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COMPUTER PROGRAMS FOR SPHERICAL SATELLITES

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Introduction

This paper describes briefly two rather specialized computer programs which solve for steady-state temperatures over a sphere. The first program deals with opaque surfaces and allows for skin conduction and black body internal radiation. This program differs from the conventional 3-D heat transfer programs in that it automatically subdivides the sphere into a specified number of nodes and computes for each node, surface area, conductances, radiation shape factors and incident solar flux. The second progam allows for transmission through partially or fully transparent external surfaces and accounts for multiple diffuse reflections at the inner surfaces, but does not include conduction.

The first program was written because a number of GSFC satellites or parts of satellites have been spherical, and it has been used directly or modified to calculate skin temperatures of ECHO-II, and Explorer 17, and of the magnetometers for IMP and OGO. The second program was written specifically for the proposed Rebound satellite, a large ECHO-type balloon which was to be constructed of a laminate of an aluminum mesh and a transparent plastic film.

Nomenclature

- A - Surface area
- A_p - Projected area
- F_{ij} - Shape factor from node i to node j
- H - Infrared energy incident on interior
- I - Solar energy incident on interior
- K - Conductivity
- P_a - Incident albedo radiation
- P_e - Incident earth emitted radiation
- Q_{cond} Conductance
- R - Radius of sphere
- s - Solar constant
- S_i - Solar input
- T - Absolute temperature

ϕ - Equatorial angle measured from the i axis

t - Thickness of shell

X - Fraction of surface that is opaque

α - Internal solar absorptivity

$\bar{\alpha}$ - External solar absorptivity

ϵ - Internal I.R. emissivity

$\bar{\epsilon}$ - External I.R. emissivity

ρ - Solar reflectivity (internal)

ρ^* - I.R. reflectivity (internal)

τ - Solar transmissivity

τ^* - I.R. transmissivity

σ - Stefan-Boltzmann constant

θ - Polar angle of spherical segment measured from spin axis

θ_s - Angle between spin axis and solar vector

γ - Angle between normal to spherical segment and solar vector

Subscripts

i - Nodes

j - Nodes

p - Transmitting material

o - Opaque material

Opaque Sphere Program

The sphere is divided into a number of concentric rings about the spin axis as shown in Figure 1. All rings or nodes subtend equal angles at the origin. The surface area of a differential element is

$$dA = R^2 \sin\theta \, d\theta \, d\Phi \tag{1}$$

which when integrated over a node gives

$$A_i = 2\pi R^2 (\cos\theta_i - \cos\theta_j) \tag{2}$$

The solar energy absorbed is proportional to the area of the node projected in the direction of the sun. The projection of an elemental area is

$$dA_p = dA \cos\gamma \tag{3}$$

where $\cos\gamma$ can be determined from the dot product of the solar vector and the normal to the elemental area. This expression must be integrated for three distinct zones (see Figure 2).

a. For $0 \le \theta \le 90 - \theta_S$ sunlight is incident on every elemental area of each node. For each node the projection is a complete ring whose area is

$$A_{p\,i} = \pi R^2 \cos\theta_S \left(\sin^2\theta_j - \sin^2\theta_i\right) \tag{4a}$$

b. For $90 - \theta_S < \theta \le 90 + \theta_S$ the projection is no longer a complete ring and it becomes necessary to limit the integration to sunlit areas only. The projected area can be demonstrated to be

$$\begin{aligned} A_{p\,i} = R^2 \Big[& \cos\theta_i \left(\sin^2\theta_S - \cos^2\theta_i\right)^{\frac{1}{2}} \\ & - \cos\theta_j \left(\sin^2\theta_S - \cos^2\theta_j\right)^{\frac{1}{2}} \\ & + \sin^{-1}\left(\cos\theta_i / \sin\theta_S\right) - \sin^{-1}\left(\cos\theta_j / \sin\theta_S\right) \\ & + \sin^2\theta_j \cos\theta_S \cos^{-1}\left(-\cot\theta_j \cot\theta_S\right) \\ & - \sin^2\theta_i \cos\theta_S \cos^{-1}\left(-\cot\theta_i \cot\theta_S\right) \Big] \end{aligned} \tag{4b}$$

c. For $90 + \theta_S < \theta \le 180$ the nodes do not see the sun and

$$A_{p\,i} = 0 \tag{4c}$$

In the expressions for the limits of θ for each of the zones, it was assumed that $\theta_s \leq 90^\circ$. For $90 < \theta_s \leq 180$, the program in effect inverts the sphere by a simple transformation of variables to avoid defining new limits of θ for each zone.

The conductance between adjacent nodes, i and i + 1 is

$$Q_{cond} = \frac{2\pi Kt}{\ln \dfrac{\tan\left(\frac{\theta_{i+1}}{2}\right)}{\tan\left(\frac{\theta_i}{2}\right)}} \tag{5}$$

where the angles θ_i and θ_{i+1} refer to the centers of the two nodes. This formula can be derived by integration of the expression for conductance between differential areas.

For a hollow sphere, the radiation shape factor is simply

$$F_{ij} = \frac{A_j}{4\pi R^2} \tag{6}$$

The thermal balance at any node in steady-state is

$$\bar{\alpha}_i A_{p\,i} S + \bar{\alpha}_i A_i P_{a\,i} + \bar{\epsilon}_i A_i P_{e\,i} = A_i \sigma \bar{\epsilon}_i T_i^{\,4} + \sum_j A_i F_{i\,j} \sigma \epsilon_i \,(T_i^{\,4} - T_j^{\,4})$$

$$+ \sum_j Q_{cond_{i\,j}} (T_i - T_j) \tag{7}$$

To solve these simultaneous equations, the T^4 terms are linearized by a Taylor expansion and then matrix methods combined with iterative procedures are used to solve for the temperatures.

It is planned in the near future to publish a more detailed description of this program.

Explorer 17 Thermal Model

The Explorer 17 was not a hollow sphere and hence the computer program had to be modified to include the internal components. The majority of the electronics were mounted on a platform located below the sphere's equator. The thermal model chosen was simply that of a sphere circumscribed about a cylinder as shown in Figure 3. The shape factors were computed by hand and read in with other data.

Transparent Sphere Program

The transparent sphere program solves for the steady-state temperature of a partially transmitting sphere. It is assumed that all reflections are diffuse, that conduction is negligible, and that there is no diffraction as the energy passes through the transparent part of the sphere.

Instead of the thermal balance consisting of one equation per node as in Equation 7, there are now three equations per node. These equations are derived by the Poljak[1] Radiosity Method and are completely general for any enclosure. However, the program is specifically written for a sphere and hence the geometric equations described for the opaque sphere program are included in this program.

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1 Eckert, E.R.G. and Drake, R. M., Jr., "Heat and Mass Transfer," McGraw-Hill Book Co., Inc., 1959, p. 405.

The internal energy is divided into I.R. and visible (solar spectrum) components. The net solar energy incident internally on a node is given by

$$I_i = S_i + P_{a\,i} + \sum_j F_{ij} \left[\chi\rho_{o\,j} + (1-\chi)\rho_{p\,j} \right] I_j \tag{8}$$

These equations are solved simultaneously for I_i by a matrix solution.

The net I.R. energy incident internally on a node is given by

$$H_i = P_i + \sum_j F_{ij} \left[\chi\epsilon_{o\,j} + (1-\chi)\epsilon_{p\,j} \right] T_j^{\,4} + \sum_j F_{ij} \left[\chi\, \rho^*_{o\,j} + (1-\chi)\rho^*_{p\,j} \right] H_j \tag{9}$$

These equations are solved simultaneously for H in terms of T^4 by a matrix solution.

Finally, the thermal balance in steady-state for each node is

$$\begin{aligned} & A_{p\,i} S \left[\chi\bar{\alpha}_{o\,i} + (1-\chi)\bar{\alpha}_{p\,i} \right] + A_i \Big\{ \left[\chi\bar{\alpha}_{o\,i} + (1-\chi)\bar{\alpha}_{p\,i} \right] P_{a\,i} + \\ & \left[\chi\bar{\epsilon}_{o\,i} + (1-\chi)\bar{\epsilon}_{p\,i} \right] P_i - \left[\chi\rho_{o\,i} + (1-\chi)\rho_{p\,i} + (1-\chi)\tau - 1 \right] I_i \Big\} \\ & = A_i \sigma T_i^{\,4} \Big\{ \left[\chi\bar{\epsilon}_{o\,i} + (1-\chi)\bar{\epsilon}_{p\,i} \right] + \left[\chi\epsilon_{o\,i} + (1-\chi)\epsilon_{p\,i} \right] \Big\} + \\ & \left[\chi\rho^*_{o\,i} + (1-\chi)\rho^*_{p\,i} + (1-\chi)\tau^* - 1 \right] H_i \end{aligned} \tag{10}$$

These equations are linear in T^4 and hence can be solved by a matrix solution to yield the steady-state temperature distribution.

A complete explanation of the derivation of the equations for the temperature of a partially transmitting sphere is being planned as a NASA Technical Note for future publication.

GEOMETRY OF SPHERE

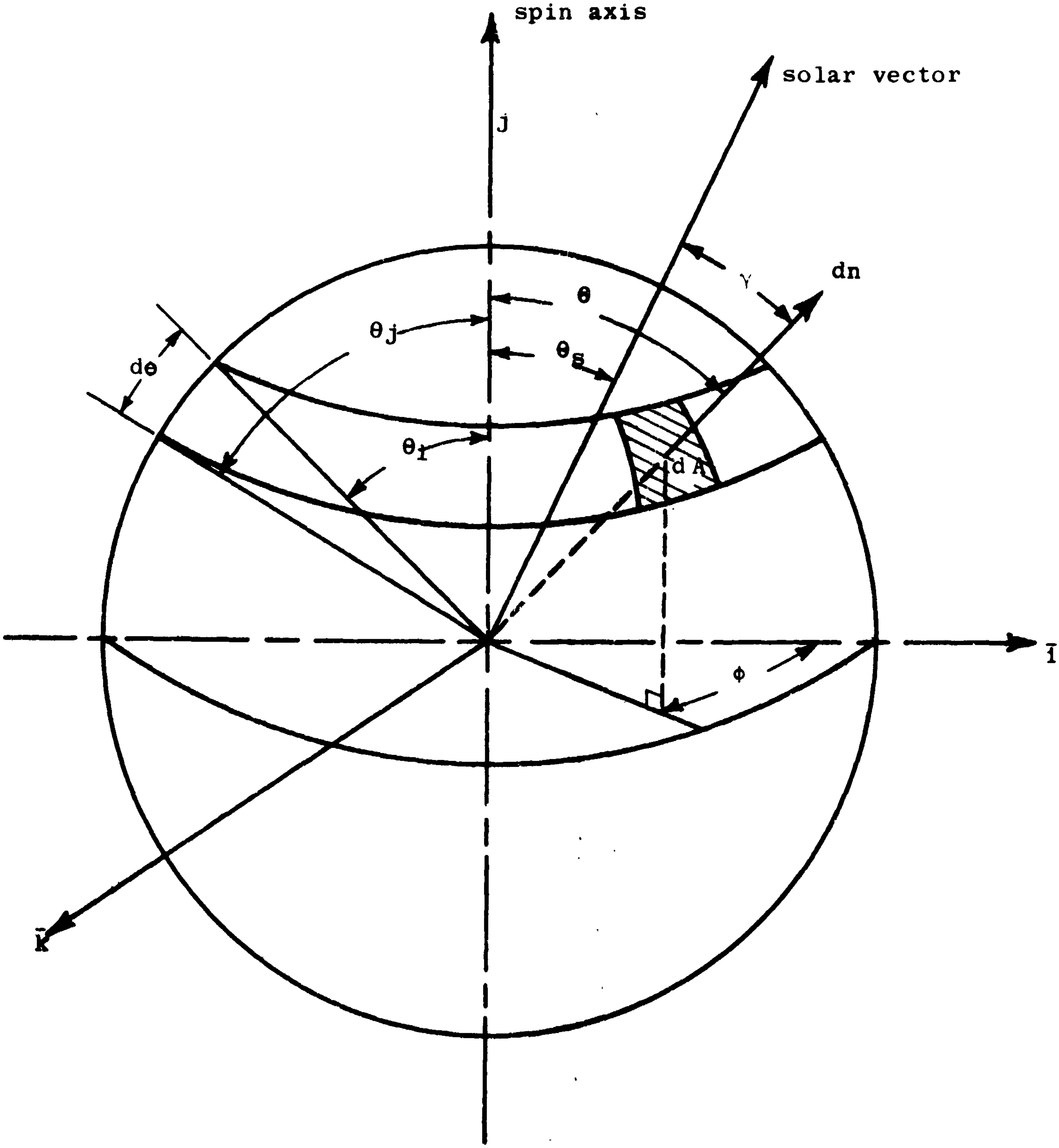

Figure 1

GEOMETRY SHOWING PROJECTED AREA ZONES

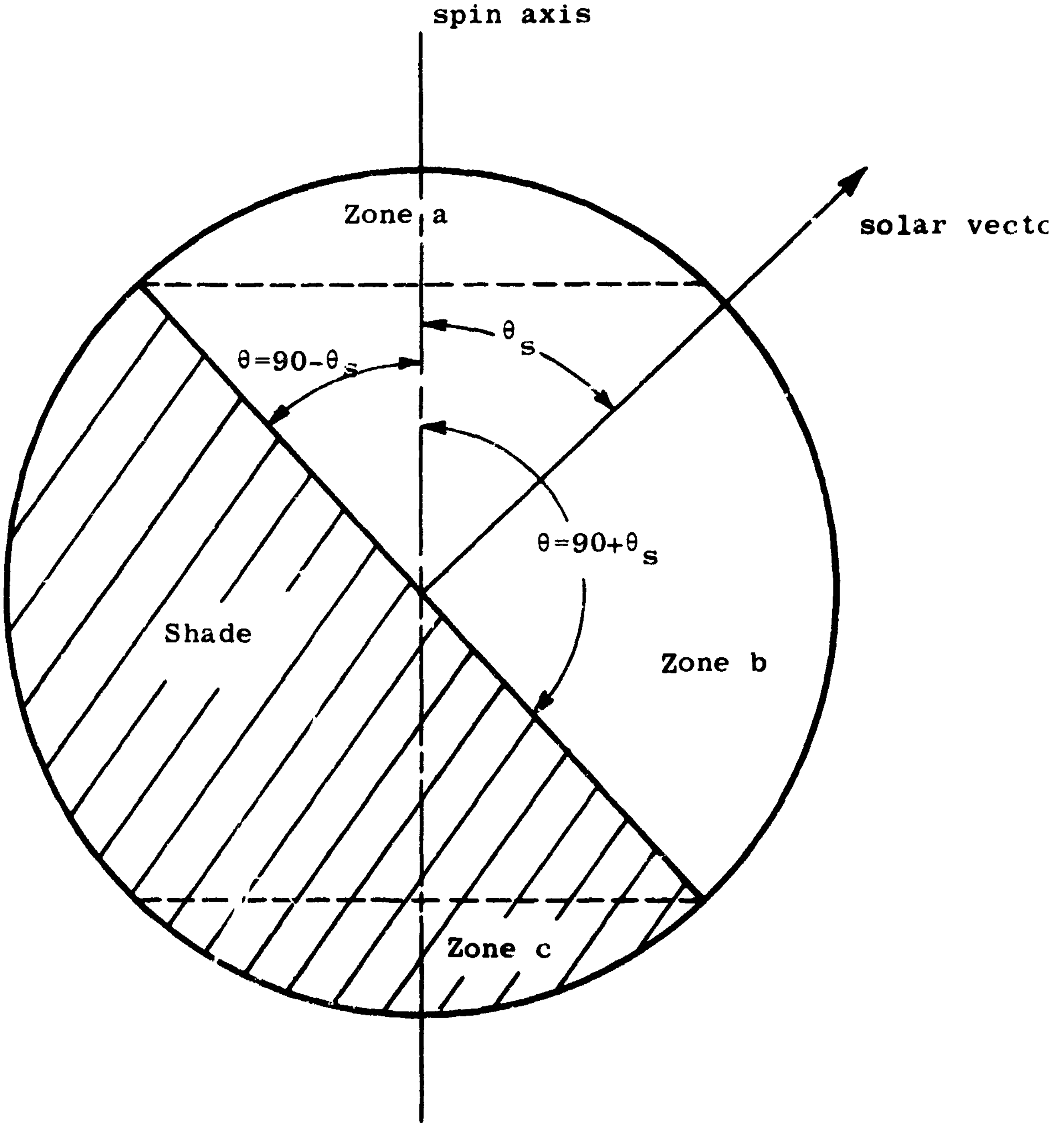

Figure 2

EXPLORER 17 THERMAL MODEL

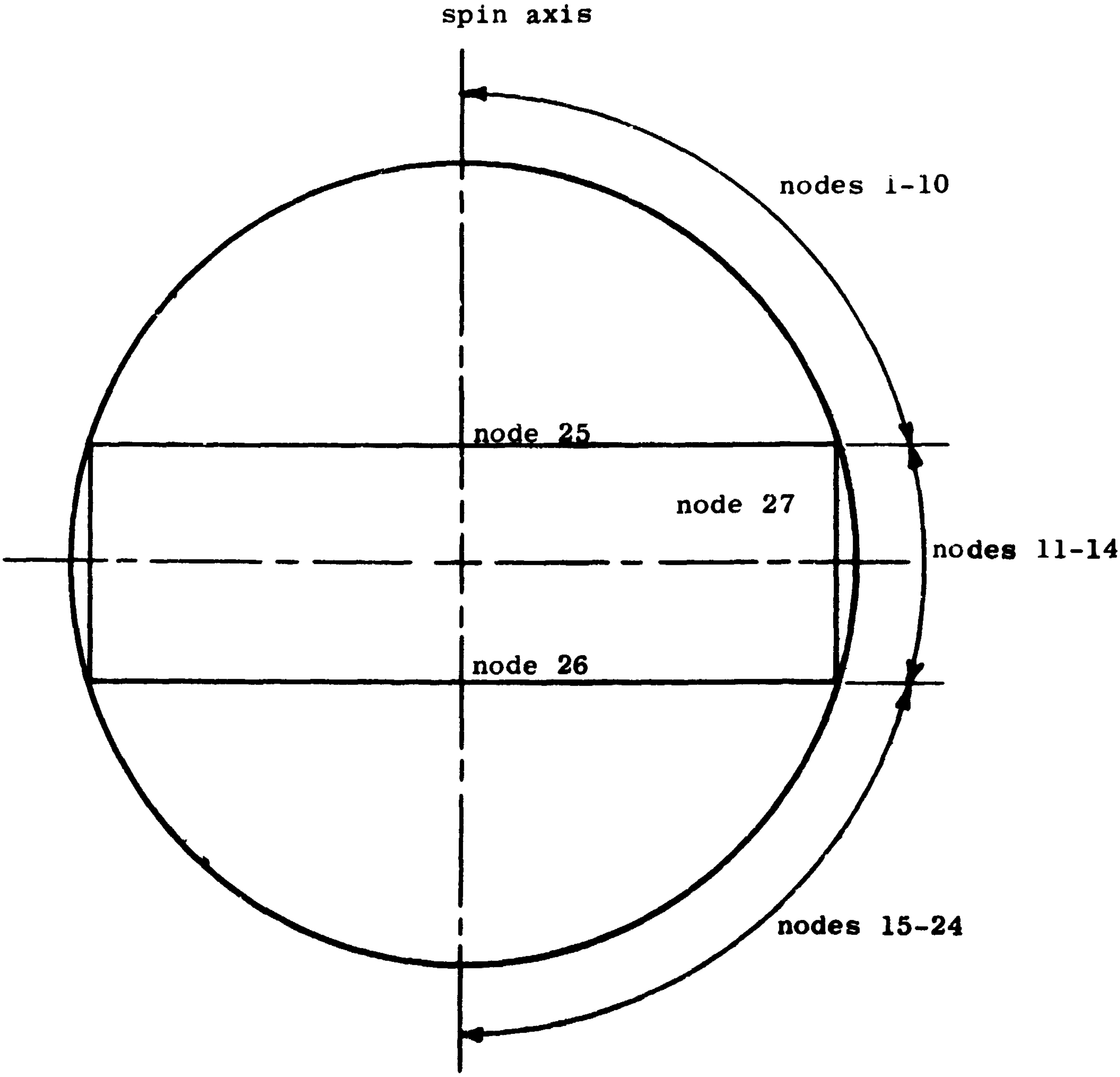

Figure 3