ANALYSIS OF THE LINEARITY CHARACTERISTICS, TAPE RECORDERS AND COMPENSATION EFFECTS IN THE FM/FM TELEMETRY SYSTEM

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SYSTEMS ENGINEERING GROUP

BUREAU OF ENGINEERING RESEARCH

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ABSTRACT

This report describes a set of exploratory experiments performed on the FM/FM Telemetry System. These experiments were performed to investigate:

1. System linearity characteristics for two major sub-systems; i.e., the package sub-system and the receiving and data reduction sub-system.
2. The effects of tape recorders as a system component.
3. The effects of tape speed compensation as related to system noise.

In general, the purpose of the linearity experiments was to determine if any non-linear effects existed within the system. Overall system tests indicated an affirmative answer. The degree of non-linearity was determined and a mathematical model which describes this effect was formulated. This experiment was performed prior to that presented in Technical Report Number 9 and served as a foundation for the design of that experiment.

The experiments related to tape recorder effects resulted in the development of a methodology for isolating the errors associated with individual components of the system. This methodology is presented in detail and the isolated errors within the experimental system summarized.

A methodology for determining the error assignable to the instability of the system was also developed. This methodology is presented and the portion of instability error associated with the various system components summarized.

Findings on the advantages offered by tape speed compensation have also been summarized. A possible explanation for the large interaction variance documented in Technical Report Number 2 is discussed as is the large contribution of the analog tape recorders to total system noise.

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SECTION I

INTRODUCTION

A. SUMMARY OF PAST EXPERIMENTS

Technical Report Number 2 (NAS8-5003), Accuracy Analysis of FM/FM Telemetry System for the Saturn Vehicle, summarized the findings of a set of low noise and a set of high noise experiments performed on the stated telemetry system. In summary form these experiments were concerned specifically with the behavior of XO-4 packages and their subcarrier oscillators under varying noise conditions. A block diagram of the system as it existed at that time is portrayed in Figure 1. It was determined that the experimental packages and subcarrier oscillators were not significantly different in behavior; however, an effect which appeared to be an interaction between packages and subcarrier oscillators was found to be highly significant. Specific estimates were made for each component of variance based on the theoretical model:

$$X_{ijk} = \mu + t_i + \theta_j + I_{ij} + e_{ijk}$$

where:

μ - a grand mean

t_i - a package effect

θ_j - a subcarrier oscillator effect

I_{ijk} - a package - SCO interaction effect

e_{ijk} - a random experimental error.

From these components a composite error term $\hat{\sigma}^2$(Response) was developed and expressed as a percent of full range. It was

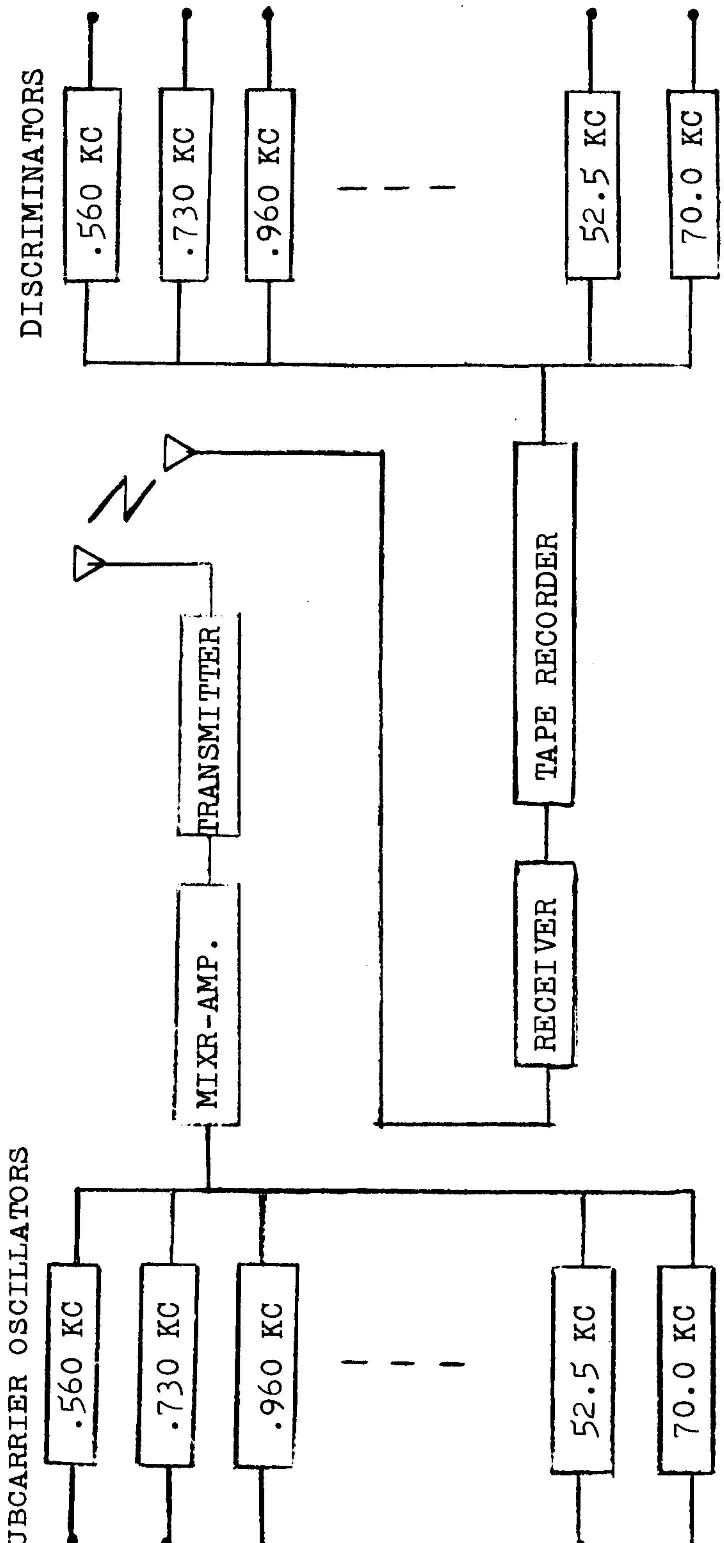

FIGURE 1. FM/FM TELEMETRY SYSTEM

estimated that the minimum and maximum precision of the FM/FM Telemetry System including the data reduction process expressed as 99% confidence limits were:

Average Minimum Precision = $\pm$ 4.74%

Average Maximum Precision = $\pm$ 3.00%.

It should be noted that at the time the foregoing experiments were conducted the data reduction process included the services of the computation laboratory. Outputs from the receiver were recorded on magnetic tape, and then transferred to the computation laboratory for demodulation and subsequent processing through the A/D converter using standardized procedures.

As would be expected, the results of the preceding experiments led to several specific recommendations on areas which warranted further study. It was on the basis of these recommendations that the experiments to be reported here were designed and ultimately performed. Each of the recommendations, and the actions which have subsequently taken place to accomplish them, will be discussed individually.

1. <u>Linearity Experiments</u>. Past experiments including the two previously summarized have been based on only five calibration levels (i.e., 0, 25, 50, 75, and 100 percent increments of a 5 volt scale). Although this reflects standard calibration procedures, it was concluded from the preceding experiments that it would be desirable to map linearity curves for the total system as well as some of its selected components at possibly twenty or more calibration points. In this way assumptions related to the linearity of the total system to the data reduction portion of the system could be verified or disproved. Since non-linearity is potentially a source of bias and consequently inaccuracy in any telemetry system it is essential that such characteristics be determined and either compensated for or eliminated. As a result of this recommen-

dation several experiments have been performed to determine the linearity of the system in question and some of its selected components. These experiments include studies of the linearity characteristics of:

a. The entire FM/FM Telemetry System from the input of known voltages generated by a DC Voltage Standard into an XO-4 package through the digitized output of a ground station printer.

b. That part of the system from the discriminators through the ground station printer.

2. Interaction Variance. An A/D converter and an on-line printer were installed in the ground station of the Telemetry Laboratory of the Marshall Space Flight Center after the completion of the Low Noise and High Noise Experiments. It was further suggested in Technical Report Number 2 that it would be desirable to perform experiments which isolated the data reduction system from the rest of the system. In this way, it was felt that a better understanding of the "cause" of the large interaction variance would result. Such a series of experiments has now been performed and the results will be reported in Sections III and IV of this report.

3. Equipment. It was also recommended that the Telemetry Laboratory obtain sufficient digital data processing equipment to conduct experimental evaluations of telemetry systems. Subsequent to the preceding recommendation a Telemetry Data Analysis System has been designed, fabricated, and installed by the Systems Engineering Laboratories of Fort Lauderdale, Florida. This system will facilitate the performance of several recommendations which appear in the final section of this report.

B. ACCURACY AND PRECISION IN SYSTEMS OF MEASUREMENT

Measurement is defined as determining the dimensions, capacity, or quantity of anything. For our purposes, it is important here to recognize that any measured value contains not only variation in the quantity measured but also errors of measurement as well. Further, specification of any measurement system must include not only the measuring device but also the procedures to be used including the manipulations of the user as well. When repeated measurements are made of a quality characteristic under constant conditions a pattern of variability usually results. The variability may be directly attributed to the measurement system being used and each measurement system will have its own unique pattern of variation. Normal procedure, then, in the study of any measuring system is to describe its characteristics in terms of accuracy and precision.

1. Accuracy. Accuracy of a measurement system refers to its lack of bias. An understanding of the accuracy of any measurement system is achieved by studying its systematic errors. To determine whether a specific measuring system or instrument has a bias it must be compared to some invariant standard. The determination of accuracy may not be as simple as one might initially expect. For one thing, the only way to determine a "true" value is by the use of some other measuring system. Naturally the system used to determine the "true" value should be a high precision system believed to be without bias. Determination of accuracy also implies the calibration of the system used which in turn gives rise to other considerations. For instance, it may be found that the systematic error varies for different "true" values throughout the total spectrum of the characteristic to be measured. In this study accuracy will refer to the constant or systematic errors of the system under study.

2. Precision. Whenever a measurement system is used repeatedly to measure an unchanging characteristic under

carefully controlled conditions the resulting variability in measurement is termed precision. In quantitative terms precision may be measured by the standard deviation of the frequency distribution generated by repeated measurements of an unchanging characteristic or generated by the measurement of a homogeneous sample in the case of destructive testing. When a specific measurement system is being used this variable or random error may be interpreted as the measure of variability within samples or the experimental (residual) error.

The consistency of the measurement pattern of variation of a particular measurement system as it is used repeatedly over a period of time is termed reproducibility. Reproducibility in a measurement system infers that its variability is in statistical control (homogeneous) which may be verified by the use of Shewart control charts. If repeated measurements exhibit an erratic pattern of variability, the measurement system used must be considered not reproducible. Consequently, a statement of the general precision of a measurement system implies that the method of measurements used is reproducible.

C. THE FM/FM (XO-4) TELEMETRY SYSTEM

The measurement system with which we are concerned in this report is the FM/FM (XO-4) Telemetry System. This system differs from the one portrayed in Figure 1 only in respect to the data reduction subsystem. In past experiments the output from the receiver has been recorded on magnetic tape, transferred to the computation laboratory for demodulation and subsequent processing through the A/D converter, and ultimately through a printer. In this series of experiments the entire process was accomplished within the Telemetry Laboratory. In addition a coaxial cable was used to connect the package to the receiver. In several of the experiments a Hewlett-Packard Pushbutton Oscillator was used to simulate the input of an XO-4 package into the data reduction system.

The input to the XO-4 Telemetry System is a voltage which ranges from a minimum reading of 0.00 to a maximum reading of 5.00 volts. Unfortunately, the output of the system is a set of dimensionless numbers. In the following experiments efforts have been made to set the range of these values at 975 counts which is less than the 1024 count full scale limit of the telemetry laboratory printer. This procedure was used to permit detection of "off scale" readings. If it is assumed that the XO-4 system is linear, then the dimensionless numbers may be functionally related to the input voltages by an appropriate transformation using the calibration levels. By assuming complete linearity the functional relationship may be stated as follows:

$$V = \frac{5}{R}(D - C_0)$$

$$= \frac{5}{975}(D - 24)$$

$$= .00513(D - 24)$$

where,

V = Voltage input.

D = Digitized output.

C_0 = Digitized output at the 0% calibration level assumed to be 24.

R = Range of digitized output from 0% to 100% calibration assumed to be 975 counts in this case.

It is obvious from the above relationship that no systematic errors are possible as long as the calibration levels are linear and the range of the digitized output remains constant. These assumptions have been tested statistically and will be discussed in Section II.

D. THE SPECIFIC PROBLEM

1. Restrictions. Evaluation of prior experiments with the FM/FM Telemetry System and consultation with the technical representatives resulted in the following restraints to be observed in the conduct of the experimental program.

a. The system calibration would be simulated by either an accurate d.c. voltage source or an accurate frequency reference standard depending on the portion of the system under study.

b. The input d.c. voltage or reference frequency would be adjusted to simulate the five levels of calibration. An exception was provided for linearity checks to permit the use of 0.25 volt intermediate levels of calibration.

c. As in the past only one channel would be under test at a time. The d.c. voltage on the remaining channels would be set at 2.500000.

d. The output from the receiver would be either directly printed or recorded on magnetic tape and in some cases both simultaneously. All demodulation and subsequent processing through the A/D converter and printer would be accomplished on the equipment available in the Telemetry Laboratory.

e. A production quality XO-4 package and other system components would be used in all experiments. Check-out procedures would be those normally in use and would be checked by Telemetry Laboratory personnel.

f. The digitization of the output signal would be at a rate of 5 samples per second. Sufficient data would be recorded to provide at least a two second sample for each calibration within each channel for all experiments.

g. Several XO-4 packages and at least five sub-carrier oscillators for each frequency would be made available as well as two Ampex Analog recorders in the Telemetry Laboratory ground station.

h. When using magnetic tape for recording data, 5 tape tracks would be used to record data, 1 track for compensation, and 1 track for voice identification.

2. Questions to be Answered.

1. Does a linear model provide an adequate fit to the data for a particular channel?

2. Does a significant non-linear effect exist in the system?

3. If there is a non-linear effect, which component (or components) is responsible for the effect?

4. Is there a significant difference in the effects of different analog tape recorders?

5. Is tape speed compensation effective in the reduction of noise when using analog tape recorders?

6. Is there an interaction effect between recorders and tape speed compensation?

7. What amount of error do the discriminators, digitizer, printer, and associated circuitry introduce into the system?

8. What amount of error is introduced into the system by the package, SCO's and associated circuitry?

9. What is the amount of error introduced into the system due to the recorder and magnetic tape?

10. How much, if any, does tape speed compensation reduce the total error of the system?

11. Is the experimental data reproducible? If not, how much reduction in system error can be expected if measures are taken to ensure reproducibility of the experimental data?

SECTION II

LINEARITY EXPERIMENT

A. THE PROBLEM

1. Introduction. The assumptions concerning linearity of the XO-4 FM/FM telemetry system acquire their importance in the data reduction process. More specifically, at present the mathematical model assumed in the data reduction process for the system relationship is a fourth degree polynomial. If, for example, the relationship between system input and output is truly linear then it is possible to introduce error into the reduction process by using a model other than a linear one.

The determination of the linearity characteristics of the FM/FM telemetry system is also important in evaluating the need for redesign. If the system is non-linear an analysis of the subsystem components could provide indications of the source of the non-linear effect.

Consequently, the linearity experiment was designed to:

(1) derive an appropriate linear relationship for each channel within the system.

(2) determine if the linear model provides an adequate relationship between system input and output.

(3) determine whether a higher degree equation provides a better model for the system.

(4) isolate the non-linear effect, if any, within the subsystem components.

The experiment was conducted under the direction of qualified ground station personnel.

2. <u>Accuracy</u>. The calibration of the FM/FM telemetry system is facilitated by a five-step calibration sequence. Such a calibration sequence occurs several times during a flight. In the data reduction process Lagrangian interpolation[1] is used to relate the digitized output values to the input voltage of the system. Loss of accuracy results only in the data reduction process when an inappropriate mathematical relationship is used to infer input values associated with given output values.

The accuracy of the FM/FM telemetry system is illustrated in Figure 2 . The solid line represents a conceptual plot of the true relation and the dashed line is an estimated relationship resulting from calibration data. If the two curves do not coincide then inaccuracy (constant bias) is introduced into the information. This inaccuracy is represented by the resulting vertical deviations of the two curves in the figure.

It is possible that a particular degree curve provides the best relationship between input and output even though the curve does not pass exactly through every calibration mean. The calibration means are computed from a sample and, therefore, will likely deviate from the true means because of the random error (precision) of the system. Again, refering to Figure 2 , the estimating relation may pass exactly through the means and still be the wrong relationship for the system.

Therefore, it is desirable to develop a procedure which considers the random error in establishing the system relation. Such a procedure could be applied equally well to the evaluation of the linearity characteristics of the subsystem components.

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[1] Lagrangian interpolation results in the fitting of an nth degree curve through n+1 points, in this case a fourth degree curve through 5 points.

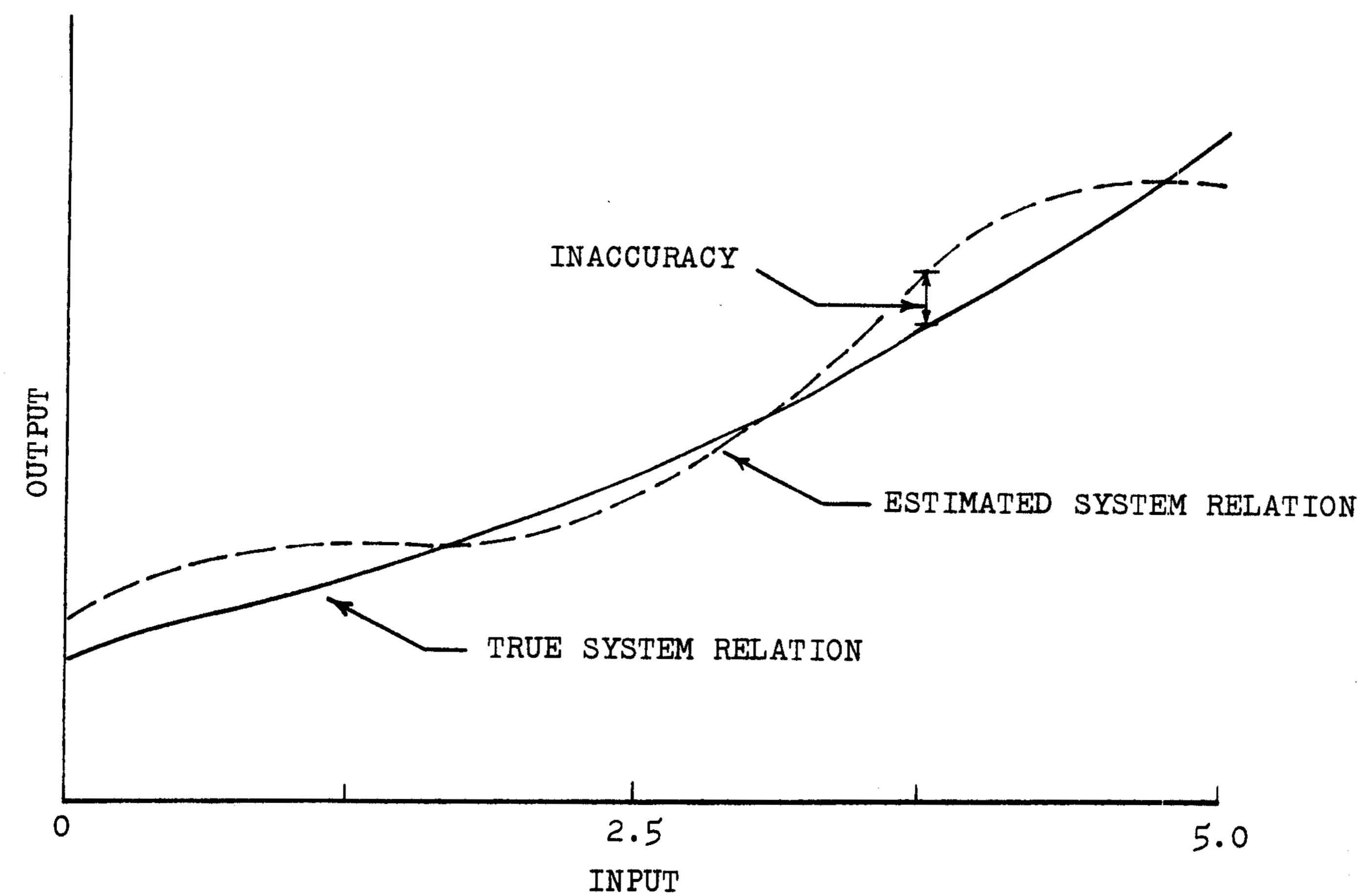

FIGURE 2. CONCEPTUAL PLOT OF SYSTEM ACCURACY

3. Present Linearity Test Procedure. One procedure currently being used to check the linearity of subsystem components is as follows[2].

(1) Input known voltages into the system ordinarily at standard calibration levels.

(2) Determine a digitized output value for each level of input.

(3) Fit the "ideal straight line" on a graph of output digits versus input voltage by drawing a straight line to connect the zero percent point to the 100 percent output point. The sample size for determining the points is not specified but the points apparently represent a mean for some time period as specified for checking a specific component.

(4) Add the greatest positive deviation and the greatest negative deviation from the "ideal straight line" and divide the sum by two to obtain an average.

(5) Divide the value determined in step (4) by the scale range of the "ideal straight line" and multiply by 100 to convert to a percentage.

(6) If the value expressed in percent is equal to or less than $\pm$ 0.15% accept the derived line as an acceptable model.

This procedure was not considered acceptable for experimental purposes for several reasons including:

(1) The line thus derived from the zero and 100 percent points does not reflect the information available from the other points. All of the points contain a certain amount of systematic and

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[2] The procedure is specified in "SCO-101D, 15 May, 1963, Specification Subcarrier Oscillator".

random errors. The "ideal straignt line" connecting the zero and 100 percent points does not use the knowledge of these errors.

(2) The line determined cannot be mathematically related to the population from which it was drawn. For example the coefficients are not necessarily unbiased and minimum variance estimates of the universe parameters.

Therefore, in this experiment it was decided to use some other procedure to obtain an acceptable model for the relation between input and output.

B. THE EXPERIMENTAL DESIGN

1. Experimental Conditions.

a. INPUT. Two different types of input were employed in the linearity experiment, voltage and frequency. For a test of the entire XO-4 (FM/FM) system a 21 step voltage function was used to simulate the input from a transducer through the XO-4 package to the receiver. The 21 steps were generated over a zero to five volt range in 0.25 volt increments by a d.c. voltage standard accurate to six decimal places.

To isolate the non-linear component, if any, associated with the package, input was provided to the receiver by a Hewlett-Packard frequency generator. In this case 11 frequency steps were generated from the lower to upper band edge of each of the 17 channels. The 11 frequency values for each channel are presented in Table 1.

The two types of input were primarily used to isolate the non-linear component, if any, in the system. A schematic diagram of the system for the two inputs is given in Figure 3. It can be seen from the figure that both the package system and frequency standard system were identically the same from the receiver through the printer.

TABLE 1

FREQUENCY DEVIATION SCHEDULE ABOUT CENTER FREQUENCY BY CHANNELS

Step	Channel								
	2	3	4	5	6	7	8	9	10
1	520	675	890	1200	1575	2125	2775	3600	5000
2	528	686	904	1220	1600	2160	2820	3660	5080
3	536	697	918	1240	1625	2195	2865	3720	5160
4	544	708	932	1260	1650	2230	2910	3780	5240
5	552	719	946	1280	1675	2265	2955	3840	5320
6	560	730	960	1300	1700	2300	3000	3900	5400
7	568	741	974	1320	1725	2335	3045	3960	5480
8	576	752	988	1340	1750	2370	3090	4020	5560
9	584	763	1002	1360	1775	2405	3135	4080	5640
10	592	774	1016	1380	1800	2440	3180	4140	5720
11	600	785	1030	1400	1825	2475	3225	4200	5800

Step	Channel							
	11	12	13	14	15	16	17	18
1	6800	9700	13,400	20,350	27,750	37,000	48,550	64,750
2	6910	9860	13,620	20,680	28,200	37,600	49,340	65,800
3	7020	10,020	13,840	21,010	28,650	38,200	50,130	66,850
4	7130	10,180	14,060	21,340	29,100	38,800	50,920	67,900
5	7240	10,340	14,280	21,670	29,550	39,400	51,710	68,950
6	7350	10,500	14,500	22,000	30,000	40,000	52,500	70,000
7	7460	10,660	14,720	22,330	30,450	40,600	53,290	71,050
8	7570	10,820	14,940	22,660	30,900	41,200	54,080	72,100
9	7680	10,980	15,160	22,990	31,350	41,800	54,870	73,150
10	7790	11,140	15,380	23,320	31,800	42,400	55,660	74,200
11	7900	11,300	15,600	23,650	32,250	43,000	56,450	75,250

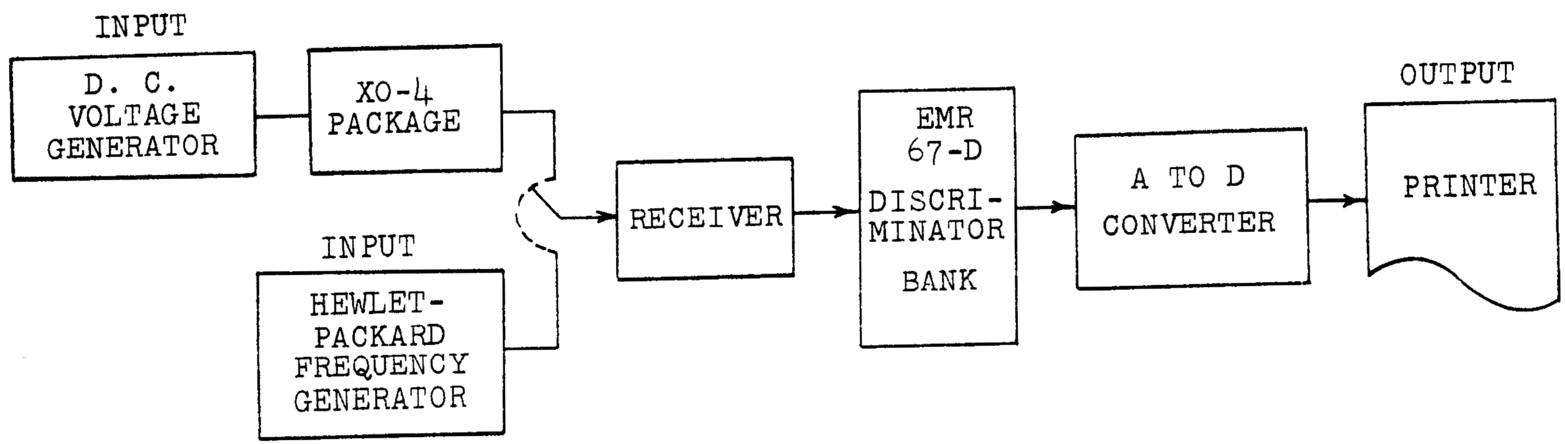

FIGURE 3. SCHEMATIC DIAGRAM OF THE SYSTEM UNDER THE TWO EXPERIMENTAL CONDITIONS

Hence, any difference in the two systems will reflect effects preceding the receiver.

b. OUTPUT. The output of the system was a set of dimensionless numbers (counts). The range of the digitization and printing equipment was from zero to 1024 counts. Therefore, for this experiment effort was made to maintain the origin at 24 counts and the range at 975 counts. This enabled the experimenters to detect any shifts at the end points. Thirty samples were taken at each level from which means were computed and retained for subsequent analysis.

2. Mathematical Models.

a. SYSTEM MODEL. Under the assumption of linearity of the FM/FM telemetry system a model relating system input to output is as follows:

$$O_i = \alpha_o + \alpha_1 I_i + e_i \qquad [1]$$

where,

$\{O_i\}$ is a set of observed output values from the system.

α_o and α_1 are the universe parameters of the appropriate linear relationship for the system.

I_i is the ith value of the controlled input to the system.

$\{e_i\}$ is a set of independent random variables which represent the unpredictable and uncontrolled effects of the system. The e_i are assumed to be drawn from a normal population with zero mean and variance σ^2, i.e. $NID(o,\sigma^2)$.

In practice the α's must be empirically estimated from experimental data for the system. Such an estimating relation might be,

$$O_i = a_o + a_1 I_i \qquad [2]$$

where,

a_o and a_1 are estimates of the universe parameters α_o and α_1.

In the data reduction process it is necessary to infer inputs for given outputs. Therefore, for data reduction equation [2] would be in the form

$$I_i = (O_i - a_o)/a_1. \qquad [3]$$

However, since either of the above equations may be used to determine the linearity characteristics of the system we will use the form of equation [2] in this report. Equation [2] can be extended to provide models for higher degree polynomials as follows:

$$O_i = a_o + a_1 I_i + a_2 I_i^2 + \ldots + a_r I_i^r. \qquad [2a]$$

Models through the fifth degree will be considered in the latter part of this chapter.

b. LEAST SQUARES. In the analysis of the experimental data the method of least squares was used to construct mathematical models for the environment. This method is more desirable than others as it gives unbiased and minimum variance estimates of the universe parameters. This method is so named because through the method of least squares we obtain the curve which minimizes the squared deviations between the actual and computed points:

$$\Sigma d^2 = \Sigma(Y - \hat{Y})^2 = \text{a minimum.}$$

The Y's are the observed (actual) points and the $\hat{Y} = a_o + a_1 bX$ are the computed points (for a first degree curve).

To illustrate the method of least squares consider the situation where it is desired to fit a straight line to a set of experimental data. To determine estimates of a_o and a_1 we form the expression,

$$\Sigma d^2 = \Sigma(Y - \hat{Y})^2 = \Sigma(Y - a_o - a_1 X)^2.$$

To minimize the above expression partial derivatives of this expression with respect to a_o and a_1 are set equal to zero. This mathematical manipulation results in the following normal equations:

$$a_o N + a_1 \Sigma X = \Sigma Y \qquad [4]$$

$$a_o \Sigma X + a_1 \Sigma X^2 = \Sigma XY \qquad [5]$$

Inasmuch as a_o and a_1 are the only unknown quantities in equations [4] and [5], their values may be determined by simultaneous solution of these two equations. To determine the least squares second degree curve for the data, it is necessary to add another term, $a_2 X^2$, to the expression for Σd^2 and rederive the set of normal equations. This may be continued for any degree curve desired. Table 2 lists a summary of the normal equations through the fifth degree.

TABLE 2

NORMAL EQUATIONS THROUGH THE FIFTH DEGREE*

		Linear	Quadratic	Cubic	Quartic	Quintic	
	$a_o N$	$+ a_1 \Sigma X$	$+ a_2 \Sigma X^2$	$+ a_3 \Sigma X^3$	$+ a_4 \Sigma X^4$	$+ a_5 \Sigma X^5$	$= \Sigma Y$
Linear	$a_o \Sigma X$	$+ a_1 \Sigma X^2$	$+ a_2 \Sigma X^3$	$+ a_3 \Sigma X^4$	$+ a_4 \Sigma X^5$	$+ a_5 \Sigma X^6$	$= \Sigma XY$
Quadratic	$a_o \Sigma X^2$	$+ a_1 \Sigma X^3$	$+ a_2 \Sigma X^4$	$+ a_3 \Sigma X^5$	$+ a_4 \Sigma X^6$	$+ a_5 \Sigma X^7$	$= \Sigma X^2 Y$
Cubic	$a_o \Sigma X^3$	$+ a_1 \Sigma X^4$	$+ a_2 \Sigma X^5$	$+ a_3 \Sigma X^6$	$+ a_4 \Sigma X^7$	$+ a_5 \Sigma X^8$	$= \Sigma X^3 Y$
Quartic	$a_o \Sigma X^4$	$+ a_1 \Sigma X^5$	$+ a_2 \Sigma X^6$	$+ a_3 \Sigma X^7$	$+ a_4 \Sigma X^8$	$+ a_5 \Sigma X^9$	$= \Sigma X^4 Y$
Quintic	$a_o \Sigma X^5$	$+ a_1 \Sigma X^6$	$+ a_2 \Sigma X^7$	$+ a_3 \Sigma X^8$	$+ a_4 \Sigma X^9$	$+ a_5 \Sigma X^{10}$	$= \Sigma X^5 Y$

* For each degree curve those terms to the left of and above the indicated lines are equated to the corresponding terms to the right of the equal sign.

3. Criteria for Model Selection.

a. INTRODUCTION. Experience indicates that the FM/FM telemetry system may be moderately non-linear. Therefore, some criterion must be established to determine the deviation permissible before the hypothesis of linearity is rejected. Further, if the linearity assumption is rejected the criterion must be applicable to the testing of the appropriateness of a higher degree model for the system. From graphs of past data it appears that the non-linear effect is only slight. Therefore it was decided that the analysis of this non-linear effect should not continue past the fitting of a fifth degree (quintic) curve.

In order to select the appropriate model from the mathematical models available some measure of effectiveness for each of the models must be obtained. Two such measures often associated with least square curves are the standard error of the estimate and the coefficient of correlation.

b. STANDARD ERROR OF THE ESTIMATE. The standard error of the estimate ($S_{Y.X}$) is a measure of the amount of variation of the actual values of a dependent variable from its estimated or computed values. It is defined more precisely as:

$$S_{Y.X} = \sqrt{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 / N} \qquad [6]$$

where,

Y_i = the observed value

$\hat{Y}_i$ = the computed value from the regression equation

N = the number of observations.

This measure indicates the variation which has not been explained by the estimating equation. As in the case of a sample variance it is necessary to unbias $S^2_{Y.X}$ to obtain a reasonable estimate of the variation expected in the total population. An unbiased estimate of $\sigma^2_{Y.X}$ of the universe is

$$\hat{S}^2_{Y.X} = S^2_{Y.X}\left(\frac{N}{N-m}\right) \qquad [7]$$

where,

m = the number of coefficients in the estimating equation.

Because of the nature of least squares regression analysis $S_{Y.X}$ must either remain constant or decrease as each higher degree curve is fitted to the data. The standard error cannot increase. If the next higher degree equation offers no better fit, then the corresponding coefficient is zero and $S_{Y.X}$ will remain constant. However, the unbiased standard error, $\hat{S}_{Y.X}$, may increase as well as remain constant or decrease.

Consideration was given to using the unbiased standard error to determine the appropriate mathematical model for the system. The appropriate model would be that degree equation for which

$$\hat{S}_{Y.X(k-1)} > \hat{S}_{Y.X(k)} \leq \hat{S}_{Y.X(k+1)} \qquad [8]$$

where,

k = the degree of the estimating equation.

However, such a measure of appropriateness of the system model was not acceptable because in some cases $\hat{S}_{Y.X(k+2)}$ may be less than $\hat{S}_{Y.X(k)}$ even though $\hat{S}_{Y.X(k+1)}$ is not.

The unbiased standard error is an acceptable measure of variation once the system model has been selected.

c. COEFFICIENT OF CORRELATION. The coefficient of correlation, r, is used to measure the degree of relationship between variables and is independent of the units or terms in which the variables are expressed. However, the coefficient of correlation is not independent of the standard error of the estimate. The relation between the two is

$$r^2 = 1 - S^2_{Y.X} / S^2_Y \quad . \qquad [9]$$

Perfect correlation is indicated by $r = \pm 1$. When there is no functional relationship between the variables then $r = 0$.

As in the case of the standard error the correlation coefficient is a biased estimate. An unbiased estimate of the universe correlation coefficient is given by

$$\bar{r}^2 = \frac{r^2(N-1) - (m-1)}{N - m} \quad . \qquad [10]$$

With the knowledge (from equation [9]) that r^2 is <u>inversely</u> proportional to $S^2_{Y.X}$ it is not surprising to find that r must continue to increase or remain constant for each higher degree equation. Analogous to $\hat{S}_{Y.X}$, $\bar{r}$ is not restricted to increase or remain constant.

As a measure of effectiveness for selection of the appropriate regression model we could choose that degree equation for which

$$\bar{r}_{(k-1)} < \bar{r}_{(k)} \geq \bar{r}_{(k+1)} \quad . \qquad [11]$$

As before such a measure was found to be unacceptable since even though equation [11] may be satisfied $\bar{r}_{(k+2)}$ could also be greater than $\bar{r}_{(k)}$.

d. ANALYSIS OF VARIANCE. The technique of analysis of variance can be used to test the significance of each of the coefficients of an estimating equation. For example, analysis of variance may be used to determine if a significant linear trend exists among the data. The analysis of variance technique may also be used to determine whether a second degree curve provides a significantly better fit to the data than a linear model. This technique together with regression analysis was used in the linearity experiment described in this section. As we are only interested in those effects through the fifth degree we will assume that those effects higher than the fifth degree are negligible. These effects will be included in the residual (error) sum of squares. By testing all of the effects at once it will be possible to circumvent certain problems associated with using analysis of variance with regression analysis[3].

The analysis of variance is presented in Table 3. There is only one degree of freedom associated with each effect. To test the significance of each effect it is necessary to divide the appropriate mean square by the residual (error) mean square term. This ratio is compared to values of the theoretical F-distribution with the appropriate degrees of freedom. Significance of a particular degree effect is indicated when its corresponding variance ratio is greater than the theoretical value of the F-distribution.

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[3]If, for example, each effect is tested against its own error term then the problem arises as to how many effects must be non-significant before we discontinue testing higher degree effects.

TABLE 3

THE ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	d.f.	Mean Square	Variance Ratio
Linear	SS_{a_1}	1	s_1^2	s_1^2/s_e^2
Quadratic	SS_{a_2}	1	s_2^2	s_2^2/s_e^2
Cubic	SS_{a_3}	1	s_3^2	s_3^2/s_e^2
Quartic	SS_{a_4}	1	s_4^2	s_4^2/s_e^2
Quintic	SS_{a_5}	1	s_5^2	s_5^2/s_e^2
Error	$\Sigma(Y-\hat{Y})^2$	N-6	s_e^2	
Total	$\Sigma(Y-Y)^2$	N-1		

4. Restrictions. The linearity experiment was performed according to the following rules and restraints:

(1) A production quality XO-4 package would be used in the total system tests. An "accurate" Hewlett-Packard frequency generator would be used in the subsystem tests.

(2) A 21 step calibration function would be input to the XO-4 package when it was under test. The 21 voltage steps would be simulated by a d.c. voltage standard accurate to six decimal places. An 11 step calibration function would be input to the receiver by the frequency generator for the subsystem tests.

(3) The linearity experiment would be performed under "low noise" conditions, i.e., all channels except the one under test would be set at center frequency.

(4) The digitization of the output signal would be under real time conditions. The printing rate would be at five samples per second. Sufficient time would be allowed to obtain 30 samples for each calibration step.

C. THE ANALYSIS

1. Introduction. The tests were performed for each of the 17 channels and each of the two conditions of input producing 34 sets of data. Tables 4 and 5 present the means of 30 samples at each level.

A curvilinear regression program written for the Univac Solid State 80 Computer was utilized to obtain the necessary quantities for the analyses. The program was run in the double precision mode and gave the quantities S_y and a value of $S_{Y.X}$ for each of the five degree curves fitted to the data.

TABLE 4

LINEARITY DATA FOR SYSTEM CONTAINING XO-4 PACKAGE

	Channel							
Volts	2	3	4	5	6	7	8	9
0.00	24	23	27	27	43	46	29	25
0.25	73	72	75	75	89	91	78	73
0.50	122	121	123	124	136	137	126	122
0.75	170	170	172	171	183	183	174	171
1.00	219	219	220	220	229	230	223	219
1.25	268	267	269	268	276	276	271	268
1.50	316	316	318	317	323	322	319	317
1.75	365	364	366	366	369	369	367	366
2.00	414	413	415	415	417	417	415	414
2.25	462	462	463	463	464	464	464	463
2.50	512	510	512	511	512	512	512	512
2.75	560	559	560	560	559	560	561	560
3.00	608	608	609	610	607	608	609	608
3.25	657	657	657	658	655	656	657	657
3.50	706	705	706	707	703	704	706	705
3.75	754	754	755	756	752	752	755	755
4.00	803	803	804	804	800	799	803	804
4.25	851	852	852	854	849	849	851	854
4.50	899	901	901	902	899	898	901	903
4.75	949	950	950	951	948	948	949	953
5.00	997	999	999	998	998	996	998	1002

TABLE 4 (continued)

	Channel								
Volts	10	11	12	13	14	15	16	17	18
0.00	24	28	32	29	33	31	28	24	18
0.25	72	77	80	77	80	80	77	73	67
0.50	121	125	128	125	128	127	125	122	117
0.75	170	173	175	174	176	176	173	171	167
1.00	219	222	224	221	224	224	221	220	217
1.25	268	270	271	270	271	271	270	269	266
1.50	317	319	319	319	319	319	318	318	316
1.75	366	367	368	367	367	369	366	367	364
2.00	415	415	416	415	415	415	414	415	415
2.25	463	464	464	463	464	464	463	463	463
2.50	512	512	513	512	512	512	511	512	514
2.75	561	560	561	561	560	561	560	561	562
3.00	609	609	609	609	609	608	608	609	611
3.25	658	658	657	658	657	658	656	658	659
3.50	707	707	706	706	706	708	706	707	709
3.75	755	755	754	755	755	755	755	755	757
4.00	804	804	803	803	803	804	804	804	806
4.25	853	853	851	852	852	852	853	852	855
4.50	902	902	901	901	901	902	902	901	903
4.75	951	951	949	949	950	951	951	950	951
5.00	999	1000	998	998	999	999	999	998	1001

TABLE 5

LINEARITY DATA
FOR SYSTEM CONTAINING FREQUENCY STANDARD

Step*	Channel							
	2	3	4	5	6	7	8	9
1	999	999	999	999	1000	996	996	1000
2	907	900	898	902	903	903	897	903
3	812	799	803	805	808	805	799	804
4	712	707	702	709	702	704	701	708
5	613	609	607	609	612	608	603	609
6	518	507	508	512	512	508	507	512
7	421	412	409	414	412	415	407	414
8	329	315	309	318	316	317	310	318
9	224	219	213	217	218	214	213	220
10	133	125	115	120	120	121	117	124
11	34	26	17	24	24	24	21	26

* Based on response characteristics of channel.

TABLE 5 (continued)

Step*	Channel								
	10	11	12	13	14	15	16	17	18
1	999	999	999	999	999	999	999	999	999
2	900	901	903	903	902	902	903	902	902
3	804	803	806	805	804	805	806	803	804
4	707	706	709	707	707	707	708	706	706
5	608	608	613	609	610	609	611	608	608
6	512	511	515	512	512	512	513	511	510
7	415	414	418	414	415	415	415	414	413
8	318	317	320	317	317	318	318	316	315
9	220	219	222	219	220	220	221	218	216
10	123	122	124	122	122	123	124	121	119
11	26	25	29	24	25	26	28	24	21

* Based on response characteristics of channel.

2. Computations. The sums of squares were obtained from the results of the regression program by utilizing a derived computational procedure. The procedure will be illustrated by an example set of data. Table 6 presents the results of the regression program for a sample of data.

TABLE 6

REGRESSION ESTIMATES FROM DATA OF XO-4 PACKAGE USING CHANNEL 2

Degree (k)	$S_{Y.X(k)}$
1	0.44516
2	0.39593
3	0.39592
4	0.38887
5	0.37318
N = 21	S_Y = 294.53174

The necessary computations are as follows.

(1) The total sum of squares

$$SS_Y = NS_Y^2 = 21\ (294.53174)^2$$

$$= 1,821,727.86327$$

(2) The linear sum of squares

$$SS_{a_1} = NS_Y^2 - NS^2_{Y.X(1)}$$

$$= 21\ (294.53174)^2 - 21\ (0.44516)^2$$

$$= 1,821,723.70170$$

(3) The quadratic sum of squares

$$SS_{a_2} = NS^2_{Y.X(1)} - NS^2_{Y.X(2)}$$

$$= 21(0.44516)^2 - 21(0.39593)^2$$

$$= 0.86961$$

(4) The cubic sum of squares

$$SS_{a_3} = NS^2_{Y.X(2)} - NS^2_{Y.X(4)}$$

$$= 21(0.39593)^2 - 21(0.39592)^2$$

$$= 0.00021$$

(5) The quartic sum of squares

$$SS_{a_4} = NS^2_{Y.X(3)} - NS^2_{Y.X(4)}$$

$$= 21(0.39592)^2 - 21(0.38887)^2$$

$$= 0.11613$$

(6) The quintic sum of squares

$$SS_{a_5} = NS^2_{Y.X(4)} - NS^2_{Y.X(5)}$$

$$= 21(0.38887)^2 - 21(0.37318)^2$$

$$= 0.25116$$

(7) The residual or error sum of squares

$$SS_e = NS^2_{Y.X(5)}$$

$$= 21(0.37318)^2$$

$$= 2.92446$$

These calculations are summarized and presented in standard form in Table 7. Since only the linear effect is significant it is concluded that the linear model provides an adequate fit for these data.

3. <u>XO-4 Package System</u>. The regression analysis and analyses of variance were performed on the XO-4 package data of Table 4. The results of the tests are presented in Table 8. In general the total system appears to be non-linear. Although a few channels exhibited cubic or higher degree effects the tests indicate a much stronger quadratic effect. All but one of the 17 channels displayed a significant quadratic effect at the α = .01 level.

If a second degree equation is used to explain the relationship between input and output of the system then the expected variation would be given by $S^2_{Y.X(2)}$. Using a mean range of 969.824 an estimate of the average varaition in percent of range becomes,

$$\overline{S}^2_{Y.X(2)} = \sum_{i=2}^{18} \hat{S}^2_{Y.X(2)i}/17 = 0.26261$$

and

$$\%\ \text{Variation} = 100\sqrt{\overline{S}^2_{Y.X(2)}}/\overline{R} = 0.053$$

TABLE 7

THE ANALYSIS OF VARIANCE FOR THE DATA OF TABLE 6

Source of Variation	Sum of Squares	d.f.	Mean Square	Variance Ratio	$F_{.05}$	$F_{.01}$
Linear	1,821,723.70170	1	1,821,723.70170	9,344,089.57	4.54	8.68
Quadratic	0.86961	1	0.86961	4.46	4.54	8.68
Cubic	0.00021	1	0.00021	.00	4.54	8.68
Quartic	0.11613	1	0.11613	.60	4.54	8.68
Quintic	0.25116	1	0.25116	1.29	4.54	8.68
Error	2.92446	15	0.19496			
Total	1,821,727.86327	20				

TABLE 8

SUMMARY OF SIGNIFICANCE TESTS
FOR XO-4 PACKAGE

Channel	Linear	Quadratic	Cubic	Quartic	Quintic
2	*				
3	*	*	*		
4	*	*			
5	*	*	*		
6	*	*	*		
7	*	*			
8	*	*			
9	*	*	*		*
10	*	*			*
11	*	*			
12	*	*			
13	*	*			
14	*	*			
15	*	*			
16	*	*		*	
17	*	*	*		
18	*	*			

* Significance at $\alpha = .01$

4. <u>Frequency Standard System</u>. The results of the regression analysis and analyses of variance of Table 5 are summarized in Table 9. Only two channels exhibited non-linear effects at the $\alpha = .01$ level. Hence, we may conclude that the frequency standard subsystem is linear.

We may determine an estimate of the variation about the linear model from the standard error, $S_{Y.X(1)}$. Using a mean range of 973.824 an estimate of the average variation becomes,

$$\bar{S}^2_{Y.X(1)} = \sum_{i=2}^{18} \hat{S}^2_{Y.X(1)}{}_i/17 = 1.70770$$

and

$$\% \text{ Variation} = 100\sqrt{\bar{S}^2_{Y.X(1)}}\Big/\bar{R} = 0.134$$

This represents the residual variation which remains unexplained after the linear model has been fitted to the means.

TABLE 9

SUMMARY OF SIGNIFICANCE TESTS
FOR FREQUENCY STANDARD

Channel	Linear	Quadratic	Cubic	Quartic	Quintic
2	*				
3	*				
4	*				
5	*				
6	*				
7	*				
8	*	*			
9	*				
10	*				
11	*				
12	*	*	*		
13	*				
14	*				
15	*				
16	*				
17	*				
18	*				

*Significance at $\alpha = .01$

5. Isolation of the Non-linear Component. The important conclusions of II-C-3 and II-C-4 were as follows.

(1) The total system was non-linear.

(2) The frequency standard subsystem was linear.

Recall, from Figure 3 , that the two systems employed identically the same network from the receiver through the printer. Therefore, we may further conclude that the non-linear effect indicated in the total (XO-4 package) system must result before the receiver. There are two pieces of equipment in the total system before the receiver: the d.c. voltage standard and the XO-4 package. Inasmuch as each setting was made on the d.c. voltage standard accurate to six decimal places this instrument was adequately linear for these analyses. This leads to the conclusion that the non-linear effect of the total system exists within the XO-4 package.

It would be meaningful to visualize the way in which the quadratic model is oriented to the data for the total system. Figure 4 exhibits a conceptual plot of the general relationship of the appropriate second degree equation to the linear model. It can be seen from the figure that the quadratic curve has a positive acceleration over the range of the data.

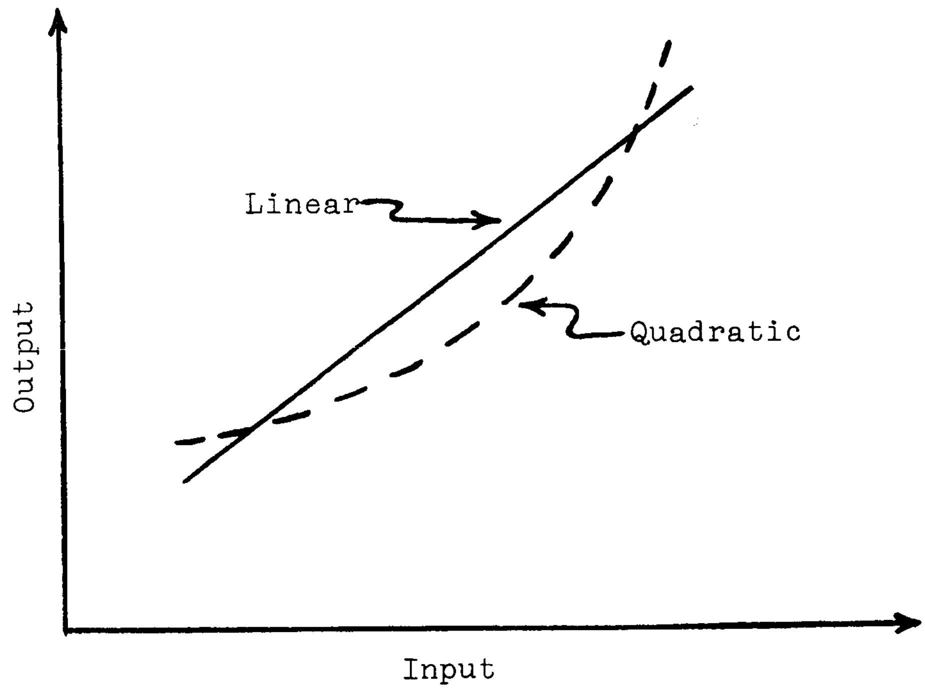

FIGURE 4 . CONCEPTUAL PLOT
OF THE ORIENTATION OF THE
NON-LINEAR EFFECT OF THE SYSTEM

The two important facts which are of importance to telemetry engineers in the redesign of the system are as follows.

(1) The non-linear effect exists in the XO-4 package.

(2) The non-linear effect is one of positive acceleration.

SECTION III

ANALYSIS OF THE EFFECTS OF ANALOG TAPE RECORDERS AND TAPE SPEED COMPENSATION

A. EXPERIMENTAL CONDITIONS

An experiment to analyze the effects of analog tape recorders and the effects of tape speed compensation on the FM/FM Telemetry System was conducted in the Telemetry Laboratory of the Marshall Space Flight Center. The experiment was supervised by experienced ground station personnel and all equipment setups were thoroughly checked before actual data recording began. Much of the actual setting up of the equipment and the regulation of the instruments was done by the personnel of the Systems Engineering Group. A schematic diagram of the telemetry system for this experiment is presented in Figure 5.

Only one package, an XO-4, was used in this experiment and it was supplied voltage by a DC voltage standard. Five sets of subcarrier oscillators (SCO's) were used on the package. In addition an AC frequency standard was used as a sixth input source of data. Each input source was tested at five voltage levels, 0.00, 1.25, 2.50, 3.75, 5.00 volts, with the voltages being stepped through in sequence from 0.00 to 5.00 volts. The frequency standard was used to simulate the voltage input levels by equating 0% full scale output to 0.00 volt, 25% full scale output to 1.25 volt, 50% full scale output to 2.50 volts, 75% full scale output to 3.75 volts, and 100% full scale output to 5.00 volts.

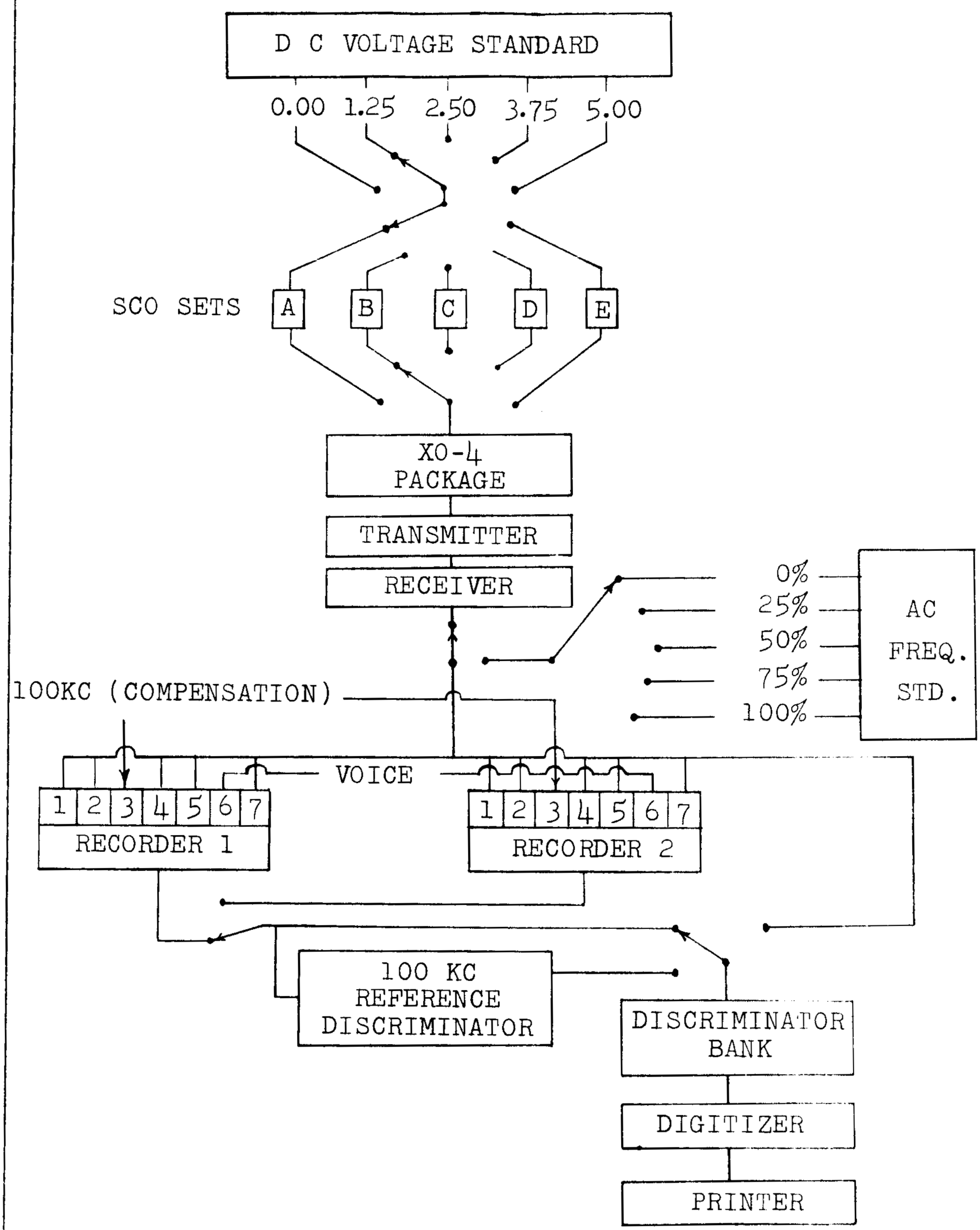

FIGURE 5. SCHEMATIC DIAGRAM FOR AN EXPERIMENT INVOLVING THE FM/FM TELEMETRY SYSTEM

Each input was also tested on each of the standard IRIG telemetry channels 2 through 18 under "low noise" conditions.[1] Thus, for each input source there were five calibration levels and 17 channels for a total of 85 sets of "real time" data.[2] Five sets of SCO's were used in this experiment for two reasons; to insure that additional data would be available if some of the data contained spurious results, and also to provide additional data if replications of SCO's were subsequently recommended. However, only one set of SCO's and the frequency standard were analyzed as sources of input.

The identification of the data was by input source, the various sets of SCO's were labeled A, B, C, D, E, and the frequency standard was labeled F. The data generated by SCO set A was chosen to be analyzed altogether with that generated by the frequency standard. It was felt that one set of SCO's and one package would suffice since the analyses performed in Technical Report No. 2[3] indicated that the average effects are constant from one set of SCO's to the next and from one package to the next.

In addition to the real time data two Ampex analog tape recorders were used to record the data simultaneously. The selection of input source and channel under test was randomized and the sequence of the tests is shown in Table 10.

---

[1] "Low noise" conditions mean that each channel not under test is supplied with 2.50 volts.

[2] "Real time" data is data which is printed out at the time of testing. Data which is recorded on tape and later reproduced is not real time data.

[3] Griffin, Marvin A. and Simpson, Richard S. Accuracy Analysis of FM/FM Telemetry System for the Saturn Vehicle. University, Alabama: University of Alabama, Bureau of Engineering Research, October, 1963.

TABLE 10

RANDOMIZED SEQUENCE OF THE TESTS

Sequence		1	2	3	4	5	6
		SCO B	SCO C	SCO E	FREQ REF STD	SCO D	SCO A
1	CHANNEL	18	16	7	12	11	2
2		4	12	2	7	4	12
3		2	4	5	10	13	6
4		17	10	12	3	2	15
5		15	8	16	2	18	3
6		10	5	14	15	14	8
7		3	3	8	5	7	11
8		11	6	4	17	16	5
9		12	17	10	13	3	4
10		5	15	18	14	15	13
11		9	14	9	18	10	7
12		16	11	11	8	12	16
13		8	13	13	11	9	10
14		7	18	6	16	17	18
15		13	7	17	9	8	17
16		6	9	3	4	5	9
17		14	2	15	6	6	14

The tapes used for recording were 7 track, 1/2 inch wide, 1-1/2 mils thick analog recording tape. Six reels of tape were used - one for each input source. Three reels of tape were Ampex brand and three were Scotch brand. A 100 KC reference sine wave was recorded on track 3 to be used for tape speed compensation. Track 6 was used for an audio identification of the data on the tapes. The actual data was recorded on the remaining tracks (1, 2, 4, 5, and 7). When the tapes were played back the data was taken from track 4 (the same data was on all of the data tracks.)

In order to analyze the effects of tape speed compensation and the effects of the tape recorders, each tape was replayed both with and without tape speed compensation. Tape speed compensation was accomplished by using a 100 KC reference discriminator and the standard compensation adjustments. When no compensation was being used this reference discriminator was completely disconnected so as to avoid any bias which it might introduce.

Each time a sample of data was being taken the printer was allowed to print for approximately five seconds. Since the printing rate was about four words per second[4], approximately 20 data words were printed on the paper. From this data a sample of 10 consecutive words were chosen near the middle of the printout. It was felt that this was the best way to obtain a representative sample.

In performing this experiment considerable care was taken to insure that the range of the data generated was kept as constant as possible. This normally required an adjustment of the potentiometer on the digitizer and/or resetting the responses of the subcarrier oscillators prior to each data run. The researchers felt unless a constant range was maintained it would be difficult to make valid comparisons.

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[4]The printer available at the time the experiment was run was a small one-channel printer.

B. THE EXPERIMENTAL DATA

The data generated in this experiment was summarized by the means and variances of the samples. Each sample consisted of 10 individual observations. All of the subsequent analyses were performed by the appropriate manipulation of these means and variances.

A large amount of data was generated in this experiment and in order to identify the data a shorthand system of coding was used. This system of coding is shown in Figure 6. As can be seen the coding system is simply a shorthand representation of the path followed by the electrical signal. For instance, SBR2WOCP identifies the signal through subcarrier oscillator set B (SB), tape recorder 2 (R2), reproduced without compensation (WOC), and then into the digitizer and printer (P).

As stated previously the range of the experimental data was kept as constant as possible so that valid comparisons might be made. Table 11 is a summary of the ranges computed from the experimental data. The ranges are only listed for SCO set A and the frequency standard. The reader should observe the stability of the range. The average range for SCO A and the frequency standard is 971.4.

				CODE IDENTIFICATION
FREQUENCY STANDARD			PRINTER	FP
	RECORDER 1	WITH COMPENSATION	PRINTER	FR1WCP
		WITHOUT COMPENSATION	PRINTER	FR1WOCP
	RECORDER 2	WITH COMPENSATION	PRINTER	FR2WCP
		WITHOUT COMPENSATION	PRINTER	FR2WOCP
SCO SET A			PRINTER	SAP
	RECORDER 1	WITH COMPENSATION	PRINTER	SAR1WCP
		WITHOUT COMPENSATION	PRINTER	SAR1WOCP
	RECORDER 2	WITH COMPENSATION	PRINTER	SAR2WCP
		WITHOUT COMPENSATION	PRINTER	SAR2WOCP

FIGURE 6. CODE IDENTIFICATION OF DATA

TABLE 11

RANGES OF THE EXPERIMENTAL DATA

CHANNEL	CODE IDENTIFICATION				
	SAP	FP	SAR1WCP	FR1WCP	SAR2WCP
2	971.0	964.2	971.0	965.0	971.7
3	984.0	978.0	984.2	980.6	982.2
4	961.0	973.8	962.2	973.9	962.0
5	954.2	971.6	952.3	952.8	951.6
6	975.5	975.6	978.5	956.2	980.7
7	935.1	945.7	987.0	974.7	986.0
8	971.9	973.1	970.2	972.9	961.6
9	917.7	975.7	977.6	974.7	973.0
10	970.7	971.3	972.4	973.9	970.2
11	975.2	969.7	971.9	953.4	976.1
12	970.0	973.7	971.2	973.6	971.8
13	966.1	974.0	967.3	976.9	966.5
14	966.7	968.6	971.6	972.3	967.4
15	969.0	974.2	967.4	974.0	971.4
16	973.5	972.1	978.8	974.0	972.9
17	980.1	972.1	982.1	980.6	979.6
18	972.9	970.4	975.5	973.7	974.5
$\bar{R}$	965.6	970.8	973.0	970.8	971.7

TABLE 11 (continued)

CHANNEL	CODE IDENTIFICATION				
	FR2WCP	SAR1WOCP	FR1WOCP	SAR2WOCP	FR2WOCP
2	965.9	968.1	964.2	969.6	960.3
3	979.4	982.8	982.4	979.0	983.1
4	974.8	960.2	958.9	962.8	977.0
5	961.1	952.7	976.0	947.6	974.9
6	956.1	977.0	975.7	975.1	977.0
7	972.5	985.3	975.7	982.5	974.2
8	964.1	972.5	980.8	970.4	961.4
9	975.4	977.4	974.3	977.8	980.0
10	974.3	970.3	971.9	967.8	974.1
11	974.0	974.3	969.6	971.2	962.0
12	971.0	972.7	974.7	972.5	969.3
13	974.4	970.0	975.7	968.0	966.9
14	974.2	971.2	975.3	974.1	986.2
15	975.5	971.8	971.6	965.1	972.5
16	977.9	974.9	967.3	971.0	974.6
17	993.6	979.6	986.2	975.4	968.6
18	973.2	978.3	970.3	978.3	960.3
$\overline{R}$	972.8	972.9	973.6	971.1	971.9

C. ANALYSIS OF THE DATA

1. Test of Means. The analysis of any data is contingent upon the information required. The requirements of this experiment are listed in Section I-D-2 of this report. The analysis of variance technique was used to test the hypothesis of equality of means as listed in the requirements of the experiment. The analyses of the means for this experiment were performed by considering two mathematical models.

a. MODEL I. The first mathematical model considered was,

$$X_{ijk} = \mu + t_i + \theta_j + I_{ij} + e_{ijk}$$

where,

X_{ijk} = the response in digitized units

μ = an overall mean

t_i = a recorder (row) effect, $(i = 1,2,\ldots,r)$

θ_j = a channel (column) effect, $(j = 1,2,\ldots,c)$

I_{ij} = a recorder-channel interaction effect

e_{ijk} = a random experimental error which is normally distributed with zero mean and standard deviation σ', $(k = 1,2,\ldots, g)$, i.e. NID $(0,\sigma'^2)$

In order for an analysis of variance to be valid σ'^2 must be homogeneous over all classifications. A Bartlett's test of homogeneity of variances was used to test this assumption. It was found that the variances were not homogeneous across channels. An example of one of the Bartlett's tests is shown in Table 12 and a summary of all the Bartlett's tests is presented in Table 13 .

Since the primary objection to Model I was that channels were considered as a main classification it was decided that the analyses of variance should be performed independently for each channel. This was the basis for Model II.

TABLE 12

AN EXAMPLE OF BARTLETT'S TEST FOR THE FREQUENCY STANDARD

Data Was Reproduced Without Compensation on Recorder 1.

Input Level - 0% Full Scale

Channel	σ_i^2	s_i^2	Log s_i^2	n_iLog s_i^2	$n_i\ s_i^2$
2	1.61	1.79	0.2529	2.2761	16.11
3	1.29	1.43	0.1553	1.3977	12.87
4	7.81	8.68	0.9385	8.4465	78.12
5	0.69	0.77	9.8865-10	88.9786-90	6.93
6	30.21	33.57	1.5260	13.7340	302.13
7	2.09	2.32	0.3655	3.2895	20.88
8	13.04	14.49	1.1611	10.4499	130.41
9	21.41	23.79	1.3764	12.3876	214.11
10	18.36	20.40	1.3069	11.7864	183.60
11	17.60	19.56	1.2914	11.6226	176.04
12	6.21	6.90	0.8388	7.5492	62.10
13	18.76	20.84	1.3189	11.8701	187.56
14	9.16	10.18	1.0078	9.0702	91.62
15	7.61	8.46	0.9274	8.3466	76.14
16	29.64	32.93	1.5176	13.6584	296.37
17	26.49	29.43	1.4687	13.2183	264.87
18	13.09	14.54	1.1626	10.4634	130.86
				238.5450-90	2250.72

$\Sigma\, n_i = 17(9) = 153 = n \qquad g = 17$

$M = 2.3026\ [n \text{ Log } (\Sigma\, n_i s_i^2/n) - \Sigma\, n_i \text{ Log } s_i^2]$

$= 2.3026\ [153 \text{ Log } (2250.72/153) - (238.5450 - 90)] = 69.30$

$C = 1 + \frac{1}{3(g-1)}\ [\Sigma\ \frac{1}{n_i} - \frac{1}{n}] = 1 + \frac{1}{3(17-1)}\ [\frac{17}{9} - \frac{1}{153}] = 1.039$

$M/C = 69.30/1.039 = 66.70 \qquad \text{Ho: } \sigma_2'^2 = \sigma_3'^2 = \ldots \sigma_{18}'^2$

$\chi^2_{.05}\ (n' = 17-1 = 16) = 26.30 \qquad$ Reject Ho since $M/C > \chi^2_{.05}$

TABLE 13

M/C VALUES FOR THE BARTLETT'S TESTS
OF HOMOGENEITY OF VARIANCES RUN ON
THE FREQUENCY STANDARD AND SUBCARRIER OSCILLATOR A

(Critical Values are $\chi^2_{.05,16} = 26.30$, $\chi^2_{.01,16} = 32.00$)

Code Identification	Level				
	0%	25%	50%	75%	100%
FR1WCP	72.19	92.00	126.53	79.13	375.11
FR1WOCP	66.70	100.95	67.72	94.52	54.52
FR2WCP	158.32	212.68	270.39	196.03	197.52
FR2WOCP	36.23	54.78	67.06	42.75	55.41
SAR1WCP	54.65	62.34	101.48	80.12	99.43
SAR1WOCP	84.40	70.77	76.54	62.44	77.22
SAR2WCP	139.68	135.09	108.39	92.67	130.64
SAR2WOCP	61.68	44.96	58.55	33.06	62.94

b. MODEL II. The second mathematical model considered was,

$$X_{ijk} = \mu + t_i + \theta_j + I_{ij} + e_{ijk}$$

where,

X_{ijk} = the response in digitized units

μ = an overall mean

t_i = a recorder (row) effect, $(i=1,2,\ldots,r)$

θ_j = a compensation (column) effect, $(j=1,2,\ldots,c)$

I_{ij} = a recorder-compensation interaction effect

e_{ijk} = a random experimental error assumed $\sim NID(o,\sigma'^2)$.

In this model it is not necessary that the error variances be homogeneous across channels because each channel is considered separately. However, when Bartlett's tests were performed on the data in the classification of Model II these tests indicated non-homogeneity of variances for most cases, but the departure from homogeneity was not as extreme as in Model I. Most statisticians agree that the requirement of homogeneous variances is not too restrictive for an analysis of variance. This means that if this assumption is not met the consequences are not serious in most cases.[5] If this assumption is not met the risk of the first type (α) is altered. It will be seen later that even large deviations in α will not affect the conclusions of the significance tests for most of the

[5] For a discussion of this point, see reference (Scheffe, pp. 351-359).

analyses of variance arrayed in Model II. For this reason, it was decided that Model II was the best model to test the assumption of equal means for the experimental data.

An example set of data for Model II is given in Table 14 and the analysis of variance calculations are summarized, complete with variance estimates, for the example data in Table 15. It can be seen that the critical F value for the main effects is $F_{.05} = 161.00$. The computed F ratio was normally very much smaller than this. For the interaction term the critical F value is $F_{.05} = 4.14$. The computed F ratio sometimes exceeded this figure but, in general, it was less. Thus, a moderate change in the critical F value would not greatly affect the results of the analyses of variance.

The analysis of variance of the data was performed in the manner described and illustrated in Technical Report No. 2. The only difference in the computations was that the number of rows and columns and the group size were different. Also, the data of Technical Report No. 2 were summarized as means and standard deviations (σ) whereas the data of this experiment were summarized as means and variances (σ^2).

A summary of the results of the tests of significance using the analysis of variance and Model II is presented in Table A-1 of Appendix A for the frequency standard and Table A-2 of the same appendix for SCO set A. All significance tests for these analyses of variance were performed at the $\alpha = 0.05$ level of significance. A composite summary of the results of the significance tests of Tables A-1 and A-2 is given in Table 16. A summary of the estimates of the components of variance is presented in Table A-3 of Appendix A for the frequency standard and Table A-4 of the same appendix for SCO set A.

TABLE 14

AN EXAMPLE SET OF DATA

FOR ANALYSIS OF VARIANCE USING MODEL II

Data Was Generated by

The Frequency Standard, Channel 2, at 0.00 Volt Input

		COMPENSATION	
		WITH	WITHOUT
Recorder 1	$\bar{X}$ σ^2	28.20 0.56	28.30 1.61
Recorder 2	$\bar{X}$ σ^2	27.00 0.00	29.90 36.29

TABLE 15

THE ANALYSIS OF VARIANCE FOR THE DATA OF TABLE 14

Source of Variation	Sum of Squares	d.f.	Mean Square	Variance Ratio	$F_{0.05}$
Recorder-Main Effect	0.40	1	s_1^2 0.40	s_1^2/s_3^2 0.02	161.00
Compensation-Main Effect	22.50	1	s_2^2 22.50	s_2^2/s_3^2 1.15	161.00
Recorder-Comp Interaction	19.60	1	s_3^2 19.60	s_3^2/s_e^2 1.84	4.14
Experimental Error	384.60	36	s_e^2 10.68		
TOTAL	427.10	39			

$\hat{\sigma}^2 = 10.68$ $\quad \hat{\sigma}_t^2 = 0.00$ $\quad \hat{\sigma}_\theta^2 = 0.14$ $\quad \hat{\sigma}_I^2 = 0.89$

$\hat{\sigma}^2_{(\text{Response})} = 11.71$ $\qquad$ RANGE = 971.7

$\hat{\sigma}_{(\text{Response})}(100)/\text{RANGE} = 0.35\%$

TABLE 16

COMPOSITE SUMMARY
OF THE SIGNIFICANCE TESTS FOR MODEL II
All Tests Performed at $\alpha = 0.05$

Source of Variation	Number of Non-Significant Ratios	Number of Significant Ratios	Total Number of Ratios
Recorder	164	6	170
Compensation	164	6	170
Interaction	135	35	170

In general, the analyses of variance summarized in Table 16 indicate that the two main effects and the interaction effect are non-significant. The average or main effect of the two recorders is not generally significant. The average or main effects of recording with compensation is not generally significantly different from the main effects of recording without compensation. The recorder-compensation interaction effect is not generally significant.

2. Tests of Variance. The analysis of variance is a method of testing for significant difference of means. It is desirable that telemetry equipment yield the same mean response for a given input. However, it is more important to determine some measure of the noise, i.e., variability, associated with the mean value of the response. Thus, the question becomes, "Does tape speed compensation significantly affect the variance of data in the data reduction stage of a telemetry system?"

The question posed above was investigated by computing a series of individual variance ratios. Each variande ratio, $\sigma^2_{WOC}/\sigma^2_{WC}$, is a ratio of the variance computed for the data without compesnation, σ^2_{WOC}, to the variance computed for the data with compensation, σ^2_{WC}. The results of the variance ratios which were performed are presented in Tables A-5 through A-8 of Appendix A. A summary of the tests of significance is presented in Table 17. For the indication of significant ratios the symbol (*) denotes that the variance ratio was significantly large ($\sigma^2_{WOC} > \sigma^2_{WC}$) and the symbol (+) denotes that the variance ratio was significantly small ($\sigma^2_{WOC} < \sigma^2_{WC}$). If neither of these symbols appears after a variance ratio then the two variances in that particular ratio are not significantly different. All tests were performed at the $\alpha = 0.05$ level of significance.

It can be seen from Table 17 that only seven times in a total of 340 tests the data reproduced with compensation was more variable than the data reproduced without compensation. Conversely, in 182 of the 340 tests the data reproduced without compensation was more variable than the data reproduced with compensation. In 151 cases tested there was no significant difference in the two variances. Thus, it is logical to conclude that tape speed compensation often helps to reduce variability in the recording process. Even in those cases where variability is not reduced neither is it usually increased. Therefore, it appears advantageous to always employ compensation when reproducing data from tape recorders of this type.

It is also of interest to determine if one recorder introduces more noise than the other recorder. As in the previous analysis this question was approached by a series of individual variance ratios. Each variance ratio, σ^2_2/σ^2_1,

TABLE 17

SUMMARY OF SIGNIFICANCE
OF VARIANCE RATIOS COMPILED IN TABLES A-5 THROUGH A-8
(The Connotation of the Symbols of Significance
Are as Defined in Text, p. 58)

		Significance			
		*	+	None	
Frequency Standard	Recorder 1	29	2	54	
	Recorder 2	59	2	24	
Subcarrier Oscillator A	Recorder 1	39	3	43	
	Recorder 2	55	0	30	
	TOTAL	182	7	151	340

is a ratio of the variance for the data from Recorder 2, σ_2^2, to the variance for the data from Recorder 1. σ_1^2. The results of these variance ratio tests are presented in Tables A-9 through A-12 of Appendix A. For the indication of significant ratios the symbol (*) denotes that the variance ratio was significantly large ($\sigma_2^2 > \sigma_1^2$) and the symbol (+) denotes that the variance ratio was significantly small ($\sigma_2^2 < \sigma_1^2$). If neither of these symbols appears after a variance ratio then the two variances in that particular ratio are not significantly different. All tests were performed at the $\sigma = 0.05$ level of significance. A summary of the tests of significance is presented in Table 18.

From Table 18 it can be seen that Recorder 1 rarely had a significantly larger variance than Recorder 2 (only

5 of 340 tests). However, in 140 of 340 tests Recorder 2 had a significantly larger variance than Recorder 1. In 195 tests the two variances were not significantly different. Thus, there is statistical evidence that Recorder 2 introduces more variability into the data reduction system than does Recorder 1.

TABLE 18

SUMMARY OF SIGNIFICANCE
OF VARIANCE RATIOS COMPILED IN TABLES A-9 THROUGH A-12
(The Connotation of the Symbols of Significance
Are as Defined in Text, p. 59)

	Compensation	Significance			
		*	+	None	
Frequency Standard	With	22	3	60	
	Without	54	0	31	
Subcarrier Oscillator A	With	22	2	61	
	Without	42	0	43	
	TOTAL	140	5	195	340

In the analyses of the two tape recorders it should be pointed out that the effect of any differences in magnetic tapes were confounded within the effect of the tape recorders. The variability in question is the effect, then, of not only the tape recorder, but also of the particular tape used on the recorder. It was assumed that there were no significant differences between various magnetic tapes. It is recommended that this assumption be statistically tested in some future experiment(s).

SECTION IV

ISOLATION OF SYSTEM ERRORS

A. INTRODUCTION

1. Reproducibility. When repeated measurements exhibit erratic patterns of variation, the method of measurement used is not reproducible. It has already been shown that the residual errors across all channels and input levels indicated a non-homogeneity of variances, therefore, any pooling of variance estimates across channels is not strictly valid since these measurements were not reproducible in that classification. In order to make a system of measurement reproducible, efforts should be made to find and eliminate assignable causes for data which are not in statistical control. Statistical control is best examined by means of Shewart control charts.

2. Theory of Control Charts. One of the primary uses of control charts is to determine whether statistical control has been maintained within a system. Statistical control is determined by the nature of the many effects which cause variation in the result on an experimental investigation.

Knowledge of the behavior of chance variations is the foundation on which control charts analysis rests. These chance variations are the sum of many complex, probabilistic causes. The effect of each of these causes is usually slight and no major part of the total variation can be traced to a single cause. Conversely, assignable causes may be thought of as deterministic in nature. These are normally relatively large variations that are attributable to special causes. An assignable cause might be a significant difference in instruments, in workers, or in their relationships to one another.

If a group of data is studied and it is found that their variation conforms to a statistical pattern that might reasonably be produced by chance causes, then it is assumed that no special assignable causes are present. The conditions which produced this variation are said to be "within control". On the other hand if the variations in the data do not conform to a pattern that might reasonably be produced by chance causes, then it is concluded that one or more assignable causes are at work. In this case the conditions producing the variation are said to be "out of control".

In control chart theory it is assumed that the chance variations, being ordered in time or in some other manner, will behave in a random manner and will form a normal distribution. The control limits are usually determined so that if chance causes alone are at work the probability that a point will fall above the upper limit is 0.001, and the probability that a point will fall below the lower limit is 0.001. If the system of chance causes produces a variation that follows the normal curve, the 0.001 probability limits are practically equivalent to 3σ limits. For a normal distribution the probability that a deviation from the mean will exceed 3σ is 0.0027 considering both positive and negative deviations. For normal variation, therefore, 3σ limits are the practical equivalent of 0.001 probability limits since these yield a probability of 0.002 that a deviation will be "detected".

3. <u>Control Charts For VARIANCES</u>. In order to construct a control chart we must compute a center line, an upper control limit (UCL) and a lower control limit (LCL). The center line for a control chart is the expected value of the statistic being plotted. For a σ^2 control chart the center line is given by,

$$E(\sigma^2) = \bar{\sigma}^2$$

or $$E(\sigma^2) = \sigma'^2(n-1)/n$$

where,

E = the expected value operator

$\bar{\sigma}^2$ = the average sample variance

σ'^2 = the universe variance.

In order to determine the control limits we must know the standard deviation of the distribution of sample variances. When the distribution of individual values is normally distributed the standard deviation of the distribution of sample variances, σ_{σ^2}, is approximately

$$\sigma_{\sigma^2} = \sigma'^2 \sqrt{2(n-1)}\ /n.$$

The control limits for a σ^2 control chart are then geven by

$$E(\sigma^2) + 2\sigma_{\sigma^2} = \sigma'^2(n-1)/n \pm 3\sigma'^2 \sqrt{2(n-1)}\ /n$$

$$= \sigma'^2(n-1)/n\ [1 \pm \sqrt{18/(n-1)}\ ]$$

$$= E(\sigma^2)\ [1 \pm \sqrt{18/(n-1)}\ ].$$

The expected value of the sample variance is equal to the average sample variance, i. e., $E(\sigma^2) = \bar{\sigma}^2$. For a sample size of $n = 10$, the limits then become,

$$E(\sigma^2)\ [1 \pm \sqrt{18/9}\ ] = \bar{\sigma}^2\ (1 \pm \sqrt{2}\ )$$

$$\text{UCL} = 2.414\ \bar{\sigma}^2 \qquad \text{LCL} = 0$$

4. <u>Method of Analysis</u>. The following section of this report will be concerned with the isolation of the error of some of the component subsystems of the FM/FM telemetry system. The methodology for the isolation of these errors can best be explained by means of an example.

As an illustration, let us consider how one might isolate the reduction in total system variability which is accomplished when tape speed compensation is used. The path followed by the electrical signal when the magnetic tape of the frequency standard data is reproduced on Recorder 1 without compensation (FR1WOCP) is represented by the block diagram of Figure 7.

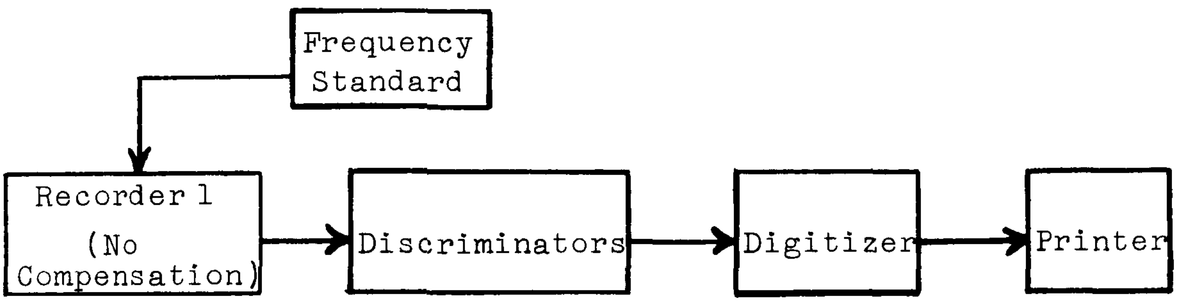

FIGURE 7. BLOCK DIAGRAM OF THE SYSTEM WHEN DATA OF THE FREQUENCY STANDARD IS REPRODUCED ON RECORDER 1 WITHOUT COMPENSATION.

The system of Figure 7 will produce data which has a variance caused by several system components. Let us denote the total variance of the data from this system as $\sigma^2_{FR1WOCP}$. One way of stating the components of $\sigma^2_{FR1WOCP}$ is,

$$\sigma^2_{FR1WOCP} = \sigma^2_{R1} + \sigma^2_{WOC} + \sigma^2_{d} + \sigma^2_{e}$$

where,

σ^2_{R1} = the variance introduced by Recorder 1, the magnetic tape and associated recording circuitry

σ^2_{WOC} = the variance which would be reduced by the introduction of the compensation network

σ^2_{d} = the variance introduced by the discriminators, the digitizer, the printer, and associated circuitry

σ^2_{e} = an experimental error.

It can be seen that σ^2_{R1} and σ^2_{d} can, in turn, be considered as containing more than one component. However, in this experiment these variances cannot be further separated. It has been assumed throughout the report that the frequency generator is a "precise" simulator of the voltage inputs and thus no component of variance is introduced by this piece of equipment. The term σ^2_{e} consists of many components - operator differences, "drift" in the electrical hardware, random errors, etc.

Next, consider the path followed by the electrical signal when the magnetic tape of the frequency standard data is reproduced on Recorder 1 with compensation (FR1WCP). This may be visualized by the block diagram of Figure 8 .

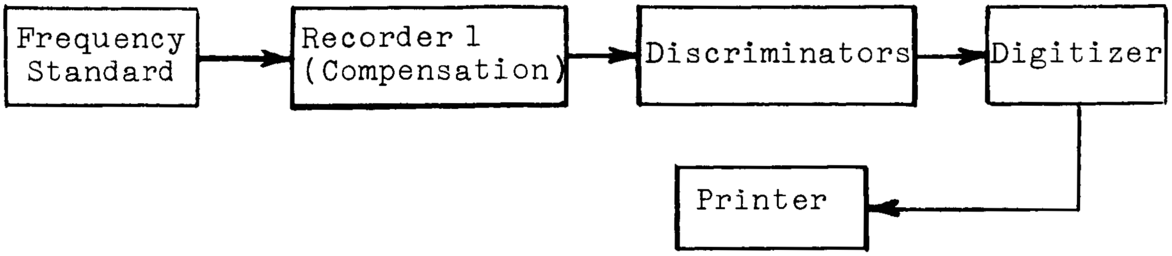

FIGURE 8. BLOCK DIAGRAM OF THE SYSTEM WHEN DATA OF THE FREQUENCY STANDARD IS REPRODUCED ON RECORDER 1 WITH COMPENSATION.

Let us denote the variance of the data from the system presented in Figure 8 as σ^2_{FR1WCP}. One way of stating the components of this variance is

$$\sigma^2_{FR1WCP} = \sigma^2_{R1} + \sigma^2_{WC} + \sigma^2_{d} + \sigma^2_{e}$$

where,

σ^2_{R1}, σ^2_{d}, and σ^2_{e} are as previously defined, and

σ^2_{WC} = the variance introduced by the compensation network.

In order to determine the improvement in variability which is accomplished by tape speed compensation we may proceed as follows:

$$\sigma^2_{FR1WOCP} - \sigma^2_{FR1WCP} = (\sigma^2_{R1} + \sigma^2_{WOC} + \sigma^2_{d} + \sigma^2_{e}) -$$

$$(\sigma^2_{R1} + \sigma^2_{WC} + \sigma^2_{d} + \sigma^2_{e}) = \sigma^2_{WOC} - \sigma^2_{WC} .$$

The term $\sigma^2_{WOC} - \sigma^2_{WC}$ represents an increase in system error which is caused by not using tape speed compensation in the reproduction of magnetic recording tape.

We will let $\sigma^2_{WOC} - \sigma^2_{WC} = \sigma^2_{C}$.

In order that the subtraction of the two variances, i.e. $\sigma^2_{FR1WOCP} - \sigma^2_{FR1WCP}$, be strictly valid each variance must be in statistical control. To determine statistical control we will use the σ^2 control chart which was discussed in Section IV - A-3. To illustrate the process we will use the actual experimental data to determine a reproducible measure of $\sigma^2_{WOC} - \sigma^2_{WC}$.

The variances of the FR1WOCP data are presented in Table 19 . The control chart computations for this data are presented in Table 20 . The average of the 85 variances of Table 19 is $\bar{\sigma}^2 = 12.10$ giving control limits of UCL = 29.21 and LCL = 0.00. Five variances are outside of these limits and were removed as being out of control. A revised average for the 80 remaining variances is $\bar{\sigma}^2 = 10.08$ and the revised control limits are UCL = 24.34 and LCL = 0.00. The data are then examined and three variances are seen to be out of control, i.e. greater than the UCL. The removal of out of control variances is continued until only 75 of the original 85 variances were retained. These variances are now in statistical control. The final revised average variance is $\bar{\sigma}^2_{FR1WOCP} = 9.02$.

The control chart analysis is also performed upon the variances of the FR1WCP data. The data are presented in Table 21 and the control chart computations are summarized in Table 22 . It can be seen from Table 22 that 46 of the original 85 variances are not in statistical control and are subsequently discarded. This indicates that the telemetry system for this experimental condition was considerably beyond the limits of reproducibility. The final revised average sample variance is $\bar{\sigma}^2_{FR1WCP} = 0.82$. This indicates that if the system could be made reproducible the sample variance would be reduced from 10.14 to 0.82 for the particular

TABLE 19

THE VARIANCES OF THE FR1WOCP DATA

	INPUT LEVEL	(VOLTS)			
Channel	0.00	1.25	2.50	3.75	5.00
2	1,61	0.61	0.41	0.44	0.85
3	1.29	0.41	1.29	0.44	0.41
4	7.81	1.00	3.60	106.09	8.96
5	0.69	1.40	6.41	5.20	3.29
6	30.21	33.89	12.36	22.45	8.44
7	2.09	1.81	1.85	3.20	4.64
8	13.04	6.44	25.29	11.56	20.96
9	21.41	13.36	5.04	17.20	10.40
10	18.36	2.45	5.24	7.44	4.09
11	17.60	8.24	1.69	14.84	13.84
12	6.21	6.64	4.89	7.41	3.00
13	18.76	11.69	4.84	14.76	20.69
14	9.16	30.16	8.44	23.84	11.85
15	7.61	16.61	10.81	16.41	13.29
16	29.64	19.80	18.85	12.41	14.01
17	26.49	18.76	21.60	16.45	15.81
18	13.09	24.45	17.84	12.76	8.04

TABLE 20

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE 19

N	$\bar{\sigma}^2$	UCL = 2.414 $\bar{\sigma}^2$	LCL = 0.00 $\bar{\sigma}^2$
85	12.10	29.21	0.00
80	10.08	24.34	0.00
77	9.38	22.65	0.00
76	9.19	22.19	0.00
75	9.02	21.77	0.00

TABLE 21

THE VARIANCES OF THE FR1WCP DATA

	INPUT LEVEL (VOLTS)				
Channel	0.00	1.25	2.50	3.75	5.00
2	0.56	0.25	0.24	0.24	0.16
3	0.69	0.09	0.24	0.16	0.25
4	0.36	0.09	0.21	1.24	0.09
5	3.64	1.76	1.09	2.36	1.04
6	19.45	16.80	16.41	8.96	22.21
7	1.56	1.45	1.04	1.80	1.09
8	10.24	7.81	22.04	11.65	18.01
9	0.96	1.24	0.76	0.69	0.69
10	2.29	1.01	0.41	0.49	1.36
11	5.76	4.89	2.61	2.44	488.16
12	1.60	0.80	0.81	2.96	0.84
13	4.04	1.20	3.49	7.21	4.25
14	5.36	4.65	4.96	6.44	6.25
15	4.84	15.49	4.36	5.16	1.84
16	9.04	5.24	5.24	6.49	4.24
17	6.96	14.65	4.20	5.65	4.04
18	4.89	4.40	5.41	4.41	1.64

TABLE 22

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE 21

N	$\bar{\sigma}^2$	UCL = 2.414 $\bar{\sigma}^2$	LCL = 0.00 $\bar{\sigma}^2$
85	10.14	24.48	0.00
84	4.45	10.74	0.00
75	2.90	7.00	0.00
70	2.49	6.01	0.00
66	2.24	5.41	0.00
64	2.13	5.14	0.00
59	1.86	4.49	0.00
54	1.59	3.84	0.00
46	1.13	2.73	0.00
43	0.97	2.34	0.00
40	0.86	2.08	0.00
39	0.82	1.98	0.00

experimental conditions (FR1WCP) using a sample size of 10 and an average range of 971.4.

The net improvement resulting from using tape speed compensation is determined as the difference between the two variances (which are now within statistical control):

$$\bar{\sigma}_C^2 = \bar{\sigma}_{FR1WOCP}^2 - \bar{\sigma}_{FR1WCP}^2 = 9.02 - 0.82 = 8.20.$$

To obtain an unbiased variance estimate, S_C^2, we correct for the sample size[1].

$$S_C^2 = (10/9)(\bar{\sigma}_C^2) = (10/9)(8.20) = 9.11$$

$$S_C = \sqrt{9.11} = 3.02$$

The error expressed as a percent of the average range is:

$$100\ S_C/\bar{R} = 100(3.02)/971.4 = 0.311\%$$

where,

$\bar{R}$ = the average range of all the experimental data from the frequency standard and SCO set A.

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[1] In this and subsequent calculations the symbol S will be used to denote an unbiased estimate of σ. Two useful formulas are $S^2 = \frac{\Sigma(X_i - \bar{X})^2}{n-1}$ and $S^2 = \sigma^2[n/(n-1)]$.

B. ERRORS OF SYSTEM COMPONENTS

1. <u>Errors of the Discriminators, Digitizer, Printer, and Associated Circuitry.</u> The real time data for the frequency standard (FP) may be used to estimate the precision of that part of the telemetry system which includes the discriminators, the digitizer, the printer, and the associated circuitry between the frequency standard and the printer. This can be visualized by considering the block diagram of Figure 9 .

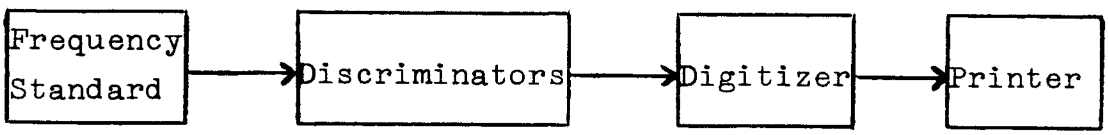

FIGURE 9. BLOCK DIAGRAM OF SYSTEM COMPONENTS FOR THE REAL TIME DATA OF THE FREQUENCY STANDARD

The frequency standard was used to simulate the output of a package and a set of SCO's. The frequencies were varied in percent of full scale as follows: 0%, 25%, 50%, 75%, and 100%. These frequency levels simulated DC voltage inputs to the package of 0.00, 1.25, 2.50, 3.75, and 5.00 volts, respectively. Thus, circumventing the package and SCO's and associated circuitry, we can directly analyze the noise contributed by that part of the system beyond the package. This consists of the discriminators, the digitizer, the printer, and associated circuitry. This analysis is valid, of course, only if the frequency standard is assumed to be errorless. This assumption was made based upon the hypothesis that it was a "good" standard.

In order to obtain a measure of the error of that part of the system described above an average variance was taken over all input levels and all channels.

This resulted in averaging 85 sample variances to obtain:

$$\bar{\sigma}_d^2 = \sum_{i=2}^{18} \sum_{j=1}^{5} \sigma_d^2 \, ij/85 = 0.212$$

where,

$\bar{\sigma}_d^2$ = the variance of the FP data averaged over all input levels and all channels.

An unbiased sample standard deviation, S_d, is obtained by correcting for the sample size[2].

$$S_d^2 = \frac{10}{9} (0.212) = 0.236$$

$$S_d = \sqrt{0.236} = 0.486$$

Expressed as a percent of full scale the error contributed by this portion of the system (assuming reproducibility) may be expressed as:

$$100 \, \bar{S}_d / \bar{R} = 100(0.486)/971.4 = 0.050\%.$$

As was previously explained the foregoing calculations were made based upon the assumption that the system is reproducible. The variance estimate, $\bar{\sigma}_d^2$, was subsequently checked to test the hypothesis of reproducibility. This variance was not in statistical control and a control chart analysis of the type illustrated in Section IV-A-4 was performed. The 85 variances are listed in Table A-13 of the Appendix and the control chart analysis is summarized in Table A-14 of the Appendix. As a result of the control chart analysis, 13 of the original 85 variances were discarded as non-homogeneous.

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[2] Ibid. pg. 70

The final estimate of the system error which would be obtained as a result of a reproducible system is computed as follows:

$$\bar{\sigma}_d^2 = 10.09/72 = 0.140$$

$$\bar{s}_d^2 = 0.140(10/9) = 0.156$$

$$\bar{s}_d = \sqrt{0.156} = 0.395$$

$$100\ \bar{s}_d/\bar{R} = 100(0.395)/971.4 = 0.041\%.$$

It can be seen that a net reduction of error of 0.050% - 0.041% = 0.009% may be obtained by stabilizing the system so that it is within statistical control. This is done by finding assignable causes for data which are out of control and adjusting the system accordingly.

It should be pointed out that the error computed in this section is not a pure error. It contains not only errors produced by the system components but also errors introduced by the operators and other small errors which combine to form an experimental error. These experimental errors are confounded in a complex and inextricable manner, therefore, no attempt will be made to isolate them. Great care was taken to attempt to minimize the experimental error of this experiment and it is felt that the errors incurred in the data are primarily system component errors. In the following sections the experimental error is not included in any estimates of error since it is subtracted in the manner illustrated by the example of Section IV-A-4.

2. <u>Error Associated with the Package, SCO's, and Associated Circuitry</u>. The real time data for SCO set A (SAP) yields a variance which contains an error contributed by the package and SCO's plus the error which was isolated in the previous section. This is illustrated by the block diagram of Figure 10 .

DC Voltage Standard → Package and SCO's → Discriminators → Digitizer → Printer

FIGURE 10. BLOCK DIAGRAM OF THE SYSTEM COMPONENTS FOR THE SAP DATA

If we subtract the variance obtained from the previous section, S_d^2, from the variance obtained from the system above, S_{SAP}^2, we may compute an estimate of the error contributed by the package-SCO unit. In the process, the experimental error will be eliminated since it is common to both variances.

The average variance for the SAP data is computed by averaging the real time data for SCO set A over the 17 channels and the five input levels:

$$\bar{\sigma}_{SAP}^2 = \sum_{i=2}^{18} \sum_{j=1}^{5} \sigma_{SAP\,ij}^2/85 = 56.50/85 = 0.665.$$

In order to obtain an unbiased estimate we correct for the sample size:

$$S_{SAP}^2 = 10/9(0.665) = 0.739.$$

To estimate the desired variance, the appropriate subtraction is performed:

$$S_p^2 = \bar{S}_{SAP}^2 - \bar{S}_d^2 = 0.739 - 0.236 = 0.503$$

where,

$\bar{S}_p^2$ = an unbiased estimate of the variance contributed by the package, the SCO's and associated circuitry.

Therefore, $S_p = \sqrt{0.503} = 0.709$ and the measure of error (assuming reproducibility) is:

$$100\ S_p/\bar{R} = 100(0.709)/971.4 = 0.073\%.$$

Again, we use the control chart approach for establishing statistical control. The control chart computations are given in Table A-16 of Appendix A. After control has been reached 25 of the original 85 variances have been discarded and the results are:

$$\bar{\sigma}_{SAP}^2 = 18.85/65 = 0.290$$

$$\bar{S}_{SAP}^2 = 0.290(10/9) = 0.322$$

$$S_p^2 = \bar{S}_{SAP}^2 - S_d^2 = 0.322 - 0.156 = 0.166$$

$$S_p = \sqrt{0.166} = 0.407.$$

The amount of error which could be expected of a reproducible system is then:

$$100\ S_p/\bar{R} = 100(0.407)/971.4 = 0.042\%.$$

The net improvement in the error which would result in placing the system within statistical control is then:

$$0.073\% - 0.042\% = 0.031\ \%$$

3. Error of the Recorder and Magnetic Tape. In order to determine the contribution of error due to the recorder and magnetic tape we may use the data which was reproduced from the recorders without tape speed compensation. The variance of this data may be subtracted from the variance of the real time data to obtain the contribution of total error due to the recorders and magnetic tape. This can be seen if we inspect Figure 11 below.

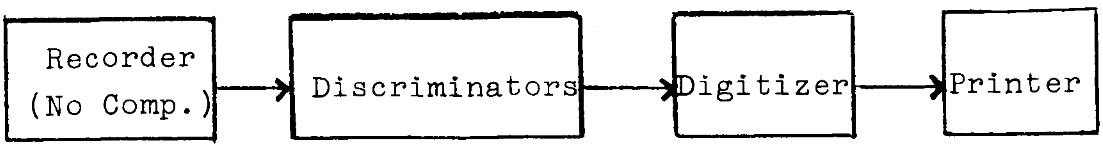

FIGURE 11. BLOCK DIAGRAM OF THE SYSTEM WHEN DATA IS REPRODUCED WITHOUT TAPE SPEED COMPENSATION

We will define the following variances:

σ_y^2 = variance obtained from the system of Figure 11 .

S_y^2 = an unbiased estimate of the universe variance obtained from the system of Figure 11.

S_R^2 = an unbiased estimate of the universe variance attributed to the recorder and magnetic tape

There are four reels of tapes which supplied data for the particular system of Figure 11 . The tapes were produced by the two tape recorders and by the frequency standard and SCO set A. To obtain an average variance we will pool the variances of all four tapes. The computations are (assuming statistical control):

$$\bar{\sigma}_y^2 = \sum_{i=2}^{18} \sum_{j=1}^{5} \sum_{k=1}^{2} \sum_{m=1}^{2} \sigma_y^2\, ijkm/340 = 12{,}463.37/340$$

$$= 36.657$$

$$s_y^2 = (10/9)\ \bar{\sigma}_y^2 = (10/9)36.657 = 40.730$$

$$s_R^2 = s_y^2 - s_d^2 = 40.730 - 0.236 = 40.494$$

$$s_R = \sqrt{40.494} = 6.363$$

$$100\ s_R = 100(6.363)\ /\ 971.4 = 0.655\%.$$

To obtain a measure of the noise which is reproducible we employ the σ^2 control chart technique. The data and the control chart computations for $\bar{\sigma}_y^2$ are presented in Tables A-17 through A-24 of Appendix A. The four variances are based upon different degrees of freedom, thus to pool these variances we do not take a simple average. The computations are (for a system in statistical control):

$$\bar{\sigma}_y^2 = (n_1\bar{\sigma}_{y1}^2 + n_2\bar{\sigma}_{y2}^2 + n_3\bar{\sigma}_{y3}^2 + n_4\bar{\sigma}_{y4}^2)\ /(n_1 + n_2 + n_3 + n_4)$$

where,

$\bar{\sigma}_{y1}^2$, $\bar{\sigma}_{y2}^2$, $\bar{\sigma}_{y3}^2$, $\bar{\sigma}_{y4}^2$ = the average variances of the four tapes pooled over all channels and all input levels.

n_1, n_2, n_3, n_4 = the sample size associated with σ_{y1}^2, σ_{y2}^2, σ_{y3}^2, and σ_{y4}^2 respectively.

$$\bar{\sigma}_y^2 = [75(9.02) + 66(42.60) + 76(11.17) + 68(34.19)]$$

$$/(75 + 66 + 76 + 68) = 6627.75/285 = 23.255$$

$$s_y^2 = (10/9)\ \bar{\sigma}_y^2 = (10/9)\ 23.255 = 25.839$$

$$s_R^2 = s_y^2 - s_d^2 = 25.839 - 0.156 = 25.683$$

$$s_R = \sqrt{25.683} = 5.068$$

$$100\ s_R\ /\ \bar{R} = 100(5.068)/971.4 = 0.522\%.$$

The improvement in recorder error which could be gained by making the system reproducible is:

$$0.655\% - 0.522\% = 0.133\%.$$

4. Reduction of Error Assignable to Compensation. As in the previous sections we can employ a subtractive process to obtain a variance which indicates the reduction of error which may be accomplished by using tape speed compensation. To obtain this variance we simply subtract the variance which is obtained from the data when tape speed compensation is used from the variance obtained when no compensation is used. The former variance is obtained from the data which follow the path indicated by the block diagram of Figure 12.

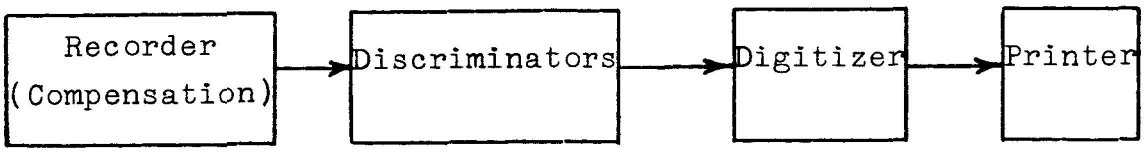

FIGURE 12. BLOCK DIAGRAM OF THE SYSTEM WHEN DATA ARE REPRODUCED WITH TAPE SPEED COMPENSATION

We will define the following variances:

σ_z^2 = variance obtained from the system of Figure 12.

s_z^2 = an unbiased estimate of the universe variance obtained from the system of Figure 12.

s_c^2 = improvement in the system variance when tape speed compensation is employed.

To get an average variance, $\bar{\sigma}_z^2$, we will pool the variances obtained from the two recorders and from the frequency standard and SCO set A. The computations are (assuming statistical control):

$$\bar{\sigma}_z^2 = \sum_{i=2}^{18} \sum_{j=1}^{5} \sum_{k=1}^{2} \sum_{m=1}^{2} \sigma_z^2\, ijkm/340 = 3918.56/340$$

$$= 11.525$$

$$s_z^2 = (10/9)\ 11.525 = 12.806$$

$$s_c^2 = s_y^2 - s_z^2 = 40.730 - 12.806 = 27.924$$

$$s_c = \sqrt{27.924} = 5.284$$

$$100\ s_c/\bar{R} = 100(5.284)/971.4 = 0.544\%.$$

Again, we employ the σ^2 control chart technique to obtain a measure of noise which is reproducible. The data and the control chart computations for $\bar{\sigma}_z^2$ are presented in Tables A-25 through A-32 of Appendix A. The computations are (for a system in statistical control):

$$\bar{\sigma}_z^2 = (n_1\bar{\sigma}_{z1}^2 + n_2\bar{\sigma}_{z2}^2 + n_3\bar{\sigma}_{z3}^2 + n_4\bar{\sigma}_{z4}^2)/(n_1 + n_2 + n_3 + n_4)$$

where,

$\bar{\sigma}_{z1}^2, \bar{\sigma}_{z2}^2, \bar{\sigma}_{z3}^2, \bar{\sigma}_{z4}^2$ = the average variances of the four tapes pooled over all channels and all input levels.

n_1, n_2, n_3, n_4 = the sample size associated with σ_{z1}^2, σ_{z2}^2, σ_{z3}^2, and σ_{z4}^2 respectively.

$$\bar{\sigma}_z^2 = [39(0.82) + 33(1.08) + 84(1.83) + 46(2.18)] / (39 + 33 + 84 + 46) = 321.62/202 = 1.592$$

$$S_z^2 = (10/9)\ \bar{\sigma}_z^2 = (10/9)\ 1.592 = 1.769$$

$$S_c = S_y^2 - S_z^2 = 25.839 - 1.769 = 24.070$$

$$S_c = \sqrt{24.070} = 4.906$$

$$100\ S_c/\bar{R} = 100(4.906)/971.4 = 0.505\%.$$

The improvement in the reduction of error by the use of compensation which could be gained by making the system reproducible is:

$$0.544\% - 0.505\% = 0.039\%.$$

SECTION V

CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY AND CONCLUSIONS: LINEARITY EXPERIMENT.

Results of the linearity experiment have revealed that the FM/FM (XO-4) telemetry system is non-linear. The tests indicated that the appropriate model for the relation between input and output was a quadratic curve. To aid telemetry engineers in the redesign of the system, analyses of subsystem components were performed. From the subsequent analyses it was determined that the non-linear effect existed in the XO-4 package. If the redesign or replacement of the XO-4 package is sufficient to eliminate its non-linear effect then the system will become linear. An estimate of the residual variation about the quadratic curve in percent of range is:

$$100\ \bar{S}^2_{Y.X(2)}/\bar{R} = 0.053\%$$

B. SUMMARY AND CONCLUSIONS: TAPE RECORDER AND COMPENSATION EXPERIMENT.

1. Effect of Analog Tape Recorders. It should be recalled that this experiment did not isolate the effect which different analog tapes might have on a specific recorder. Thus, any conclusions to be drawn concerning tape recorders will necessarily be based upon the assumption that the difference in various magnetic analog recording tapes (of the same specifications) is not significant. Making this assumption, the conclusions concerning different analog tape recorders for this experiment may be summarized as follows:

(a) Analog tape recorders of the type used in this experiment do not differ significantly in their mean response.

(b) The BETWEEN recorder variation may be estimated as:

$$\hat{\sigma}_t^2 = \sum_{i=2}^{18} \sum_{j=1}^{5} \sum_{k=1}^{2} \sigma_t^2 \; ijk/170 = 10.10$$

$$\hat{\sigma}_t = \sqrt{10.10} = 3.178$$

$$100\ \hat{\sigma}_t/\bar{R} = 100(3.178)/971.4 = 0.327\%$$

Since the difference between recorders was not significant, $\hat{\sigma}_t$ may also be assumed to be zero.

(c) Analog tape recorders of the type used in this experiment do tend to differ significantly in the variance of their response. With a sample size of 10 and a mean range of 971.4 the ratio of two recorder variances may be as much as 100 in some cases.

The first conclusion indicates that the linearity characteristics for the type of recorders used are the same. Thus, a mean digitized output of, say 999, from one recorder represents the same voltage input as a digitized output of 999 from the other recorders. This could be important in making calibrations or linearity checks since these are determined on a basis of mean values. If one recorder affects the linearity of the data in a certain manner, e.g., if it has a cubic response, then the other recorders will add the same bias. This could also be important in redesign of recorders for one must simply select a random sample of this type of recorder in order to determine where non-linearity is being introduced.

The second conclusion indicates how much variance may be expected between the mean response of the recorders. Since it was determined previously that the recorders were not significantly different one valid estimate of the variance of the mean response would be zero. However, the analysis of variance is based upon two risks: α, the risk of rejecting a true hypothesis, and β, the risk of accepting a false hypothesis. The first risk, α, is set at 0.05 for this experiment, but β is an unknown subject to the actual conditions under which the experiment was run. Consequently, statements with uncertainty cannot be made but we can make "best estimates" of certain parameters. The best estimate of the variance between recorder means is 0.327 percent of full scale. This means that if the recorders are used over a long period of time, each time randomly selecting a recorder, the variance of the mean response of the recorders is 0.327 percent of full scale.

The third conclusion simply states that the two recorders differ significantly in the noise of their responses. Although the mean value of the responses of the recorders is essentially the same, the variability centered around this mean response is different.

2. Effect of Tape Speed Compensation. Tape speed compensation is used in the recording process in an attempt to control speed errors (e.g., wow and flutter) on a tape recorder. This is accomplished by recording a 100 KC reference sine wave on one track of the magnetic tape simultaneously with the recording of data. When the tape is reproduced the recorder "follows" the sine wave as a reference. An improvement in reproduction will result only if the reference sine wave is a "good" sine wave, i.e., high precision, and if the addition of the extra components needed for compensation (a 100 KC discriminator, extra circuitry, etc.) does not add more error to the system than it compensates. There may be additional errors in the recorder other than those attributable to tape speed and these errors are not theoretically affected by tape speed compensation. Thus, it is

impossible to isolate a pure compensation effect but we can isolate a differential effect between results when compensation is used and when it is not used. Keeping this point in mind, the conclusions concerning tape speed compensation for this experiment may be summarized as follows:

(a) The mean response obtained from an analog signal for a given input is essentially the same whether or not tape speed compensation is used.

(b) The variation BETWEEN the effects of using compensation and not using compensation may be estimated as:

$$\hat{\sigma}_\theta^2 = \sum_{i=2}^{18} \sum_{j=1}^{5} \sum_{k=1}^{2} \sigma_\theta^2 \; ijk/170 = 5.09$$

$$\sigma_\theta = \sqrt{5.09} = 2.256$$

$$100\,\hat{\sigma}_\theta/\bar{R} = 100(2.256)/971.4 = 0.232\%.$$

Since the difference between compensation and no compensation was not significant, $\hat{\sigma}_\theta$ may also be assumed to be zero.

(c) The use of tape speed compensation eliminates more error than it introduces in the analog reproduction process. For a sample size of 10 and a mean range of 971.4 the variance obtained by not using compensation may be as much as several hundred times as large as the variance obtained by using compensation.

C. SUMMARY AND CONCLUSIONS: ISOLATION OF SYSTEM ERRORS.

One of the primary objectives of this experiment was to isolate some of the component subsystems of the FM/FM telemetry system and determine their individual contributions to the overall error of the system. It is believed that some of these errors may have contributed to the high package - SCO interaction effect in Technical Report No. 2.

The isolation of the error of several component subsystems was illustrated in Section IV-B. The component errors which were isolated may be summarized in two categories. The first category is the errors which were actually computed from the experimental data. The second category is the estimated errors which would result if the system were in statistical control. The summary of errors follows.

Estimates of Errors Computed from the Experimental Data

(a) Error associated with the discriminator bank, the digitizer, and associated circuitry:

$$100\ S_d/\overline{R} = 0.050\%$$

(b) Error associated with the package, the SCO's, and associated circuitry:

$$100\ S_p/\overline{R} = 0.073\%$$

(c) Error associated with the analog recorders and analog magnetic tapes:

$$100\ S_r/\overline{R} = 0.655\%$$

(d) Net reduction of error which is accomplished when compensation is used versus when compensation is not used:

$$100\ S_c/\overline{R} = 0.544\%$$

Estimates of Errors Assuming the System Could be Made to Conform to Statistical Control.

(a) Error associated with the discriminator bank, the digitizer, and associated circuitry:

$$100\ S_d/\overline{R} = 0.041\%$$

(b) Error associated with the package, the SCO's, and associated circuitry:

$$100\ S_p/\overline{R} = 0.042\%$$

(c) Error associated with the analog recorders and analog magnetic recording tapes:

$$100\ S_r/\overline{R} = 0.522\%$$

(d) Net reduction of error which is accomplished when compensation is used versus when compensation is not used:

$$100\ S_c/\overline{R} = 0.505\%$$

From the foregoing summary it should be quite obvious that if best results are to be obtained it is important that tape speed compensation be used in the data reduction process. It is also apparent that of the 1.000% error attributed to the FM/FM system in Technical Report No. 2 that a large amount of this error may be attributed to the effect of the analog tape recorders. This latter contribution of error is quite possibly one reason for the large interaction variance which was documented in Technical Report No. 2.

D. RECOMMENDATIONS

1. Linearity Experiment. It is suggested that subsequent research be undertaken to provide insight into the

following areas.

a. XO-4 PACKAGE. Experiments should, if possible, be designed and performed to isolate the non-linear effect within an XO-4 package. Separate consideration should be given to the subcarrier oscillators, the mixer-amplifier, the transmitter, and the internal circuitry of the physical XO-4 package. Results of such experiments might reduce the design requirements to eliminate the non-linear effect in future packages.

b. MATHEMATICAL MODEL. It is desired to develop a method to eliminate the separation of all degree effects at once. Such a method should have the following characteristics.

(1) It should be a sensitive test.

(2) It should indicate at each stage of the testing whether the appropriate model has been obtained.

It is recommended that such a method be developed for determining the appropriate mathematical model for the system. Such a method would prove useful in the data reduction process.

c. ACCURACY ESTIMATE. In Section II-A-1 of this report an indication was presented of how inaccuracy may result in the system. However, no estimate of accuracy has been given in the analysis. It is felt that the standard error associated with the fitting of the mathematical model to the means provides an indication of the accuracy. However, the effects of accuracy are confounded with the effects of sampling error (precision) in the standard error for the means. Therefore, it is recommended that further research be initiated to develop a method of separating an effect which can be attributed to accuracy alone.

2. Tape Recorder and Compensation Experiment. There are recommendations for further study indicated by the tape recorder and compensation experiment. The first recommendation is that a study be conducted to determine if there is any significant difference between analog recording tapes of the type used in the ground station. It will be recalled that in this experiment the effect of magnetic tapes was confounded in the effects of the tape recorders. A study should be conducted to isolate tape effects, if any, from recorder effects. A by-product of an experiment of this nature should be the analysis of any interaction between tapes and tape recorders.

The second recommendation is that an experiment of the nature reported herein be performed and the data tapes then sent to the computation laboratory for reduction. In this way it would be possible to estimate what amount of error is contributed to the system by the data reduction process of the computation laboratory. This experiment would not necessarily be as large scale as the one of this report and it might be performed in conjunction with the analog tape experiment suggested above.

The third recommendation is that the methodology initiated in this report be extended to analyze existing and future ground station systems. It is fully recognized that the experiment of this report utilized several system components which are no longer in operation in the telemetry ground station. It is nonetheless felt that some very pertinent and useful information has been obtained through this experiment and the method of analysis can be extended to analyze future systems.

APPENDIX A

SPECIAL TABLES

TABLE A-1

SUMMARY OF THE TESTS OF SIGNIFICANCE FOR THE FREQUENCY STANDARD USING MODEL II AND $\alpha = 0.05$

(Non-Significant Effects are Denoted by a Blank Space and Significant Effects are Denoted by the Symbol X.)

Channel	Level	Recorder	Compensation	Rec X Comp
	0			
	25			
2	50			
	75			
	100			
	0			X
	25			
3	50			
	75			
	100			
	0			
	25	X	X	
4	50			
	75			
	100			X
	0			
	25			X
5	50			
	75			
	100			X
	0			
	25			
6	50			X
	75			
	100			

TABLE A-1 (continued) FREQUENCY STANDARD

Channel	Level	Recorder	Compensation	Rec X Comp
	0			
	25			
7	50			
	75			
	100			
	0			
	25			
8	50			
	75			X
	100			X
	0			
	25			
9	50		X	
	75			
	100		X	
	0			
	25			X
10	50			
	75			
	100			
	0			
	25	X		
11	50			
	75			X
	100			X
	0			X
	25			X
12	50			X
	75			X
	100			X

TABLE A-1 (continued) FREQUENCY STANDARD

Channel	Level	Recorder	Compensation	Rec X Comp
	0			
	25			
13	50			
	75			
	100			
	0			X
	25			
14	50			
	75			
	100			
	0			
	25			
15	50			
	75			
	100			
	0			
	25			X
16	50			
	75			
	100			
	0			X
	25			
17	50			
	75			X
	100			X
	0			
	25			
18	50			
	75			
	100			

TABLE A-2

SUMMARY OF THE TESTS OF SIGNIFICANCE FOR SCO SET A USING MODEL II AND $\alpha = 0.05$

(Non-Significant Effects are Denoted by a Blank Space and Significant Effects are Denoted by the Symbol X.)

Channel	Level	Recorder	Compensation	Rec X Comp
	0			X
	25			X
2	50			X
	75			
	100			X
	0			
	25			
3	50			X
	75	X	X	
	100			X
	0			
	25			
4	50			
	75			X
	100	X	X	
	0			
	25			
5	50			
	75			
	100	X		
	0			
	25			
6	50			
	75			
	100			

TABLE A-2 (continued) SCO A

Channel	Level	Recorder	Compensation	Rec X Comp
	0			
	25			X
7	50			
	75			X
	100			
	0			X
	25			X
8	50			
	75			
	100			
	0			
	25			
9	50			
	75			
	100			
	0			
	25	X		
10	50			
	75			
	100			
	0			
	25			
11	50		X	
	75			
	100			
	0			
	25			X
12	50			
	75			
	100			

TABLE A-2 (continued) SCO A

Channel	Level	Recorder	Compensation	Rec X Comp
	0			
	25			X
13	50			
	75			
	100			
	0			
	25			
14	50			
	75			
	100			
	0			
	25			
15	50			X
	75			
	100			X
	0			
	25			
16	50			
	75			
	100			
	0			
	25			
17	50			
	75			
	100			
	0			
	25			
18	50			
	75			
	100			

TABLE A-3

SUMMARY OF COMPONENTS OF VARIANCE COMPUTED FROM THE ANALYSES OF VARIANCE OF MODEL II FOR THE FREQUENCY STANDARD

Chan.	Level (%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2$ (Res)	$\frac{100\ \hat{\sigma}'\text{(Res)}}{\text{Range}}$
2	0	10.68	0.00	0.14	0.89	11.71	0.35
	25	9.32	0.00	0.12	0.00	9.44	0.32
	50	5.66	0.00	0.12	0.87	6.65	0.27
	75	11.80	0.00	0.00	0.38	12.18	0.36
	100	6.79	0.34	0.94	0.32	8.39	0.30
3	0	11.87	0.00	2.75	3.87	18.49	0.44
	25	4.09	2.97	0.07	0.00	7.13	0.27
	50	8.98	4.94	0.00	0.00	13.92	0.38
	75	11.79	1.69	2.79	0.00	16.27	0.41
	100	4.21	0.00	0.00	1.27	5.48	0.24
4	0	21.71	5.90	0.00	0.82	28.43	0.55
	25	26.31	2.64	7.22	0.00	36.17	0.62
	50	37.80	3.74	9.90	0.00	51.44	0.74
	75	70.04	0.00	28.50	0.00	98.54	1.02
	100	8.97	0.00	11.92	43.32	64.21	0.82
5	0	32.65	0.00	70.20	0.00	102.85	1.04
	25	12.22	37.17	73.14	10.00	132.53	1.18
	50	51.81	0.00	13.91	11.22	76.94	0.90
	75	27.94	1.42	17.10	3.97	50.43	0.73
	100	17.74	0.32	16.49	8.15	42.70	0.67

TABLE A-3 (continued)

Chan.	Level(%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2$ (Res)	$\frac{100\ \hat{\sigma}' (Res)}{Range}$
6	0	41.14	140.42	178.54	7.45	367.55	1.97
	25	43.47	166.14	43.47	3.77	256.85	1.65
	50	38.95	199.17	18.99	22.11	279.22	1.72
	75	31.55	167.09	0.00	6.77	205.41	1.47
	100	35.94	153.00	0.00	3.70	192.64	1.43
7	0	7.64	0.00	0.00	2.30	9.94	0.32
	25	14.47	0.00	0.00	0.00	14.47	0.39
	50	12.67	1.26	0.15	0.83	14.91	0.40
	75	7.51	0.00	0.00	1.35	8.86	0.31
	100	11.39	0.00	0.00	0.82	12.21	0.36
8	0	38.87	13.65	10.96	0.74	64.22	0.82
	25	100.05	36.92	4.68	0.00	141.65	1.22
	50	161.74	14.40	0.91	0.00	177.05	1.37
	75	86.30	0.00	0.00	29.81	116.11	1.11
	100	81.89	7.95	0.00	47.31	137.15	1.21
9	0	36.60	7.82	14.00	1.40	59.82	0.80
	25	39.32	9.23	28.28	1.13	77.96	0.91
	50	43.42	1.19	5.60	0.00	50.21	0.73
	75	25.94	0.87	7.54	0.00	34.35	0.60
	100	21.22	0.88	6.63	0.00	28.73	0.55
10	0	17.37	0.54	0.00	0.98	18.89	0.45
	25	10.19	0.00	0.00	5.48	15.67	0.41
	50	5.96	0.08	1.62	0.00	7.66	0.28
	75	10.87	0.00	0.00	0.00	10.87	0.34
	100	4.93	0.00	0.00	0.07	5.00	0.23

TABLE A-3 (continued)

Chan.	Level(%)	$\hat{\sigma}'^2$	$\hat{\sigma}_t'^2$	$\hat{\sigma}_\theta'^2$	$\hat{\sigma}_I'^2$	$\hat{\sigma}'^2(\text{Res})$	$100\,\hat{\sigma}'(\text{Res})/\text{Range}$
11	0	30.92	0.00	0.42	0.91	32.25	0.58
	25	0.22	1.89	0.05	0.00	2.16	0.15
	50	88.22	1.26	1.60	0.00	91.08	0.98
	75	49.99	6.54	22.40	24.16	103.09	1.04
	100	170.32	0.00	0.00	242.18	412.50	2.09
12	0	6.68	10.81	0.00	16.14	33.63	0.60
	25	8.36	0.00	0.00	27.79	36.15	0.62
	50	7.16	0.00	0.00	592.21	599.37	2.52
	75	11.75	0.00	0.00	23.33	35.08	0.61
	100	5.96	0.00	0.00	29.65	35.61	0.61
13	0	23.54	0.81	1.71	4.15	30.21	0.57
	25	13.57	0.18	0.00	0.00	13.75	0.38
	50	14.38	0.00	4.42	3.40	22.20	0.48
	75	26.49	0.72	3.19	0.00	30.40	0.57
	100	16.50	3.74	0.54	0.00	20.78	0.47
14	0	13.38	19.11	19.80	7.36	59.65	0.79
	25	49.77	1.84	3.45	0.00	55.06	0.76
	50	27.97	0.35	0.54	0.00	28.86	0.55
	75	35.15	2.55	3.33	0.00	41.03	0.66
	100	34.39	0.00	0.00	0.00	34.39	0.60
15	0	28.46	0.00	0.00	2.68	31.14	0.57
	25	26.41	4.20	7.00	0.00	37.61	0.63
	50	11.65	3.60	0.00	0.00	15.25	0.40
	75	35.61	0.03	0.75	0.00	36.39	0.62
	100	26.12	0.00	0.00	1.59	27.71	0.54

TABLE A-3 (continued)

Chan.	Level(%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2_{(Res)}$	$\frac{100\ \hat{\sigma}'(Res)}{Range}$
16	0	26.06	22.78	0.00	2.23	51.07	0.74
	25	47.96	0.00	0.00	30.61	78.57	0.91
	50	63.30	7.35	17.40	14.37	102.42	1.04
	75	55.60	0.00	10.40	1.73	67.73	0.85
	100	48.14	1.00	19.09	0.00	68.23	0.85
17	0	25.57	0.00	0.00	39.04	64.61	0.83
	25	17.76	5.94	0.69	0.00	24.39	0.51
	50	21.81	0.73	0.00	0.00	22.54	0.49
	75	20.55	0.00	0.00	23.45	44.00	0.68
	100	21.34	0.00	0.00	76.19	97.53	1.02
18	0	24.25	0.00	3.90	4.60	32.75	0.59
	25	32.76	0.00	0.00	0.00	32.76	0.59
	50	67.79	6.11	5.06	9.62	88.58	0.97
	75	20.65	0.00	0.14	0.00	20.79	0.47
	100	55.05	7.48	7.04	0.00	69.57	0.86

TABLE A-4

SUMMARY OF COMPONENTS OF VARIANCE COMPUTED FROM THE ANALYSES OF VARIANCE OF MODEL II FOR SCO SET A.

Chan.	Level (%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2$ (Res)	$\frac{100\ \hat{\sigma}'\text{(Res)}}{\text{Range}}$
2	0	6.15	0.00	0.00	3.00	9.15	0.31
	25	2.72	0.00	0.00	1.29	4.01	0.21
	50	3.73	0.00	0.00	2.52	6.25	0.26
	75	6.04	0.09	0.54	0.00	6.67	0.27
	100	1.61	0.00	1.29	2.09	4.99	0.23
3	0	5.69	0.90	1.17	0.15	7.91	0.29
	25	10.56	0.00	2.66	0.38	13.60	0.38
	50	2.20	0.00	7.36	3.98	13.54	0.38
	75	4.02	0.45	4.65	0.00	9.12	0.31
	100	4.55	0.37	6.67	2.61	14.20	0.39
4	0	10.32	6.60	0.00	1.37	18.29	0.44
	25	6.74	6.12	0.62	1.15	14.63	0.39
	50	5.16	3.82	1.22	1.44	11.64	0.35
	75	16.69	0.00	0.00	9.22	25.91	0.52
	100	8.86	3.77	1.89	0.00	14.52	0.39
5	0	23.59	0.61	1.07	1.25	26.52	0.53
	25	18.17	4.86	0.35	0.00	23.38	0.50
	50	9.58	10.72	4.48	0.00	24.78	0.51
	75	24.21	14.96	0.00	0.00	39.17	0.64
	100	9.77	12.96	0.13	0.00	22.86	0.49

TABLE A-4 (continued)

Chan.	Level(%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2(\text{Res})$	$\frac{100\ \hat{\sigma}'(\text{Res})}{\text{Range}}$
6	0	9.59	1.98	6.11	0.14	17.82	0.43
	25	9.38	2.47	0.30	0.00	12.15	0.36
	50	17.58	0.25	2.07	0.00	19.90	0.46
	75	12.77	1.47	0.00	1.78	16.02	0.41
	100	13.43	1.70	0.00	0.00	15.13	0.40
7	0	42.09	135.24	1.54	0.00	178.87	1.38
	25	45.86	103.02	0.00	21.94	170.82	1.34
	50	43.87	88.40	18.36	6.50	157.13	1.29
	75	33.06	52.72	0.00	37.65	123.43	1.14
	100	49.16	103.53	0.00	2.65	155.34	1.28
8	0	36.46	0.00	17.50	19.88	73.84	0.88
	25	24.71	0.00	0.00	27.78	52.49	0.74
	50	20.67	0.00	0.00	6.34	27.01	0.53
	75	16.56	1.93	0.00	1.23	19.72	0.46
	100	13.62	0.00	0.92	1.20	15.74	0.41
9	0	65.08	0.00	0.00	0.00	65.08	0.83
	25	29.36	0.63	0.00	0.00	29.99	0.56
	50	34.78	0.38	1.60	0.00	36.76	0.62
	75	24.82	0.00	0.72	0.00	25.54	0.52
	100	19.25	0.32	2.34	0.00	21.91	0.48
10	0	18.98	5.80	0.40	1.71	26.89	0.53
	25	32.25	3.90	0.38	0.00	36.53	0.62
	50	43.25	6.00	1.50	0.00	50.75	0.73
	75	15.41	1.76	0.50	0.00	17.67	0.43
	100	25.86	0.00	0.00	0.48	26.34	0.53

TABLE A-4 (continued)

Chan.	Level(%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2(\text{Res})$	$\frac{100\ \hat{\sigma}'(\text{Res})}{\text{Range}}$
11	0	23.29	0.00	0.00	1.04	24.33	0.51
	25	16.47	1.10	1.26	0.00	18.83	0.45
	50	31.63	0.40	1.28	0.00	33.31	0.59
	75	16.23	0.05	2.12	0.00	18.40	0.44
	100	12.96	0.00	1.11	1.11	15.18	0.40
12	0	11.15	0.10	0.38	0.00	11.63	0.35
	25	9.36	0.00	0.00	11.31	20.67	0.47
	50	8.06	0.00	0.00	0.88	8.94	0.31
	75	4.26	0.00	0.00	0.30	4.56	0.22
	100	2.28	0.06	0.00	0.13	2.47	0.16
13	0	43.48	0.00	6.40	0.00	49.88	0.73
	25	26.61	0.00	0.00	9.94	36.55	0.62
	50	22.78	0.00	0.00	5.56	28.34	0.55
	75	20.05	0.00	0.85	5.28	26.18	0.53
	100	16.94	1.42	0.00	5.07	23.43	0.50
14	0	51.02	0.00	0.00	3.90	54.92	0.76
	25	32.96	0.00	0.00	2.23	35.19	0.61
	50	28.51	0.00	0.00	2.44	30.95	0.57
	75	25.93	0.00	0.00	6.41	32.34	0.59
	100	20.68	1.05	0.76	0.00	22.49	0.49
15	0	11.54	0.00	5.03	1.74	18.31	0.44
	25	15.22	0.00	0.00	3.54	18.76	0.45
	50	8.47	5.04	11.48	7.56	32.55	0.59
	75	18.26	2.52	8.52	3.70	33.00	0.59
	100	10.55	0.00	3.69	12.27	26.51	0.53

TABLE A-4 (continued)

Chan.	Level(%)	$\hat{\sigma}'^2$	$\hat{\sigma}'^2_t$	$\hat{\sigma}'^2_\theta$	$\hat{\sigma}'^2_I$	$\hat{\sigma}'^2(\mathrm{Res})$	$\frac{100\ \hat{\sigma}'(\mathrm{Res})}{\mathrm{Range}}$
16	0	46.61	17.32	7.54	0.00	71.47	0.87
	25	38.45	0.00	0.00	3.18	41.63	0.66
	50	26.54	0.00	0.00	2.64	29.18	0.56
	75	29.53	11.00	8.75	0.00	49.28	0.72
	100	18.58	0.54	0.66	0.00	19.78	0.46
17	0	41.03	0.68	2.90	0.00	44.61	0.69
	25	48.71	0.00	0.00	0.00	48.71	0.72
	50	34.90	0.77	1.19	0.00	36.86	0.62
	75	33.89	1.10	0.00	0.00	34.99	0.61
	100	41.12	1.36	0.00	0.00	42.48	0.67
18	0	44.78	2.40	0.58	0.00	47.76	0.71
	25	33.58	0.00	0.00	1.48	35.06	0.61
	50	49.76	0.00	0.56	2.31	52.63	0.75
	75	40.41	0.00	0.00	4.08	44.49	0.69
	100	30.83	1.20	2.18	0.00	34.21	0.60

TABLE A-5

VARIANCE RATIOS ($\sigma^2_{WOC}/\sigma^2_{WC}$) COMPUTED FOR THE DATA GENERATED BY THE FREQUENCY STANDARD AND RECORDED ON RECORDER 1.

	Input Level (Voltage)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	2.88	2.44	1.71	1.83	5.31*
3	1.87	4.56*	5.37*	2.75	1.64
4	21.69*	11.11*	17.14*	85.56*	99.56*
5	0.19+	1.26	5.88*	2.20	3.16
6	1.55	2.02	0.75	2.51	0.38
7	1.34	1.25	1.78	1.78	4.26*
8	1.27	0.82	1.15	0.99	1.16
9	22.30*	10.77*	6.63*	24.93*	15.07*
10	8.02*	2.43	12.78*	15.18*	3.01
11	3.06	1.69	0.65	6.08*	0.03+
12	3.88	8.30*	6.04*	2.50	3.57
13	4.64*	9.74*	1.39	2.05	4.87*
14	1.71	6.49*	1.70	3.70	1.90
15	1.57	1.07	2.48	3.18	7.22*
16	3.28	3.78	3.60	1.91	3.30
17	3.81	1.28	5.14*	2.91	3.91
18	2.68	5.56*	3.30	2.89	4.90*

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-6

VARIANCE RATIOS ($\sigma^2_{WOC}/\sigma^2_{WC}$) COMPUTED FOR THE DATA

GENERATED BY THE FREQUENCY STANDARD

AND RECORDED ON RECORDER 2.

CHANNEL	Input Level (Voltage) 0.00	1.25	2.50	3.75	5.00
2	∞ *	50.06*	43.84*	68.68*	12.88*
3	98.43*	16.54*	153.05*	464.00*	22.77*
4	91.10*	109.12*	205.68*	688.61*	57.10*
5	1.92	0.76	0.41	0.19+	1.98
6	0.73	0.27	0.75	0.91	0.83
7	0.98	0.23+	2.70	0.55	0.26
8	1.36	0.69	0.63	1.15	0.43
9	72.40*	225.73*	143.74*	33.16*	35.08*
10	28.08*	17.45*	4.56*	25.90*	5.51*
11	32.83*	55.70*	50.79*	129.16*	91.63*
12	7.08*	34.39*	30.39*	22.47*	8.73*
13	11.66*	5.04*	3.60	6.57*	9.74*
14	3.20	7.76*	13.92*	18.68*	20.79*
15	12.31*	4.30*	2.28	9.05*	11.31*
16	11.97*	9.54*	12.96*	12.29*	12.79*
17	7.67*	1.23	9.52*	5.01*	3.61
18	3.66	14.91*	30.36*	8.28*	35.18*

$F_{.025}(9,9) = 4.03$ *Ratio is singificantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-7

VARIANCE RATIOS (σ^2_{WOC} / σ^2_{WC}) COMPUTED FOR THE DATA GENERATED BY SUBCARRIER OSCILLATOR SET A AND RECORDED ON RECORDER 1.

CHANNEL	Input Level (Voltage) 0.00	1.25	2.50	3.75	5.00
2	4.00	1.00	1.50	1.88	1.21
3	4.80*	1.17	7.81*	1.98	∞ *
4	1.45	8.64*	7.81*	0.05+	1.36
5	0.05+	0.92	6.09*	1.22	2.60
6	0.24+	3.99	0.59	2.25	0.63
7	2.79	2.39	0.87	1.33	2.52
8	0.53	1.00	0.91	13.89*	1.08
9	5.53*	2.01	3.99	8.80*	19.00*
10	7.75*	25.28*	11.34*	6.07*	24.05*
11	6.47*	2.56	9.92*	21.23*	20.14*
12	1.31	3.95	5.89*	0.75	2.55
13	10.36*	9.17*	10.72*	10.05*	3.31
14	2.37	3.47	4.91*	25.91*	19.59*
15	6.22*	6.97*	2.44	43.29*	1.23
16	4.02	7.68*	7.89*	8.73*	5.08*
17	7.27*	2.23	3.44	5.13*	2.91
18	2.59	11.83*	1.62	5.72*	7.45*

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-8

VARIANCE RATIOS (σ^2_{WOC} / σ^2_{WC}) COMPUTED FOR THE DATA GENERATED BY SUBCARRIER OSCILLATOR SET A AND RECORDED ON RECORDER 2.

	Input Level (Voltage)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	15.23*	8.34*	6.96*	3.11	8.38*
3	5.07*	73.49*	5.44*	6.86*	19.47*
4	59.15*	14.20*	12.49*	52.01*	34.61*
5	0.91	0.38	1.93	0.78	0.88
6	0.65	0.55	0.73	1.56	0.36
7	0.81	1.01	1.07	0.76	3.51
8	0.40	1.52	0.58	0.91	0.71
9	64.34*	82.62*	26.69*	83.01*	23.67*
10	37.91*	43.78*	2.27	18.96*	53.06*
11	25.44*	39.19*	31.28*	31.60*	364.89*
12	14.73*	3.74	14.46*	2.00	2.50
13	17.20*	25.58*	4.44*	10.90*	5.69*
14	36.70*	8.24*	7.72*	3.56	3.83
15	2.82	18.29*	3.01	12.94*	3.13
16	9.26*	7.02*	13.26*	7.04*	7.99*
17	8.14*	16.15*	5.04*	45.89*	13.56*
18	14.51*	6.68*	15.26*	10.16*	20.11*

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-9

VARIANCE RATIOS (σ_2^2 / σ_1^2) COMPUTED FOR THE DATA

GENERATED BY THE FREQUENCY STANDARD

AND REPRODUCED WITH TAPE SPEED COMPENSATION

	Input Level (Voltage)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	0.00+	2.56	1.83	2.50	10.56*
3	0.59	9.00*	0.83	0.56	2.44
4	2.11	9.44*	3.05	0.17+	4.44*
5	10.66*	16.54*	116.51*	33.06*	19.19*
6	2.93	4.95*	3.88	4.81*	2.43
7	7.72*	2.75	11.12*	7.91*	25.69*
8	4.82*	26.23*	14.89*	11.47*	9.94*
9	1.55	0.45	1.37	3.20	2.62
10	0.63	1.78	6.93*	2.37	1.39
11	0.45	0.49	2.32	0.51	.00+
12	1.26	0.84	0.79	0.46	2.15
13	1.21	4.97*	2.70	1.34	0.76
14	1.49	3.55	1.18	0.76	0.78
15	1.40	0.77	1.87	2.06	3.48
16	0.47	2.67	2.79	2.10	2.65
17	0.97	0.93	1.19	1.53	3.06
18	3.04	1.27	1.30	1.40	3.18

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-10

VARIANCE RATIOS (σ_2^2 / σ_1^2) COMPUTED FOR THE DATA

GENERATED BY THE FREQUENCY STANDARD

AND REPRODUCED WITHOUT TAPE SPEED COMPENSATION

	Input Level (Voltage)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	22.54*	52.52*	47.05*	93.66*	25.60*
3	31.29*	32.68*	23.73*	94.91*	33.88*
4	8.87*	92.76*	36.57*	1.36	2.55
5	107.84*	10.05*	8.11*	2.89	12.02*
6	1.37	0.67	3.87	1.74	5.31*
7	5.65*	4.98*	16.84*	2.44	1.57
8	5.16*	21.91*	8.18*	13.31*	3.66
9	5.04*	9.46*	29.66*	4.26*	6.10*
10	2.20	12.82*	2.47	4.04*	2.55
11	4.85*	16.29*	181.80*	10.88*	7.94*
12	2.29	3.31	3.98	4.12*	5.27*
13	3.04	2.57	7.03*	4.32*	1.51
14	2.80	4.24*	9.65*	3.83	8.51*
15	10.94*	3.08	1.72	5.85*	5.45*
16	1.72	6.75*	10.04*	13.51*	10.26*
17	1.96	1.11	2.21	2.63	2.82
18	4.16*	3.41	11.98*	4.00	22.80*

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-11

VARIANCE RATIOS (σ_2^2 / σ_1^2) COMPUTED FOR THE DATA
GENERATED BY SUBCARRIER OSCILLATOR SET A
AND REPRODUCED WITH TAPE SPEED COMPENSATION

	Input Level (Volts)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	5.38*	2.34	6.71*	2.85	2.33
3	5.07*	0.71	6.31*	2.64	∞ *
4	1.36	2.21	5.90*	0.03+	0.56
5	3.89	3.89	15.69*	8.57*	6.24*
6	1.97	6.00*	1.83	3.72	2.81
7	18.39*	9.70*	3.58	3.80	5.08*
8	9.19*	1.49	7.47*	17.98*	5.27*
9	1.11	0.40	2.07	0.29	3.58
10	0.77	0.89	0.55	0.69	1.19
11	0.64	0.62	2.11	2.52	0.14+
12	0.58	1.21	1.51	2.12	0.81
13	2.06	1.00	2.17	1.39	0.88
14	0.85	1.04	1.70	7.85*	7.07*
15	2.23	1.00	1.64	6.45*	1.38
16	2.60	2.83	1.23	5.11*	1.24
17	5.31*	1.00	2.25	0.74	1.13
18	1.37	3.87	2.29	2.33	0.90

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-12

VARIANCE RATIOS (σ_2^2 / σ_1^2) COMPUTED FOR THE DATA GENERATED BY SUBCARRIER OSCILLATOR SET A AND REPRODUCED WITHOUT TAPE SPEED COMPENSATION

	Input Level (Volts)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	20.47*	19.54*	31.14*	4.72*	16.17*
3	5.35*	44.46*	4.39*	9.12*	18.27*
4	55.45*	3.64	9.45*	29.66*	14.13*
5	72.16*	1.63	4.97*	5.45*	2.12
6	5.44*	0.83	2.29	2.58	1.61
7	5.33*	4.12	4.40*	2.17	7.09*
8	7.03*	2.27	4.72*	1.18	3.49
9	12.94*	16.52*	13.86*	2.76	4.46*
10	3.78	1.53	4.99*	2.17	2.63
11	2.52	9.43*	6.65*	3.75	2.51
12	6.59*	1.14	3.71	5.70*	0.79
13	3.41	2.80	0.90	1.51	1.51
14	13.22*	2.46	2.66	1.08	1.38
15	1.01	2.62	2.03	1.93	3.51
16	5.99*	2.59	2.07	4.13*	1.95
17	5.95*	7.22*	3.30	6.59*	5.24*
18	7.66*	2.19	21.54*	4.15*	2.43

$F_{.025}(9,9) = 4.03$ *Ratio is significantly large at $\alpha = 0.05$

$F_{.975}(9,9) = 0.25$ +Ratio is significantly small at $\alpha = 0.05$

TABLE A-13

THE VARIANCES OF THE FP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	0.69	0.24	0.00	0.09	0.21
3	0.09	0.76	0.24	0.21	0.09
4	0.16	0.16	0.16	0.09	0.00
5	0.24	0.21	0.16	0.00	0.00
6	0.21	0.09	0.20	0.24	0.09
7	0.16	0.16	0.09	0.21	0.09
8	0.29	0.09	0.16	0.16	0.00
9	0.21	0.09	0.00	0.09	0.24
10	0.24	0.21	0.00	0.21	0.29
11	0.89	0.20	0.24	0.16	0.44
12	1.04	0.40	0.09	0.21	0.09
13	0.00	0.00	0.21	0.24	0.40
14	0.24	0.00	0.00	0.36	0.00
15	0.16	0.16	0.25	0.00	0.40
16	0.21	0.45	0.00	0.25	0.64
17	0.49	0.21	0.21	0.00	0.00
18	1.01	0.21	0.21	0.16	0.21

TABLE A-14

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-13

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	0.212	0.51	0.00
79	0.165	0.40	0.00
73	0.143	0.35	0.00
72	0.140	0.34	0.00

TABLE A-15

THE VARIANCES OF THE SAP DATA

	Input Level (Volts)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	0.20	0.24	0.85	1.61	0.00
3	0.49	0.25	0.45	0.25	0.29
4	0.69	0.84	0.09	0.00	0.09
5	0.09	0.16	0.21	0.09	0.21
6	0.21	1.81	0.21	0.29	0.36
7	0.09	0.21	0.16	1.00	8.60
8	1.24	0.24	0.61	0.21	0.25
9	0.25	0.29	0.09	0.00	0.36
10	0.69	0.41	0.24	0.41	0.76
11	1.01	0.25	0.21	0.25	0.25
12	8.41	1.21	0.65	0.16	0.21
13	0.09	0.36	0.21	0.16	0.56
14	0.41	0.65	0.20	0.09	0.40
15	0.56	1.29	0.64	0.20	0.36
16	0.41	0.29	1.16	0.76	0.36
17	0.84	1.16	2.20	0.29	0.85
18	0.29	1.21	0.84	0.40	0.56

TABLE A-16

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-15

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	0.66	1.59	0.00
80	0.42	1.01	0.00
74	0.36	0.87	0.00
72	0.34	0.82	0.00
67	0.30	0.72	0.00
65	0.29	0.70	0.00

TABLE A-17

THE VARIANCES OF THE FR1WOCP DATA

	Input Level (Volts)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	1.61	0.61	0.41	0.44	0.85
3	1.29	0.41	1.29	0.44	0.41
4	7.81	1.00	3.60	106.09	8.96
5	0.69	1.40	6.41	5.20	3.29
6	30.21	33.89	12.36	22.45	8.44
7	2.09	1.81	1.85	3.20	4.64
8	13.04	6.44	25.29	11.56	20.96
9	21.41	13.36	5.04	17.20	10.40
10	18.36	2.45	5.24	7.44	4.09
11	17.60	8.24	1.69	14.84	13.84
12	6.21	6.64	4.89	7.41	3.00
13	18.76	11.69	4.84	14.76	20.69
14	9.16	30.16	8.44	23.84	11.85
15	7.61	16.61	10.81	16.41	13.29
16	29.64	19.80	18.85	12.41	14.01
17	26.49	18.76	21.60	16.45	15.81
18	13.09	24.45	17.84	12.76	8.04

TABLE A-18

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-17

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	12.10	29.21	0.00
80	10.08	24.34	0.00
77	9.38	22.65	0.00
76	9.19	22.19	0.00
75	9.02	21.76	0.00

TABLE A-19

THE VARIANCES OF THE FR2WOCP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	36.29	32.04	19.29	41.21	21.76
3	40.36	13.40	30.61	41.76	13.89
4	69.24	92.76	131.64	144.61	22.84
5	74.41	17.69	52.01	15.01	39.56
6	41.44	22.61	47.81	39.09	44.84
7	11.81	9.01	31.16	7.81	7.29
8	67.29	141.09	206.85	153.84	76.81
9	107.89	126.41	149.49	73.29	63.49
10	40.44	31.41	12.96	30.04	10.41
11	85.36	134.25	307.25	161.45	109.96
12	14.24	22.01	19.45	30.56	15.81
13	57.04	30.01	34.01	63.69	31.25
14	25.64	127.89	81.44	91.36	100.84
15	83.24	51.09	18.61	96.01	72.49
16	50.89	133.60	189.20	167.64	143.81
17	51.84	16.85	47.69	43.25	44.60
18	54.44	83.49	213.76	51.01	183.29

TABLE A-20

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-19

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	68.77	166.01	0.00
79	57.94	139.87	0.00
73	50.45	121.79	0.00
68	44.55	107.54	0.00
66	42.60	102.83	0.00

TABLE A-21

THE VARIANCES OF THE SAR1WOCP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	0.96	0.41	0.36	2.80	0.29
3	2.69	0.81	1.25	1.21	0.81
4	0.64	4.84	1.64	1.21	1.96
5	0.49	8.21	3.96	6.09	5.96
6	1.81	9.41	7.80	8.09	5.61
7	11.41	17.21	14.81	17.56	16.84
8	4.81	15.44	4.96	16.81	4.76
9	16.36	5.80	8.01	22.89	12.16
10	13.64	44.49	25.41	16.09	25.01
11	22.24	5.36	14.29	11.89	13.09
12	4.56	12.49	5.65	1.41	3.29
13	33.24	23.84	37.41	26.24	19.84
14	12.24	29.29	24.36	38.60	26.64
15	16.41	14.01	7.41	21.21	5.89
16	21.29	33.89	28.49	18.24	19.76
17	18.89	19.24	23.96	15.40	21.25
18	16.84	33.61	7.29	25.40	30.09

TABLE A-22

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-21

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	13.63	32.90	0.00
79	11.86	28.63	0.00
77	11.40	27.52	0.00
76	11.17	26.96	0.00

TABLE A-23

THE VARIANCES OF THE SAR2WOCP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	19.65	8.01	11.21	13.21	4.69
3	14.40	36.01	5.49	11.04	14.80
4	35.49	17.61	15.49	35.89	27.69
5	35.36	13.40	19.69	33.21	12.64
6	9.85	7.84	17.84	20.89	9.04
7	60.80	70.84	65.21	38.09	119.45
8	33.81	35.04	23.41	19.84	16.61
9	211.69	95.84	111.04	63.09	54.21
10	51.56	68.29	126.81	34.89	65.80
11	55.96	50.56	95.09	44.56	32.84
12	30.05	14.24	20.96	8.04	2.60
13	113.49	66.76	33.56	39.69	29.89
14	161.85	72.16	64.89	41.56	36.84
15	16.60	36.76	15.05	40.89	20.69
16	127.44	87.64	59.01	75.29	38.44
17	112.41	138.89	79.04	101.41	111.29
18	129.00	73.45	157.05	105.29	73.21

TABLE A-24

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-23

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	52.13	125.84	0.00
78	43.31	104.55	0.00
72	37.58	90.72	0.00
69	34.97	84.42	0.00
68	34.20	82.56	0.00

TABLE A-25

THE VARIANCES OF THE FR1WCP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	0.56	0.25	0.24	0.24	0.16
3	0.69	0.09	0.24	0.16	0.25
4	0.36	0.09	0.21	1.24	0.09
5	3.64	1.76	1.09	2.36	1.04
6	19.45	16.80	16.41	8.96	22.21
7	1.56	1.45	1.04	1.80	1.09
8	10.24	7.81	22.04	11.65	18.01
9	0.96	1.24	0.76	0.69	0.69
10	2.29	1.01	0.41	0.49	1.36
11	5.76	4.89	2.61	2.44	488.16
12	1.60	0.80	0.81	2.96	0.84
13	4.04	1.20	3.49	7.21	4.25
14	5.36	4.65	4.96	6.44	6.25
15	4.84	15.49	4.36	5.16	1.84
16	9.04	5.24	5.24	6.49	4.24
17	6.96	14.65	4.20	5.65	4.04
18	4.89	4.40	5.41	4.41	1.64

TABLE A-26

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-25

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	10.14	24.48	0.00
84	4.45	10.74	0.00
75	2.90	7.00	0.00
70	2.49	6.01	0.00
66	2.24	5.41	0.00
64	2.13	5.14	0.00
59	1.86	4.49	0.00
54	1.59	3.83	0.00
46	1.13	2.72	0.00
43	0.97	2.34	0.00
40	0.86	2.07	0.00
39	0.82	1.98	0.00

TABLE A-27

THE VARIANCES OF THE FR2WCP DATA

	Input Level (Volts)				
CHANNEL	0.00	1.25	2.50	3.75	5.00
2	0.00	0.64	0.44	0.60	1.69
3	0.41	0.81	0.20	0.09	0.61
4	0.76	0.85	0.64	0.21	0.40
5	38.81	23.16	127.00	78.01	19.96
6	57.00	83.21	63.64	43.09	53.89
7	12.05	39.84	11.56	14.24	28.00
8	49.36	204.84	328.09	133.65	179.01
9	1.49	0.56	1.04	2.21	1.81
10	1.44	1.80	2.84	1.16	1.89
11	2.60	2.41	6.05	1.25	1.20
12	2.01	0.64	0.64	1.36	1.81
13	4.89	5.96	9.44	9.69	3.21
14	8.01	16.49	5.85	4.89	4.85
15	6.76	11.89	8.16	10.61	6.41
16	4.25	14.01	14.60	13.64	11.24
17	6.76	13.69	5.01	8.64	12.36
18	14.89	5.60	7.04	6.16	5.21

TABLE A-28

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-27

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	22.27	53.76	0.00
75	7.80	18.83	0.00
68	5.04	12.17	0.00
60	3.81	9.20	0.00
53	2.87	6.93	0.00
49	2.46	5.94	0.00
43	1.91	4.61	0.00
36	1.28	3.09	0.00
34	1.13	2.73	0.00
33	1.08	2.61	0.00

TABLE A-29

THE VARIANCES OF THE SAR1WCP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	0.24	0.41	0.24	1.49	0.24
3	0.56	0.69	0.16	0.61	0.00
4	0.44	0.56	0.21	22.29	1.44
5	10.04	8.96	0.65	5.00	2.29
6	7.69	2.36	13.29	3.60	8.84
7	4.09	7.20	17.01	13.21	6.69
8	9.09	15.44	5.44	1.21	4.41
9	2.96	2.89	2.01	2.60	0.64
10	1.76	1.76	2.24	2.65	1.04
11	3.44	2.09	1.44	0.56	0.65
12	3.49	3.16	0.96	1.89	1.29
13	3.21	2.60	3.49	2.61	6.00
14	5.16	8.44	4.96	1.49	1.36
15	2.64	2.01	3.04	0.49	4.80
16	5.29	4.41	3.61	2.09	3.89
17	2.60	8.61	6.96	3.00	7.29
18	6.49	2.84	4.49	4.44	4.04

TABLE A-30

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-29

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	3.98	9.61	0.00
79	3.12	7.53	0.00
73	2.67	6.45	0.00
68	2.36	5.70	0.00
67	2.30	5.55	0.00

TABLE A-31

THE VARIANCES OF THE SAR2WCP DATA

CHANNEL	Input Level (Volts) 0.00	1.25	2.50	3.75	5.00
2	1.29	0.96	1.61	4.25	0.56
3	2.84	0.49	1.01	1.61	0.76
4	0.60	1.24	1.24	0.69	0.80
5	39.05	34.84	10.20	42.84	14.29
6	15.16	14.16	24.36	13.41	24.85
7	75.21	69.84	60.89	50.16	34.01
8	83.56	23.04	40.61	21.76	23.24
9	3.29	1.16	4.16	0.76	2.29
10	1.36	1.56	1.24	1.84	1.24
11	2.20	1.29	3.04	1.41	0.09
12	2.04	3.81	1.45	4.01	1.04
13	6.60	2.61	7.56	3.64	5.25
14	4.41	8.76	8.41	11.69	9.61
15	5.89	2.01	5.00	3.16	6.61
16	13.76	12.49	4.45	10.69	4.81
17	13.81	8.60	15.69	2.21	8.21
18	8.89	11.00	10.29	10.36	3.64

TABLE A-32

CONTROL CHART COMPUTATIONS FOR THE DATA OF TABLE A-31

N	$\bar{\sigma}^2$	UCL=2.414 $\bar{\sigma}^2$	LCL=0.00 $\bar{\sigma}^2$
85	11.82	28.54	0.00
75	6.32	15.25	0.00
69	4.94	11.93	0.00
62	3.93	9.49	0.00
55	3.09	7.46	0.00
49	2.44	5.89	0.00
47	2.26	5.46	0.00
46	2.18	5.27	0.00

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