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ANALYSIS OF THE TRANSIENT FLOW OF HYDROGEN IN HEATED PASSAGES IN THE LAMINAR-TURBULENT TRANSITION REGION

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SUMMARY

A transient analysis has been conducted of the flow characteristics in a single heated tube of circular cross section. The study was made in association with phenomena occurring in the nuclear rocket reactor core. Twenty-one cases were considered involving the following: twelve perturbations of heat flux into the tube wall, five perturbations in inlet pressure, and four perturbations of pressure drop. The results of the analysis are depicted in terms of the phase plane that generalizes the results and illustrates all flow behavior of the laminar instability condition.

The following four external parameters, which influence the transient behavior of the system, were considered: inlet pressure, outlet pressure, inlet enthalpy, and heat flux into the tube wall.

The analysis has shown that there are five kinds of transients. The first type of transient corresponds to an initial laminar flow rate on the steady-state curve which when perturbed goes to a turbulent equilibrium point. The second type of transient starts from the laminar flow rate on the steady-state curves and terminates at zero flow rate. A third sort of transient begins from an operating point corresponding to a turbulent flow rate on the steady-state curve and proceeds to an equilibrium point at a higher turbulent flow rate. The fourth kind of transient is initiated from a turbulent point on the steady-state curves and reaches a stable equilibrium point at a lower turbulent flow rate. Finally, the last kind of transient begins at a turbulent steady-state point and terminates at zero flow. These various transients, which are approximately independent of past initial conditions, can be more simply described by a family of curves in a phase plane plot.

The "stability criterion" near equilibrium points is simply that the four external parameters give an operating point with a positive pressure drop as a function of flow rate slope on the characteristic curves. If this slope is negative or if there is no operating point, then the condition is unstable.

INTRODUCTION

The shutdown transient for the nuclear rocket after a period of full power operation lasts for a period of several days to remove the decay heat. Hydrogen propellant is used as a coolant. It is essential to use a minimum propellant for this purpose in order to increase the useful payload. When the minimum flow rate of coolant consistent with the maximum safe material temperature limits of the reactor core is used, a possible flow problem may result.

The core of a nuclear rocket consists of thousands of closely spaced parallel passages. To simulate the problem that may arise in the nuclear rocket, it has become conventional practice to use a single tube with a plenum chamber at each end as a model. It has been demonstrated analytically in reference 1 that in the laminar-turbulent transition region two possible mass flow rates may exist in a single heated tube for a given pressure drop and heat flux in the steady state. Also, the negative slope of the characteristic curve is associated with a potential instability. These two aspects define the problem that is referred to as the "laminar instability problem." The characteristic curves have been verified experimentally in tests on a single heated tube (ref. 2).

Since it is possible to have two distinct flow rates for a given pressure drop across the reactor core, it is conceivable that one passage may have turbulent flow entering and changing to laminar flow while an adjacent passage may have laminar flow entering and leaving. Even if the flow is stable, a problem exists. The heat-transfer coefficients are different for turbulent and laminar flows so that the tube walls will be at different temperatures. This condition will give rise to thermal stresses and may result in a failure of the reactor core.

A steady-state analysis was conducted to define the influence of the external parameters. A transient analysis was conducted to find the kinds of transients occurring and to reveal the general nature of the flow stability problem; the analysis was accomplished by perturbing heat flux into the tube wall, inlet pressure, and pressure drop. This report presents the results of these analytical analyses.

ANALYSIS

Steady-State Analysis

The steady-state analysis was performed in order to determine the influence of the several parameters on the characteristic curves. The geometry of the tube consisted of a length of 52 inches, an inner diameter of 0.1160 inch, and an outer diameter of 0.1476 inch. This analysis used a test section consisting of an 80-percent nickel - 20-percent chromium alloy, which has a density of 0.304 pound per cubic inch.

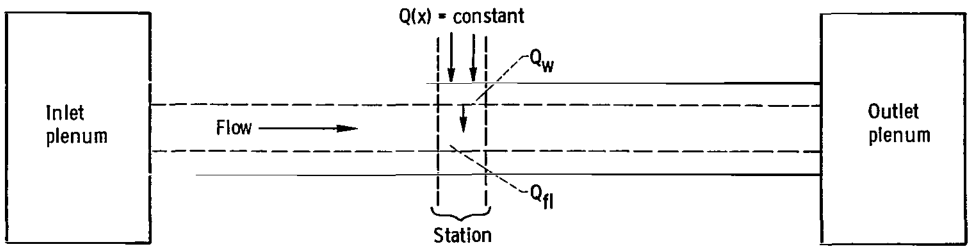


Figure 1. - Schematic of simulated physical system.

Figure 1 shows the physical system being considered. It consists of a single heated tube with a plenum chamber at each end. For the purposes of analysis, the tube length was considered divided into 26 stations each 2 inches long.

The basic assumptions of the calculations include the following. First, it was assumed that axial heat transfer along the tube wall by conduction is negligible. Therefore, the agreement between the analytical results of this study and the real physical situation improves as the tube wall thickness is decreased. Second, it is assumed that the overall pressure drop is comprised of two elements. One element arises due to the friction drop while the other occurs by virtue of the momentum pressure drop:

$$\Delta P_O = \Delta P_F + \Delta P_M \tag{1}$$

(Symbols are defined in appendix A.) An iterative technique is used to calculate the overall pressure drop.

It is further assumed that all the heat entering the tube wall goes into an increase of the enthalpy of the fluid:

$$\Delta H = \Delta LQ/\dot{W} \tag{2}$$

The wall temperature is computed from Newton's law of convective heat transfer:

$$T_W(x) = \Delta LQ/(hA) + T_b(x) \tag{3}$$

A more detailed description of the steady-state analysis is included in appendix B.

Transient Analysis

The transient analysis involves a quasi-steady state method of approach. It is assumed that steady-state flow conditions exist at any instant of time corresponding to the wall temperature distribution, inlet pressure and enthalpy, and outlet pressure. The resulting flow and fluid state throughout the tube results in a heat-transfer rate into the fluid which gives a new wall temperature distribution at the next instant of time. The basic assumptions of the transient analysis are the same as those of the steady-state analysis.

Pressure drop is obtained by an iterative technique from the basic equation

$$\Delta P_O = \Delta P_F + \Delta P_M \tag{1}$$

After the pressure drop is computed for a station, the heat-transfer problem is solved for that station. It is then assumed that the difference between the heat flux entering the tube wall and that transferred to the fluid goes into an increase of temperature of the tube wall:

$$Q_w(x) = Q(x) - Q_{fl}(x) \tag{4}$$

Finally, the new wall temperature distribution is computed:

$$T_{w,new}(x) = T_{w,old}(x) + \Delta L Q_w(x)\theta/(\rho_m V_m C_{p,m} N) \tag{5}$$

This procedure is continued until the entire tube length is completed.

A more detailed explanation of the transient analysis is given in appendix C. A digital computer program served as a tool to perform the actual numerical computations.

Correlations

The friction factor and heat-transfer correlations common to both analyses are the following:

Friction factors:

$$f_\ell = 16/Re_b$$

$$f_t = 0.0014 + 0.125\Big/Re_b^{0.32}$$

Heat-transfer coefficients:

$$h_\ell = 1.75\,\frac{K}{D}\left[\left(\dot{W}C_{P,fi}\right)\Big/(KL)\right]^{0.33}$$

$$h_t = 0.021 \frac{K}{D} Re_b^{0.8} Pr_b^{0.4} \left(\frac{T_b}{T_w}\right)^{0.525}$$

RESULTS AND DISCUSSION

The steady-state parametric study and the transient analysis were both performed with a composition of 100 percent parahydrogen as the working fluid.

The results of the steady-state analysis are shown in figures 2 to 4. The influence on the characteristic curves of varying the heat flux into the tube wall while holding inlet pressure and enthalpy constant at the respective values of 10.0 pounds per square inch absolute and 351.0 Btu per pound is shown in figure 2. The influence on the characteristic curves of varying the inlet enthalpy while holding the wall heat flux constant at 0.0016 Btu per second per inch and the inlet pressure constant at 10 pounds per square inch absolute is shown in figure 3. The influence on the characteristic curves of varying the inlet pressure while holding wall heat flux and inlet enthalpy constant at 0.0016 Btu per second per inch and 351.0 Btu per pound is shown in figure 4. The single curve of figure 4 shows that both inlet and outlet pressure effects are included in the parameter $P_{in}\Delta P$. The $P_{in}\Delta P$ term was chosen as an ordinate for figures 2, 3, and 4 because it brought together the characteristic curves for various inlet pressures and thereby simplified that part of the transient analysis.

The characteristic curves are shown to demonstrate the same shape for all parameters: wall heat flux, inlet pressure, and inlet enthalpy. Because of this, it is possible to obtain two distinct flow rates for a given pressure drop.

The characteristic curves are shown for flow rates from 2×10^{-6} to 2×10^{-4} pound per second. The range of interest in the nuclear rocket application is easily contained within this span. The lower limit of the characteristic curves is determined by wall material temperature limitations of the reactor core. The wall heat flux varied from 0.0012 to 0.002 Btu per second per inch, which is a typical range of values encountered during the shutdown of the nuclear rocket. The upper limit of flow rate of the characteristic curves for values greater than 2×10^{-4} pound per second was determined by the sonic velocity limit of the working fluid.

The three classes of problems subjected to the transient analysis were defined as follows:

Class 1 - The effect of perturbations in wall heat flux

Class 2 - The effect of perturbations in inlet pressure

Class 3 - The influence of perturbations in pressure drop

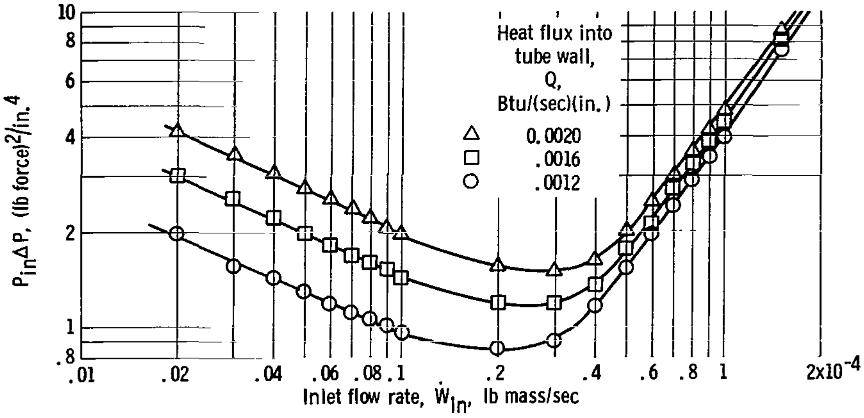


Figure 2. - Effect of wall heat flux on steady-state characteristic curve. Inlet pressure, 10 psia; inlet enthalpy, 351.0 Btu per pound mass.

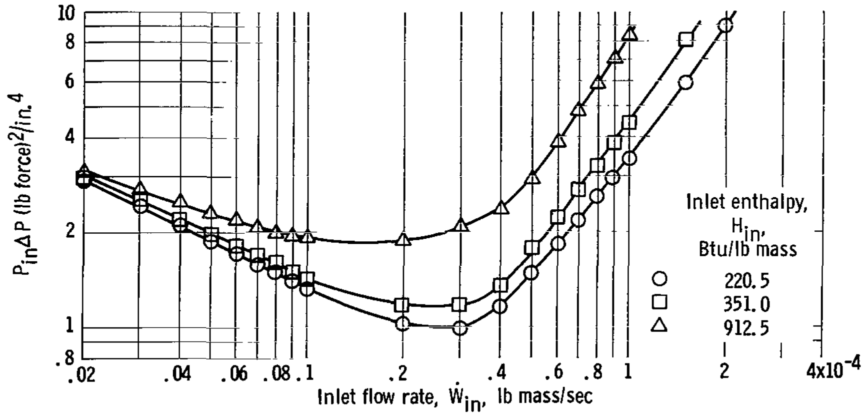


Figure 3. - Effect of inlet enthalpy on steady-state characteristic curve. Inlet pressure, 10 psia; heat flux into tube wall, 0.0016 Btu per second per inch.

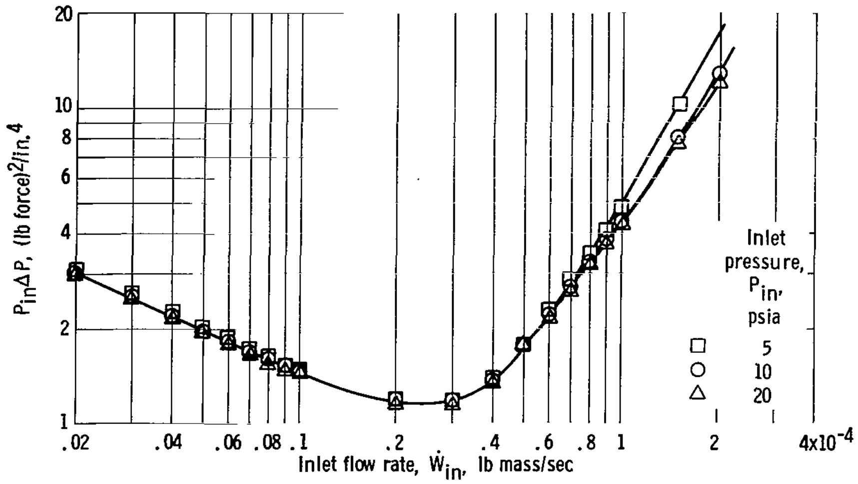


Figure 4. - Effect of inlet pressure on steady-state characteristic curve. Inlet enthalpy, 351.0 Btu per pound mass; heat flux into tube wall, 0.0016 Btu per second per inch.

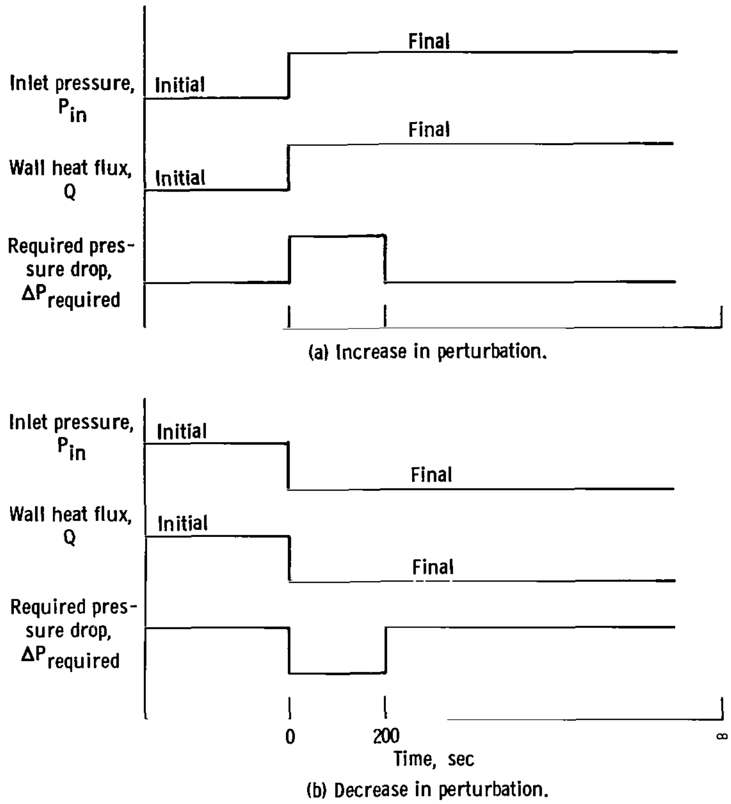


Figure 5. - Form of perturbations in inlet pressure, wall heat flux, and required pressure drop.

Figure 5 shows the two types of perturbations used in the analysis. Perturbations in wall heat flux and inlet pressure consisted of a pulse of infinite width. Perturbations of required pressure drop consisted of a pulse of finite width. The external wall heat flux distribution for both the steady-state and transient analysis was constant.

The results of the transient analysis for all cases are presented in terms of the phase plane. In general, such a plot consists of a time derivative of a variable against the variable (ref. 3). The phase plane representation was selected since it provided a convenient method of presenting and summarizing on a single plot the results of several transient cases. The variable common to all stations, flow rate and its time derivative, is used.

A typical flow rate as a function of time response is shown plotted in figure 6 for the first class of perturbations. The accuracy of the curve is determined by the size of the multiplier of the time constant specified. A multiplier of one was used throughout this analysis.

The phase plane figures were generated by graphically differentiating curves of the type shown in figure 6. The accuracy of this method was checked by analytically determining the derivatives. This was accomplished in the following manner. A least-squares

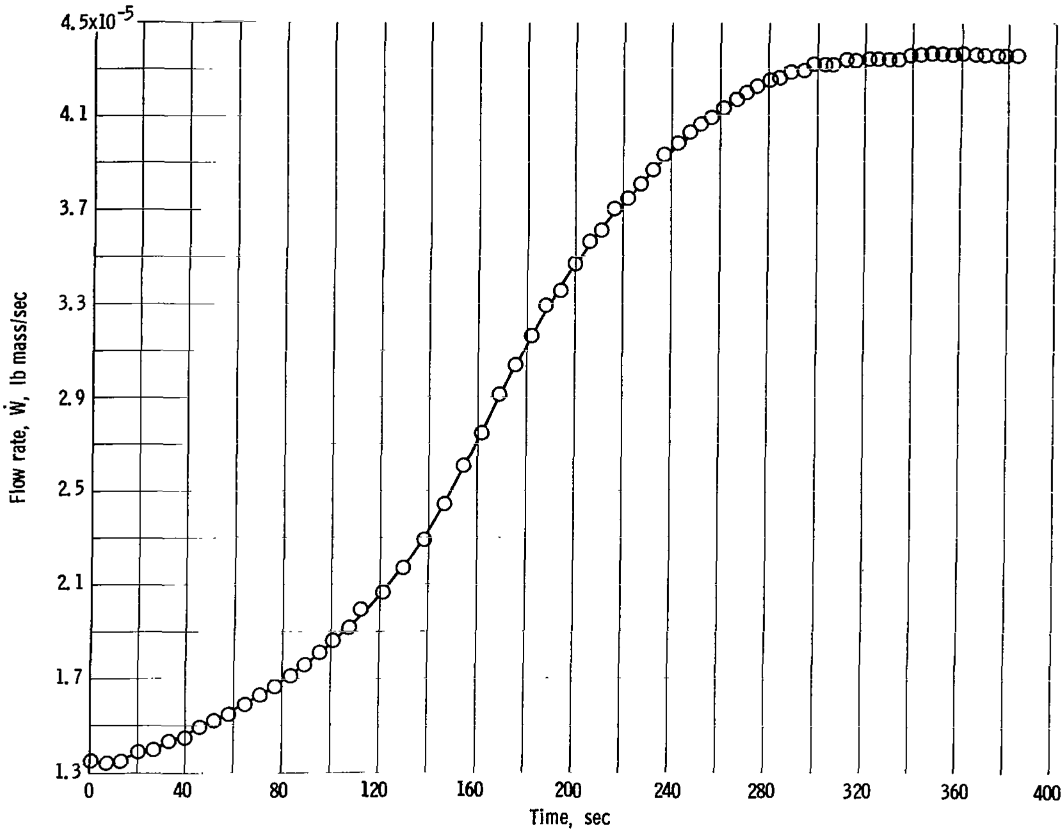
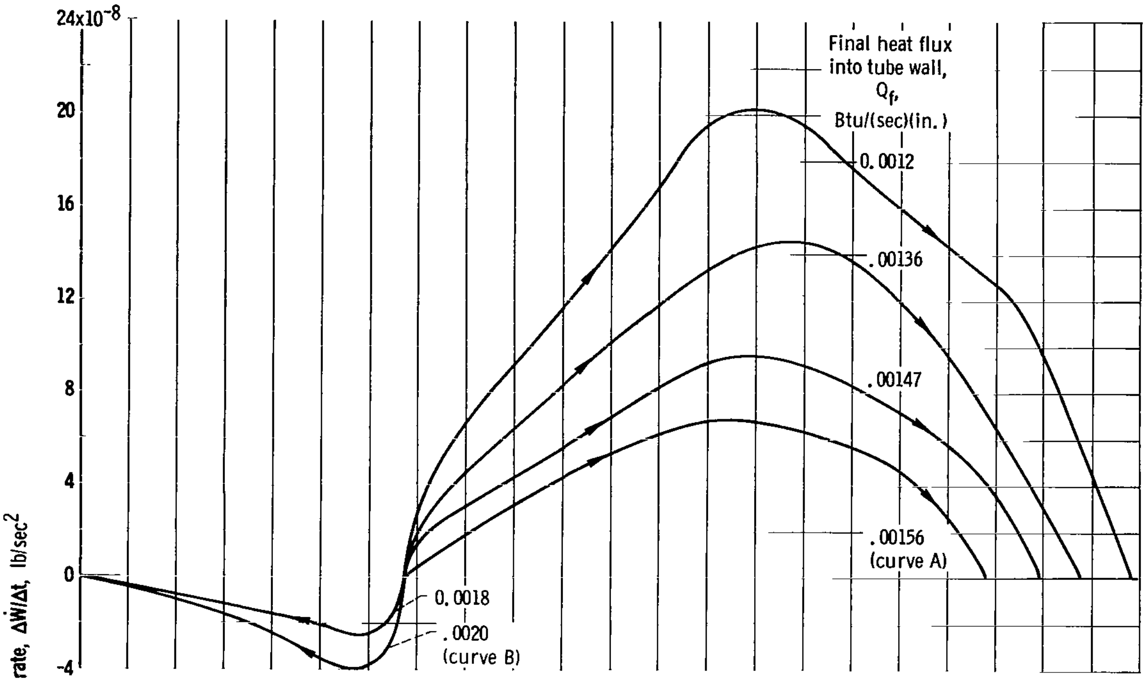


Figure 6. - Flow rate as function of time response. Initial conditions: heat flux into tube wall, 0.0016 Btu per second per inch; flow rate, 0.1345×10^{-4} pound mass per second.

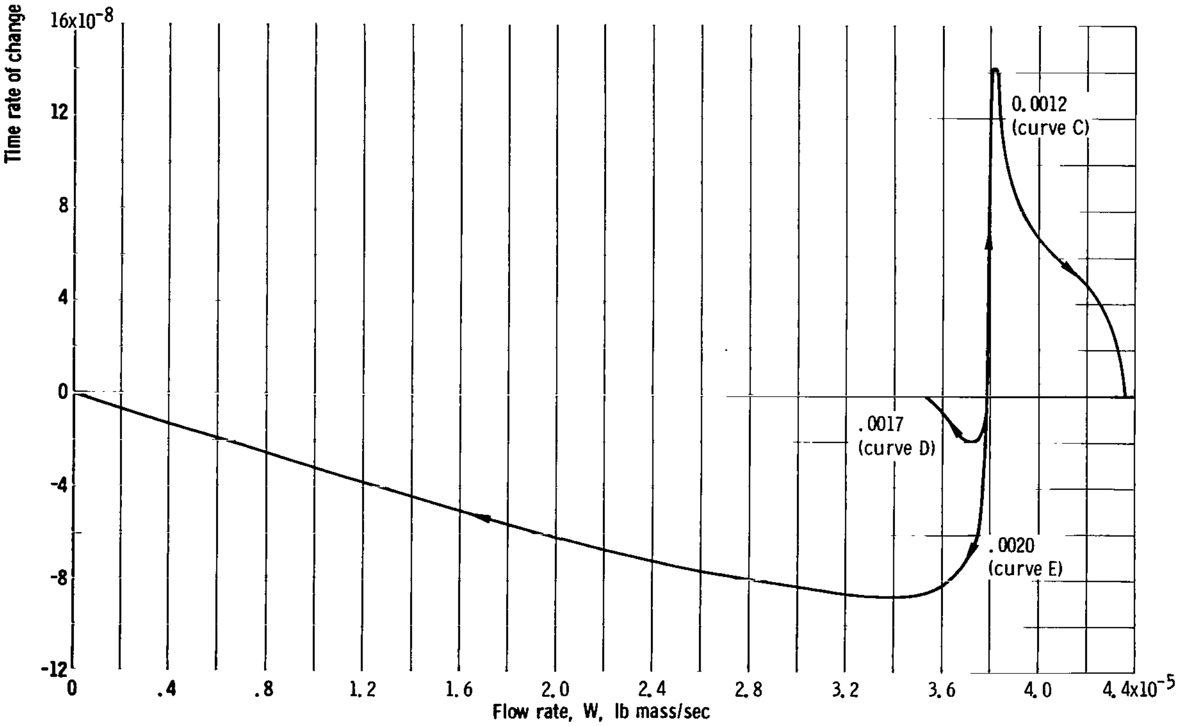
curve fit was used to approximate the data of a flow rate as a function of time response. A hyperbolic tangent whose argument was time was the principal function. From the hyperbolic term a corrective term consisting of a cubic polynominal in time was subtracted. This function fit the data well except at time zero where a maximum error of 6 percent occurred. The result was differentiated with respect to time and the graphical analytical development of the phase planes compared. Satisfactory enough agreement was shown between the two methods to justify the graphical technique.

Twelve cases of perturbations involving wall heat flux were studied (class 1), and the results are shown in figures 7 and 8. The phase plane of figures 7(a) and (b) was determined for the restriction of a constant pressure drop of $\Delta P = 0.13$ pound per square inch absolute. The pressure drop was held constant to simulate the process occurring in the reactor core of the nuclear rocket. The curves are parametric with the final value of external wall heat flux.

The interrelatedness of the steady-state and transient analyses becomes apparent at this point of the discussion. The effect of perturbing a steady-state operating point in figure 2 (p. 6) by a perturbation involving a decrease in wall heat flux while holding the

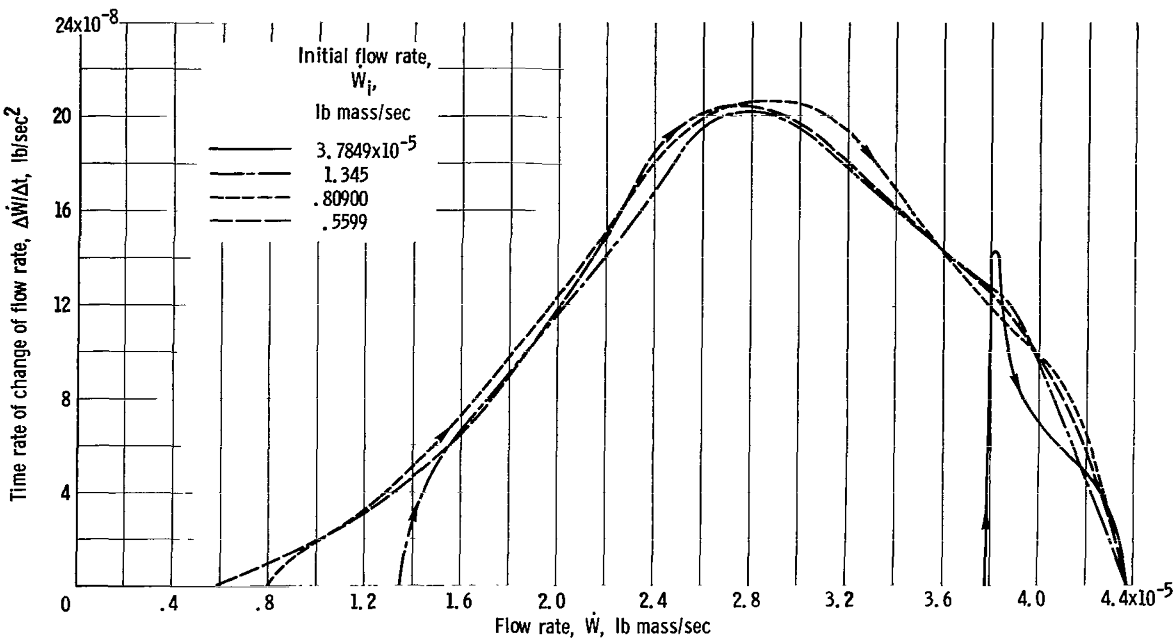


(a) Transients originate from laminar initial mass flow rate. Initial conditions: heat flux into tube wall, 0.0016 Btu per second per inch; 1.345×10^{-5} pound mass per second.



(b) Transients originate from turbulent initial mass flow rate. Initial conditions: heat flux into tube wall, 0.0016 Btu per second per inch; flow rate, 3.7849×10^{-5} pound mass per second.

Figure 7. - Phase plane representation of transient analysis for wall heat flux perturbations. Constant inlet pressure and enthalpy of 10 psia and 351.0 Btu per pound, respectively; pressure drop, 0.130 psia.



(c) Transients originate from various mass flow rates. Initial conditions: heat flux into tube wall, 0.0012 Btu per second per inch.

Figure 7. - Concluded.

pressure drop constant is considered. A new stable point will be reached on the turbulent branch where the final wall heat flux curve intersects the constant pressure drop line. This transient is shown as curve A in figure 7(a).

An increasing perturbation in wall heat flux from the initial condition on the laminar branch was unstable in all cases. This resulted because the line of constant pressure drop does not intersect the perturbed wall heat flux curve in figure 2. The flow rate went to zero as shown by transient B in figure 7(a).

A perturbation representing a decrease in wall heat flux from the turbulent initial condition in figure 2 reached a stable solution at a higher turbulent flow. The transient is shown as curve C on figure 7(b).

One of two possibilities exists if the perturbation represents an increase of wall heat flux for the turbulent initial condition. If the perturbation is small enough so that the line of constant pressure drop is intersected by the perturbed wall heat flux curve in figure 2, a stable operating point can be reached on the turbulent branch at a lower flow rate. This type of transient is shown as curve D on figure 7(b). If the perturbation is so large that this condition is not met, the system drives to zero flow rate, that is, is unstable. This condition is shown by transient E in figure 7(b). Indeed, there are five kinds of transients as shown by figures 7(a) and (b) collectively. Two of these transients correspond to laminar initial conditions, while the remaining three correspond to turbulent initial conditions of flow rate.

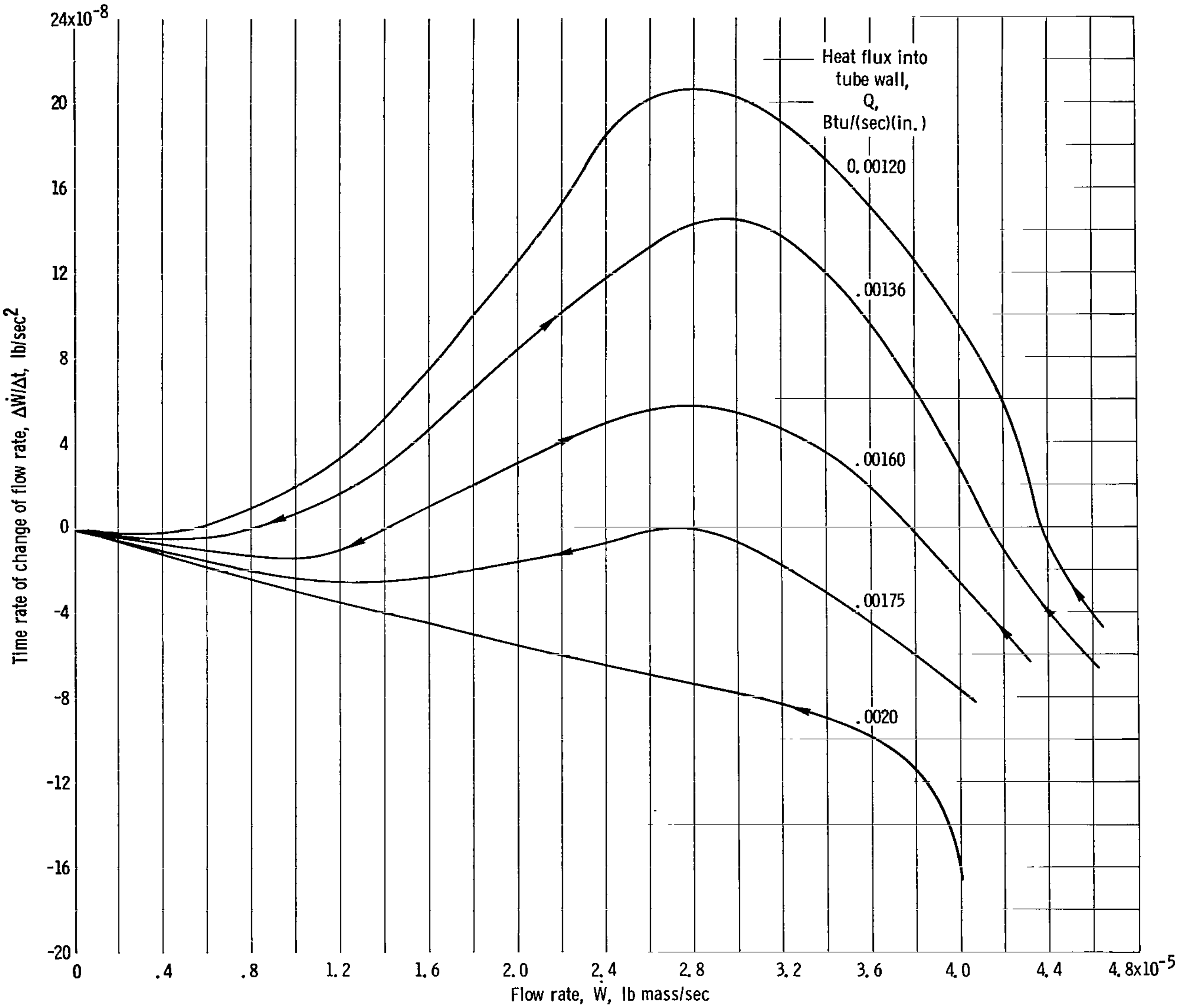


Figure 8. - Phase plane for wall heat flux perturbations.

Figure 7(c) shows that, corresponding to the four external parameters, a unique curve exists in the phase plane. A specific case in point is the curves in figure 7(c) determined by the following four external parameters: inlet pressure of 10 pounds per square inch absolute, outlet pressure of 9.87 pounds per square inch absolute, inlet enthalpy of 351.0 Btu per pound, and final external wall heat flux of 0.0012 Btu per second-inch. Curves for four initial conditions corresponding to flow rates of $\dot{W}_1 = 0.55990\times10^{-5}$, $\dot{W}_2 = 0.80900\times10^{-5}$, $\dot{W}_3 = 1.345\times10^{-5}$, and $\dot{W}_4 = 3.7849\times10^{-5}$ pound per second are seen to coalesce into a single curve after the effects of initial conditions have fairly quickly settled out.

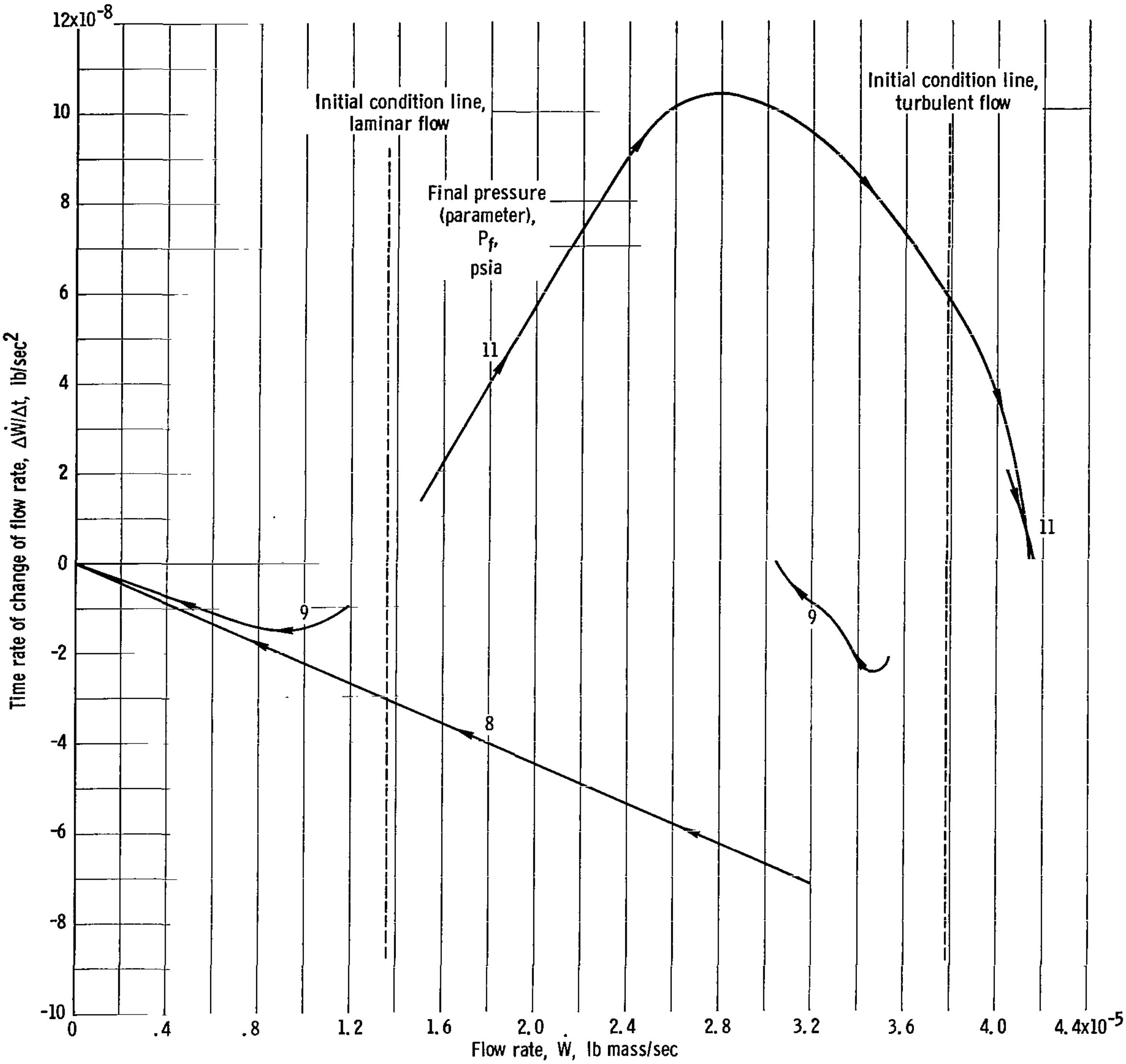


Figure 9. - Phase plane plot for inlet pressure perturbations. Pressure drop, 0.13 psia; heat flux into tube wall, 0.0016 Btu per second per inch; enthalpy, 351 Btu per pound mass.

Thus, if the initial part of the transients in figure 7 are ignored, a more general plot, independent of initial or past conditions, can be obtained. Figure 8 is thus obtained. The part of the curves of figure 7 that are ignored are filled in by using the equilibrium conditions; that is, the zero crossings of the curves are known. This procedure is necessary because the generation of that part of the curve would be very time consuming using the computer code.

If the external heat flux, inlet pressure and enthalpy, and outlet pressure are given, the transient will follow the corresponding curve of figure 8. An initial arbitrary wall temperature distribution will cause an initial deviation from those curves. This deviation is rather quickly settled out because of the strong coupling between stations along

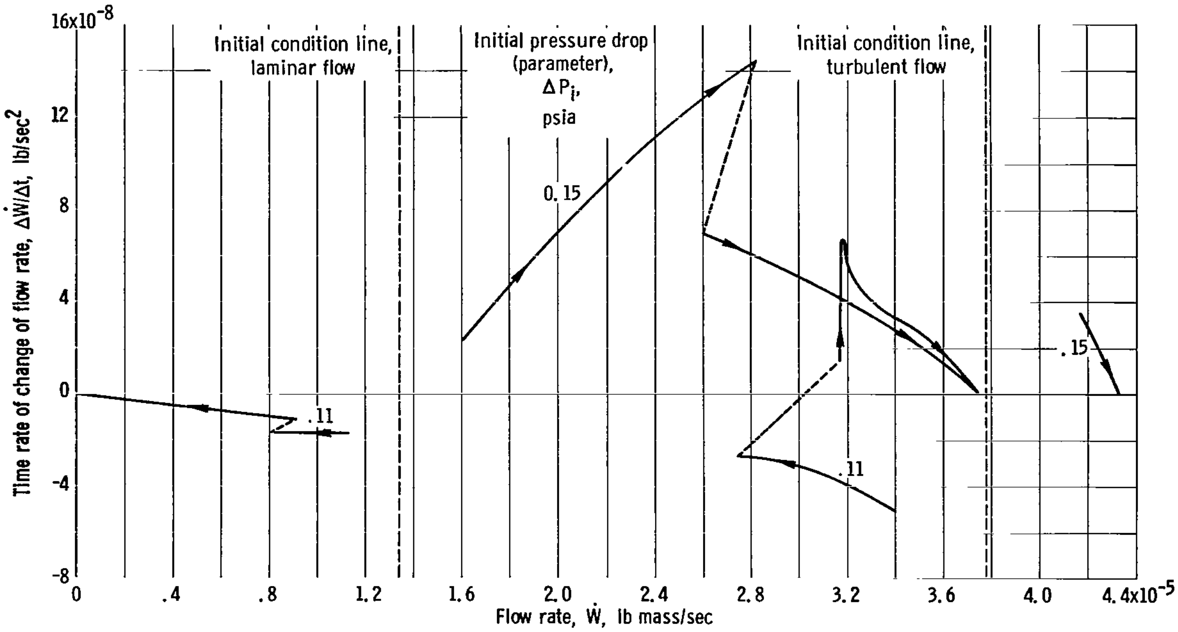


Figure 10. - Phase plane plot for required pressure drop perturbations. Heat flux into tube wall, 0.0016 Btu per second per inch; inlet enthalpy, 351 Btu per pound mass.

the tube.

The time sense on the phase plane plot is obtained by noting that the time interval to go from one point to another is the area under the reciprocal of the curve. Thus, because of the nonlinearity of the curves of figure 7 or 8, the time to go from the left equilibrium point to the right equilibrium point, for example, is much longer at relatively higher heat fluxes.

Figure 9 is the phase plane plot for the transients generated by perturbing the inlet pressure. Indeed, the five cases shown represent the five kinds of transients noted before. The similarity of figure 9 to figures 7 and 8 is apparent. These curves can be considered as retracing the same kind of phase plane curves as figure 8 after initial condition effects have settled out.

Figure 10 shows plots for the transients generated by pulse-type perturbations on outlet pressure. Each branch follows a piece of the curves of figure 8. Figure 10 shows how a more general perturbation (a pulse) may be traced out using figure 8. It also further supports the contention that the plot of figure 8 is generally true and can be used to predict transient behavior and stability characteristics.

SUMMARY OF RESULTS

The results of the analyses described herein yielded the following principal conclusions:

1. The nonequilibrium, nonlinear "laminar instability" behavior can be illustrated on a phase plane diagram like figure 8.

2. The curves in the phase plane showing the transient behavior are of two families: one family intersects the flow rate axis at two points and the origin, and the other lies below the axis and intersects it at only the origin.

3. Actual transients, which can start from arbitrary wall temperature distributions, will approach unique curves in the phase plane corresponding to the four external parameters of inlet pressure, outlet pressure, inlet enthalpy, and external wall heat flux after the effects of initial wall temperature distribution have settled out.

4. Each point of the curve in the phase plane represents, approximately, a corresponding quasi-equilibrium wall temperature distribution.

5. Five kinds of transients were obtained:

(a) Those beginning at laminar steady-state initial condition go to turbulent equilibrium point.

(b) Those beginning at laminar steady-state initial condition go to zero flow rate.

(c) Those beginning at turbulent steady-state initial condition go to a higher turbulent flow rate equilibrium point.

(d) Those beginning at turbulent steady-state initial condition go to lower turbulent flow rate equilibrium point.

(e) Those beginning at turbulent steady-state initial condition go to zero flow rate.

6. There were no instabilities of constantly increasing flow rate.

7. All phase plane curves are probably similar, qualitatively, to those shown in this report.

8. The stability criteria near equilibrium points is simply that the four external parameters give an operating point on the characteristic curve with a positive slope. If this slope is negative or if there is no operating point, then the condition is unstable.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 10, 1966,
121-30-02-03-22.

APPENDIX A

SYMBOLS

A area, in.^{2}

$C_{p,m}$ specific heat of nickel chromium alloy, Btu/(lb mass)(^oR)

D diameter, in.

f friction factor

G mass flow per area in square feet, lb mass/(sec)(ft^2)

g_c acceleration due to gravity, 32.2 (lb mass-ft)/(lb force)(sec^2)

H enthalpy, Btu/lb mass

ΔH change in enthalpy

h heat-transfer coefficient, Btu/(sec)(in.^{2})(^oR)

J mechanical equivalent of heat constant, 778 (ft-lb force)/Btu

K thermal conductivity based on film temperature

L length, in.

ΔL station length, in.

N multiplier of time constant

Nu Nusselt number

P pressure, psia

Pr Prandtl number

ΔP (inlet pressure - outlet pressure), psia pressure drop

Q heat flux into tube wall, Btu/(sec)(in.)

Q_{fl} heat transfer to fluid, Btu/(sec)(in.)

Q_w heat stored in wall, Btu/(sec)(in.)

Re Reynolds number

T temperature, ^oR

t time, sec

V_m volume of tube wall material, in.^{3}

$\dot{W}$ flow rate, lb mass/sec

θ time constant, θ = minimum $\left[\frac{\rho_m V_m C_{p,m}}{hA}\right]$

Δt time increment $\Delta\theta/N$, sec

ρ_{fl} fluid density, lb mass/ft^3

ρ_m density of nickel chromium alloy, lb mass/in.^{3}

Subscripts:

avg average

b bulk

calc calculated

F friction

f final

fi film

fl fluid

i initial

in inlet

j j^{th} iteration

ℓ	laminar	o	outlet
M	momentum	p	plenum
m	material (nickel chromium alloy)	t	turbulent
O	overall	w	wall

APPENDIX B

DETAILS OF STEADY-STATE ANALYSIS

The purpose of this analysis was to provide a computational procedure for pressure drop and wall temperature distribution for a single heated tube when the flow rate, geometry, inlet pressure, inlet enthalpy, and heat flux into the tube wall are known.

The tube length is divided into discrete finite intervals called stations to facilitate computing the conditions that exist at a definite axial position. Since the inlet pressure and temperature are known, the remaining fluid properties at the inlet may be determined immediately. The computed Reynolds number is based on properties at the inlet to the station. The pressure at the outlet of a station is found by an iterative technique. For the first trial, the outlet pressure of the station is assumed equal to the inlet pressure of the station. Outlet enthalpy is calculated as

$$H_o = H_{in} + \frac{\Delta L Q}{\dot{W}} \tag{B1}$$

These values of enthalpy and pressure are used to define the state of the fluid at the outlet and so they define the remaining properties there.

A new exit pressure is next calculated as

$$P_{o,new} = P_{in} - \frac{G^2}{144\, g_c}\left(\frac{1}{\rho_o} - \frac{1}{\rho_{in}}\right) - \frac{4fL}{144D}\ \frac{G^2}{2g\rho_{avg}} \tag{B2}$$

If this new value of outlet pressure and the previously calculated outlet enthalpy are used, a new outlet state of the fluid is defined from which the remaining thermodynamic properties can be found.

Iteration is continued using equation (B2) until the outlet pressure converges to some prescribed tolerance. Once the outlet pressure has converged, it and the other properties at the outlet of the current station become inlet conditions to the next station. Again the iterative technique described previously is used to find the outlet conditions for that station. This procedure is repeated until all the stations comprising the tube length have been treated.

The fluid bulk temperature is adjusted to relate to the center of each station. Finally, the wall temperature distribution is computed from

$$T_w(x) = \frac{\Delta L Q}{hA} + T_b(x) \tag{B3}$$

where the appropriate laminar or turbulent heat-transfer coefficient is used, depending on the Reynolds number for that station.

A computer program exists to perform the computational procedure described previously. The program employed a subroutine, described in reference 4, to obtain the fluid properties.

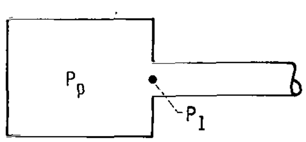
APPENDIX C

TRANSIENT ANALYSIS

The purpose of this analysis was to provide a computational procedure that would permit flow rate, wall temperature, and other variables to be computed as a function of time. It was assumed that a quasi-steady-state technique could be employed. In addition, it was assumed that inlet pressure, inlet temperature, wall heat flux, overall pressure drop, geometry, and initial wall temperature distribution are known.

The analysis treats the flow at a given instant of time assuming steady-state conditions prevail along the tube in the manner described in the following steps:

(1) Static pressure is computed as follows (see sketch (a)):

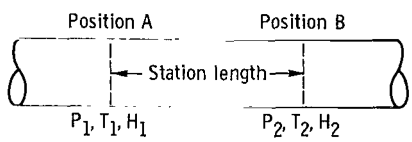


(a)

$$P_1 = P_p - 1.2 \frac{G^2}{2g_c\rho_p} \tag{C1}$$

(2) The static enthalpy at the entrance of the tube is obtained as follows:

$$H_1 = H_p - \frac{G^2}{2g_c J\rho_p^2} \tag{C2}$$



(b)

(3) The model is shown in sketch (b) for each station.

If the station is the first station of the tube, P_1 equals P calculated in step 1, H_1 equals H calculated in step 2, and T_1 is the fluid temperature in the plenum. If the station is not the first, properties are equal to the properties at position B from the previous station.

(4) The Nusselt modulus is calculated based on the conditions at position A.

(5) The heat-transfer coefficient and heat-transfer are calculated:

$$h = \frac{NuK}{D} \tag{C3}$$

$$Q_{fl} = hA(T_w - T_b) \tag{C4}$$

(6) The enthalpy at position B is calculated:

$$H_2 = H_1 + \frac{\Delta L \, Q_{fl}}{\dot{W}} \tag{C5}$$

(7) It is assumed that $P_2 = P_1$, and H_2 from step 6 is used to define the state at position B. The remaining properties at B are determined by the use of the subroutine program.

(8) The pressure drop is calculated from

$$P_2 = P_1 - \frac{1}{144}\left\{4f \frac{L}{D} \frac{G^2}{2g_c\rho_{avg}} - \frac{G^2}{g_c}\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\right\} \tag{C6}$$

(9) Using P_2 from step 8 and H_2 from step 6 define a new state at position B from which T_2 and remaining properties can be obtained.

(10) The average T_b and T_{fi} are calculated:

$$\left.\begin{aligned} T_{b,avg} &= \frac{(T_2 + T_1)}{2} \\ T_{fi,avg} &= \frac{(T_w + T_{b,avg})}{2} \end{aligned}\right\} \tag{C7}$$

(11) Go back to step 4 and base the calculations of the Reynolds and Prandtl numbers on the new temperatures computed in step 10. Steps 5 to 10 are repeated until two Q_{fl}'s calculated in succession in step 5 agree within some prescribed tolerance. When this tolerance is reached, the station is completed. The exit pressure, temperature, and enthalpy become entrance properties for the next station.

(12) The ΔP_{calc} is calculated:

$$\Delta P_{calc} = P_{in} - P_o \tag{C8}$$

(13) The flow rates are then adjusted by the equation

$$\dot{W}_{new} = \dot{W}_{old}\left(\frac{\Delta P_{required}}{\Delta P_{calc}}\right)^{X_n} \tag{C9}$$

where

$$X_n = \frac{\ln\left(\frac{W_j + 1}{W_j}\right)}{\ln\left(\frac{\Delta P_j + 1}{P_j}\right)}$$

(14) The calculation procedures of steps 1 to 14 are repeated until $\Delta P_{required}$ agrees with ΔP_{calc} to some prescribed tolerance.

(15) After the pressure drops, bulk temperatures, and heat-transfer coefficients are obtained, the time constant for each station and new wall temperature distribution are obtained as follows:

$$\theta = \frac{\rho_m V_m C_{p,m}}{hA} \tag{C10}$$

$$Q_w(x) = Q(x) - Q_{fl}(x) \tag{C11}$$

$$T_{w,new}(x) = T_{w,old}(x) + \frac{\Delta L\ \theta Q_w(x)}{\rho_m V_m C_{p,m} N} \tag{C12}$$

A new ΔP based on the $T_{w,new}$ obtained in equation (C12) is calculated by returning to step 1 and using the final flow rate calculated in step 13.

(16) The calculations are continued until all T_w's change less than a prescribed tolerance.

A computer program employing the mechanics described serves as a tool to perform the numerical calculations.

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