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Technical Report

DETERMINATION OF GAS BEARING STABILITY BY RESPONSE TO A STEP-JUMP

by

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ABSTRACT

The stability of a gas bearing is treated by a new procedure in which the bearing film is characterized by its responses to step-jump displacements. Duhamel's theorem is invoked to generalize these step responses in a system of dynamical equations. Stability is determined by calculation of a "growth factor" for each degree of freedom.

author

NOMENCLATURE

A_n = nth LaGuerre coefficient (equation 8)

B_j = jth LaGuerre coefficient (equation 10)

C = Ground-in clearance

F_{ij} = the force in the "j" direction due to a displacement in the "i" direction

δF_{ij} = the difference in F_{ij} between time t and equilibrium at time zero (equation 4)

$H(t-\tau)$ = the response function observed at time t produced by stimulus at time τ (equation 4)

I_T, I_p = shaft transverse and polar moments of inertia

L = length of bearing

L_1, L_2 = distances from shaft mass center to bearings one, two

$L_n(x)$ = nth LaGuerre polynomial (equation 6)

M = shaft mass

p_a = ambient pressure

R = shaft radius

$r(t)$ = a response function (equation 3)

$s(t)$ = a stimulus (equation 3)

t = time variable

W_i = Gauss integration weighting factors

δx, δy = small displacements in x and y directions

α = attenuation constant

α_1, α_2 = shaft angular coordinates

β = growth factor (equation 14)

NOMENCLATURE (Cont.)

γ = growth frequency (equation 15)

ε = eccentricity ratio

$\Lambda = \frac{6\mu\Omega}{p_a}\left(\frac{R}{C}\right)^2$

μ = viscosity

τ = dummy variable

Ω = shaft angular speed

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1. INTRODUCTION

Recent interest in gas hybrid journal bearings has promoted a closer look at the stability of rotor-bearing systems and, in particular, at the methods by which stability might be predicted. In general, two different methods have been used to handle the mathematical stability problem. The first method treats small perturbations from a hypotesized steady-state mode of operation and determines whether these perturbations grow or diminish. The Routh-Hurwitz criterion is used in this connection. The second method consists of direct digital computation of all dynamical and fluid film equations and is known as the "orbit" method. It can handle linear, as well as non-linear, aspects of the problem. Both procedures have been employed extensively in earlier gas-bearing stability work at The Franklin Institute [(1,2,3)*] and elsewhere.

The foregoing methods of stability analysis have their advantages and disadvantages. The advantage of the perturbation method is principally that of any linearized analysis; namely, that superposition is possible and results are easily generalized. It has the disadvantage that unusual geometries are not easily accommodated and that in multi-degree-of-freedom systems the characteristic equation is exceedingly complicated. The second method has great flexibility, and can incorporate grooves and other aspects of bearing design quite readily. It gives shaft and film behavior in great detail. It is excellent for delineating the performance of a particular design, but the lack of generality of its solution makes parametric investigations expensive. Its principal disadvantage is its consumption of considerable computer time.

A new procedure for stability analysis is presented here which utilizes the strong points of both the orbit and the linearized approaches. The procedure obviates the necessity for a solution of a large character-

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*Number in parenthesis refer to references.

istic equation on the one hand, while avoiding repetitious calculations of fluid-film pressure distributions on the other. Briefly, the method consists of using an orbit program to give the responses to step-jump displacements in each degree of freedom of a system. By means of Duhammel's theorem(4) these step responses can be used in a system of dynamical equations. A possibility then exists of running linearized orbit programs without the necessity of detailed fluid-film calculations for every case studies. The computing time of the original orbit program is thereby greatly lessened.

2. TECHNICAL DISCUSSION

2.1 Response To Step-Jump

In a linear system, superposition of forcing functions leads to superposition of responses. If the system stimulation is sinusoidal in character, the methods of Fourier synthesis can be used to predict responses to generalized forcing functions. The same sort of generalization is also possible if the response to step-jump stimulus is known, and because this type of response is more readily obtained from an orbit program, the analysis here will be based upon it.

Generalization of the response to step-jump, can be accomplished by means of Duhamel's Theorem. A brief heuristic derivation is as follows: Suppose that r(t), a response, is linearly related to s(t), a stimulus. Let H(t-τ) denote the r-function observed by time, t, as produced by unit increase of the s-function at time, τ. Then we can consider the more general response occasioned by a more general stimulus to be obtained by superimposed step-jumps as shown in Figure 2-1. The jagged contour can be made to approximate the smooth curve with arbitrarily high precision by reduction of Δτ.
Clearly,

$$r(t) = s(o)\ H(t) + \sum_n (\Delta s)_n\ H(t - n\Delta\tau)\ ,$$

$$= s(o)\ H(t) + \sum_n \left(\frac{\Delta s}{\Delta\tau}\right)_n H(t - n\Delta\tau)\Delta\tau. \qquad [1]$$

With $n\Delta\tau = \tau$, and $n \to \infty$, $\Delta\tau \to o$, this equation becomes

$$r(t) = s(o)\ H(t) + \int_o^t \dot{s}(\tau)\ H(t-\tau)d\tau. \qquad [2]$$

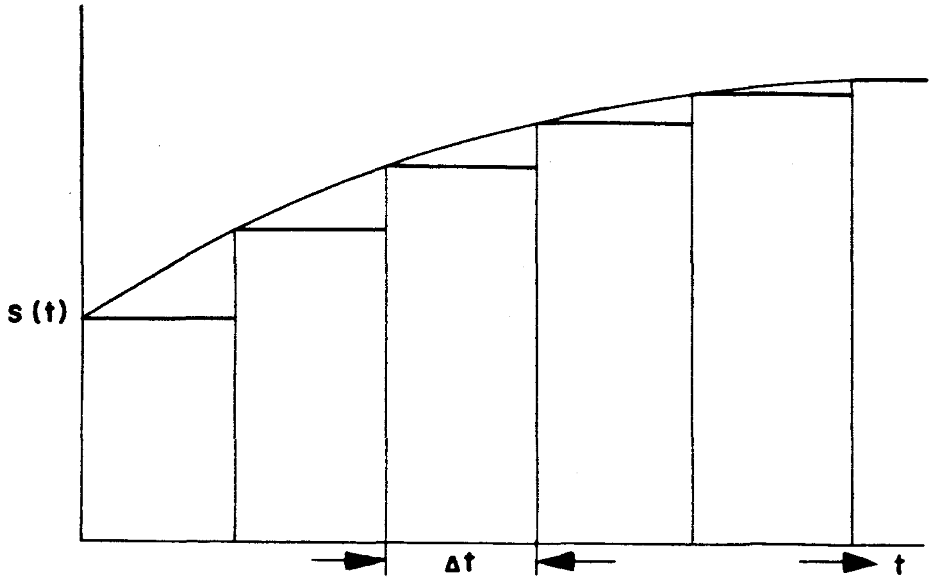

FIG.2-1 APPROXIMATION OF "FORCING FUNCTION" BY SUCCESSIVE STEP-JUMPS

Alternatively, integration by parts gives:

$$r(t) = H(o)\ s(t) + \int_o^t s(\tau)\ \dot{H}(t-\tau)d\tau. \qquad [3]$$

This second form is found more useful in present applications.

2.2 Gas-Bearing Response Functions

To illustrate the character of the response to step-jump in a typical gas-bearing application, let us consider the forces on an infinitely-long gas-lubricated journal bearing, as shown in Figure 2.2. Corresponding to some vertical loading, the shaft center will, if stable, assume some equilibrium position (x_o, y_o). In this case the integrated fluid film forces become: $F_x = 0$, F_y = load. Now if the shaft is suddenly given a small x-wise displacement, δx, and held there, both F_x and F_y will be affected. There will be transient force responses to the step-jump in "x" and new steady-state forces will asymptotically be

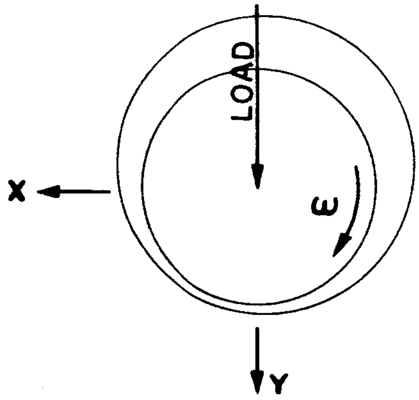

FIG. 2-2. INFINITELY-LONG GAS-LUBRICATED JOURNAL BEARING

achieved. Similar results can be found for any small y-displacement, δy. Typically, the results due to unit δx_i at $t = o$ can be expressed as:

$$\delta F_{ij} = F_j(t) - F_j(x_o, y_o);\; H_{ij} = C\, \delta F_{ij}/p_a RL. \qquad [4]$$

Orbit programs are well suited to provide responses for the kind of displacement just hypothesized. Figures 2-3 and 2-4 give computer results for an infinite journal bearing operating with $\varepsilon = 0.6$, $\Lambda = 1.46$. It should be noted that the H_{ij} curves give total dimensionless shaft forces -- not fluid film details -- and that these same curves always apply for small deviations from the specified operating condition, regardless of the rest of the shaft dynamics. The near-antisymmetry, $H_{ij} \cong -H_{ji}$ is reminiscent of journal bearings with a continuous film of incompressible fluid(5). In fact, at time zero, when the gas is "trapped" by the sudden small displacement (so that ph = constant at each point in the bearing) the antisymmetry is exactly true.

For computer purposes, it is preferable to have the H_{ij} in analytical, rather than in tabular, form. Asymptotically, it may be expected that

$$H_i \rightarrow H_i(\infty) + (\text{constant})e^{-\alpha t}. \qquad [5]$$

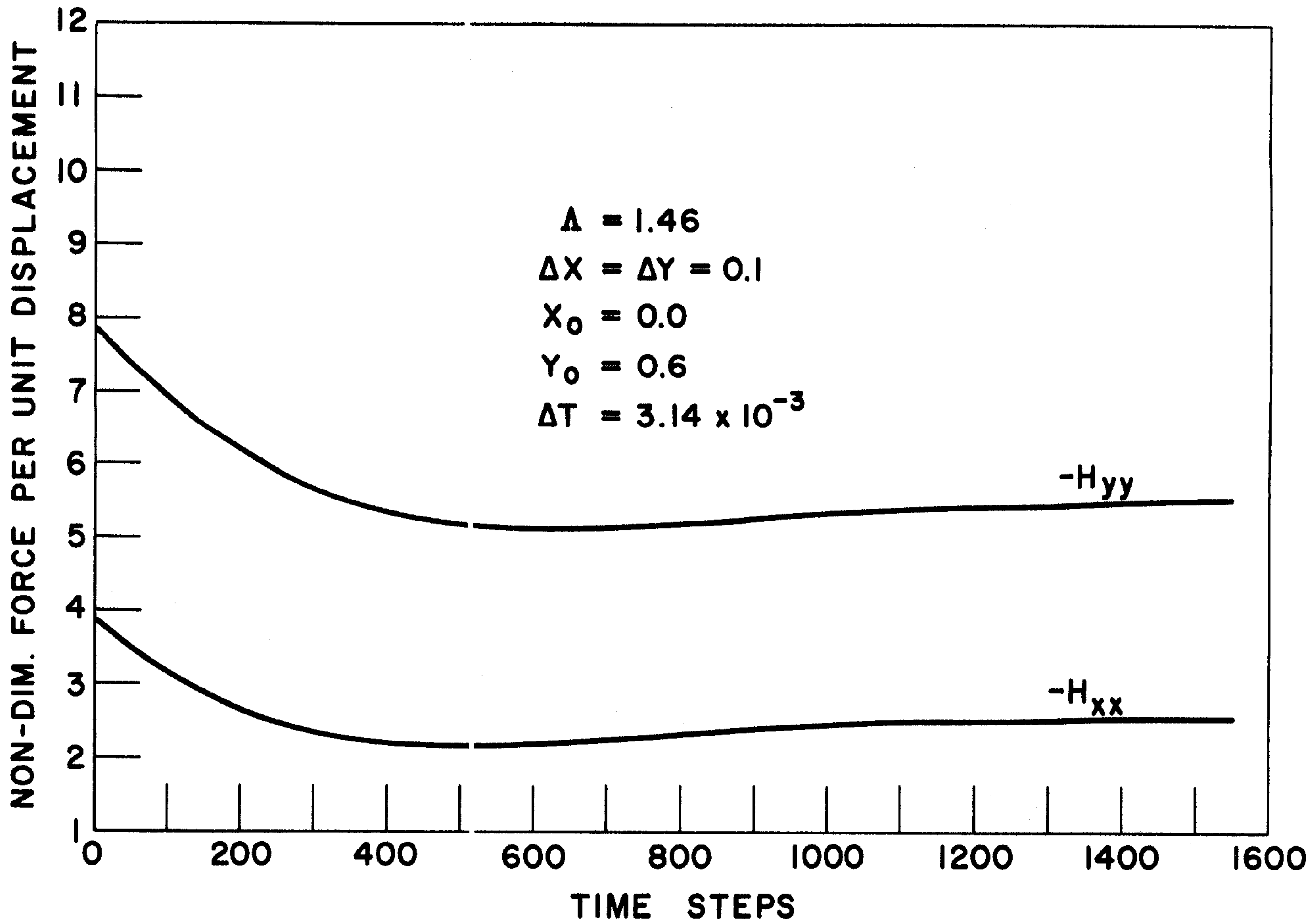

FIG. 2-3. *FORCE RESPONSE TO STEP-JUMP FOR AN INFINITELY LONG JOURNAL BEARING — H_{xx} AND H_{yy} VS. TIME*

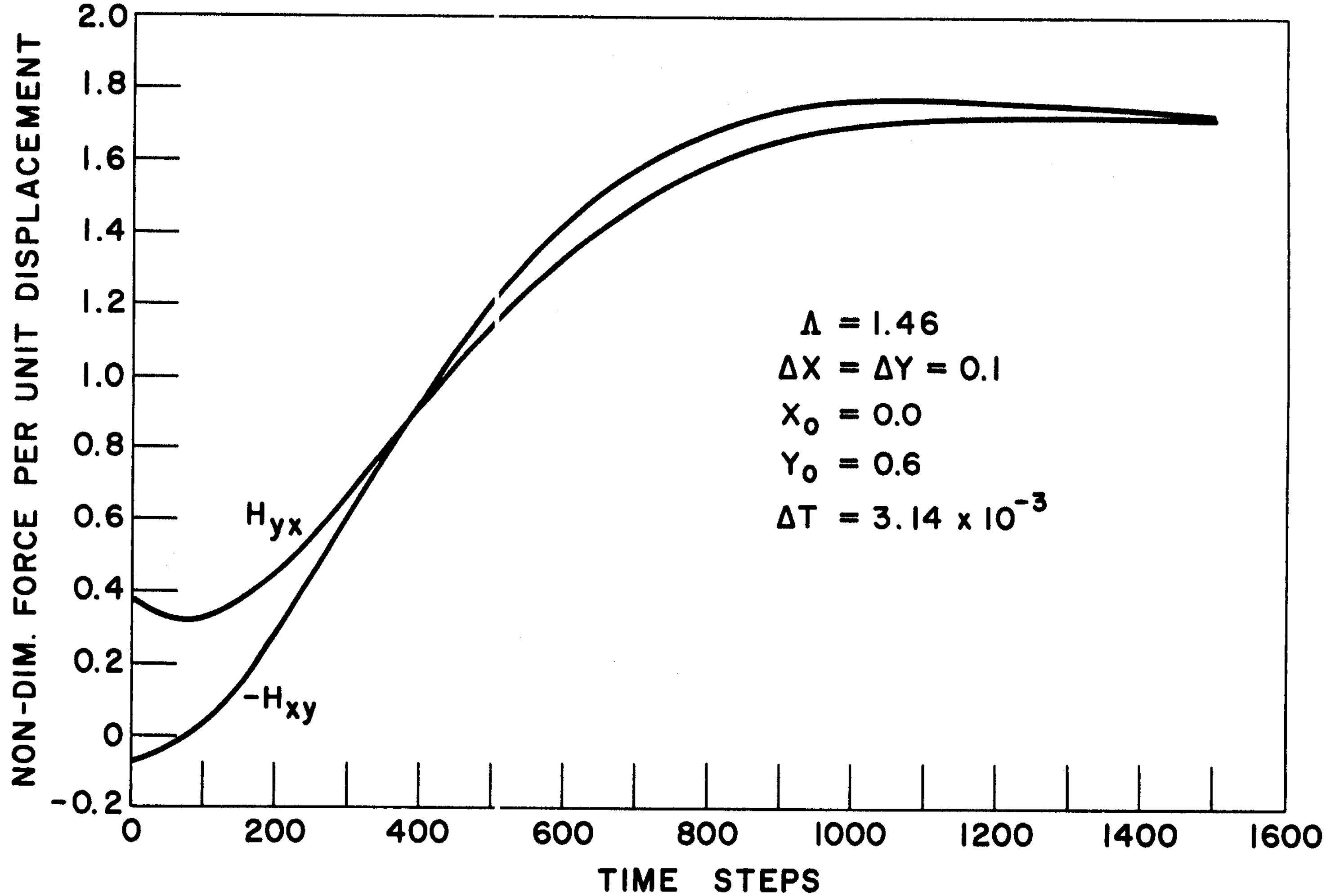

FIG. 2-4. FORCE RESPONSE TO STEP-JUMP FOR AN INFINITELY LONG JOURNAL BEARING — H_{xy} AND H_{yx} VS. TIME

To represent intermediate behavior, an expansion in LaGuerre's polynomials is used. These polynomials are chosen because they are orthogonal in the interval zero to infinity with a exponential weighting factor. As a consequence, the coefficients found for these polynomials are "best" in the least-squares sense.
They have the form:(6)

$$L_n(x) = \sum_{k=o}^{n} \frac{n!}{(n-k)!k!} \frac{(-x)^k}{k!},$$
$$L_0(x) = 1,$$
$$L_1(x) = 1 - x, \qquad [6]$$
ect.

and

$$\int_o^\infty e^{-x} L_n(x)\ L_m(x)\ dx = \delta_{mn}.$$

The series approximation:

$$H(t) - H(\infty) = \sum_{n=o}^{\infty} A_n L_n(\alpha t) e^{-\alpha t}, \qquad [7]$$

is used. The coefficients A_n are determined by multiplication of both sides of this last equation by $L_m(\alpha x)$ and integrating. Thus

$$\int_o^\infty L_m(\alpha t)\ [H(t) - H(\infty)]dt = \int_o^\infty \sum_{n=o}^{\infty} A_n L_n(\alpha t)\ L_m(\ t) e^{-\alpha t} dt$$
$$= A_m/\alpha. \qquad [8]$$

Prior to the running of a linearized orbit, an accurate value of the attenuation coefficient "α" is not known and one must be guessed. Fortunately, a choice is not critical, inasmuch as any "error" in the guessed value will be absorbed by the LaGuerre coefficients. However, if the attenuation coefficient is optimally chosen, the coefficients of the LaGuerre series will approach zero most rapidly. To convert to a new attenuation coefficient, it is not necessary to rerun the orbit program. Instead, the following conversion relation can be used.
Thus:

$$\sum B_k L_k(\beta t) e^{-\beta t} = \sum A_k L_k(\alpha t) e^{-\alpha t}, \qquad [9]$$

where:

$$B_j = \sum_{n=o}^{j} (-1)^{j+n} \frac{(\beta-\alpha)^{j-n} \quad \beta^{n+1}}{\alpha^{j+1}} \frac{j!}{n!\ (j-n)!} A_n . \qquad [10]$$

To approximate the results in Figures 2-3 and 2-4, an $\alpha = 1.0$ was used. When ten LaGuerre polynomials are used therewith, the numerical results are indetectibly different on the scale shown.

2.3 Stability Characteristics of the Infinitely Long Self-Acting Gas Journal Bearing

The foregoing theory was first applied to calculated the stability threshold of an infinitely long self-acting gas journal bearing operating with a steady load appropriate to $\varepsilon = 0.6$, $\Lambda = 1.46$. Information on this geometry and operating condition is available from several sources(2,7). The procedure for using the information from step-jump responses is straight forward. Dynamical equations are written in the form:

$$m\delta\ddot{x} = \delta F_{xx} + \delta F_{yx} ,$$
$$m\delta\ddot{y} = \delta F_{xy} + \delta F_{yy} , \qquad [11]$$

with

$$\delta F_{yx} = H_{yx}(o)\ \delta y(t) + \int_o^t \delta y(\tau)\ \dot{H}_{yx}(t-\tau)d\tau \quad \text{etc.} \qquad [12]$$

Typical initial conditions assumed in the present case were:

$$\delta x(o) = -1 \qquad \delta\dot{x}(o) = 0$$
$$\delta y(o) = 0 \qquad \delta\dot{y}(o) = -1$$

The corresponding lineareized orbits were computed numerically. Eventual growth of the displacements δx and δy was taken to indicate instability, with contrary results being taken to indicate stability. Figure 2-5 shows a linearized orbit deemed to be stable, Figure 2-6 shows one deemed to be marginally stable, and Figure 2-7 shows one deemed to be highly unstable. Physically, the difference between these cases lies in the mass associated with the shaft.

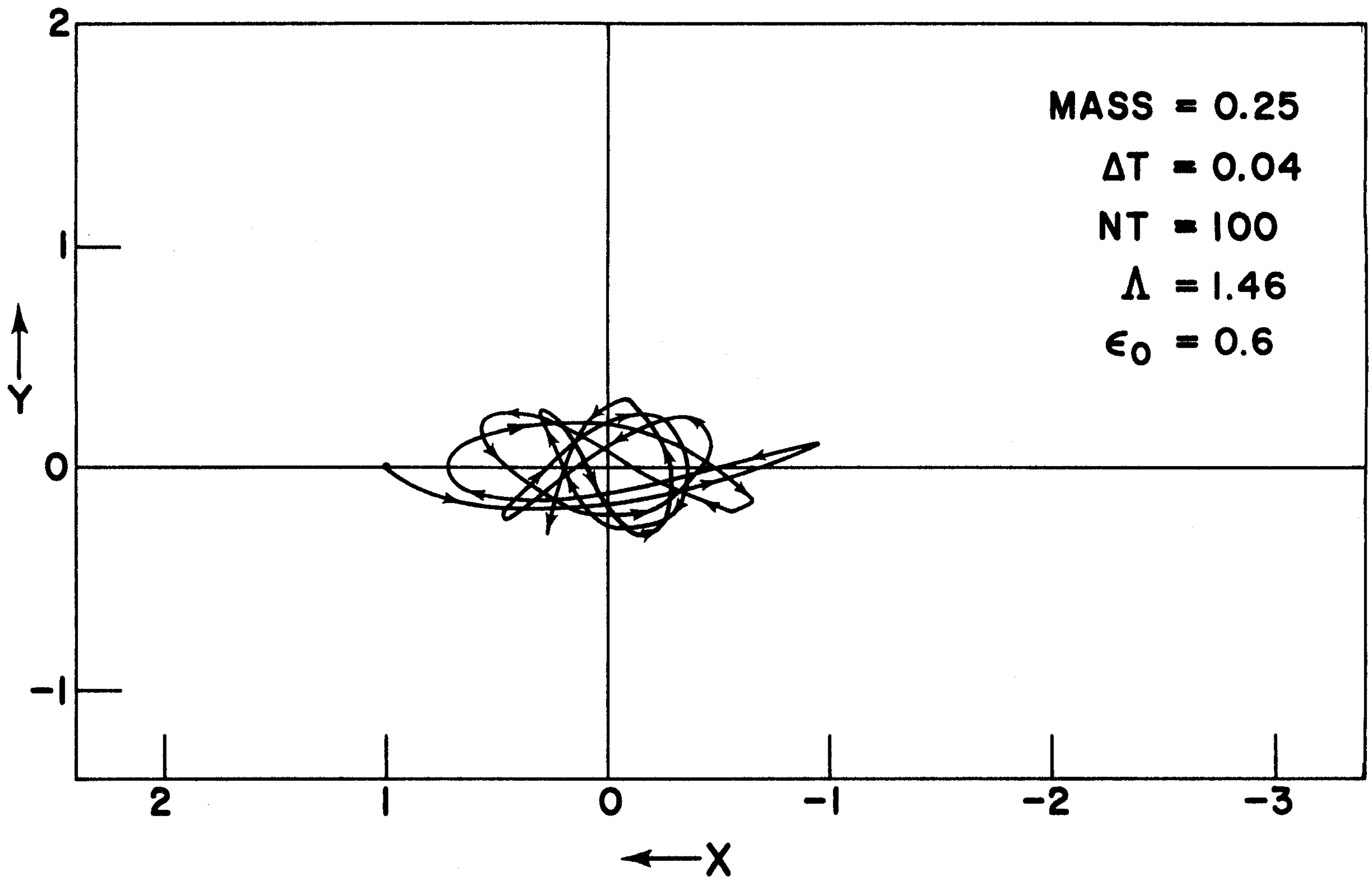

FIG. 2-5. RESPONSE TO SMALL PERTURBATION ABOUT AN EQUILIBRIUM POSITION FOR INFINITELY LONG JOURNAL BEARING ORBIT PLOT OF SHAFT MASS CENTER COORDINATES

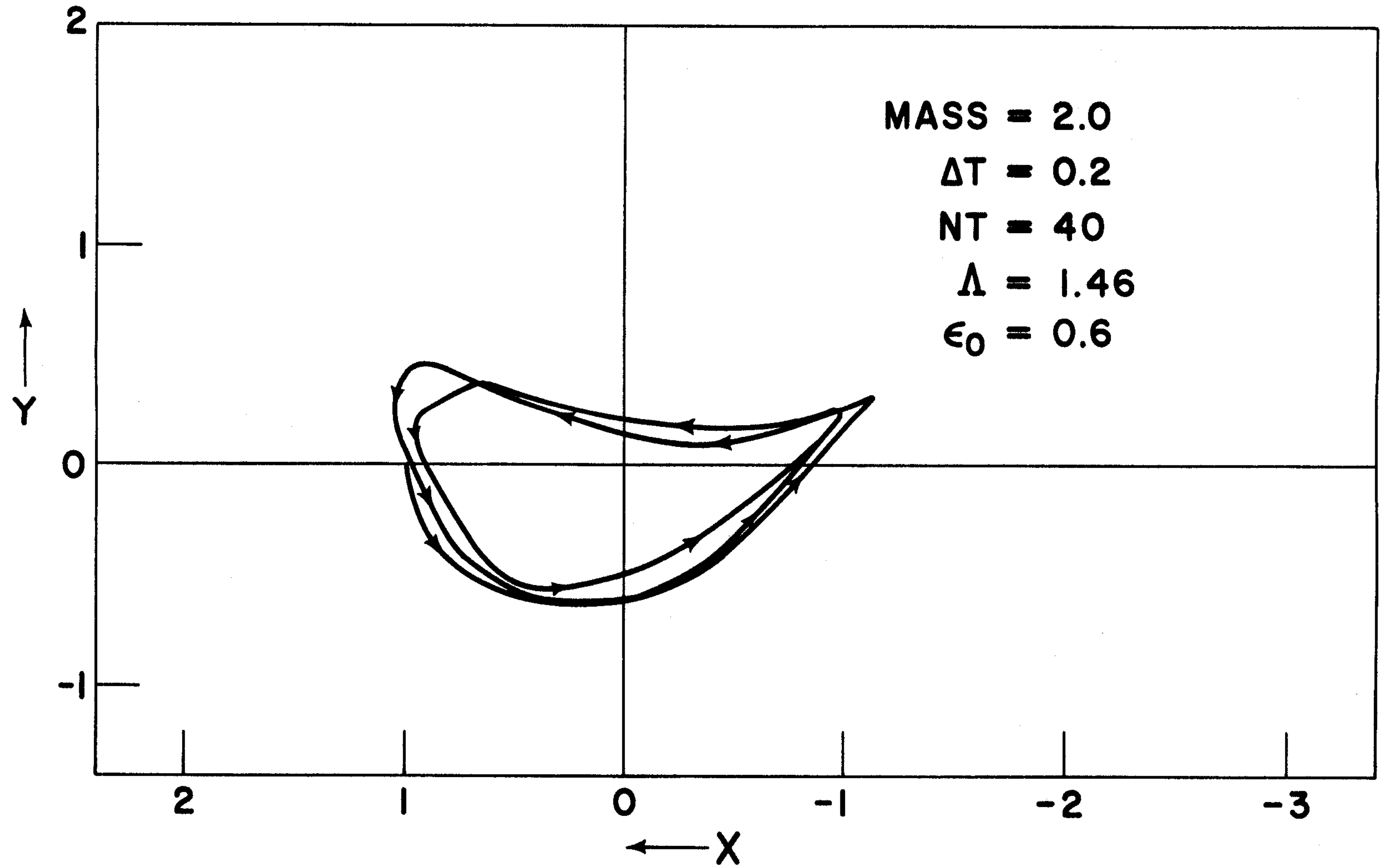

FIG. 2-6. RESPONSE TO SMALL PERTURBATION ABOUT AN EQUILIBRIUM POSITION FOR INFINITELY LONG JOURNAL BEARING ORBIT PLOT OF SHAFT MASS CENTER COORDINATES

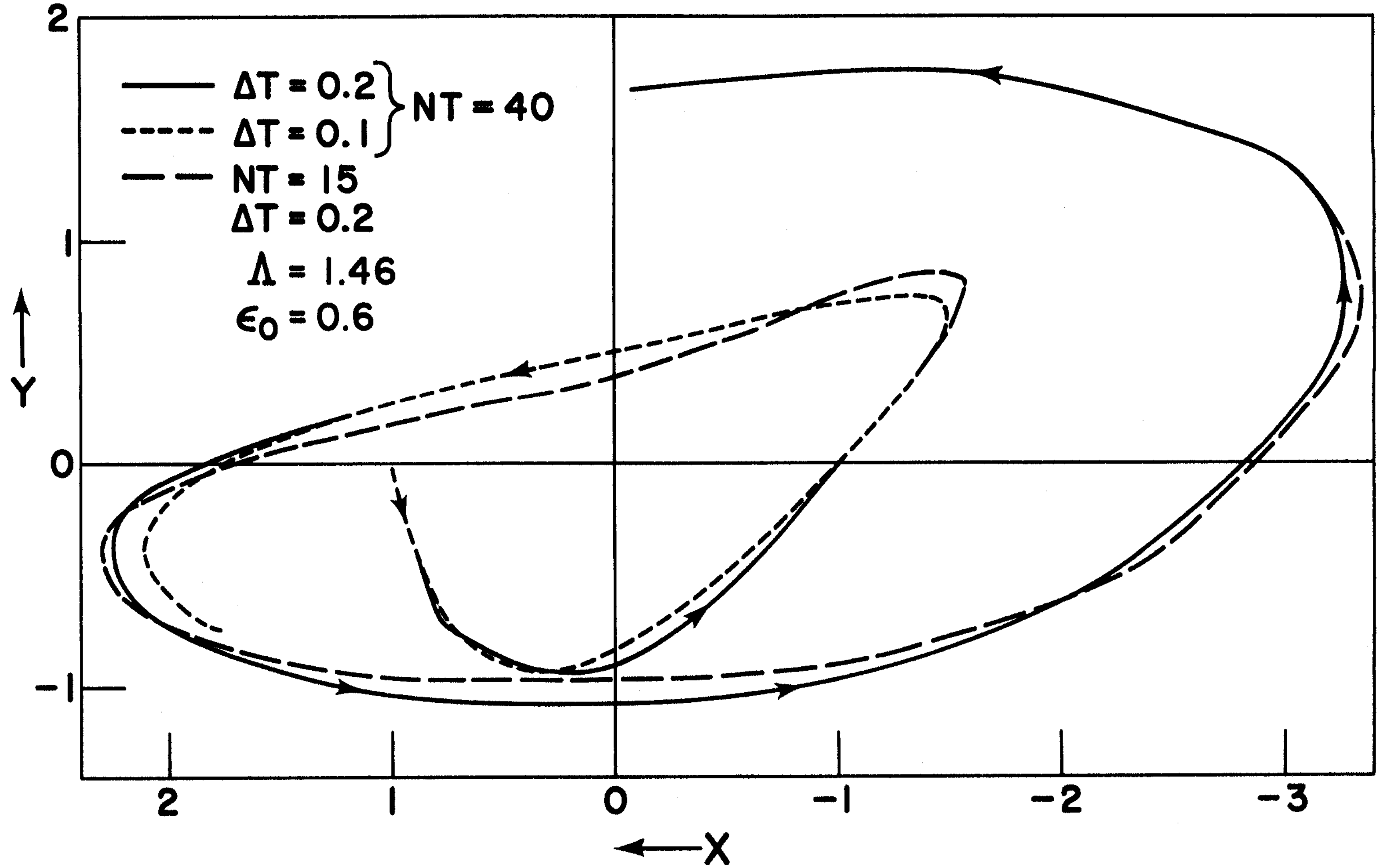

FIG. 2-7. *RESPONSE TO SMALL PERTURBATION ABOUT AN EQUILIBRIUM POSITION FOR INFINITELY LONG JOURNAL BEARING ORBIT PLOT OF SHAFT MASS CENTER COORDINATES*

To remove as much as possible the personal judement factor in setting the stability threshold, a growth factor was calculated from the orbit results. Asymptotically δy was assumed to possess the form:

$$\delta y(t) = Ae^{\beta t} \sin (\gamma t + \phi), \qquad [13]$$

and the growth factor was computed from four successive values of δy (spaced by Δt).

Thus:

$$e^{2\beta\Delta t} = \frac{\delta y_3 \ \delta y_1 - \delta y_2^2}{\delta y_2 \ \delta y_0 - \delta y_1^2} . \qquad [14]$$

The associated frequency "γ" was also of interest:

$$\cos (\gamma\Delta t) = \frac{\delta y_0 + \delta y_2 \ e^{-2\beta\Delta t}}{2 \ \delta y_1 \ e^{-\beta\Delta t}} . \qquad [15]$$

Figure 2-8 shows the growth-rate found for the given operating condition $\varepsilon = 0.6$, $\Lambda = 1.46$, and various values of dimensionless mass. The critical value of 2.17 converts to $\frac{MC \ \omega^2}{4\pi p_a L} = 0.831$. In Figure 2-9 this last value is compared with the results of Marsh and of Castelli and Elrod. The ratio of the critical value of "γ" as obtained from eq. [15] is conpared in Figure 2-10 with Marsh's work. Agreement is excellent in each case.

Computer runs to provide individual points on the curve in Figure 2-8 can be performed very quickly (approx. 30 secs on a Univac - 1107 computer). Part of the speed achievable is due to a special integration procedure used in the convolution integral. To obviate the need for using data at every time step, a modified Gauss integration rule was adopted for which the locations and ordinate weighting factors W_i are given below.

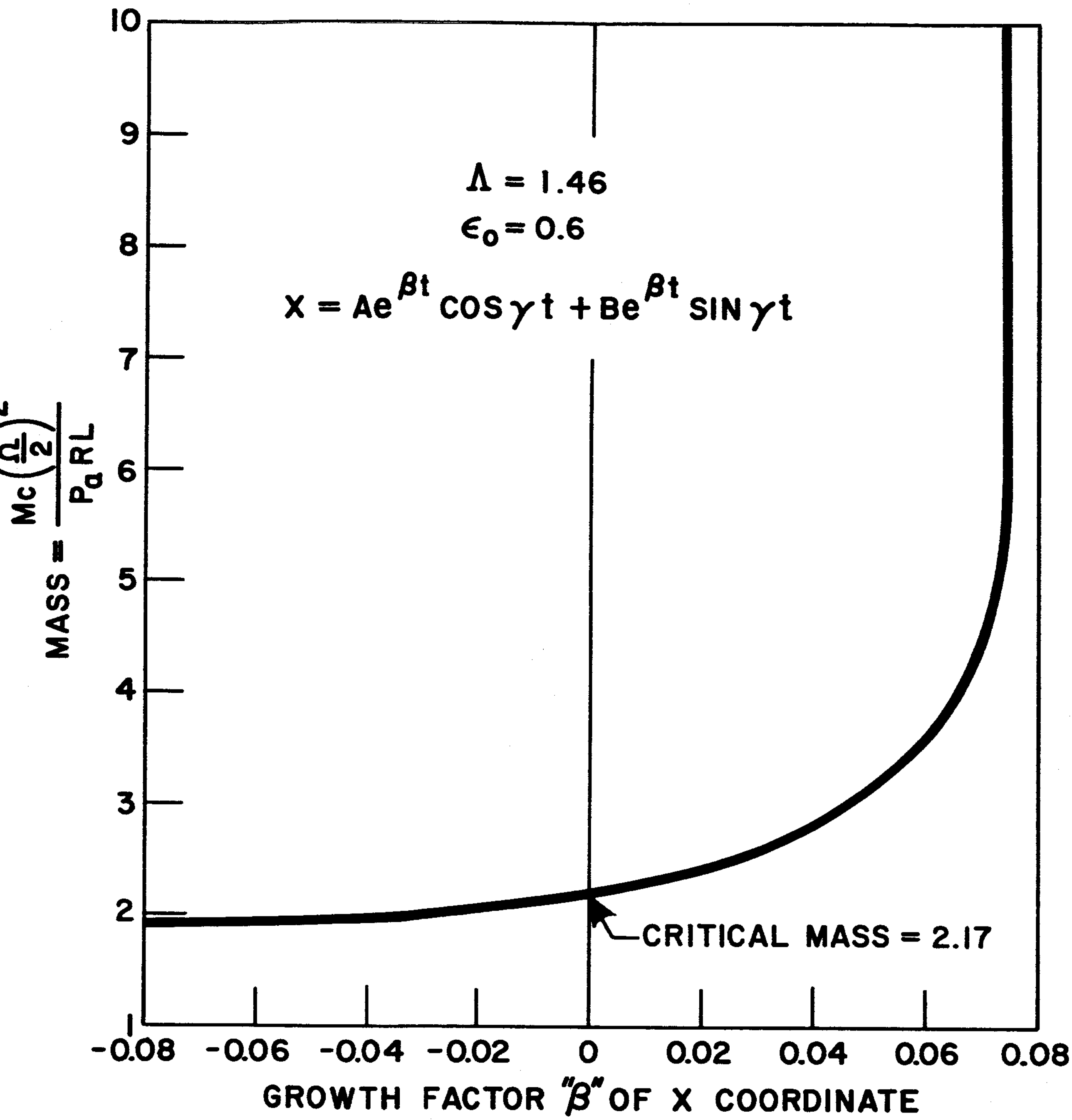

FIG. 2-8. SINGLE BEARING - TRANSLATIONAL STABILITY

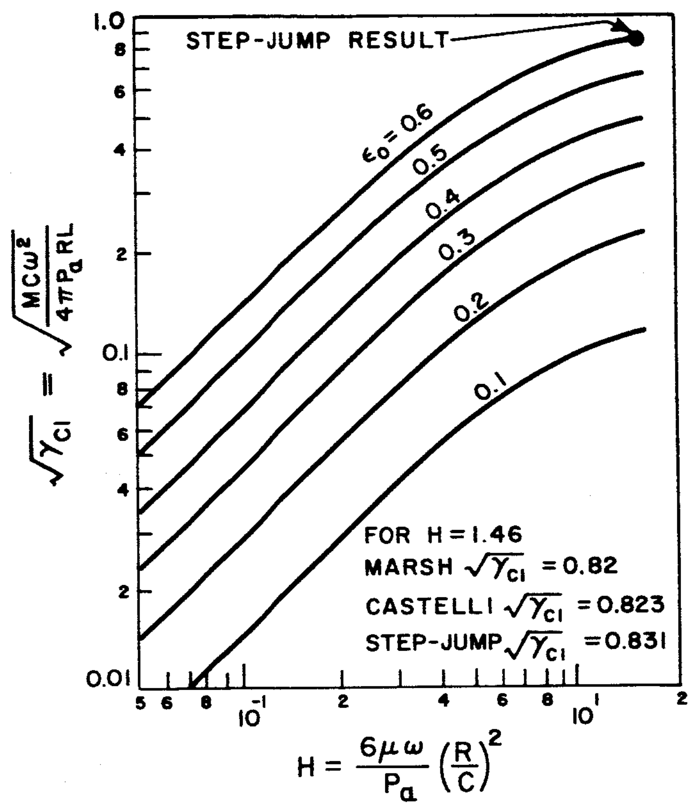

FIG. 2-9. THE CRITICAL TRANSLATIONAL STABILITY RATIO, SINGLE BEARING, L/D → ∞ (AFTER H. MARSH)

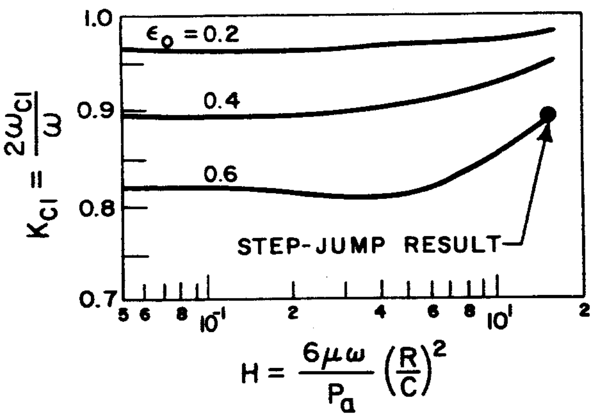

FIG. 2-10. THE TRANSLATIONAL CRITICAL FREQUENCY RATIO, SINGLE BEARING, L/D → ∞ (AFTER H. MARSH)

$\pm X_i$	W_i
0	.197197636
8/40	.199459835
15/40	.139711837
19/40	.062229510

The rule is exact for sixth-degree polynomials and nearly exact for polynomials up to thirteenth degree. (For example, it gives $\int_0^1 x^{13}dx =$ 0.07136 instead of 0.07143).

2.4 Stability Characteristics of a Two-Bearing System

To show the versatility of the new step-jump technique, a two-bearing system was next studied. This system was conceived to consist of two equally-loaded long bearings each similar to the single bearing discussed in Section 2.3. Figure 2-11 shows the geometrical arrangement and defines linear and angular coordinates appearing in the dynamical equations. The shaft rotates with angular velocities ($-\alpha_2$, α_1 and Ω) about the x, y and z) axes respectively through the shaft center of gravity and their time derivatives are of disturbance magnitude.

$$M\delta\ddot{x}_M = \delta F_{x,1} + \delta F_{x,2} ,$$
$$M\delta\ddot{y}_M = \delta F_{y,1} + \delta F_{y,2} . \qquad [16]$$

Here $\delta_{x,1}$ represents the force in the x-direction on the shaft by bearing #1, etc. The average linear coordinates of the shaft within the bearings are:

$$\delta x_1 = \delta x_M + L_1\ \delta\alpha_1 \qquad \delta x_2 = \delta x_M - L_2\ \delta\alpha_1$$
$$\delta y_1 = \delta y_M + L_1\ \delta\alpha_2 \qquad \delta y_2 = \delta y_M - L_2\ \delta\alpha_1 \qquad [17]$$

The separation of the bearings is presumed large enough to neglect the effects of conical misalignment on forces or torques. Therefore:

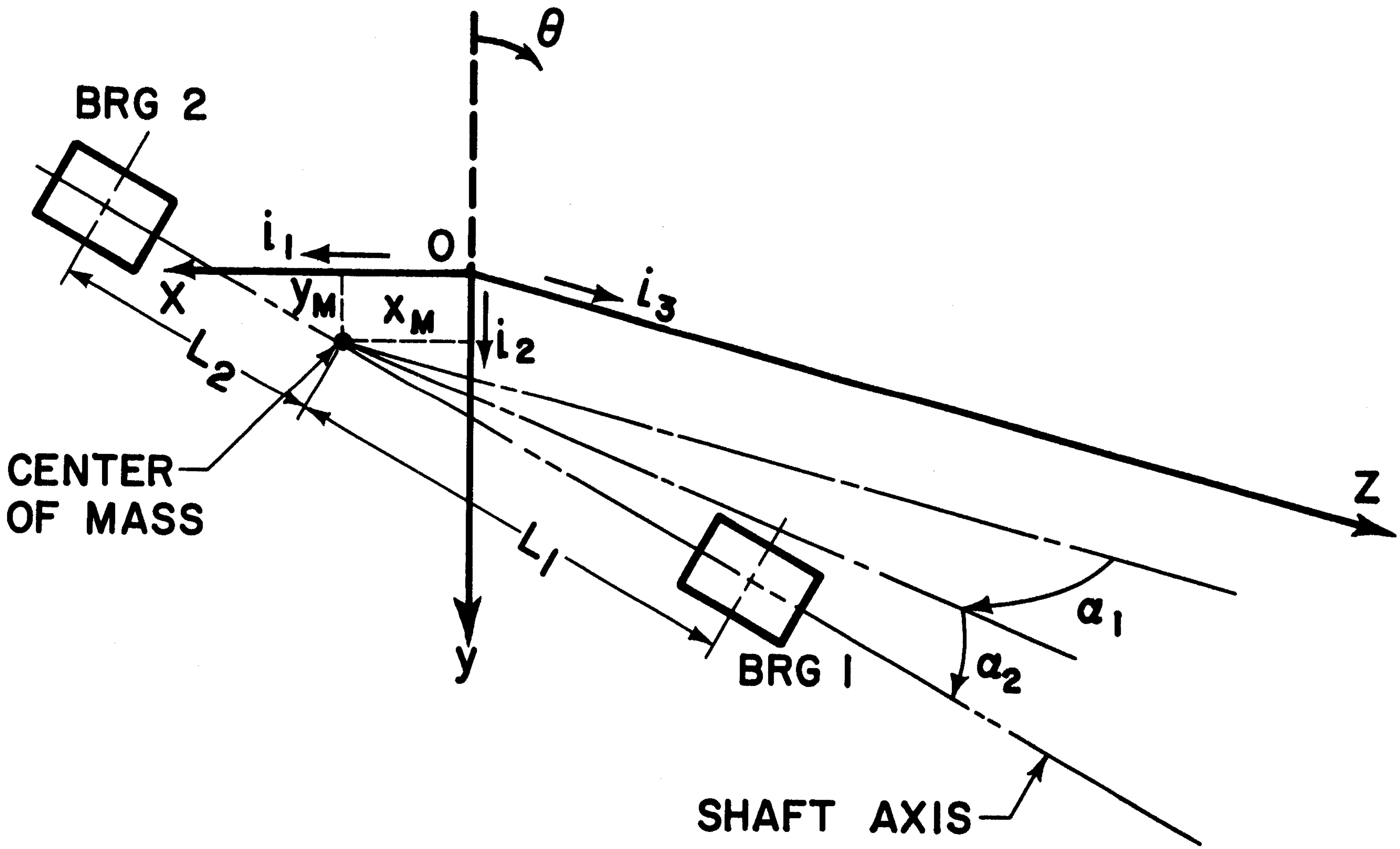

FIG. 2-11. JOURNAL COORDINATE SYSTEM

$$\delta F_{x,1} = H_{xx}(o)\ \delta x_1(t) + \int_o^t \delta x_1(\tau)\ \dot{H}_{xx}(t - \tau)d\tau$$
$$+ H_{yx}(o)\ \delta y_1(t) + \int_o^t \delta y_1(\tau)\ \dot{H}_{yx}(t - \tau)d\tau \qquad [18]$$

etc., where the H-functions here are the <u>same</u> as for the single bearing of section 2.3. The angular acceleration equation becomes:

$$I_T\ddot{\alpha}_1 = I_p\Omega\dot{\alpha}_2 + \left(L_1\ \delta F_{x,1} - L_2\ \delta F_{x,2}\right),$$
$$I_T\ddot{\alpha}_2 = -I_p\Omega\dot{\alpha}_1 + \left(L_2\ \delta F_{y,2} - L_1\ \delta F_{y,1}\right). \qquad [19]$$

Here I_T and I_p are the transverse polar moments of inertia.

For the brief, illustrative study of two-bearing stability, a system was taken which has marginal translational (as opposed to conical) stability. A dimensionless mass (as per Figure 2-8) of 2.0 was chosen. For large enough bearing separation, the results of Section 2.3 are duplicated. As the bearing locations are brought together, the immunity of the system to conical whirl is reduced and the conical stability threshold is transgressed. These features are illustrated by Figures 2-12 to 2-15.

For the response shown in Figures 2-12 and 2-13 the total bearing separation is 20 , and the bearing system is stable in both the translational and conical modes. On the other hand, when the bearing separation is reduced to 4, all other operating conditions remaining fixed, the translational modes remain stable, while the conical modes become unstable. This fact is shown in Figures 2-14 and 2-15. Figure 2-16 shows the conical orbit of this unstable condition, and Figure 2-17 shows the determination of the stability threshold by means of a plot of bearing separation versus exponential growth factor. The critical bearing separation differs from that given by Marsh's approximate formula by less than 8%.

Listings of the digital computer programs used to implement the above analyses are given in Appendix A.

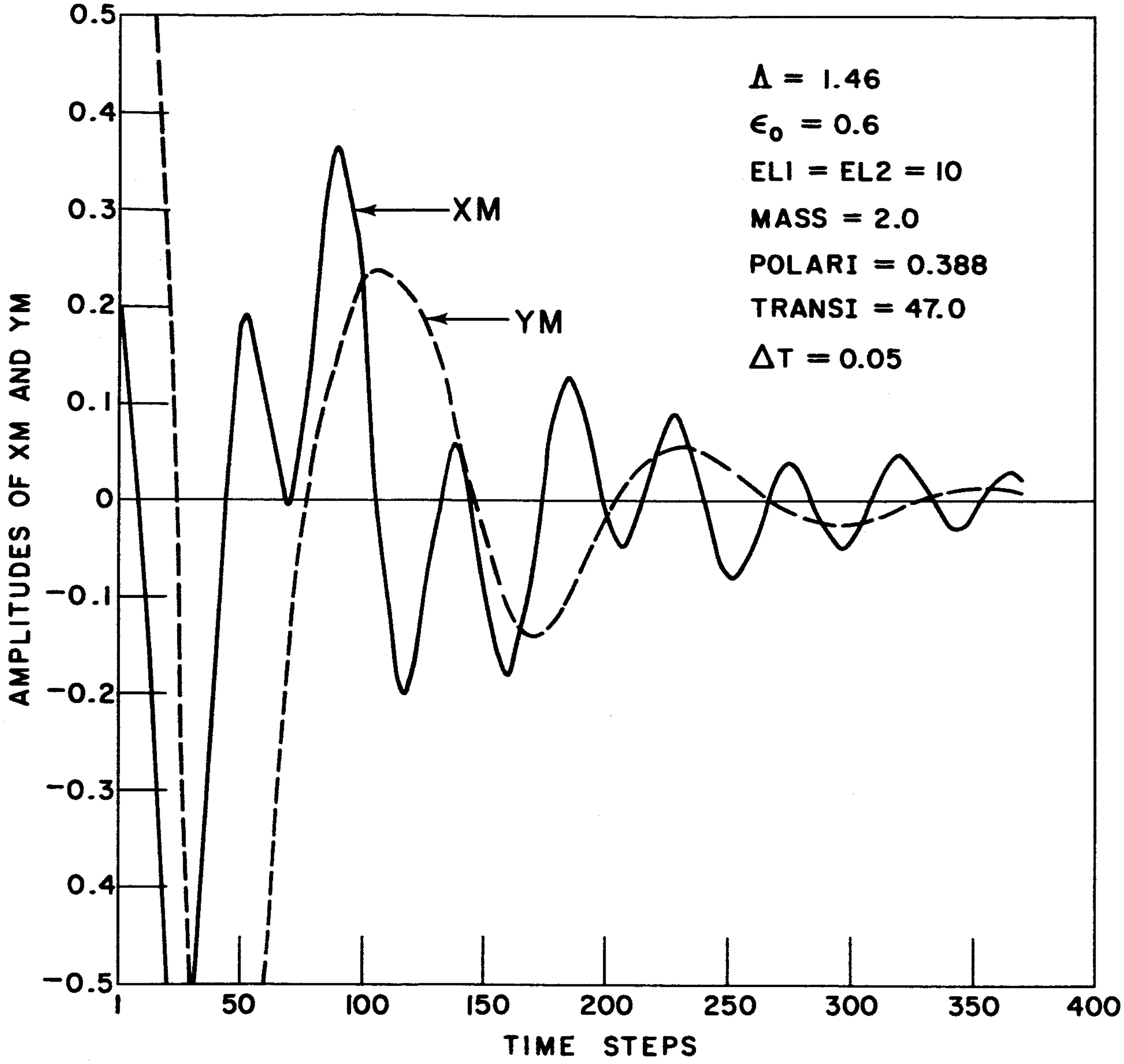

FIG. 2-12. TRANSLATIONAL MOTION OF TWO BEARING SYSTEM SHAFT MASS CENTER COORDINATES VS TIME

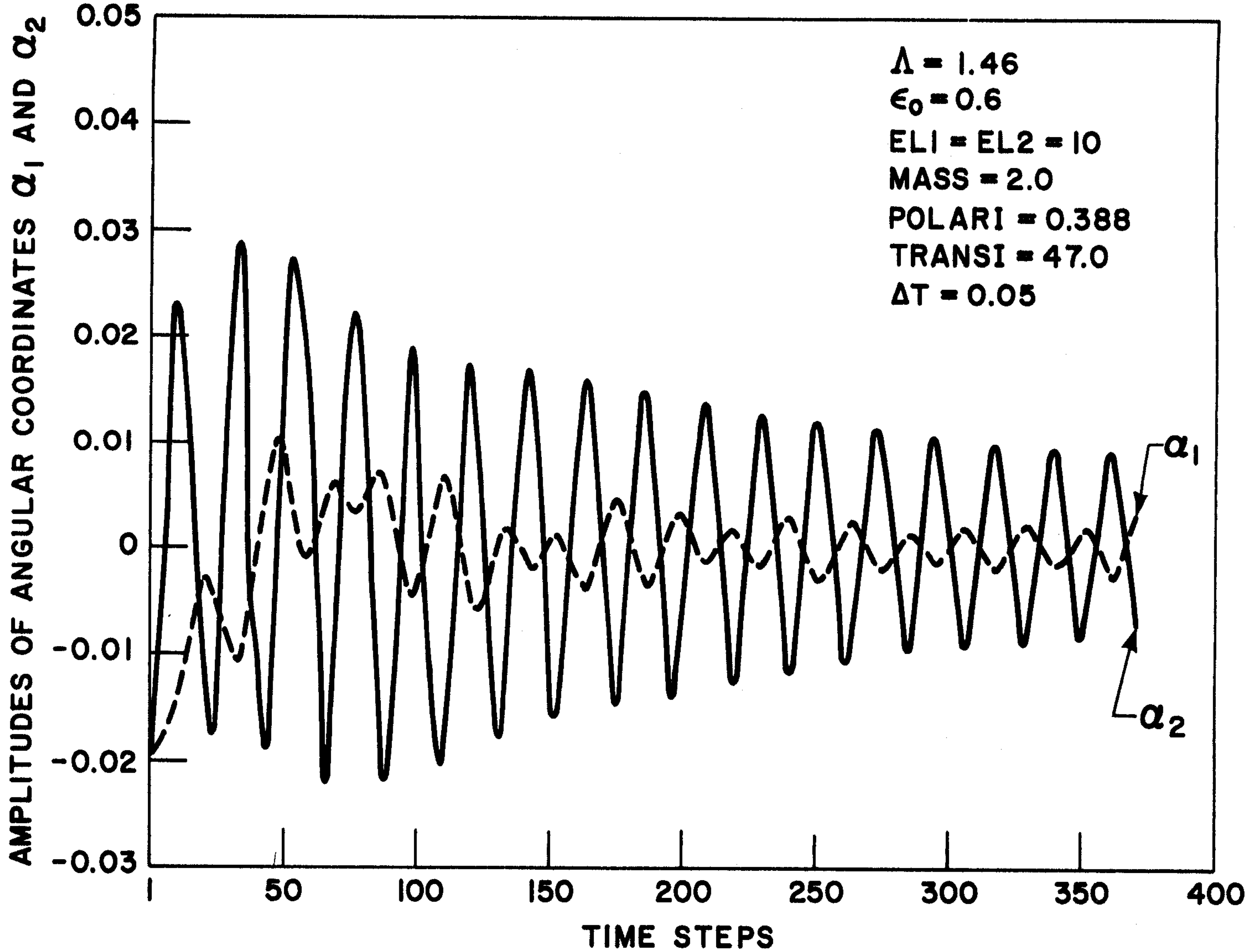

FIG. 2-13. CONICAL MOTION OF TWO BEARING SYSTEM SHAFT ANGULAR COORDINATES VS TIME

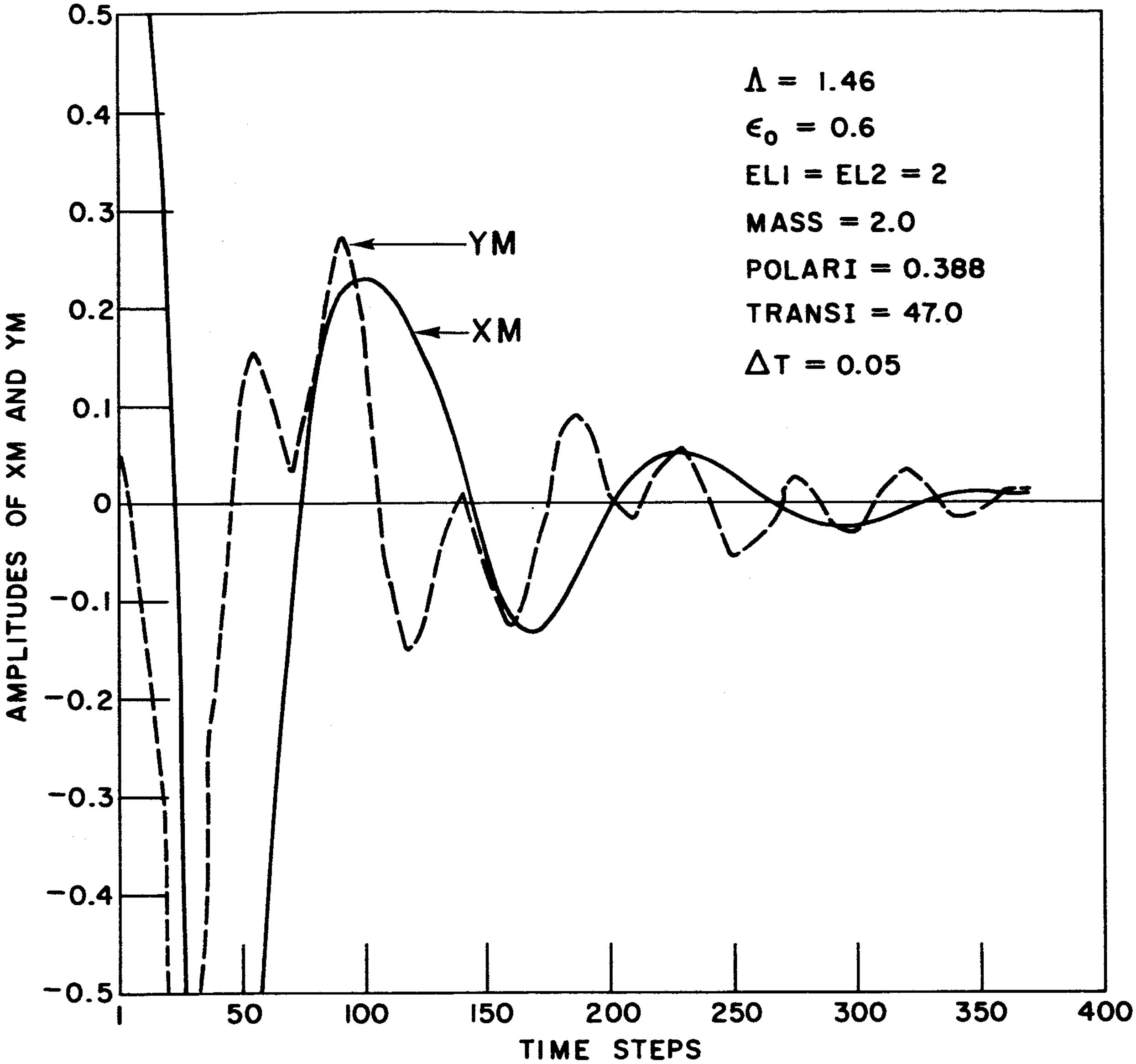

FIG. 2-14. TRANSLATIONAL MOTION OF TWO BEARING SYSTEM SHAFT MASS CENTER COORDINATES VS TIME

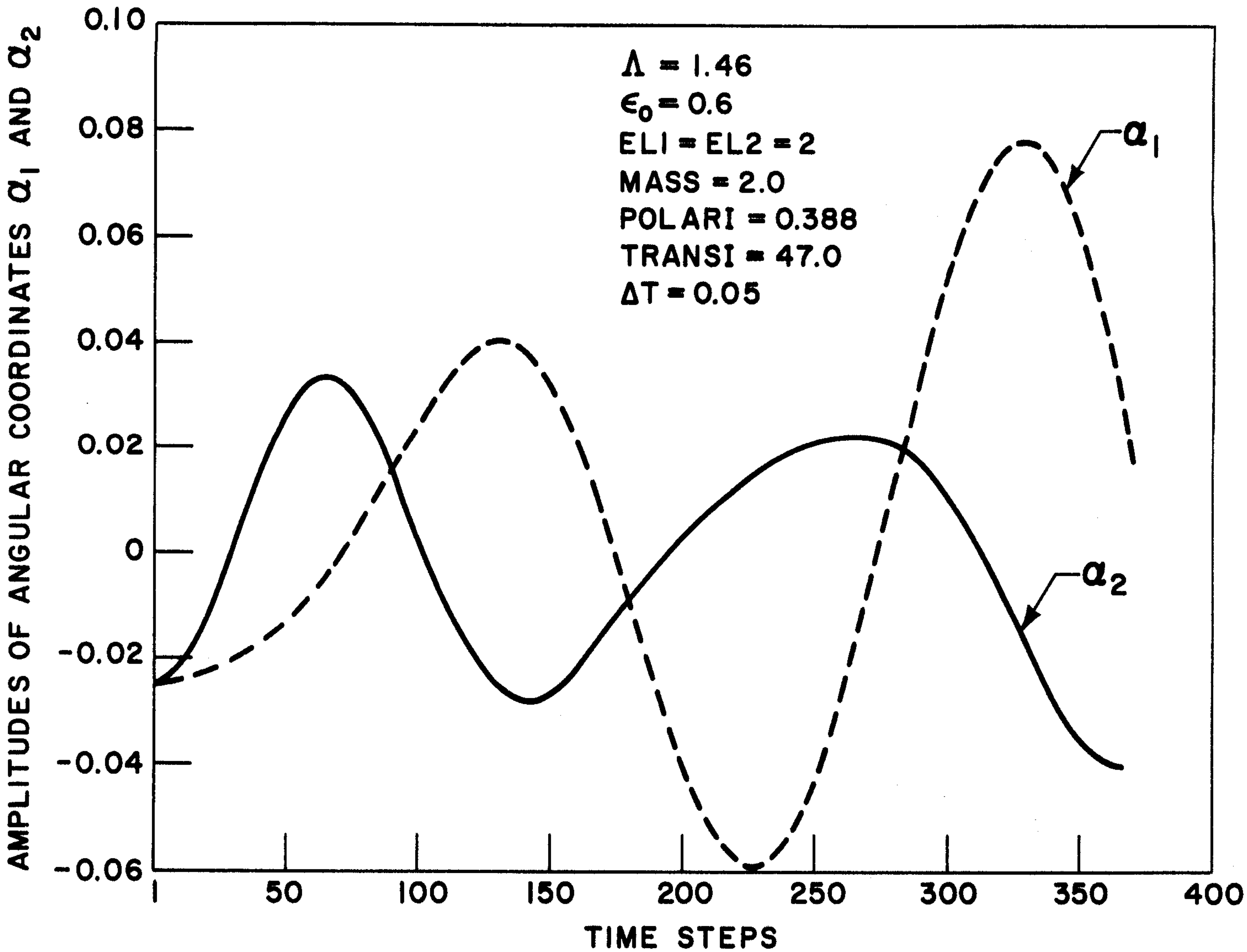

FIG. 2-15. CONICAL MOTION OF TWO BEARING SYSTEM SHAFT ANGULAR COORDINATES VS TIME

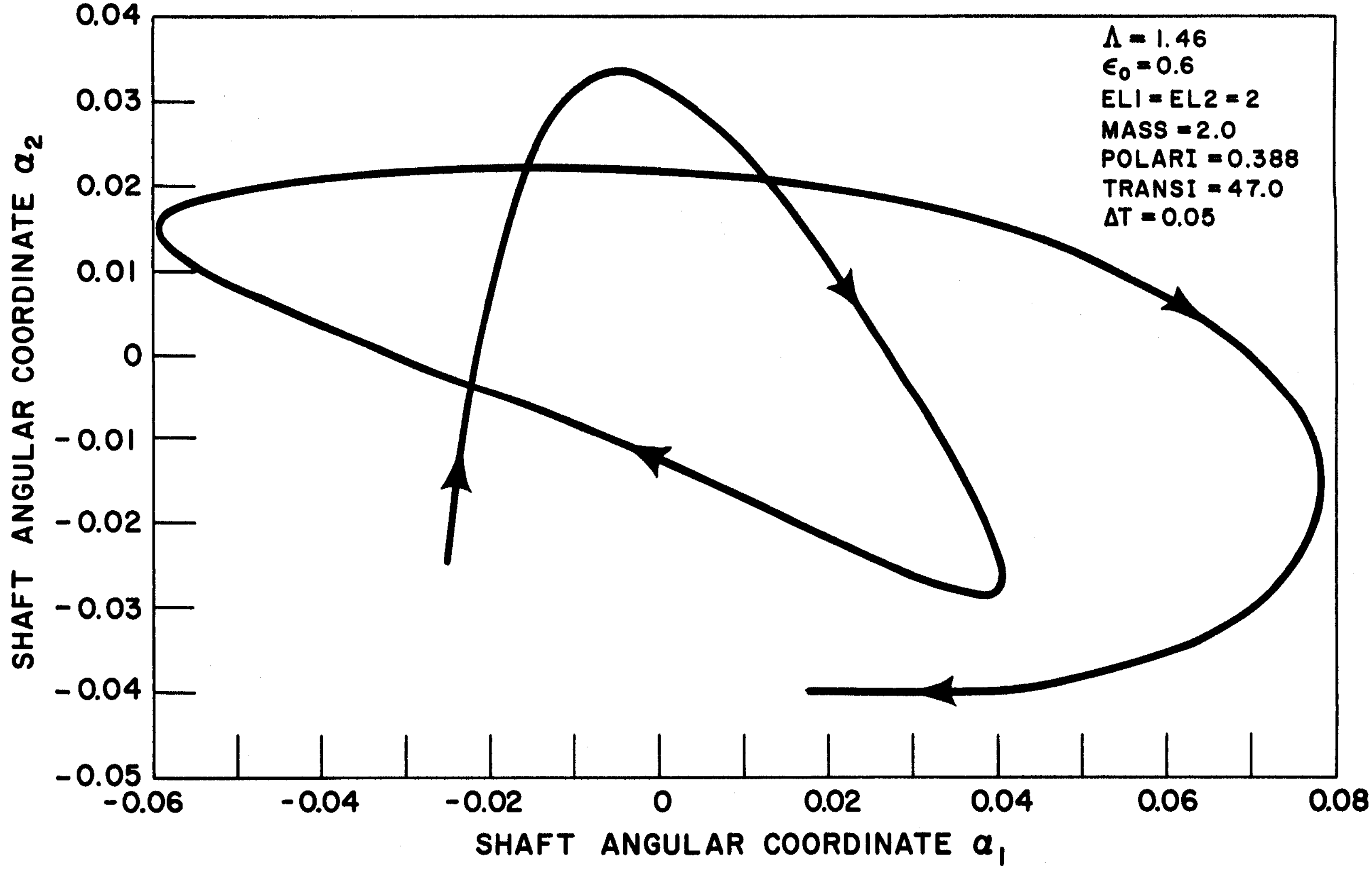

FIG. 2-16. CONICAL MOTION OF TWO BEARING SYSTEM ANGULAR COORDINATES OF SHAFT — α_1 VS. α_2

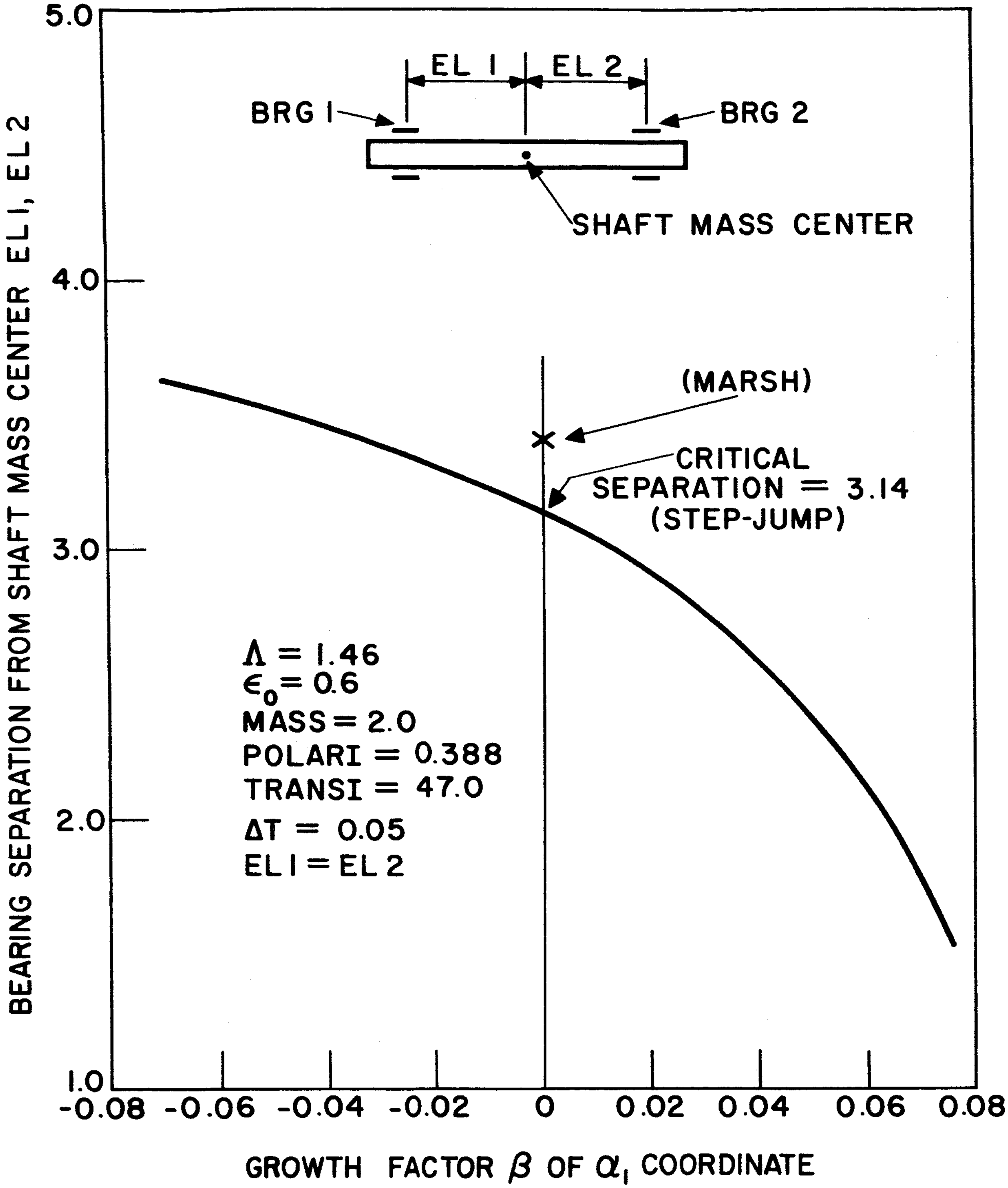

FIG. 2-17. TWO BEARING SYSTEM—CRITICAL BEARING SEPARATION

3. CONCLUSIONS

The utility of Duhamel's method has been demonstrated for numerical investigations of stability and dynamics of bearing systems. This new "step-response" method complements bearing orbit-programs by permitting rapid parametric examinations of stability-in-the small. In many instances, the method would appear to be preferable to methods employing complex variable in that (a) computed quantities have easily interpreted physical counterparts and (b) the complexity of the procedure augments only slightly with system size.

4. RECOMMENDATIONS

1. As a consequent of the implementation of the step response method, it appears desirable to standardize sections of the analysis, such as the manner by which the response functions are obtained, the determination of the LaGuerre coefficients, the optimization of the attenuation factor, etc. so that these sections can be used as library routines for other types of bearing configurations.

2. The method described in this report should be used to study the stability of other types of bearings. In particular, the externally pressurized thrust bearing and hybrid journal bearings. With the appropriate organization of the component parts of the analysis, the stability analysis of these more complex bearings can be done in a straight forward manner.

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APPENDIX A

Fortran Program Listings

The program used to produce the LaGuerre coefficient (ROSIE) for a long, plane journal bearing was compiled on an IBM 7094 in FORTRAN IV and uses "NAMELIST" for input. This program is an adaption from a more generalized program and, as a consequence, has certain input that are not applicable for the type of problem treated in this report

ROSIE contains the following routines:

MAIN
SUBROUTINE CUREAL (KAY)
SUBROUTINE SET 1
SUBROUTINE ALFA (KK)
SUBROUTINE FILM
SUBROUTINE FORCE (K)
SUBROUTING QQO
FUNCTION ALAGER (N, ALPHAT)

The MAIN program require the following input in NAMELIST form :

SXM = 0.0
SYM = eccentricity
$\left.\begin{array}{l}\text{SA1} = 0.0\\ \text{SA2} = 0.0\end{array}\right\}$ no shaft rotation
$\left.\begin{array}{l}\text{SB1} = 0.0\\ \text{SB2} = 0.0\end{array}\right\}$ no bearing rotation
M = no. of circumferential grid intervals
N = no. of axial grid intervals
PLAMDA = $\frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$
ROVL = R/L
DT = time step
TMAX = maximum allowable no. of time steps before termination
INF = FALSE
ORDER = order of the LaGuerre Poly. (an integer)
ALPHA = the attenuation constant "α"
NK = 3
NCASE = case no. (an integer)

SUBROUTINE CUREAL is specially written for each type of problem and contains a specification for the step-displacement from equilibrium; DELDEG is the size of the step taken.

For each degree of freedom, the LaGuerre coefficient are punched out in a loop which goes from K = 1, ORDER. The information on each card is

K, ORDER, XM, YM, AX(K), AY(K),

where XM and YM are the coordinates; AX and AY are the coefficient representing the forces in the X and Y directions. The FORMAT is

2I3, 2F7.3, 2E18.8, 26 X 2HØP

The Dynamics program which reads the punched card output listed above was compiled in FORTRAN IV on a UNIVAC 1107. The routines used are

ELRO (Main program)

SUBROUTINE LAGUER

FUNCTION ALAGER (N, ALPHAT)

The input consist of

1. READ:NDEG, NORDER, KSTEP, ALPHA

   FORMAT 3I6, F10.0

where

NDEG = no. of degrees of freedom (2)

NORDER = order of LaGuerre polynomial

KSTEP = the interval at which the growth factors are to be printed out (10)

2. For each degree of freedom:

   READ: punched card output described above

3. READ: $(H(\infty)_{ij}$, i = 1,2) J = 1,2)

   FORMAT 4E15.8

   (this input must be punched from printed output of coefficient program)

4. READ:NT, NTMAX, DELTAT

   FORMAT 2I6, F10.0

where

NT = integration interval

NTMAX = maximum no. of time steps

DELTAT = the time step DT x NT

5. READ:KLUE, AMASS, EL1, EL2, TRANSI, POLARI, ASYMM

FORMAT I6, 6F10.0

where

KLUE = 1, go back to point 5 READ

= 2, go back to point 4 READ

= 3, Stop

AMASS = shaft mass (non-dimensional) $\frac{MC\ \Omega^2}{4\ p_a RL}$

EL1, EL2 = distance from shaft mass center to center line of brg, 1 and 2 divided by length of bearing

TRANSI, POLARI = shaft transverse and polar moment of inertia (non-dimensional

$$\left.\begin{matrix} I_T \\ I_p \end{matrix}\right\} \frac{C\Omega^2}{4\ p_a RL^3}$$

ASYMM = initial displacement of brg. 1 relative to brg. 2

The program listings follow.

```
      COMMON SXM,SYM,SA1,SA2,SR1,SR2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1       A2,B1,B2,H[63,17],S[63],C[63],HTHE[63,17],HETA[63,17],
     2       Q[63,17],PLAMDA,RROVLL,DT,QQ[63,17],TMAX,INF,EQ[63,17],
     3       P[63,17],FORCEX[51],FORCEY[51],TORKY[51],TORKY[51],ORDER,
     4       ALPHA,AX[20],AY[20],AXZ[20],AYZ[20],AINF[20],NK,KLUE,
     5       DFLDEG,KOUNT
      COMMON/FACTOR/FE,FFE,FFFE,FFFFF
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      NAMELIST/INPUT/SXM,SYM,SA1,SA2,SR1,SR2,M,N,     PLAMDA,ROVL,DT,
     1TMAX,INF,ORDER,ALPHA,NK,NCASE
      IN=5
      OUT=6
      KAY=1
      FAIL = .FALSE.
      PASS = .FALSE.
      KLUE=1
   10 READ[IN,INPUT]
      WRITE[OUT,1]
    1 FORMAT[1H1]
      WRITE[OUT,INPUT]
      RROVLL = ROVL**2
      CALL SET1
      CALL FILM
      FE = .5/DTHE
      FFE = 1./DTHE**2
      FFFE = .5/DETA
      FFFFF = 1./DETA**2
      DO 15 I=M1,M2
      Q[I,1] = H[I,1]
   15 Q[I,NN]= H[I,NN]
      IF[FAIL]GO TO 200
      IF[PASS] GO TO 25
      DO 20 I=M1,M2
      DO 20 J=2,N
   20 Q[I,J] = H[I,J]
      IK=1
   25 CONTINUE
      DO 30 J=2,N
      Q[1,J]=Q[M+1,J]
   30 Q[M+3,J]=Q[3,J]
      IF[INF] GO TO 35
      GO TO 34
   35 DO 36 I=M1,M2
      Q[I,1] = Q[I,2]
   36 Q[I,NN]= Q[I,N]
   34 CALL QQQ
      IF [FAIL] GO TO 200
      IF[T .LE. TMAX] GO TO 40
      GO TO 50
   40 T = T+DT
   42 FORMAT[7E15.7]
      IK=IK+1
      GO TO 25
   50 DO 60 I=M1,M2
      DO 60 J=1,NN
```

```
      KS=1
      CALL FORCE[KS]
      WRITE[OUT,16] T
      WRITE[OUT,42][[P[I,J],J=1,17],I=1,39]
      WRITE[OUT,61]FORCEX[1],FORCEY[1],TORKX[1],TORKY[1]
   61 FORMAT[/19H EQUILIBRIUM FORCES // 5X 4F18.8]
   70 CALL  SET1
      CALL CUREAL[KAY]
   72 GO TO [71,10,150],KLUE
   71 IF[PASS] GO TO 90
      DO 80 I=M1,M2
      DO 80 J=1,NN
   80 Q[I,J]=EQ[I,J]
   90 CALL FILM
      DO 92 I=M1,M2
      Q[I,1]=H[I,1]
   92 Q[I,NN]=H[I,NN]
   91 DO 100 ITER=1,50
      DO  95 J=2,N
      Q[1,J]=Q[M+1,J]
   95 Q[M+3,J]=Q[3,J]
      IF[INF] GO TO 97
      GO TO 98
   97 DO 96 I=M1,M2
      Q[I,1]=Q[I,2]
   96 Q[I,NN]=Q[I,N]
   98 CALL QCQ
      IF [FAIL] GO TO 200
      CALL FORCE[ITER]
      CALL ALFA[ITER]
  100 T=T+DT
      WRITE[OUT,105]
  105 FORMAT[1H1 3X 1HT 12X 6HFORCEX 12X 6HFORCEY 13X 5HTORKX 13X
     1 5HTORKY ]
      WRITE[OUT,16] T
   16 FORMAT [F15.8]
      DO 110 L=1,50
      LL= 50*[KOUNT-1]+L
  110 WRITE[OUT,111] LL,FORCEX[L],FORCEY[L],TORKX[L],TORKY[L]
  111 FORMAT[1X I4,4F18.8]
      WRITE[OUT,120]
  120 FORMAT[1H1 50X 20HLAGUER COEFFICIENTS // 5X 1HT 16X 2HAX 16X 2HAY
     115X 3HAXZ 15X 3HAYZ 14X 4HAINF]
      DO 130 L=1,ORDER
  130 WRITE[OUT,131]LL,AX[L],AY[L],AXZ[L],AYZ[L],AINF[L]
  131 FORMAT[2X I4,5F18.8]
      KOUNT = KOUNT + 1
      IF[T .LE. TMAX] GO TO 91
      KAY=KAY+1
      XX= ALPHA/DELDEG
      DO 160 K=1,ORDER
      AX[K]=[AY[K]-AINF[K]*FORCEX[50]]* XX
      AY[K]=[AY[K]-AINF[K]*FORCEY[50]]* XX
      WRITE[OUT,161] K,ORDER,XM,YM,AX[K],AY[K]
  161 FORMAT[2I3,2F7.3,2F18.8,26X 2H$P ]
      AXZ[K]=[AXZ[K]-AINF[K]*TORKX[50]]*XX
```

```
      WRITE(OUT,120)
      DO 170 L=1,ORDER
  170 WRITE(OUT,131)LL,AX(L),AY(L),AXZ(L),AYZ(L),AINF(L)
      IF(KAY .LE. NK) GO TO 70
      WRITE(OUT,140) KAY
  140 FORMAT(1X4HKAY= I4,   6H   GOOF )
  200 WRITE(OUT,201)NCASE
  201 FORMAT(48X21HCLEAR. ZERO ,CASE NO. I5 )
  150 STOP
      END
$IBFTC KUREAL   LIST,SDD
      SUBROUTINE CUREAL(KAY)
      COMMON SXM,SYM,SA1,SA2,SB1,SB2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1       A2,B1,B2,H(63,17),S(63),C(63),HTHE(63,17),HETA(63,17),
     2       Q(63,17),PLAMDA,RBOVLL,DT,QO(63,17),TMAX,INF,EQ(63,17),
     3       P(63,17),FORCEX(51),FORCEY(51),TORKX(51),TORKY(51),ORDER,
     4       ALPHA,AX(20),AY(20),AXZ(20),AYZ(20),AINF(20),NK,KLUE,
     5       DELDEG,KOUNT
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      GO TO (1,2,3,4,5,6),KAY
    1 KLUE = 1
      YM=.7
      DELDEG=.1
      TMAX=4.7124
      GO TO 100
    2 KLUE = 1
      XM=.1
      DELDEG=.1
      GO TO 100
    3 KLUE = 3
      GO TO 100
    4 CONTINUE
    5 CONTINUE
    6 CONTINUE
  100 RETURN
      END
$IBFTC SSET1
      SUBROUTINE SET1
      COMMON SXM,SYM,SA1,SA2,SB1,SB2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1       A2,B1,B2,H(63,17),S(63),C(63),HTHE(63,17),HETA(63,17),
     2       Q(63,17),PLAMDA,RBOVLL,DT,QO(63,17),TMAX,INF,EQ(63,17),
     3       P(63,17),FORCEX(51),FORCEY(51),TORKX(51),TORKY(51),ORDER,
     4       ALPHA,AX(20),AY(20),AXZ(20),AYZ(20),AINF(20),NK,KLUE,
     5       DELDEG,KOUNT
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      XM = SXM
      YM = SYM
      A1 = SA1
      A2 = SA2
      B1 = SB1
      B2 = SB2
      T = 0.
      KOUNT = 1
      DO 10 K=1,ORDER
```

```
      AY[K]=0.0
      AXZ[K]=0.0
      AYZ[K]=0.0
   10 AINF[K]=0.0
      RETURN
      END
$IBFTC AALFA    LIST,SDD
      SUBROUTINE ALFA[KK]
      COMMON SXM,SYM,SA1,SA2,SB1,SB2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1          A2,B1,B2,H[63,17],S[63],C[63],HTHE[63,17],HETA[63,17],
     2          Q[63,17],PLAMDA,RROVLL,DT,QQ[63,17],TMAX,INF,EQ[63,17],
     3          P[63,17],FORCEX[51],FORCEY[51],TORKX[51],TORKY[51],ORDER,
     4          ALPHA,AX[20],AY[20],AXZ[20],AYZ[20],AINF[20],NK,KLUE,
     5          DELDEG,KOUNT
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      ALPHAT = ALPHA*T
      DO 10 K=1,ORDER
      POLYN = ALAGER[K-1,ALPHAT]
      AX[K] = AX[K] + DT*POLYN*FORCEX[KK]
      AY[K] = AY[K] + DT*POLYN*FORCEY[KK]
      AXZ[K] = AXZ[K]+DT*POLYN*TORKX[KK]
      AYZ[K] = AYZ[K]+DT*POLYN*TORKY[KK]
   10 AINF[K] = AINF[K]+DT*POLYN
      RETURN
      END
$IBFTC FFILM    LIST,SDD
      SUBROUTINE FILM
      COMMON SXM,SYM,SA1,SA2,SB1,SB2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1          A2,B1,B2,H[63,17],S[63],C[63],HTHE[63,17],HETA[63,17],
     2          Q[63,17],PLAMDA,RROVLL,DT,QQ[63,17],TMAX,INF,EQ[63,17],
     3          P[63,17],FORCEX[51],FORCEY[51],TORKX[51],TORKY[51],ORDER,
     4          ALPHA,AX[20],AY[20],AXZ[20],AYZ[20],AINF[20],NK,KLUE,
     5          DELDEG,KOUNT
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      PIE = 3.14159265
      RAD = PIE/180.0
      DTHE = 360.0*RAD/FLOAT[M]
      DETA = 1.0/FLOAT[N]
      M1=2
      M2=M+2
      NN=N+1
      DO 10 J=1,NN
      Z = -.5 +FLOAT[J-1]*DETA
      XPRIM = XM + [A1-B1]*Z
      YPRIM = YM + [A2-B2]*Z
      DO 10 I=M1,M2
      ARG = DTHE*FLOAT[I-M1]
      S[I]=SIN[ARG]
      C[I]=COS[ARG]
      H[I,J] = 1.0 + XPRIM*S[I] + YPRIM*C[I]
      IF[H[I,J] .LE. 0.0] GO TO 20
      HTHE[I,J] = XPRIM*C[I]- YPRIM*S[I]
   10 HETA[I,J] = [A1-B1]*S[I]+ [A2-B2]*C[I]
   30 RETURN
```

```
      RETURN
      END
$IBFTC FFORCE  LIST,SDD
      SUBROUTINE FORCE[K]
      DIMENSION SAV1[5],SAV2[5],SAV3[5],SAV4[5]
      COMMON SYM,SYM,SA1,SA2,SB1,SB2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1       A2,B1,B2,H[63,17],S[63],C[63],HTHE[63,17],HETA[63,17],
     2       Q[63,17],PLAMDA,RROVLL,DT,QQ[63,17],TMAX,INF,EQ[63,17],
     3       P[63,17],FORCEX[51],FORCEY[51],TORKX[51],TORKY[51],ORDER,
     4       ALPHA,AX[20],AY[20],AXZ[20],AYZ[20],AINF[20],NK,KLUE,
     5       DELDEG,KOUNT
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      DO 20 J=1,NN
      SX=0.0
      SY=0.0
      SXZ=0.0
      SYZ=0.0
      Z = -.5 + FLOAT[J-1]*DETA
      MN=M+1
      DO 10 I=M1,MN
      P[I,J] = Q[I,J]/H[I,J]
      SX = SX + P[I,J]*S[I]*DTHE
      SY = SY + P[I,J]*C[I]*DTHE
      SXZ= Z*SX
   10 SYZ= Z*SY
      IF[J.EQ.1] GO TO 30
      DZ = DETA
      GO TO 40
   30 DZ = 0.0
   40 FORCEX[K] = CLCINT[1,DZ,SX,SAV1]
      FORCEY[K] = CLCINT[1,DZ,SY,SAV2]
      TORKX[K]= CLCINT[1,DZ,SXZ,SAV3]
      TORKY[K]= CLCINT[1,DZ,SYZ,SAV4]
   20 CONTINUE
      RETURN
      END
$IBFTC QZA     LIST,SDD
      SUBROUTINE QQQ
      COMMON SYM,SYM,SA1,SA2,SB1,SB2,T,M,N,M1,M2,NN,DTHE,DETA,XM,YM,A1,
     1       A2,B1,B2,H[63,17],S[63],C[63],HTHE[63,17],HETA[63,17],
     2       Q[63,17],PLAMDA,RROVLL,DT,QQ[63,17],TMAX,INF,EQ[63,17],
     3       P[63,17],FORCEX[51],FORCEY[51],TORKX[51],TORKY[51],ORDER,
     4       ALPHA,AX[20],AY[20],AXZ[20],AYZ[20],AINF[20],NK,KLUE,
     5       DELDEG,KOUNT
      COMMON/FACTOR/FE,FFE,FFFF,FFFFF
      LOGICAL FAIL,PASS,INF
      INTEGER OUT,ORDER
      DO 10 I=M1,M2
      DO 10 J=2,N
      QT=[Q[I+1,J]-Q[I-1,J]]*FE
      QTT = [Q[I+1,J]+Q[I-1,J]-2.*Q[I,J]]*FFE
      QZ = [Q[I,J+1]-Q[I,J-1]]*FFFF
      QZZ = [Q[I,J+1] + Q[I,J-1]-2.*Q[I,J]]*FFFFF
      DQ = - QT + [Q[I,J]*[-Q[I,J]*[1.0-H[I,J]]+H[I,J]*QTT-QT*HTHE[I,J]
     1    +RROVLL*[QZZ*H[I,J]-QZ*HETA[I,J]]]+H[I,J]*[QT**2+RROVLL*
```

```
    2       Q7**2))/PLAMDA
      QQ(I,J) = Q(I,J)+DQ*DT
      IF(QQ(I,J) .GT. 100.) GO TO 25
   10 CONTINUE
      DO 20 I = M1,M2
      DO 20 J = 2,N
   20 Q(I,J) = QQ(I,J)
   21 RETURN
   25 WRITE(OUT,26)
   26 FORMAT( 9H BLOW UP  )
      FAIL = .TRUE.
      RETURN
      END
$IBFTC LAG      LIST,SDD
      FUNCTION ALAGER(N,ALPHAT)
      S=1.0
      NN=N+1
      DO 10 K=1,NN
      S=-S*ALPHAT*FLOAT(N-K+1)/FLOAT(K*K)
   10 ALAGER=ALAGER+S
      RETURN
```

```
FLT FLRO.1.660406, 41048
      COMMON SX(2),SDX(2),X(4,1000),DEX(4,1000),DDX(2),H(2,2,100),
     1    HDWT4(2,2),HDWT5(2,2),HDWT6(2,2),HDWT7(2,2),
     1            DELTAT,A(2,2,10),HINF(2,2),AX(10),AY(10),NT,ALPHA,DT,
     2       NDEG,NORDER,HDOT(2,2,100),HDWT1(2,2),HDWT2(2,2),HDWT3(2,2)
     3,SUM(4),XM(1000),YM(1000),ALPHA1(1000),ALPHA2(1000)
     1,BETA(4)
      COMMON MK1,MK2,MK3,MK4,MK5,MK6,MK7
      READ(5,10)NDEG,NORDER,KSTEP,ALPHA
   10 FORMAT(3I6,F10.0)
      DO 20 J = 1,NDEG
      DO 20 L = 1,10
      READ(5,11) KQ,OQ,XQ,YQ,AX(KQ),AY(KQ)
   11 FORMAT(2I3,2F7.3,2F18.8)
      A(J,1,L) =AX(L)
   20 A(J,2,L) =AY(L)
      READ(5,15) ((HINF(J,I),I=1,2),J=1,2)
   15 FORMAT(4F15.8)
      WRITE(6,23) ((HINF(J,I),I=1,2),J=1,2)
   23 FORMAT(24H HINF 1,1 1,2 2,1 2,2 = 4F18.8 )
      WRITE(6,19)
   19 FORMAT(//6H ORDER6X 6HA(1,1), 12X 6HA(1,2) , 12X 6HA(2,1) , 12X
     16HA(2,2) /  )
      DO 21 N=1,NORDER
   21 WRITE(6,22) N,A(1,1,N),A(1,2,N),A(2,1,N),A(2,2,N)
   22 FORMAT(2X I2, 4F18.8)
  400 READ(5,410) NT,NTMAX,DELTAT
  410 FORMAT(2I6,F10.0)
      FTMAX = NTMAX
      FNT = NT
      NNTT=NT+5
      DT = DELTAT/FNT
   30 READ(5,13)KLUE,AMASS,FL1,FL2,TRANSI,POLARI,ASYMM
   13 FORMAT(I6,6F10.0)
      FL=FL1+FL2
      WRITE(6,9)NDEG,NT,DT,NORDER,NTMAX,ALPHA,KLUE,AMASS,POLARI,TRANSI,
     1           FL1,FL2,ASYMM
    9 FORMAT(26H NO. OF DEG. OF FREEDOM = I2//23H INTEGRATION INTERVAL =
     1I3,16H WITH TIME STEP F15.8 // 23H LAGUERRE POLY ORDER = I2,
     212H MAX TIME = I4,   10H  ALPHA = F15.8,9H  KLUE = I1//8H MASS =
     3F15.8,5X 9HPOLARI = F15.8,5X 9HTRANSI = F15.8//
     4 6HFL1 = F15.8, 5X 6HFL2 = F15.8, 5X 8HASYMM =  F15.8//)
      READ(5,12) (SX(K),SDX(K),K=1,NDEG)
   12 FORMAT(4F12.0)
      DO 50 I=1,NDEG
      NTT = NT+1
      DO 60 L=1,NTT
      X(I,L) = SX(I)
      X(I+2,L)=SX(I)+ASYMM
      DEX(I+2,L)=0.0
   60 DEX(I,L) = 0.0
      DEX(I+2,1)=SDX(I)
   50 DEX(I,1) = SDX(I)
      DXM=FL2*DEX(1,1)/FL+FL1*DEX(3,1)/FL
      DYM = FL2*DEX(2,1)/FL + FL1*DEX(4,1)/FL
      DALPH1=(DEX(1,1)-DEX(3,1))/FL
      DALPH2=(DEX(2,1)-DEX(4,1))/FL
```

```
      DO 51 L=1,NTT
      YM[L]=FL2*X[1,1]/FL+FL1*X[3,1]/EL
      YM[L] = FL2*X[2,1]/EL+FL1*X[4,1]/EL
      ALPHA1[L]=[X[1,1]-X[3,1]]/FL
   51 ALPHA2[L]=[X[2,1]-X[4,1]]/FL
      T= 0.0
      N1=1
      N2=50
      L=NT+1
      KINT=NT/40
      K1=KINT
      K2=5*KINT
      K3=12*KINT
      K4=20*KINT
      K5=28*KINT
      K6=35*KINT
      K7=39*KINT
      MK1 = K1+1
      MK2 = K2+1
      MK3 = K3+1
      MK4=K4+1
      MK5=K5+1
      MK6=K6+1
      MK7=K7+1
      CALL LAGUER
      DDELT=DELTAT*DT
      WRITE[6,1000]
 1000 FORMAT[//4H   L8X 7HHD[1,1], 12X 7HHD[1,2], 12X 7HHD[2,1], 12X
     1       7HHD[2,2] /   ]
      DO 1001 K=1,NT
 1001 WRITE[6,1002] K,HDOT[1,1,K],HDOT[1,2,K],HDOT[2,1,K],HDOT[2,2,K]
 1002 FORMAT[2X I2, 4F18.8]
      DO 80 I=1,NDEG
      DO 80 J=1,NDEG
      HDWT1[I,J] = HDOT[I,J,MK1]*DDELT*.06222951
      HDWT2[I,J]= HDOT[I,J,MK2]*DDELT * .13971184
       HDWT3[I,J] = HDOT[I,J,MK3] * DDELT*.19945984
      HDWT4[I,J]=HDOT[I,J,MK4]*DDELT*.19719764
      HDWT5[I,J]=HDOT[I,J,MK5]*DDELT*.19945983
      HDWT6[I,J]=HDOT[I,J,MK6]*DDELT*.13971184
      HDWT7[I,J]=HDOT[I,J,MK7]*DDELT*.06222950
      H[I,J,1] = [H[I,J,1]+HINF[I,J]]*DT
   80 CONTINUE
   95 LK1 = L-K1
      LK2 = L-K2
      LK3 = L-K3
      LK4=L-K4
      LK5=L-K5
      LK6=L-K6
      LK7=L-K7
      DO 210 I=1,NDEG
      SUM[I]=0.0
      SUM[I+2]=0.0
      DO 150 J=1,NDEG
      SUM[I]=SUM[I]+X[J,LK1]*HDWT1[J,I]+
     1X[J,LK2]*HDWT2[J,I]+
     1X[J,LK3]*HDWT3[J,I]+
```

```
   1X[J,LK4]*HDWT4[J,I]+
   1X[J,LK5]*HDWT5[J,I]+
   1X[J,LK6]*HDWT6[J,I]+
   1X[J,LK7]*HDWT7[J,I]+
   1X[J,L]*H[J,I,1]
    SUM[I+2]=SUM[I+2]+X[J+2,LK1]*HDWT1[J,I]+
   1X[J+2,LK2]*HDWT2[J,I]+
   1X[J+2,LK3]*HDWT3[J,I]+
   1X[J+2,LK4]*HDWT4[J,I]+
   1X[J+2,LK5]*HDWT5[J,I]+
   1X[J+2,LK6]*HDWT6[J,I]+
   1X[J+2,LK7]*HDWT7[J,I]+
   1X[J+2,L]*H[J,I,1]
150 CONTINUE
210 CONTINUE
    DXMNU=DXM+[SUM[1]+SUM[3]]/AMASS
    XM[L+1]=XM[L]+.5*[DXMNU+DXM]*DT
    DYMNU=DYM+[SUM[2]+SUM[4]]/AMASS
    YM[L+1]=YM[L]+.5*[DYMNU+DYM]*DT
    DANU1=DALPH1 + [POLAR]*2.0*DALPH2*DT+EL1*SUM[1]-FL2*SUM[3]]/TRANSI
    ALPHA1[L+1]=ALPHA1[L]+DT*[DANU1+DALPH1]*.5
    DANU2=DALPH2+[-POLAR]*2.0*DALPH1*DT+FL1*SUM[2]-EL2*SUM[4]]/TRANSI
    ALPHA2[L+1]=ALPHA2[L]+DT*[DANU2+DALPH2]*.5
    DXM=DXMNU
    DYM=DYMNU
    DALPH1=DANU1
    DALPH2=DANU2
    L = L+1
    X[1,L]=XM[L]+FL1*ALPHA1[L]
    X[2,L]=YM[L]+FL1*ALPHA2[L]
    X[3,L]=XM[L]-FL2*ALPHA1[L]
    X[4,L]=YM[L]-FL2*ALPHA2[L]
    T = T + DT
    IF[L/50*50-L .EQ.0 .OR. L.GE.NTMAX] GO TO 320
    GO TO 95
320 WRITE[6,311]
311 FORMAT[1H1 15X 4HSTEP 12X 2HXM 18X 2HYM 15X 6HALPHA1 14X 6HALPHA2]
    DO 350 K = N1,N2
    IF[[K/KSTEP*KSTEP-K.EQ.0].AND.[K.GE.NNTT.AND.K.LE.NTMAX]]GO TO 700
    GO TO 705
700 Q1=[YM[K-1]*XM[K-3]-YM[K-2]**2]/[XM[K]*XM[K-2]-XM[K-1]**2]
    Q2=[YM[K-1]*YM[K-3]-YM[K-2]**2]/[YM[K]*YM[K-2]-YM[K-1]**2]
    Q3=[ALPHA1[K-1]*ALPHA1[K-3]-ALPHA1[K-2]**2]/
   1[ALPHA1[K]*ALPHA1[K-2]-ALPHA1[K-1]**2]
    Q4=[ALPHA2[K-1]*ALPHA2[K-3]-ALPHA2[K-2]**2]/
   1[ALPHA2[K]*ALPHA2[K-2]-ALPHA2[K-1]**2]
    IF[Q1.GT.0.0] BETAXM=-ALOG[Q1]/[2.*DT]
    IF[Q2.GT.0.0]BETAYM=-ALOG[Q2]/[2.*DT]
    IF[Q3 .GT. 0.0] BETA1=-ALOG[Q3]/[2.*DT]
    IF[Q4 .GT. 0.0] BETA2=-ALOG[Q4]/[2.*DT]
    Y0X= XM[K-3]*EXP[BETAXM*[  3.*DT]]
    Y1X= XM[K-2]*EXP[BETAXM*[  2.*DT]]
    Y2X= XM[K-1]*EXP[BETAXM*[  DT]]
    Y0Y= YM[K-3]*EXP[BETAYM*[  3.*DT]]
    Y1Y= YM[K-2]*EXP[BETAYM*[  2.*DT]]
    Y2Y= YM[K-1]*EXP[BETAYM*[  DT]]
```

```
      Y1A1  = ALPHA1(K-2)*EXP(BETA1*(  2.*DT))
      Y2A1  = ALPHA1(K-1)*EXP(BETA1*(  DT))
      Y0A2  = ALPHA2(K-3)*EXP(BETA2*(  3.*DT))
      Y1A2  = ALPHA2(K-2)*EXP(BETA2*(  2.*DT))
      Y2A2  = ALPHA2(K-1)*EXP(BETA2*(  DT))
      ARG1  = (Y0X+Y2X)/(2.*Y1X)
      ARG2  = (Y0Y+Y2Y)/(2.*Y1Y)
      ARG3  = (Y0A1  +  Y2A1)/(2.*Y1A1)
      ARG4  = (Y0A2  +  Y2A2)/(2.*Y1A2)
      IF(ARG1 .GE. 1. .OR. ARG2 .GE. 1.) GO TO 712
      IF(ARG3 .GE. 1. .OR. ARG4 .GE. 1.) GO TO 712
      GAMMAX=ACOS(ARG1)/DT
      GAMMAY=ACOS(ARG2)/DT
      GAMA1=ACOS(ARG3)/DT
      GAMA2=ACOS(ARG4)/DT
      GO TO 710
  712 GAMMAX = 0.
      GAMMAY = 0.
      GAMA1  = 0.
      GAMA2  = 0.
  710 WRITE(6,711)K,BETAXM,BETAYM,BETA1,BETA2,GAMMAX,GAMMAY,GAMA1,GAMA2
  711 FORMAT(5X12HTIME STEP = I3/10X 18HGROWTH FACTOR X = E15.8/10X
     1 18HGROWTH FACTOR Y =  E15.8/10X 23HGROWTH FACTOR ALPHA1 = E15.8/
     210X 23HGROWTH FACTOR ALPHA2 = E15.8/10X  9HFREQ X = E15.8,
     310X 9HFREQ Y = E15.8/10X10HFREQ A1 = E15.8,10X10HFREQ A2 = E15.8)
  705 WRITE(6,312)K,XM(K),YM(K),ALPHA1(K),ALPHA2(K)
  350 CONTINUE
  312 FORMAT(15X I4,4E20.8)
      N1 = N1 + 50
      N2 = N2 + 50
      IF( L.GT. NTMAX) GO TO 500
      GO TO 95
  500 GO TO (30,400,600),KLUF
  600 STOP
      END
  ELT DLAGUE,1,660303, 36661
      SUBROUTINE LAGUER
C
      COMMON SX(2),SDX(2),X(4,1000),DEX(4,1000),DDX(2),H(2,2,100),
     1    HDWT4(2,2),HDWT5(2,2),HDWT6(2,2),HDWT7(2,2),
     1            DELTAT,A(2,2,10),HINF(2,2),AX(10),AY(10),NT,ALPHA,DT,
     2        NDEG,NORDER,HDOT(2,2,100),HDWT1(2,2),HDWT2(2,2),HDWT3(2,2)
     3,SUM(4),XM(1000),YM(1000),ALPHA1(1000),ALPHA2(1000)
     1,BETA(4)
      COMMON MK1,MK2,MK3,MK4,MK5,MK6,MK7
      KORDER = NORDER-1
      DO 100 LL=1,8
      GO TO (1,2,3,4,5,6,7,8),LL
    1 L=1
      GO TO 9
    2 L=MK1
      GO TO 9
    3 L=MK2
      GO TO 9
    4 L=MK3
      GO TO 9
```

```
    5 L=MK4
      GO TO 9
    6 L=MK5
      GO TO 9
    7 L=MK6
      GO TO 9
    8 L=MK7
    9 TT=FLOAT[L-1]*DT
      ALPHAT = ALPHA*TT
      DO 80 I=1,NDEG
      DO 80 J=1,NDEG
      HH=0.0
      DO 50 K=1,NORDER
      POLYN = ALAGER[K-1,ALPHAT]
   50 HH = HH+POLYN*A[I,J,K]
      H[I,J,L] = HH*EXP[-ALPHAT]
   80 CONTINUE
      ALPHAT = ALPHAT+.00001
      DO 200 I=1,NDEG
      DO 200 J=1,NDEG
      HD =0.0
      DO 250 K=1,KORDER
  250 HD=HD+A[I,J,K+1]*FLOAT[K]*[ALAGER[K,ALPHAT]-ALAGER[K-1,ALPHAT]]
     1        /ALPHAT
      HDOT[I,J,L] = ALPHA*[HD*EXP[-ALPHAT]-H[I,J,L]]
  200 CONTINUE
  100 CONTINUE
      WRITE[6,110]
  110 FORMAT[//4H   L8X 6HH[1,1], 12X 6HH[1,2], 12X 6HH[2,1], 12X
     1         6HH[2,2] /  ]
      DO 120 L=1,NT
  120 WRITE[6,121] L,H[1,1,L],H[1,2,L],H[2,1,L],H[2,2,L]
  121 FORMAT[2X I2, 4F18.8]
      RETURN
      END
 ELT ALAGER,1,651230, 34045
      FUNCTION ALAGER[N,ALPHAT]
      S=1.0
      ALAGER=1.0
      NN=N+1
      DO 10 K=1,NN
      S=-S*ALPHAT*FLOAT[N-K+1]/FLOAT[K*K]
   10 ALAGER=ALAGER+S
      RETURN
      END
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13. ABSTRACT

The Stability of a gas bearing is treated by a new procedure in which the bearing film is characterized by its responses to step-jump displacements. Duhamel's theorem is invoked to generalize these step responses in a system of dynamical equation. Stability is determined by calculation of a "growth factor" for each degree of freedom.

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