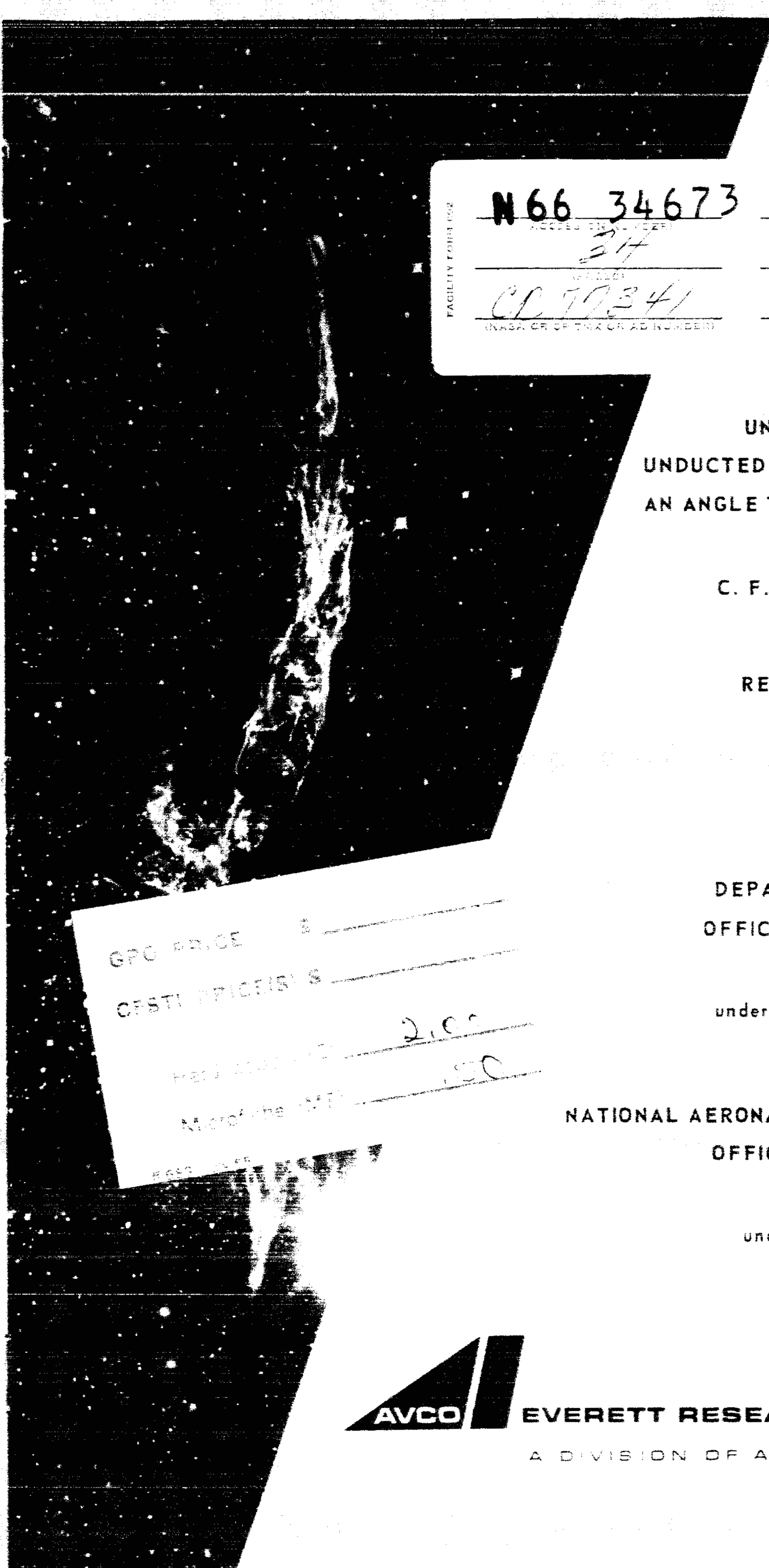

UNSTABLE GROWTH OF UNDUCTED WHISTLERS PROPAGATING AT AN ANGLE TO THE GEOMAGNETIC FIELD

C. F. Kennel and R. M. Thorne

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ABSTRACT

The local growth rate for whistler waves propagating at an angle to the geomagnetic field is combined with model ray path calculations to estimate the wave growth along the whistler path. This growth rate depends critically upon the equatorial plane distribution of electrons in the few hundreds of electron volt energy range. Under a variety of conditions apparently satisfied by the magnetosphere electron velocity distribution, a region of whistler growth near the equatorial plane exists and is sufficient to make a whistler mode unstable overall as it bounces back and forth across the equatorial plane between reflection points. Only unducted whistlers generated by resonant particle instabilities are treated.

Author

1. INTRODUCTION

Following work by Dragt (1961), Wentzel (1961), Cornwall (1965), Chang and Pearlstein (1964), Andronov and Trakhtengerts (1964) and others, Kennel and Petschek (1966), (hereinafter called KP) suggested that non-linear wave-particle scattering limited the intensity of trapped particles in the magnetosphere to the critical intensity for instability of the waves bouncing back and forth in the magnetosphere. Ion cyclotron waves were shown to couple strongly to ions, whistler waves to electrons. Recently, Cornwall (private communications) has reached the same conclusion for the trapped proton distribution.

The arguments in KP were based on the properties of waves propagating parallel to the magnetic lines of force. Since the index of refraction of both whistler and ion cyclotron waves is anisotropic with respect to wave normal angle to the magnetic field, it seems very likely that, although the group velocity of these waves remains directed essentially along the magnetic field, the propagation vector $\underset{\sim}{k}$ will not remain parallel to the magnetic lines of force, as the waves propagate. Consequently, Thorne and Kennel (1966), (hereinafter called T), undertook a study of the propagation of whistlers in the magnetosphere with the aim of clarifying this question. They suggested that the wave normal angle to the magnetic field Θ increases as unducted whistlers propagate away from the geomagnetic equatorial plane (where they are generated by the whistler cyclotron instability (KP)) and may propagate nearly normal to the field at high latitudes. In addition, Kennel (1966) (hereinafter called K) generalized the whistler mode instability analysis of KP to whistlers propagating at an angle to the field. When Θ becomes sufficiently large, whistler mode waves are damped by resonant interactions with low energy electrons, which we call Landau electrons. This interaction is small when Θ is small, and the critical angle at which Landau damping dominates cyclotron growth is a strong function of the energy spectrum of resonant particles. This note couples the propagation analysis of T with the instability analysis of K. We have not exhausted all aspects of this question but have concentrated on establishing the conditions for which whistler mode wave growth will occur along a realistic unducted wave trajectory which includes changes in wave normal angle θ, number of resonant electrons, and so on.

Landau electrons, which have energies of a few hundred electron volts in the equatorial plane, play the critical role in determining stability. We therefore devote some attention to the questions of their intensity and distribution in velocity space. Whatever effects Landau electrons may have will probably be most pronounced for the auroral lines of force, where considerable fluxes of low energy (1 keV) electrons have been observed (O'Brien and Taylor, 1964, Sharp et al., 1964, 1965). Landau electrons are probably not numerous in the region of stably trapped fluxes of relatively high energy electrons (> 40 keV) located at lower latitudes than the auroral zone (Frank, 1965). This region we shall call the Van Allen zone. To our knowledge, detailed experimental data on the Landau electron distribution is still somewhat sparse.

Nevertheless, even if Landau electrons are reasonably numerous, whistler mode growth can still occur along a realistic unducted wave trajectory. KP argued that the maximum incremental wave growth occurs for parallel propagating whistlers at the equatorial plane. We therefore calculate the growth along the ray paths computed in T for whistlers starting in the equatorial plane propagating nearly parallel to the magnetic field. Making the relatively pessimistic assumption that the electron energy spectrum varies as $1/E^2$ between $\approx$ 40 keV and $\approx$ 100 eV, we find that net wave growth occurs along these whistler ray trajectories. However, a $1/E^3$ spectrum shows damping overall, so that unducted whistler growth depends critically on the Landau electron intensity.

In Section 1.1, we review very briefly the propagation studies of T. In Section 1.2, we discuss qualitatively the condition for net wave growth along a ray trajectory, and in Section 1.3, we describe the analysis in K of the whistler mode growth rate. For more details, the reader should refer to the original papers. In Section 2.2, we discuss in a preliminary way what properties can be inferred about magnetospheric Landau electrons. In Section 2.3 we present numerical calculations of the growth rate as a function of location along the ray path.

1.1 Unducted Whistler Mode Propagation in Cold Plasma Geometrical Optics Limit

For a review of the general whistler mode propagation problem see the book by Helliwell (1965). Here we discuss the results of the analysis of T. For the index of refraction, n, they took the cold plasma quasi-longitudinal expression (Allis and Buchsbaum, 1963) in the limit of a whistler frequency ω well below the electron cyclotron frequency Ω :

$$n^2 = \frac{k^2c^2}{\omega^2} = \frac{\omega_{p-}^2}{\Omega\omega\cos\Theta} \qquad (1.1)$$

where c = speed of light, $\underset{\sim}{k}$ = wave number $\omega_{p-} = \sqrt{\frac{4\pi Ne^2}{m^-}}$, the electron plasma frequency, and $\cos\Theta$ is the angle between the wave-vector $\underset{\sim}{k}$ and the magnetic field B. Notice that n^2 is proportional to $N/B \cos\Theta$. The geomagnetic field strength B variation with geomagnetic latitude λ was approximated by a dipole,

$$\frac{B(\lambda)}{B(0)} = \frac{\sqrt{1 + 3\sin^2\lambda}}{\cos^6\lambda} \qquad (1.2)$$

where $B(0)$ is the magnetic field strength in the equatorial plane $\lambda = 0$. In the equatorial plane, the density N was taken as

$$\frac{N(0)}{B(0)} = \text{constant everywhere} \qquad (1.3)$$

This distribution is roughly that found from whistler dispersion measurements (Helliwell, 1965). On the other hand N was assumed constant along the magnetic lines of force, the density distribution for an isothermal plasma in magnetostatic equilibrium. Thus N/B decreases away from the equatorial plane. This distribution is justified in somewhat more detail in T. Their results probably hold at least qualitatively for all those siutuations where N/B is a maximum in the equatorial plane on a given line of force.

One of the principal conclusions drawn from the above propagation studies is that unducted whistlers which are initially generated propagating parallel to the lines of force by an instability near the equatorial plane can only propagate away from the equatorial plane by changing their wave normal angle. Consider the following intuitive argument in geometrical optics; waves propagate into regions of increasing index of refraction n. N/B decreases with increasing $|\lambda|$ so that $\cos\Theta$ must decrease, and Θ increase as the wave propagates away from the equatorial plane to higher latitudes. This conclusion has significant consequences for the study of resonant particle wave growth, since new sets of particles can resonate with non-parallel whistlers. In particular, the reader should refer to the discussion of the Landau resonance in Section 1.3 following.

One of the conclusions of this paper is that the region of gain for a whistler wave is restricted to near the equatorial plane, and thus that whistlers propagate out of the region of gain. If a reflection mechanism returns some portion of the wave intensity back and forth across the equatorial plane, large whistler amplitudes can build up by this convective feedback process. One such mechanism, involving the lower hybrid resonance, is discussed in T. If the intensity of trapped electrons is sufficiently intense to produce gain enough to overcome losses of wave energy due to imperfect reflection or other processes whistler growth will occur. We shall simulate all wave losses by an effective reflection coefficient, and write the approximate condition that the whistler intensity remain constant.

1.2 Condition for a Steady Wave Distribution

The following equation describes the changes in intensity $W(\omega)$ of a given mode following a ray trajectory in a weakly spatially inhomogeneous medium

$$(\partial/\partial t + \underset{\sim}{V}_G \cdot \underset{\sim}{\nabla})W = (2\gamma - \nu)W \tag{1.4}$$

where $\underset{\sim}{V}_G(\omega, \underset{\sim}{x})$ is the local group velocity of a given wave packet, $\gamma(\omega, \Theta, \underset{\sim}{x})$ is the growth or damping rate along the ray path, and $\nu(\omega, \underset{\sim}{x})$

represents schematically the loss of whistler energy from the magnetospheric cavity due to imperfect containment along the lines of force, imperfect reflection and any other effects of this nature.

Equation (1.4) corresponds to the lowest order WKB solution for wave propagation. Effects due to singularities in the WKB solution, such as turning points where $\underset{\sim}{k}(\underset{\sim}{x}) = 0$, are not included in (1.4). Moreover, all effects due to the non-linear coupling between different whistlers have systematically been disregarded. This assumption was justified for the Van Allen Zone in KP, though it remains to be proven for the intense auroral zone whistler flux which probably can be inferred from the high precipitation rates there.

The condition for a steady state wave distribution, $\partial W/\partial t = 0$, is immediately

$$\frac{\partial \ln W}{\partial s} = \frac{2\gamma(s) - \nu(s)}{V_G(s)} \tag{1.5}$$

where $ds = \underset{\sim}{V_G} \cdot \underset{\sim}{dx}$ is an element of ray path and $V_G(s)$ is the magnitude of the group velocity. Suppose that unducted whistlers bounce back and forth between imperfect reflection points $+s_1$ and $-s_1$. We simulate $\nu(s)$ by a term $\ln 1/R\ [\delta(s - s_1) + \delta(s + s_1)]$, where R is the effective reflection coefficient. When the wave trajectory is approximately closed and whistlers return to the same location in the equatorial plane the condition that W be single valued and that the gain $\gamma(s)$ make up for wave losses at the reflection points is

$$2 \oint \frac{\gamma(s)ds}{V_G(s)} = 2\ln 1/R \tag{1.6}$$

or

$$\gamma \simeq \frac{V_G}{L} \ln 1/R \tag{1.7}$$

where γ and V_G are typical values, and L is roughly the length of the ray path between reflection points. Thus, when $R < 0$, the growth rate integrated along a ray path must be positive to maintain a steady wave distribution. Instability occurs when $\int \frac{\gamma(s)\, ds}{V_G} > \ln 1/R$.

1.3 Stability of Whistler Waves Propagating at an Angle to the Magnetic Field

Here we review the results of a stability analysis by Kennel (1966). When the plasma velocity distribution consists of a dense cold plasma on which is superimposed a tenuous high-energy component, the propagation and polarization may be treated as above by cold-plasma theory, whereas the resonant growth or damping is a small correction, which involves the details of the velocity distribution. Cold-plasma theory predicts that as the angle between the wave vector and the magnetic field increases from zero, the whistler polarizaiton changes from pure right-hand circular to elliptical. In the pure right-hand case, those electrons in the diffuse high-energy tail having velocities along the lines of force which Doppler-shift the wave frequency to their gyrofrequency maintain a constant phase with respect to the rotating wave fields, and therefore exchange energy with the wave efficiently. This resonance we call the principal cyclotron resonance. With elliptical polarization, a Doppler-shift to any multiple of the gyrofrequency produces a similar, though somewhat weaker, resonance. We call all these cyclotron resonances. The condition that an electron be in cyclotron resonance is $k_{\parallel} v_{\parallel} = \omega + m\Omega$, where $v_{\parallel}$ is the velocity component parallel to the magnetic field, Ω the electron gyrofrequency, and m a non-zero integer. This expression may be translated using (1.1) into one involving the energy in parallel motion necessary for cyclotron resonance, $E_c(m)$

$$E_c(m) = \frac{m^- v_{\parallel}^2}{2} = \frac{B^2}{8\pi N} \cdot \frac{m^2 \beta}{\cos\Theta} \; ; \qquad \beta \equiv \frac{\Omega}{\omega} \qquad (1.8)$$

where m^- is the electron mass, $B^2/8\pi N$ is the magnetic energy per particle. Evidently $E_c(m)$ is the minimum total energy necessary for cyclotron resonance, since resonant electrons may have any energy whatsoever in motion perpendicular to the magnetic field. Since β is ordinarily a large parameter, the whistler cyclotron resonances affect primarily the high-energy component of the Van Allen electron distribution, with energies well above $B^2/8\pi N$.

When $\Theta \neq 0$, there can then be a resonance of an entirely different character from the cyclotron resonances. Those electrons travelling along the lines of force with the parallel phase velocity of the wave, $\omega/k_{\parallel} = \omega/k \cos\Theta$, stay in phase with the sinusoidal electromagnetic field structure of the wave parallel to the magnetic field and can exchange energy with the wave. This resonance, analogous to that found in plasmas without external magnetic fields, we call the Landau resonance. The resonance condition, $k_{\parallel} v_{\parallel} - \omega = 0$, may be transformed to an expression involving the energy E_L necessary for Landau resonance

$$E_L = \frac{\bar{m}}{2} \left(\frac{\omega}{k_{\parallel}}\right)^2 = \frac{1}{\beta\cos\Theta} \frac{B^2}{8\pi N} \qquad (1.9)$$

The ratio of Landau to cyclotron energy for a given frequency wave is

$$\frac{E_L}{E_c(m)} = \frac{1}{m^2\beta} \qquad (1.10)$$

At very low frequencies, $\beta \to \infty$, the Landau energy is very much smaller than the cyclotron energies.

The growth rate when $\Theta \neq 0$ is a sum of the partial growth rates due to interactions with each set of resonant particles. Each partial growth rate in turn depends upon an appropriate velocity space gradient in the distribution of electrons at each resonance and a positive definite weighting function which depends primarily upon Θ. The magnitude of the weighting function picks out the important resonances. The sign of each partial growth rate is fixed by the velocity space gradient alone, while the magnitude of each partial growth rate is fixed by both the number of resonant particles and the weighting function there. We discuss intuitively first the behavior of the partial growth rates at the cyclotron resonances, and then at the Landau resonance.

For parallel propagation, $\Theta = 0$, the above mentioned weighting functions are zero at all but the $m = -1$ principal cyclotron resonance. For small angles Θ, this resonance continues to dominate all others. The sign of the partial growth rate at this resonance is determined by the gradient of the pitch angle distribution of resonant electrons (Sagdeev and Shafranov (1961)). When there are more particles with flat pitches (velocity nearly perpendicular to the field) than with small (nearly parallel), this resonance yields growth. An anisotropy of this sign exists in the magnetosphere, due to the loss of small pitch particles to the atmosphere.

The sign of the partial growth rate at each cyclotron resonance, in fact, is fixed by the pitch angle anisotropy. For a typical magnetospheric (mirror) electron distribution, the negative ($m < 0$) cyclotron resonances contribute growth, while the positive ($m > 0$) cyclotron resonances contribute damping. However, the weighting functions weight negative resonances more heavily, so that for the above pitch angle distribution, the net effect of all cyclotron resonances is wave growth.

The partial growth rate at the Landau resonance is proportional to the gradient with respect to parallel velocity $\partial F/\partial |v_{\parallel}|$ of the resonant electron distribution times a positive definite weighting function which approaches 0 as $\Theta \rightarrow 0$. Parallel waves have no Landau resonance. We repeat: the sign of the Landau growth rate does not depend upon a pitch angle gradient, as in the cyclotron case, but essentially on a gradient with respect to the parallel velocity component, $\partial F/\partial |v_{\parallel}|$. If $\partial F/\partial |v_{\parallel}| > 0$ at some phase velocity, this partial growth rate will be positive (unstable). $\partial F/\partial |v_{\parallel}| < 0$ implies a damping contribution. If the pitch angle distribution is not too skewed, $\partial F/\partial |v_{\parallel}|$ will behave sensibly like the gradient with respect to particle energy. Then, since we expect the electron energy distribution to be monotonic and decreasing, the Landau resonance should ordinarily contribute damping.

The net growth rate is a competition between the damping Landau resonance and unstable cyclotron resonances. Since the Landau energy is much lower than the cyclotron energies, there will be many more Landau than cyclotron electrons, emphasizing Landau damping. On the other hand, the Landau weighting function tends to zero as $\Theta \rightarrow 0$, and the (unstable) cyclotron resonance behavior dominates. Evidently there is a critical angle Θ beyond which Landau damping predominates. The magnitude of the critical angle is critically dependent upon the electron energy spectrum in the plasma, i.e. the relative number of particles at each resonance.

Reproduced in Fig. 1 is a plot, taken from Kennel (1966), of the growth rate $\gamma(\Theta)$, normalized to its value at $\Theta = 0$, for electron velocity distribution of the form

$$f(v,\alpha) = \begin{cases} v^{-2p} \ln\{\sin\alpha / \sin\alpha_o\} & , \ \alpha_o \leqq \alpha \leqq \pi/2 \\ 0 & , \ 0 \leqq \alpha \leqq \alpha_o \\ v^{-2p} \ln\{\sin(\pi-\alpha) / \sin(\pi-\alpha_o)\} & , \ \pi/2 \leqq \alpha \leqq \pi-\alpha_o \\ 0 & , \ \pi-\alpha_o \leqq \alpha \leqq \pi \end{cases} \qquad (1.11)$$

where

v = particle speed = $\sqrt{v_\perp^2 + v_\parallel^2}$

$\alpha = \tan^{-1} v_\perp / v_\parallel$ = pitch angle

α_0 = opening angle of the magnetospheric loss cone,

$\alpha_0 \approx 1/20$ radian

The above pitch angle distributions are solutions of the approximate pitch angle diffusion equations which are satisfied when a finite whistler intensity is present (KP, 1966). Thus the growth rate calculated from this distribution is consistent with the hypothesis of a steady non-zero turbulent wave amplitude. For details of the solution, refer to KP. Because of precipitation into the loss cone, pitch angle diffusion automatically creates an anisotropy which is unstable at the cyclotron resonances.

The growth rate is plotted for values of $p = 2, 3, 5$. For the spectrum commonly found for Van Allen electrons (McDiarmid et al. (1964)), the whistler mode will be unstable over a cone of some $10^o - 40^o$. A $p = 5$ spectrum, on the other hand, is unstable only for wave normal angles less than one degree to the field, thus illustrating the sensitivity

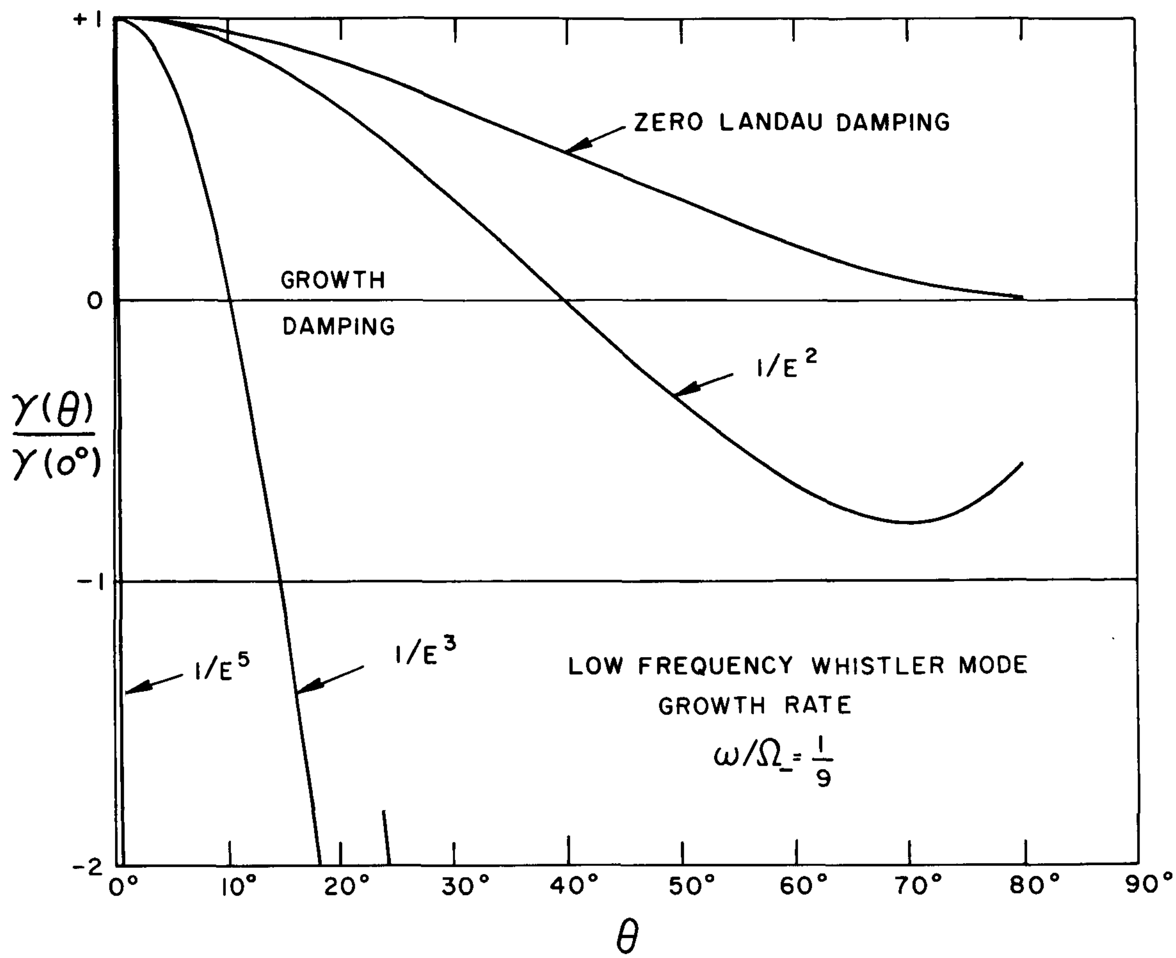

Fig. 1 Whistler Growth Rate as Function of Wave Normal Angle

The growth at any angle Θ has been normalized to its value for strictly parallel propagation. When Landau electrons damp the wave, the growth rate peaks at $\Theta = 0$; the maximum angle at which growth occurs is a strong function of the energy spectrum. When Landau damping can be ignored, wave growth occurs for all angles.

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to the electron energy distribution. Also shown is the case when Landau damping may be neglected altogether, and the mirror pitch angle distribution provides instability at all wave normal angles.

In summary, pitch angle diffusion into the loss cone creates a pitch angle distribution, (1.11), which is unstable at the cyclotron resonances. The cone of angles for which the unstable cyclotron resonances dominate Landau damping will be significant only when:

1) Landau electrons can be neglected altogether,

or when

2) The distribution of electrons with energy (between the two resonances) is not too steep.

Of course, if $\partial F/\partial |v_{\parallel}| > 0$, so that the Landau electrons also create a positive partial growth rate, the whistler will be unstable for all angles to the field.

2. GROWTH RATE OF QUASI-TRAPPED UNDUCTED WHISTLER RADIATION

2.1 <u>Introduction</u>

In the absence of external wave sources, such as lightening-generated whistlers, the necessary condition that whistler turbulence exist is that the net growth rate integrated along the ray path, $\int \gamma d_s / V_G$ be positive. Since the magnitude of the growth rate is proportional to the number of resonant electrons, a sufficiently intense flux of resonant electrons will ensure satisfaction of the full criterion (1.6), once the net growth has been shown to be positive.

For the pitch angle distributions appropriate to mirror configurations, cyclotron electrons will always be destabilizing everywhere. Therefore, there will always be at least local regions of positive wave growth where the waves propagate locally parallel to the magnetic field. Kennel and Petschek argued on intuitive grounds that these regions would dominate the net growth. However, as whistlers propagate further, their wave normal angle necessarily changes, and Landau damping can become important. Thus, whether or not whistlers can grow hinges on the role played by Landau electrons.

We discuss briefly in Section 2.2 the observational question of whether appreciable fluxes of Landau electrons exist near the equatorial plane. We conclude partly that they probably are not important, but mostly that more observations are needed. In Section 2.3, we assume pessimistically that a $1/E^p$ energy spectrum extends between the cyclotron and Landau resonance, making a damping Landau resonance, and show that wave growth will occur when $p < 3$.

2.2 Do Many Equatorial Landau Electrons Exist?

Since different groups of Landau electrons are associated with the precipitation into the atmosphere of cyclotron electrons of different energies, it is convenient to define Landau electrons in terms of the cyclotron electrons also in resonance with a given whistler. When $\omega/\Omega \approx 1/2$, the Landau and cyclotron energies are comparable (roughly $B^2/8\pi N$) and when $\omega/\Omega > 1/2$, the Landau energy exceeds the cyclotron energy. We exclude from this discussion those whistlers with $\omega/\Omega > 1/2$ so that here the cyclotron energy always exceeds the Landau energy.

The estimates of the resonant energy for a given wave depend upon the magnetic field strength and the total number density through the parameter $B^2/8\pi N$, which will undoubtedly vary with time and space, particularly in the auroral regions. KP estimated in a schematic way the variation of $B^2/8\pi N$ with radial distance from the Earth, and for the sake of clarity we will use these estimates here. For instance, in the Van Allen radiation zone, $L \approx 3$-5, the magnetic energy per particle at the equator was estimated to be 1-2 keV, while on the night side auroral zone, $B^2/8\pi N$ at $L \approx 7$ was ≈ 400 eV. Because of uncertainty in the density, this last value which was based on a magnetic field of 10^{-3} gauss and a total density of roughly $50/cm^3$ may well be in error. Recently, Watanabe (1965) has suggested that the actual densities may in fact be considerably lower than this. However, $B \approx 10^{-3}$ gauss, is the undistorted dipole value, and is probably an overestimate, (Ness, 1965), so that both B and N may well need to be revised but both downwards. The exact value of $B^2/8\pi N$ is not critical for the arguments to follow, unless $\omega/\Omega \approx 1/2$, and so we use the above estimates, in full realization that they are subject to further experimental revision.

As has become clear, the magnetosphere has several distinct regions where the (quasi-) trapped particles have different properties. Low energy electrons, the order of a few hundred eV to a few keV, are found on the day-side only beyond the magnetospheric boundary (Bonetti, et al., 1963) and are thus not of interest here, and on the night-side on the auroral lines of force, only at distances greater than roughly six Earth radii.(Gringauz, 1961, Freeman, 1964). Within the auroral zone surrounding the Earth at distances less than 6-8 Earth radii is the so-called Van Allen stably trapped zone. Here fluxes of > 40 keV electrons are observed to have quasi-stationary properties (Frank, 1965). Gurnett and Fritz (1965) observed a sharp spatial transition between regions of high fluxes of low energy (in their case, > 10 keV) electrons and high energy (> 40 keV) electrons near the Earth. The > 10 keV fluxes were only intense in the auroral regions where the > 40 keV fluxes became relatively weak. It is natural therefore to consider the auroral and Van Allen regions separately. Furthermore, since the auroral regions are probably the more likely candidate for having significant Landau effects, we treat them first.

As an extension of the picture proposed by KP for > 40 keV Van Allen electrons, to lower energy auroral (≈ 1-5 keV) electrons, we tentatively identify the heavy auroral precipitation background (Sharp et al., 1964, O'Brien and Taylor, 1964) with equatorial plane pitch angle diffusion at the dominant cyclotron resonance of the whistler mode. While this step requires further justification, we concentrate here only on the role of Landau electrons, assuming that the auroral cyclotron electrons do contribute to wave growth. If $B^2/8\pi N$ is 400 eV, an electron with a 1-2 keV parallel energy will have a cyclotron resonance with whistlers with $\Omega/\omega \approx 4$ (Eq. 1.8). Then, substituting $\Omega/\omega \approx 4$ into equation 1.10, the Landau energy for the same whistler is thus roughly 100 eV.

Thus, interesting Landau interactions probably occur near the equatorial plane with electrons in the hundred volt energy range. Unfortunately, experimental information on this energy range is sparse. However, the existing evidence suggests that hundred volt electrons may not be very numerous in the auroral zone. Sharp et al. (1965) found that the integral electron energy spectrum is flat between 180 eV and 1 keV in the auroral regions near the Earth. In other words, the fluxes of electrons with energies of a few hundred eV and very small pitches in the equatorial plane are quite small. If the pitch angle distribution is not grossly anisotropic, we may extend this inference to the whole equatorial distribution function in this energy range. Since turbulence does not guarantee an isotropic distribution of Landau particles, this must remain an assumption. Similarly, O'Brien and Taylor (1964) concluded that the number flux of 10 eV electrons precipitated in an aurora is no greater than that of 1 keV electrons, suggesting at least a flat number distribution of precipitated electrons in the $10\text{-}10^3$ eV energy range.

O'Brien and Taylor (1964) estimated the average energy flux deposited in the auroral zone to be 3-5 ergs/cm^2-sec, though occasionally this flux may rise as high as roughly 1000 ergs/cm^2-sec. (O'Brien and Laughlin, 1962, Sharp et al., 1964). The bulk is in the 1-10 keV energy range (Sharp, et al., 1965). Since this is a precipitation flux, we may assume that a pitch angle diffusion process keeps the pitch angle distribution reasonably isotropic. Therefore 3-5 ergs/cm^2-sec is probably a reasonable estimate for the energy flux in space, barring significant energization along the lines of force between the equatorial plane and the auroral zone.

If the mean energy of the precipitating electrons is taken to be 2 keV, then 3-5 ergs/cm^2-sec corresponds roughly to a flux J(>2 keV) above 2 keV of roughly $1\text{-}2 \times 10^9$/cm^2-sec which is in reasonable agreement with the available measurements in space. (For a review of these measurements, see the article by Ness (1965)).

Suppose we pick a distribution function of the form, (1.11)

$$f \sim \frac{1}{v^{2p}} \; g(\rho) \qquad (2.1)$$

where ρ denotes the velocity space solid angle. For this distribution,

$J(>v)$, the flux above the speed v is proportional to v^{4-2p}; $\mathcal{E}(>v)$, the fractional energy density above v, is also proportional to v^{4-2p}, while $n(>v)$ the fractional number density $> v$ is proportional to v^{3-2p}. Note that $n(>v) = J(>v)/v$. The flux $J(>2\text{ keV})$ of $1\text{-}2 \times 10^9/\text{cm}^2$-sec corresponds to a number density $n(>v)$ of $(1/2\text{-}1)/\text{cm}^3$. At the velocity corresponding to 100 eV, $n(>v)$ would therefore be roughly $(1/2\text{-}1) \times (20)^{(3p-3)/2}$ using the above form for the distribution. When $p = 3$ this yields roughly 85 electrons/cm^3 above 100 eV, probably unacceptably high in view of Watanabe's (1965) result for the total density. Certainly, the $p = 3$ spectrum cannot extend to 10 eV, and $p = 4$ is definitely out of the question. We conclude, both from auroral zone data (Sharp, et al., 1965) and also these rough estimates that p is most likely smaller than three in the auroral zone.

The Van Allen zone should therefore also be relatively free of Landau electrons. If $B^2/8\pi N \approx 2$ keV typically, the whistler in cyclotron resonance with an electron having 40 keV parallel energy will have $\Omega/\omega \approx 20$ so that the Landau electrons again have energies the order of a hundred eV. Typical equatorial plane fluxes of Van Allen electrons > 40 keV are roughly $3 \times 10^7/\text{cm}^2$-sec (Frank, 1965) corresponding to a fractional density of $3 \times 10^{-3}/\text{cm}^3$ and a mean energy density of perhaps $\mathcal{E}(>40\text{ keV}) \approx 2 \times 10^{-10}$ ergs/cm^3. Extrapolating the distribution function to 100 eV, we find $\mathcal{E}(>100\text{ eV}) \approx 2 \times 10^{-10}(400)^{p-2}$ ergs/cm^3. When $p = 3$, $\mathcal{E}(>100\text{ eV}) \approx 10^{-6}$ ergs/cm^3. This would create a diamagnetic distortion the order of 500 γ in the geomagnetic field, which is unacceptably high, and so p is probably less than 3 in the Van Allen zone.

Sharp, et al. (1965) found an upper limit of at most 10^{-1} ergs/cm^2-sec precipitated to the atmosphere in the form of electrons > 180 eV at latitudes below the auroral zone. Extrapolating this flux to the equatorial plane and comparing with the energy flux implied by the number flux estimate $J(>40\text{ keV}) \approx 3 \times 10^7/\text{cm}^2$-sec, roughly 1 erg/$\text{cm}^2$-sec, suggests that $p \approx 0$ in the Van Allen regions.

These extremely crude arguments indicate that the distribution with energy of electrons between the cyclotron and Landau energies is probably harder than $p = 3$ for both the Van Allen and the auroral zones. The whistler mode growth rate is independent of the detailed behavior of the distribution between the Landau and cyclotron energy. The schematic choice $1/v^{2p}$ for the dependence on the particle speed only weights the relative number of particles at the two resonances. As we shall show, any distribution for which the relative number of Landau and cyclotron electrons can be parametrized by a $p < 3$ will probably be unstable to unducted whistler growth in the magnetosphere.

2.3 Numerical Computation of the Growth Rate Along a Ray Path

Referring to equations (1.8) and (1.9), we see that the cyclotron energy will vary as $B^3/N \cos \Theta$ and the Landau energy as $B/N \cos \Theta$, as measured following the ray path. Since the group velocity never diverges greatly from magnetic field direction, the ray path is approximately along the line of force. Then, if N is roughly constant along the lines of force, the resonant energies for a given wave will be lowest in the equatorial plane and increase rapidly up the line of force away from the equatorial plane. If the electron distribution is monotonic decreasing with increasing energy, the wave will encounter the greatest intensity of locally resonant electrons as it crosses the equatorial plane. On the other hand if the energy distribution has a maximum somewhere between the minimum Landau energy along the ray path and the cyclotron energy range, it is possible that the wave will encounter the greatest number of Landau electrons not in the equatorial plane but at some higher latitude. However, then it is likely that $\partial F/\partial \, |v_{\parallel}|$ will be positive, and the Landau electrons will thus be unstable over an appreciable portion of the ray path. Therefore, it seems reasonable that an electron distribution of the form $1/v^{2p}$, $p > 0$, gives a pessimistic estimate of stability since it has large numbers of low energy damping Landau electrons. Since we are interested in proving that some whistlers are unstable, it is natural to choose for consideration those which are propagating nearly parallel to the magnetic field in the equatorial plane because unstable cyclotron effects are strongest for this case. We then follow the ray path to higher latitudes to evaluate the effects of the change in the wave normal angle Θ and the progressively more important Landau effects. If the net growth along a ray path is positive, instability is sure to occur when the trapped fluxes are sufficiently intense. Therefore, it is not necessary to fix the normalization of f.

The ratio of the total whistler mode growth (or damping) rate as a function of wave normal angle Θ to the dominant $m = -1$ cyclotron growth rate at $\Theta = 0$ for the turbulent electron distribution (1.11) was evaluated by K. The local growth rate for the wave at $\Theta = 0$ varies with space as $(B^3/N)^{-p}$, since it is proportional to the number of particles in cyclotron resonance, and since $f \sim 1/v^{2p}$. We normalize the parallel growth rate at any point to that at the equator. Both ω/Ω and $\cos\Theta$ vary along the ray path, fixing the energy of the Landau particles and the magnitude of their contribution. We take both ω/Ω and $\cos\Theta$ from the cold plasma ray path calculations discussed above (T) and use them to calculate $\gamma(s,\Theta)$, the local growth increment. For a whistler propagating from the equator along a cold-plasma ray path to a given latitude λ', presumably that of a reflection point, the net growth Γ along the path is

$$\Gamma = \int_0^{s(\lambda')} \frac{\gamma(s,\Theta(s))}{V_G(s)}\, ds = \frac{L_o\gamma(0,0)}{V_G(0)} \int_0^{\lambda'} \gamma(\Theta,\lambda)\frac{V_G(0)}{V_G(\lambda)}\left[\frac{(R\cos\lambda/L_o)^6}{(1+3\sin^2\lambda)^{3/2}}\right]\left\{\frac{ds}{d\lambda}\right\} d\lambda \qquad (2.2)$$

Here we have normalized the group velocity V_G and the growth rate to their values at $\Theta = 0$ and $\lambda = 0$. $(ds/d\lambda)$ is the rate of change of path length s with latitude λ.

Choosing $p = 2, 3$ and initial equatorial wave normal angles $\Theta_0 = -20^o, 0^o, +20^o$, we have plotted on Fig. 2 computations of the integrand of (2.6), the growth in one element of ray path, as a function of magnetic latitude. A wave with negative Θ points towards the Earth in the meridian plane, and vice versa for $\Theta > 0$. The integrals will clearly be positive (unstable) for $p = 2$, negative (damping) for $p = 3$. If the equatorial cyclotron electron fluxes are sufficiently intense, growth is assured.

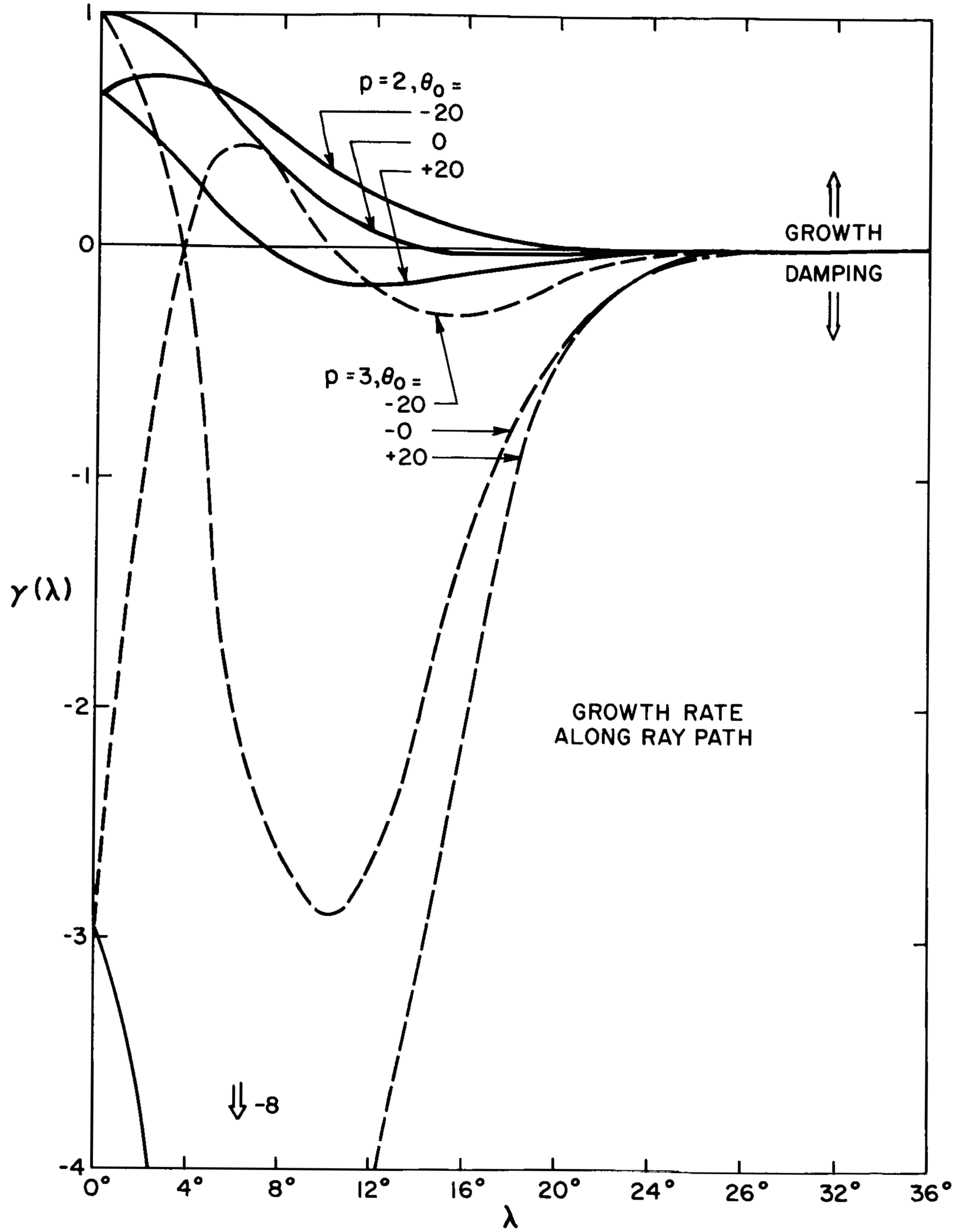

Fig. 2 Local Growth Rate along Unducted Whistler Ray Path as Function of Geomagnetic Latitude

The curves converge to zero rapidly because the number of electrons locally in resonance diminishes with increasing latitude. If p = 2, the area under these curves is positive indicating growth, p = 3 implies damping. The waves chosen here with initial equatorial wave normal angles Θ_0 of -20°, 0, $+20^\circ$ (where $\pm$ denotes pointing away or towards the Earth respectively in the meridian plane) are expected to have important cyclotron growth effects.

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Landau interactions do not dominate the cyclotron resonances until the wave normal angle has some finite non-zero value. This in turn implies that the whistler must propagate some distance from the equatorial plane. But then the number of resonant electrons decreases, and also the wave group velocity increases in this model so that whistlers do not feel the resonant interactions for as long a time as at the equator. These effects reduce the strength of high latitude resonance interactions. For $p = 3$, the angle at which Landau damping dominates is 10°. Here the $\Theta_0 = -20^{\circ}$ and 0° waves have only very small regions of growth, large regions of damping. The $p = 2$ spectrum has a critical angle of 40°, and the large cyclotron growth region near the equator is sufficient to make the whistler unstable overall.

3. DISCUSSION

Due to the anisotropic loss-cone pitch angle distribution characteristic of mirror magnetic configurations, cyclotron electrons always make whistlers unstable over that portion of their ray path where the waves propagate sufficiently parallel to the magnetic field. If this segment of the ray path is sufficiently long that the growth on this portion exceeds the damping elsewhere on the path, and any other non-resonant losses of wave energy, overall convective whistler growth will ensue. If the parallel segment of the ray path is located in the equatorial plane, the local whistler growth increment will be largest since there are the most cyclotron electrons at the equator. The length of the growth segment depends on the critical wave normal angle Θ_c at which Landau damping dominates cyclotron growth, and the rate at which the wave normal angle approaches the critical angle Θ_c as the whistler propagates. Θ_c depends primarily upon the energy spectrum between $\approx 100 - 10^4$ eV, while the rate of change of Θ depends upon the index of refraction, which in turn depends upon the magnetic field strength B and the number density N.

There are observational uncertainties connected with both the Landau electron component and also the total number density N. The number density of hundred volt Landau electrons near the equatorial plane is not well known. They are apparently not precipitated to any significant extent in the auroral zone and presumably would make at best a small contribution to the total density measurements made by the whistler mode technique. Note that it is not sufficient to know the number density of Landau electrons, but it is also necessary to have information on the velocity-space gradients of the electron distribution in this energy range. As Yakimenko (1963) and others have pointed out, quasi-linear diffusion due to plasma turbulence smooths out the velocity distribution and reduces the Landau damping rate. In a steady diffusion process, it is thus important to identify the sources and sinks in velocity space of Landau electrons. These, in turn, would fix the growth (or damping) rate. If it could be established that the distribution of Landau electrons increases with increasing parallel velocity $(\partial F/\partial |v_{\parallel}| > 0)$, the Landau electrons would also be a source of whistler instability.

Since the Landau effects only become important when the whistler has a finite wave normal angle to the geomagnetic field, ducted whistlers would probably not be affected by Landau electrons since the ducts keep the wave normal angle fairly small. For very steep electron energy spectra, $p > 5$, however, the Landau damping dominates cyclotron growth at wave-normal angles less than a degree, and in this case, may be important even for ducted propagation.

The total electron density near the equatorial plane can be measured by whistler mode techniques (Helliwell, 1965). However, there seems to be some uncertainty about the variation along the lines of force. The model of T, N = constant along the lines of force, gives, at least intuitively, the fastest rate of change of wave normal angle along the ray path. It is unlikely that N decrease towards the Earth. Thus, N = constant yields the most rapid physically acceptable decrease of N/B. Θ then will probably change the most rapidly with propagation to compensate. This suggests that the unstable path lengths will be shortest when N is constant. On the other hand, the group velocity increases the most rapidly away from the equatorial plane when N is constant so that the higher latitude damping effects at large wave normal angles are not weighted as heavily. These effects should be given further consideration.

Since unducted whistler growth probably depends to some extent on the properties of low energy electrons, any increase in the low energy component without a corresponding increase in the cyclotron electrons is a possible candidate for enhanced Landau effects. Such an increase, the so-called "knee," is observed in the region $L \approx 2.5 - 3.5$ in the magnetosphere (Carpenter (1963)). While many questions remain to be answered, the fact that VLF emissions tend to occur preferentially on the low density side of the "knee" is tantalizing.

The calculations contained in Section 2.3, based on what appear to be fairly stringent assumptions, suggest that there is at least a thin region near the equatorial plane, roughly 20^{o} in latitude width for which whistler growth occurs. When the energy spectrum is sufficiently hard, in this case $1/E^2$, this growth dominates the Landau damping which takes place along the rest of the ray path. Thus it appears that a finite whistler intensity can be generated and that the conclusions of KP carry over to the possibly more realistic case of an unducted whistler distribution which has a variety of wave normal angles. Depending on the Landau electron distribution, their conclusions may be valid for the auroral as well as the Van Allen radiation zones. Information on the distribution of hundred volt electrons would resolve this question.

We have concentrated throughout on the conditions for which a steady laser action of the magnetosphere trapped electron and whistler population can occur. In this case, whistler radiation which is reflected back towards the equatorial plane and reamplified there is the dominant component of the wave distribution. One reflection mechanism, involving the lower hybrid resonance, was suggested in T. This necessarily occurs well away from the equator, and the Landau damping must be small on the high latitude portions of the ray path to ensure net wave growth. This mechanism, when it occurs, efficiently produces large wave amplitudes and therefore precipitation. One would expect the smallest critical trapped electron intensities for the onset of precipitation in this regime.

Sudan (1965) has shown that cyclotron electrons with a large pitch angle anisotropy are non-convectively unstable to parallel propagating whistlers. In effect, whistlers with small group velocities are always amplifying. (One expects on intuitive grounds that this will be so for those off-angle waves for which Landau damping is negligible, since the unstable electrons are those propagating in the opposite direction to the wave.) We have concentrated on the convective parts of the whistler instability on the premise that the convective growth rates are likely to be larger than the non-convective, and with some feedback or external source, will therefore probably dominate. However, the medium itself is probably unstable to whistler growth whether or not reflection occurs.

As the density of low energy Landau electrons is increased, the path length of the region of wave growth decreases, and that of Landau damping increases to become the dominant effect. The waves generated at the equator are then absorbed elsewhere on the line of force, and laser action is probably not possible. However, this does not mean that whistler-driven electron precipitation cannot occur. All that is needed is a sufficiently large whistler amplitude somewhere along the line of force. The natural fluctuations of the plasma radiation fields, which should be themselves well above thermal levels because of the large non-thermal tail of the velocity distribution, are an input source for whistler growth. When the trapped electron intensity is sufficiently high, a large whistler amplitude, and therefore precipitation rate, can be achieved starting with the fluctuation source and propagating through the growth region, however narrow. Since the increment in wave amplitude depends only logarithmically on the trapped electron intensity, orders of magnitude increases in electron intensity are not needed. It seems likely that at least the very high fluxes of auroral electrons can precipitate in this way, perhaps on a non-steady basis. The essential point is that whistler growth always occurs for parallel propagating waves. Coupled with the fact that unducted whistlers change their wave normal angles, this means that wave growth is likely to occur over some fraction of the ray path. Thus, an upper limit to stably trapped cyclotron electron intensities is bound to exist.

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The local growth rate for whistler waves propagating at an angle to the geomagnetic field is combined with model ray path calculations to estimate the wave growth along the whistler path. This growth rate depends critically upon the equatorial plane distribution of electrons in the few hundreds of electron volt energy range. Under a variety of conditions apparently satisfied by the magnetosphere electron velocity distribution, a region of whistler growth near the equatorial plane exists and is sufficient to make a whistler mode unstable overall as it bounces back and forth across the equatorial plane between reflection points. Only unducted whistlers generated by resonant particle instabilities are treated.

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