N66 34434

E-2570

APPLICATIONS OF THE METHOD OF MONTE CARLO TO PROBLEMS IN THERMAL RADIATION

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ABSTRACT

A summary of the work involving the Monte Carlo method in the solution of problems in thermal radiation transfer is presented, which indicates general methods previously used for solving problems in which radiation is coupled with other modes of energy transfer. Previous work involving radiation in absorbing-emitting media is included.

An example is outlined to indicate the use of the Monte Carlo method in the design of a space radiator. Suggestions are given for solution of a complex case incorporating the effects of coupled conduction, convection and radiation, wavelength dependent and selective surfaces, nonisothermal conditions, and strongly directional or nondiffuse emitting and reflecting surfaces.

A discussion is given of the factors that may affect convergence, running time, and accuracy of the Monte Carlo solutions and of the advantages and disadvantages of this approach for practical problems. Also discussed are the case of programming for complex problems and the probable machine time requirements of the method.

INTRODUCTION

In many thermal radiation problems, integral equations of a type that are amenable to a Monte Carlo solution appear. One example of current interest from a design standpoint is a space radiator system. A segment of such a system might be as shown in figure 1.

A number of analyses of such a system using conventional approaches (refs. 1 to 4, for example) have been performed, but these are limited by certain common factors. The general formulation consists of using "interchange" ("shape", "view", or "configuration") factors, which are derived through an auxiliary analysis to give the fraction of energy leaving a given area element and arriving at another. Inherent in such factors is the assumption of a diffuse surface, which follows the cosine law for emitted and reflected energy.

These fractions of arriving energy are then summed by an appropriate integration, and the net energy flux for each area element is determined. Although it is possible to account for such factors as wavelength dependent properties, temperature dependent properties, and coupling with conduction or convection in the system studied with this type of analysis, it becomes quite cumbersome to evaluate the multiple integral terms that appear. These terms may involve integration over three dimensions and the wavelength interval of interest.

Recently, a number of papers have appeared in the open literature applying the method of Monte Carlo to problems involving thermal radiation exchange (refs. 5 to 8). Reference 5, which is concerned with radiant transfer through absorbing-emitting media, discusses the case of the wavelength- and temperature-dependent radiation absorption coefficient. The advantage of this method, as pointed out by this paper, is the simple evaluation of an otherwise difficult-to-solve set of integral equations.

In reference 6, the case of transients in the temperature of an absorbing-emitting gas with simultaneous heat conduction is treated extensively. In references 7 and 8 a Monte Carlo technique is used to solve for the steady-state, axial heat flux distribution in a rocket nozzle, considering the effects of flow and convection. In reference 8 real equilibrium propellant properties were used, and temperature, pressure, and wavelength effects on the propellant radiation absorption coefficient are demonstrated.

The latter three papers all involve the solution of a set of nonlinear integro-differential equations, generally using a Monte Carlo technique for evaluation of radiation integral terms. A standard method such as Newton-Raphson is used for evaluation of the remaining set of equations, which are put in finite-difference form.

SAMPLE MONTE CARLO PROGRAM

In essence, the Monte Carlo technique removes the evaluations of multiple integrals from the computer program and replaces it with a simple summation. As an example of this approach, we may write the energy balance on a volume element dV_R of a typical radiator fin (fig. 1) as

$$A_k k\nabla^2 T + 2\int_\lambda \int_A \alpha_{\lambda,a}\epsilon_{\lambda,e}e_{\lambda,a}F_{a-e}\, dA_a\, d\lambda = 2\int_\lambda \epsilon_{\lambda,a}e_{\lambda,a}\, dA_a\, d\lambda \qquad (1)$$

where the first term is the energy conducted into the volume element, the second term is the radiant energy absorbed by the two exposed surface elements of dV_R (assuming symmetry about the plane of the fin), and the term to the right of the equality is the energy radiated from the two exposed area elements. For an element on a radiator tube, a convection term must be added.

If the integral term can be evaluated separately, the complete equation can be written in finite difference form with the integral replaced by a constant for each iteration. The coupled set of equations that results (one for each element considered) can then be solved by any of a number of standard techniques. A new temperature distribution results that can be used to reevaluate the integral terms. This procedure is continued until the temperatures converge, that is, until the old and new temperatures are the same.

The remainder of this section will be devoted to the evaluation of the integral terms by the Monte Carlo procedure, leaving the methods of solution of the entire matrix of equations and the attendant problems of convergence and accuracy to be settled by the knowledge, experience, and intuition of the interested programmer.

Monte Carlo Procedure

The procedure to be presented is a simple and straightforward "engineering" approach to the problem. It is aimed at giving the general methods that can be applied with no attempt at giving the more complex but often faster running "shortcuts" in programming with which the literature on Monte Carlo abounds.

The method for this type problem reduces to this: a number N_{dA} of "bundles" of energy are assumed to be emitted from each area element on the radiator. The energy per bundle, C_b, is taken as constant for all bundles within the system to simplify bookkeeping, and it can also be specified as a program input. The rate of bundles to be emitted from a given element must then be given by

$$N_{dA} = \frac{\left[\int_{\lambda} \epsilon_\lambda e_\lambda \, d\lambda\right] dA}{C_b} \tag{2}$$

If the direction of emission of the bundle is (η, θ) as shown in figure 2, then for a surface with directional emissivity ϵ_η independent of wavelength, the direction of an individual bundle is given by

$$R_1 = \frac{\int_0^{\eta} \epsilon_\eta \cos \eta \sin \eta \, d\eta}{\int_0^{\pi/2} \epsilon_\eta \cos \eta \sin \eta \, d\eta} \tag{3}$$

$$\theta = 2\pi R_2 \tag{4}$$

where R_1 and R_2 are numbers chosen at random from a set of numbers evenly distributed in the range $0 \leq R \leq 1$. The derivation of equations (3) and (4) is given in reference 5. These equations assume an emissivity that is symmetrical about the normal to the surface such as we might expect for a real etched or plated surface. For a surface with a constant ϵ_η, equation (3) reduces to

$$\cos \eta = \sqrt{R_1} \tag{5}$$

If ϵ_η is a nonanalytic function, equation (3) may be integrated numerically to find a curve of η as a function of R_1, which may then be curve fit to put a function, perhaps of the form

$$\eta = A + BR_1 + CR_1^2 + \ldots \tag{6}$$

directly into the machine.

Directional effects might be examined for their possible use in increasing radiator efficiency in this manner. Directional properties for a grooved surface with very strong directional effects are given in reference 9. Such grooving could be used to minimize interchange of radiation between segments of the radiator by directing the radiation preferentially toward space.

Returning to the problem at hand, we may specify the wavelength of the bundle from the equation

$$R_3 = \frac{\int_{\lambda_{min}}^{\lambda} \epsilon_\lambda e_\lambda \, d\lambda}{\int_{\lambda_{min}}^{\lambda_{max}} \epsilon_\lambda e_\lambda \, d\lambda} \tag{7}$$

where λ_{min} and λ_{max} are the limits of the wavelength range of interest. For ease of use, we may again fit a curve of the form of equation (6) to R_3 as a function of λ for use in the program.

It should be noted that ϵ_η has been assumed independent of wavelength and ϵ_λ independent of direction of emission. If this were not so, equation (3) would have to be carried out for many values of λ, and equation (6) would then give a family of curves with λ as the parameter for use in the final program.

Similarly, a family of curves with η as parameter would result from equation (7).

It is seen from the relations derived previously that η, θ, and λ may be found by picking three random numbers R_1, R_2, and R_3 and using them in simple relations. If the direction of emission is known, simple geometry determines whether the bundle will be radiated to space or will strike another point on the radiator surface. For example, if the element of bundle emission is on the fin, it can be seen from figure 3 that for x taken as the distance to the center of the nearest tube, the energy bundle must be lost to space if

$$\cos \eta > \frac{r}{x} \tag{8}$$

However, if the inequality is not satisfied, the bundle will not necessarily strike a surface. Such simple tests are useful if a large fraction of bundles are lost to space, as they save many useless calculations of final position of the bundle.

If the bundle does strike another surface, it may be either reflected or absorbed. To determine this, another number R_4 is chosen and compared to the spectral absorptivity α_λ evaluated at the bundle wavelength. If $R_4 > \alpha_\lambda$, the bundle will be reflected; otherwise, it will be absorbed.

If it is reflected, the new direction must be chosen again by selecting at random from the correct distribution of possible angles. For a diffusely reflecting surface, these are given by equations (4) and (5). For a specular surface, we need only to find the incident angle of the bundle by geometry, and then the angles of reflection are given by

$$\left.\begin{aligned} \eta_{\text{refl.}} &= \eta_{\text{incident}} \\ \theta_{\text{refl.}} &= \theta_{\text{incident}} + \pi \end{aligned}\right\} \tag{9}$$

For a directional surface, the direction of reflection will in general depend on the angle of incidence. A family of curves with the angle of incidence η' must then be specified of the form

$$R_6 = \left[\frac{\int_0^{\eta} \rho_{\eta',\eta} \cos \eta \sin \eta \, d\eta}{\int_0^{\pi/2} \rho_{\eta',\eta} \cos \eta \sin \eta \, d\eta}\right]_{\eta'} \tag{10}$$

where $\rho_{\eta',\eta}$ is the reflectivity in direction η for energy incident from η'. As a practical matter, such data is scarce. One source is reference 10 for an idealized grooved surface.

The reflected bundle is now followed through any successive reflections until it is either absorbed at a surface or lost to space. Each bundle emitted from each element is followed in the same way.

Absorptions at each area element are tallied as they occur. Multiplying the number of bundles finally absorbed by the energy per bundle gives the total radiant energy absorbed at each area element, and we have thus completed evaluation of the integral term in equation (1).

If solar radiation is to be considered, the direction of incidence must be specified, and sufficient bundles are fired at our radiator to equal the incident solar energy. The wavelength of each incident bundle is again chosen by equation (8) using solar emissivities and temperatures. The bundles are followed as outlined previously.

Running time for such a program is difficult to predict. It would certainly depend on many factors; however, the major ones are probably the astuteness of the original temperature distribution guess and the number of energy bundles followed. For this type of program, the running time would be nearly proportional to the number of bundles followed. For the mathematically similar but more cumbersome rocket nozzle program of reference 8, four to five iterations took less than 10 minutes time, following about 75,000 energy bundles on the final few. A Newton-Raphson technique gave good convergence of the set of equations for each iteration. On the IBM 7094, the four to five iterations brought convergence to within 0.1 percent for all temperatures in the distribution. The program outlined here would almost certainly run faster because the multiple absorptions in the nozzle propellant are not present, and these absorptions represent a major fraction of the running time.

Advantages and Disadvantages of the Method

The main work involved in programming this method is the initial evaluation of the functions describing wavelength and directional dependence. These may involve numerical integration, which needs to be carried out only once. The remainder of the programming then consists of setting up a series of "do" loops around the simple algebraic equations derived above.

Many techniques are available (ref. 11) for estimating error in Monte Carlo calculations, but accuracy can usually be increased by following more energy bundles (decreasing the energy per bundle). Often it is difficult to predict or deduce the error present in a standard multiple numerical integration.

The advantages of the Monte Carlo method for these programs might be summarized as

(1) The ability to handle complex problems in a simple and straightforward manner

(2) The simplicity of the resulting computer program, with the attendent ease of checking

(3) The ability to control to some extent and to accurately estimate the probable errors in the solutions

There are also certain disadvantages. It is possible that for certain problems sufficient accuracy cannot be obtained from the straightforward methods outlined without excessive computer times. More sophisticated approaches would then be required, with some loss of the advantages outlined previously. These approaches are well developed in reference 11.

Because of the simplicity of the equations and the independence of each occurrence along the path of a bundle, it is sometimes difficult to pinpoint or even detect the presence of program errors. Judgements made are usually on the basis of the final summations, which may show little obvious effect of error unless gross mistakes are present. Standard approaches generally provide a number of possible intermediate checkpoints, lending some feeling of security to the programmer.

CONCLUSIONS

The Monte Carlo method may offer considerable savings in programming complexity and time for thermal radiation problems in which variations with wavelength and direction are encountered in surface radiative properties. The method appears to offer short computer running times even where complex variations occur.

SYMBOLS

A	area
C_b	energy per Monte Carlo bundle
e_λ	Planck spectral energy distribution
F_{a-e}	view factor between area elements a and e
k	thermal conductivity
N_{dA}	rate of bundles emitted from a given surface element
r	outside radius of tube on tube and fin radiator
R	random number from evenly distributed set in range 0 to 1
T	absolute temperature
V	volume
x	normal distance from area element on fin to centerline of nearest tube
α	surface absorptivity
ϵ	surface emissivity
λ	wavelength
η,θ	angles defined in fig. 2
$\rho_{\eta',\eta}$	reflectivity in angle η for energy incident at angle η'

Subscripts:

λ	spectrally dependent
a,e	surface area element index, absorbing or emitting element, respectively
η,θ	dependent on angle η or θ

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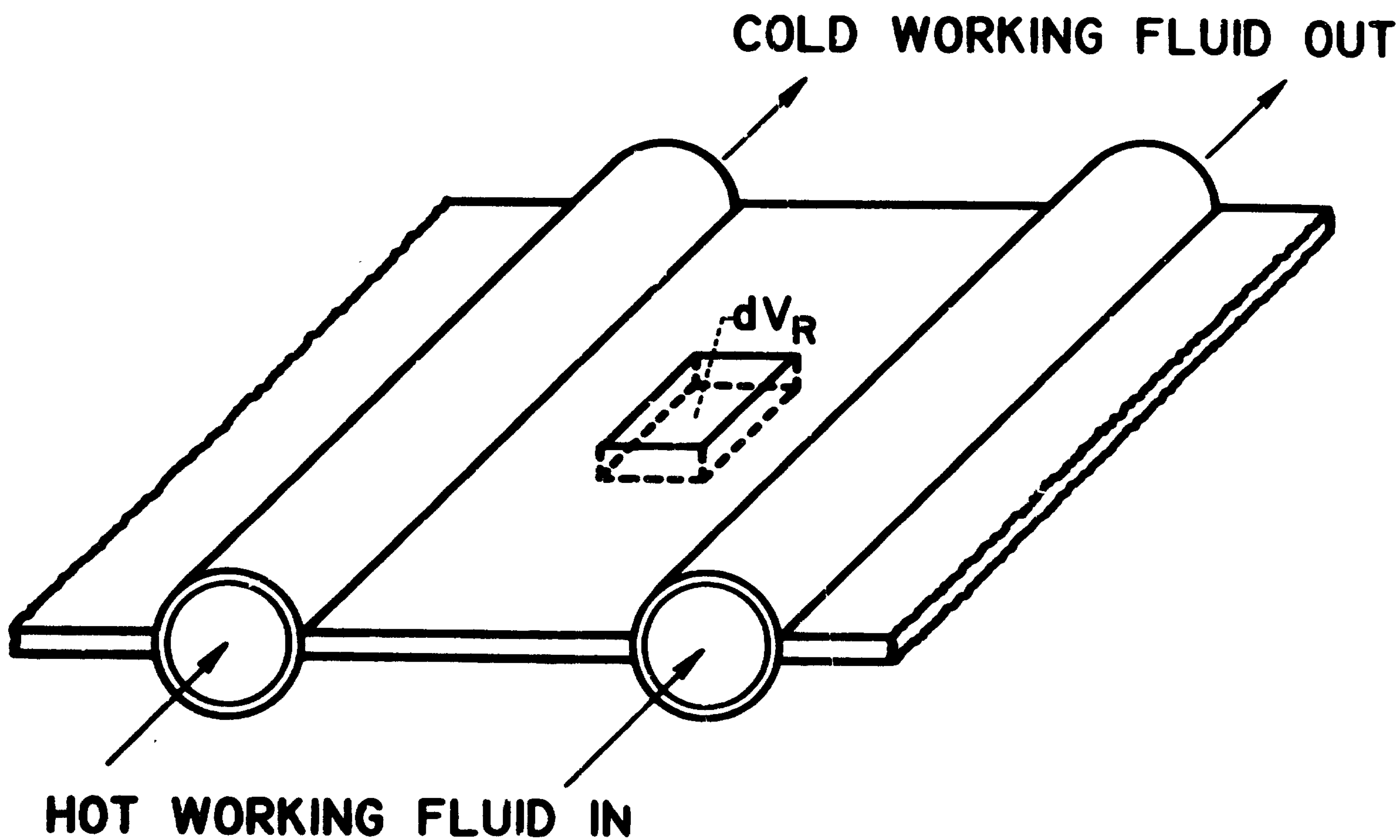

Fig. 1. - Fin-tube space radiator segment.

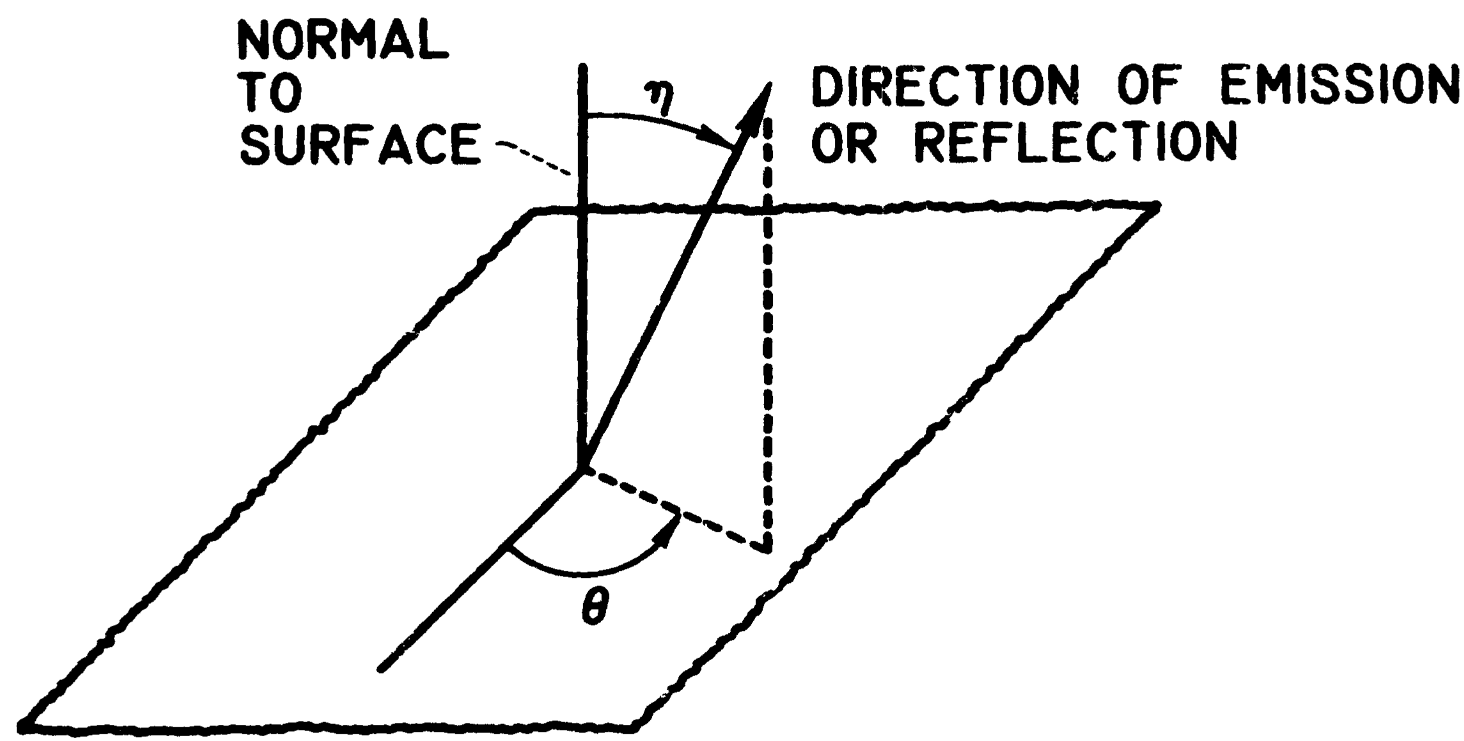

Fig. 2. - Geometry of bundle emission.

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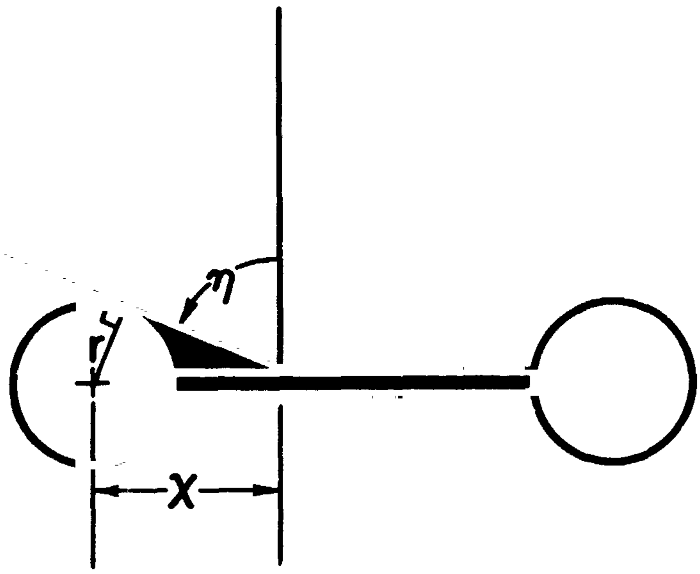

Fig. 3. - Criterion for loss to space.