A Nomenclature of Symbols Relevant to the Probability of Contaminating Mars

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A nomenclature of symbols, useful in developing expressions for the probability of contaminating Mars under a variety of conditions, is given below. This nomenclature is similar in many respects to that prepared earlier by Schalkowsky and Cooley of Exotech Incorporated. This is advantageous since the notation presented by those authors is used fairly extensively. However, in line with suggestions made in a telephone conversation with Schalkowsky, the use of subscripts has been curtailed because of difficulties that have arisen in the typing of subscripts, particularly for subscripts which are capital letters. The definitions have been developed systematically in such a way that it is easy to distinguish between symbols which pertain to a single mission or capsule and those which pertain to the whole period of biological exploration, between symbols pertaining to capsules and those pertaining to organisms, those pertaining to objects or organisms which have not been subjected to decontamination treatment and those pertaining to things for which sterilization has been attempted.

First let us look at symbols designating probability concepts. Suppose that A and B are any events. Then

$P(A)$ = probability that the event A occurs, regardless of whether or not the event B occurs.

$P(A+B)$ = probability that the event A occurs or the event B occurs (or both occur).

$P(A \cdot B)$ = probability that the events A and B both occur.

$P(A|B)$ = probability that the event A occurs given that the event B occurs. When a probability is conditional on more than one given event, these events are listed after the $|$ symbol and are separated by commas.

The symbol P refers to true, though usually unknown, probability. An estimate of a probability P is denoted by $\hat{P}$. Two basic rules for dealing with probabilities such as those defined above are

Rule 1: $P(A+B) = P(A) + P(B) - P(A \cdot B)$,

Rule 2: $P(A \cdot B) = P(A|B)P(B) = P(B|A)P(A)$.

If A and B are mutually exclusive events, $P(A \cdot B) = 0$ and the first of these rules reduces to the special case

$$P(A+B) = P(A) + P(B).$$

If A and B are stochastically independent, $P(A|B) = P(A)$, $P(B|A) = P(B)$ and the second rule reduces to

$$P(A \cdot B) = P(A) \cdot P(B);$$

A third rule deals with the event $\bar{A}$ = "not A", where A is any event.

Rule 3: $P(\bar{A}) = 1 - P(A)$.

Another operational symbol related to probability is E, where E means "expected value of". Thus if m denotes a number of viable organisms after decontamination treatment and if P(s) is the probability that a single such organism survives a space journey, then $E(m|s) = mP(s)$ = the expected number of decontaminated treated organisms which will survive a space journey.

Next let us look at conventions regarding the designation of events. An event denoted by a capital letter involves a spacecraft. For instance, the symbol C denotes contamination of Mars by spacecraft. Small letters denote events for which the basic unit is an individual organism. For instance, g will denote the growth of an organism. A subscript on a spacecraft event symbol denotes that the event involves a particular decontaminated lander or intended flyby which has not been decontaminated. The absence of a subscript on a spacecraft event symbol means that the event is defined relative to the entire period of biological exploration. The total number of decontaminated landers during the period of biological exploration is denoted by N and the dummy subscript used to indicate a series of decontaminated landers will be i, $i = 1,2,\ldots,N$. Similarly, the total number of non-decontaminated orbiters not intended for landing is denoted by N' and the dummy subscript used to indicate a series of such flybys is j, $j = 1,2,\ldots,N'$. In general, a prime on an event symbol means that the event involves a spacecraft or organism which has not been subjected to decontamination treatment, while the absence of a prime means the event involves a spacecraft or organism for which sterilization has been attempted. The symbol m is used to denote a number of viable organisms before a space journey. The rules given for subscripts and primes pertain to m as well as to event symbols, so m'_j denotes the number of viable organisms initially on the jth non-decontaminated orbiter.

Next let us define several basic event symbols.

C = contamination of Mars, during the entire period of biological experimentation being considered, with decontaminated spacecraft.

s = survival of organisms on (or in) a decontaminated spacecraft through a space journey, including a landing on Mars or, at least, the arrival at a position where it can possibly contaminate Mars.

r = release of a viable organism from a decontaminated spacecraft on the surface of Mars.

g = growth and spread of an organism from a decontaminated spacecraft on Mars, that is contamination by a viable organism from a decontaminated spacecraft.

Let us also depart from the conventions defined earlier and define the symbol P when used by itself to be such that

P = probability of contaminating Mars during the entire period of biological exploration under consideration regardless of the source of contamination or its exposure to attempted sterilization.

Many pertinent probability symbols can now be developed from these basic event symbols using the conventions and operations defined earlier. Several of these are listed below. These are given not with the intent of giving an exhaustive list but to illustrate the use of the definitions given earlier. Some relationships between probabilities derived using Rules 1, 2 and 3 are also given.

$P(C)$ = probability of contaminating Mars with decontaminated spacecraft during the entire period under consideration.

$P(C')$ = probability of contaminating Mars with non-decontaminated spacecraft during the entire period under consideration.

Note that $P = P(C+C') = P(C) + P(C') - P(C \cdot C')$ by Rule 1.

$P(s)$ = probability than an organism on (or in) a decontaminated spacecraft will survive through a space journey.

$P(s')$ = probability that an organism on (or in) a non-decontaminated spacecraft will survive through a space journey.

$P(\bar{s})$ = probability that an organism on (or in) a decontaminated spacecraft will not survive through a space journey

$= 1 - P(s)$ by Rule 3.

$P(r|s)$ = probability that a viable organism from a decontaminated spacecraft will be released on the surface of Mars given that it has survived through a space journey

= $P(r \cdot s)/P(s)$ by Rule 2.

$P(r)$ = probability that a viable organism from a decontaminated spacecraft will be released on the surface of Mars

= $P(r \cdot s) + P(r \cdot \bar{s})$ by Rule 1 since $r \cdot s$ and $r \cdot \bar{s}$ are mutually exclusive events

= $P(r \cdot s)$ since $P(r \cdot \bar{s}) = 0$.

$\hat{P}(r)$ = an estimate of the probability that a viable organism from a decontaminated spacecraft will be released on the surface of Mars.

$P(r'|s')$ = probability that a viable organism from a non-decontaminated spacecraft will be released on the surface of Mars given that it has survived through a space journey

= $P(r' \cdot s')/P(s')$.

$P(g|r,s)$ = probability that an organism from a decontaminated spacecraft will grow and spread on Mars given that it has survived a space journey and has been released on the surface of Mars as a viable organism

= $P(g \cdot r \cdot s)/P(r \cdot s) = P(g \cdot r \cdot s)/P(r|s)P(s)$ by Rule 2.

$P(g|s)$ = probability that an organism from a decontaminated spacecraft will grow and spread on Mars given that it has survived a space journey

$= P(g \cdot r|s) + P(g \cdot \bar{r}|s)$ by Rule 1 since $g \cdot r|s$ and $g \cdot \bar{r}|s$ are mutually exclusive events

$= P(g \cdot r|s)$ since $P(g \cdot \bar{r}|s) = 0$

$= P(g \cdot r \cdot s)/P(s)$ by Rule 2

$= P(g|r \cdot s)P(r \cdot s)/P(s)$ by Rule 2.

$P(\bar{g}'|s')$ = probability that an organism from a non-decontaminated spacecraft will not grow and spread on Mars given that it has survived a space journey

$= 1 - P(g'|s')$ by Rule 3.

$P(g_i|s_i)$ = probability that an organism from the ith decontaminated spacecraft will grow and spread on Mars given that it has survived a space journey.

$\hat{P}(g_j'|s_j')$ = an estimate of the probability that an organism from the jth non-decontaminated spacecraft will grow and spread on Mars given that it has survived a space journey.

$E(m_i|s_i)$ = expected number of viable organisms on the ith decontaminated spacecraft after a space journey.

$E(m_j'|r_j', s_j')$ = expected number of viable organisms from the jth non-decontaminated spacecraft on Mars after a space journey and release on Mars.

An advantage of this system of applying operations to basic event symbols to develop probability notation is that it is easy to incorporate new events into the definitions either by operating on events already defined or by introducing new basic event notation. This system of notation is also very flexible and therefore will be likely to retain its utility as new decontamination probability models are introduced.