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MEASUREMENTS OF SATELLITE RANGE WITH A RUBY LASER

C. G. Lehr, L. A. Maestre, and P. H. Anderson

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ABSTRACT

Range measurements on the GEOS-I and BE-C satellites were made with an experimental optical radar located at the Smithsonian astrophysical observing station in New Mexico. The radar's transmitter was a pulsed ruby laser. The receiver incorporated a time-interval counter with an accuracy of ± 10 nsec, which gives a resolution of ± 1.5 m.

The most distant range measured was 2.6 Mm. The signals returned from the retroreflecting satellites were more than 16 db below the values predicted by the radar-range equation.

The measured ranges were compared with values obtained from orbits computed with field-reduced data of the Baker-Nunn observing stations. The measured and computed ranges were consistent to within a few hundreds of meters, the error that might be expected in the field-reduced orbits.

MEASUREMENTS OF SATELLITE RANGE WITH A RUBY LASER[1]

C. G. Lehr, L. A. Maestre, and P. H. Anderson

1. INTRODUCTION

We present some new results from the continuing laser experiment at the Smithsonian astrophysical observing station, Organ Pass, New Mexico. The experimental system has already been described, and previous results have been reported (General Electric Co., 1965; Anderson et al., 1965). We consider here the experiments performed in the two periods December 8 to 23, 1965 and January 24 to February 5, 1966. The new features of this work are our first range measurements on GEOS-I and the use of a new time-interval counter, whose 10 nsec accuracy corresponds to a range resolution of 1.5 m.

We also use the data from all our experiments to compare the performance of the laser system with that predicted by the range equation. We have too few points for a good statistical sample, so the conclusions we present are tentative.

We compare a number of range measurements with values derived from field-reduced orbits. These range measurements were obtained with the new counter and with the laser in the Q-switched mode of operation. The comparison is not as meaningful as one with precisely reduced

[1] This work was supported by Grant No. NsG 87-60 from the National Aeronautics and Space Administration.

orbits will be when these have been computed. All we show at present is that no large discrepancies between the laser measurements and the Baker-Nunn measurements are presently apparent.

2. THE RANGE EQUATION

In comparing measurements and calculations for our laser system, we use the range equation (Lehr, 1966, Section 3):

$$\frac{S}{E} = \frac{1}{R^4} \cdot \frac{A_S A_R}{\Omega_T \Omega_S} \cdot T^2 \cdot \frac{10^{19}}{2.86} \frac{\text{photons}}{\text{J}} , \tag{1}$$

where S is the received signal, in photons, R is the range of the satellite, A_S is the effective area of the satellite's retroreflector, A_R is the effective area of the receiver's light collector, Ω_T is the solid angle of the transmitted laser beam, Ω_S is the solid angle of the beam reflected from the satellite, T is the atmospheric extinction, and E is the transmitted energy of the laser. The quantities in equation (1) take on the following numerical values (Lehr, 1966, Appendices C, D, E, F, and G);

$A_S^*(m^2)$	8.0×10^{-3}	BE-B, C	9.35×10^{-2}	GEOS-I
Ω_S(sterad)	2.8×10^{-9}	BE-B, C	7.3×10^{-9}	GEOS-I
$A_R^*(m^2)$	0.177 (laser receiver)		9.04×10^{-2} (Baker-Nunn camera)	
Ω_T(sterad)	7.8×10^{-7}			
T	0.7			

*These are effective values that take account of losses in the optical systems.

$$
E(J) \begin{cases} 0.5 \text{ (Q switched)} \\ 11 \text{ (not Q switched, June 1965 experiments)} \\ 36 \text{ (not Q switched, other experiments)} \end{cases}
$$

Using these values we obtain the following expressions from equation (1):

Laser receiver; GEOS-I: $E = 0.5J$ $\quad \log S = 6.40 - 4 \log R \quad , \qquad (2a)$

Laser receiver; BE-B, C: $E = 0.5J$ $\quad \log S = 5.74 - 4 \log R \quad , \qquad (2b)$

B-N camera; BE-B, C: $E = 11J$ $\quad \log S = 6.79 - 4 \log R \quad , \qquad (2c)$

where S is in photons and R is in Mm. Equations (1) and (2) do not allow for any reduction in received-signal strength that may result from velocity aberration (Plotkin, 1964); nor do they include a number of other effects that may also reduce the signal strength. We prefer to account for them as deviations from equations (1) and (2).

In comparing our experimental results with equation (2), we must convert the values of S in millivolts, obtained from oscilloscopic records, to numbers of photons. The following calculation gives the necessary conversion factor.

The resolution of the oscilloscope is specified to be 30 nsec; thus a pulse from the photomultiplier, no matter how much narrower, will have this duration. The photoelectronic charge is 1.60×10^{-19} coulomb and the gain of the photomultiplier is 10^7; consequently the output current is

$$\frac{1.60 \times 10^{-19}}{30 \times 10^{-9}} \times 10^7 = 5.3 \times 10^{-5} \text{ amp/photoelectron} \quad .$$

For a load resistance of 50 ohms, we obtain 2.7 mv/photoelectron, which, for a quantum efficiency of 0.03, becomes 11.2 photons/mv.

Figure 1 is an oscillogram of randomly emitted photoelectrons. Although the heights of the pulses vary, their average approximates the calculated value of 2.7 mv for 1 photoelectron.

The length of the laser's transmitted pulse is about twice the resolution time of the receiver's oscilloscope; therefore, the oscilloscope integrates only over half the returned pulse. When a strong signal is received, many photoelectrons are generated. If these photoelectrons are equally distributed in time, the conversion factor of 2.7 mv/photoelectron gives only half the true signal strength. Despite this consideration, 2.7 mv/photoelectron will be used for both strong and weak signals. For strong signals this procedure introduces an error of 3 db.

Figures 2, 3, and 4 show typical oscillograms. Only one transmitted pulse is shown; the others were similar. Figures 3 and 4 show the variations in intensity that sometimes occur in the oscilloscopic record of the returned pulse. The cause of these intensity variations has not been determined. Some of the photographs were not as distinct as those in Figures 2 to 4: the top of the return pulse faded into the background and the measurement of pulse height was difficult and possibly subject to error.

The strength of the returns from GEOS-I are plotted against range in Figure 5. For comparison, equation (2a) is shown in Figure 5a. All the experimental signal strengths are more than 20 db below the calculated values. No significant dependence of the signal strength on the satellite's elevation or azimuth was apparent. The cause of the discrepancies has not been ascertained. A similar graph for BE-C is presented in Figure 6. Here data from previous reports have been used. One point corresponding to non-Q-switched operation was included. In this case the return was barely discernible in the background noise, so the energy of a single spike in the 1-msec returning pulse was assumed to be equivalent to 1 photoelectron. The transmitted energy per spike was taken to be the 0.5 J transmitted in Q-switched operation. For this satellite the received signal strength comes within 19 db of the value calculated from the range equation.

Figure 7 compares the strength of a photographic return from the BE-B satellite with equation (2c), the range equation that is obtained when the Baker-Nunn camera is used as a receiver. The strength of the signal was estimated from the appearance of the point image on the film. The signal turns out to be about 16 db below the calculated value. No photoelectric returns from Q-switched operation have been obtained as yet for the BE-B satellite.

3. THE RANGE

Table 1 gives the results of the range measurements obtained between December 20, 1965 and February 5, 1966. These measurements were obtained by using a Model 783G El Dorado time-interval counter. This counter has an accuracy of ± 10 nsec plus the error in the time base. Our time base was a 1-Mc signal obtained from the station's EECo clock. The error it introduces into the time-interval measurement should be negligible.

The readings of the time-interval counter were increased by 0.19 μsec to compensate for time delays in the transmission lines and amplifiers used with the transmitted and received pulses. The correction was obtained by ranging on a target at a surveyed distance of 1535.74 m from the laser to the target and back to the receiver. The direction of the target was roughly south of the laser.

An atmospheric correction was made by assuming that the index of refraction n has the following variation with H (in kilometers), the height above the earth:

$$n = 1 + 292 \times 10^{-6} \exp(-0.1385H) \quad .$$

From this expression we obtain the following formula (see Anderson et al., 1965, Appendix):

$$d = \frac{2.10}{\sin \alpha} \text{ m} \quad , \tag{3}$$

where d is the difference between the optical path in the atmosphere and in vacuum, and α is the elevation angle.

The value of the velocity of light[2] used in converting time-interval measurements to range measurements was

$$c = 2.997928 \times 10^8 \text{ m sec}^{-1} \quad .$$

The relative positions of the transmitter, receiver, and camera are depicted in Figure 8. A correction derived in the Appendix was applied to all the laser measurements; consequently the values reported are referred to the station coordinates of the Baker-Nunn camera.

The residuals probably do not represent errors in the range measurements. When orbits are computed from precisely reduced data, the range measurements can be checked more accurately.

Table 2 summarizes the data that were used in plotting Figures 5 and 6 but that were not included in Table 1.

Figure 9 shows that the transmitted pulse length was 50 to 60 nsec. Figure 10 shows the structure of a laser pulse returned from a satellite. An expanded sweep was used. The irregularities in the shape are

[2] This value comes from p. 7-3 of the American Institute of Physics Handbook (1963). The error given there is ± 4 in the last figure.

probably produced by the detector; only about 16 photoelectrons were produced. Obtaining presentations with an expanded scale required an accurate delay of the start of the sweep. Unfortunately, no expanded traces could be obtained for returns that showed intensity variations like those of Figures 3 and 4.

4. ACKNOWLEDGMENTS

The authors acknowledge with pleasure the contributions of Gordon L. Snyder of the General Electric Company; J. T. Williams and his staff at Smithsonian's observing station in New Mexico; Kaare Aksnes, Daniel R. Hanlon, George A. Westgate, and Mrs. Beatrice R. Miller of SAO's headquarters in Cambridge, Massachusetts.

Table 1. Summary of range measurements

Date (UT)	Time[a] (UT)	Satellite	Azimuth[b] (degrees)	Elevation[b] (degrees)	Strength of received signal (photons)	Time[c] interval (msec)	Atmospheric correction (m)	Range[d] (Mm)	Deviation from field-reduced orbit (m)
20 Dec. '65	$11^h 26^m 00^s.0079$	65089-01	24.5	66.4	1230	8.92563	2.3	1.337,914	+ 84
20 Dec. '65	11 27 00.0066	65089-01	73.3	67.4	2460	8.70612	2.3	1.305,004	+310
20 Dec. '65	11 29 00.0075	65089-01	119.0	44.3	-	10.48073	2.9	1.571,001	+620
25 Jan. '66	13 00 00.0073	65032-01	241.3	30.9	-	10.68792	4.1	1.602,098	***
28 Jan. '66	12 55 30.0057	65032-01	341.1	57.4	-	7.39632	2.5	1.108,687	+ 93
29 Jan. '66	03 51 10.0110	65089-01	301.2	33.4	-	15.17640	3.8	2.274,908	-190
29 Jan. '66	03 52 10.0106	65089-01	293.5	40.0	-	13.52004	3.3	2.026,624	-190
29 Jan. '66	03 53 10.0088	65089-01	281.6	46.8	345	12.16234	2.9	1.823,109	-200
29 Jan. '66	03 54 10.0080	65089-01	263.2	52.4	403	11.22275	2.7	1.682,265	-190
29 Jan. '66	12 13 50.0063	65032-01	291.0	58.3	2240	7.26998	2.5	1.089,758	+ 43
29 Jan. '66	12 14 50.0050	65032-01	339.2	66.3	-	6.80542	2.3	1.020,112	+ 65
29 Jan. '66	12 15 50.0061	65032-01	24.5	57.2	-	7.32696	2.5	1.098,276	+ 72
31 Jan. '66	01 56 30.0079	65089-01	9.7	44.3	381	13.41336	3.0	2.010,600	+ 24
31 Jan. '66	01 57 30.0098	65089-01	25.0	48.9	-	12.45320	2.8	1.866,681	+ 27
31 Jan. '66	01 58 30.0080	65089-01	44.6	51.0	430	11.93189	2.7	1.788,533	+ 50
31 Jan. '66	01 59 30.0073	65089-01	65.3	49.3	656	11.91522	2.8	1.786,029	+ 63
1 Feb. '66	01 59 20.0088	65089-01	346.4	38.4	242	14.80128	3.4	2.218,663	-180
1 Feb. '66	02 00 20.0095	65089-01	354.1	45.7	493	13.21434	2.9	1.980,785	-140
1 Feb. '66	02 01 20.0074	65089-01	6.6	53.5	932	11.92157	2.6	1.786,998	- 92
1 Feb. '66	02 03 20.0092	65089-01	57.5	61.6	1820	10.67500	2.4	1.600,163	+ 42
1 Feb. '66	02 04 20.0086	65089-01	84.7	56.6	1790	10.89405	2.5	1.632,960	+ 42
1 Feb. '66	02 05 20.0074	65089-01	102.0	47.9	1030	11.66770	2.8	1.748,923	+120
1 Feb. '66	12 08 40.0066	65032-01	22.4	39.0	-	9.18286	3.3	1.376,468	+190
1 Feb. '66	12 10 50.0084	65032-01	47.4	22.8	-	12.69731	5.4	1.903,258	+150
1 Feb. '66	12 11 20.0089	65032-01	50.5	19.7	-	13.70945	6.1	2.054,972	+230
3 Feb. '66	02 09 20.0080	65089-01	325.1	53.4	-	11.93067	2.6	1.788,375	+ 48
3 Feb. '66	02 10 30.0099	65089-01	324.7	68.2	-	10.42652	2.3	1.562,904	- 97
4 Feb. '66	02 14 00.0071	65089-01	303.8	54.5	-	11.65264	2.6	1.746,701	- 51
4 Feb. '66	02 16 30.0069	65089-01	221.5	72.7	470	9.80048	2.2	1.469,061	+ 39
4 Feb. '66	02 17 30.0071	65089-01	185.8	63.1	4840	10.13840	2.4	1.519,707	+ 16
4 Feb. '66	02 18 00.0073	65089-01	177.4	56.9	2630	10.53922	2.5	1.579,786	+ 2
4 Feb. '66	02 19 00.0079	65089-01	168.1	45.1	-	11.72798	3.0	1.757,972	- 10
5 Feb. '66	02 18 40.0075	65089-01	282.8	50.9	-	11.92574	2.7	1.787,642	- 7
5 Feb. '66	02 19 10.0085	65089-01	273.9	54.3	-	11.41870	2.6	1.711,637	+ 58
5 Feb. '66	02 19 40.0084	65089-01	262.6	57.1	-	11.034407	2.5	1.653,981	+ 60
5 Feb. '66	02 20 10.0087	65089-01	249.2	58.6	1610	10.78603	2.4	1.616,800	+ 50
5 Feb. '66	02 20 40.0074	65089-01	234.8	58.5	1460	10.68499	2.5	1.601,651	+ 50
5 Feb. '66	02 21 10.0072	65089-01	221.0	56.8	-	10.73600	2.5	1.609,294	+ 37
5 Feb. '66	02 21 40.0082	65089-01	209.2	453.7	1340	10.93769	2.6	1.639,525	+ 24
5 Feb. '66	02 22 10.0085	65089-01	199.7	49.7	650	11.28258	2.7	1.691,219	+ 7
5 Feb. '66	02 22 40.0079	65089-01	192.4	45.4	256	11.75848	3.0	1.762,551	- 6

[a]Time of reflection of laser pulse from satellite.

[b]Predicted.

[c]Corrected for time delays in system.

[d]Includes the atmospheric correction and is referred to the coordinates of the Baker-Nunn camera.

[e]The very large residual obtained from this observation indicated an experimental error of some sort.

Table 2. Summary of received signal strengths for observations that did not yield precise range measurements.

Date (UT)	Time (UT)	Satellite	Azimuth (degrees)	Elevation (degrees)	Predicted range (Mm)	Strength of received signal (photons)
19 June '65[a]	$4^h25^m16^s$	64064-01	226	63	1.19	$\sim 10^5$
31 July '65[b]	03 05 30	65032-01	16	27	2.140	~30
8 Oct. '65	02 18 50	65032-01	187.4	47.8	1.611	85
8 Oct. '65	02 20 50	65032-01	139.2	57.1	1.442	70
8 Oct. '65	02 21 50	65032-01	112.9	52.6	1.492	102
8 Oct. '65	02 22 50	65032-01	95.6	44.4	1.629	80
9 Oct. '65	01 40 40	65032-01	127.3	45.9	1.639	171
9 Oct. '65	01 41 40	65032-01	109.7	41.5	1.727	190
12 Oct. '65	01 35 00	65032-01	70.2	40.1	1.769	640
12 Oct. '65	03 25 30	65032-01	323.8	49.7	1.551	538
13 Oct. '65	02 47 20	65032-01	26.4	47.4	1.587	605
14 Oct. '65	02 05 40	65032-01	15.4	58.4	1.447	426
14 Oct. '65	02 06 40	65032-01	33.9	49.1	1.581	403
16 Dec. '65	02 34 00	65089-01	125.5	48.6	2.644	202
19 Dec. '65	11 23 30	65089-01	84.0	49.1	1.524	280
20 Dec. '65	02 53 20	65089-01	22.4	77.2	2.136	146
21 Dec. '65	02 54 00	65089-01	244.1	62.1	2.358	168
27 Jan. '66	03 43 20	65089-01	320.9	45.4	1.912	176
27 Jan. '66	03 44 20	65089-01	318.6	56.3	1.674	297
31 Jan. '66	02 01 30	65089-01	95.3	37.5	2.018	242
4 Feb. '66	02 11 30	65089-01	316.0	33.2	2.379	132
5 Feb. '66	02 23 40	65089-01	182.1	36.6	1.951	64

[a]Photographic return.

[b]Not Q-switched.

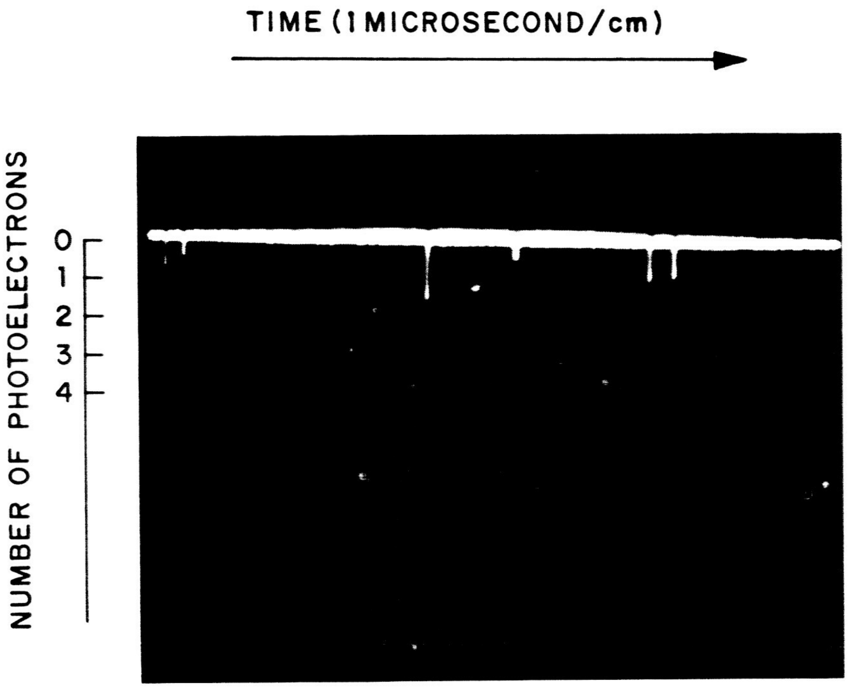
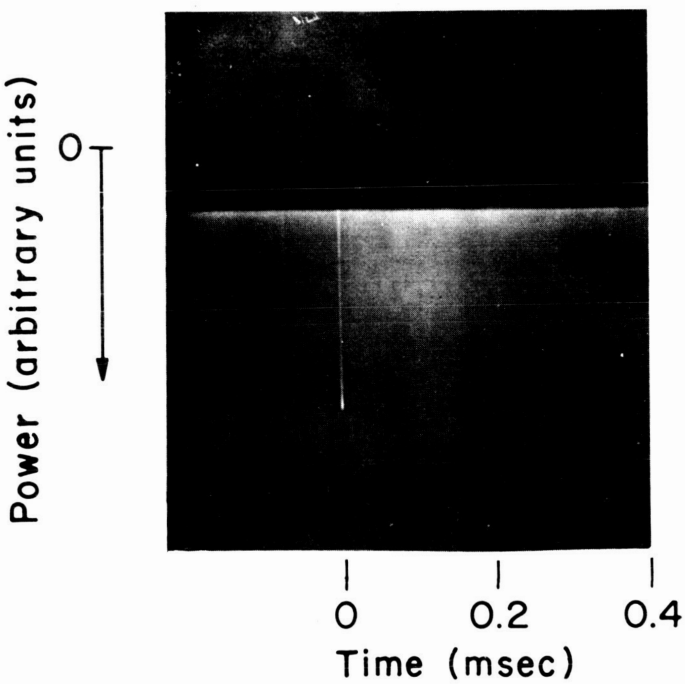


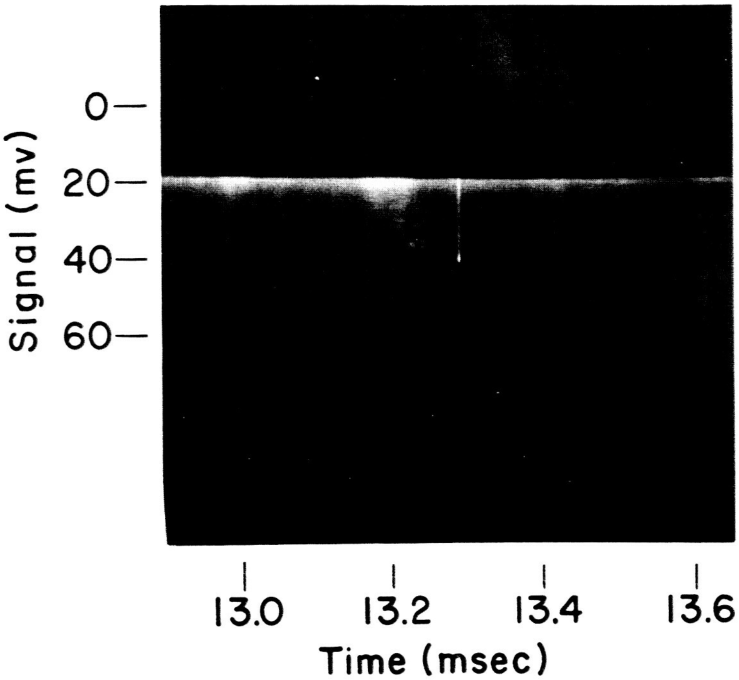
Figure 1. Noise of the night sky. The vertical scale is the one used throughout this report in determining the strength of the received signal. The appropriateness of this scale may be estimated from the knowledge that, with high probability, each of the six pulses represents the emission of a single photoelectron. (This photograph was furnished by the General Electric Company.)



(a) Transmitted

Figure 2. Laser pulse.

Date and time (UT)	21 Dec. '65 $2^h06^m10^s$
Object	6503201 (BE-C)
Elevation (degrees)	62
Received signal (photons)	490
Predicted range (Mm)	1.988



(b) Received

Figure 2. (Cont.)

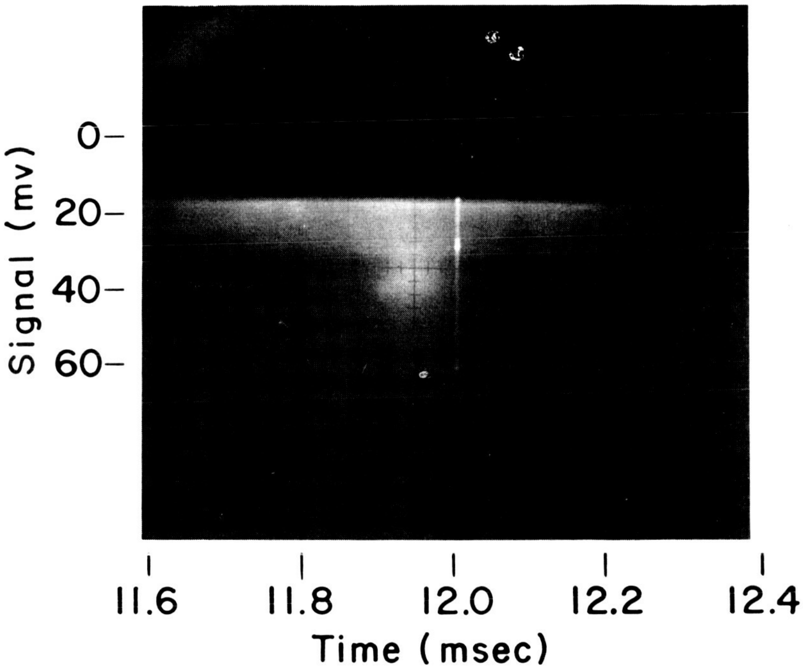


Figure 3. Received laser pulse.

Date and time (UT)	31 Jan. '66 $01^{h}59^{m}30^{s}$
Object	6508901 (GEOS-I)
Elevation (degrees)	49
Received signal (photons)	660
Predicted range (Mm)	1.798

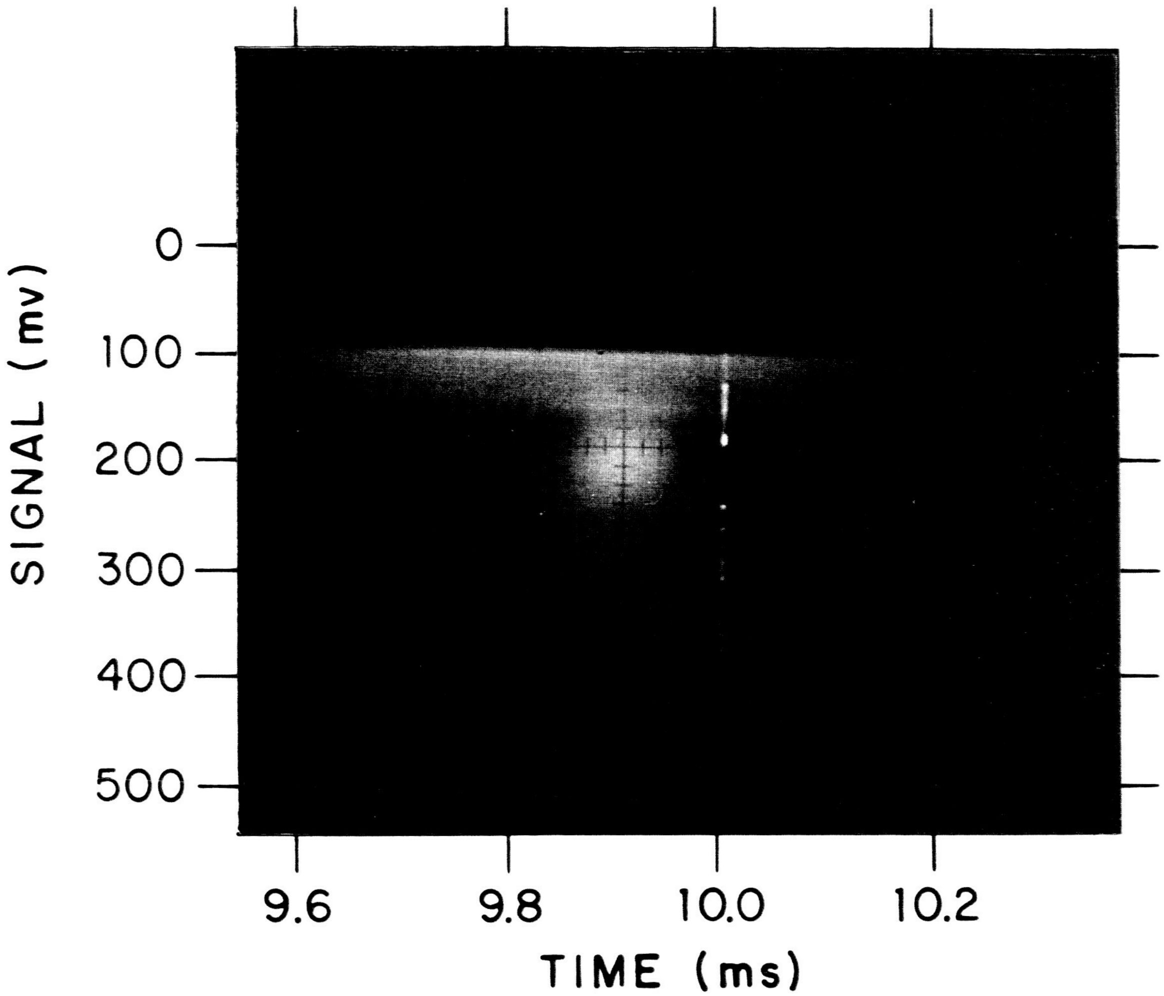


Figure 4. Received laser pulse.

Date and time	4 Feb. '66 $02^{h}17^{m}30^{s}$
Object	6508901 (GEOS-I)
Elevation (degrees)	63
Received signal (photons)	4800
Predicted range (Mm)	1.517

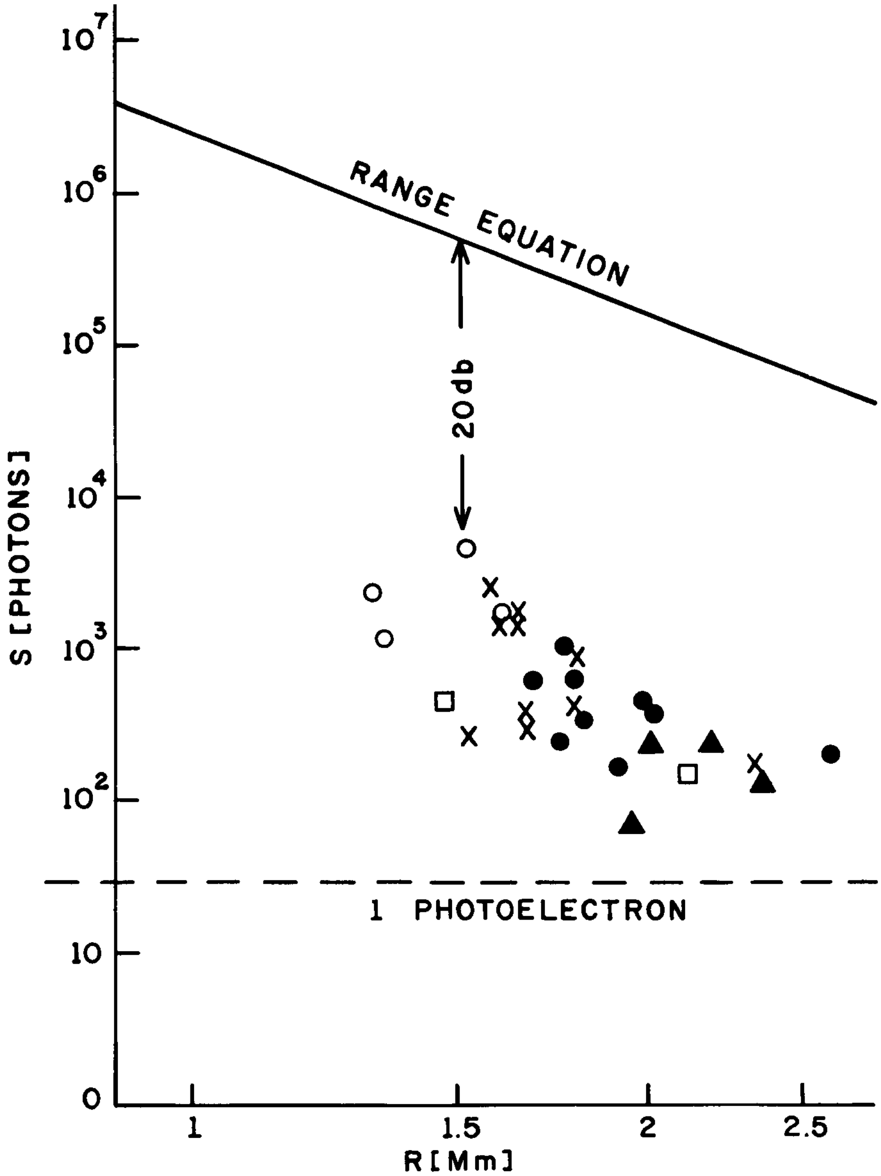


Figure 5a. Signal strength versus range (GEOS-I satellite; laser Q-switched, output 1/2 J; elevation a parameter).

▲ 30° - 39°

● 40° - 49°

X 50° - 59°

O 60° - 69°

□ 70° - 79°

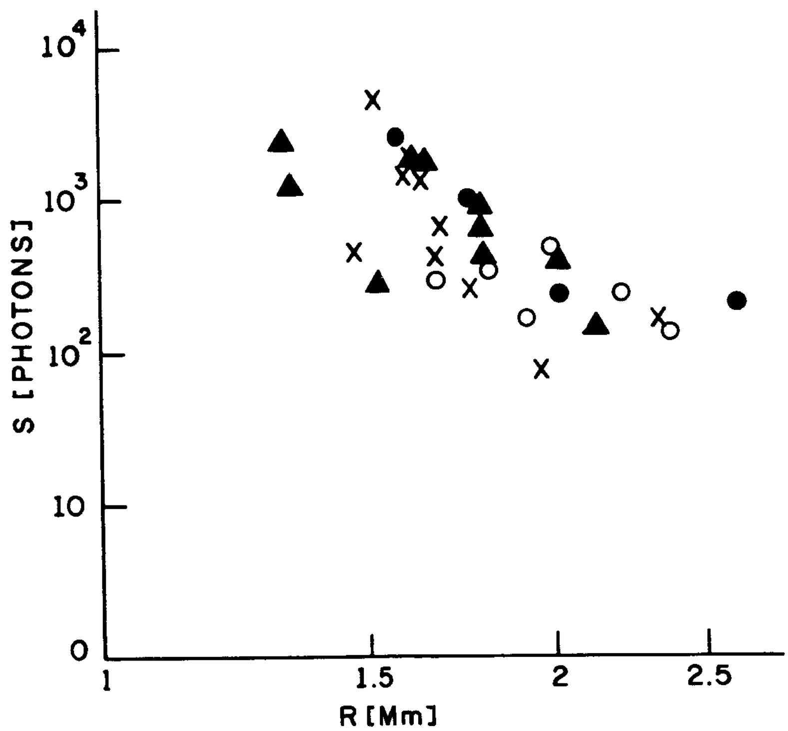


Figure 5b. Signal strength versus range (GEOS-I satellite; laser Q-switched, output 1/2 J; azimuth a parameter).

▲ 0° - 89°

● 90° - 179°

X 180° - 269°

O 270° - 359°

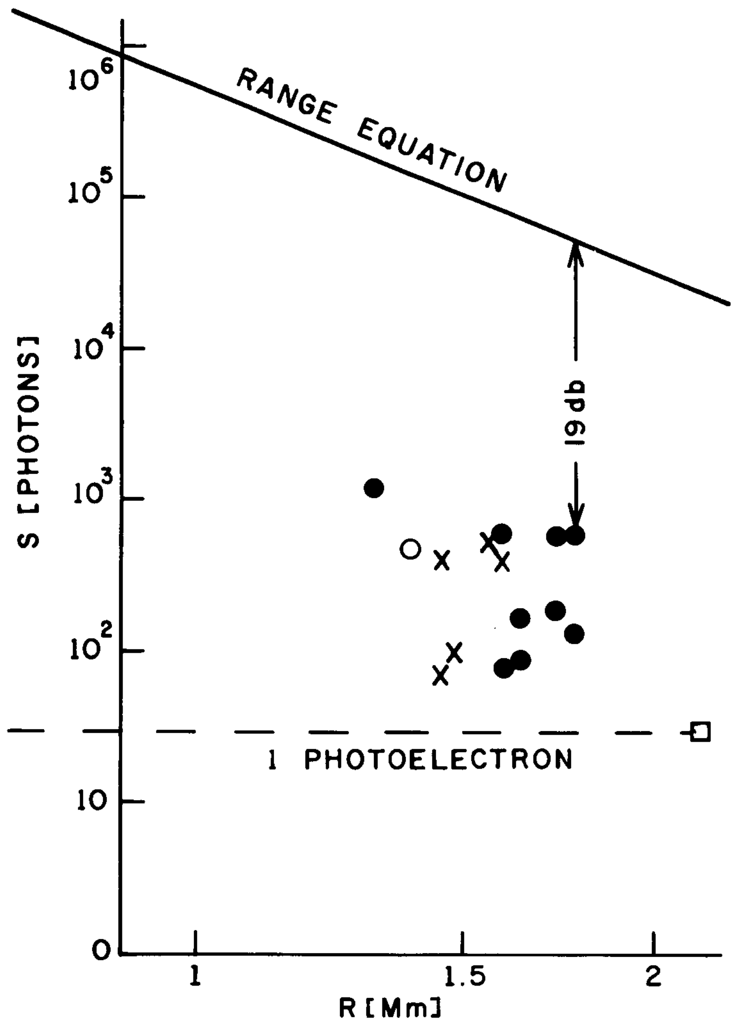


Figure 6a. Signal strength versus range (BE-C satellite).

● Laser Q-switched; output 1/2 J; elevation 40° - 49°
X Laser Q-switched; output 1/2 J; elevation 50° - 59°
O Laser Q-switched; output 1/2 J; elevation 63°
□ Laser not Q-switched, output 36 J; elevation 27°

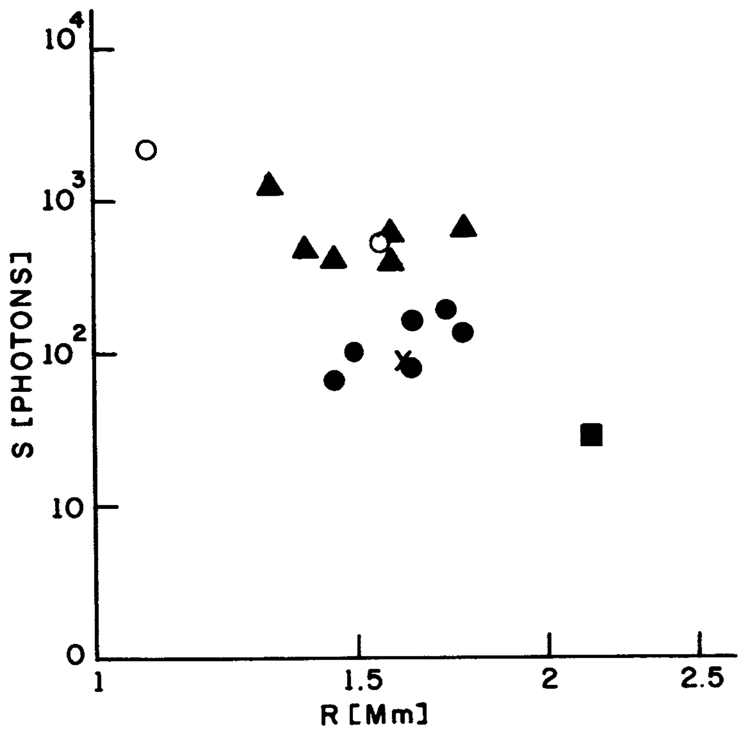


Figure 6b. Signal strength versus range (BE-C satellite).

■ Laser not Q-switched, output 36 J; azimuth 16°
▲ Laser Q-switched, output 1/2 J; azimuth 0° - 89°
● Laser Q-switched, output 1/2 J; azimuth 90° - 179°
X Laser Q-switched, output 1/2 J; azimuth 180° - 269°
O Laser Q-switched, output 1/2 J; azimuth 270° - 359°

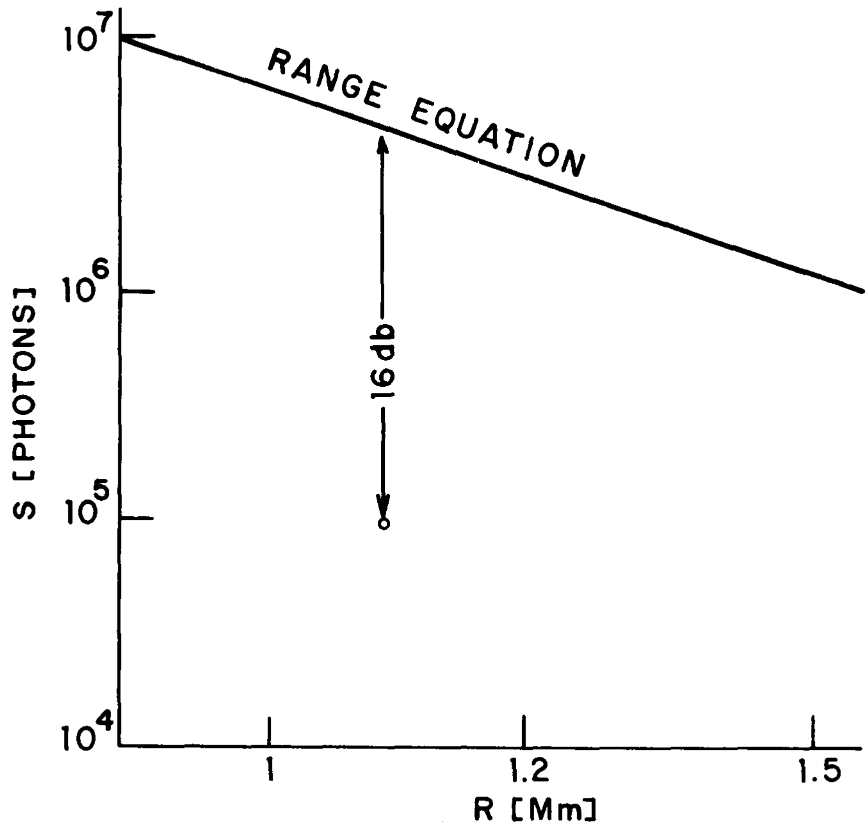


Figure 7. Signal strength versus range (BE-B satellite). Photographic return using Baker-Nunn camera. Laser not Q-switched; output 11 J, elevation 60°. Density of signal estimated from film to be about 0.3 above fog.

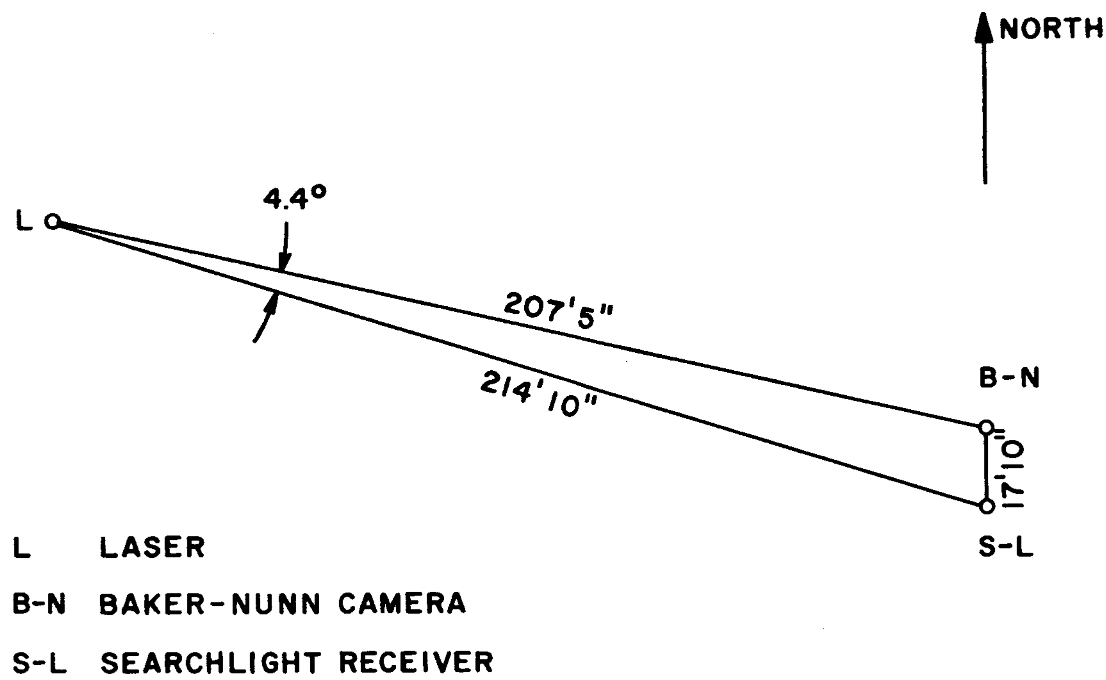


Figure 8. Layout of laser system. Distances measured to ± 4"; angle, to ± 0°.2. The triangle is approximately horizontal, the height of its plane being approximately that of the Baker-Nunn camera.

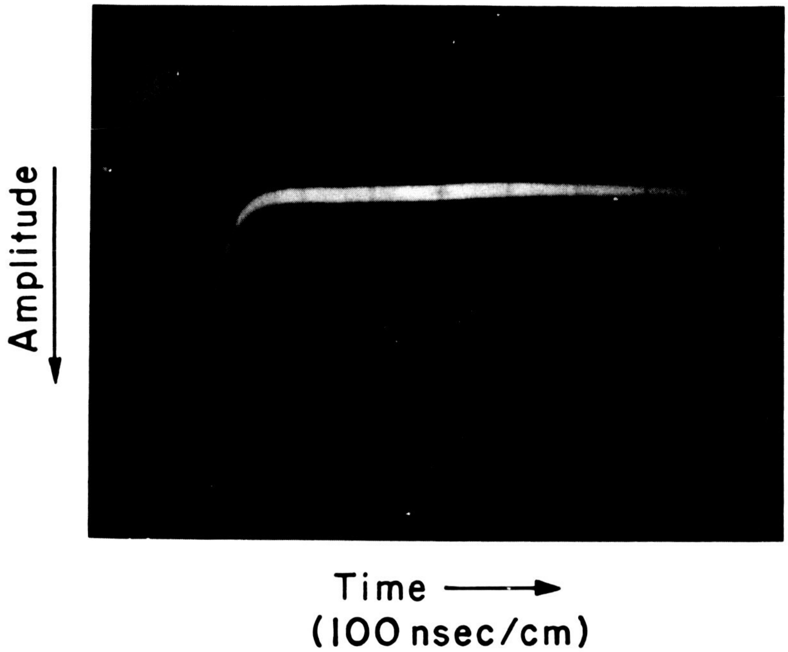


Figure 9a. Transmitted Q-switched pulse. Pulse width (nsec): at base, 120; at half-power point, 60.

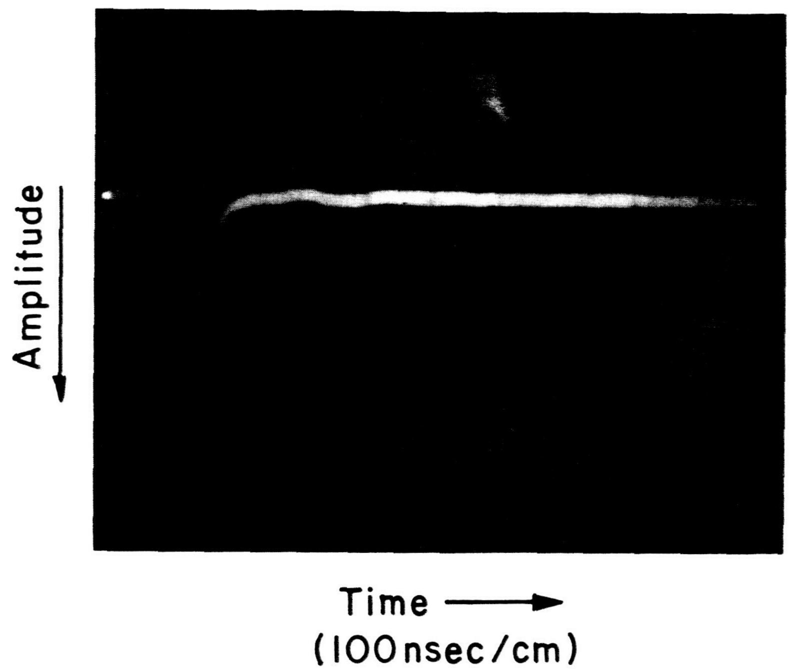


Figure 9b. Received Q-switched pulse corresponding to (a), from a diffuse target about 1/2 mile from the laser. Pulse width (nsec): at base, 90; at half-power point, 50.

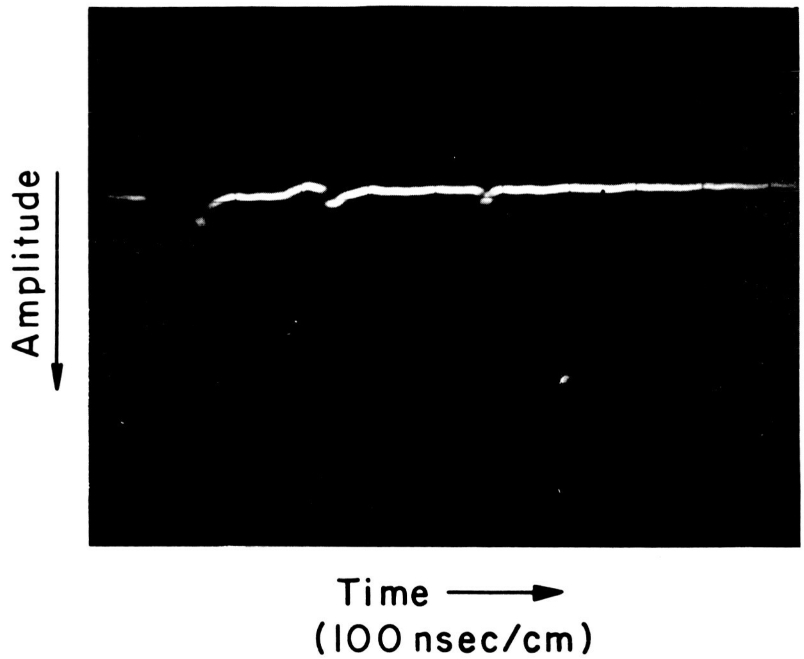


Figure 10. Structure of returned laser pulse (same return as Figure 2 on an expanded sweep). Pulse width (nsec): at base, 80; at half-power point, 30. Rise time (nsec), 50; rate of rise, 1 mv/nsec.(The small pulses following the main return come from reflections in the coaxial line connecting the photomultiplier and the oscilloscope.)

5. REFERENCES

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APPENDIX

The satellite range we obtain from laser measurements is half the distance from the laser to the satellite and back to the receiver. This range does not correspond exactly to the range from the Baker-Nunn camera to the satellite. The difference results from the relative displacements of the laser, the receiver, and the Baker-Nunn camera. This difference must be accounted for when laser-range measurements are compared to ranges derived from Baker-Nunn orbits. These computed ranges are usually referred to the location of the camera. For a valid comparison, the laser measurements must be referred to the same position. The appropriate correction is derived below.

Figure A-1 shows the vectors that connect the points involved in the analysis that will follow. In this figure, $\underline{a}$ is the vector of length a that extends from the receiver to the laser, $\underline{b}$ is the vector of length b that extends from the receiver to the Baker-Nunn camera. $\underline{r}'$ is the vector of length r' that extends from the receiver to the satellite, and $\underline{r}$ is the vector of length r that extends from the Baker-Nunn camera to the satellite

$$\cos \gamma = \frac{\underline{a} \cdot \underline{r}'}{a r'} \tag{A-1}$$

$$\cos \delta = \frac{\underline{b} \cdot \underline{r}'}{b r'} \tag{A-2}$$

These vectors may be expressed in Cartesian coordinates where $\underline{i}$ is a unit vector directed toward the east, $\underline{j}$ is a unit vector directed toward the north, and $\underline{k}$ is a unit vector directed toward the zenith.

If α and ϕ are the elevation and azimuth of the satellite,

$$\underline{r}' = r'(-\underline{i}\cos\alpha\sin\phi + \underline{j}\cos\alpha\cos\phi + \underline{k}\sin\alpha) \quad . \tag{A-3}$$

If the vectors $\underline{a}$ and $\underline{b}$ are written in terms of their Cartesian coordinates, we have the following expressions:

$$\left.\begin{aligned} \underline{a} &= a(-\underline{i}\cos\alpha_a\sin\phi_a + \underline{j}\cos\alpha_a\cos\phi_a + \underline{k}\sin\alpha_a) \\ \underline{b} &= b(-\underline{i}\cos\alpha_b\sin\phi_b + \underline{j}\cos\alpha_b\cos\phi_b + \underline{k}\sin\alpha_b) \end{aligned}\right\} \tag{A-4}$$

where the notation is similar to that of equation (A-3). From the data given in Figure 8 we see that the vector $\underline{a}$ has a length of 64.5 m, an elevation of 0°, and an azimuth of 73°.1; the vector $\underline{b}$ has a length of 5.44 m, and both its elevation and azimuth are 0°. Consequently (A-3) can be written as follows for the present location of the equipment in New Mexico

$$\left.\begin{aligned} \underline{a} &= 64.5(-\underline{i}\sin 73^\circ.1 + \underline{j}\cos 73^\circ.1) \\ &= 64.5(-0.956\,\underline{i} + 0.290\,\underline{j}) \\ \underline{b} &= 5.44\,\underline{j} \end{aligned}\right\} \tag{A-5}$$

Let r_m be the measured range (i.e., let $2r_m$ be the distance from the laser to the satellite and back to the receiver). Then, to a good approximation,

$$2r_m = 2r' - a\cos\gamma$$

or

$$r' = r_m + \frac{a\cos\gamma}{2} \quad . \tag{A-6}$$

The distance between the Baker-Nunn camera and the satellite is

$$r = r' - b \cos \delta \quad . \qquad \text{(A-7)}$$

The substitution of (A-6) into (A-7) gives the following expression for converting laser measurements to range measurements from the Baker-Nunn camera.

$$r = r_m + \frac{a \cos \gamma}{2} - b \cos \delta \quad . \qquad \text{(A-8)}$$

The use of equations (A-1) through (A-5) gives the following expressions for cos γ and cos δ.

$$\left.\begin{aligned} \cos \gamma &= \cos \alpha_a \cos \alpha(\sin \phi_a \sin \phi + \cos \phi_a \cos \phi) \\ &\qquad + \sin \alpha_a \sin \alpha \\ &= (0.956 \sin \phi + 0.290 \cos \phi) \cos \alpha \\ \cos \delta &= \cos \alpha_b \cos \alpha(\sin \phi_b \sin \phi + \cos \phi_b \cos \phi) \\ &\qquad + \sin \alpha_b \sin \alpha \\ &= \cos \alpha \cos \phi \end{aligned}\right\} \qquad \text{(A-9)}$$

The expression for the range from the Baker-Nunn camera is obtained by substituting equation (A-9) into equation (A-8). For the present locations of the equipment we obtain

$$r = r_m + \cos \alpha(-30.8 \sin \phi + 3.90 \cos \phi) \quad , \qquad \text{(A-10)}$$

the expression that has been applied to the laser measurements presented in this report.

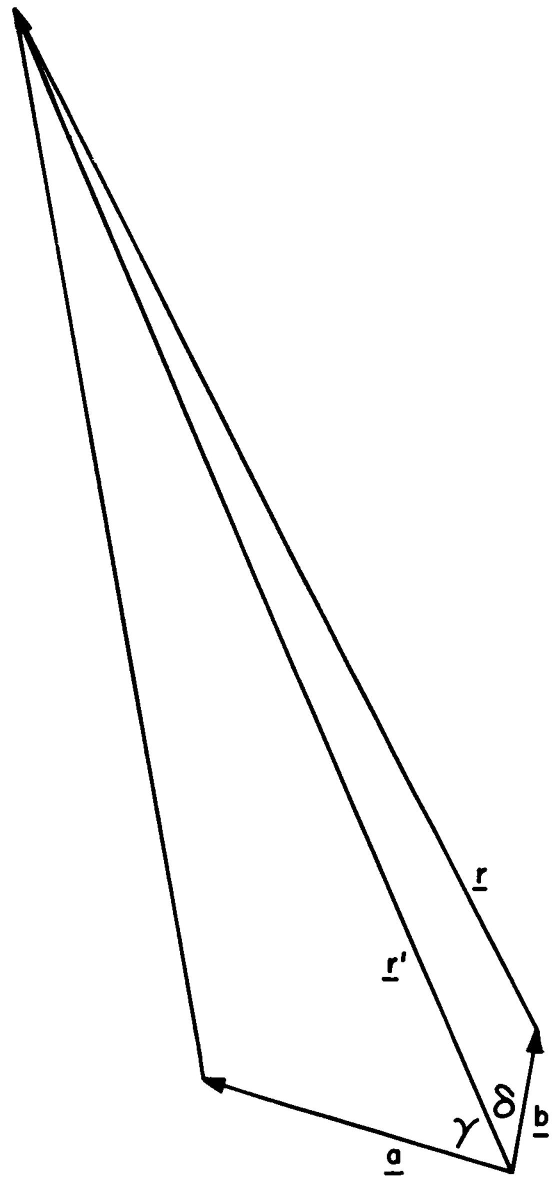

Figure A-1. Vector relation between the satellite and tracking instruments at the ground station.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

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