NASA - Langley

SIMPLIFIED COMPUTATION OF EARTH THERMAL AND ALBEDO RADIATION INCIDENT ON A SPACECRAFT

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ABSTRACT

The integral used for the computation of the earth thermal and albedo flux on a section of a satellite is directly integrable if the projected area is represented by a truncated Fourier series in the aspect angle. A great saving in computer time results from this representation. The series for a general surface of revolution, whose axis coincides with the axis of rotational symmetry of the satellite, is obtained by dividing the surface into elemental conical frustums and averaging the series for a cone over the generating curve.

SYMBOLS

$A_P(\eta)$	projected, or apparent, area of a surface viewed at an aspect angle η
A_T	total area of a surface
a_s	mean solar absorptivity of a surface
e	thermal emissivity of a surface
i_k	coefficient of the kth term in the Fourier cosine series for $A_P/A_T(\eta)$
I_k	kth coefficient of the power series in cosine η which approximates $A_P/A_T(\eta)$
q	heat input to a surface per unit time
r_e	mean albedo of the earth
S	solar radiation constant
T_e	black-body temperature of the earth
(α,ϕ)	angular coordinates of an element of surface area on the earth, with the origin at the satellite
α_o	half the angle subtended at the satellite by the earth
o	semivertex angle of a cone or frustum
η	aspect angle - the angle between the spin axis of a satellite and the line of view
η_e	value of η at the point on earth directly beneath the satellite
θ_S	angle between the earth-satellite line and the earth-sun line
μ	symbol for A_P/A_T
σ	Stefan-Boltzmann constant

Computation of the heat input to a spacecraft surface due to earth thermal and albedo radiation involves evaluation of a double integral and extensive tabulation of the projected area of the surface. However, a good closed-form approximation for the integral (requiring no numerical integration and no tabulation of projected areas) is obtained by substituting a truncated Fourier series for the projected area.

The two heat input rates are given by

$$q_{\substack{\text{earth}\\ \text{thermal}}} = A_T e \sigma T_e^4 F(\alpha_o, \eta_e)$$

$$q_{\text{albedo}} \cong A_T S r_e a_s \cos \vartheta_S F(\alpha_o, \eta_e)$$

where F, the so-called shape factor, is given by

$$F(\alpha_o, \eta_e) = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\alpha_o} \frac{A_P}{A_T} (\alpha, \phi, \eta_e) \sin \alpha \, d\alpha \, d\phi$$

Expanding the projected area in a Fourier series

$$\frac{A_P}{A_T} (\eta) \cong i_o + i_1 \cos \eta + i_2 \cos 2\eta + i_4 \cos 4\eta$$

By use of trigonometric identities, this series is converted to

$$\frac{A_P}{A_T} (\eta) \cong I_o + I_1 \cos \eta + I_2 \cos^2 \eta + I_4 \cos^4 \eta$$

where

$$\begin{cases} I_o = i_o - i_2 + i_4 \\ I_1 = i_1 \\ I_2 = 2i_2 - 8i_4 \\ I_4 = 8i_4 \end{cases}$$

Taking the scalar product of the unit vectors along the satellite axis and the view line, we get

$$\cos \eta = \cos \eta_e \cos \alpha + \sin \eta_e \sin \alpha \cos \phi$$

Making use of this relation and the series for $\frac{A_P}{A_T}$, we find

$$\begin{aligned} F(\eta_e,\alpha_o) \cong\ & (1 - \cos \alpha_o)\left[2I_o + I_2 \sin^2\eta_e + \tfrac{3}{4} I_4 \sin^4\eta_e\right] \\ & + (1 - \cos^2\alpha_o)\left[I_1 \cos \eta_e\right] \\ & + (1 - \cos^3\alpha_o)\left[I_2\left(\tfrac{2}{3} - \sin^2\eta_e\right) + I_4 \sin^2\eta_e\left(2 - \tfrac{5}{2} \sin^2\eta_e\right)\right] \\ & + (1 - \cos^5\alpha_o)\ I_4\left(\tfrac{2}{5} - 2 \sin^2\eta_e + \tfrac{7}{4} \sin^4\eta_e\right) \end{aligned}$$

Thus, the problem is reduced to finding the coefficients I_k in the expansion for $A_P/A_T(\eta)$. For a satellite spinning about its axis of symmetry, the projected areas of some surfaces are given by simple expressions such as these:

$$\mu_{plate} = \left(\frac{A_P}{A_T}\right)_{plate} = \begin{cases} \cos \eta & 0 \leq \eta \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \eta \leq \pi \end{cases}$$

$$\mu_{plate} \cong \frac{1}{5\pi} + \frac{1}{2} \cos \eta + \frac{12}{5\pi} \cos^2\eta - \frac{16}{15\pi} \cos^4\eta$$

and

$$\mu_{cylinder} = \frac{1}{\pi} \left|\sin \eta\right| \cong \frac{46}{15\pi^2} - \frac{8}{15\pi^2} \cos^2\eta - \frac{32}{15\pi^2} \cos^4\eta$$

$$\mu_{sphere} = \frac{1}{4}$$

$$\mu_{hemisphere} = \frac{1}{4} + \frac{1}{4} \cos \eta$$

Other surfaces are much more difficult to work with. However, since any surface of revolution can be divided into elemental frustums of cones, finding the I_k's of a cone as a function of its semivertex angle δ will make possible a simple general solution. Dividing the cone into infinitesimal plane elements and integrating the series for the projected area of a plate around the cone from 0 to 2π, the following result is obtained:

$$\mu_{cone} \cong \left[\frac{1}{5\pi} + \frac{6}{5\pi}\cos^2\delta - \frac{2}{5\pi}\cos^4\delta\right] + \left[\frac{1}{2}\sin\delta\right]\cos\eta$$

$$+ \left[\frac{12}{5\pi} - \frac{34}{5\pi}\cos^2\delta + \frac{4}{\pi}\cos^4\delta\right]\cos^2\eta$$

$$+ \left[-\frac{16}{15\pi} + \frac{16}{3\pi}\cos^2\delta - \frac{14}{3\pi}\cos^4\delta\right]\cos^4\eta$$

For a general body of revolution, the projected area is found by summing infinitesimal frustums

$$\mu_{body} = \frac{A_P}{A_T} = \frac{\int \mu_{frust.}\, dA}{\int dA} = \bar{\mu}_{frust.}$$

For a generating curve, $y = f(x)$, revolved about the X-axis, the semivertex angle δ of an infinitesimal frustum is given by

$$\tan \delta = \left|\frac{df}{dx}\right|$$

From this, the I_k's of the infinitesimal frustum can be found as functions of x. Then,

$$(I_k)_{body} = \frac{\int_{x_1}^{x_2} \left[I_k(x)\right]_{frust.} \, dA}{\int_{x_1}^{x_2} dA} = (\overline{I_k})_{frust.}$$

where

$$dA = 2\pi f(x)\sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx$$

In polar coordinates, revolving the curve $r = r(\theta)$ about the axis $\theta = 0$ gives

$$\tan \delta = \left| \frac{r \cos \theta + \frac{dr}{d\theta} \sin \theta}{r \sin \theta - \frac{dr}{d\theta} \cos \theta} \right|$$

and

$$dA = 2\pi r^2 \sin \theta \sqrt{1 + \left(\frac{1}{r}\frac{dr}{d\theta}\right)^2}\, d\theta$$

The surfaces of revolution will, of course, be split into sections small enough to justify the assumption of equal temperature throughout a section.

This technique for surfaces of revolution can be applied also to surfaces not physically circular in cross section, but optically so due to their spin.

For a satellite tumbling about an axis perpendicular to the symmetry axis, with some residual spin about the latter, the projected area is given by

$$\mu_{tumbling} = \left[\frac{A_P}{A_T}(\eta)\right]_{tumbling} = \frac{1}{2\pi}\int_0^{2\pi} \mu_{spin}^{(\eta_1)}\ d\phi$$

where

$$\cos\eta_1 = \sin\eta\cos\phi$$

Substituting

$$\mu_{spin}(\eta_1) \cong \sum_{k=0}^{4} (I_k)_{spin}(\sin\eta\cos\phi)^k$$

we get

$$\mu_{tumbling} \cong (I_0)_{spin} + \frac{1}{2}(I_2)_{spin}\sin^2\eta + \frac{3}{8}(I_4)_{spin}\sin^4\eta$$

or

$$\begin{cases} (I_0)_{tumble} = \left[I_0 + \frac{1}{2}I_2 + \frac{3}{8}I_4\right]_{spin} \\ (I_1)_{tumble} = 0 \\ (I_2)_{tumble} = -\left[\frac{1}{2}I_2 + \frac{3}{4}I_4\right]_{spin} \\ (I_4)_{tumble} = \frac{3}{8}(I_4)_{spin} \end{cases}$$