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NEW SYSTEM OF ELEMENTS AND TABLES OF MOTION OF JUPITER'S X SATELLITE

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(USSR)

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ABSTRACT

Tables have been composed, giving the jovicentric coordinates of the tenth satellite of Jupiter. Determined to that effect were all the solar perturbations that provide the geocentric positions (α, δ) of the satellite within 5". The Delaunay theory of motion of the Moon was applied to the determination of solar perturbations of Jupiter's X satellite. The theoretical values of node's and perijove's secular motions have been computed.

A beforehand improvement of the Herget system of elements has been performed on the basis of satellite's observations from 1938 to 1942, taking all these perturbations into consideration. The resulting corrections of elements proved to be rather substantial. The new system of elements represents the observations ten times better than does the Herget system of elements.

The tables have been compiled on the basis of this new system of elements.

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The Xth satellite of Jupiter was discovered by Nicholson in 1938 at the Mount-Wilson Observatory by the photographical method. The satellite is weak; its brightness is of approximately the 19th stellar magnitude (Discovery of two new satellites..., 1938).

Wilson computed the orbit by three approximate positions. It represented observations with errors not exceeding 4". Then, by a series

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* NOVAYA SISTEMA ELEMENTOV I TABLITSY DVIZHENIYA X SPUTNIKA YUPITERA.

of observations between 6 July and 21 November 1938, he performed orbit correction without taking into account the perturbations, thus obtaining a new system of elements little differring from the first. This orbit represented observations with errors of 12 – 13" for α and δ (Wilson, 1939).

Herget computed simultaneously with Wilson an orbit, effecting two differential corrections taking into account the solar perturbations, thus obtaining the following system of elements (UAI Circ. No. 727):

1938 Epoch July 27.3128

$$M_0 = 211^{\circ}.0221 \quad n = 1.382030 \quad a = 0.078601 \text{ a.u.} \quad e = 0.13244 \quad P = 260.5\ d$$

$$\left.\begin{aligned} i &= 28^{\circ}.266 \\ \omega &= 247.044 \\ \Omega &= 82.507 \end{aligned}\right\} \text{Eclipt. and equin. 1950.0} \qquad \text{(I)}$$

This system is in good agreement with the Wilson's system of elements.

The system of elements of Jupiter's X satellite, obtained by Herget, allowed to provide ephemerides and conduct observations up to 1943. During that period 36 satellite observations were published.- All were obtained by Nicholson at Mount-Wilson.

Later on given were the ephemerides computed by Musen on the basis of temporarily improved Herget elements.

After 1943 the observations of the satellite ceased, and only in 1951, Nicholson obtained again a few observations. However, these were obtained on single photoplates alongside with the positions of a new object near Jupiter, which, according to subsequent clarfications, was found to be the new Jupiter's XII satellite. The positions of both bodies were found to be proximate, but which of those belong to the X satellite and which ones to the new object, is something that could not be ascertained at once.* The last observations of the satellite were obtained in 1954 by Van Biesbroeck [10] at the MacDonald Observatory.

There is basis to assume that Nicholson observed the Jupiter's X satellite in 1952, 1953, 1954 and 1955.** - It is unfortunate that these observations were not published.

* UAI Circ. Nos- 1332, 1333; Nachrichtenblatt der Astronom.Zentralstelle, vorlaufige Mitteilung, No. 134. ** (Harvard Ann.Card NOs.1189, 1234, 1277).

Such is the history of the question of discovery and study of the motion of Jupiter's X satellite.

One may feel that the system of elements obtained by Herget according to a rather small number of observations encompassing an insignificant time interval is unsatisfactory for the calculation of ephemerides that would allow to identify the observed positions of the satellite. This may be the cause of the fact that after 1943 systematic observations of the satellite are absent.

As to the investigation of the motion of Jupiter's X satellite, as far as is known not a single work devoted to this question has been available to date.

The present work is the beginning of the study of Jupiter's X satellite's motion, with the view of creating an analytical theory for it. To that effect it is necessary to study in the first place the character of perturbations conditioned by the Sun. When resolving this problem, we shall make use of the analytical motion theory of the Moon by Delaunay 1860, 1867 [5]. This theory, created as a result of twenty-year labor, provides the fullest literal solution of a complex problem of celestial mechanics, that of the motion of the Moon.

Developing the Euler idea, Delaunay applied the method of elliptic element variation to the integration of equations of perturbed motion of the Moon. Subsequently, he integrated the equations of motion expressed in canonical form, taking at each approximation in the right-hand part of the equations only separate terms of the perturbation function, rather than the function as whole. In this way Delaunay obtained general expressions for all solar inequalities up to the 7th order relative to small parameters. The final formulas for the coordinates of the Moon were obtained by Delaunay in the general form; they constitute four-argument trigonometric series with literal coefficients depending upon lunar orbit parameters (for longitude and latitude the series have 400 – 500 terms; for the parallax – 100 terms).

The solution of the problem of Moon's motion, obtained by Delaunay in the general form, has a successful application in some other satellite problems, such, for example, as at construction of the theory of motion of Jupiter's VI and VII satellites and some of Saturn's satellites.

The successful application of the lunar theory to other satellites is explained by the fact that the observations of major planets' satellites are conducted from the Earth and not from their central bodies. At the same time the knowledge of the coordinates relative to central body are not required with a precision compulsory for the Moon, where the central body is the Earth. The Delaunay method is currently applied for the construction of the theory of motion of Earth's artificial satellites.

On the basis of the Herget system of elements we shall determine by the Delaunay theory all the inequalities in the motion of Jupiter's X satellite, conditioned by the perturbing action of the Sun, which permit to assure a precision in geocentric positions to 5". In order to obtain the coefficients of solar inequalities, we compiled on the basis of Delaunay expansions the tables (2 – 4) in a form allowing quickly so to reconvert these coefficients that they correspond to another (improved) system of elements. The tables (2 – 4) may serve as initial data for the entire subsequent work connected with the construction of more precise tables of Jupiter's X satellite motion.

We determined by the Delaunay formulas of 1872 [6] the secular motions of the node and perijove, obtaining afterward the values of all arguments entering in the series by which the jovicentric coordinates of the satellite are represented. On the basis of the initial tables and having at our disposal the values of all the required arguments, we could compile the tables (5, 6), allowing to obtain rapidly the jovicentric coordinates (longitude, latitude and the reciprocal of the radius-vector) of the satellite at any moment of time.

Taking advantage of the compiled tables, we effected the differential correction of the Herget orbit after comparing the obtained preliminary theory with the observations. It was then ascertained that the observations of 1938 - 1942 could not be linked with those of 1951 and 1954 by a unique system of elements obtained as a result of a single differential correction.

Inasmuch as the 1951 observations give rise to doubts as to the appurtenance to Jupiter's X satellite and are distributed over a very small time interval, just as are the observations of 1954, they were not included in the orbit improvement. Thus, the obtained new system of elements was based upon observations within the 1938 – 1942 time interval, which constitutes approximately six revolutions of the satellite.

The new system of elements, representing observations by a factor of 10 better than the Herget system of elements, was laid at the basis of tables (13, 14) for the determination of jovicentric coordinates of the satellite. In these tables all the coefficients of solar inequalities are fully recomputed.

Tables (13, 14) may serve for the calculation of satellite's ephemerides. It should be borne in mind that in order to obtain the geocentric positions of the satellite with a precision to 1", the compiled tables should be expanded three times, that is, 3 times more terms must be taken into account in the Delaunay expansions.

A further work for the creation of the theory of motion of Jupiter's X satellite will be dependent upon the availability of new observations.

1. – SOLAR INEQUALITIES IN THE MOTION OF JUPITER'S X SATELLITE

The expansions for the coordinates of the Moon, obtained by Delaunay, have the following form:

$$\left.\begin{aligned} V &= \varepsilon_0 + nt + \sum A e^p e'^{p'} \gamma^q \alpha^r m^s \sin(iD + jl + j'l' + kF), \\ U &= \sum B e^p e'^{p'} \gamma^q \alpha^r m^s \sin(iD + jl + j'l' + kF), \\ \frac{a}{r} &= \sum C e^p e'^{p'} \gamma^q \alpha^r m^s \cos(iD + jl + j'l' + kF), \end{aligned}\right\} \qquad (1)$$

where V, U and $\frac{a}{r}$ are respectively the longitude, latitude and the parallax of the Moon.

In their application to Jupiter's satellites these quantities are the jovicentric longitude and latitude, while the parallax is utilized for the calculation of radius-vector's reciprocal, by which the satellite's radius-vector itself can be found.

In the series (1) A, B, C are numerical coefficients; p, p', q, r and s are whole numbers assuming values from 0 to ∞ ; i, j, j', k vary from $-\infty$ to $+\infty$, assuming whole values.

e, e', γ, α, m are Delaunay parameters, of which the significance is : e and e' are respectively the orbit eccentricities of the satellite and Jupiter; $\gamma = \sin\frac{I}{2}$, where I is the mutual inclination angle of Jupiter and the satellite; $\alpha = \frac{a}{a'}$, where a and a' are the semiaxes if satellite's and Jupiter's orbits; $m = \frac{n'}{n}$, where n and n' are the mean motions of the satellite and Jupiter.

D, l, l', F are Delaunay arguments, of which the expressions through the usual angular elements have the form :

$$\left.\begin{aligned} l &= M_0 + \left(n - \frac{d\pi}{dt}\right)t, \\ l' &= M_0' + n't, \\ D &= \varepsilon_0 - \varepsilon_0' + (n - n')t, \\ F &= \varepsilon_0 - \Omega_0 + \left(n - \frac{d\Omega}{dt}\right)t. \end{aligned}\right\} \qquad (2)$$

The physical sense of these quantities is as follows : l, l' are the mean anomalies of the satellite and of the Sun; F is the mean argument of satellite's latitude; M_0, M'_0 are the mean anomalies respectively in the satellite's and Sun's epoch; ε_0, ε_0' are the mean longitudes respectively in the satellite's and Sun's epoch; $\frac{d\pi}{dt}$ and $\frac{d\Omega}{dt}$ are the mean motions of satellite's perijove and node.

Algebraic expressions were obtained by Delaunay for the determination of these mean motions and of Moon's theory (Delaunay, 1872 [6], Proskurin, 1955 [1]). We took advantage of these expressions for the determination of annual variations of the perijove and node of Jupiter's X satellite. The results will be presented below.

Presented in Table 1 are the values of the Delaunay parameters by which expansions are performed for the Moon and for X, VI and VII satellites of Jupiter.

TABLE 1*

Paramet. / Object	e	e'	γ	α	m
Moon......	0.05490	0.01677	0.04489	0.00256	0.07480
X Jup. sat..	0.13244	0.04840	0.23364	0.01511	0.06012
VI " "	0.15798	0.04840	0.24561	0.01475	0.05783
VII " "	0.20719	0.04840	0.23980	0.01508	0.05993

* The values of satellite's X parameters are obtained on the basis of the system of elements (I).

It may be seen from Table 1 that the parameters of all the three satellites are mutually proximate. This means that the satellite moves along close orbits with nearly identical velocities. Consequently, the character of the motion of Jupiter's VI, VII and X satellites is identical and this is why they may be related to a single group.

Comparison of the Delaunay parameters for this group with those of the Moon shows that they are several times greater than the lunar parameters. If the satellite observations were conducted from Jupiter instead of the Earth, the series, obtained by Delaunay, would be found insufficient for the determination of precise positions, so that the creation of new exact theories of satellite motion would be required. But inasmuch as the observations are conducted from Earth, the precision, with which jovicentric coordinates of the satellites of this group can be obtained by applying the Delaunay theory, will provide sufficiently precise geocentric positions.

S. S. Tokmalayeva [4] has shown in 1956 that the coefficients of the Delaunay expansions are practically correct only to 0.001 in longitude and latitude and to 0.0001 in the parallax when applied to VII (and consequently to VI and X) Jupiter's satellite.

According to this we shall estimate the error with which the application of the Delaunay theory will enable us to obtain the geocentric positions of Jupiter's X satellite.

The geocentric coordinates of the satellite are linked with its jovicentric rectangular coordinates by the following equations

$$\left.\begin{aligned} \rho\cos\alpha\cos\delta &= x + X_{\odot} + X_J, \\ \rho\sin\alpha\cos\delta &= y + Y_{\odot} + Y_J, \\ \rho\sin\delta &= z + Z_{\odot} + Z_J, \end{aligned}\right\} \qquad (3)$$

where x, y, z are the rectangular jovicentric coordinates of the satellite; $X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$ are the rectangular geocentric coordinates of the Sun; X_J, Y_J, Z_J are the rectangular heliocentric coordinates of Jupiter.

The first ones are computed by the formulas

$$\left.\begin{aligned} x &= r\cos V\cos U, \\ y &= r\sin V\cos U, \\ z &= r\sin U, \end{aligned}\right\} \qquad (4)$$

where V, U and r are the jovicentric longitude, latitude and radius-vector, obtained by the Delaunay theory.

The quantities $X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$ are known from the theory of motion of the Earth. The quantities X_J, Y_J, Z_J are known from the theory of motion of Jupiter; they are given in the tables "Planetary Coordinates".

In order to determine the errors in α and δ as functions of the errors admitted in the values of V, U, r, let us differentiate the equations (3) assuming $X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$, X_J, Y_J, Z_J as constants, and let us resolve them relative to $d\alpha$ and $d\delta$. We shall obtain

$$\left.\begin{aligned} \cos\delta\, d\alpha &= -\sin\alpha \frac{dx}{\rho} + \cos\alpha \frac{dy}{\rho}, \\ d\delta &= -\sin\delta\cos\alpha \frac{dx}{\rho} - \sin\delta\sin\alpha \frac{dy}{\rho} + \cos\delta \frac{dz}{\rho}. \end{aligned}\right\} \tag{5}$$

The differentiation of the equations (4) gives the expressions dx, dy, dz as functions of dV, dU and dr. Let us substitute the differentials dx, dy, dz into the first of the equations (5) and estimate the value of $\cos\delta\, d\alpha$ by module. We shall obtain

$$|\cos\delta\, d\alpha| \leqslant \left|2\frac{r}{\rho}\, dV\right| + \left|2\frac{r}{\rho}\, dU\right| + \left|2\frac{1}{\rho}\, dr\right|. \tag{6}$$

Since V and U are determined with errors of one order, (6) may be rewritten as follows :

$$|\cos\delta\, d\alpha| \leqslant 4\frac{r}{\rho}|dV| + \frac{2}{\rho}|dr|. \tag{7}$$

The Delaunay theory does not provide us with the radius-vector, but with the value of $\frac{a}{r}$. Let us express <u>dr</u> through $d\left(\frac{a}{r}\right)$

$$d\left(\frac{a}{r}\right) = -\frac{a}{r^2}\, dr,$$

hence

$$dr = -\frac{r^2}{a}\, d\left(\frac{a}{r}\right).$$

The quantities a and r are of same order, and therefore

$$|dr| = r\left|d\left(\frac{a}{r}\right)\right|. \tag{8}$$

Substituting (8) into the correlation (7), we shall obtain the estimate of the error in α as a function of errors in the values of V, U and $\frac{a}{r}$

$$|\cos\delta\, d\alpha| \leqslant 4\frac{r}{\rho}|dV| + 2\frac{r}{\rho}\left|d\left(\frac{a}{r}\right)\right|. \tag{9}$$

The estimate of the error in δ is obtained analogously

$$|d\delta| \leqslant 5\frac{r}{\rho}|dV| + 3\frac{r}{\rho}\left|d\left(\frac{a}{r}\right)\right|. \qquad (10)$$

For the Jupiter's X, VI, VII satellites $r \approx 0.08$, $\rho \approx 4.00$. Therefore, if the Delaunay theory allows to determine V and U for a given group of satellites with a precision to 0.001 and $\frac{a}{r}$ with a precision to 0.0001, by substituting these values into the correlations (9) and (10), we shall obtain

$$|\cos\delta d\alpha| \leqslant 1\overset{''}{.}1,$$
$$|d\delta| \leqslant 1\overset{''}{.}6.$$

Consequently, the application of the lunar theory of Delaunay to the motion of the Jupiter's X satellite allows to determine its geocentric positions with an error $\approx 1.5''$.

With the view of diminishing the volume of operation when computing the coefficients of solar inequalities, we limited ourselves in the present work by the 5" precision in the positions, for which it was sufficient to determine V and U with a precision to 0.01° and not 0.001° (thus lowering the work volume by a factor of 3). We estimate that at this stage of the work the precision is sufficient, inasmuch as the system of elements, laid at the basis of the determination of solar inequalities, requires correction. There is no doubt that when a good system of satellite orbit's elements is obtained, it will be necessary to improve also the precision of computation of solar inequalities' coefficients.

In order to obtain V and U with a precision to 0.01°, it is prerequisite to select in series (1) about 60 terms, and to obtain $\frac{a}{r}$ with a precision to 0.0001 — 50 terms.

In order to avoid the selection of terms of required precision by means of cumbersome calculations of all the coefficients of Delaunay expansions, we took advantage of the Proskurin work [1]. By using the tables of the said work, we noted all the terms of required precision for our purpose. Thus the solar inequality coefficients for Jupiter's X satellite were computed by the Delaunay expansions only for these terms. This was possible, inasmuch as the parameters of both satellites are very close in their values. To compile the tables it is practical to represent the coefficients of the trigonometric quantities in series (1), as follows:

../..

$$A' = e^p e'^{p'} \gamma^q \alpha^r \sum_{s=0}^{n} a_s m^s, \qquad B' = e^p e'^{p'} \gamma^q \alpha^r \sum_{s=0}^{n} b_s m^s, \qquad C' = e^p e'^{p'} \gamma^q \alpha^r \sum_{s=0}^{n} c_s m^s, \tag{11}$$

For the comparison of the tables for the coefficients of solar inequalities it is practical to represent the coefficient of trigonometric quantities in series (1) in the following form:

$$\left.\begin{aligned} V &= \varepsilon_0 + nt + \sum A' \sin(iD + jl + j'l' + kF), \\ U &= \sum B' \sin(iD + jl + j'l' + kF), \\ \frac{a}{r} &= \sum C' \cos(iD + jl + j'l' + kF). \end{aligned}\right\} \tag{12}$$

The system of elements of satellite's orbit, obtained by Herger, was laid at the basis of table composition for the coefficients A', B', C'. Let us bring forth this system here, and also the Jupiter's system of elements corresponding to it.

Epoch 1938 July 27. 3128

X SATELLITE			JUPITER	
$M_0 = 211.0221^\circ$			$\lambda' = 329.20254$	
$n = 1.382030^\circ$			$n' = 0.083091$	
$a = 0.078601$ a.u.			$a' = 5.202561$	
$e = 0.13244$		(I)	$e' = 0.048398$	(II)
$i = 28.266^\circ$	Eclipt.		$i' = 1.30614^\circ$	
$\omega = 247.044$	and equi.		$\omega' = 273.57223$	
$\Omega = 82.507$	1950.0		$\Omega' = 99.92939$	

(λ' is the longitude of Jupiter in the orbit corresponding to initial epoch).

The values of Delaunay parameters of Jupiter's X satellite are the following :

$$\begin{aligned} m &= 0.0601224 \\ \alpha &= 0.0151081 \\ \gamma &= 0.2336352 \\ e &= 0.13244 \\ e' &= 0.048398. \\ g' &= \lambda' - \omega' - \Omega'. \end{aligned} \tag{13}$$

Tables 2 — 4 give the values of the coefficients A', B', C'. Let us elucidate the arrangement of these tables. The upper line of Tables 2 and 3 are the values of m' expressed in degrees; at the same time the power of m does not exceed four. In the first column we placed the number of the

TABLE 2

Coefficients of Solar Inequalities (Longitude)

Numbers after Delaunay					$s=0$	$s=1$	$s=2$	$s=3$	$s=4$	
	p	p'	q	r \ m^s	57°296	3°445	0°207	0°012	0°001	A'
2	0	1	0	0	—	−0.1452	—	+2.2233	+15.2570	−0°46
2	0	1	2	0	—	+0.0360	−0.0390	—	—	+11
2	2	1	0	0	—	−0.0030	−0.0780	−0.7580	−5.3480	−4
3		2			—	−0.0050	—	—	—	−2
7	1				+0.2649	—	—	—	—	+15.18
7	3				−0.0006	—	—	—	—	−3
8	1	1			—	+0.0340	+0.2470	+1.4940	+8.7390	+19
8	1	1	2		—	−0.0110	−0.0400	—	—	−5
12	1	1			—	−0.0340	−0.1440	−0.6190	—	−15
12	1	1	2		—	+0.0110	—	—	—	+4
16	2				+0.0219	—	—	—	—	+1.25
16	2		2		−0.0012	+0.0040	—	—	—	−5
16	4				−0.0001	—	—	—	—	−1
16	2		4		−0.0006	+0.0040	—	—	—	−2
17	2	1			—	+0.0056	+0.0403	—	—	+3
17	2	1	2		—	−0.0021	—	—	—	−1
20	2	1			—	−0.0056	—	—	—	−2
20	2	1	2		—	+0.0021	—	—	—	+1
23	3				+0.0025	—	—	—	—	+14
23	3		2		−0.0003	—	—	—	—	−2
23	3		4		−0.0001	—	—	—	—	−1
28	4				+0.0003	—	—	—	—	+2
37			2		−0.0546	—	+0.1501	−1.9702	—	−3.12
37			4		−0.0030	—	—	—	—	−17
37	2		2		−0.0022	+0.0202	—	—	—	−6
37			6		−0.0002	—	—	—	—	−1
37	2		4		−0.0012	+0.0079	—	—	—	−4
37	4		2		+0.0002	—	—	—	—	+1
44	1		2		−0.0145	—	+0.0343	—	—	−82
44	1		4		−0.0008	—	—	—	—	−5
44	3		4		−0.0002	—	—	—	—	−1
45	1	1	2		—	−0.0024	—	—	—	−1
47	1	1	2		—	+0.0024	—	—	—	+1
49	2		2		−0.0031	—	—	—	—	−18
49	2		4		−0.0001	—	—	—	—	−1
54	3		2		−0.0006	—	—	—	—	−3
57	4		2		−0.0004	—	—	—	—	−2
58	1		2		−0.0217	+0.1220	—	—	—	−82
58	1		4		−0.0071	+0.0466	—	—	—	−25
58	3		2		+0.0010	−0.0080	—	—	—	+3
58	1		6		−0.0024	—	—	—	—	−14
58	3		4		+0.0006	—	—	—	—	+3

[1] Underlined spots in Tabl. 2 - 4 mean that the given coefficients are either zero or small, and that at multiplication by the respective multiplier m', they have values less than 0.01 in Tables 2 and 3 and less than 0.0001 in Table 4.

Table 2 (continuation)

Таблица 2 (*продолжение*)

№ по Делоне	p	p'	q	$r \diagdown m^s$	$s=0$ 57°296	$s=1$ 3°445	$s=2$ 0°207	$s=3$ 0°012	$s=4$ 0°001	A'
63	2		2		+0.0005	+0.0081	—	—	—	+0°06
63	2		4		+0.0005	+0.0015	—	—	—	+ 3
68	3		2		+0.0001	—	—	—	—	+ 1
68	3		4		+0.0002	—	—	—	—	+ 1
73			4		+0.0015	—	—	—	—	+ 9
73			6		+0.0002	—	—	—	—	+ 1
73	2		4		+0.0004	—0.0029	—	—	—	+ 1
78	1		4		+0.0008	—	—	—	—	+ 5
78	3		4		+0.0001	—	—	—	—	+ 1
81	2		4		+0.0003	—	—	—	—	+ 2
82	1		4		+0.0012	—0.0067	—	—	—	+ 5
82	1		6		+0.0005	—	—	—	—	+ 3
89			2		—	—0.0409	—0.1604	—	—	— 17
89	2				—	+0.0822	+0.3017	1.1009	—	+ 36
89			4		—	—0.0067	—	—	—	— 2
89	2		2		—	—0.0075	—0.0527	—	—	— 4
89	0	0	0	0	—	—	+1.3750	+4.9167	12.4028	+ 36
90		1	2		—	—0.0046	—0.0345	—	—	— 2
90	2	1			—	+0.0093	+0.0602	—	—	+ 4
90		1			—	—	+0.2329	+1.4489	+ 5.7102	+ 7
94		1	2		—	+0.0020	—	—	—	+ 1
94	2	1			—	—0.0040	—	—	—	— 1
94		1			—	—	—0.0333	—	—	— 1
98	1		2		—	—0.0108	—0.0371	—	—	— 4
98	3				—	+0.0142	+0.0482	—	—	+ 6
98	3		2		—	—0.0019	—	—	—	— 1
98	1				—	—	+0.2814	+0.9326	—	+ 7
99	3	1			—	+0.0016	—	—	—	+ 1
99	1	1			—	—	+0.0477	—	—	+ 1
105	2		2		—	—0.0023	—	—	—	— 1
105	4				—	+0.0025	—	—	—	+ 1
105	2				—	—	+0.0521	—	—	+ 1
118	1				—	+0.4967	+2.1770	+8.3150	27.0245	+2.29
118	1		2		—	—0.0434	—0.3244	—1.4992	— 6.2882	— 24
118	1	2			—	—0.0029	—	—	—	— 1
119	1	1			—	+0.0561	+0.3607	+1.5819	—	+ 29
119	1	1	2		—	—0.0049	—0.0450	—	—	— 3
120	1	2			—	+0.0049	+0.0416	—	—	+ 3
123	1	1			—	—0.0240	—0.0347	+0.8367	+11.7456	— 7
127	2				—	+0.0493	+0.2324	+1.5022	+ 7.3276	+ 24
127	2		2		—	—0.0014	—	—	—	0
128	2	1			—	+0.0056	+0.0306	—	—	+ 3
134	3				—	+0.0076	+0.0364	—	—	+ 3
139	4				—	—	—	—	—	0
148			4		—	+0.0022	—	—	—	+ 1
148	2		2		—	—0.0117	—0.0528	—	—	— 5
148			2		—	—	—0.0751	—	—	— 2

(Table 2) continuation

Таблица 2 (*продолжение*)

№ по Делоне	p	p'	q	r \ m^s	$s=0$ 57°296	$s=1$ 3°445	$s=2$ 0°207	$s=3$ 0°012	$s=4$ 0°001	A'
153	3		2		—	−0.0035	—	—	—	−0°01
153	1		2		—	—	−0.0352	—	—	— 1
161	1		2		—	−0.0271	−0.1374	—	—	— 12
162	1	1	2		—	−0.0030	−0.0238	—	—	— 2
166	2		2		—	−0.0072	—	—	—	— 2
166	2		4		—	−0.0018	—	—	—	— 1
179	1		4		—	+0.0015	—	—	—	+ 1
183			2		—	+0.1228	−0.3002	—	—	+ 36
183			4		—	−0.0045	+0.0292	—	—	— 1
183	2		2		—	−0.0090	—	—	—	— 3
183	2		4		—	−0.0018	—	—	—	— 1
184		1	2		—	+0.0139	−0.0291	—	—	+ 4
187		1	2		—	−0.0059	—	—	—	— 2
188	2		2		—	+0.0067	—	—	—	+ 2
190	1		2		—	−0.0298	+0.0261	—	—	— 10
190	1		4		—	−0.0109	+0.0386			— 3
191	1	1	2		—	−0.0034	—	—	—	— 1
195	2		2		—	−0.0054	—	—	—	— 2
195	2		4		—	—	—	—	—	0
204	1		2		—	+0.0108	−0.1102	—	—	+ 1
204	3		2		—	−0.0020	—	—	—	— 1
209	2		2		—	−0.0018	−0.0380	—	—	— 1
214	3		2		—	—	—	—	—	0
218			4		—	−0.0045	—	—	—	— 2
253	3				—	—	+0.0265	—	—	+ 1
253	1				—	—	—	+0.5276	—	+ 1
258	2				—	—	+0.0771	+0.6336	—	+ 2
342				1	—	−0.0283	−0.1756	−0.8129	—	— 14
342			2	1	—	+0.0170	+0.1077	+0.6566	—	+ 9
342	2			1	—	−0.0017	−0.0222	—	—	— 1
346		1		1	+0.0018	−0.0082	+0.0505	—	—	+ 9
346		1	2	1	−0.0003	−0.0032	—	—	—	— 3
346	2	1		1	+0.0001	+0.0014	—	—	—	+ 1
349	1			1	—	−0.0047	−0.0293	—	—	— 2
349	1		2	1	—	+0.0034	—	—	—	+ 1
352	1	1		1	+0.0003	−0.0014	—	—	—	+ 1
364	1			1	—	−0.0103	−0.0572	—	—	— 5
364	1		2	1	—	+0.0051	+0.0326	—	—	+ 2
367	1	1		1	+0.0003	−0.0030	—	—	—	+ 1
369	2			1	—	−0.0018	—	—	—	— 1
396			2	1	—	−0.0077	+0.0213	—	—	— 2
432	2			1	—	−0.0014	—	—	—	0
448			2	1	—	−0.0026	—	—	—	— 1

Таблица 3 (Table 3)

Коэффициенты солнечных неравенств (широта)

№ по Делоне	p	p′	q	r \ m^s	s = 0 57°296	s = 1 3°445	s = 2 0°207	s = 3 0°012	s = 4 0°001	B′
1			1		+0.4673	—	—	—	—	+26°77
1	2		1		—0.0082	—	—	—	—	— 47
1			5		—0.0002	—	—	—	—	— 1
2		1	1		—	+0.0085	—	—	—	+ 3
2		1	3		—	—0.0056	—	—	—	— 2
6		1	1		—	—0.0085	—0.0244	—	—	— 3
6		1	3		—	+0.0056	—	—	—	+ 2
10	1		1		+0.0619	—	—	—	—	+ 3.55
10	3		1		—0.0014	—	—	—	—	— 8
10	1		5		+0.0002	—	—	—	—	+ 1
11	1	1	1		—	+0.0090	+0.0570	—	—	+ 4
11	1	1	3		—	—0.0033	—	—	—	— 1
14	1	1	1		—	—0.0090	—0.0379	—	—	— 4
14	1	1	3		—	+0.0033	—	—	—	+ 1
17	2		1		+0.0092	—	—	—	—	+ 53
17	2		3		—0.0001	—	—	—	—	— 1
17	4		1		—0.0002	—	—	—	—	— 1
18	2	1	1		—	+0.0025	—	—	—	+ 1
20	2	1	1		—	—0.0025	—	—	—	— 1
22	3		1		+0.0014	—	—	—	—	+ 8
27	4		1		+0.0002	—	—	—	—	+ 1
31	1		1		—0.0619	—	+0.1827	—	—	— 3.51
31	1		3		—0.0085	+0.0285	—	—	—	— 39
31	3		1		+0.0007	—0.0023	—	—	—	+ 3
31	1		5		—0.0023	+0.0124	—	—	—	— 9
31	3		3		+0.0004	—0.0033	—	—	—	+ 1
32	1	1	1		—	+0.0067	+0.0461	—	—	+ 3
35	1	1	1		—	—0.0067	—0.0416	—	—	— 3
38	2		1		—0.0061	+0.0173	+0.0324	—	—	— 28
38	2		3		—0.0022	+0.0146	—	—	—	— 8
38	4		3		+0.0001	—	—	—	—	+ 1
43	3		1		—0.0008	+0.0023	—	—	—	— 4
43	3		3		—0.0003	+0.0018	—	—	—	— 1
48	4		1		—0.0001	—	—	—	—	— 1
53			3		—0.0042	—	+0.0351	—	—	— 23
53			5		—0.0002	—	—	—	—	— 1
53	2		3		—0.0018	+0.0085	—	—	—	— 7
53	2		5		—0.0005	—	—	—	—	— 3
53	4		3		+0.0001	—	—	—	—	+ 1
58	1		3		—0.0017	—	—	—	—	— 10
58	3		3		—0.0004	+0.0020	—	—	—	— 2
63	2		3		—0.0005	—	—	—	—	— 3
63	4		3		—0.0001	—	—	—	—	— 1
66	3		3		—0.0001	—	—	—	—	— 1
67	1		3		—0.0068	+0.0285	+0.0314	—	—	— 28
67	1		5		—0.0018	+0.0109	—	—	—	— 7
67	3		3		+0.0005	—0.0022	—	—	—	+ 2

Table 3 (continuation) Таблица 3 (продолжение)

№ по Делоне*	p	p'	q	$r \backslash m^s$	$s=0$ 57°296	$s=1$ 3°445	$s=2$ 0°207	$s=3$ 0°012	$s=4$ 0°001	B'
72	2		3		+0.0004	+0.0024	—	—	—	+0°03
81	1		5		+0.0002	—	—	—	—	+ 1
82	2		5		+0.0001	—	—	—	—	+ 1
83			3		—	−0.0048	−0.0363	—	—	− 2
83	2		1		—	+0.0346	+0.1235	+0.4559	—	+ 15
83	2		3		—	—	—	—	—	0
83			1		—	—	+0.3212	+1.1488	—	+ 8
84	2	1	1		—	+0.0039	+0.0254	—	—	+ 2
84		1	1		—	—	+0.0544	—	—	+ 1
87	2	1	1		—	−0.0017	—	—	—	− 1
90	1		3		—	−0.0019	—	—	—	− 1
90	3		1		—	+0.0081	+0.0269	—	—	+ 3
90	1		1		—	—	+0.1083	—	—	+ 2
95	4		1		—	+0.0018	—	—	—	+ 1
95	2		1		—	—	+0.0272	—	—	+ 1
103	1		1		—	+0.1160	+0.4661	+1.8031	+5.8800	+ 52
103	1		3		—	−0.0139	−0.0705	—	—	− 6
103	3		1		—	−0.0028	—	—	—	− 1
104	1	1	1		—	+0.0131	+0.0792	—	—	+ 6
104	1	1	3		—	−0.0016	—	—	—	− 1
107	1	1	1		—	−0.0056	—	—	—	− 2
110	2		1		—	−0.0019	−0.0249	—	—	− 1
124	2		3		—	−0.0018	—	—	—	− 1
132	1		3		—	−0.0032	−0.0252	—	—	− 2
132	3		3		—	−0.0017	—	—	—	− 1
137	2		3		—	−0.0038	—	—	—	− 1
143			1		—	+0.1752	+0.3652	+0.8995	—	+ 69
143			3		—	+0.0143	−0.0697	—	—	+ 3
143	2		1		—	+0.0069	+0.0271	—	—	+ 3
144		1	1		—	+0.0198	+0.0901	—	—	+ 9
148		1	1		—	−0.0085	−0.0406	—	—	− 4
152	1		1		—	+0.0232	+0.0445	—	—	+ 9
152	1		3		—	−0.0051	—	—	—	− 2
152	3		1		—	+0.0021	—	—	—	+ 1
152	1		5		—	−0.0025	—	—	—	− 1
153	1	1	1		—	+0.0026	—	—	—	+ 1
159	2		1		—	+0.0035	—	—	—	+ 1
173	1		1		—	+0.0928	+0.4061	+1.7797	+5.9407	+ 43
173	1		3		—	−0.0057	−0.0660	—	—	− 3
174	1	1	1		—	+0.0105	+0.0640	—	—	+ 5
177	1	1	1		—	−0.0045	—	—	—	− 2
180	2		1		—	+0.0188	+0.1043	+0.5101	—	+ 9
180	2		3		—	—	—	—	—	0
181	2	1	1		—	+0.0021	—	—	—	+ 1
185	3		1		—	+0.0045	+0.0230	—	—	+ 2

* The numbers are after Delaunay

Table 3 (continuation)

Таблица 3 (*продолжение*)

№ по Делоне*	p	p'	q	r \ m'	s=0	s=1	s=2	s=3	s=4	B'
					57°296	3°445	0°207	0°012	0°001	
194			3		—	+0.0239	−0.0363	—	—	+0°07
194	2		3		—	−0.0046	—	—	—	− 2
195		1	3		—	+0.0027	—	—	—	+ 1
199	1		3		—	−0.0070	—	—	—	− 2
199	1		5		—	−0.0020	—	—	—	− 1
208	1		3		—	+0.0089	—	—	—	+ 3
208	3		3		—	−0.0016	—	—	—	− 1
240	2		1		—	—	+0.0324	—	—	+ 1
266	1		1		—	—	+0.0435	—	—	+ 1
317			1	1	—	−0.0066	−0.0366	—	—	− 3
317			3	1	—	+0.0040	+0.0257	—	—	+ 2
320		1	1	1	+0.0004	−0.0019	—	—	—	+ 2
322	1		1	1	—	−0.0020	—	—	—	− 1
349			1	1	—	−0.0066	−0.0227	—	—	− 3
349			3	1	—	+0.0018	+0.0229	—	—	+ 1
352		1	1	1	+0.0004	−0.0019	—	—	—	+ 2
354	1		1	1	—	−0.0028	—	—	—	− 1

TABLE 4 Таблица 4 Coeff.of Solar Inequ.

Коэффициенты солнечных неравенств $\left(\frac{a}{r}\right)$

№ по Делоне*	p	p'	q	r \ m'	s=0	s=1	s=2	s=3	s=4	C'
					$10.000000 \cdot 10^{-1}$	$0.601224 \cdot 10^{-1}$	$0.036147 \cdot 10^{-1}$	$0.002173 \cdot 10^{-1}$	$0.000131 \cdot 10^{-1}$	
1	0	0	0	0	+1.00000	—	+0.16667	—	—	+1.0006
2		1			—	—	−0.07260	—	—	− 3
2		1	2		—	—	+0.02378	—	—	+ 1
4	1				+0.13244	—	−0.07721	−0.58976	—	+ 1320
4	3				−0.00029	—	—	—	—	− 3
4	1		4		+0.00099	—	—	—	—	+ 10
4	3		2		−0.00016	—	—	—	—	− 2
5	1	1			—	+0.01683	+0.11147	+0.65480	—	+ 16
5	1	1	2		—	−0.00551	—	—	—	− 3
7	1	1			—	−0.01683	−0.08383	−0.34107	—	− 14
7	1	1	2		—	+0.00551	—	—	—	+ 3
9	2				+0.01754	—	−0.01461	—	—	+ 175
9	4				−0.00010	—	—	—	—	− 1
10	2	1			—	+0.00361	+0.03080	—	—	+ 3
12	2	1			—	−0.00361	−0.02093	—	—	− 3
14	3				+0.00261	—	—	—	—	+ 26

* The numbers are after Delaunay

Table 4 (continuation) **Таблица 4 (продолжение)**

№ по Делоне*	p	p'	q	r \ m'	$s=0$	$s=1$	$s=2$	$s=3$	$s=4$	C'
					$10.000000\cdot10^{-1}$	$0.601224\cdot10^{-1}$	$0.036147\cdot10^{-1}$	$0.002173\cdot10^{-1}$	$0.000131\cdot10^{-1}$	
15	3	1			—	+0.00100	—	—	—	+ 1
16	3	1			—	—0.00100	—	—	—	— 1
17	4				+0.00041	—	—	—	—	+ 4
18	5				+0.00007	—	—	—	—	+ 1
19	2		2		—0.00479	+0.01616	—	—	—	— 38
19			2		—	—	+0.10917	—	—	+ 4
22	3		2		—0.00107	—	—	—	—	— 11
22	1		2		—	—	+0.02982	—	—	+ 1
23	1		2		—0.01807	+0.06100	—	—	—	— 144
23	1		4		—0.00395	—	—	—	—	— 40
23	3		2		+0.00059	—	—	—	—	+ 6
24	1	1	2		—	+0.00164	—	—	—	+ 1
25	1	1	2		—	—0.00164	—	—	—	— 1
26	3		2		—0.00008	—	—	—	—	— 1
27	2				—	+0.06578	+0.20720	+0.71826	—	+ 49
27	2		2		—	—0.00718	—	—		— 4
27	0	0	0	0	—	—	+1.00000	+3.16667	+7.27778	+ 44
27			2		—	—	—0.10917	—0.26381	—	— 5
28	2	1			—	+0.00743	+0.04239	—	—	+ 6
28		1			—	—	+0.16939	+0.94981	—	+ 8
29		2			—	—	+0.01991	—	—	+ 1
30	2	1			—	—0.00318	—	—	—	— 2
30		1			—	—	—0.02420	—	—	— 1
32	3				—	+0.01470	+0.04571	—	—	+ 10
32	1				—	—	+0.27322	+0.83609	—	+ 12
32	1		2		—	—	—0.02982	—	—	— 1
33	3	1			—	+0.00166	—		—	+ 1
33	1	1			—	—	+0.04627	+0.28679	—	+ 2
36	4				—	+0.00308				+ 2
36	2				—	—	+0.06139	—	—	+ 2
40	1				—	+0.24833	+0.77398	+2.54470	+8.34902	+ 184
40	1		2		—	—0.02711	—0.14549	—	—	— 22
40	1	2			—	—0.00145	—	—	—	— 1
41	1	1			—	+0.02804	+0.12709	+0.37336	—	+ 22
41	1	1	2		—	—0.00306	—	—	—	— 2
42	1	2			—	+0.00247	+0.01455	—	—	+ 2
43	1	1			—	—0.01202	—	—	—	— 7
43	1	1	2		—	+0.00131	—	—	—	+ 1
45	2				—	—	—0.06578	—0.24667	—	— 3
48	3				—	—0.00381	—0.02330	—	—	— 3
51	4				—	—0.00106	—	—	—	— 1
52	1		2		—	—	—0.03727	—	—	— 1
53	2		2		—	—0.00898	—	—	—	— 5
54			2		—	—	—0.16376	—	—	— 6

* The numbers are after Delaunay

TABLE 4 (continuation and end)

№ по Делоне					$s=0$	$s=1$	$s=2$	$s=3$	$s=4$	C'
	p	p'	q	r \ m^s	$10.000000 \cdot 10^{-1}$	$0.601224 \cdot 10^{-1}$	$0.036147 \cdot 10^{-1}$	$0.002173 \cdot 10^{-1}$	$0.000131 \cdot 10^{-1}$	
55		1	2		—	—	—0.02774	—	—	—0.0001
57	1		2		—	—0.01491	+0.02841	—	—	— 8
58	1	1	2		—	—0.00168	—	—	—	— 1
60	2		2		—	—0.00395	—	—	—	— 2
61	1		2		—	—0.01898	—0.02508	—	—	— 12
62	1	1	2		—	—0.00214	—	—	—	— 1
64	2		2		—	—0.00503	—	—	—	— 3
69	3				—	—	+0.02756	—	—	+ 1
69	1				—	—	—	+0.51215	—	+ 1
72	2				—	—	+0.06167	+0.43783	—	+ 3
78				1	—	—0.01417	—0.07649	—0.34330	—	— 12
78			2	1	—	+0.00851	—	—	—	+ 5
81		1		1	+0.00091	—0.00411	+0.02526	—	—	+ 8
81		1	2	1	—0.00015	—	—	—	—	— 2
83	1			1	—	—0.00375	—0.02214	—	—	— 3
85	1	1		1	+0.00024	—0.00109	—	—	—	+ 2
89	2			1	—	+0.00090	—	—	—	+ 1
91			2	1	—	+0.00464	—	—	—	+ 3
100			2	1	—	—0.00129	—	—	—	— 1

expansion term according to Delaunay. Given in columns 2 — 5 are the values of the exponents of the parameters e, e', γ, α. In the column 6 — 10 we placed the values of the expressions $a_s e^p e'^{p'} \gamma^q \alpha^r$, $b_s e^p e'^{p'} \gamma^q \alpha^r$. Finally in the last column the values of the coefficients A' and B' are given, which were obtained according to expressions (11). Table 4 has quite an analogous arrangement, except that the values of m' are expressed in radians and not in degrees; accordingly the coefficients C' are expressed likewise.

As already stated, these tables may serve as initial data for the entire subsequent work linked with the construction of the tables of Jupiter's X satellite. Their presence allows the reconversion of the coefficients of solar inequalities for any new system of elements without having to resort to cumbersome computations, that is, directly by Delaunay expansions. We shall illustrate this by an example.

Assume that a new system of elements has been obtained. We shall compute for it the new values of the Delaunay parameters. We shall denote them by e_{*}, γ_{*}, α_{*}, m_{*}; e' does not vary. We shall compare the ratios of the new parameters to the old, thus obtaining new values m_{*}'. For example, in order to recompute the coefficient of the term No. 58 with the characteristic $e\gamma^2$ in Table 2, we must compute the conversion factor from the old system to the new. It will be equal to

$$\frac{e_{*}}{e}\left(\frac{\gamma_{*}}{\gamma}\right)^2)$$

and to recompute the corresponding values of the columns 6 — 10. Then we shall obtain the new value of the coefficient A' at once.

2. - DETERMINATION OF THE JOVICENTRIC COORDINATES OF THE SATELLITE

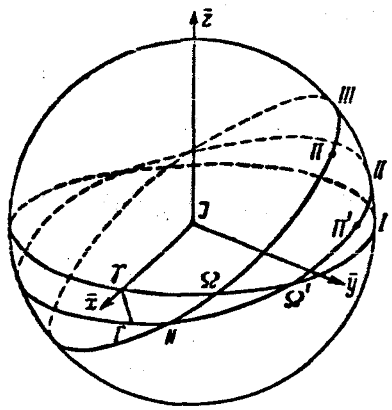
In the Delaunay expansions each term is characterized by its own argument. This is why we may unify in Tables 2 — 4 the terms with identical arguments, that is, the terms of identical numbers. Moreover, for the computation of satellite's coordinates, the values of arguments must be available for all the terms of Tables 2 — 4.

The expressions of the arguments according to the Delaunay theory are represented by formulas (2). In order to obtain the values of these arguments, it is necessary to compute the secular motions of satellite's perijove and node. According to the formulas by Delaunay and Porskurin [6, 1], these motions are as follows:

$$\frac{d\pi}{dt} = +1^{\circ}.5924 \text{ per annum}$$

$$\frac{d\Omega}{dt} = -1^{\circ}.2587 \text{ per annum along Jupiter's orbit}$$

It is necessary to bear in mind that we shauld take for the basic plane that of Jupiter's orbit. In our case the elements of the system (1) Ω, ω and i are referred to the ecliptic. This is why, before computing the arguments we must obtain the values of angular elements Ω_1, ω_1 and i_1, referred to Jupiter's orbit plane. Let the plane I (Fig), passing through Jupiter, be parallel to the ecliptic, the plane II — the Sun's orbit plane, and the plane III — the satellite's orbit plane. The direction Jγ is parallel to the direction from the Sun to the point of vernal equinox. Γ



is the point of intersection of the orbit of the Sun with the great circle passing through the point γ and the pole of Sun's orbit. The angle reading will be conducted from the point Γ. Then, relative to the orbit plane of Jupiter

$\Omega_1 = \Gamma N$ is the longitude of satellite's ascending node;

$\omega_1 = N\Pi$ is the distance of the perijove to the node;

Π is the point of satellite's perijove.

Known to us are the ecliptic longitude of the node $\Omega = \gamma\Omega$, the distance of satellite's perijove from the node $\omega = \Omega\Pi$, and also $\Omega' = \gamma\Omega'$ which is the longitude of the Sun's node. From the spherical triangles $N\Omega\Omega'$ and $\gamma\Gamma\Omega'$ we find $N\Omega$, $N\Omega'$ and $\Gamma\Omega'$, and then

$$\Omega_1 = \Gamma N = \Gamma\Omega' - N\Omega,$$
$$\omega_1 = N\Pi = N\Omega + \omega.$$

As to the inclination of satellite's orbit relative to Jupiter plane it is determined from the spherical triangle $N\Omega\Omega'$. The computations give the following results:

$$\left.\begin{aligned} \Omega_1 &= 81,\ 74702^\circ \\ \omega_1 &= 247.90472 \\ i_1 &= 27.02238 = I_0 \end{aligned}\right\} \quad \begin{array}{l} \text{Epoch 1938} \\ \text{July 27,3128} \\ \text{equin. 1950.0} \end{array}$$

In accord with formulas (2) we obtain the following system of satellite arguments:

$$\left.\begin{aligned} l &= 211^\circ 02210 + 1^\circ 377670\, t, \\ l' &= 315.86046 + 0.0^\circ 3091\, t, \\ D &= 31.30923 + 1.298939\, t, \\ F &= 98.92682 + 1.385476\, t, \end{aligned}\right\} \qquad (14)$$

at the same time, t is expressed in days from epoch 1938 July 27.3128 = = 2429107.3128 Julian days.

Assembling the terms with identical arguments in Table 2 — 4, we computed the values of the arguments for all these terms. Note that the expansion of the longitude and of the quantity $\frac{a}{r}$ have identical arguments. This allows to compose tables for the determination of jovicentric coordinates of the satellite in a more compact manner. This is attained by way of unifying the tables 2 and 4 into a single table 5. In it, the values of all arguments are placed at the center, the coefficients for the calculation of perturbations of the longitude

$$\delta V = \sum A' \sin (iD + jl + j'l' + kF),$$

are given to the left, while the expansion coefficients of the quantity

$$\frac{a}{r} = \sum C' \cos (iD + jl + j'l' + kF).$$

are brought out to the right.

The argument indices are brought out in the columns 4 — 7. Table 6 serves for the computation of the jovicentric latitude.

In order to obtain the jovicentric coordinates by means of Tables 5 and 6 for some moment of time, it is prerequisite :

1) to compute the arguments for that moment of time, counting <u>t</u> in days from 1938 July 27.3128;

2) to find the sines and the cosines of these arguments and to multiply them by the corresponding coefficients;

3) to sum up the terms by the entire table.

The sum thus obtained will provide the values of the quantities δV, U and $\frac{a}{r}$.

A correction must be introduced into these values, namely: all must be multiplied by $\left(1 - \frac{J}{m'}\right)$, where J is the mass of Jupiter and m' that of the Sun.

The fact is that in the theory of motion of the Moon, the motion of the Sun is viewed as a motion along an ellipse, of which the great semiaxis is determined by the correlation

$$a'^3 n'^2 = k^2 (M + m'),$$

where M is the mass of the Earth plus that of the Moon.

However, for simplicity, in the perturbation function in the equations of motion of the Moon it is usually assumed $k^2 m' = a'^3 n'^2$, that is, the masses of

Таблица 5 (TABLE 5)

&

δV и величина $\frac{a}{r}$

№ по пор. *	№ по Делоне **	A′	i	j	j′	k	Argument Аргумент	№ по пор. *	№ по Делоне **	C′
			0	0	0	0		1	1	+1.0006
1	2	− 0°39	0	0	1	0	315°860 + 0°083091t	2	2	− 2
2	3	− 2	0	0	2	0	271.720 + 0.166182t			
3	7	+15.15	0	1	0	0	211.022 + 1.377670t	3	4	+ 1325
4	8	+ 14	0	1	−1	0	255.162 + 1.294579t	4	5	+ 13
5	12	− 11	0	1	1	0	166.883 + 1.460761t	5	7	− 11
6	16	+ 1.17	0	2	0	0	62.044 + 2.755340t	6	9	+ 174
7	17	+ 2	0	2	−1	0	106.184 + 2.672249t	7	10	+ 3
8	20	− 1	0	2	1	0	17.905 + 2.838431t	8	12	− 3
9	23	+ 11	0	3	0	0	273.066 + 4.133010t	9	14	+ 26
			0	3	−1	0	317.206 + 4.049919t	10	15	+ 1
			0	3	1	0	228.927 + 4.216101t	11	16	− 1
10	28	+ 2	0	4	0	0	124.988 + 5.510680t	12	17	+ 4
			0	5	0	0	335.111 + 6.888350t	13	18	+ 1
11	37	− 3.39	0	0	0	2	197.854 + 2.770952t	14	19	− 34
12	44	− 88	0	1	0	2	48.876 + 4.148622t	15	22	− 10
13	45	− 1	0	1	−1	2	93.015 + 4.065531t			
14	47	+ 1	0	1	1	2	4.736 + 4.231713t			
15	49	− 19	0	2	0	2	259.898 + 5.526292t			
16	54	− 3	0	3	0	2	110.920 + 6.903962t			
17	57	− 2	0	4	0	2	321.942 + 8.281632t			
18	58	− 1.15	0	−1	0	2	346.832 + 1.393282t	16	23	− 178
			0	−1	−1	2	30.971 + 1.310191t	17	24	+ 1
			0	−1	1	2	302.692 + 1.476373t	18	25	− 1
19	63	+ 9	0	−2	0	2	135.809 + 0.015612t			
20	68	+ 2	0	−3	0	2	284.787 − 1.362058t	19	26	− 1
21	73	+ 11	0	0	0	4	35.707 + 5.541904t			
22	78	+ 6	0	1	0	4	246.729 + 6.919574t			
23	81	+ 2	0	2	0	4	97.751 + 8.297244t			
24	82	+ 8	0	−1	0	4	184.685 + 4.164234t			
25	89	+ 49	2	0	0	0	62.618 + 2.597878t	20	27	+ 84
26	90	+ 9	2	0	−1	0	106.758 + 2.514787t	21	28	+ 14
			2	0	−2	0	150.898 + 2.431696t	22	29	+ 1
27	94	− 1	2	0	1	0	18.479 + 2.680969t	23	30	− 3
28	98	+ 8	2	1	0	0	273.641 + 3.975548t	24	32	+ 21
29	99	+ 2	2	1	−1	0	317.780 + 3.892457t	25	33	+ 3
30	105	+ 1	2	2	0	0	124.663 + 5.353218t	26	36	+ 4
31	118	+ 2.04	2	−1	0	0	211.596 + 1.220208t	27	40	+ 161
32	119	+ 26	2	−1	−1	0	255.736 + 1.137117t	28	41	+ 20
33	120	+ 3	2	−1	−2	0	299.875 + 1.054026t	29	42	+ 2
34	123	− 7	2	−1	1	0	167.457 + 1.303299t	30	43	− 6
35	127	+ 24	2	−2	0	0	0.574 − 0.157462t	31	45	− 3
36	128	+ 3	2	−2	−1	0	44.714 − 0.240553t			
37	134	+ 3	2	−3	0	0	149.552 − 1.535132t	32	48	− 3
			2	−4	0	0	298.530 − 2.912802t	33	51	− 1
38	148	− 6	2	0	0	2	260.472 + 5.368830t			
39	153	− 2	2	1	0	2	111.494 + 6.746500t			
40	161	− 12	2	−1	0	2	49.450 + 3.991160t	34	52	− 1
41	162	− 2	2	−1	−1	2	93.590 + 3.908069t			
42	166	− 3	2	−2	0	2	198.428 + 2.613490t	35	53	− 5
43	179	+ 1	2	−1	0	4	247.304 + 6.762112t			
44	183	+ 31	2	0	0	−2	224.765 − 0.173074t	36	54	− 6
45	184	+ 4	2	0	−1	−2	268.904 − 0.256165t	37	55	− 1
46	187	− 2	2	0	1	−2	180.625 − 0.089983t			
47	188	+ 2	0	0	2	−2	136.486 − 0.006892t			
48	190	− 13	2	1	0	−2	75.787 + 1.204596t	38	57	− 8
49	191	− 1	2	1	−1	−2	119.926 + 1.121505t	39	58	− 1
50	195	− 2	2	2	0	−2	286.809 + 2.582266t	40	60	− 2
			2	−1	0	−2	13.743 − 1.550744t	41	61	− 12
			2	−1	−1	−2	57.882 − 1.633835t	42	62	− 1

* Number by order; ** Number by Delaunay

(Table 5 continuation)

Таблица 5 *(продолжение)*

№ по пор. *	№ по Делоне **	A′	i	j	j′	k	Аргумент Argument	№ по пор. *	№ по Делоне **	C′
51	209	−0°01	2	−2	0	−2	162°721 − 2°928414t	43	64	−0.0003
52	218	− 2	2	0	0	−4	26.911 − 2.944026t			
53	253	+ 2	4	−1	0	0	274.215 + 3.818086t	44	69	+ 2
54	258	+ 2	4	−2	0	0	63.193 + 2.440416t	45	72	+ 3
55	342	− 6	1	0	0	0	31.309 + 1.298939t	46	78	− 7
56	346	+ 7	1	0	1	0	347.170 + 1.382030t	47	81	+ 6
57	349	− 1	1	1	0	0	242.331 + 2.676609t	48	83	− 3
58	352	+ 1	1	1	1	0	198.192 + 2.759700t	49	85	+ 2
59	364	− 3	1	−1	0	0	180.287 − 0.078731t			
60	367	+ 1	1	−1	1	0	136.148 + 0.004360t			
61	369	− 1	1	−2	0	0	329.265 − 1.456401t	50	89	+ 1
62	396	− 2	1	0	0	−2	193.456 − 1.472013t	51	91	+ 3
63	448	− 1	3	0	0	−2	256.074 + 1.125865t	52	100	− 1

Таблица 6 (TABLE 6)

U

№ по пор. *	№ по Делоне **	B′	i	j	j′	k	Аргумент Argument
1	1	+26°29	0	0	0	1	98°927 + 1°385476t
2	2	+ 1	0	0	−1	1	143.066 + 1.302385t
3	6	− 1	0	0	1	1	54.787 + 1.468567t
4	10	+ 3.48	0	1	0	1	309.949 + 2.763146t
5	11	+ 3	0	1	−1	1	354.088 + 2.680055t
6	14	− 3	0	1	1	1	265.809 + 2.846237t
7	17	+ 51	0	2	0	1	160.971 + 4.140816t
8	18	+ 1	0	2	−1	1	205.111 + 4.057725t
9	20	− 1	0	2	1	1	116.831 + 4.223907t
10	22	+ 8	0	3	0	1	11.993 + 5.518486t
11	27	+ 1	0	4	0	1	223.015 + 6.896156t
12	31	− 3.95	0	−1	0	1	247.905 + 0.007806t
13	32	+ 3	0	−1	−1	1	292.044 − 0.075285t
14	35	− 3	0	−1	1	1	203.765 + 0.090897t
15	38	− 35	0	−2	0	1	36.883 − 1.369864t
16	43	− 5	0	−3	0	1	185.861 − 2.747534t
17	48	− 1	0	−4	0	1	334.838 − 4.125204t
18	53	− 33	0	0	0	3	296.780 − 4.156428t
19	58	− 12	0	1	0	3	147.803 + 5.534098t
20	63	− 4	0	2	0	3	358.825 + 6.911768t
21	66	− 1	0	3	0	3	209.847 + 8.289438t
22	67	− 33	0	−1	0	3	85.758 + 2.778758t
23	72	+ 3	0	−2	0	3	234.736 + 1.401088t
24	81	+ 1	0	−1	0	5	283.612 + 5.549710t
25	82	+ 1	0	−2	0	5	72.590 + 4.172040t
26	83	+ 21	2	0	0	1	161.545 + 3.983354t
27	84	+ 3	2	0	−1	1	205.685 + 3.900263t
28	87	− 1	2	0	1	1	117.406 + 4.066445t
29	90	+ 4	2	1	0	1	12.567 + 5.361024t
30	95	+ 2	2	2	0	1	223.589 + 6.738694t
31	103	+ 45	2	−1	0	1	310.523 + 2.605684t
32	104	+ 5	2	−1	−1	1	354.663 + 2.522593t
33	107	− 2	2	−1	1	1	266.384 + 2.688775t
34	110	− 1	2	−2	0	1	99.501 + 1.228014t
35	124	− 1	2	0	0	3	359.399 + 6.754306t
36	132	− 3	2	−1	0	3	148.377 + 5.376636t
37	137	− 1	2	−2	0	3	297.355 + 3.998966t
38	143	+ 75	2	0	0	−1	323.692 + 1.212402t

* Number by order; ** Number according to Delaunay

TABLE 6 (end)

No. order	No. Delaun.	B'	i	j	j'	k	Argument
39	144	+0°09	2	0	−1	−1	7°831 + 1°129311t
40	148	− 4	2	0	1	−1	279.552 + 1.295493t
41	152	+ 7	2	1	0	−1	174.714 + 2.590072t
42	153	+ 1	2	1	−1	−1	218.853 + 2.506981t
43	159	+ 1	2	2	0	−1	25.736 + 3.967742t
44	173	+ 40	2	−1	0	−1	112.670 − 0.165268t
45	174	+ 5	2	−1	−1	−1	156.809 − 0.248359t
46	177	− 2	2	−1	1	−1	68.530 − 0.082177t
47	180	+ 9	2	−2	0	−1	261.647 − 1.542938t
48	181	+ 1	2	−2	−1	−1	305.787 − 1.626029t
49	185	+ 2	2	−3	0	−1	50.625 − 2.920608t
50	194	+ 5	2	0	0	−3	125.838 − 1.558550t
51	195	+ 1	2	0	−1	−3	169.978 − 1.641641t
52	199	− 3	2	1	0	−3	336.860 − 0.180880t
53	208	+ 2	2	−1	0	−3	274.816 − 2.936220t
54	240	+ 1	4	−2	0	1	162.120 + 3.825892t
55	266	+ 1	4	−1	0	−1	175.288 + 2.432610t
56	317	− 1	1	0	0	1	130.236 + 2.684415t
57	320	+ 2	1	0	1	1	86.097 + 2.767506t
58	322	− 1	1	1	0	1	341.258 + 4.062085t
59	349	− 2	1	0	0	−1	292.382 − 0.086537t
60	352	+ 2	1	0	1	−1	248.243 − 0.003446t
61	354	− 1	1	1	0	−1	143.405 + 1.291133t

the Earth and of the Moon are neglected. This inaccuracy must be taken into account in the final results (Delaunay, 1860 [5], pp.20, 21). It will be entirely taken into account if all the coefficients of solar inequalities are multiplied by the quantity

$$\left(1-\frac{M}{m'}\right).$$

In our case, applying this to Jupiter satellites,

$$1-\frac{J}{m'}=0.9990452$$

the mass of the satellites being valued zero.

3. - COMPARISON OF THEORY WITH OBSERVATIONS

According to the theory created by us, we shall now compute the geocentric positions of the satellite for all the available moments of observations and compare them with the observed positions.

The summary of all the available observations from the time of satellite discovery up until now, and the results of comparison of theory with the observations are compiled in Table 7. All the observations are reduced to a unique system of coordinates — the axes 1950.0.

TABLE 7

Representation of Observations

No. by order	Universal		α (1950.0)	δ (1950.0)	Computed-observed Δα	Computed-observed Δδ	Place of Obser.	Sources
1	1938 July	6.3743	$22^h17^m15^s.21$	−11°13′32″.3	+ 3ˢ.51	+ 13″.4	Mount Wilson	
2		6.4667	22 17 13.69	−11 13 41.6	+ 3.44	+ 10.5		
3		9.4229	22 16 22.89	−11 18 23.4	+ 2.93	+ 8.9	"	
4		9.4521	22 16 22.38	−11 18 26.0	+ 2.97	+ 8.6	"	
5		27.3361	22 09 13.67	−12 00 14.9	+ 1.48	− 15.4	"	
6		28.3174	22 08 45.05	−12 03 06.3	+ 1.47	− 13.8	"	A. J., 48, 1114
7		29.4667	22 08 10.91	−12 06 30.2	+ 1.29	− 17.1	"	
8	Aug.	24.4167	21 54 07.40	−13 33 16.2	+ 2.23	− 80.9	"	
9		25.2129	21 53 41.88	−13 36 00.3	+ 2.70	− 79.1	"	
10	Oct.	18.1285	21 39 13.15	−15 25 53.6	+ 15.92	− 97.5	"	
11		20.1257	21 39 25.45	−15 25 54.0	+ 16.75	− 93.5	"	
12		23.1319	21 39 50.17	−15 25 15.1	+ 17.07	− 84.1	"	
13	Nov.	21.1188	21 49 30.06	−14 41 12.0	+ 11.97	− 13.8	"	
14	1939 Jul.	15.4205	0 31 26.11	+ 1 36 35.3	+ 14.51	− 109.7	"	
15		15.4528	0 31 26.49	+ 1 36 36.4	+ 14.73	− 109.2	"	
16	Aug.	16.4365	0 33 47.03	+ 1 33 14.1	+ 20.52	+ 76.4	"	A. J., 49, 1123
17		16.4806	0 33 46.72	+ 1 33 11.8	+ 18.90	+ 76.4	"	
18	Oct.	8.3998	0 15 53.39	+ 0 00 48.7	+ 0.97	+ 84.4	"	
19	Dec.	15.0885	0 01 26.93	− 0 48 17.8	− 7.72	− 159.8	"	
20	1940 Sep.	8.3736	2 57 26.16	+16 02 05.9	− 17.44	− 105.9	"	
21		8.4438	2 57 25.74	+16 02 03.0	− 17.43	− 107.7	"	
22	Oct.	25.3488	2 38 37.74	+14 14 20.0	− 28.15	− 432.5	"	
23		25.3731	2 38 36.88	+14 14 15.2	− 27.94	− 430.7	"	
24	Nov.	1.2160	2 34 23.98	+13 49 17.5	− 26.68	− 458.6	"	
25		1.4181	2 34 16.35	+13 48 33.3	− 26.87	− 457.3	"	
26	1941 Dec.	23.2569	4 56 06.78	+22 24 12.2	+ 15.54	+ 245.3	"	A. J., 50, 1147
27	1942 Feb.	17.2342	4 44 28.41	+22 23 09.5	− 16.27	− 96.3	"	
28		18.2618	4 44 26.92	+22 23 19.4	− 16.70	− 104.9	"	
29	Nov.	8.4951	7 53 12.49	+21 30 26.6	+ 7.67	− 121.3	"	
30		9.5090	7 53 17.86	+21 30 12.4	+ 7.00	− 126.9	"	
31	1943 Jan.	6.2400	7 32 51.54	+21 57 36.7	− 62.89	− 380.4	"	
32		6.3230	7 32 48.03	+21 57 40.5	− 63.02	− 381.2	"	
33	1951 Sep.	29.26245	0 40 06.62	+ 2 19 43.3	+143.75	+1718.7	"	UAI Circ., № 1333; Nbl. AZ Vor. Mit., № 134 1951 Hов6. 9
34		29.39782	0 40 03.19	+ 2 19 28.0	+143.35	+1723.4	"	
35		30.29101	0 39 41.51	+ 2 17 48.3	+142.39	+1739.7	"	
36		30.30663	0 39 41.08	+ 2 17 47.1	+142.32	+1741.4	"	
37	Oct.	1.31775	0 39 16.21	+ 2 15 53.7	+140.64	+1757.0	"	UAI Circ., № 1332
38		2.22609	0 38 53.74	+ 2 14 11.4	+139.12	+1772.4	"	
39		2.24814	0 38 53.15	+ 2 14 09.5	+139.06	+1773.1	"	
40		2.26428	0 38 52.77	+ 2 14 07.2	+139.00	+1773.1	"	
41		4.38116	0 37 59.71	+ 2 10 12.1	+134.70	+1800.0	"	UAI Circ., № 1333
42	1954 Feb.	3.30866	5 04 25.78	+22 59 04.3	+ 79.64	+ 700.3	Macdonald	
43		4.23182	5 04 22.11	+22 59 21.6	+ 77.44	+ 683.1	To же	A. J., 60, 1225
44		5.23260	5 04 18.81	+22 59 36.0	+ 74.84	+ 663.8	"	
45		6.25208	5 04 16.17	+22 59 42.3	+ 72.45	+ 630.5	"	

TABLE 8

Jovicentric coordinates of the satellite at moments of observ.

No. of obs.	δV	U	$\frac{a}{r}$	No. of obs.	δV	U	$\frac{a}{r}$
1	$-1°.91$	$+24°.57$	0.8608	24	$-12°.19$	$+12°.07$	0.9694
2	-1.96	$+24.58$	0.8606	25	-12.23	$+11.94$	0.9700
3	-2.79	$+25.06$	0.8608	26	$+8.03$	$+15.22$	0.9237
4	-2.80	$+25.07$	0.8608	27	-4.80	$+27.26$	0.8835
5	-7.20	$+26.43$	0.8692	28	-5.02	$+27.22$	0.8844
6	-7.41	$+26.38$	0.8699	29	-6.46	$+26.83$	0.8815
7	-7.67	$+26.38$	0.8710	30	-6.71	$+26.73$	0.8824
8	-12.74	$+22.55$	0.9169	31	-16.20	$+4.22$	1.0379
9	-12.84	$+22.31$	0.9188	32	-16.19	$+4.17$	1.0381
10	-16.21	-7.78	1.1313	33	$+1.54$	-25.44	1.1976
11	-15.97	-9.22	1.1401	34	$+1.64$	-25.40	1.1977
12	-15.16	-11.46	1.1517	35	$+2.30$	-25.11	1.1979
13	$+2.23$	-26.13	1.1993	36	$+2.31$	-25.11	1.1979
14	-14.92	-15.10	1.1478	37	$+3.09$	-24.77	1.1979
15	-15.12	-15.10	1.1479	38	$+3.74$	-24.47	1.1975
16	$+5.57$	-26.07	1.1876	39	$+3.76$	-24.46	1.1975
17	$+5.60$	-26.06	1.1875	40	$+3.77$	-24.46	1.1974
18	$+18.09$	$+2.00$	0.9874	41	$+5.31$	-23.60	1.1962
19	-0.35	$+25.95$	0.8652	42	$+9.22$	$+19.51$	0.9552
20	-1.71	$+26.81$	0.8753	43	$+9.10$	$+19.91$	0.9526
21	-1.68	$+26.81$	0.8750	44	$+9.02$	$+20.27$	0.9501
22	-10.91	$+15.56$	0.9500	45	$+8.88$	$+20.73$	0.9478
23	-10.89	$+15.54$	0.9500				

When comparing the theory with observations for the accounting of the abberation time $A\rho$, we took for ρ the geocentric distance of Jupiter, for the geocentric distances of the satellite are unknown. Such an error seems quite acceptable in the first approximation. We consider also quite admissible to ignore the influence of the parralax, inasmuch as it does not exceed 5" in α and δ.

In order to take into account the abberation effect at the time of observation t, instead of computing the coordinates of the satellite on the moment of time t we do so for the time $t-A\rho$, where $A=498^s.58=0^d.0057706$.

The values of δV, U and $\frac{a}{r}$ at the times of observations are computed by the tables 5 and 6 and the results are compiled in Table 8.

The jovicentric longitude of the satellite, V, is determined by the formula

$$V=\varepsilon_0+nt+\delta V,$$

where ε_0 is the mean longitude in the epoch; t is the number of days from the epoch 1938 July 27.3128.

We determine the rectangular jovicentric coordinates of the satellite (x, y, z) by formulas (4). Its geocentric coordinates α, δ and ρ are determined by formulas (3). In the right-hand parts of these formulas the rectangular coordinates of the satellite, the Sun and Jupiter must be referred to the equatorial plane. The theory of motion of Jupiter's X satellite provides us with the values of the coordinates x, y, z in relation to Jupiter's orbit plane. This is why it is necessary to pass from rectangular jovicentric coordinates x, y, z to rectangular jovicentric coordinates referred to the plane passing through the center of Jupiter, parallelwise to the equator; we shall denote them by x', y', z. Then

$$\left.\begin{aligned} \bar{x} &= \alpha_1 x + \alpha_2 y + \alpha_3 z, \\ \bar{y} &= \beta_1 x + \beta_2 y + \beta_3 z, \\ z &= \gamma_1 x + \gamma_2 y + \gamma_3 z, \end{aligned}\right\} \tag{15}$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3$ are the direction cosines of the axes x, y, z relative to axes x, y, z. They are determined by the formulas

$$\left.\begin{aligned} \alpha_1 &= \cos(\bar{x}x) = \cos\Omega' \cos\Gamma\Omega' + \sin\Omega' \sin\Gamma\Omega' \cos i', \\ \alpha_2 &= \cos(\bar{x}y) = \cos\Omega' \sin\Gamma\Omega' - \sin\Omega' \cos\Gamma\Omega' \cos i', \\ \alpha_3 &= \cos(\bar{x}z) = \sin\Omega' \sin i', \\ \beta_1 &= \cos(\bar{y}x) = \sin\Omega' \cos\Gamma\Omega' - \cos\Omega' \sin\Gamma\Omega' \cos i', \\ \beta_2 &= \cos(\bar{y}y) = \sin\Omega' \sin\Gamma\Omega' + \cos\Omega' \cos\Gamma\Omega' \cos i', \\ \beta_3 &= \cos(\bar{y}z) = -\cos\Omega' \sin i', \\ \gamma_1 &= \cos(zx) = -\sin\Gamma\Omega' \sin i', \\ \gamma_2 &= \cos(zy) = \cos\Gamma\Omega' \sin i', \\ \gamma_3 &= \cos(zz) = \cos i'. \end{aligned}\right\} \tag{16}$$

In formulas (16)

$$\Omega' = 99^\circ\!.92939 \qquad i' = 1.30614$$

are the heliocentric elliptical coordinates of Jupiter in the epoch 1938 July 27.3128 and equinox 1950.0 (see the system of Jupiter's elements (II)).

$\Gamma Q'$ is determined from the spherical triangle $\gamma\Gamma Q'$ (Fig.). Computations give for the same epoch and equinox $\Gamma Q' = 99.93192$. The values of direction cosines $\alpha_1, \ldots, \gamma_3$, corresponding to the brought out values Ω', i' and $\Gamma Q'$, are as follows

$$\left.\begin{aligned} \alpha_1 &= +0.999747, & \beta_1 &= -0.000088, & \gamma_1 &= -0.022452, \\ \alpha_2 &= 0.000000, & \beta_2 &= +0.999991, & \gamma_2 &= -0.003931, \\ \alpha_3 &= +0.022453, & \beta_3 &= +0.003930, & \gamma_3 &= +0.999740. \end{aligned}\right\} \tag{17}$$

The transfer to the system of axes x', y', z' is achieved by way of rotation of the axes, $\bar{x}$, $\bar{y}$, $\bar{z}$ by the angle ε (inclination of the ecliptic to equator). We have :

$$\left.\begin{aligned} \rho\cos\alpha\cos\delta &= x' + X_{\odot} + X_J, \\ \rho\sin\alpha\cos\delta &= y' + Y_{\odot} + Y_J, \\ \rho\sin\delta &= z' + Z_{\odot} + Z_J, \end{aligned}\right\} \tag{18}$$

Having obtained the coordinates x', y', z', we determine α and δ of the satellite by the formulas

$$\left.\begin{aligned} x' &= \bar{x}, \\ y' &= \bar{y}\cos\varepsilon - \bar{z}\sin\varepsilon, \\ z' &= \bar{y}\sin\varepsilon + \bar{z}\cos\varepsilon. \end{aligned}\right\} \tag{19}$$

where $X_{\odot}$, $Y_{\odot}$, $Z_{\odot}$ are the rectangular geocentric equatorial coordinates of the Sun; X_J, Y_J, Z_J are the rectangular heliocentric equatorial coordinates of Jupiter.

The results of comparison of theory with observations show (Table 7) that the deviation of computed coordinates from those observed $\Delta\alpha(O-C)$ and $\Delta\delta(O-C)$ increase with time and attain a substantial value. It is quite evident that the satellite's system of elements (I) requires correction.

4. - ELEMENT IMPROVEMENT OF JUPITER'S X SATELLITE

The correction of orbit's system of elements is performed by the Eckert - Brouwer method [7]. Six osculating orbit elements are corrected : M_0, a, e, i, Ω_0, ω_0. In our case the elements Ω and ω are not constant and vary proportionally to the time

$$\left.\begin{aligned} \Omega &= \Omega_0 + \frac{d\Omega}{dt}(t - t_0), \\ \omega &= \omega_0 + \frac{d\omega}{dt}(t - t_0), \end{aligned}\right\} \tag{20}$$

where $\frac{d\Omega}{dt}$ and $\frac{d\omega}{dt}$ are the secular motions of the node and perijove.

Because of the secular motion of the perijove the mean anomaly is determined by the formula

$$M = M_0 + \left(n - \frac{d\pi}{dt}\right)(t - t_0). \tag{21}$$

Let us denote

$$\frac{d\pi}{dt} = g, \quad \frac{d\Omega}{dt} = \nu, \quad \frac{d\omega}{dt} = \frac{d\pi}{dt} - \frac{d\Omega}{dt} = g - \nu. \tag{22}$$

The accounting of secular motions of the node and perijove introduces certain changes in the formulas for the differential coefficients of the conditional equations. We shall derive these formulas here.

The variation of rectangular coordinates are linked with the variations of orbit elements by the following correlations:

$$\delta x = \frac{\partial x}{\partial M}\delta M_0 + \frac{\partial x}{\partial a}\delta a + \frac{\partial x}{\partial e}\delta e + \frac{\partial x}{\partial i}\delta i + \frac{\partial x}{\partial \Omega}\delta\Omega + \frac{\partial x}{\partial \omega}\delta\omega \tag{23}$$

and two analogous expressions for δy and δz.

Following the Eckert-Brouwer method, the coefficients $\frac{\partial x}{\partial M}$, $\frac{\partial x}{\partial a}$, ..., $\frac{\partial x}{\partial \omega}$ of the equations (23) will be expressed by x, y, z and their derivatives $\dot{x}$, $\dot{y}$, $\dot{z}$, and we shall take the variations of coordinates in the form, in which the unknown correction shall be those to elements Ω, ω, i, and not to elementary rotations near the equatorial axes. The coefficients at $\delta\Omega$ and $\delta\omega$ in case of ecliptic elements have the following expressions by rectangular lar coordinates (Samoylova-Yakhnotova, 1945 [2])

$$\left.\begin{aligned} \frac{\partial x}{\partial \Omega} &= -z\sin\varepsilon - y\cos\varepsilon, & \frac{\partial x}{\partial \omega} &= R_y z - R_z y, \\ \frac{\partial y}{\partial \Omega} &= x\cos\varepsilon, & \frac{\partial y}{\partial \omega} &= R_z x - R_x z, \\ \frac{\partial z}{\partial \Omega} &= x\sin\varepsilon, & \frac{\partial z}{\partial \omega} &= R_x y - R_y x, \end{aligned}\right\} \tag{24}$$

$$\left.\begin{aligned} R_x &= \sin i \sin\Omega, \\ R_y &= -\sin i\cos\Omega, \\ R_z &= \cos i. \end{aligned}\right\} \tag{25}$$

Inasmuch as Ω and ω are expressed by the formulas (20), the coefficients at $\delta\Omega_0$ and $\delta\omega_0$ have the form (24) and, moreover, additional terms are obtained, which allow to find the corrections $\delta(g-\nu)$ and $\delta\nu$. The coefficients, standing before them, constitute the expressions (24) multiplied by $(t - t_0)$.

Let us find the expressions for the derivatives $\frac{\partial x}{\partial M}$, $\frac{\partial y}{\partial M}$, $\frac{\partial z}{\partial M}$. - Inasmuch as Ω and ω are functions of time, the derivative of the coordinates over t will be

$$\frac{dx}{dt} = \dot{x} = \frac{\partial x}{\partial M}\cdot\frac{dM}{dt} + \frac{\partial x}{\partial \Omega}\cdot\frac{d\Omega}{dt} + \frac{\partial x}{\partial \omega}\cdot\frac{d\omega}{dt} = (n-g)\frac{\partial x}{\partial M} + \nu\frac{\partial x}{\partial \Omega} + (g-\nu)\frac{\partial x}{\partial \omega}. \tag{26}$$

Hence we obtain $\frac{\partial x}{\partial M}$ and by the same way $\frac{\partial y}{\partial M}$ and $\frac{\partial z}{\partial M}$

.../..

$$\left.\begin{aligned}\frac{\partial x}{\partial M}&=\frac{\dot x}{n-g}-\frac{\nu}{n-g}\cdot\frac{\partial x}{\partial\Omega}-\frac{g-\nu}{n-g}\cdot\frac{\partial x}{\partial\omega},\\ \frac{\partial y}{\partial M}&=\frac{\dot y}{n-g}-\frac{\nu}{n-g}\cdot\frac{\partial y}{\partial\Omega}-\frac{g-\nu}{n-g}\cdot\frac{\partial y}{\partial\omega},\\ \frac{\partial z}{\partial M}&=\frac{\dot z}{n-g}-\frac{\nu}{n-g}\cdot\frac{\partial z}{\partial\Omega}-\frac{g-\nu}{n-g}\cdot\frac{\partial z}{\partial\omega},\end{aligned}\right\}\tag{27}$$

at the same time $\frac{\partial x}{\partial\Omega}, \frac{\partial x}{\partial\omega}, \ldots, \frac{\partial z}{\partial\omega}$ are computed by formulas (24). As to the derivatives of $\dot x$, $\dot y$, $\dot z$, if no tables for values of rectangular coordinates are available so as to find these quantities at the moments of observations, we may utilize the formulas of elliptical motion

$$\left.\begin{aligned}x&=aP_x(\cos E-e)+bQ_x\sin E,\\ y&=aP_y(\cos E-e)+bQ_y\sin E,\\ z&=aP_z(\cos E-e)+bQ_z\sin E,\end{aligned}\right\}\tag{28}$$

where $b=a\sqrt{1-e^2}$.

Here P_x, ... Q_z are the direction cosines of orbital axes, depending on the elements of orbit orientation in space as follows :

$$\left.\begin{aligned}P_x&=\cos\omega\cos\Omega-\sin\omega\sin\Omega\cos i,\\ P_y&=(\cos\omega\sin\Omega+\sin\omega\cos\Omega\cos i)\cos\varepsilon-\sin\omega\sin i\sin\varepsilon,\\ P_z&=(\cos\omega\sin\Omega+\sin\omega\cos\Omega\cos i)\sin\varepsilon+\sin\omega\sin i\cos\varepsilon,\\ Q_x&=-\sin\omega\cos\Omega-\cos\omega\sin\Omega\cos i,\\ Q_y&=(-\sin\omega\sin\Omega+\cos\omega\cos\Omega\cos i)\cos\varepsilon-\cos\omega\sin i\sin\varepsilon,\\ Q_z&=(-\sin\omega\sin\Omega+\cos\omega\cos\Omega\cos i)\sin\varepsilon+\cos\omega\sin i\cos\varepsilon.\end{aligned}\right\}\tag{29}$$

Differentiating (28) over time, we obtain

$$\left.\begin{aligned}\dot x&=\frac{a}{r}(n-g)(-aP_x\sin E+bQ_x\cos E)+a(\cos E-e)\dot P_x+b\sin E\dot Q_x,\\ \dot y&=\frac{a}{r}(n-g)(-aP_y\sin E+bQ_y\cos E)+a(\cos E-e)\dot P_y+b\sin E\dot Q_y,\\ \dot z&=\frac{a}{r}(n-g)(-aP_z\sin E+bQ_z\cos E)+a(\cos E-e)\dot P_z+b\sin E\dot Q_z,\end{aligned}\right\}\tag{30}$$

where $\dot P_x, \dot P_y, \ldots, \dot Q_z$ are the derivatives of direction cosines relative to time

On the basis of formulas (29)

$$\left.\begin{aligned}\dot P_x&=(g-\nu)Q_x-\nu(P_y\cos\varepsilon+P_z\sin\varepsilon),\\ \dot Q_x&=-(g-\nu)P_x-\nu(Q_y\cos\varepsilon+Q_z\sin\varepsilon),\\ \dot P_y&=(g-\nu)Q_y+\nu P_x\cos\varepsilon,\\ \dot Q_y&=-(g-\nu)P_y+\nu Q_x\cos\varepsilon,\\ \dot P_z&=(g-\nu)Q_z+\nu P_x\sin\varepsilon,\\ \dot Q_z&=-(g-\nu)P_z+\nu Q_x\sin\varepsilon.\end{aligned}\right\}\tag{31}$$

Therefore, the expressions $\frac{\dot{x}}{n-g}$, $\frac{\dot{y}}{n-g}$ and $\frac{\dot{z}}{n-g}$, entering into formulas (27), are determined by formulas (30). The quantities $P_x, Q_x, \ldots, Q_z$, $\dot{P}_x, \dot{Q}_x, \ldots, \dot{Q}_z$ are functions of time and that is why they should be computed for each observation or normal spot enetring into orbit improvement.

Quite analogously to what was done in the first volume of Subbotin's "Course of Celestial Mechanics" [3] , p. 304, derivatives relative to a and the eccentricity

$$\left.\begin{aligned} a\frac{\partial x}{\partial a} &= x-\frac{3}{2}n(t-t_0)\frac{\partial x}{\partial M}, \\ a\frac{\partial y}{\partial a} &= y-\frac{3}{2}n(t-t_0)\frac{\partial y}{\partial M}, \\ a\frac{\partial z}{\partial a} &= z-\frac{3}{2}n(t-t_0)\frac{\partial z}{\partial M}, \end{aligned}\right\} \qquad (32)$$

$$\frac{\partial x}{\partial e} = -Hx + K\left[\frac{\dot{x}}{n-g} - a(\cos E - e)\frac{\dot{P}_x}{n-g} - b\sin E\frac{\dot{Q}_x}{n-g}\right].$$

are obtained. It is easy to show that there takes place the following equality

$$a(\cos E - e)\frac{\dot{P}_x}{n-g} + b\sin E\frac{\dot{Q}_x}{n-g} = \frac{\nu}{n-g}\cdot\frac{\partial x}{\partial \Omega} - \frac{g-\nu}{n-g}\cdot\frac{\partial x}{\partial \omega}$$

and two analogous equalities for the coordinates y and z. That is why we have

$$\left.\begin{aligned} \frac{\partial x}{\partial e} &= -Hx + K\frac{\partial x}{\partial M}, \\ \frac{\partial y}{\partial e} &= -Hy + K\frac{\partial y}{\partial M}, \\ \frac{\partial z}{\partial e} &= -Hz + K\frac{\partial z}{\partial M}, \end{aligned}\right\}. \qquad (33)$$

$$\begin{aligned} H &= -\frac{\cos E - e}{1-e^2}, \\ K &= \frac{(2-e^2-e\cos E)\sin E}{1-e^2}. \end{aligned} \qquad (34)$$

In formula (32) and (33) the quantities $\frac{\partial x}{\partial M}, \frac{\partial y}{\partial M}, \frac{\partial z}{\partial M}$ are determined according to (27). The derivatives over the inclination in case of elliptic elements have the form

$$\left.\begin{aligned} \frac{\partial x}{\partial i} &= N_y z - N_z y, \\ \frac{\partial y}{\partial i} &= N_z x - N_x z, \\ \frac{\partial z}{\partial i} &= N_x y - N_y x, \\ N_x &= \cos\Omega, \\ N_y &= \sin\Omega\cos\varepsilon, \\ N_z &= \sin\Omega\sin\varepsilon. \end{aligned}\right\} \qquad (35)$$

Thus, formulas (24), (27), (32), (33) and (35) determine the coefficients of the equations (23). The variations of spherical coordinates α and δ are linked with the variations of triangular coordinates by the following correlation

$$\begin{Bmatrix} \rho\cos\delta\Delta\alpha \\ \rho\Delta\delta \end{Bmatrix} = \begin{Bmatrix} \delta x \\ \delta y \\ \delta z \end{Bmatrix} \begin{Bmatrix} -\sin\alpha & -\cos\alpha\sin\delta \\ \cos\alpha & -\sin\alpha\sin\delta \\ 0 & \cos\delta. \end{Bmatrix} \tag{36}$$

Proceeding with the improvement of the system of elements (1), we assembled all the available observations in normal places. 17 normal sites have been formed (Table 9). We estimate that all the normal sites have an identical weight.

In the last column of Table 9 the number of observations is given which were entered in the normal place.

At the beginning the improvement of the system of elements was performed by observations encompassing the time interval from 1938 to 1943 (the first 15 normal places. As a result of the solution of the system of 15 conditional equations, the ninth and the fifteenth normal places had to be excluded, for upon improvement for these moments, $\Delta\alpha$ and $\Delta\delta$ did not agree with the remaining ones. There is basis to assume that these observations are erroneous.

Eliminating these normal places, we again resolved the system of already 13 conditional equations, obtaining a satisfactory result. Tables 10 and 11 give for this variant solution a system of conditional equations and of normal equations corresponding to it.

Round parentheses denote in Table 10 the equations for right ascensions and the brackets point to equations for declination.

As a result of the solution of the system of normal equations, the corrections to elements have been obtained :

$$\left.\begin{aligned} \delta M_0 &= +5^\circ\!.6707 \pm 0^\circ\!.2985, \\ \delta n &= +0^\circ\!.006758 \pm 0^\circ\!.000205, \\ \delta a &= -0.000256 \pm 0.000008, \\ \delta e &= -0.02505 \pm 0.00112, \\ \delta i &= +0^\circ\!.5393 \pm 0^\circ\!.0816, \\ \delta\Omega &= -1^\circ\!.7712 \pm 0^\circ\!.2503, \\ \delta\omega &= -3^\circ\!.2335 \pm 0^\circ\!.3655. \end{aligned}\right\} \tag{37}$$

TABLE 9

Normal Places

No. Order	Universal Time			α (1950.0)	δ (1950.0)	Number of observ.
1	Jul. 1938	Июль	7.9290	$22^h16^m49^s39$	$-11°15'55''4$	4
2			28.3734	22 08 43.38	−12 03 17.6	3
3	Aug	Авг.	24.8148	21 53 54.61	−13 34 36.6	2
4	Oct	Окт.	20.4620	21 39 27.65	−15 25 50.7	3
5	Nov.	Нояб.	21.1188	21 49 30.06	−14 41 12.0	1
6	Jul. 1939	Июль	15.4367	0 31 26.30	+ 1 36 35.7	2
7	Aug	Авг.	16.4586	0 33 47.67	+ 1 33 13.1	2
8	Oct.	Окт.	8.3998	0 15 53.39	+ 0 00 48.7	1
9	Dec.	Дек.	15.0885	0 01 26.93	− 0 48 17.8	1
10	Sep 1940	Сент.	8.4087	2 57 25.88	+16 02 04.4	2
11	Oct	Окт.	28.8390	2 36 29.26	+14 01 38.8	4
12	Dec 1941	Дек.	23.2569	4 56 06.78	+22 24 12.2	1
13	Feb 1942	Февр.	17.7480	4 44 32.55	+22 23 14.2	2
14	Nov	Нояб.	9.0021	7 53 15.26	+21 30 19.3	2
15	Jan. 1943	Янв.	6.2815	7 32 49.78	+21 57 38.8	2
16	Oct 1951	Окт.	1.2995	0 39 15.54	+ 2 15 53.8	9
17	Feb. 1954	Февр.	4.7563	5 04 20.28	+22 59 25.7	4

TABLE 10

Conditional Equations

№ по пор.	$\delta M_0 \cdot 10^{-2}$	$\delta i \cdot 10^{-2}$	$\delta \Omega_0 \cdot 10^{-2}$	$\delta \omega \cdot 10^{-2}$	$\frac{\delta a}{a}$	$\delta e \cdot 10^{-2}$	l
(1)	−1.55854	−0.34417	−1.82566	−2.12204	−0.01398	−0.04621	$0^s013117$
(2)	−1.36712	−0.32779	−1.69584	−1.90902	−0.01085	0.54617	0.005728
(3)	−0.53065	−0.18891	−0.89441	−0.95184	−0.01372	0.37283	0.010005
(4)	1.78871	0.04824	1.07897	1.26400	−0.06340	−1.22119	0.066601
(5)	1.68694	0.12754	1.03228	1.17275	−0.06573	0.13031	0.048246
(6)	1.09913	0.40303	0.33407	0.68269	−0.15152	0.25793	0.060892
(7)	1.91014	0.74134	1.24823	1.36343	−0.26572	1.46967	0.082096
(8)	0.22979	−0.05456	0.32425	−0.11362	−0.01860	0.59535	0.004042
(10)	−0.79822	−1.29970	−0.91800	−1.05547	0.23570	1.48389	−0.069829
(11)	−1.39920	−0.72254	−1.91402	−1.58107	0.41336	2.78879	−0.110742
(12)	1.20482	−0.64406	1.61726	1.32263	−0.53252	2.44622	0.059892
(13)	−0.30783	−1.08781	−0.32397	−0.31561	0.15974	1.44319	−0.063517
(14)	0.59377	−0.36359	0.76935	0.90653	−0.32057	0.71285	0.028447
[1]	−0.32967	1.82607	−0.70076	−0.44962	0.00647	0.82039	0.002889
[2]	−0.59579	2.01111	−0.62876	−0.74303	0.00689	1.11779	−0.004278
[3]	−0.76770	1.65973	−0.29902	−0.88932	0.01006	1.49597	−0.022222
[4]	−0.20666	−0.52850	0.39388	−0.24981	0.00113	0.73358	−0.025472
[5]	0.44891	−1.18531	0.35691	0.32241	−0.02305	0.65969	−0.003833
[6]	−0.20023	−0.65736	0.18471	−0.29100	0.01667	1.10215	−0.030417
[7]	0.93271	−1.19818	0.54350	0.68386	−0.13705	1.49096	0.021222
[8]	1.04648	0.09470	0.08085	0.92716	−0.15743	2.15854	0.023444
[10]	−0.23563	1.52624	−0.29437	−0.31536	0.07959	1.28552	−0.029667
[11]	−1.30384	0.86968	−0.61467	−1.37682	0.39286	2.63823	−0.123556
[12]	0.85301	0.92553	0.17332	0.94279	−0.37759	1.70119	0.068139
[13]	−0.13796	1.50383	−0.03923	−0.11863	0.07585	0.99393	−0.027944
[14]	−0.26738	1.71773	−0.13166	−0.29862	0.15724	0.69604	−0.034472

TABLE 11

Normal Equations

№ по пор.	$\delta M_0 \cdot 10^{-2}$	$\delta i \cdot 10^{-2}$	$\delta \Omega_0 \cdot 10^{-2}$	$\delta\omega \cdot 10^{-2}$	$\frac{\delta a}{a}$	$\delta e \cdot 10^{-2}$	l
1	25.90433	− 1.13076	20.57947	25.18006	−3.73242	− 2.89608	1°022799
2		27.78604	− 1.35885	− 1.36466	0.12850	4.81099	0.060892
3			20.93983	21.96846	−3.09337	− 4.75575	0.724416
4				26.75958	−3.75224	− 5.68715	0.948548
5					1.11933	− 0.01118	−0.243737
6						49.29283	−0.575466

A few more variants of the solutions were performed with the view of tying up the observations of 1951 and 1954 by a single system; they included the normal places 16 and 17. However, the solutions of these variants obtained as a result of one-time orbit correction, were to be worse than the solution of the initial variant with corrections (37), brought up above. That is why for the further investigation of the motion of Jupiter's X satellite it was decided to admit the system of elements obtained by us and based upon the observations from 1938 to 1942, encompassing about six satellite revolutions. Let us bring forth this system :

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$$\begin{array}{ll}
M_0 & 216^\circ\!.6928 \pm 0^\circ\!.2985 \\
n & 1^\circ\!.388788 \pm 0^\circ\!.000205 \\
a & (0.078345 \pm 0.000008)\ \text{a.} \\
e & 0.10739 \pm 0.00112 \\
i & 28^\circ\!.8053 \pm 0^\circ\!.0816 \\
\Omega & 80.7358 \pm 0.2503 \\
\omega & 243.8105 \pm 0.3655 \\
g & +1^\circ\!.54549 \\
\nu & -1^\circ\!.22360
\end{array} \qquad \text{(III)}$$

(g and ν being the theoretical values of secular motions of the perijove and node, corresponding to the new system of elements).

The new Delaunay parameters and arguments are :

$$\left.\begin{aligned}
m &= 0.0598299, \\
\alpha &= 0.0150589, \\
\gamma &= 0.238320, \\
e &= 0.10739,
\end{aligned}\right\} \qquad (38)$$

$$\left.\begin{aligned}
l &= 216^\circ\!.6928 + 1^\circ\!.384557\, t, \\
l' &= 315.8605 + 0.083091\, t, \\
D &= 31.9871 + 1.305697\, t, \\
F &= 101.4309 + 1.392138\, t.
\end{aligned}\right\} \qquad (39)$$

5.- TABLES OF MOTION OF JUPITER'S X SATELLITE

The representation of normal places prior and after the improvement is shown in Table 12 hereafter. It may be seen from that table that the new system of elements (III) provides a substantial improvement in the representation of satellite observations by comparison with the Herget's system of elements. However, the maximum misfit (observed vs computed) constitutes 36" (prior to improvement it constituted 445"). Corrections to elements were obtained great. All this is evidence that the new system of elements requires further correction, quite desirably on the basis of new observations.

TABLE 12

Representation of normal places

No order	Universal t.		Prior to improvement		After improvement	
			$\cos\delta\Delta\alpha$	$\Delta\delta$	$\cos\delta\Delta\alpha$	$\Delta\delta$
1	1938	7.9290	+ 47".2	+ 10".4	+ 3".1	+ 8".0
2		28.3734	+ 20.6	— 15.4	+ 3.4	— 3.0
3		24.8148	+ 36.0	— 80.0	+ 9.8	— 6.0
4		20.4620	+239.8	— 91.7	+16.1	+ 4.6
5		21.1188	+173.7	— 13.8	+ 8.3	+ 3.6
6	1939	15.4367	+219.2	—109.5	+ 0.8	+ 9.8
7		16.4586	+295.6	+ 76.4	—26.8	— 8.4
8		8.3998	+ 14.6	+ 84.4	+ 5.6	+12.3
10	1940	8.4087	—251.4	—106.8	+ 9.5	+23.9
11	"	28.8390	—398.7	—444.8	—16.9	— 5.5
12	1941	23.2569	+215.6	+245.3	— 7.6	— 8.0
13	1942	17.7480	—228.7	—100.6	+20.1	+15.6
14		9.0021	+102.4	—124.1	+35.9	+ 4.3
			211".4		15".3	

TABLE 13

δV and $\frac{a}{r}$

No. by order		A'	i	j	j'	k	Argument			C'
			0	0	0	0		1	1	+1.0006
1	2	— 0°.37	0	0	1	0	315°.8605 + 0°.083091 t	2	2	— 2
2	3	— 2	0	0	2	0	271.7210 + 0.166182 t			
3	7	+12.29	0	1	0	0	216.6928 + 1.384557 t	3	4	+ 1077
4	8	+ 12	0	1	—1	0	260.8323 + 1.301466 t	4	5	+ 10
5	12	— 10	0	1	1	0	172.5533 + 1.467648 t	5	7	— 8
6	16	+ 78	0	2	0	0	73.3856 + 2.769114 t	6	9	+ 115
7	17	+ 2	0	2	—1	0	117.5251 + 2.686023 t	7	10	+ 2

continued../..

Table 13 (continuation) Таблица 13 *(продолжение)*

№ по пор.	№ по Делоне	A'	i	j	j'	k	Аргумент Argument	№ по пор.	№ по Делоне	C'
8	20	− 1	0	2	1	0	29°2461 + 2°852205 t	8	12	− 2
9	23	+ 5	0	3	0	0	290.0784 + 4.153671 t	9	14	+ 14
10	28	+ 1	0	4	0	0	146.7712 + 5.538228 t	10	17	+ 2
11	37	− 3.50	0	0	0	2	202.8618 + 2.784276 t	11	19	− 22
12	44	− 74	0	1	0	2	59.5546 + 4.168833 t	12	22	− 5
13	45	− 1	0	1	−1	2	103.6941 + 4.085742 t			
14	47	+ 1	0	1	1	2	15.4151 + 4.251924 t			
15	49	− 13	0	2	0	2	276.2474 + 5.553390 t			
16	54	− 2	0	3	0	2	132.9402 + 6.937947 t			
17	57	− 1	0	4	0	2	349.6330 + 8.322504 t			
18	58	− 1.00	0	−1	0	2	346.1690 + 1.399719 t	13	23	− 154
			0	−1	−1	2	30.3085 + 1.316628 t	14	24	+ 1
			0	−1	1	2	302.0295 + 1.482810 t	15	25	− 1
19	63	+ 7	0	−2	0	2	129.4762 + 0.015162 t			
20	68	+ 2	0	−3	0	2	272.7834 − 1.369395 t			
21	73	+ 11	0	0	0	4	45.7236 + 5.568552 t			
22	78	+ 5	0	1	0	4	262.4164 + 6.953109 t			
23	81	+ 1	0	2	0	4	119.1092 + 8.337666 t			
24	82	+ 7	0	−1	0	4	189.0308 + 4.183995 t			
25	89	+ 35	2	0	0	0	63.9742 + 2.611394 t	16	27	+ 68
26	90	+ 8	2	0	−1	0	108.1137 + 2.528303 t	17	28	+ 12
			2	0	−2	0	152.2532 + 2.445212 t	18	29	+ 1
27	94	− 1	2	0	1	0	19.8347 + 2.694485 t	19	30	− 2
28	98	+ 5	2	1	0	0	280.6670 + 3.995951 t	20	32	+ 14
29	99	+ 1	2	1	−1	0	324.8065 + 3.912860 t	21	33	+ 3
			2	2	0	0	137.3598 + 5.380508 t	22	36	+ 2
30	118	+ 1.64	2	−1	0	0	207.2814 + 1.226837 t	23	40	+ 129
31	119	+ 21	2	−1	−1	0	251.4209 + 1.143746 t	24	41	+ 16
32	120	+ 2	2	−1	−2	0	295.5604 + 1.060655 t	25	42	+ 2
33	123	− 5	2	−1	1	0	163.1419 + 1.309928 t	26	43	− 5
34	127	+ 16	2	−2	0	0	350.5886 − 0.157720 t	27	45	− 2
35	128	+ 2	2	−2	−1	0	34.7281 − 0.240811 t			
36	134	+ 2	2	−3	0	0	133.8958 − 1.542277 t	28	48	− 2
37	148	− 4	2	0	0	2	266.8360 + 5.395670 t			
38	153	− 2	2	1	0	2	123.5288 + 6.780227 t			
39	161	− 10	2	−1	0	2	50.1432 + 4.011113 t	29	52	− 1
40	162	− 1	2	−1	−1	2	94.2827 + 3.928022 t			
41	166	− 2	2	−2	0	2	193.4504 + 2.626556 t	30	53	− 4
42	183	+ 34	2	0	0	−2	221.1124 − 0.172882 t	31	54	− 6
43	184	+ 4	2	0	−1	−2	265.2519 − 0.255973 t	32	55	− 1
44	187	− 2	2	0	1	−2	176.9729 − 0.089791 t			
45	188	+ 2	2	0	2	−2	132.8334 − 0.006700 t			
46	190	− 11	2	1	0	−2	77.8052 + 1.211675 t	33	57	− 7
47	191	− 1	2	1	−1	−2	121.9447 + 1.128584 t	34	58	− 1
48	195	− 1	2	2	0	−2	294.4980 + 2.596232 t	35	60	− 2
			2	−1	0	−2	4.4196 − 1.557439 t	36	61	− 10
			2	−1	−1	−2	48.5591 − 1.640530 t	37	62	− 1
49	207	− 1	2	−2	0	−2	147.7268 − 2.941996 t	38	64	− 2
50	218	− 2	2	0	0	−4	18.2506 − 2.957158 t			
			4	−1	0	0	271.2556 + 3.838231 t	39	69	+ 2
51	258	+ 2	4	−2	0	0	54.5628 + 2.453674 t	40	72	+ 2
52	342	− 6	1	0	0	0	31.9871 + 1.305697 t	41	78	− 7
53	346	+ 7	1	0	1	0	347.8476 + 1.388788 t	42	81	+ 6
54	349	− 1	1	1	0	0	248.6799 + 2.690254 t	43	83	− 2
55	352	+ 1	1	1	1	0	204.5404 + 2.773345 t	44	85	+ 1
56	364	− 2	1	−1	0	0	175.2943 − 0.078860 t			
57	396	− 2	1	0	0	−2	189.1253 − 1.478579 t	45	91	+ 3
58	448	− 1	3	0	0	−2	253.0995 + 1.132815 t	46	100	− 1

TABLE 14 Таблица 14

U

No. by order	No. by Delaun	B'	i	j	j'	k	Аргумент Argument
1	1	+26°98	0	0	0	1	101°4309 + 1°392138 t
2	2	+ 1	0	0	−1	1	145.5704 + 1.309047 t
3	6	− 1	0	0	1	1	57.2914 + 1.475229 t
4	10	+ 2.89	0	1	0	1	318.1237 + 2.776695 t
5	11	+ 3	0	1	−1	1	2.2632 + 2.693604 t
6	14	− 2	0	1	1	1	273.9842 + 2.859786 t
7	17	+ 34	0	2	0	1	174.8165 + 4.161252 t
8	18	+ 1	0	2	−1	1	218.9560 + 4.078161 t
9	20	− 1	0	2	1	1	130.6770 + 4.244343 t
10	22	+ 5	0	3	0	1	31.5093 + 5.545809 t
11	27	+ 1	0	4	0	1	248.2021 + 6.930366 t
12	31	− 3.29	0	−1	0	1	244.7381 + 0.007581 t
13	32	+ 3	0	−1	−1	1	288.8776 − 0.075510 t
14	35	− 3	0	−1	1	1	200.5986 + 0.090672 t
15	38	− 24	0	−2	0	1	28.0453 − 1.376976 t
16	43	− 3	0	−3	0	1	171.3525 − 2.761533 t
17	53	− 33	0	0	0	3	304.2927 + 4.176414 t
18	58	− 10	0	1	0	3	160.9855 + 5.560971 t
19	63	− 2	0	2	0	3	17.6783 + 6.945528 t
20	66	− 1	0	3	0	3	234.3711 + 8.330085 t
21	67	− 30	0	−1	0	3	87.5999 + 2.791857 t
22	72	+ 2	0	−2	0	3	230.9071 + 1.407300 t
23	81	+ 1	0	−1	0	5	290.4617 + 5.576133 t
24	82	+ 1	0	−2	0	5	73.7689 + 4.191576 t
25	83	+ 15	2	0	0	1	165.4051 + 4.003532 t
26	84	+ 2	2	0	−1	1	209.5446 + 3.920441 t
27	90	+ 3	2	1	0	1	22.0979 + 5.388089 t
28	103	+ 37	2	−1	0	1	308.7123 + 2.618975 t
29	104	+ 5	2	−1	−1	1	352.8518 + 2.535884 t
30	107	− 2	2	−1	1	1	264.5728 + 2.702066 t
31	110	− 1	2	−2	0	1	92.0195 + 1.234418 t
32	132	− 1	2	−1	0	3	151.5741 + 5.403251 t
33	137	− 1	2	−2	0	3	294.8813 + 4.018694 t
34	143	+ 76	2	0	0	−1	322.5433 + 1.219256 t
35	144	+ 9	2	0	−1	−1	6.6828 + 1.136165 t
36	148	− 4	2	0	1	−1	278.4038 + 1.302347 t
37	152	+ 4	2	1	0	−1	179.2361 + 2.603813 t
38	153	+ 1	2	1	−1	−1	223.3756 + 2.520722 t
39	159	+ 1	2	2	0	−1	35.9289 + 3.988370 t
40	173	+ 33	2	−1	0	−1	105.8505 − 0.165301 t
41	174	+ 4	2	−1	−1	−1	149.9900 − 0.248392 t
42	177	− 1	2	−1	1	−1	61.7110 − 0.082210 t
43	180	+ 6	2	−2	0	−1	249.1577 − 1.549858 t
44	185	+ 1	2	−3	0	−1	32.4649 − 2.934415 t
45	194	+ 7	2	0	0	−3	119.6815 − 1.565020 t
46	195	+ 1	2	0	−1	−3	163.8210 − 1.648111 t
47	199	− 3	2	1	0	−3	336.3743 − 0.180463 t
48	208	+ 3	2	−1	0	−3	262.9887 − 2.949577 t
49	266	+ 1	4	−1	0	−1	169.8247 + 2.446093 t
50	317	− 1	1	0	0	1	133.4180 + 2.697835 t
51	320	+ 2	1	0	1	1	89.2785 + 2.780926 t
52	322	− 1	1	1	0	1	350.1108 + 4.082392 t
53	349	− 2	1	0	0	−1	290.5562 − 0.086441 t
54	352	+ 2	1	0	1	−1	246.4167 − 0.003350 t
55	354	− 1	1	1	0	−1	147.2490 + 1.298116 t

Substantial variations of satellite parameters also entailed substantial variations in solar perturbations; thus, for example, the greatest term in the longitude expansion varied from 15.15° to 12.29. Consequently, the further work for the improvement of the system of elements is impossible without recomputing the perturbations. This is why the coefficients of solar inequalities were completely recomputed on the basis of the new system of elements, just as were the arguments. They are compiled in Tables 13 and 14. These tables represent the motion of the satellite along the new orbit. The arrangement of Tables 13 and 14 is exactly the same as for Tables 5 and 6.

In conclusion, I consider it to be my pleasant duty to express my gratitude to my scientific advisor, M. F. Subbotin, member-correspondent of the USSR Academy of Sciences, for his valuable counsel in connection with the performance of the present work, and also to our senior scientific co-worker, V. F. Proskurin, for the subject proposed.

**** THE END ****

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