LONGITUDINAL VIBRATION OF RING STIFFENED CYLINDRICAL SHELLS CONTAINING LIQUIDS

by

Daniel D. Kana
H. Norman Abramson

Technical Report No. 7
Contract No. NAS8-11045
CONTROL NO. TP 3-85175 & S-1 (IF)
SwRI Project No. 02-1391

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama

15 June 1966

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ABSTRACT

Axisymmetric responses of longitudinally excited ring stiffened cylinders are studied experimentally in the laboratory, and are compared with the predictions of the Bleich theory for the case of tanks having rigid flat bottoms; tanks with flexible bottoms of both flat and elliptical geometry are also studied experimentally. Further, the effects of supporting the entire system on a flexible spring mount are investigated. It is found for such cases that the natural modes become infinite in number, and the lowest mode can be considerably different from that obtained when the liquid is considered to be a rigid mass. In general, it is found that the predictions of the Bleich theory compare well with the experiments, for those cases where it is applicable.

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INTRODUCTION

The response of thin cylindrical shells to longitudinal vibration is a very important aspect of the dynamics of large, liquid fueled space vehicle boosters. It is recognized that these responses can be very complicated, depending on the type of loading that is applied to the container. For space vehicle applications, the cylindrical shell walls of the tank can be subjected to both longitudinal and radial loads; however, for this study, we will consider only radial loading on the tank walls, such as that which results from the radial pressure loading of the liquid column in a longitudinally excited tank. Under these conditions, the shell can respond in both axisymmetric and nonaxisymmetric modes, depending on the input parameters as well as on the amount of ring stiffening used on the thin walled tank.

Axisymmetric responses of longitudinally vibrated cylinders have been studied analytically by Bleich [1]* and Reissner [2], employing a circumferential membrane theory approximation to determine the elastic tank effects on pressure waves in the liquid. Approximate provisions for elastic bottoms were included, but liquid free surface effects were neglected in these studies. Later, Shmakov [3] analyzed free axisymmetric vibrations of a partially liquid filled cylinder having a flat membrane bottom; Vlasov shell theory was used, and free surface effects were included. At

*Numbers in [ ] refer to List of References.

about the same time, Kana and Dodge [4] studied axisymmetric pressure waves in liquids contained in elastic cylinders as part of an investigation of the behavior of small bubbles in longitudinally excited elastic tanks.

Recently, several investigators [5, 6, 7] have studied several aspects of the axisymmetric modes of a partially liquid filled shell with various elastic and rigid bottoms, using numerical techniques to solve the coupled liquid-shell equations. In general, it was found that, in the ranges of practical parameters for current vehicle systems, axial bending effects in the shell walls of tanks vibrating in axisymmetric modes, as well as liquid surface and liquid compressibility effects, should be negligible. Further, Palmer and Asher [7] found experimentally that the axisymmetric modes were very difficult to determine in a representative tank system as they were largely obscured by the strong presence of nonaxisymmetric responses. Thus, circumferential bending effects can be very important in an unstiffened tank. Finally, the effects of ellipsoidal elastic bottoms on the equivalent longitudinal spring constants of a propellant tank have been approximated by Pinson [8]; however, this study was related to longitudinal forces within the tank wall, and considered the entire liquid mass to be a lumped frozen solid. It will be shown in the present study that such approximations can be in considerable error, within certain ranges of parameters.

In view of the above mentioned findings, it would appear that for a representative model space vehicle tank, the circumferential membrane theory of Bleich [1] should give a good approximation to the symmetric modes in the partially filled, longitudinally vibrated tank having a rigid flat bottom, provided that ring stiffeners are employed to minimize circumferential bending effects. Thus, the basic purpose of this laboratory investigation has been to obtain experimental frequency data from a representative cylindrical shell containing liquid in order to correlate with predictions of the Bleich theory. The effects of ring stiffeners are included in the investigation, their presence being accounted for by means of an approximate overall correction factor applied to the tank wall elastic modulus. Further, liquid compressibility effects are determined by comparing theory and experiment in several small elastic tanks, covering a range of stiffnesses. Finally, the effects of supporting the liquid-tank system on a spring of finite stiffness were also studied as such information should be useful for spring-mass synthesis of actual space vehicle systems. An analytical expression for predicting the coupled modes of the liquid-elastic tank system on a support spring was derived from the Bleich theory, and the results are compared with those obtained from a more exact analysis by Eulitz and Glaser [9], as well as with experimental results.

Direct correlation of laboratory experimental data with the Bleich theory is given for the case of a tank having a rigid flat bottom. Although a study of this configuration might at first appear somewhat academic, it does serve as a valuable check on the basic formulation of the problem. Experiments were then conducted using tanks having elastic bottoms of other and more practical geometries, and the results compared to the above limiting case.

MODIFICATION OF BLEICH THEORY

Results from Bleich Theory

Bleich's analysis [1] of a longitudinally excited circular cylindrical elastic membrane shell having a rigid flat bottom, and containing an inviscid compressible liquid of depth H, leads to several expressions relating the significant parameters in the system.

It is readily shown that, for the case where the coordinate system is chosen as shown in Figure 1, the analysis results in a velocity potential of the form

$$\phi = \left[A_0 \sin\left(\bar{\mu}_0 \frac{x}{a}\right) I_0 \left(\mu_0 \frac{r}{a}\right) + \sum_{n=1}^{\infty} A_n \sinh\left(\bar{\mu}_n \frac{x}{a}\right) J_0 \left(\mu_n \frac{r}{a}\right) \right] \sin \omega t \tag{1}$$

where

$$\bar{\mu}_0^2 = \mu_0^2 + \frac{\omega^2 a^2}{c_0^2} \quad , \quad \bar{\mu}_n^2 = \mu_n^2 - \frac{\omega^2 a^2}{c_0^2}$$

and I_0 and J_0 are, respectively, the modified and unmodified Bessel functions of the first kind, a is the tank radius, and c_0 is the sonic velocity in the fluid. Further, μ_0 is the single real root (when it exists), and μ_n are the imaginary roots of the frequency equation

$$\frac{I_0(\mu)}{\mu I_1(\mu)} = \frac{h\rho_t}{a\rho_L}\left(\frac{\omega_s^2}{\omega^2} - 1\right) \tag{2}$$

Here, h is shell thickness, ρ_t is tank density, ρ_L is liquid density, ω is the excitation frequency, and ω_s is the membrane shell frequency, given by:

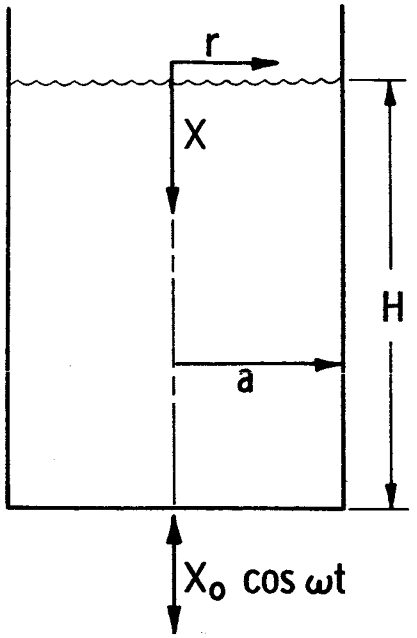

Figure 1. Coordinate system

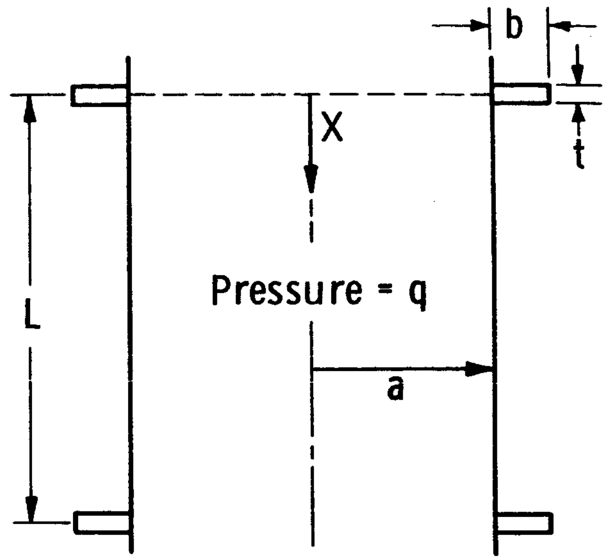

Figure 2. Details of ring stiffeners

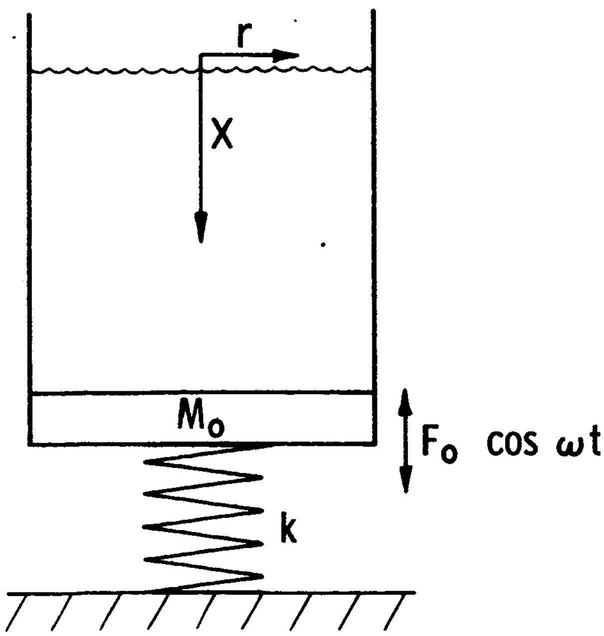

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Figure 3. Liquid filled tank mounted on elastic support

$$\omega_s^2 = \frac{E_t}{a^2 \rho_t} \tag{3}$$

where E_t is the tank modulus of elasticity. Bleich, and others, pointed out that a good approximation for the velocity potential can often be obtained by using only the first term in Eq. (1). Thus, by equating tank velocity and liquid velocity at the bottom of a tank harmonically excited with an amplitude $y = x_0 \cos \omega t$, one can evaluate the constant A_0 and obtain an approximate expression for the pressure in the tank as

$$q = 2 \frac{\rho_L a x_0 \omega^2 I_1(\mu_0) I_0\left(\mu_0 \frac{r}{a}\right) \sin\left(\bar{\mu}_0 \frac{x}{a}\right)}{\bar{\mu}_0 \mu_0 [I_0^2(\mu_0) - I_1^2(\mu_0)] \cos\left(\bar{\mu}_0 \frac{H}{a}\right)} \cos \omega t \tag{4}$$

The details of the derivation of the above expressions may be obtained from Bleich's paper [1].

For an incompressible fluid, so that $c_0 \rightarrow \infty$, we have $\bar{\mu}_0 = \mu_0$, $\bar{\mu}_n = \mu_n$, and the approximate pressure given by Eq. (4) can therefore be written as

$$\frac{q}{\rho_L x_0 \omega^2 H} = \left(\frac{a}{H}\right) \frac{2 I_1(\mu_0) I_0\left(\mu_0 \frac{r}{a}\right) \sin\left(\mu_0 \frac{x}{a}\right)}{\mu_0^2 [I_0^2(\mu_0) - I_1^2(\mu_0)] \cos\left(\mu_0 \frac{H}{a}\right)} \tag{4a}$$

If, in addition, the tank inertia is small, i.e., $\rho_t \sim 0$, then Eq. (2) can be written as

$$\frac{\omega^2 a^2}{E_t / \rho_L} = \frac{\gamma h}{a} \frac{\mu_0 I_1(\mu_0)}{I_0(\mu_0)} \tag{2a}$$

where γ is a stiffness factor to be derived later. Thus, for an incompressible fluid in a thin tank, the pressure response q at any input frequency ω can be calculated from Eqs. (2a) and (4a). Unfortunately, the factor μ_0 cannot readily be eliminated from these equations.

Approximate Ring Stiffener Factor

The results from Bleich's theory given in the preceding paragraphs are valid only for a shell of uniform wall thickness. Therefore, to apply the theory to a ring stiffened shell (Fig. 2), a correction factor will be derived for the tank modulus, so that the ring stiffened tank is approximated by a tank of uniform wall thickness having a slightly greater elastic modulus. This approximation can be derived by assuming that the effect of the stiffener rings on the tank is the same as that of a line load, P pounds per unit circumference. According to Timoshenko [10], the deflection due to the line load P in an unpressurized tank, if L is very large,* is

$$w_1 = \frac{Pe^{-\beta x}}{8\beta^3 D} (\sin \beta x + \cos \beta x)$$

where

$$D = \frac{E_t h^3}{12(1 - \nu^2)} \quad , \quad \beta = \left[\frac{3(1 - \nu^2)}{a^2 h^2}\right]^{1/4}$$

The tank wall deflection in an unstiffened tank due to an internal pressure q is

$$w_2 = \frac{a^2}{E_t h} q$$

*L is "very large" if $e^{-\beta L} < 0.01$.

The total deflection is $w = w_1 - w_2$ so that substituting in the expressions for w_1, w_2, D, and β we have

$$w = \frac{a^2}{E_t h} q - \frac{a^2 P \beta e^{-\beta x}}{2E_t h} (\sin \beta x + \cos \beta x) \tag{5}$$

The radial deflection of the stiffener rings due to a pressure $\frac{P}{t}$ is

$$w_r = \frac{a^2}{E_r b} \cdot \frac{P}{t} \tag{6}$$

This must equal w at x = 0; thus, using Eqs. (5) and (6)

$$\frac{a^2}{E_r b} \cdot \frac{P}{t} = \frac{a^2}{E_t h} q - \frac{a^2}{2E_t h} \beta P$$

and solving for P we obtain

$$P = q \left[\frac{1}{\frac{E_t h}{E_r bt} + \frac{\beta}{2}} \right] \tag{7}$$

We now consider the change in volume of the section of the tank between $x = 0$ and $x = \frac{L}{2}$. Thus

$$2\pi a \int_0^{\frac{L}{2}} w\, dx = 2\pi a \int_0^{\frac{L}{2}} \frac{a^2}{E_t h} q\, dx - 2\pi a \int_0^{\frac{L}{2}} \frac{Pe^{-\beta x}}{8\beta^3 D} (\sin \beta x + \cos \beta x)\, dx$$

$$= \frac{2\pi a^3 q \left(\frac{L}{2}\right)}{E_t h} - 2\pi a \int_0^{\frac{L}{2}} \frac{Pe^{-\beta x}}{8\beta^3 D} (\sin \beta x + \cos \beta x)\, dx$$

In the second integral, it can be assumed that $\frac{L}{2} \to \infty$ with the same accuracy as the previous assumption that L is "very large." Therefore,

$$\Delta\text{volume} = \frac{\pi a^3 q L}{E_t h} - \frac{\pi a P}{4\beta^4 D} = \frac{\pi a^3 q L}{E_t h} - \frac{\pi a^3 P}{E_t h} \qquad (8)$$

Without stiffener rings, but with a suitable E_{eff}, the change in volume of a pressurized tank is

$$\Delta\text{volume} = \frac{\pi a^3 q L}{E_{eff} h} \qquad (9)$$

The two changes in volume must be equal; hence, using Eqs. (8) and (9), we obtain

$$\frac{Lq}{E_{eff}} = \frac{Lq}{E_t} - \frac{P}{E_t}$$

from which, using Eq. (7), we obtain

$$E_{eff} = E_t \left[\frac{\frac{E_t h L}{E_r b t} + \frac{\beta L}{2}}{\frac{E_t h L}{E_r b t} + \frac{\beta L}{2} - 1} \right] = E_t \gamma$$

where γ is the ring stiffener correction factor.

Resonant Frequencies

Liquid in Elastic Tank. It is recognized that the coupled natural frequencies of the axisymmetric oscillations in the present system can be derived by reverting to the wave equation and assuming solutions in the form of normal modes of the system compatible with boundary conditions for free oscillations. However, the Bleich theory in its given form may also be readily used for this purpose, and, in particular, it can more readily be adapted to the problem of vibration of the tank on an elastic spring support, as will be shown in the next section.

Thus, in order to determine the resonant frequencies using the formulation just given, we note in Eq. (4) those values of $\bar{\mu}_0$ for which the pressure q becomes large, i.e.,

$$\bar{\mu}_{0n} = (2n - 1)\frac{\pi a}{2H} \qquad n = 1, 2, \ldots \tag{10}$$

The resonant frequencies for the longitudinally excited ring stiffened shell containing liquid can then be obtained from Eq. (2), rewritten as

$$\frac{\gamma E_t/\rho_t}{\omega_n^2 a^2} = 1 + \frac{\rho_L a}{\rho_t h}\,\frac{I_0(\mu_{0n})}{\mu_{0n} I_1(\mu_{0n})} \tag{11}$$

where γ is the ring correction factor derived previously, and

$$\mu_{0n}^2 = \bar{\mu}_{0n}^2 - \frac{\omega_n^2 a^2}{c_0^2} \tag{12}$$

Equations (11) and (12) form a highly transcendental frequency equation which can be solved either graphically or by some computer iteration technique. It can readily be shown that the mode shapes of the pressure wave assume the form of the normal modes of the system (i.e., those that result from a separation of variables in the wave equation) for the values of $\bar{\mu}_{0n}$ given in Eq. (10). Substituting these values of $\bar{\mu}_{0n}$ into Eq. (4), we obtain:

$$q_n = \left\{ \frac{2\rho_L a x_0 \omega^2 I_1(\mu_{0n})}{\mu_{0n}\bar{\mu}_{0n}[I_0^2(\mu_{0n}) - I_1^2(\mu_{0n})] \cos\left(\bar{\mu}_{0n}\frac{H}{a}\right)} \right\} I_0\left(\mu_{0n}\frac{r}{a}\right) \sin\left(\bar{\mu}_{0n}\frac{x}{a}\right) \cos \omega t \tag{13}$$

which, of course, represents a standing wave of infinite amplitude and has an odd number of quarter sine waves extending from the liquid surface to the tank bottom.

We now consider several simplifications of the frequency expressions, Eqs. (11) and (12), which will be appropriate for space vehicle applications. If the tank wall inertia is small, $\rho_t \sim 0$, and Eq. (11) can be written as

$$\frac{\omega_n^2 a^2}{E_t/\rho_L} = \frac{\gamma h}{a} \frac{\mu_{0n} I_1(\mu_{0n})}{I_0(\mu_{0n})} \tag{14}$$

and if the liquid is further considered incompressible, then $\bar{\mu}_{0n} = \mu_{0n}$, and Eqs. (10) and (14) can readily be used to calculate the frequencies ω_n directly.

One further limiting case will be considered, in order to determine the range of tank stiffnesses for which liquid compressibility becomes important. It is convenient to do this in a tank having a larger value of $\frac{H}{a}$ so that we will have $\bar{\mu}_{0n} \ll 1$. Then we can write

$$\frac{I_1(\mu_{0n})}{I_0(\mu_{0n})} \sim \frac{\mu_{0n}}{2}$$

and Eq. (14) becomes

$$\frac{\omega_n^2 a^2}{E_t/\rho_L} = \frac{\gamma h}{a} \frac{\mu_{0n}^2}{2}$$

Using Eqs. (10) and (12), this can be written as

$$\frac{\omega_n^2 a^2}{E_t/\rho_L} = \frac{\gamma h(2n - 1)^2 \pi^2}{8a} \left[1 + \frac{\gamma E_t h}{2ak}\right]^{-1} \left(\frac{a}{H}\right)^2 \tag{15}$$

where the liquid compressibility is

$$k = \rho_L c_0^2$$

System with Elastic Support. We now consider the partially liquid filled tank having a flat rigid bottom of mass M_0, and supported elastically, as shown in Figure 3. This is intended to simulate, for example, the interstage structure of a space vehicle. The coupled natural frequencies for such a system are given by

$$\omega_n^2 = \frac{k}{M_0 + \pi a^2 m_L} \tag{16}$$

where m_L is the apparent mass of the liquid (per unit area of the tank bottom) and is determined by the liquid pressure at that point. Here, we consider the apparent mass corresponding to oscillatory motion.

We assume the liquid to be incompressible, and the tank wall inertia is neglected so that Eqs. (2a) and (4a) will be utilized to determine the mass loading. Corresponding to the assumptions of Bleich, we can write

$$\frac{\partial v_x}{\partial t} = \frac{\partial}{\partial t}\left(- \frac{\partial \phi}{\partial x}\right) = - \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial t}\right) = - \frac{1}{\rho_L}\frac{\partial q}{\partial x}$$

since, from the linearized Bernoulli equation

$$q = \rho_L \frac{\partial \phi}{\partial t}$$

Substituting Eq. (4a) into the above expression, we obtain (at x = H)

$$\frac{\partial v_x}{\partial t} = \frac{\partial^2 y}{\partial t^2} = - \frac{2y_0\omega^2 I_1(\mu_0) I_0\left(\mu_0 \frac{r}{a}\right)}{\mu_0[I_0^2(\mu_0) - I_1^2(\mu_0)]} \cos \omega t$$

or

$$\frac{\partial^2 y}{\partial t^2} = - \frac{1}{\rho_L} \frac{\mu_0}{a} \cot \left(\mu_0 \frac{H}{a}\right) q$$

Thus,

$$q = - \rho_L \frac{a}{\mu_0} \tan \left(\mu_0 \frac{H}{a}\right) \frac{\partial^2 y}{\partial t^2}$$

and

$$m_L = \frac{\rho_L a}{\mu_0} \tan \left(\mu_0 \frac{H}{a}\right) \tag{17}$$

Substituting this expression into Eq. (16) and rearranging, we obtain

$$\frac{\omega_m^2 a^2}{E_t/\rho_L} = \frac{\omega_0^2 a^2}{E_t/\rho_L} \left[1 + \frac{\rho_L \pi a^3}{\mu_{0m} M_0} \tan \left(\mu_{0m} \frac{H}{a}\right)\right]^{-1} \tag{18}$$

where $\omega_0^2 = \frac{k}{M_0}$. This equation must be solved in conjunction with Eq. (14), written as

$$\frac{\omega_m^2 a^2}{E_t/\rho_L} = \frac{\gamma h}{a} \frac{\mu_{om} I_1(\mu_{om})}{I_0(\mu_{0m})} \tag{19}$$

where ω_m, m = 1, 2, 3, ... are the coupled natural modes of the combined system. These equations are again highly transcendental and must be solved either graphically or numerically on a computer.

COMPARISONS WITH EXPERIMENTAL DATA

Description of Experimental Apparatus

The experiments were performed by longitudinally exciting thin wall, ring stiffened cylindrical elastic tanks, containing water at various depths. The excitation was provided by an electrodynamic shaker, and the coupled symmetric modes of the system were determined by observing the pressure at various points in the liquid, rather than by monitoring the tank wall motion. This procedure was necessary since it was found that, for the coupled symmetric modes, pressures became large while, simultaneously, the symmetric wall motion was too small to measure relative to nonsymmetric wall motions that would also occur.

All of the test tanks employed in these experiments were 28.45 cm long and 22.9 cm in diameter, with a wall thickness of 0.127 mm. Both Mylar ($E = 7.8 \times 10^5$ psi) and brass ($E = 16 \times 10^6$ psi) were used for the model tank materials while Lucite plastic ($E = 8 \times 10^5$ psi) was used for the various patterns of ring stiffeners. The ring stiffeners were 1.59 mm thick by 4.77 mm wide and were spaced uniformly along the tank length. The main purpose in using Mylar as the model tank material was so that relatively flexible springs could then be used for testing the tanks in the elastic support condition. Even an approximate modeling of the stiffness factors corresponding to the interconnecting section of an actual vehicle would require the model springs to be quite stiff, and, of course, the stiffness factor of such model springs would be greatly influenced by the material of the model tank. Therefore, in order to work at relatively low frequencies, which would avoid resonances of the shaker and its support system, Mylar proved to be a suitable model material and was used for most of the tests. A photograph of the overall experimental setup is shown in Figure 4, in which a model Mylar tank is depicted mounted on one of the support springs.

Flat Rigid Bottom: Rigid Support

Experimental data for these conditions are shown in Figures 5 through 9, and are compared with calculated results from Bleich's theory. It should be pointed out that by "rigid support" we simply mean one in which no spring is used – the condition to which the original Bleich theory is directly applicable.

Pressure Response to Forced Excitation. Figure 5 shows a comparison of experimental data with theoretical calculations for the pressure response to forced vibrations at the center of the tank bottom, for three different liquid levels in a Mylar tank having eleven stiffener rings. The theoretical curves were obtained from a simultaneous solution of Eqs. (2a) and (4a). It can be seen that the results compare very well, up through the first two modes, for the two largest liquid depths. At the lowest liquid depth, approximately one-half of a tank radius, the comparison is not quite so good, particularly in the vicinity of the natural mode; this is not surprising, since Eqs. (2a) and (4a) represent only an approximation to the pressure because only the first term of the series Eq. (1) was retained. Furthermore, liquid

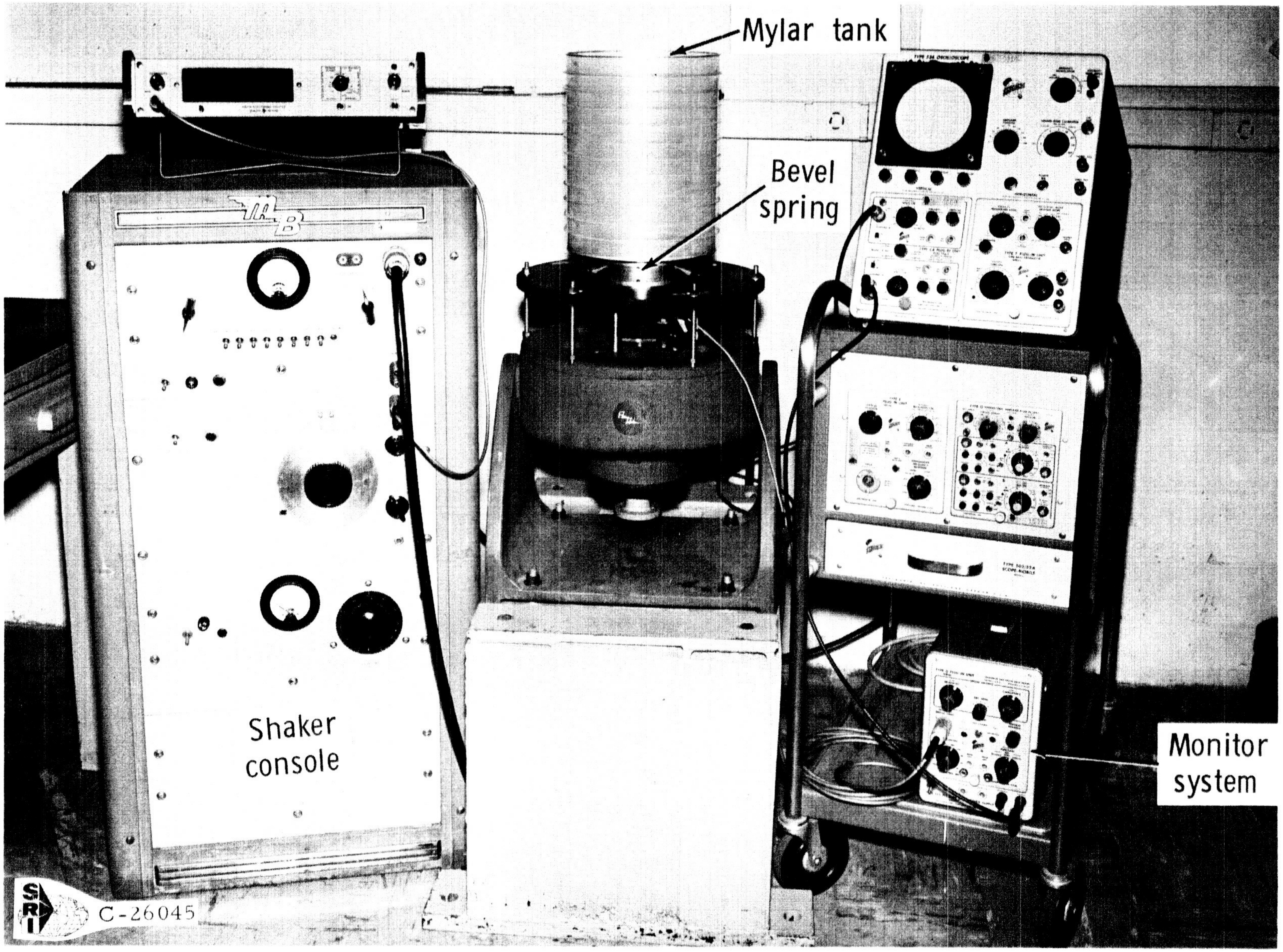

Figure 4. Photograph of experimental set-up

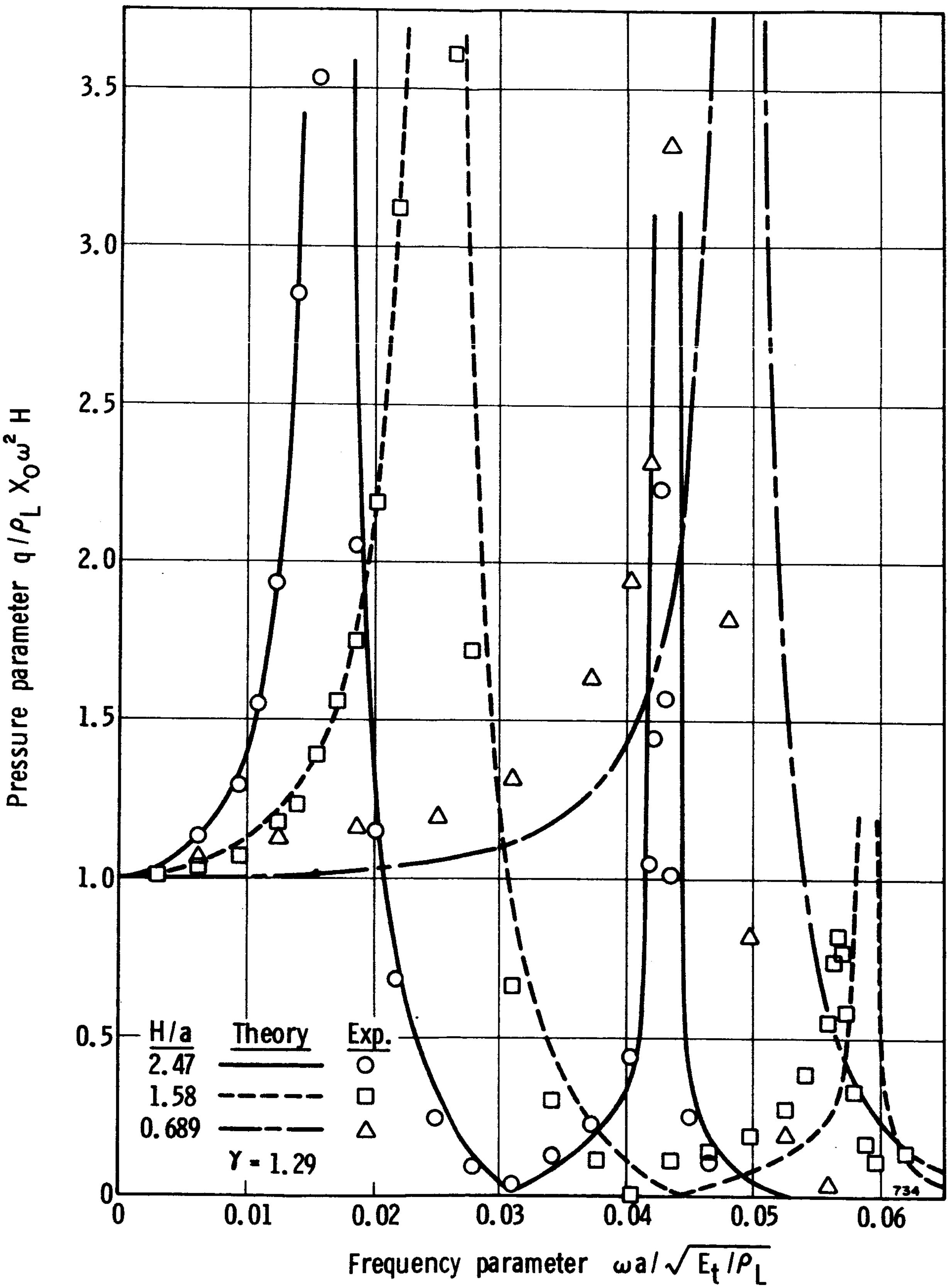

Figure 5. Pressure response to forced vibration for various liquid depths

surface response is neglected in Bleich's theory, but it is anticipated that such effects would become important at shallow depths.

Natural Frequency Variation with Depth. Figure 6a shows experimental data compared with the Bleich theory [Eqs. (10) and (14) with $\overline{\mu}_{0n} = \mu_{0n}$], in terms of the variation of the first mode frequency with liquid depth. As indicated, several uniformly spaced ring stiffener patterns were used on the Mylar tanks, and one pattern on the brass tank. A similar comparison for the second mode is shown in Figure 6b. It can be seen from these two figures that good agreement between the theory and experiments exist for either the eleven-ring Mylar or brass tank configurations, particularly at the greater liquid depths. It should be noted, however, that the data for the brass tank appear to correlate with the $\gamma = 1.0$ (no stiffener rings) curve, rather than the $\gamma = 1.08$ (eleven Lucite rings) curve. In effect, this indicates that the rings had essentially no stiffening effect at all on the brass tank. This result is expected since the tank material is much stiffer than that of the rings, and only minute displacements are experienced; in such a case, the rings cannot readily be bonded well enough to exert uniform pressure, especially for such small displacements.

The comparison for seven rings on the Mylar tanks is fair, while that for three rings is poor. In fact, it can be seen from Figure 6a that the experimental frequencies are slightly lower than those predicted for a tank having no stiffeners at all. The immediate and definite conclusion from the trends observed in these data is that the unstiffened, or even slightly

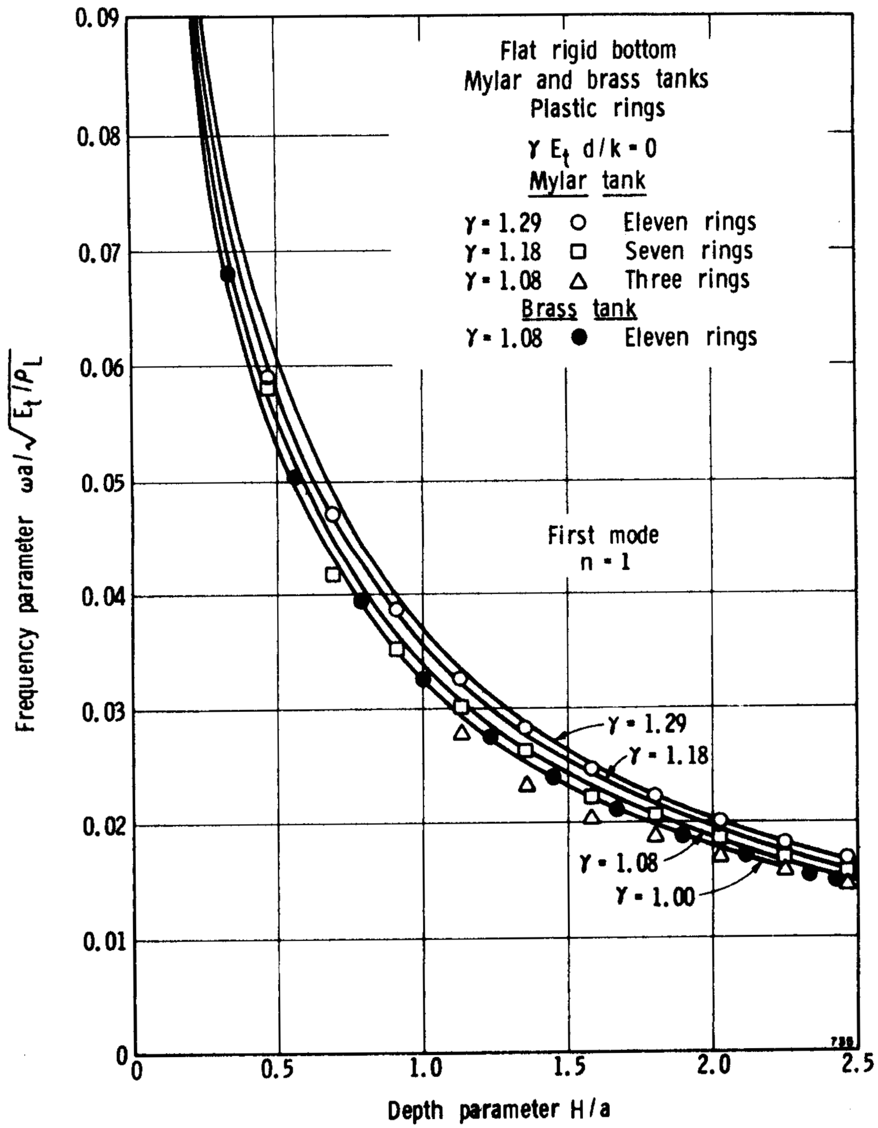

Figure 6a. Variation of natural frequency with liquid depth

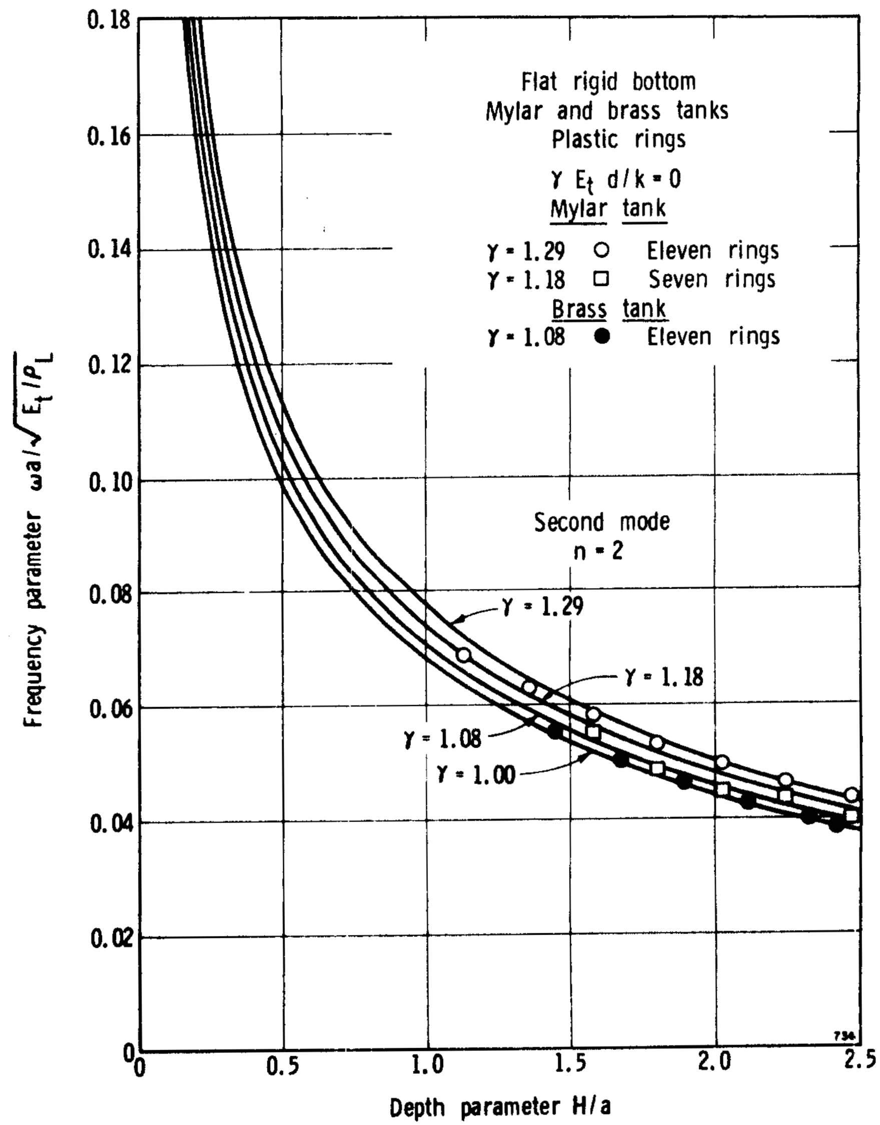

Figure 6b. Variation of natural frequency with liquid depth

stiffened, tank simply does not respond predominantly in axisymmetric modes; instead, it was found that such tanks more readily respond in non-axisymmetric wall modes.* Distortions in pressure resulting from the presence of these modes were also readily observed. Therefore, the presence of the nonaxisymmetric wall motions altered the internal pressure so that the experimental data do not agree with the completely axisymmetric behavior assumed in the theory. It appears that, for the model tanks employed in this program, the eleven-ring configuration is necessary in order that the Bleich theory yield a satisfactory prediction of frequencies.

The reciprocal of the frequency parameter for the first mode only in the brass and Mylar tanks is plotted against liquid depth and compared with experimental results in Figure 7. Theoretical curves are given for an incompressible fluid, Eqs. (10) and (14), as well as for a compressible fluid in a long tank, Eq. (15). It can be seen that the results corresponding to the parameters of the present tank configurations only begin to approach the long tank conditions when the tank is nearly full. Therefore, it appears that the incompressible fluid, short tank theory is the more adequate.

Pressure Distributions. Axial pressure distributions for the first and second axisymmetric modes, in the eleven-ring Mylar tank, are shown in Figure 8. Data were obtained for several liquid depths and are given in normalized form, appearing to be essentially sinusoidal. No appreciable

*This could be observed quite readily by means of a wall displacement transducer, and has been previously noted and reported [11].

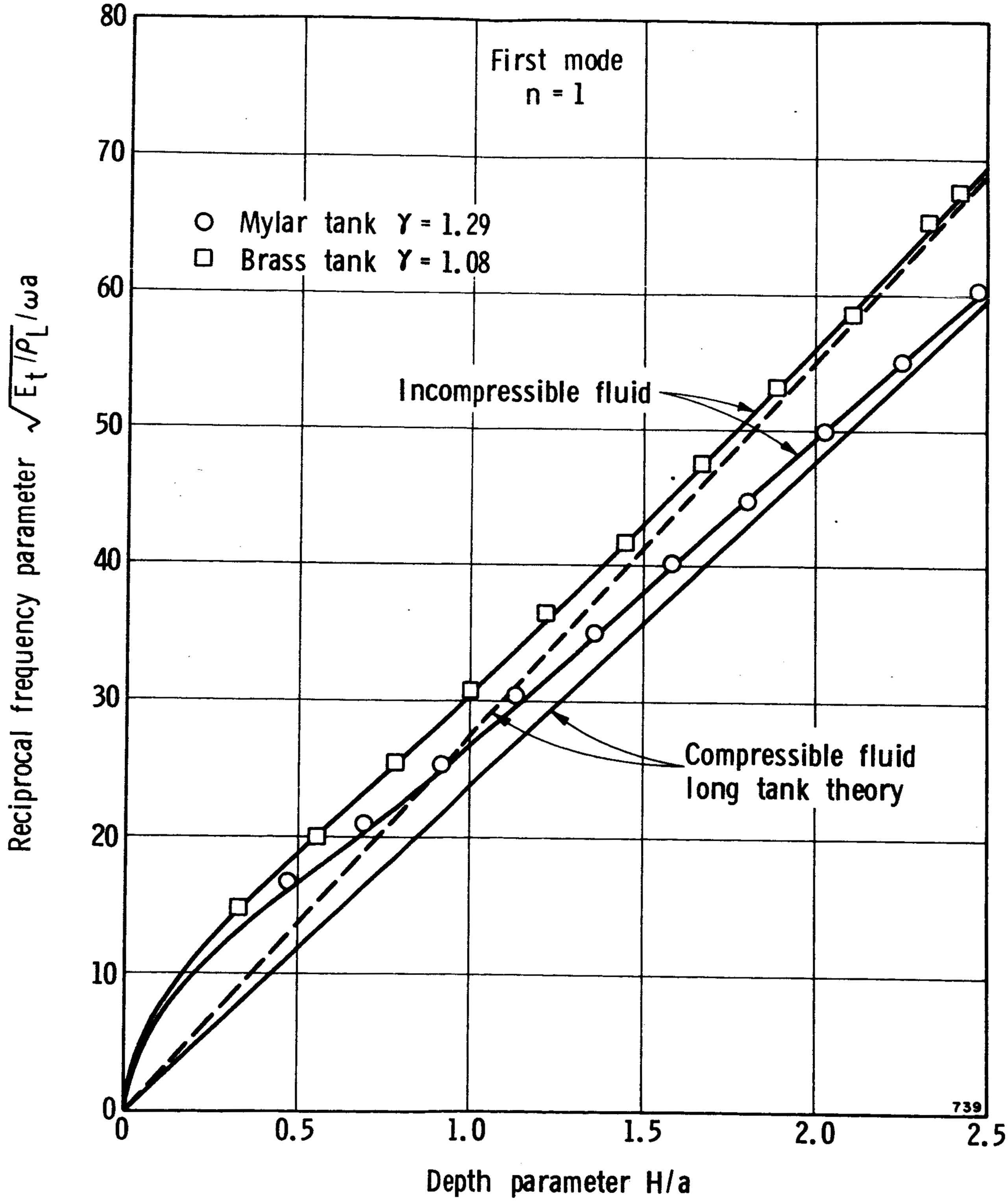

Figure 7. Variation of natural frequency with liquid depth for Mylar and brass tanks

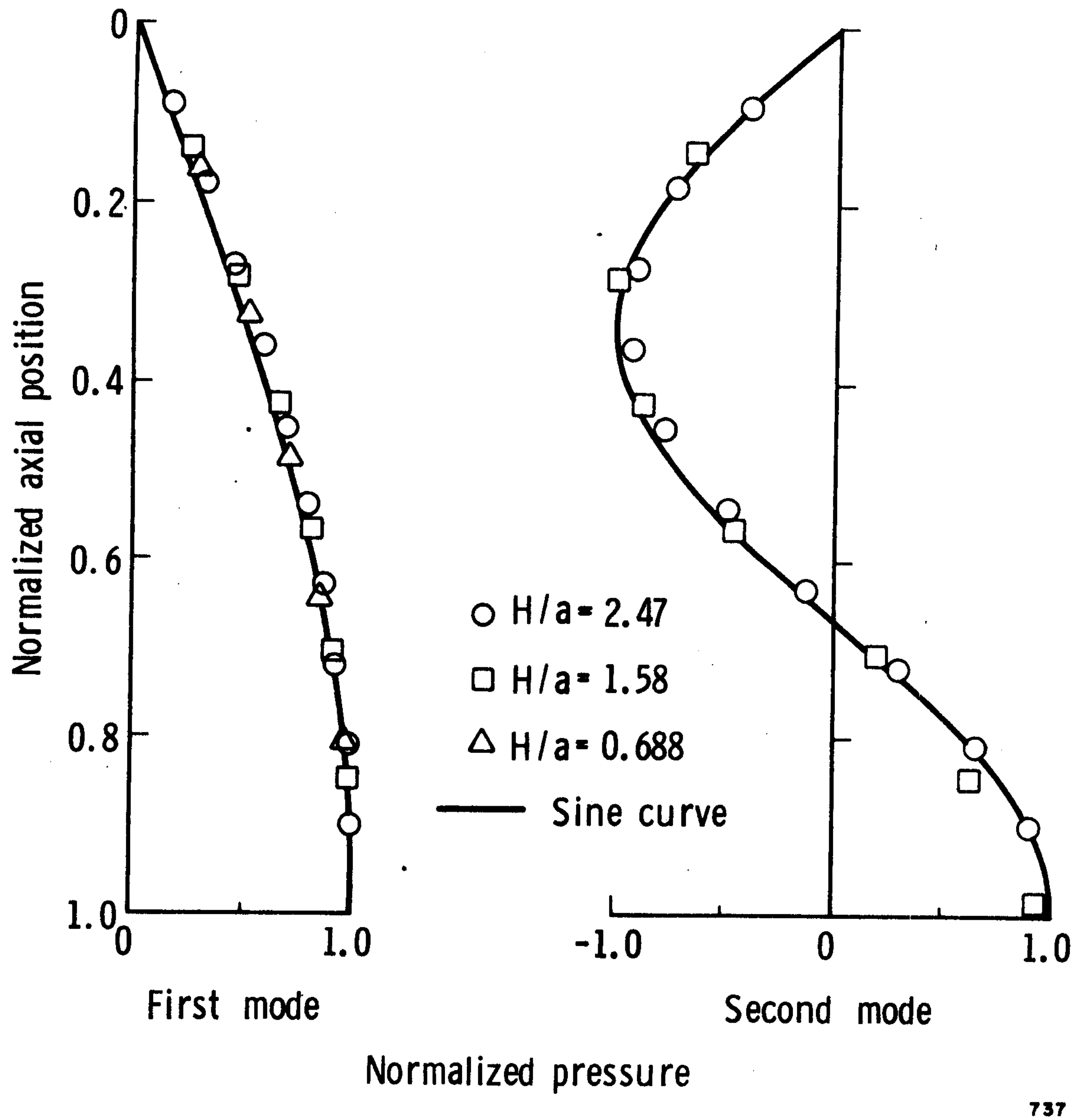

Figure 8. Normalized axial pressure distribution

radial variation of the pressure at a given axial position was present for this tank; however, such a variation in pressure did occur for the tanks with fewer rings, particularly near the walls. This again demonstrates the influence of nonaxisymmetric motions on pressures. Of course, it would be anticipated that more significant radial variations in pressure would be encountered for higher modes.

Flat Rigid Bottom: Elastic Support

The same Mylar tank (having a flat rigid bottom) employed in the experiments described above, for the eleven-ring configuration only, was modified so that it could be supported on a rather stiff spring mount, as shown in Figures 3 and 4. The mass M_0 represents that of the rigid bottom, spring, and shaker armature, in combination.

Experimental data for the coupled frequencies corresponding to the first two modes of the tank on two different springs, along with those given earlier for a rigid mount ($k = \infty$), are shown plotted against liquid depth in Figure 9. Theoretical results are given for the one-term Bleich analysis, Eqs. (10) and (14), and the modified one-term analysis, Eqs. (18) and (19), for the two different spring mounts. Also, for the two spring mounted conditions, theoretical results are also given from a more exact analysis of Eulitz and Glaser [9], based on what is essentially a ten-term approximation for the expansion in Eq. (1).

As might be expected, the coupled frequencies generally decrease for decreasing values of spring constant k. It may also be noted that the

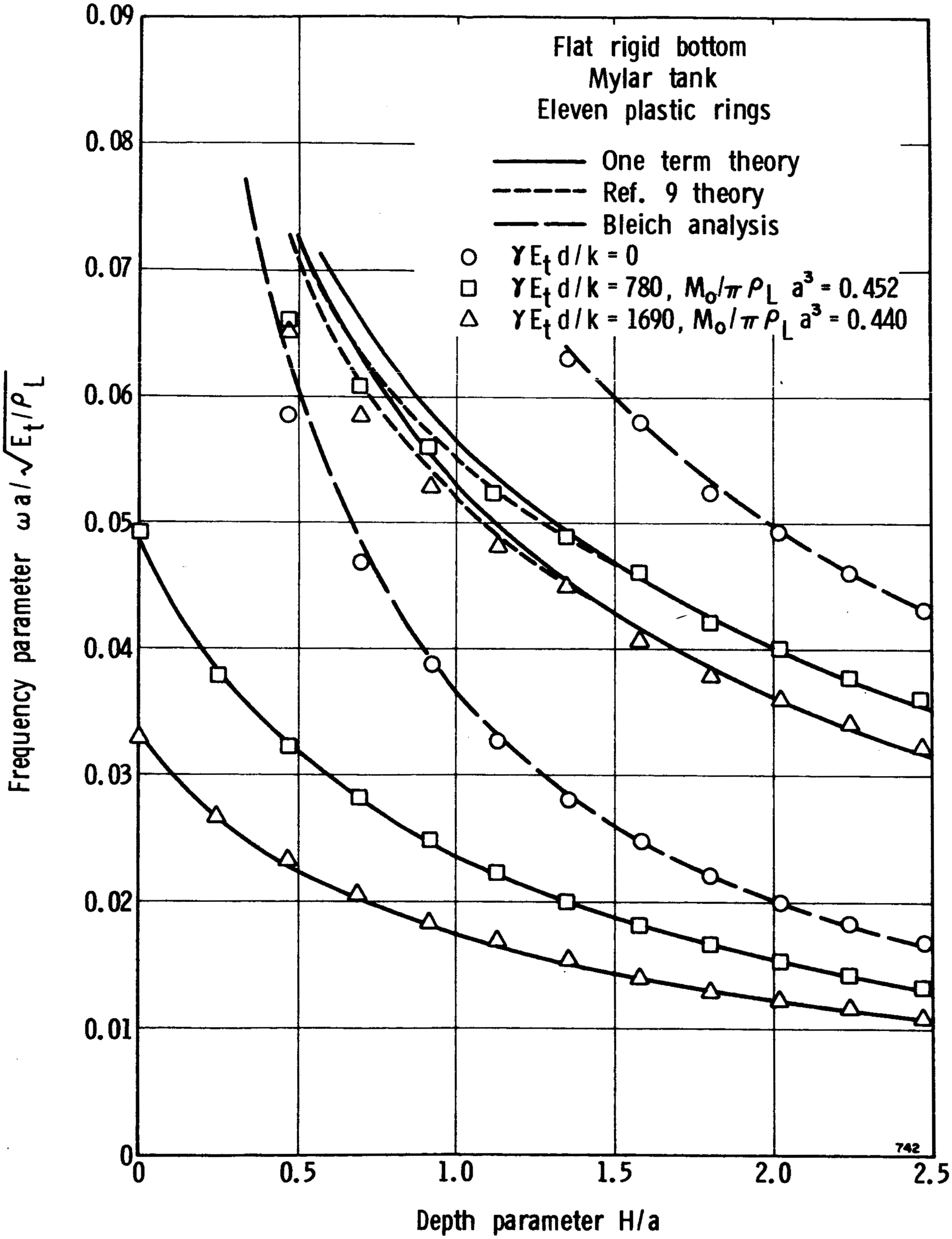

Figure 9. Variation of natural frequency with liquid depth for tanks with flat rigid bottom and elastic supports

frequencies decrease with liquid depth for the rather large values of k used. Further, it can be seen that the agreement between theory and experiment is excellent throughout the depth range for the first mode with each spring, while some deviation for the second mode may be noted at the shallow depths. This result appears to reflect the same behavior shown in the pressure response of Figure 5. Of course, for finite values of $\omega_0^2 = \frac{k}{M_0}$, the frequency parameter for all modes would converge to the same finite value at $\frac{H}{a} = 0$; for ω_0^2 infinite, the convergent value is infinity.

Finally, it can be seen that for the range of parameters considered in this study, the results based on the ten-term approximation [9] are virtually identical with those of the one-term approximation, except at lower depths in the second mode (it might be anticipated that the deviation would be even greater for higher modes). Therefore, the one-term approximation appears to give at least a good description of the system behavior in the lowest mode, throughout the depth range.

The above results for the two spring mounted cases are shown again in Figure 10; however, in this instance, they are compared with the case in which the liquid is considered to be a frozen mass, so that the wall elasticity also has no effect in the problem. It can be seen that considerable error can result in the first mode frequencies, in addition to the absence of all higher modes. The error appears to be worse for the stiffer of the two systems, with about a 30% error at full depth, for the cases studied.

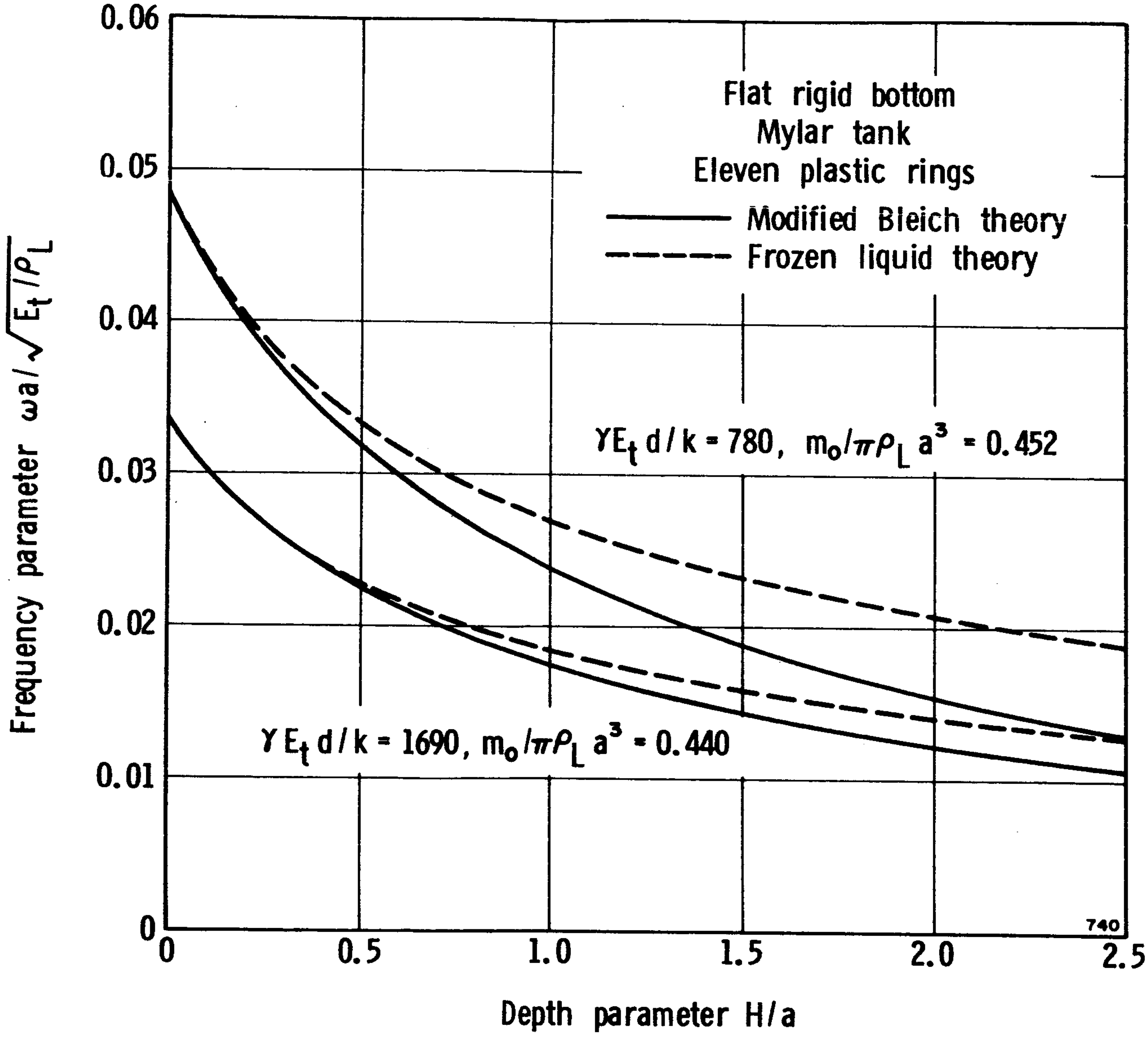

Figure 10. Comparisons of results of modified Bleich theory and frozen mass theory

Elastic Bottoms: Rigid and Elastic Supports

The frequencies of the first two symmetric modes of the same eleven-ring Mylar tank employed previously, but now modified with two different elastic bottom configurations, were determined experimentally for the same rigid and elastic support conditions. These results are given in Figures 11 and 12.

Figure 11 presents data for the tank having a 0.127 mm thick, flat Mylar (elastic) bottom (same material as shell walls). It must be emphasized that the ordinate scale in Figure 11 is expanded five times over that in Figure 9; thus, the frequencies of both modes for this case are very low. The flat bottom is so flexible, compared to the support springs, that the frequencies for both modes are essentially the same for all values of the support spring investigated. Further, it can be noted that for this low effective bottom stiffness the first mode decreases in frequency only very slightly with increasing depth, while the second mode increases with depth.

Similar data for a tank having a 0.127 mm thick elliptical Mylar bottom are shown in Figure 12. The ellipse had a major to minor axis ratio of 2.0. It should be noted that the ordinate scale used here is the same as in Figure 9, so that the frequencies of the corresponding modes for this bottom are closer to those of the flat rigid bottom than to those of the flat elastic bottom; the elliptical geometry is clearly quite efficient in providing effective stiffness, as is well known. Since this stiffness is

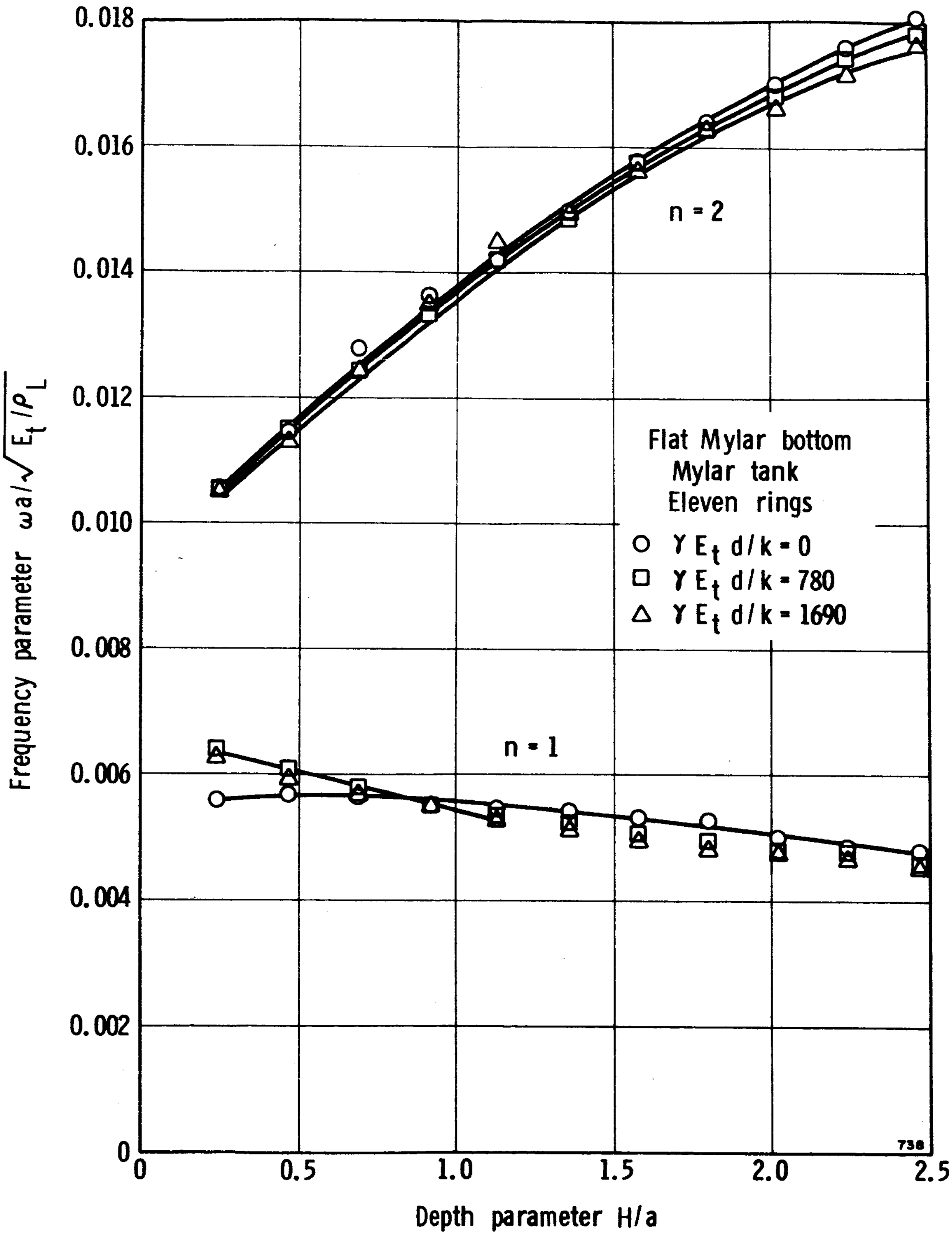

Figure 11. Variation of natural frequency with liquid depth for tanks with flat elastic bottoms and elastic supports

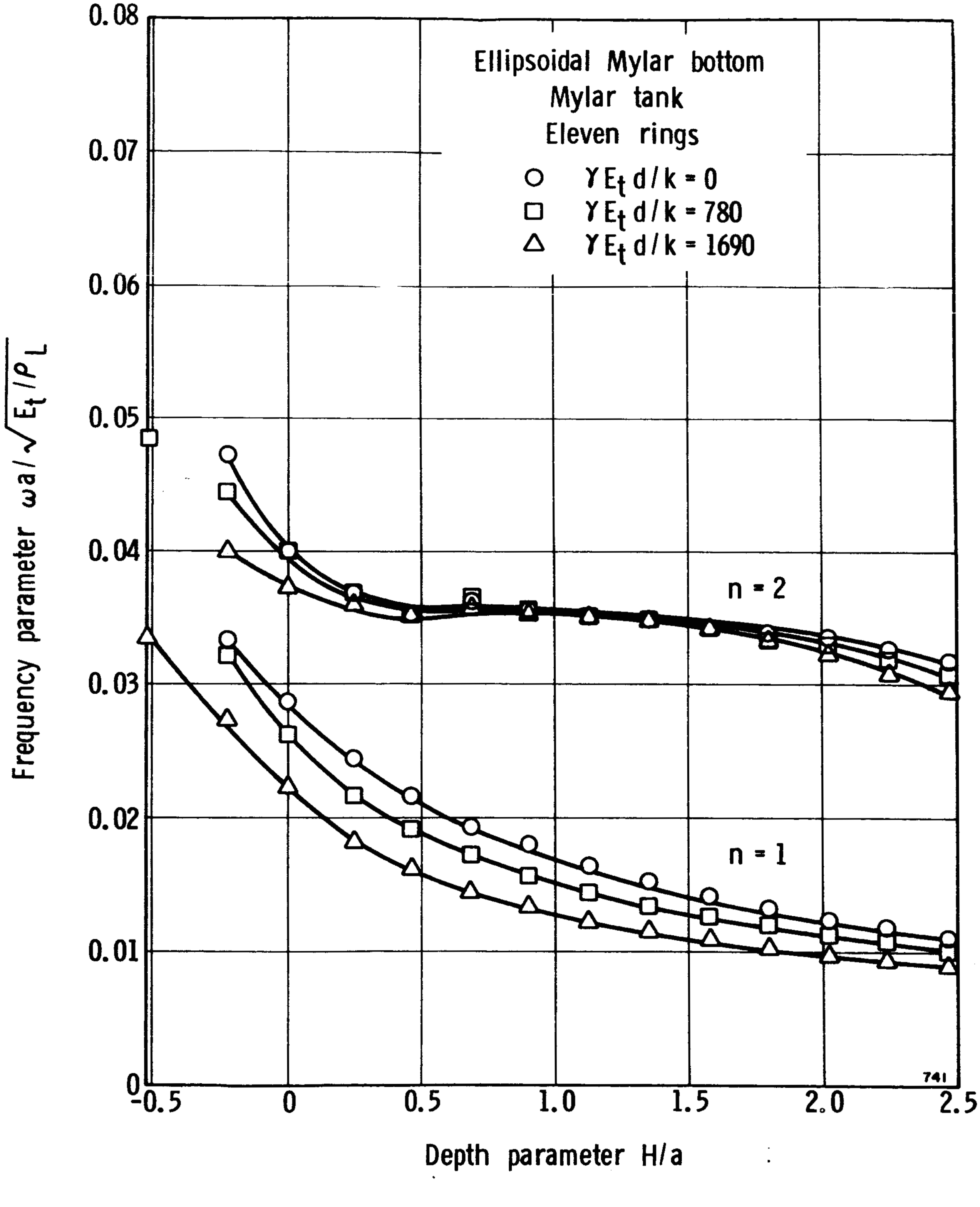

Figure 12. Variation of natural frequency with liquid depth for tanks with ellipsoidal elastic bottoms and elastic supports

intermediate between the other two, one should expect the data to behave accordingly. That is, not only are the frequency values intermediate, but a stronger influence of the support spring is evident, particularly in the first mode for the elliptical bottom tank. Also, the second mode is almost independent of depth in this tank, again a result that is consistent with the intermediate bottom stiffness for this tank. These results, insofar as general trends are concerned, are of course greatly dependent upon the relative stiffnesses of the tank elastic bottoms and the elastic spring support; greatly different values of either might result in completly different trends.

It should be mentioned that various of the higher modes could readily be observed in both of the elastic bottom tank configurations, particularly the nonaxisymmetric modes. However, we have shown here only results for the symmetric modes that could readily be observed; these modes are most significant as they have an average bottom pressure that is nonzero and becomes large near resonances.

CONCLUSIONS

For the ranges of parameters investigated in this program, it appears that the Bleich analysis incorporating a one-term approximation is adequate for predicting axisymmetric shell displacement and pressure responses in a thin, ring stiffened, partially liquid filled, flat bottom cylinder, except at very low liquid levels. Further, the incorporation of a spring mounted system into this theory likewise predicts similarly accurate results for the coupled frequencies of that system. As expected, the inclusion of additional terms into the expansion of the solution further improves the predictions for the latter system, especially for higher modes and lower liquid levels where the one-term approximation is least applicable.

Considerably more work should be done to obtain a reasonably simple approximation for the effects of bottom geometry and elasticity on the pressure response in a longitudinally excited tank. Based on the results of the present experiments, it appears that both wall elasticity and bottom elasticity have an equally significant effect on the pressure response in the contained liquid column. Therefore, for ranges of parameters that are typical of current actual vehicle tanks, both effects must be studied simultaneously, and the coupling of the support structure must also be included. Considering the liquid to be represented by an equivalent solid mass can result in significant error in predicting even the lowest coupled axisymmetric mode in the elastically supported tank, besides neglecting all higher modes.

Another uncertain factor arising from the present work and of possible importance in the design or analysis of actual vehicles is the effect of ring stiffeners in eliminating nonaxisymmetric shell vibrations. While it appears that the eleven-ring pattern used in the present study was adequate to eliminate effectively such modes for these model tanks, just how well the present stiffened tanks modeled the stiffness of an actual vehicle tank is uncertain. Such determinations should be made on the basis of orthotropic shell considerations, rather than on the approximate procedures employed here for convenience and simplicity.

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