by considering all possible sequences of 0 and 1 of length 2. This is illustrated in figures 3(c) and (d), where E represents an original channel error caused by noise in the bit detector and E' corresponds to an error in the reconstruction from NRZ-M to the original data form. Figures 3(e), (f), (g), and (h) show the effects of two and three successive errors in the bit detector.

NRZ-M differs from NRZ-L in several important respects when data are grouped in simple parity checked words. In order to detect single channel errors, parity is computed only on alternate bits on the level data (NRZ-L) prior to being converted to NRZ-M as illustrated in figures 4(a) and (b). The net effect of the error carryover in the NRZ-M format is to produce an interword influence with respect to errors (see figs. 4(e) and (f)). This effect and the performance degradation will be noted later in the report.

The difference in deletion rate and word error rate for NRZ-M data as compared to NRZ-L data (see fig. 2) is due to the error carryover effect from one word, W1 to the immediately following word, W2. For example, when a channel error occurs in the parity bit of W1, then there is the carryover error into W2. It follows that all single channel errors in W2 will be undetected errors which would have been detected by the parity check in NRZ-L data. Also, if no channel error occurs in W2 preceded by a parity error in W1, then the word W2 will be discarded due to parity of W2; hence, the deletion rate is increased.

In order to include these carryover effects in the formulations of deletion rate and word error probability (rate), consider that portion of the words which is not influenced by adjacent single and double errors, namely, $(1 - \epsilon)$ where $\epsilon = P_e$. If it is assumed that P_e is small enough that the occurrences of other than single and double errors in words are negligible, ϵ may be closely approximated by $\epsilon = (1/7)P(1E) + (2/7)P(2E)$. This form is useful for the simplification of some equations to be developed.

The words that will be deleted as a result of parity tagging include

(1) Tagged words deleted because of one channel error, and not affected by error carryover. The probability of occurrence is $P(1E)(1 - \epsilon)$.

(2) Words with no errors and words with two channel errors which are deleted because of the error carryover effect. (The previous word has an error in the parity bit.) The probability of occurrence is $\epsilon[P(0E) + P(2E)]$. Hence the deletion rate is

$$R_D = (1 - \epsilon)P(1E) + \epsilon[P(0E) + P(2E)] \tag{7}$$

In the words remaining, the undetected errors contributing to the word error probability are

(1) The protion of words with two channel errors and not affected by carryover $(1 - \epsilon)P(2E)$

(2) The portion of words with single channel errors preceded by errors that occurred in parity, $\epsilon P(1E)$. Hence, the word error probability is

$$P_{WE} = \frac{(1 - \epsilon)P(2E) + \epsilon P(1E)}{1 - R_D} \tag{8}$$

The situation is shown schematically in rows 1 to 3 of figure 5, and P_{WE} versus R_D is plotted in figure 2.

THE LIKELIHOOD DELETION STRATEGY

In the previous section the concepts of word error rate and word deletion rate were introduced as being natural consequences of organizing data in words, each word being parity checked. In the following sections the likelihood deletion strategy will be developed. It will be shown by means of a mathematical model that deletion versus error rate trade-offs can be made by using the individual bit error probabilities determined from the data demodulation process.

Statistical Model of Received Telemetry Data

The following two assumptions will be made in the development of a statistical model of the received telemetry data. First, the telemetry data are biphase modulated before transmission. This binary signaling alphabet is theoretically optimum (see ref. 4, ch. 8). Second, independent white Gaussian noise is assumed to be added onto the received telemetry signal with coherent matched filter detection techniques used at the receiver. Experimental results in a later section will verify the second assumption to be approximately valid for the Pioneer VI telemetry. Under these assumptions, the probability density function of the matched filter output is normal and is shown in figure 6(a), where $p(v_0)$ is the probability density of the matched filter output when the true value of the transmitted bit is 0, and $p(v_1)$ is the density when the true value is 1. Henceforth, the matched outputs, v, will be called "integrate" and "dump" levels (I and D levels). The shaded area under the curve represents the probability of a channel bit error, P_e, in particular, the probability that a transmitted 0 will be interpreted as a 1 in the hard decision process following the I and D output.

Assuming there is no bias to 0 or 1 errors and the variances are equal, the analysis is simplified when the statistical model is represented by a single standarized Gaussian probability distribution where the mean is 0 and the variance is 1. Hence, in terms of the standardized variable $x = v - \bar{v}/\sigma$ with mean and variance $\mu_x = 0$ and $\sigma_x^2 = 1$, respectively, the normalized distribution for v_0, $p(x)$, is shown in figure 6(b). The standardized distribution for $P(v_1)$ is the mirror image of that for $P(v_0)$. Figure 6(b) has been redrawn in two distributions, figures 7(a) and 7(b). These figures show the distributions for the correct bits and the error bits, respectively. Figure 7 also includes the effect of truncation due to the finite dynamic range of the circuits. The effect is minimal, it simply reflects the small proportion of levels which would fall outside the range into the levels of largest absolute amplitude.

The I and D levels now will be quantized for convenience of analysis as well as simplicity of application. The probability that a received bit, r_j, is in error, given its quantized I and D level v_j, is:

$$P(E_j) = \frac{P_{e_j}}{P_{e_j} + P_{c_j}} \tag{9}$$

and the probability that the bit is correct is:

$$P(C_j) = \frac{P_{c_j}}{P_{e_j} + P_{c_j}} \tag{10}$$

where

$$\left.\begin{aligned} P_{c_j} &= \frac{1}{\sqrt{2\pi}} \int_{T_{-j}}^{T_{-j+1}} e^{-x^2/2}\, dx \\ P_{e_j} &= \frac{1}{\sqrt{2\pi}} \int_{T_{j-1}}^{T_j} e^{-x^2/2}\, dx \end{aligned}\right\} \tag{11}$$

A quantization scheme is illustrated in figure 7 where equally spaced intervals over 3/12 of the linear range of the I and D output are given, with the fourth interval taking in the remaining 9/12. As will be shown later, this is approximately an optimum method of quantizing for the given number of quantization intervals, namely 4.

It stands to reason that the levels which are more likely to be in error should be more finely quantized (ref. 5). For future reference, this quantization scheme of dividing the levels into n intervals is defined as follows: the scheme will be called n/m level quantization when there are altogether n intervals on each side of zero of the distribution, and the $n - 1$ levels close to zero are equally spaced intervals each of $1/m$ of the total I and D output dynamic range. The scheme of figure 7 would therefore be called a 4/12 level scheme.

Criterion for the Detection and Deletion Method

The statistical model will now be applied to develop a criterion for deleting questionable words given their I and D levels.

Because there is no parity check provided to detect an even number of channel errors in a word, no procedure can be established that could detect such errors with certainty. An even number of channel errors in a word can, therefore, only be detected on a probabilistic basis. Since for the range of P_e of interest, the error contribution due to three or more errors is negligible compared to single and double errors, the following is considered a measure of the occurrence of double errors in the NRZ-L case. Given that not one error occurs in a word (i.e., a word without a parity tag), and given the I and D levels, the probability that two or more errors occur in a word has been derived in appendix A to be

$$P(\geq 2E/\neq 1E) = 1 - \frac{\prod_{k=1}^{7} [1 - P(E_k)]}{1 - \sum_{i=1}^{7} P(E_i) \prod_{\substack{j=1 \\ j\neq i}}^{7} [1 - P(E_j)]} \tag{12}$$

This equation is the basis for deleting questionable words. For each word that has not been tagged by the parity check, the probability,

$P(\geq 2E/\neq 1E)$, could be computed. If the value exceeds a preassigned threshold, the word is marked and discarded as a word with most likely two errors in it. Actually, this criterion will also discard a small percentage of words with no errors. Thus, there results as a function of threshold a probability of removal of words having two errors, $P(R/2E)$, and a probability of removal of words having no errors, $P(R/OE)$. These can be used to determine the resulting deletion rates and word error probabilities. Figure 8 shows $P(R/2E)$ and $P(R/OE)$ versus threshold and the method of determining the parameters is described in appendix B.

Deletion Rate and Word Error Probility for NRZ-L Data

Without special processing of the data, only the single error tagged words would be removed. With processing, the new deletion rate and word error probability are functions of the deletion threshold of the likelihood double error detection scheme as shown below.

The number of words removed is the sum of the following three terms.

(1) Number of parity tagged words

(2) Number of double error words removed by double error detection

(3) Number of zero error words removed by double error detection.

Dividing each term by the total number of words received one obtains the deletion rate

$$R_D = P(1E) + P_{ave}(\geq 2E/\neq 1E)P(R/2E) + P(OE)P(R/OE) \tag{13}$$

where

$P(R/2E)$ = probability of a word being removed, given that a double channel error exists in a word

and

$P(R/OE)$ = probability of a word being removed, given that there is no error in a word

Both $P(R/2E)$ and $P(R/OE)$ are functions of the deletion threshold and $P_{ave}(\geq 2E/\neq 1E)$ is the a priori probability of two errors given that there is not one error. (This probability is derived in appendix A. Note the contrast with $P(\geq 2E/\neq 1E)$ in the previous section where the I and D levels are assumed given.)

The word error probability is simply the number of the double error words not removed by application of the likelihood deletion scheme divided by the number of remaining words.

$$P_{WE} = \frac{P(2E)[1 - P(R/2E)]}{1 - R_D} \tag{14}$$

P(R/2E) and P(R/OE) have been computed for various thresholds (figs. 8(a), (b), and (c)) and from them R_D and P_{WE} were computed. In figures 9 and 10 P_{WE} has been plotted against R_D for various deletion thresholds and bit error rates. These parameters were computed from equations (13) and (14) where P(R/OE) and P(R/2E) were determined by the Monte Carlo method. In figure 10 P_{WE} versus R_D are compared for a 4/12 level scheme with the theoretical computations discussed in a later section. The data show good agreement, thus proving the validity of the Monte Carlo method.

Deletion Rate and Word Error Probability for NRZ-M Data

In the previous discussions, the results were based on receiving NRZ-L data which produce no error carryover. The error carryover effect for NRZ-M data makes the simple application of the likelihood deletion method less effective. Therefore a somewhat more complicated overall strategy has been developed. This strategy consists of a sequence of three strategies: S0, deletion of single error words by parity check; S1, to be discussed in the following paragraphs; and S2, the likelihood deletion strategy which is identical to that described for the NRZ-L data.

The largest contribution of word errors which causes NRZ-M to be worse than NRZ-L is made by single error words which are untagged because they follow other single error words with their error in the parity bit. The additional strategy S1 has been developed to alleviate this carryover effect. For example, if P_e is 1.5 percent, this error contribution is $1/7\ P^2(1E) = 1.32 \times 10^{-3}$ and may be compared to the probability of a double channel error, which is $P(2E) = 4.97 \times 10^{-3}$. Thus, even if there existed a perfect double error reduction strategy (all double error words removed but no others) the error rate would only have been reduced to 21 percent of the original error probability. Therefore, before operating on untagged words with the double error reduction strategy (which is S2), the relatively effective strategy, S1, will be applied in order to remove most of the error carryover untagged single error words. Strategy S1 is defined as follows:

> Remove the untagged word following a tagged word if among the I and D level classes occupied by the digits of W1, the tag bit of W1 is in the lowest class, but do not remove the untagged word, W2, if it is preceded by two tagged words.

The last part of the strategy is designed to keep the deletion rate low by excluding the relatively frequent case of two zero-channel error words following a single error word with the error in the parity bit, an event which has the probability $1/7\ P(1E)P^2(0E)$. Strategy S1 removes most of the previously undetected error words, and it also removes some correct words.

The two probabilities required for calculating the effectiveness of S1 have been determined by the Monte Carlo method, by generating many single error words with the error bit in a known position and tabulating their properties. These probabilities are

P_{G_1} = probability of detecting the error in the parity bit, given that there is an error in the parity bit

$$= \frac{\text{number of words where the error bit is in the lowest occupied class of I and D levels}}{\text{total number of single error words}} \tag{15}$$

and

P_{B_1} = probability of unnecessarily removing a word following a single error word, given that the error in W1 is not in the parity bit

= probability that the I and D level of the error in W1 is not in the low class and the I and D level parity bit is in the low class

$$= \left(\frac{\text{number of words with the error not alone in the low class of I and D levels}}{\text{total number of single error words}} \cdot \frac{\text{average number of low level bits in such words (except for the error bit)}}{n - 1} + \frac{\text{number of words with the error not in the low class}}{\text{total number of single error words}} \cdot \frac{\text{average number of low level bits in such words}}{n - 1} \right) \tag{16}$$

The numerical subscripts on P_G and P_B indicate the number of errors in W1. The number $(n - 1)$ in these equations comes from the fact that the low level bits are distributed among the $(n - 1)$ correct bits in the n bit word. The following example for an 0.8-percent bit error probability and $n = 7$ shows the effectiveness of strategy S1:

$$P_{G_1} = \frac{19149}{20000} = 0.9575$$

$$P_{B_1} = \frac{2399}{20000} \times \frac{1.442}{6} + \frac{851}{20000} \times \frac{2.13}{6} = 0.0439$$

P_{G_1} and P_{B_1} were evaluated by the Monte Carlo method and are shown in figure 11 for a range of channel error probabilities. It is noted that while the finely quantized system S1 has a lower deletion rate than that of the coarsely quantized system, the error detection for the finely quantized system is worse.

For the formulation of the final equations for R_D and P_{WE}, it will be necessary to enumerate all the conditions of one and two errors in word number 1, W1, and the resultant error situations that can occur in the adjacent following word number 2, W2. The enumeration is shown graphically in figure 5 which will be explained in the following text. In the figure it is assumed that zero, one and two errors in a word essentially exhaust all possibilities; indeed, for a bit error probability of 1 percent, P(1E) + P(2E) + P(0E) = 0.987. Rows 1 through 3 of figure 5 have been explained in the section on parity error check for NRZ-M data. Rows 4, 5, and 6 detail strategy S1. Row 4 separates the part of S1 which says that untagged words are not removed if they are preceded by two tagged words. (There are some other combinations which cause a single tag to appear due to an error combination of three words. They have not been considered here, since their probability of occurring is negligible.) Besides giving the equation for the probability of each subevent the numerical value for P_e = 1 percent is given as a means for judging which terms are negligible. Note that upon each branching, the probabilities of the subevents add to 1. Under each block, the joint probability of the subevent is shown. It is obtained simply by multiplying the joint probability of the more general subevent above the block by the probability of the subevent. In row 5 the tagged words which were not preceded by another tagged word are subdivided, according to whether they caused adjacent error influence or not. (Tagged zero error words, of course, are not subdivided.) In row 6, a further subclassification takes place according to the first part of S1 which says, "remove the untagged word following a tagged word, if the parity bit of W1 is in the lowest I and D level class." The values of P_{G_2} and P_{B_2} are not shown, but they must be certainly less than 1. Comparison of the joint probabilities shown below row 6 shows that the contribution to the word removal of the events involving P_{G_2} and P_{B_2} is negligibly small; however, P_{B_0} is relatively large, 0.48. It was calculated from the theory program discussed in the theoretical formulation section

$$P_{B_0} = \frac{\sum_{k=1}^{N} \binom{n_k}{7} P_k(0E) P_{ev_k}}{P_{ave}(0E)} \tag{17}$$

when n_k are the number of bits in the lowest occupied class of I and D level for the given type of sample. The number of untagged words removed by strategy S1 is shown on row 7. One observes that for the 1 percent P_e example, 76 percent of the previously undetected single error words are removed at the small cost of removing 0.2 percent of the zero error words. Row 8 shows the remaining words after strategy S1. Each class simply consists of the total number of words in that class reduced by the number removed by strategy S1. Row 9 represents the final splitting by application of strategy S2 on the remaining words. Numbers are not shown since they depend on the selected threshold.

The effects of the three strategies can now be read off the figure. The following expressions are in per unit of the total number of words.

Total words remaining after S0:

$$\epsilon P(1E) + (1 - \epsilon)[P(0E) + P(2E)]$$

Untagged error words remaining after S0:

$$\epsilon P(1E) + (1 - \epsilon)P(2E)$$

Zero error words removed by S1:

$$\left\{(1 - \epsilon)P(1E)\left[\frac{P(0E) + \frac{5}{7}P(0E)}{P(0E) + \frac{5}{7}P(2E) + \frac{6}{7}P(1E)}\right]\left(\frac{6}{7}\right)P_{B_1} + \epsilon P(0E) + \frac{2}{7}\frac{P(2E)}{\epsilon}P_{B_0}\right\}P(0E)$$

$$\triangleq P(S1_B)P(0E)$$

where the expression in braces defines $P(S1_B)$.

Untagged double error words removed by S1 are:

$$P(S1_B)P(2E)$$

Untagged single error words removed by S1:

$$\left\{(1 - \epsilon)P(1E)\left[\frac{P(0E) + \frac{5}{7}P(2E)}{P(0E) + \frac{5}{7}P(2E) + \frac{6}{7}P(1E)}\right]\frac{1}{7}P_{G_1}\right\}P(1E) \triangleq P(S1_G)P(1E)$$

which defines $P(S1_G)$.

Words remaining after S1:

$$[\epsilon - P(S1_G)]P(1E) + [(1 - \epsilon) - P(S1_B)][P(0E) + P(2E)]$$

Error words remaining after S1:

$$[\epsilon - P(S1_G)]P(1E) + [(1 - \epsilon) - P(S1_B)]P(2E)$$

Deletion rate after S1 only (1 - words remaining after S1):

$$R_{D1} = 1 - [\epsilon - P(S1_G)]P(1E) + [(1 - \epsilon) - P(S1_B)][P(0E) + P(2E)] \tag{18}$$

Word error probability (total words remaining after S1):

$$P_{WE_1} = \frac{[\epsilon - P(S1_G)]P(1E) + [(1 - \epsilon) - P(S1_B)]P(2E)}{1 - R_{D_1}} \tag{19}$$

Words remaining after S2:

$$[\epsilon - P(S1_G)]P(1E)[1 - P(R/1E)] + [(1 - \epsilon) - P(S1_B)]\{P(0E)[1 - P(R/0E)] + P(2E)[1 - P(R/2E)]\}$$

Error words remaining after S2:

$$[\epsilon - P(S1_G)]P(1E)[1 - P(R/1E)] + [(1 - \epsilon) - P(S1_B)]P(2E)[1 - P(R/2E)]$$

Deletion rate (1 - number of words remaining after S2):

$$R_{D_2} = 1 - \Big([\epsilon - P(S1_G)]P(1E)[1 - P(R/1E)] + [(1 - \epsilon) - P(S1_B)]\{P(0E)[1 - P(R/0E)] + P(2E)[1 - P(R/2E)]\}\Big) \tag{20}$$

Word error probability $\left(\frac{\text{error words remaining after S2}}{\text{total number of words remaining after S2}}\right)$:

$$P_{WE_2} = \frac{[\epsilon - P(S1_G)]P(1E)[1 - P(R/1E)] + [(1 - \epsilon) - P(S1_B)]P(2E)[1 - P(R/2E)]}{1 - R_{D_2}} \tag{21}$$

Since all parameters of the above equations have been previously computed, one can now plot the effectiveness of the likelihood deletion strategy for NRZ-M data. The results are shown in figures 9 and 10. Comparison of these two figures shows that the performance of the likelihood deletion strategy is not degraded by any significant amount by going to the simpler 4/12 quantization. Also, when comparing any two curves for the same channel bit error probability for NRZ-L and NRZ-M data one notices the effect of strategy S1, which reduces the word error probability for NRZ-M to nearly that of NRZ-L at a small cost of additional deletion rate. Figures 9 and 10 also show the trade-offs between word error probability and deletion rate.

THEORETICAL FORMULATION OF DATA WORD QUALITY CLASSIFICATION FROM QUANTIZED I AND D LEVELS

While the Monte Carlo method is sufficient for obtaining the performance of the probabilistic error detection method, it is useful to have an explicit theory. As will be shown in a later section, the results of the theory allow one to construct a simple logic network to classify words as to their quality.

In the statistical model, the word error probability of a given word does not depend on the sequential order of the various I and D levels in a word but only on their absolute magnitudes. Thus, when we have a word length n and the number of quantization levels is r, we are sampling from a multinomial distribution which has N sample points

$$N = \binom{n + r - 1}{r - 1} \tag{22}$$

To give some examples, for 7-bit words with 4 quantization levels there are 120 samples, which are easily enumerated, while for the same length of word but with 32 quantization levels, 12,620,256 different samples can occur. The event probability for a given sample type k is

$$P_{ev_k} = \frac{n!}{n_1! n_2! \; . \; . \; . \; n_r!} P_1^{n_1} P_2^{n_2} \; . \; . \; . \; P_r^{n_r} \tag{23}$$

where n_j is the number of bits for which the I and D levels occur in the voltage interval j, n is $\sum_{j=1}^{r} n_j$, and P_j is the probability that the level of a given bit occurs in that interval. This probability is the sum of the probabilities of the level being in the interval due either to an error or to a correct bit, namely

$$P_j = P_{e_j} + P_{c_j}$$

The probability that no errors occur in a given sample of type k, for $1 \leq k \leq 120$, is

$$P_k(0E) = [P(C_1)]^{n_1} [P(C_2)]^{n_2} \; . \; . \; . \; [P(C_r)]^{n_r} \tag{24}$$

where n_j is the number of bits in the quantum interval j, and $P(C_j)$ is defined in equation (10). The probability that a single channel error occurs in a sample of type k is

$$P_k(1E) = P_k(0E) \left[n_1 \frac{P(E_1)}{P(C_1)} + n_2 \frac{P(E_2)}{P(C_2)} + \; . \; . \; . \; + n_r \frac{P(E_r)}{P(C_r)} \right] , \tag{25}$$

and the probability that two or more errors occur in a particular sample, given that there is not exactly one error in the sample is expressed as

$$P_k(\geq 2E/\neq 1E) = \frac{1 - P_k(0E) - P_k(1E)}{1 - P_{ave}(1E)} \tag{26}$$

The correctness of the equations is checked by summing them over all possible samples weighted by the event probabilities and confirming that they add to the average event probabilities:

$$P_{ave}(0E) = (1 - P_e)^n = \sum_{k=1}^{N} P_k(0E)P_{ev_k} \tag{27}$$

$$P_{ave}(1E) = 7P_e(1 - P_e)^6 = \sum_{k=1}^{N} P_k(1E)P_{ev_k} \tag{28}$$

$$P_{ave}(\geq 2E/\neq 1E) = 1 - \frac{P_{ave}(0E)}{1 - P_{ave}(1E)} = \sum_{k=1}^{N} P_k(\geq 2E/\neq 1E)P_{ev_k} \tag{29}$$

For any given threshold X_{T_j} on $P_k(\geq 2E/\neq 1E)$, each sample k falls into one of two classes depending on $P_k(\geq 2E/\neq 1E) \geq X_{T_j}$ or $P_k(\geq 2E/\neq 1E) < X_{T_j}$. Actually in practice several thresholds X_{T_j} may be set so that all nonparity tagged words will be categorized according to these thresholds. This classification of the words may be considered as a quality of word assignment.

The above probabilities are shown in table I for a 1-percent bit error probability. Also shown are classifications of the word types into four quality categories beginning with 1 for the lowest probability of two or more errors, given not one error, and 4 for words with the highest error probability. This allows one to calculate the deletion rate and the word error rate for NRZ-L data exactly,[1] where untagged words above a selected error probability are eliminated.

$$R_{D_k} = P_{ave}(1E) + \sum_{k}^{N} (P_k(0E) + P_k(\geq 2E/\neq 1E)P_{ev_k}$$

$$\text{all } k \text{ such that } P_k(\geq 2E/\neq 1E) > X_{T_j} \tag{30}$$

$$P_{WE_k} = [P(\geq 2E/\neq 1E) - \sum_{k}^{N} P_k(\geq 2E/\neq 1E)P_{ev_k}] \frac{1 - P(1E)}{1 - R_{D_k}}$$

$$\text{all } k \text{ such that } P_k(\geq 2E/\neq 1E) > X_{T_j} \tag{31}$$

---

[1]For NRZ-M data the probabilities $P_k(xE)$ become conditional because of error carryover effects. Hence, this theoretical formulation is not easily extended to express R_D and P_{WE}.

where X_{T_j} is the jth decision threshold. The word error rates versus deletion rates as functions of P_e are compared in figure 10 with the rates determined by the Monte Carlo method. In addition, the theoretical formulation was used to determine the optimum four level quantizing scheme (see fig. 12). The 4/12 scheme proved to be optimum over a wide deletion range.

As can be seen from table I, a large majority of the 120 sample types fall in category 4, the high error probability category. This circumstance allows the construction of a relatively simple word classifier, which needs to distinguish only samples in types 1 to 3, calling all other words class 4. At first glance, it would seem that the word classification logic would have to be changed for any change in bit error probability. Figure 13 shows that this is not necessary. It shows the various deletion rates for constant X_T with the quality of word assignments calculated separately for each error probability. Figure 13 also shows the deletion rates if the quality of word assignments for $P_e = 1$ percent are used for deletion of words with other error probabilities.

APPLICATION OF THE LIKELIHOOD DELETION STRATEGY

As shown in the previous sections, the 4/12 quantization closely approximates the detection performance of a 32/32 quantization. With the four quantization regions the required circuitry for a classification scheme of nontagged words becomes feasible. There are two reasons for establishing only four categories in which to classify the nontagged words. The first reason is that one needs only a 2-bit qualifier for each data word; the second is that the system is much simpler than a system with more categories.

The theory program, discussed in the previous section, computes $P(\geq 2E/\neq 1E)$ for each of the 120 word classifications. In theory one can choose as many thresholds, X_{T_j}, as desired to categorize the words of all level classifications. In practice four X_{T_j} are chosen to result in a reasonable range of error reduction, where X_{T_1} represents no deletion. Then by selecting a suitable X_{T_j}, the experimenter can choose, within limits, the error rate he desires.

A functional block diagram of a 'quality-of-word' categorizer is shown in figure 14. The leftmost block of this figure is a rectifier. It is needed to discard the polarity information of the levels which must be removed before further processing. The level detector which follows classifies all bits of a word (according to their I and D levels) into four quantization regions. Included in this functional block are four counters which total the number of bits per word having levels within each of the four quantization regions. Thus, after bit 7 of each word, the counter values associate the word with 1 of the 120 categories itemized in the first four columns of table I. Using the corresponding "Quality of word category" of table I, one can easily write the logic design equations for translating the 120 allowable states of the 4 counters into 4 word quality categories. These decision logic functions are represented by the "Word classification logic" block in figure 14. To implement strategy S1, the decision logic is modified when an error carryover is suspected which would mask a single error in the word. This function is indicated by the block "Lowest level detector and memory for strategy S1" in figure 14. In summary, the categorizer makes use of the I and D levels of each nontagged telemetry word and classifies it according to its likelihood of being in error.

EXPERIMENTAL VERIFICATION

Experimental data were obtained and analyzed for two reasons; to verify the assumptions made for the statistics of the data demodulator matched filter output, and to compare the performance of an actual system with that predicted by the likelihood deletion strategy.

Test Configuration

To verify experimentally the analysis by using modulation and demodulation hardware, data were obtained employing the Deep Space Instrumentation Facility (DSIF) receiving station at Goldstone, California. The station equipment was used in conjunction with spacecraft and ground instrumentation developed for Pioneer VI. A block diagram of the test setup is shown in figure 15. To provide a known bit stream the Pioneer Data Format Generator was used as a data source for the Pioneer S-band test transmitter. The output of the transmitter fed an adjustable attenuator to provide the input to the receiver at Goldstone. Since the transmission channel noise of a deep space probe at S-band is predominantly the thermal noise of the receiver, this setup permitted a realistic test.

In the ordinary operation of the ground station, several tracks of data are simultaneously recorded. For this experiment, the raw input signal to the demodulator and the decision outputs of the on-site biphase demodulator/bit-synchronizer were recorded. The raw biphase signal track was later used as a signal source for an off-line biphase demodulator identical to the one at Goldstone. To this data demodulator an I and D hold circuit, whose function is illustrated in figures 4(c) and (d), was added which permitted the preservation of the sampled matched filter level for a full bit time. The hold circuit simplified the sampling of the levels required for digitization of the I and D levels. The data demodulator decisions were recorded on digital tape for further analysis.

Computer programs were written to use this digital tape as input in order to obtain the following information. First, the NRZ-L bit error sequence was determined. This was possible since the data format generator at Goldstone produces a known bit sequence. Second, the NRZ-L bit error sequence was converted to the demodulated NRZ-M error pattern. From these data the performance of the likelihood deletion strategy for NRZ-M data was determined. Third, various statistical parameters of the I and D levels were gathered in order to determine how well the postulated statistical model fit the actual data.

Test Results

Since the receiver and data demodulator are only an approximation of the ideal matched filter, the I and D levels deviate slightly from the postulated properties. The postulated properties are: (1) The levels are samples from a Gaussian distribution; (2) Successive I and D levels are statistically independent. The experimental data were analyzed in order to determine how well these postulated properties are met by a real system.

For each of the two channel bit error probabilities of 0.8 and 1.3 percent, the I and D level range was divided according to the 4/12 quantization model. For each level category, the I and D value of the following bit was classified according to the four categories with the error bit and the correct bit values separately tabulated. This permits the plotting of the cumulative I and D level distributions given the previous level. The results appear in figures 16(a) and (b) plotted on normal probability graph paper. Although they were not separately plotted, the distributions of negative and positive levels were coincident except for sign, so that no bias or nonsymmetric circuit gains were noted in the data demodulator or I and D hold circuit output. It should be noted that the normalized negative I and D voltages represent errors, and the cumulative distributions up to zero represent the error probabilities P_e.

Regarding the first postulate, the distributions are seen from figure 16 to be normal well into the tails. As for the second postulate, the figures show that for levels following the highest level error class (see curves labeled I in fig. 16) the error probabilities are high, $P_e = 4.1$ percent and 2.0 percent compared to the equivalent values for the entire distribution $P_e = 1.30$ and $P_e = 0.80$ percent (see curves labeled III in fig. 16). Curves labeled II in figure 16 are the distributions for bits following error bits which are in the lowest level quantization class. These curves show an intermediate error probability, which indicates a functional dependence of increasing bit error probability for I and D levels immediately following erroneous I and D levels of increasing magnitude. Thus, one concludes that for the two error probabilities analyzed, the statistics of the bit levels were dependent on the magnitude of the levels preceding them, thus forming a weak Markov chain. The net result of the adjacent bit level correlation is to give a higher double channel error probability at the expense of fewer single errors as compared to the binomial model. Thus, the actual word error probability is higher than would be computed directly from the channel bit error rate for an independent-bit decision model.

The data from the actual system were also used to determine the effectiveness of the hardware realization of the likelihood deletion scheme. More than a total of 1.5 million bits of data for two channel bit error rates of $P_e = 1.30$ and $P_e = 0.80$ percent were used for the analysis. The results of error detection by the likelihood deletion strategy for NRZ-L data are plotted in figure 17, curves I and II. The performance is to be compared with that of the independent-bit decision model of curves III and IV. The same data were converted to the NRZ-M format and the results are given in figure 18, curves I and II, and compared to the postulated model, curves III and IV. For both NRZ-L and NRZ-M data the detection scheme is seen to be somewhat less effective than that predicted by the model because of the intersymbol influence of adjacent I and D levels. However, for the Pioneer telemetry application this difference from the predicted performance still permits effective trade-offs to be achieved between word error probability and deletion rate.

CONCLUSIONS

A likelihood error detection scheme for parity error checked telemetry data was derived. The scheme uses the bit quality information contained in the matched filter (data demodulator) level output. This level information permits a quality of word assignment to be made for all words without parity error tags. The quality assignments can be used to remove those words which have the highest error probability, thus allowing experimenters to reduce the undetected word error rate at a given communication range, or to increase communication range at a given word error probability before a discrete reduction of the bit rate becomes necessary.

In the interest of a simple physical realization of the described scheme, it was determined that a four-level matched filter output quantization as well as a four-category specification of the word quality was sufficient to approach the full power of the method.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., April 26, 1966
125-23-02-02

APPENDIX A

PROBABILITY OF TWO OR MORE ERRORS IN A WORD

For the double error detection scheme, the probability of two or more errors in a 7-bit word, given there is not one error, is

$$P(\geq 2E/\neq 1E) = P\ (2 \text{ or more errors}/\text{not } 1 \text{ error})$$

$$= \frac{P\ (2 \text{ or more errors})}{P\ (\text{not } 1 \text{ error})}$$

But the probability of two or more errors is

[1 - P (no errors) - P (exactly 1 error)]

$$= 1 - \prod_{k=1}^{7} [1 - P(E_k)] - \sum_{i=1}^{7} P(E_i) \prod_{\substack{j=1 \\ j\neq i}}^{7} [1 - P(E_j)]$$

$$= 1 - \prod_{k=1}^{7} P(C_k) - \prod_{j=1}^{7} P(C_j) \sum_{i=1}^{7} \frac{P(E_i)}{P(C_i)}$$

The probability of not one error is

$$1 - P\ (\text{exactly } 1 \text{ error}) = 1 - \sum_{i=1}^{7} P(E_i) \prod_{\substack{j=1 \\ i\neq j}}^{7} [1 - P(E_j)]$$

Then

$$P(\geq 2E / \neq 1E) = 1 - \frac{\prod_{k=1}^{7} [1 - P(E_k)]}{1 - \sum_{i=1}^{7} P(E_i) \prod_{\substack{j=1 \\ j \neq i}}^{7} [1 - P(E_j)]}$$

$$= 1 - \frac{\prod_{k=1}^{7} P(C_k)}{1 - \prod_{j=1}^{7} P(C_j) \sum_{i=1}^{7} \frac{P(E_i)}{P(C_i)}}$$

APPENDIX B

MONTE CARLO METHOD FOR THE QUANTIZED I AND D LEVEL SYSTEM SIMULATION

The purpose of the Monte Carlo method is twofold: it allows study of the improvement when more quantization levels are used, and it allows investigation of the extension of the error correcting method for NRZ-M data for which a complete theory would be cumbersome.

The following method was used to generate words with a specific number of errors to test the correcting efficiency of the method against various decision thresholds. A unit interval was divided in the proportion of P_c and another interval in the proportion of P_e (see fig. 7). A random number generator with uniform distribution between zero and one was used to provide a pointer to select quantized levels for the correct bits from P_c, and for the required number of error bits from the P_e. In this manner many zero, single, and double error words were constructed with various error probabilities and $P(\geq 2E/\neq 1E)$ was computed for these groups in order to obtain a measure of the effectiveness of this criterion. The probability of removing a word suspected of double error is plotted versus the threshold X_T in figure 8.

For $P_e = 0.01$ (fig. 8(b)) and $X_T = 0.01$, the probability of removing a word with no errors is about 0.03 while the probability of removing a word with two errors is approximately 0.83. In figure 8, the data are shown for both the finely quantized divisions 32/32, and the optimum coarse division, 4/12. Figure 8 shows that the degradation in performance in going from a five bit accuracy to a two bit accuracy in quantizing is minimal. The main difference is that for the coarsely quantized divisions $P(R/xE)$ tends to decrease in steps instead of decreasing smoothly.

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TABLE I.- 4/12 I AND D LEVEL CLASSIFICATIONS WITH ASSOCIATED EVENT AND ERROR PROBABILITIES

K	Number of bits per word in a given I and D level classification				P_{ev_k}	$P_k(\geq 2E/\neq 1E)$	Quality of word category	$\sum_{1}^{k} P_{ev_k}$	$P_k(0E)$	$P_k(1E)$
	n_1	n_2	n_3	n_4						
1	7	0	0	0	0.4029E-00	0.3290E-05	1	0.4029E-00	0.9973E 00	0.2656E-02
2	6	1	0	0	0.2009E-00	0.2999E-04	1	0.6038E 00	0.9864E 00	0.1356E-01
3	6	0	1	0	0.1170E-00	0.1586E-03	1	0.7208E 00	0.9338E 00	0.6609E-01
4	6	0	0	1	0.7325E-01	0.7102E-03	1	0.7940E 00	0.7080E 00	0.2913E-00
5	5	2	0	0	0.4292E-01	0.1847E-03	1	0.8370E 00	0.9756E 00	0.2423E-01
6	5	1	1	0	0.4998E-01	0.9299E-03	1	0.8870E 00	0.9235E 00	0.7560E-01
7	5	1	0	1	0.3130E-01	0.4125E-02	2	0.9183E 00	0.7003E 00	0.2959E-00
8	5	0	2	0	0.1455E-01	0.4645E-02	2	0.9328E 00	0.8742E 00	0.1214E-00
9	5	0	1	1	0.1823E-01	0.2057E-01	3	0.9510E 00	0.6629E 00	0.3179E-00
10	5	0	0	2	0.5707E-02	0.9109E-01	4	0.9567E 00	0.5026E 00	0.4123E-00
11	4	3	0	0	0.5095E-02	0.4646E-03	1	0.9618E 00	0.9649E 00	0.3466E-01
12	4	2	1	0	0.8899E-02	0.1813E-02	1	0.9707E 00	0.9134E 00	0.8490E-01
13	4	2	0	1	0.5573E-02	0.7593E-02	2	0.9763E 00	0.6926E 00	0.3003E-00
14	4	1	2	0	0.5181E-02	0.6066E-02	2	0.9815E 00	0.8647E 00	0.1297E-00
15	4	1	1	1	0.6490E-02	0.2430E-01	3	0.9880E 00	0.6556E 00	0.3217E-00
16	4	1	0	2	0.2032E-02	0.9593E-01	4	0.9900E 00	0.4971E-00	0.4133E-00
17	4	0	3	0	0.1006E-02	0.1291E-01	3	0.9910E 00	0.8185E 00	0.1694E-00
18	4	0	2	1	0.1889E-02	0.4225E-01	4	0.9929E 00	0.6206E 00	0.3399E-00
19	4	0	1	2	0.1183E-02	0.1192E-00	4	0.9941E 00	0.4706E-00	0.4180E-00
20	4	0	0	3	0.2470E-03	0.2191E-00	4	0.9943E 00	0.3568E-00	0.4385E-00
21	3	4	0	0	0.3628E-03	0.8669E-03	1	0.9947E 00	0.9543E 00	0.4486E-01
22	3	3	1	0	0.8450E-03	0.2805E-02	2	0.9955E 00	0.9034E 00	0.9398E-01
23	3	3	0	1	0.5292E-03	0.1111E-01	3	0.9961E 00	0.6850E 00	0.3046E-00
24	3	2	2	0	0.7380E-03	0.7583E-02	2	0.9968E 00	0.8552E 00	0.1377E-00
25	3	2	1	1	0.9244E-03	0.2807E-01	3	0.9977E 00	0.6484E 00	0.3253E-00
26	3	2	0	2	0.2895E-03	0.1008E-00	4	0.9980E 00	0.4917E-00	0.4142E-00
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
119	0	0	1	6	0.3589E-10	0.6197E 00	4	0.1000E 01	0.1195E-00	0.3016E-00
120	0	0	0	7	0.3211E-11	0.6956E 00	4	0.1000E 01	0.9063E-01	0.2596E-00

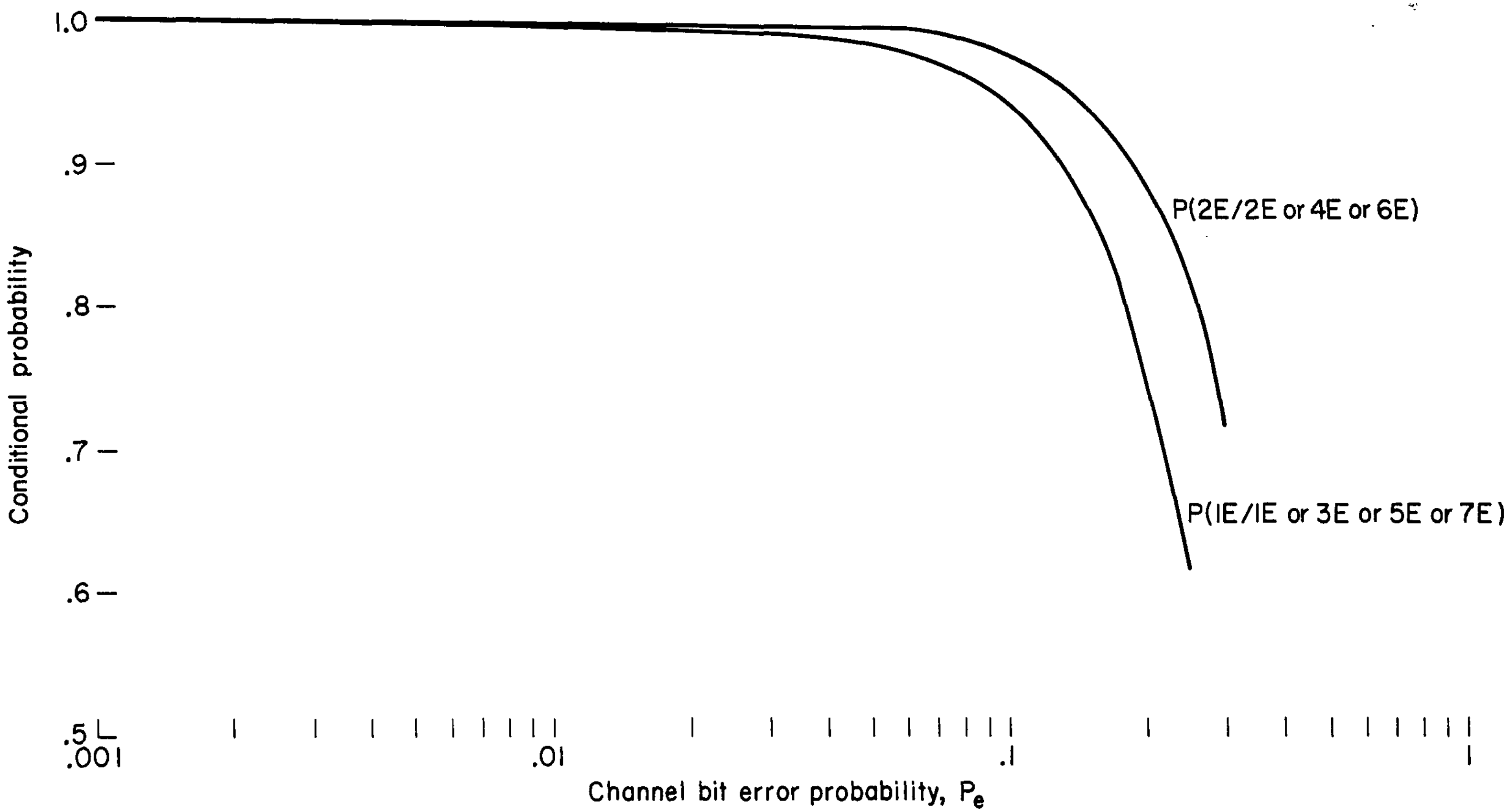


Figure 1.- Probability of one error given an odd number of errors and probability of two errors given and even number of errors versus channel bit error probability for a seven bit word.

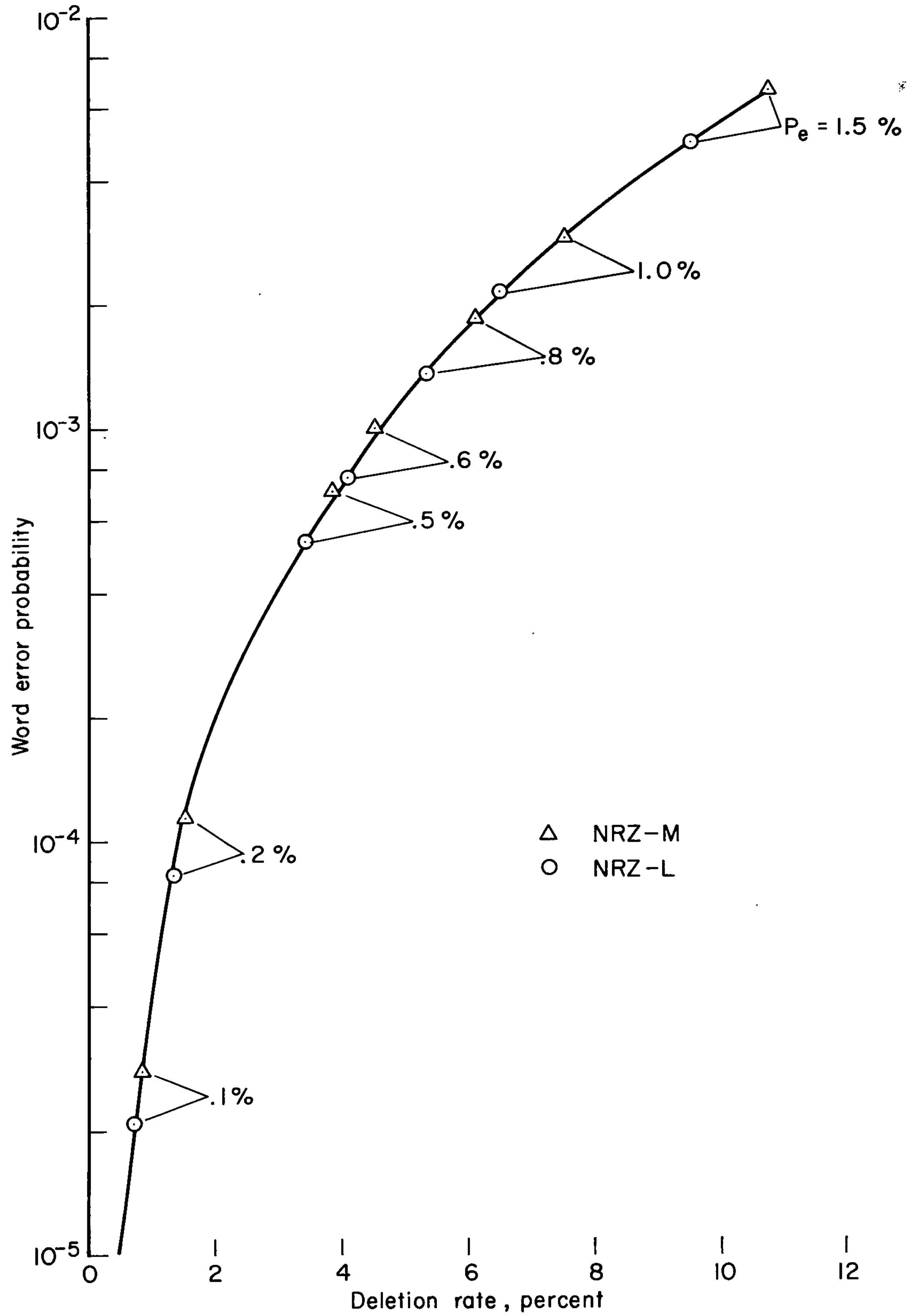


Figure 2.- Performance of parity error detection.

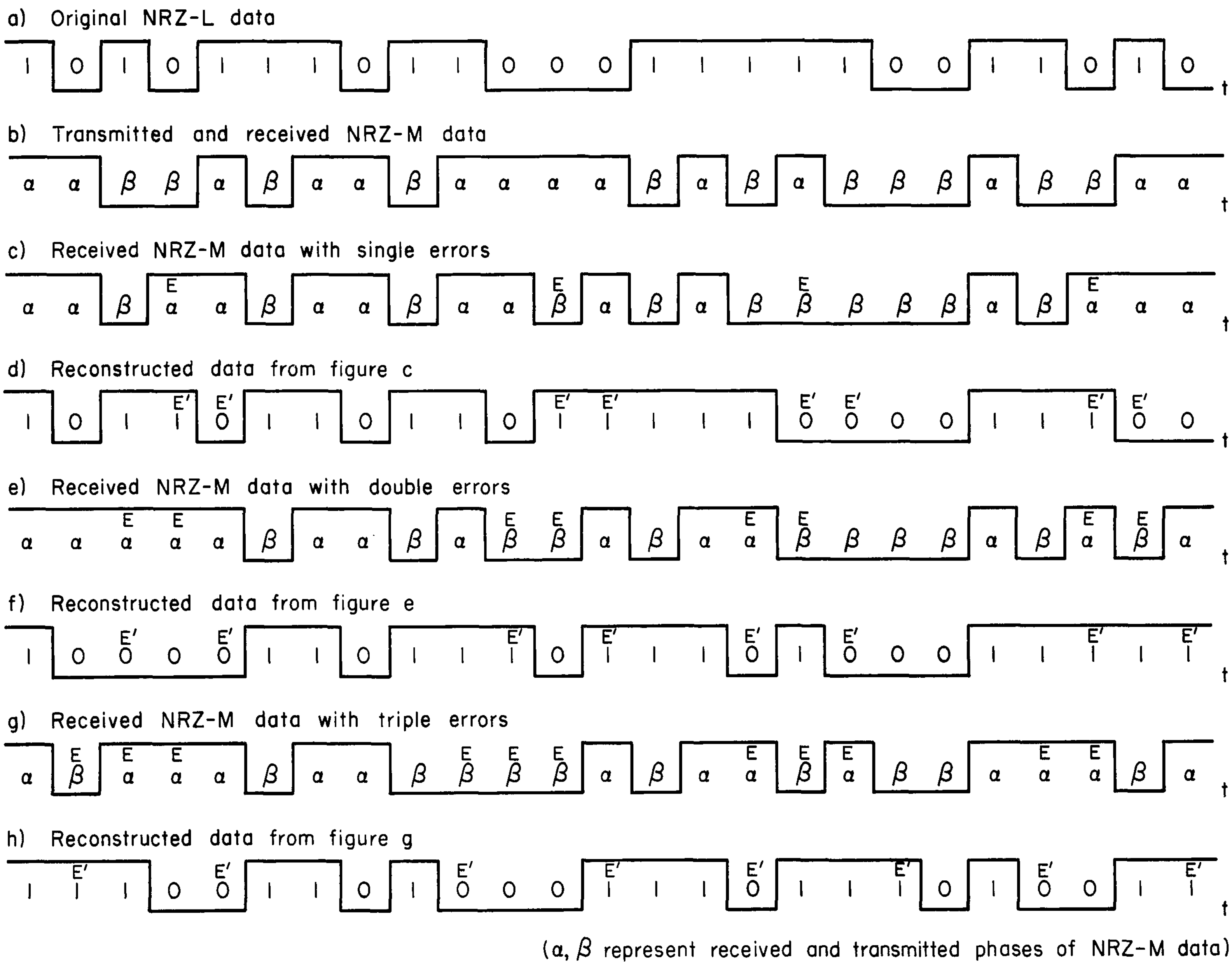


Figure 3.- Relationship between NRZ-L and NRZ-M data in a serial bit stream.

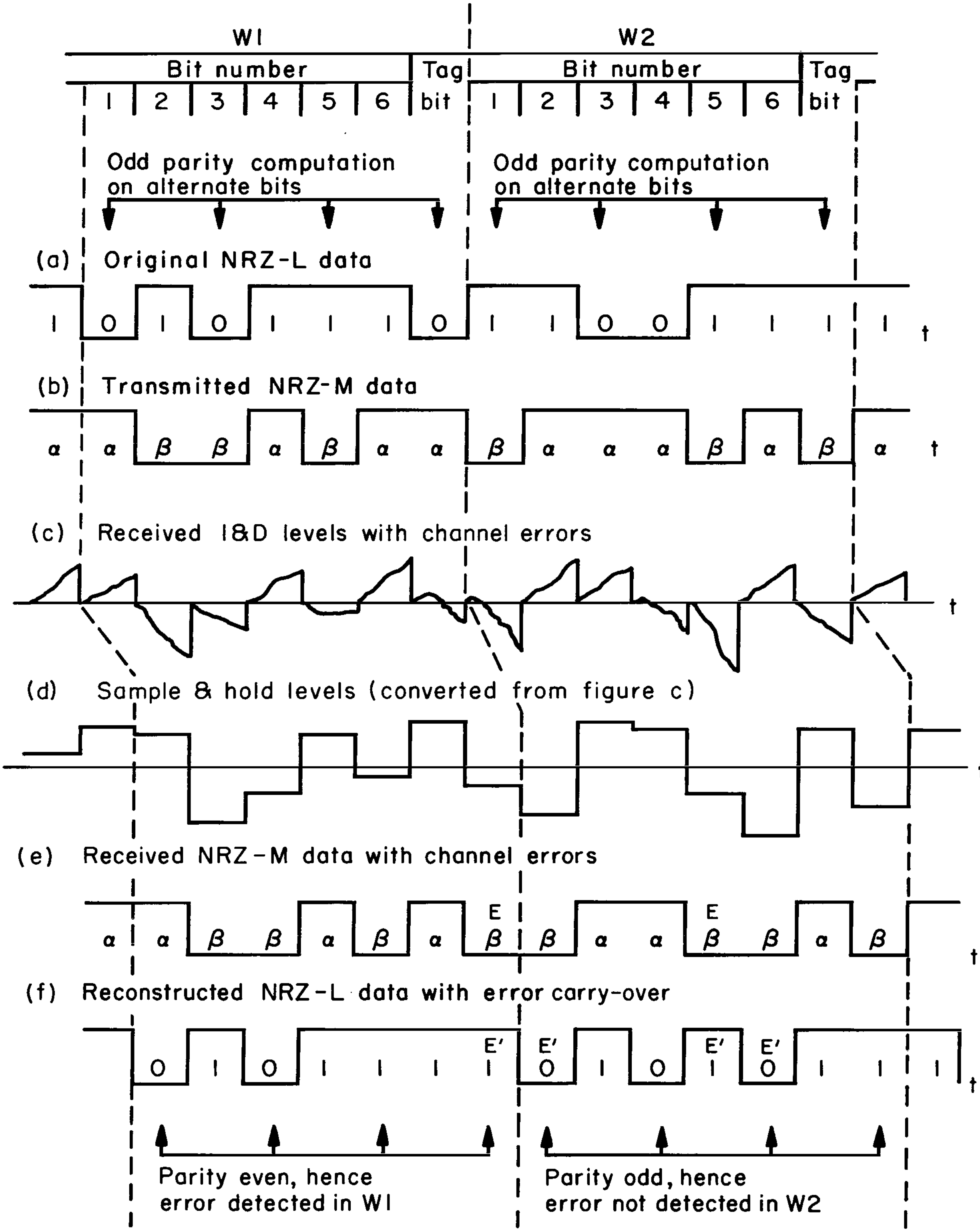


Figure 4.- Relationship between NRZ-L and NRZ-M data in a seven-bit word with parity (tag).

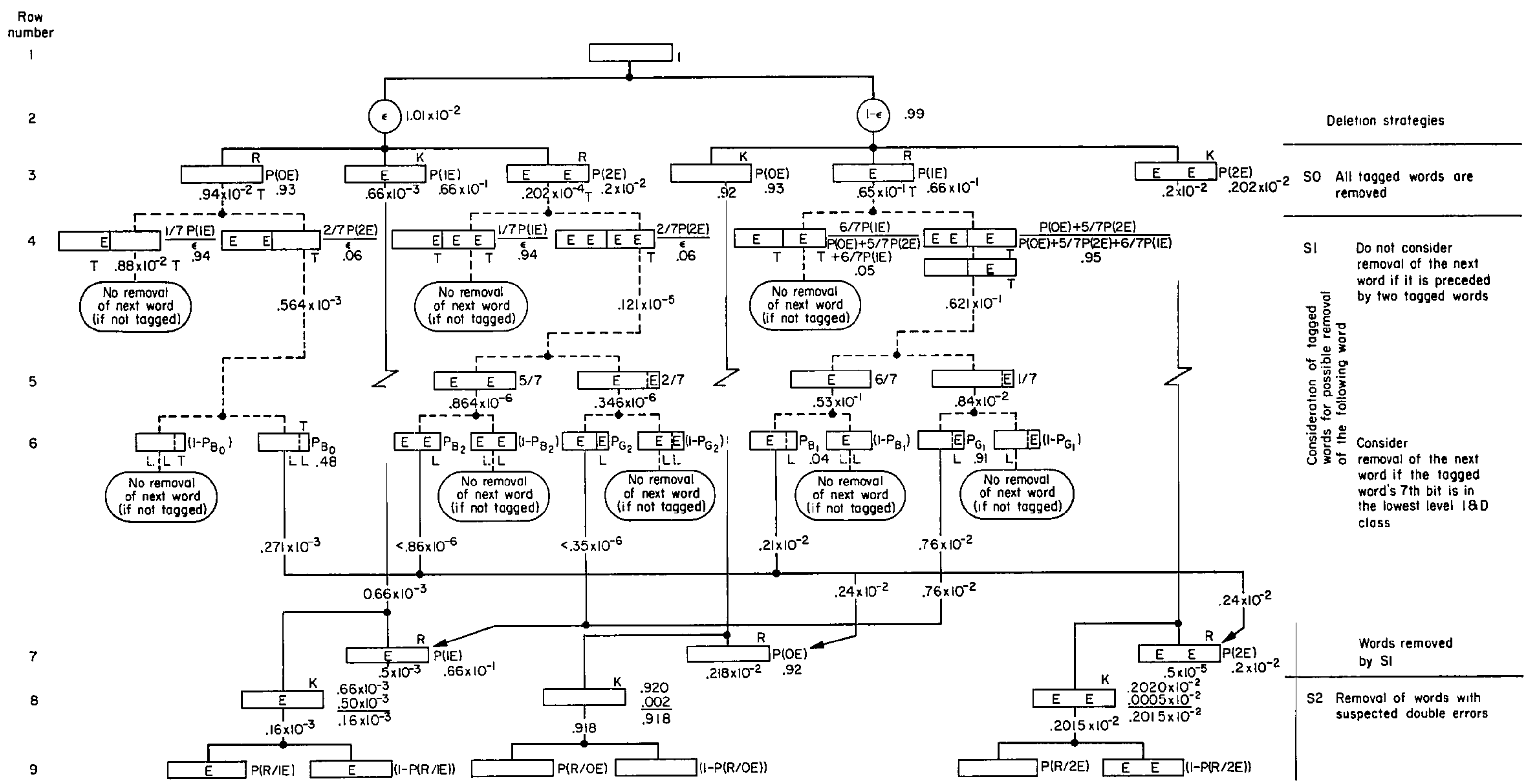
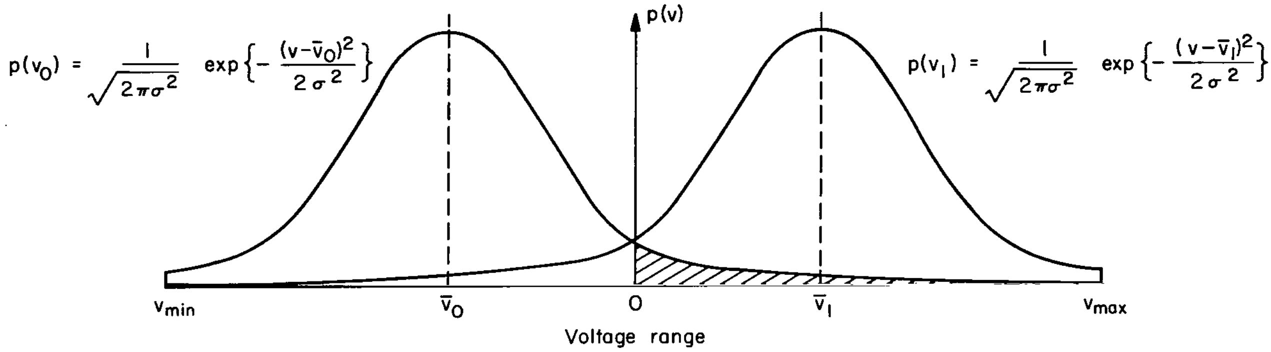
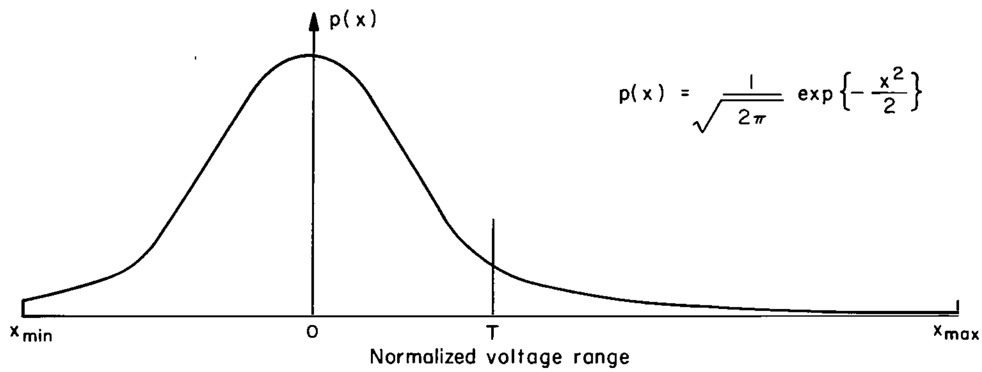


Figure 5.- Diagrammatic representation of the liklihood deletion strategy for NRZ-M data.

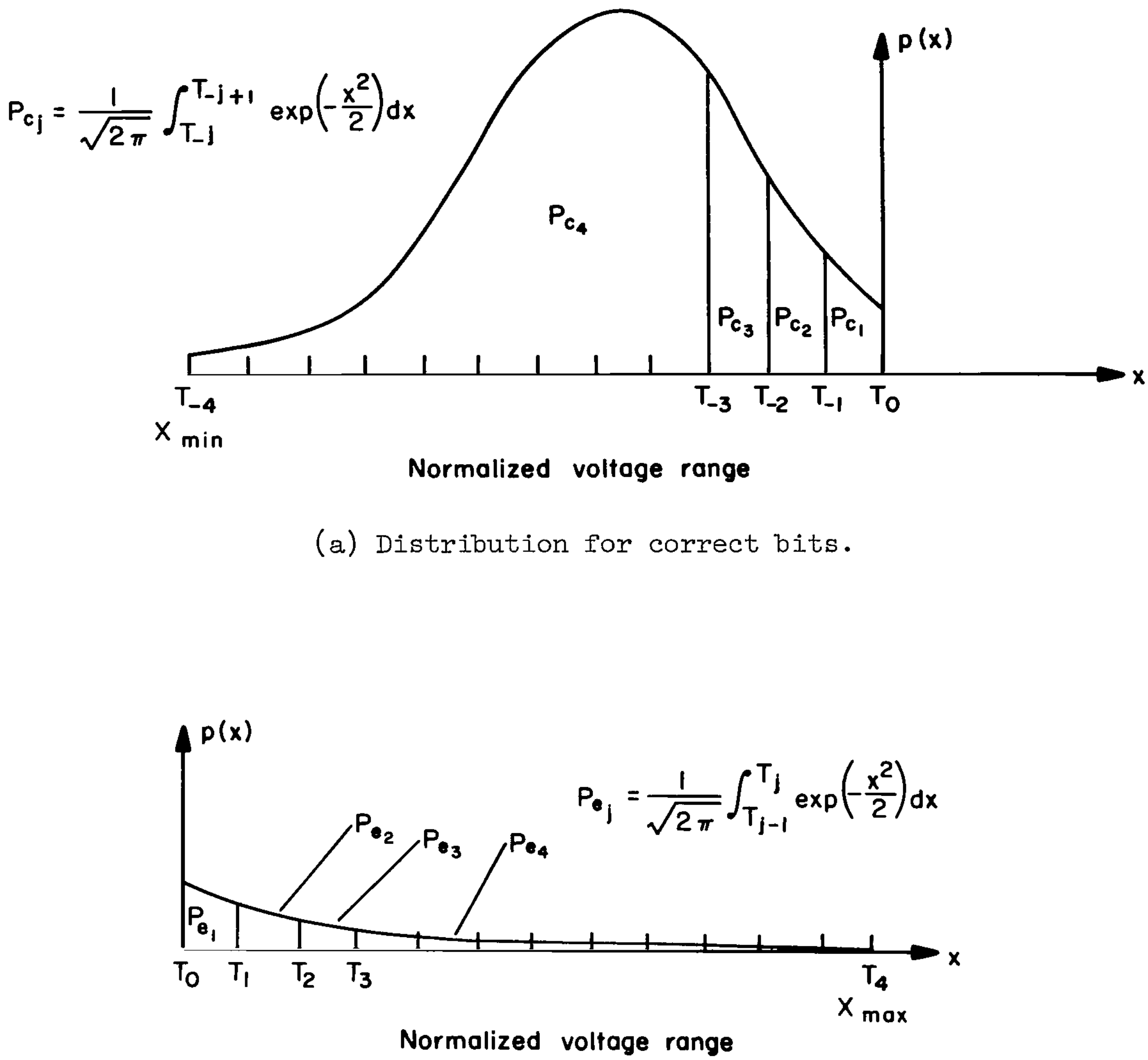


(a) Actual probability distributions.



(b) Standard probability distributions.

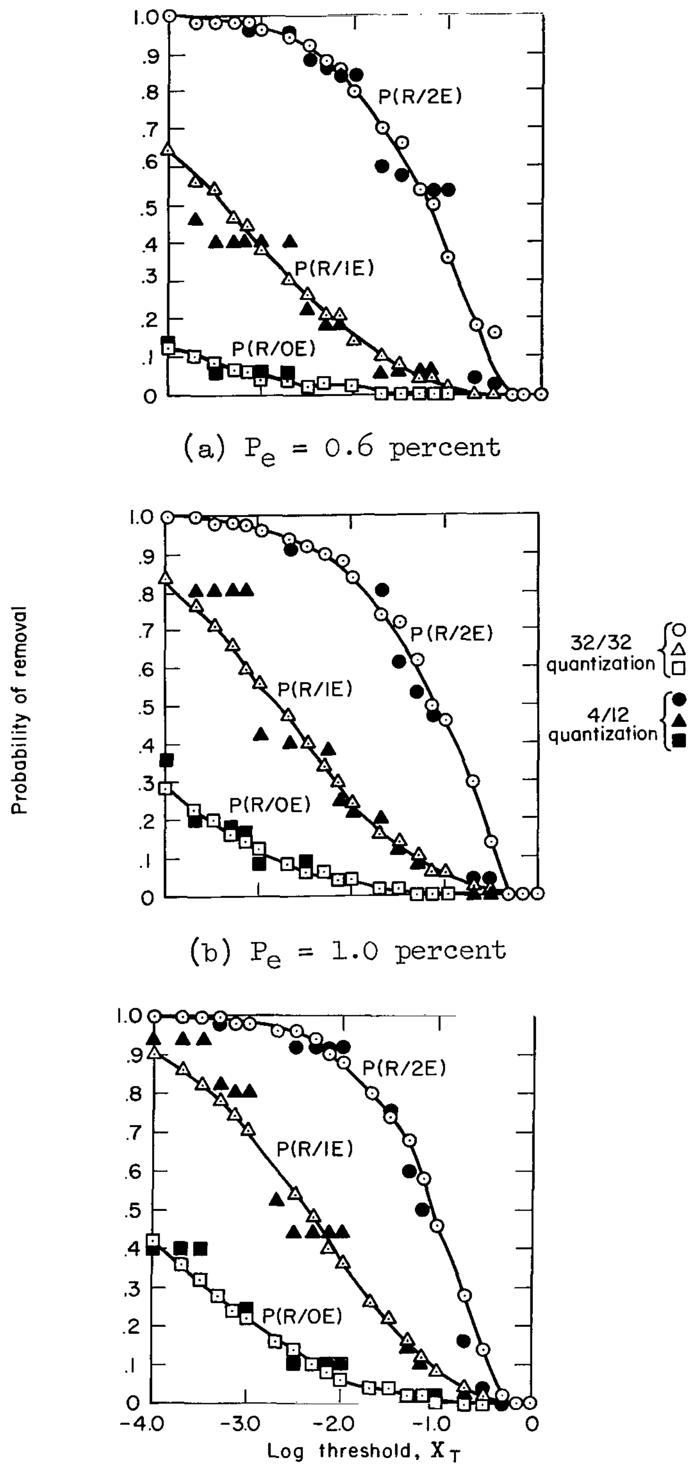
Figure 6.- Probability distributions for the integrate and dump levels.



(a) Distribution for correct bits.

(b) Distribution for error bits.

Figure 7.- Integrate and dump level probability distributions (4/12 quantization).



(a) P_e = 0.6 percent

(b) P_e = 1.0 percent

(c) P_e = 1.5 percent

Figure 8.- Probabilities of word removal for strategy S2 with 0, 1, or 2 errors in a seven-bit word for different channel bit error probabilities.

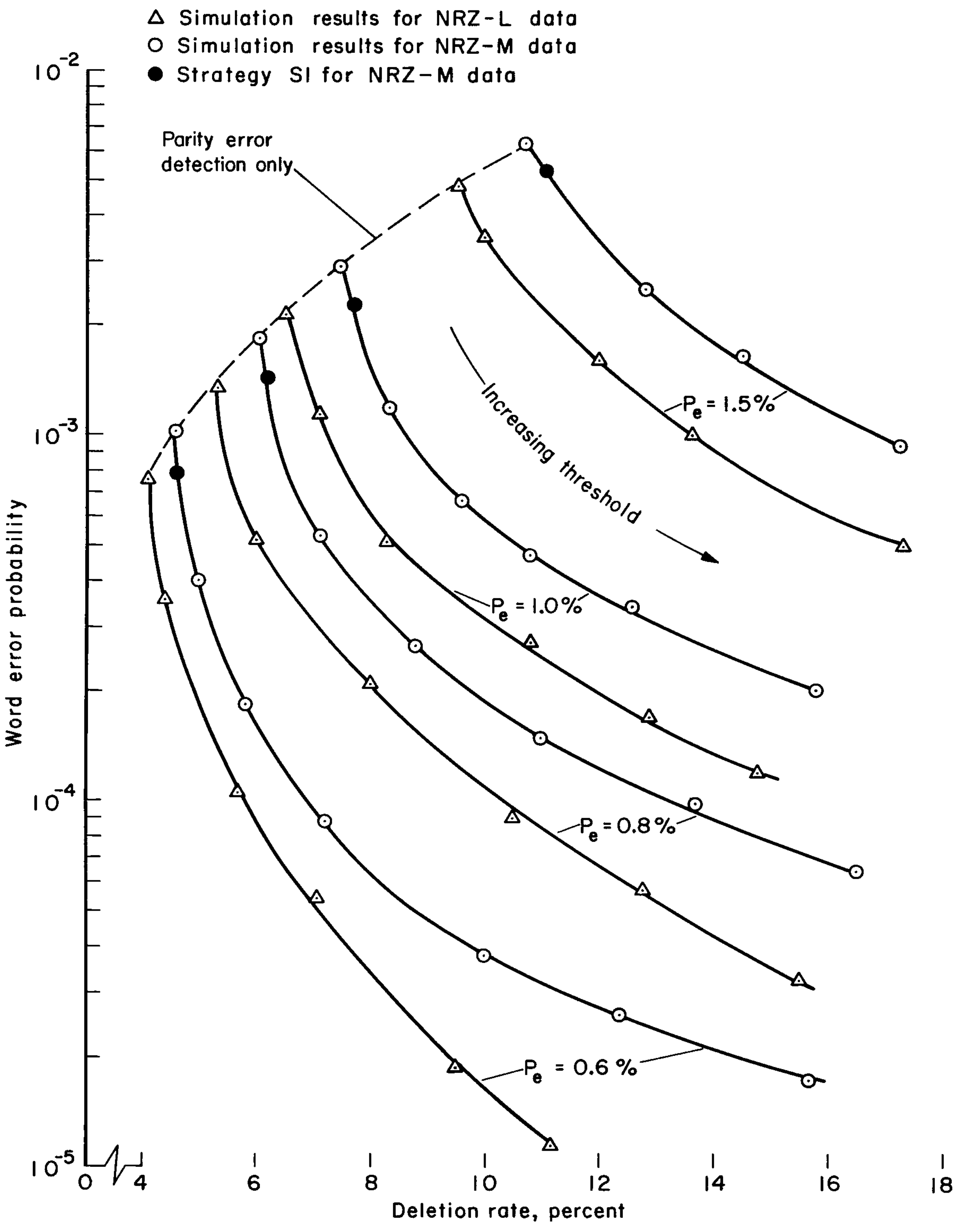


Figure 9.- Performance of the likelihood deletion strategy for NRZ-L and NRZ-M data using the 32/32 quantization; data points are calculated values for different thresholds.

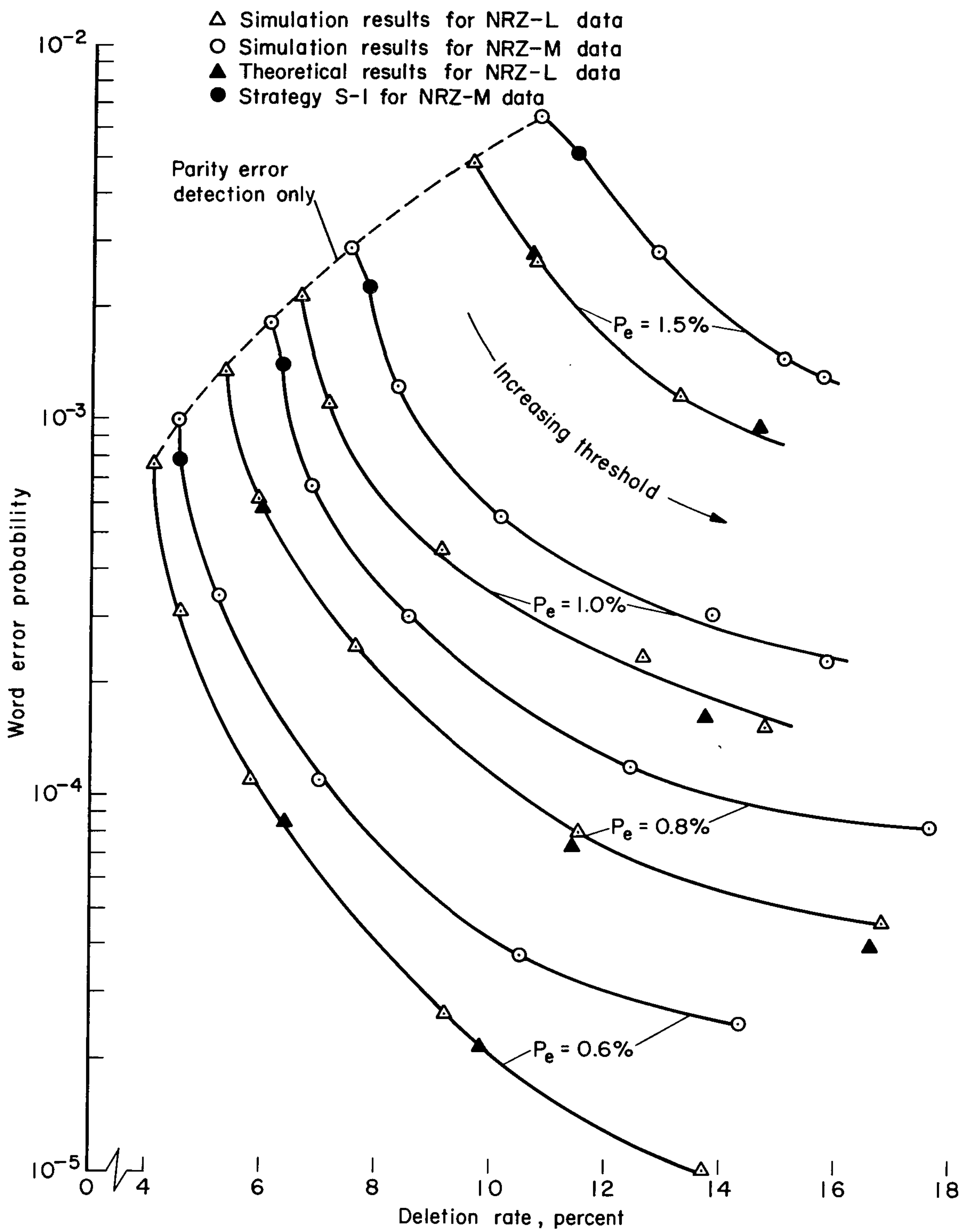


Figure 10.- Performance of the likelihood deletion strategy for NRZ-L and NRZ-M data using the 4/12 quantization; data points are calculated values for different thresholds.

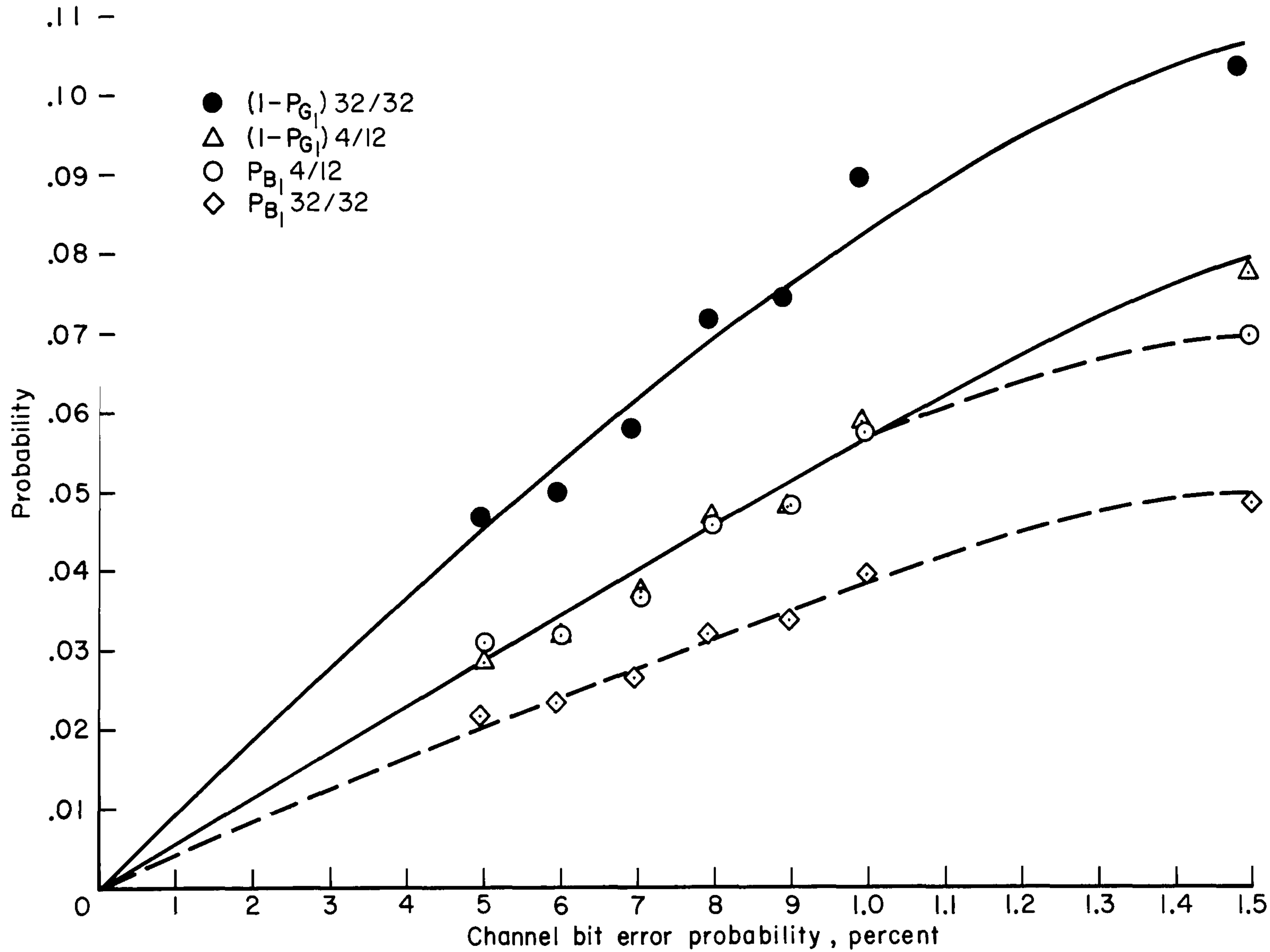


Figure 11.- Word removal probabilities for strategy S1.

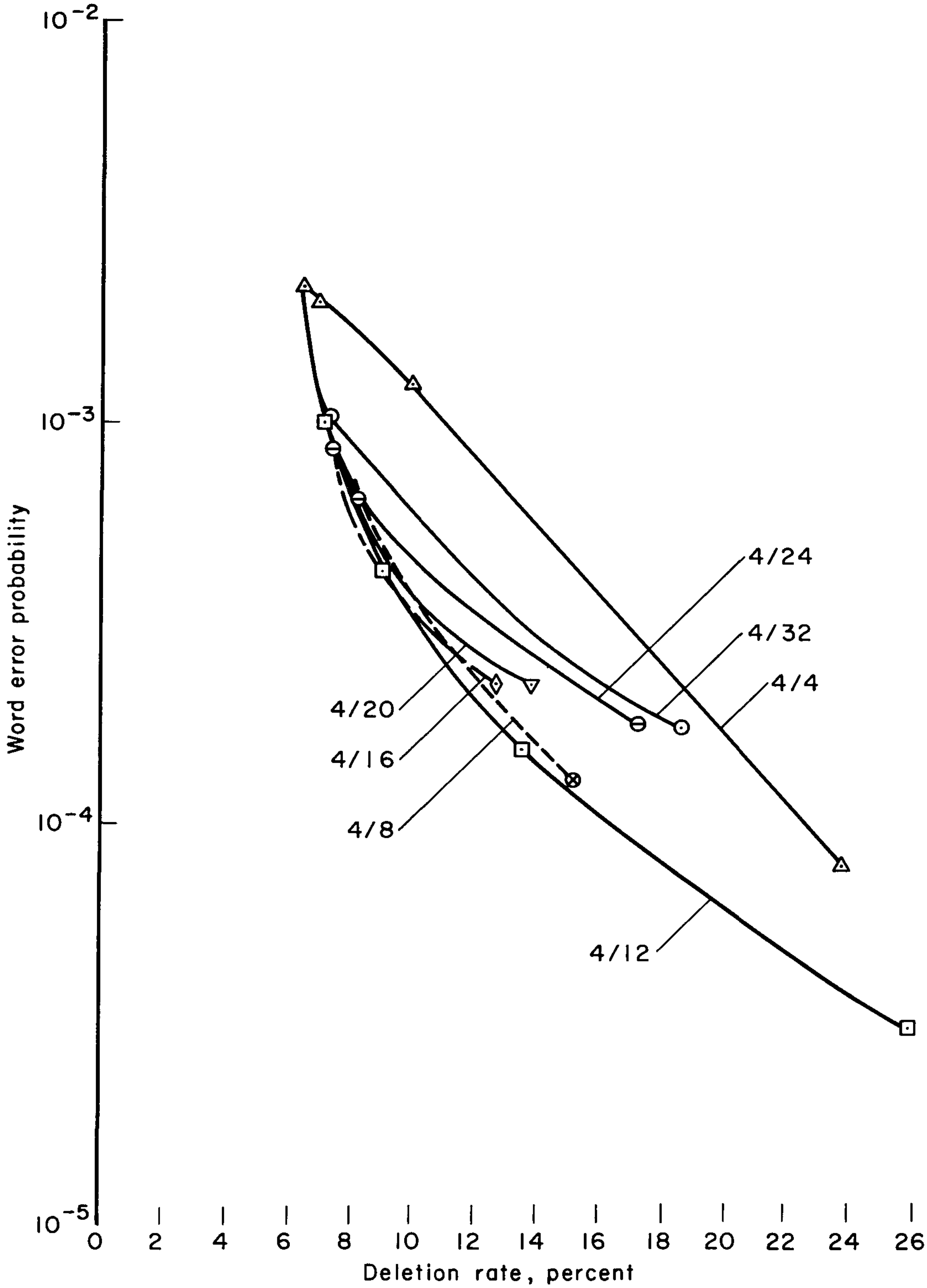


Figure 12.- NRZ-L performance for 1-percent channel bit error probability for different 4/m quantization schemes.

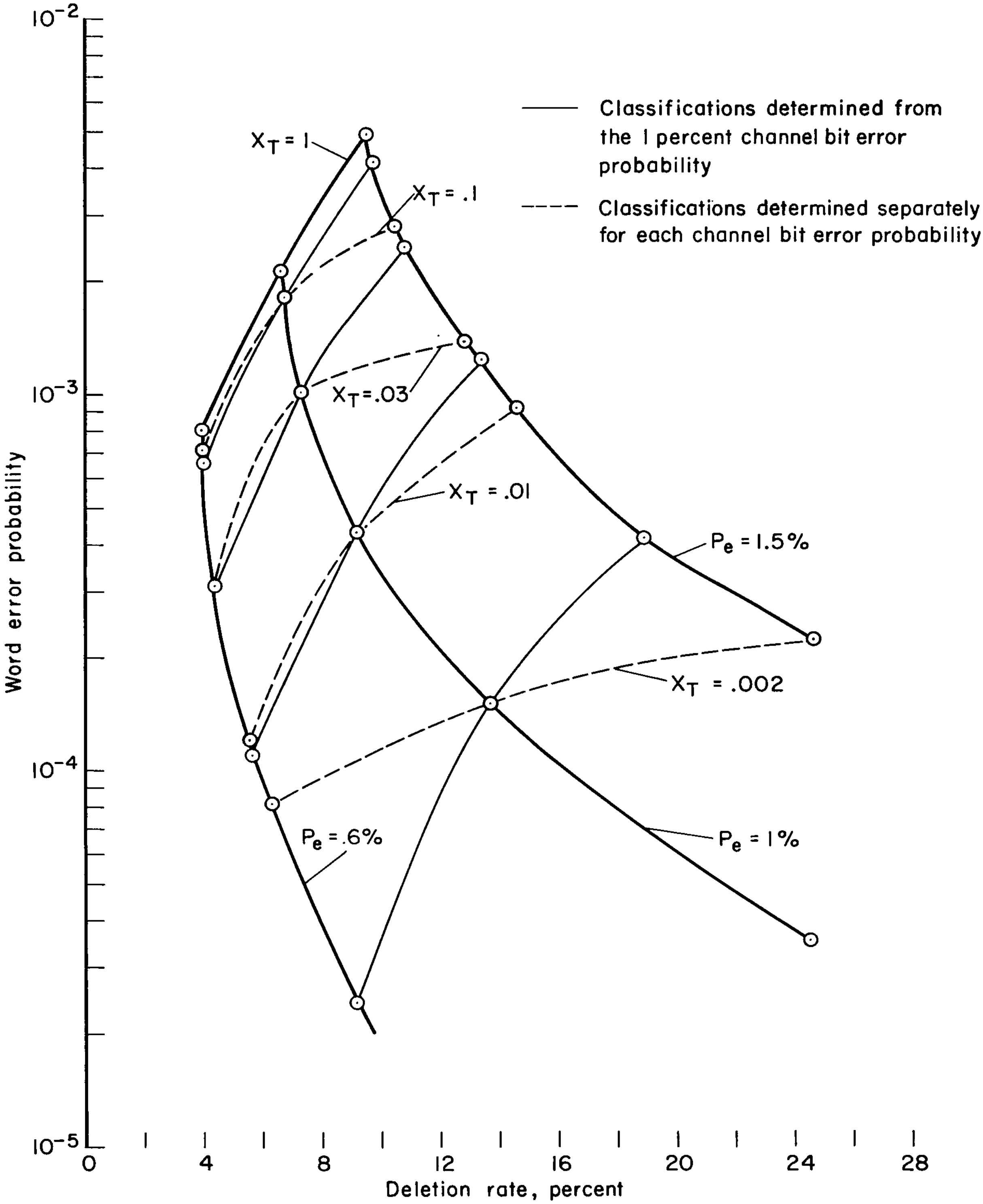


Figure 13.- NRZ-L performance from the theory program for the 4/12 quantization.

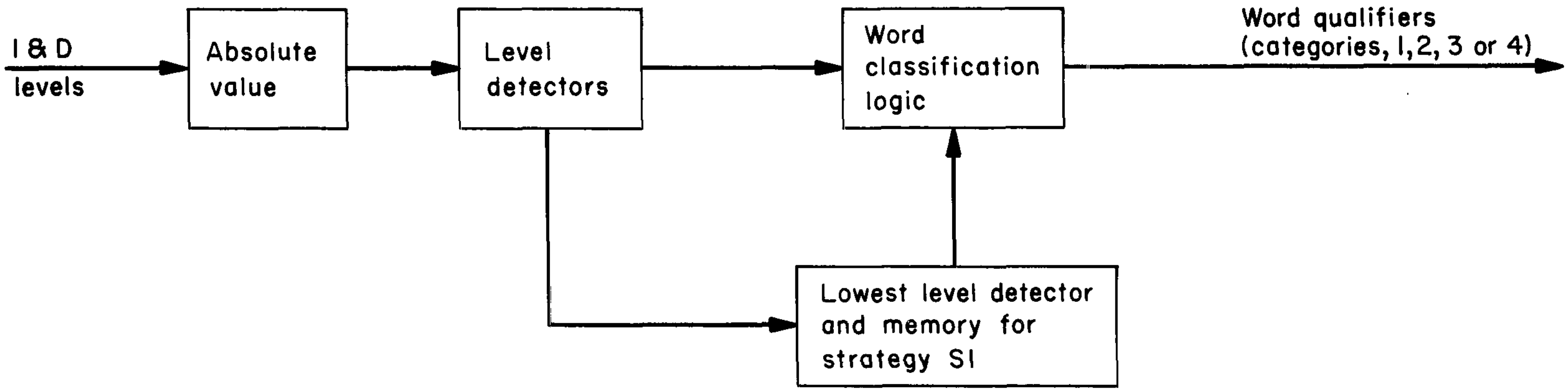
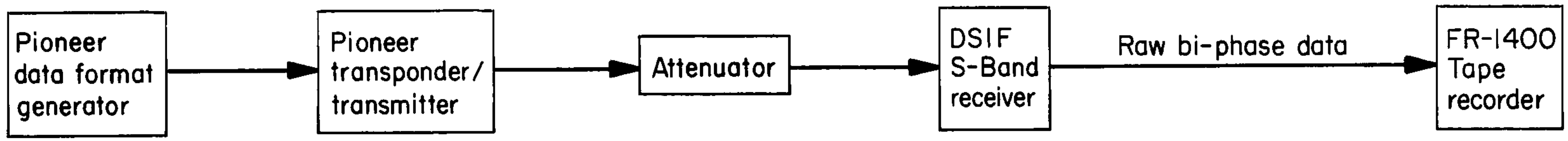
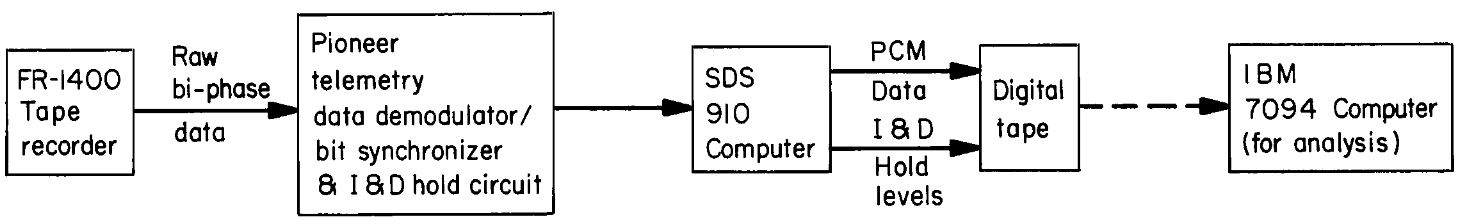


Figure 14.- Functional block diagram of quality of word categorizor.

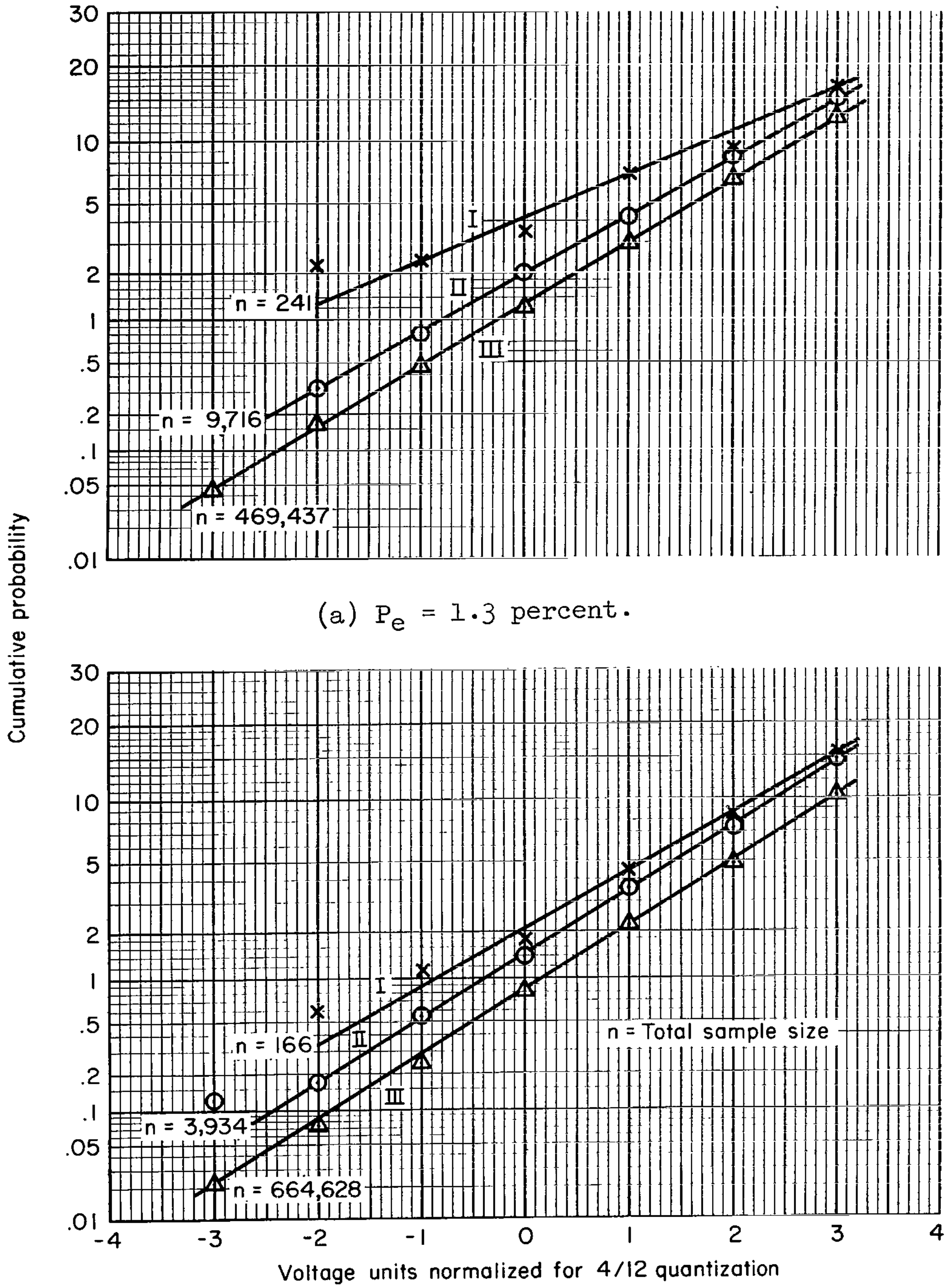


(a) Block diagram of experimental test setup at Goldstone (DSIF).



(b) Block diagram of analysis test setup at Ames Research Center.

Figure 15.- Test setup for experimental verification.



(a) P_e = 1.3 percent.

(b) P_e = 0.8 percent.

Figure 16.- Cumulative probability distributions for the experimental I and D level data plotted on normal probability graph paper.

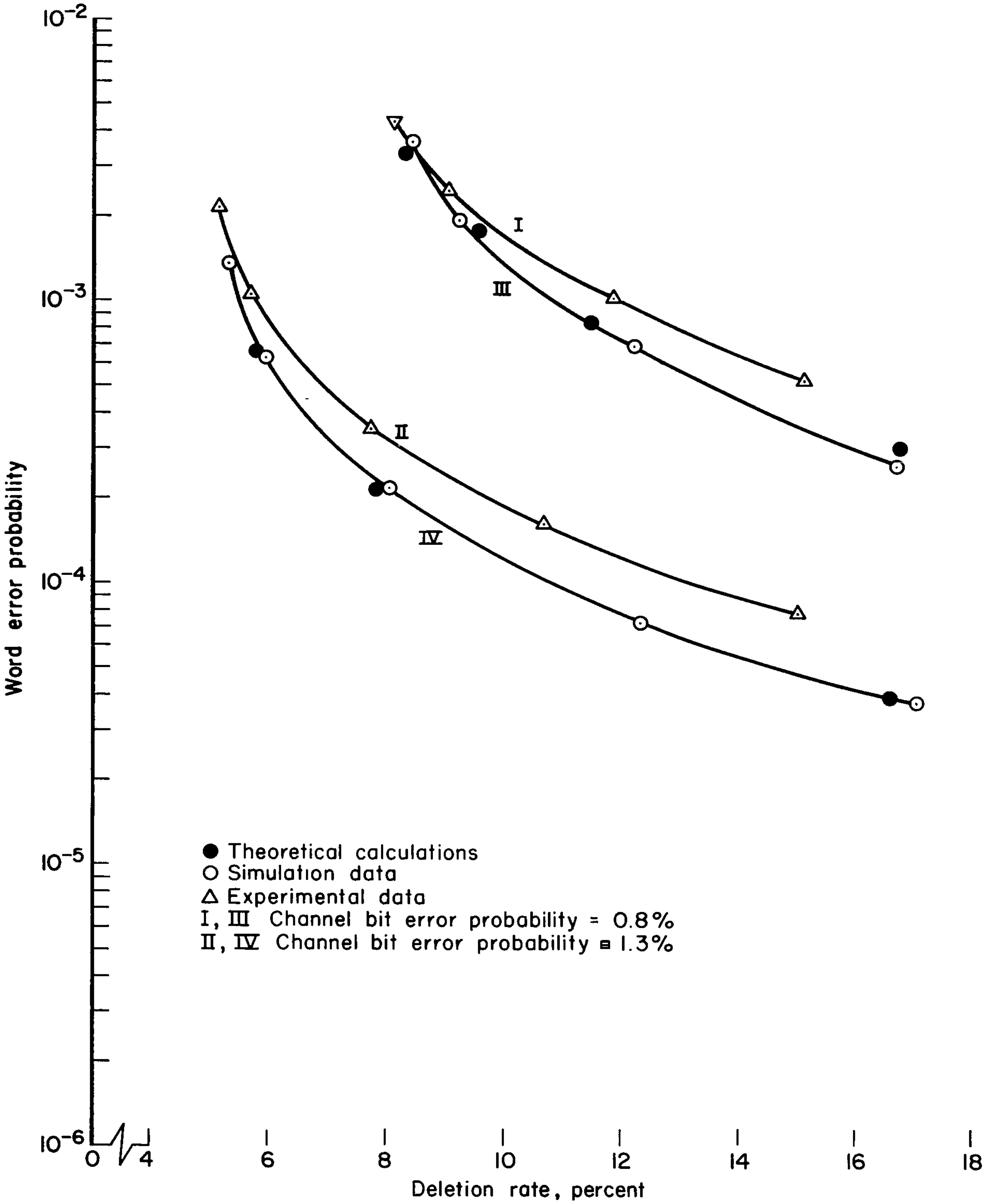


Figure 17.- Comparison between simulation and experimental performance for the likelihood deletion strategy for NRZ-L data using 4/12 quantization.

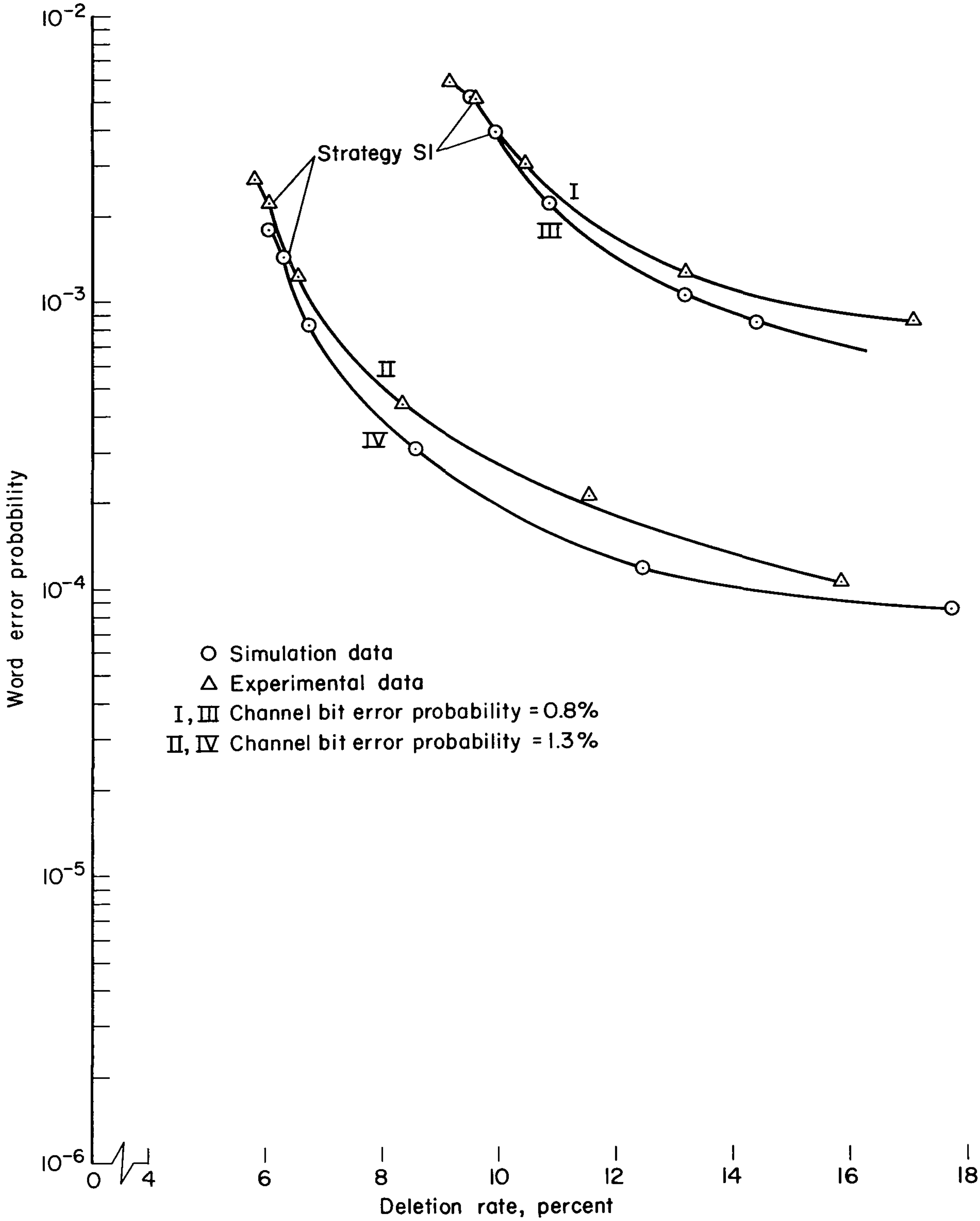


Figure 18.- Comparison between simulation and experimental performance for the likelihood deletion strategy for NRZ-M data using 4/12 quantization.

NASA-Langley, 1966 A-2348

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