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ENERGY DISSIPATION ASSOCIATED WITH GAS-PUMPING IN STRUCTURAL JOINTS

G. Maidanik

Contract No. NAS 5-9694
BBN Job No. 11234

17 May 1966

Submitted to:

Office of Director, Code 100
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ABSTRACT

Recent experimental work dealing with the vibratory energy dissipation of plates with beams riveted to them has indicated that the dominant damping mechanism is associated with "gas-pumping" in the space between the plate and the beam. Based on this deduction, a semi-phenomenological theory is developed which predicts satisfactorily not only the order of magnitude of the loss factors, but also their dependence on frequency and on gas pressure. This theory attributes the damping to viscous forces associated with gas motion tangential to the plane of the plate, resulting from the relative flexural motions between the adjacent plate and beam surfaces.

Author

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I. INTRODUCTION

A recent extensive study of joint damping in riveted structures has revealed that "gas-pumping" in the spaces between the surfaces in joints may contribute significantly to the total damping of such structural systems [1,2,3]. This report represents an attempt to account, at least semi-phenomenologically, for the experimentally observed data on riveted joint damping. The primary aim of the work presented here was to determine whether it is possible to find a gas-pumping mechanism that would lead to damping values (loss factors) agreeing in order of magnitude with those observed experimentally. It was also hoped that such an analysis might reveal the dependence of the damping on the ambient pressure, so that one could then predict the changes in the damping of riveted structures due to ambient pressure changes.

The specific structure under consideration is a plate to which a beam is fastened with uniformly spaced rivets [see Fig. 1]. This type of composite structure corresponds to that which was examined experimentally[1] and has damping characteristics which are typical of structural components in many practical constructions. The analytical approach taken in this present report is based in part on the work of J. Dimeff, J. W. Lane and G. M. Coon[4]. Their analysis was devised to account for data obtained on a gas pressure gauge. The operation of this gauge is based on the energy dissipation induced by the gas (whose ambient pressure is to be determined) on a vibrating diaphragm. They concluded that the dissipation is predominantly associated with the tangential motion of the gas with respect to the plane of the diaphragm. This motion is induced by externally induced harmonic changes in the curvature of the diaphragm. The tangential motion generates viscous forces that bring about dissipation. In this way the above researchers were able to find reasonable agreement between theoretically predicted values and the experimental data[4/]. However, the system they analyzed was well defined and, as such, was amenable to precise and detailed analysis. The system we wish to analyze, on the other hand, is more complex, and its dynamical parameters may escape precise definition. To circumvent this difficulty we apply a rather crude analysis, where parameters are defined "statistically" in order to remove some of the inherent complexity. (Such an analysis has been successfully applied to the response of and to the acoustic radiation from complex structures[5/].)

The analysis presented here is restricted to joint damping associated with multi-point-fastened riveted, (bolted, spot-welded) plate-beam systems. We begin by analyzing the dissipation pertaining to gas flows in the space between two adjacent surfaces; these flows are induced by harmonically varying pressure gradients. We show these pressure gradients to be generated in the riveted plate-beam system as a result of relative flexural motions between the adjacent plate and beam surfaces. We then develop the relationship between this relative motion and the pressure gradients generated by it. Finally, we derive an expression for the loss factor associated with this type of dissipation and show this loss factor to be a function of frequency, the critical frequency of the plate, ambient pressure, the mass impedance of the plate, the ratio of the area of the beam to that of the plate, the width of the beam, and the average separation between the plate and beam surfaces. In this report the dependences of the loss factor on ambient pressure and on frequency are of particular interest. It is shown that the expression for the loss factor is in satisfactory agreement with experimental findings.

II. DISSIPATION ASSOCIATED WITH GAS FLOWS INDUCED BY HARMONICALLY VARYING PRESSURE GRADIENTS

We study the motion of a gas occupying the space between two parallel surfaces separated by a distance h. Consider an x-y plane halfway between the two surfaces (see Fig. 2). We assume that the gas motion is induced by pressure gradients parallel to the x-y plane; the pressure in the z-direction is assumed uniform. We assume the pressure gradient field to have a temporal dependence of the form $\exp(i\omega t)$ and a spatial dependence of the form $\exp(-i\underset{\sim}{k}.\underset{\sim}{x})$. Here $\omega/2\pi$ denotes the frequency, $\underset{\sim}{k}=\{k_x,k_y\}$ denotes a wavevector, and $\underset{\sim}{x} = \{x,y\}$ denotes a position vector in the x-y plane. We take the equation of state of the gas to be of the form:

$$p\rho^{-n} = \text{constant} \quad , \tag{2.1}$$

where p, ρ denote, respectively, the pressure and the density of the gas, and n is a constant. (n is unity for an isothermal process and is equal to the ratio of the specific heats for an adiabatic process.)

We assume that the Mach numbers associated with the gas motion are much smaller than unity; then we may approximate the equations of motion of the gas, [the equations of conservation of mass and momentum] by:

$$(np_o)^{-1} \frac{\partial p}{\partial t} - \text{div}\, \underset{\sim}{u} = 0 \quad , \tag{2.2}$$

$$\frac{\partial \underset{\sim}{u}}{\partial t} = - \frac{c_o^{\,2}}{np_o} \left(1 + \frac{\xi}{c_o^{\,2}} \frac{\partial}{\partial t}\right) \text{grad } p + \nu\nabla^2 \underset{\sim}{u} \quad , \tag{2.3}$$

where $\underset{\sim}{u} = [u,v,w]$ represents the velocity vector, $\text{grad } p = \left\{ \frac{\partial p}{\partial x} , \frac{\partial p}{\partial y} \right\}$ the pressure gradient, p_o the ambient pressure of the gas, c_o the ambient sound speed in the gas, ν the kinematic viscosity of the gas, and ξ the kinematic viscosity of the second kind. We assume that ξ is of the order of magnitude of ν. As usual, t denotes time and ∇^2 the Laplace operator.

Imposing the boundary conditions $\{u,v\} = 0$ at $z = \pm \frac{1}{2} h$, we obtain from Eq. (2.3)[6]:

$$\{u,v\} \simeq - \frac{c_o^{\,2}}{i\omega np_o} \left(1 + \frac{i\omega\xi}{c_o^{\,2}}\right) (\text{grad } p) \left[1 - \frac{\cos\,(\kappa z)}{\cos(\frac{1}{2}\,\kappa z)}\right] \quad , \tag{2.4}$$

where

$$\kappa^2 \simeq - (2i/\delta^2) - k^2 \quad , \tag{2.5}$$

$$\delta = (2\nu/\omega)^{1/2} \quad , \tag{2.6}$$

δ represents the boundary layer thickness, the depth of penetration of an oscillating motion[6].

We have assumed here that edge effects are negligible. This assumption is valid only if the separation h between the surfaces is small compared with any linear dimension of the overlapping area of the surfaces in the x-y plane.

We limit our consideration to cases where $(\delta k)^2$ and $(\omega\xi/c_o^2)$ are small compared with unity; it can readily be deduced that these restrictions hold in the ranges of frequency and ambient pressure which are of interest in any practical situation to which this report is directed.

Again, because of the small Mach numbers involved in the flow, the expression for the power P_a dissipated in the region of overlap between the two surfaces may be approximated[6]:

$$P_a \simeq 2A_b \frac{p_o n}{c_o^2} \nu \int \langle|\frac{\partial \tilde{u}}{\partial z}|^2\rangle dz \quad , \tag{2.7}$$

where A_b denotes the area of the overlap of the two surfaces. The symbol $\langle\ \rangle$ indicates averaging over the appropriate area, in Eq. (2.7) over A_b.

From Eqs. (2.4) and (2.7) we obtain:

$$P_a \simeq \frac{hc_o^2 A_b}{2np_o\omega} \langle |\text{grad } p|^2 \rangle H\_(\theta) \quad , \tag{2.8}$$

where

$$\theta = h/\delta \quad , \tag{2.9}$$

$$H\_(\theta) = [\sinh(\theta) - \sin(\theta)] \left\{ \theta \left[\cosh(\theta) + \cos(\theta)\right] \right\}^{-1} . \tag{2.10}$$

The function $H\_(\theta)$ in Eq. (2.8) embodies the dependence of the dissipated power on the ratio θ of the separation h to the boundary layer thickness δ. A plot of $H\_(\theta)$ is presented in Fig. 3. We note that $H_{-}(\theta)$ has a maximum at about $\theta = 2$ and varies monotonically on both sides of this maximum.

This behavior of $H_{-}(\theta)$ can readily be interpreted with the aid of Fig. 4. A given pressure gradient parallel to the x-y plane spends itself partly in generating particle velocity gradients, and partly in generating particle velocities. The balance between these two quantities is determined by the distance of the plane from the physical surface. The dependence of the particle velocity gradient and the particle velocity on the normalized distance from a flat surface are illustrated in Fig. 4. The power dissipated in a given plane is proportional to the product of the particle velocity gradient and the particle velocity in this plane. As indicated in Fig. 4, the energy dissipated in a plane parallel to the surface varies with distance from the surface and is maximum for a plane spaced about $\frac{1}{2}\delta$ from the surface. It is now apparent that $H_{-}(\theta)$ is small if $\theta \ll 1$, because the driving pressure gradients then generate large particle velocity gradients, but low particle velocities, leading, on balance, to relatively little energy dissipation per unit driving pressure gradient. If $\theta \gg 1$, most of the flow is governed by essentially free flow of the fluid. Little dissipation occurs except in a small region near the surfaces; the substantial part of the driving pressure gradient is spent in imparting momentum to the gas. For $\theta \simeq 2$ the pressure gradient generates particle velocity gradients and particle velocities which combine most favorably to produce relatively high dissipation.

III. THE PRESSURE GRADIENTS AND THE LOSS FACTOR

If we can show that harmonically varying pressure gradients are generated in the gas that occupies the space between the surfaces of the plate and the beam, then we know that the power dissipation in a beam-plate system will obey Eq. (2.8), with A_b here representing the area of the beam surface in contact with the plate. For Eq. (2.8) to be valid, however, the width of the beam must be substantially greater than the separation between the adjacent surfaces of the beam and the plate. This is usually satisfied in practical cases, and we assume this inequality to hold throughout this report. Of course, in a practical case, the actual separation distance may vary from one location to another; indeed, if the beam is riveted one would expect the separation to be greater away from a rivet than in the vicinity of a rivet. We shall, nevertheless, assume that a typical "average" separation distance can be specified.

Our present task thus is to determine $\langle|\text{grad } p|^2\rangle$ and to show that (grad p) is of the appropriate form in the case of a riveted plate-beam system. We limit our consideration to a plate-beam system in which the area of the plate is much larger than the beam contact area. Further, we confine ourselves to the frequency range in which the flexural wavelength of the vibration on the plate is substantially smaller than a typical plate dimension. [The analysis presented here can be modified to account for less restricted cases. However, our purpose here is to present results in as simple a form as possible.] With the foregoing restrictions (and in the plate-beam experiments presented below), the vibrational field on the plate is substantially diffuse, and thus can be specified in terms of the spatial average of the mean-square velocity field on the plate, $<|V_{po}|^2>$. This quantity is readily measurable in a given experiment. Moreover, this quantity is directly related to the energy E_p stored in the plate-beam system,

$$E_p \simeq A_p m_p <|V_{po}|^2> \quad , \tag{3.1}$$

where A_p denotes the area of the plate, $A_p >> A_b$, and m_p represents the plate mass per unit area. Eq. (2.8) and the definition of loss factor, in terms of energy stored and energy dissipated per cycle, permit us to obtain an expression for the loss factor [1,2,3] η_a associated with the power dissipated by the gas motion in the space between the plate and the beam:

$$\eta_a \simeq \frac{P_a}{\omega A_p m_p <|V_{po}|^2>} = \frac{A_b h c_o^2}{2 A_p m_p n p_o \omega^2} \frac{<|\text{grad } p|^2>}{<|V_{po}|^2>} H_-(\theta) \quad . \tag{3.2}$$

It has been established, by experiments on plate-beam systems of the form considered here, that the loss factor η_a is independent of $<|V_{po}|^2>$, provided that "slapping" between the two surfaces is avoided. We thus expect that $<|\text{grad } p|^2>$ is directly related to $<|V_{po}|^2>$. We show next that such a relationship actually exists.

IV. THE STRUCTURAL MODEL

A schematic representation of the structural system under consideration is given in Fig. 1. We have stipulated earlier that we consider the vibrational field of the plate to be diffuse. Thus we may formulate our problem in terms of a single typical flexural wave on the plate, and then obtain from this simpler analysis results which apply to a diffused set of waves with frequencies lying within a narrow band (e.g. third octave) centered on the frequency of this single wave. The velocity V_{po} denotes the amplitude of the flexural velocity field on the plate, except at and near the region covered by the beam, which region is hereafter called the "strip." The velocity field on the plate in the strip will, in general, be different from V_{po} because of the beam, which acts on the plate via the rivets and via the gas occupying the space between the surfaces of the plate and the beam. The effect of the rivets on the velocity field in the strip has been studied recently by Ungar[1]. He has shown that (in absence of gas effects) equally spaced rivets produce a velocity field amplitude V_s that may be substantially different from V_{po} essentially only for $k_p d \lesssim 1$, where d is the spacing between the rivets and k_p the wavenumber of the typical wave on the plate. If $k_p d > 5$, then the difference between V_s and V_{po} is negligible. This result is reasonable, since the "effective area" over which a rivet extends its influence may be expected to be a disk having a radius of the order of k_p^{-1}. Ungar's results[1] can be summarized as:

$$\langle |V_s|^2 \rangle = \langle |V_{po}|^2 \rangle \; S(k_p d) \quad , \tag{4.1}$$

where $S(k_p d)$ is a complicated function of $k_p d$ when $k_p d \lesssim 1$, but becomes essentially unity when $k_p d \gtrsim 5$.

In addition to the rivets, the gas in the space between the surfaces of the plate and the beam influences the plate velocity field in the strip. One may invoke the principle of superposition to account for both effects simultaneously. Unfortunately, the effect of the rivets is associated with the point impedance of the plate,[1/] and, as we shall see, the effect of the gas is associated with the line impedance of the plate. This difference between the effects renders a comprehensive analysis rather difficult. To circumvent this difficulty we assume that the effects are statistically independent. This assumption eliminates the possibility of accounting for resonances and anti-resonances that may occur between the two mechanisms; only gross properties can be ascertained under this assumption. However, when $k_p d \gtrsim 5$ the conclusion should hold that the influence on the vibrational field on the strip by the rivets is negligible; in this case the influence of the gas, if substantial, is the only effect remaining. We proceed now to account for this latter effect.

In most practical systems the beam has a length much larger than its width, and is much heavier and stiffer than the plate strip. Therefore, there occurs a "bias" towards x-wise incidence of flexural waves on the strip[5/]; in addition, the beam motion can be essentially neglected. [The analysis can be made less restrictive, but at the expense of higher complexity[7/]. Since most practical plate-beam systems satisfy the above restrictions, we shall not present a more general analysis here.]

We may consider the typical flexural wave on the plate as being incident on the strip in the x-direction. This wave produces a relative motion between the surfaces of the plate and the beam that is uniform in the y-direction and has an associated wavenumber in the x-direction equal to $k = k_p$.

If $k_p h \ll 1$ and $k_o h \ll 1$, where k_o is the acoustic wavenumber in the gas at the appropriate frequency, this relative motion between the beam and the plate strip will generate a pressure field having a spatial and temporal component in the x-y plane which is closely related to the relative structural motion[8]. This pressure field will be independent of the z coordinate, in consequence of the inequality that $k_o h \ll 1$, which we shall assume to hold throughout this report.

We consider two infinitesimally narrow plate strips, both having their long axes parallel to the y-direction. One strip is centered at position x' and the other at position x on the x-axis, as indicated in Fig. 5. We may choose position x at the origin of our coordinate system so that x' is measured relative to x (see Fig. 5). The differential pressure on the strip at position x' relative to that at x is given by $[(p-p_o) \exp(-ik_p x)] \exp(-ik_p x')$. The effect of this pressure field on the strip at position x can be computed with the aid of a spatial plate transfer function $T(x'|x)$, defined by[9]:

$$T(x'|x) = \delta(x') + \frac{k_p}{\pi} \exp([i-\eta(x')]k_p x') \quad , \tag{4.2}$$

such that the effective pressure differential, $p_{eff}(x)$ at x is given by:

$$p_{eff}(x) = (p-p_o) \exp(-ik_p x) \int_b T(x'|x) \exp(-ik_p x')\, dx' , \qquad (4.3)$$

where the time dependence has been suppressed and

$$\eta(x') = \begin{cases} \eta & x'>o \\ -\eta & x'<o \end{cases} .$$

Here η represents the total loss factor of the plate, and the integral in Eq. (4.3) is taken over the width b of the beam. Note that p_{eff} (x) dx is the effective force acting on the strip at x. The delta function in Eq. (4.2) takes account of the self-transfer of the local pressure field at position x. From Eqs. (4.2) and (4.3) we obtain for the effective pressure differential amplitude:

$$p_{eff} = (p-p_o) \left(1 + \frac{k_p b}{\pi}\right) \equiv (p-p_o)\, \beta \quad , \qquad (4.4)$$

where we assumed that $\eta k_p b \ll 1$. The expression represented by β must be modified if the foregoing inequality is violated; see Eq. (4.3).

Because of the assumption that the effects of the rivets and the gas are uncorrelated, we may consider the effect of the gas starting with a strip velocity V_s. This velocity now plays the role of a dummy variable whose mean-square value is directly related to the mean-square velocity on the plate via the function

$S(k_p d)$, see Eq. (4.1). The effect of the gas on the plate strip velocity is to induce a change in the velocity amplitude V_s to a value which we denote by V_p. The change in the velocity amplitude in the strip, from V_s to V_p, can be related to the effective pressure that produces it in terms of the line impedance of the plate[5]:

$$\beta\ (p-p_o) = Z_L\ (V_s-V_p) \quad , \tag{4.5}$$

where[10]

$$Z_L = 4\omega m_p/(1-i) \quad . \tag{4.6}$$

The velocity amplitude V_p may be eliminated from Eq. (4.5) by use of Eq. (2.2), averaged over the coordinate z, and application of Eq. (2.4)[8]. This procedure yields the equation:

$$\frac{i\omega h}{np_o} \left\{ 1 - \frac{k_p{}^2 c_o{}^2}{\omega^2} \left[1 - H_+(\theta) + iH_-(\theta) \right] \right\} (p-p_o) \simeq V_p \tag{4.7}$$

where

$$H_+(\theta) = \left[\sinh(\theta) + \sin(\theta) \right] \left\{ \theta \left[\cosh(\theta) + \cos(\theta) \right] \right\}^{-1} . \tag{4.8}$$

Noting that $k_p{}^2 = \omega_g \omega / c_o{}^2$, where $\omega_g/2\pi$ is the critical frequency[11] of the plate, we obtain, from Eqs. (2.8), (4.1), (4.5) and (4.6):

$$\eta_a - \frac{A_b n p_o}{A_p m_p \omega^3 h} \cdot \left(\frac{S}{G} \right) \cdot \left(\frac{\omega_g}{\omega} \right) H_-(\theta) \quad , \tag{4.9}$$

where

$$G \equiv \left[1 - \frac{np_o\beta}{4\omega^2 m_p h} - \frac{\omega_g}{\omega}(1 - H_+) \right]^2 + \left[\frac{np_o\beta}{4\omega^2 m_p h} + \frac{\omega_g}{\omega} H_- \right]^2 . \qquad (4.10)$$

It is clear that η_a is independent of the level of excitation of the plate-beam system (provided that impacts between the surfaces of the plate and the beam are avoided). By the nature of our derivation, the loss factor η_a, as stated in Eq. (4.9), refers to a narrow frequency band centered about ω[5].

The factor (S/G) is simply related to the ratio of the spatial average of the velocity field amplitude in the strip to that on the entire plate. The factor ω_g/ω, which appears in η_a (and G), is of particular interest. It multiplies those terms that are concerned with the inertial motion of the gas, and is a measure of the normalized time that is available for this motion to be established and maintained. This aspect becomes clear when one remembers that ω_g/ω is equal to $(c_o/c_p)^2$, where c_p is the phase speed of the flexural wave. The time variables in the gas are related to the sound speed c_o, while those in the plate are related to c_p. Thus, if ω_g/ω is large, the inertial motions in the gas are accentuated and the effects of these motions are amplified.

Two significant aspects of the function G deserve special mention. The first is that the value of the function G is governed by a parameter that is the ratio of the impedance of the gas to that of the plate. In particular, the former impedance depends on the ambient pressure. If the ambient pressure is sufficiently low, G is approximately unity, indicating that the motion of the plate in the strip is not affected by the presence of the gas. On the other hand, if the ambient pressure is sufficiently high, the value of G exceeds unity. Then the gas acts so as to inhibit the relative motion between the surfaces of the plate and the beam, an effect which tends to diminish the value of the loss factor η_a. The second aspect of the function G is that it indicates that a resonance condition between the motion of the gas and that of the plate strip can occur. However, because of the real part in the line impedance of the plate and the dissipative term in the inertial motion of the gas, this resonance is weak and does not appreciably affect the value of η_a.

Eq. (4.9) constitutes the central result of this report. However, before it can be used to yield practical results, one must examine more closely the parameter $\theta = h/\delta$ where $\delta = (2\nu/\omega)^{1/2}$.

V. THE DEPENDENCE OF THE KINEMATIC VISCOSITY ON THE SEPARATION DISTANCE

If the linear dimensions of the space between the beam and plate are large compared with the molecular mean free path, the kinematic viscosity obeys:

$$\nu = \frac{1}{2} \bar{c}\Lambda \quad , \tag{5.1}$$

where $\bar{c}$ denotes the average speed of the molecules and Λ denotes the mean free path. [In air at 20°C $\bar{c} \simeq 4.6 \times 10^4$ cm/sec and $\Lambda = (5.5 \times 10^{-3}/p_o)$ cm (for p_o in mm Hg).]

However, if the gas is confined to a space which has a linear dimension nearly equal to or smaller than the mean free path between intermolecular collisions, then the expression for the kinematic viscosity depends on this dimension[4/]. One may argue that the nature of the kinematic viscosity does not change except that now the molecular-surface collisions are in control of the gas dynamics rather than the intermolecular collisions[12/]. Since the former collision process is associated with a mean free path that depends on the separation h, the kinematic viscosity would also be a function of h. Unfortunately, there exists at the present time no firm theoretical basis for ascertaining the point of departure from one regime to the other. However, on the basis of previous work[4/] dealing with a closely related problem, it appears that, when the mean free path of intermolecular collisions becomes greater than about a thirtieth of the separation distance h, the kinematic viscosity reaches an asymptotic value and the expression for this quantity is given by:

$$\nu \simeq \frac{1}{2}\,\bar{c}\,(hs^{-1}) \quad , \tag{5.2}$$

where s is a dimensionless constant equal to about 30.

Eqs. (5.1) and (5.2) can be conveniently combined into a single expression:

$$\nu = \frac{1}{2}\,\bar{c}\,h\,(s + h\Lambda^{-1})^{-1} = \begin{cases} \frac{1}{2}\,\bar{c}\Lambda & ; \quad s \ll h\Lambda^{-1} \\ \frac{1}{2}\,\bar{c}\,(s^{-1}h); & s \gg h\Lambda^{-1} \end{cases} \quad . \tag{5.3}$$

From Eqs. (2.6), (2.9) and (5.3) we obtain:

$$\theta = \left\{ \frac{h\omega}{\bar{c}}\,[\,s + h\Lambda^{-1}] \right\}^{1/2} \quad . \tag{5.4}$$

We are now in a position to compare our theoretical predictions with the available experimental data. However, before we do so, it may be helpful to examine some of the asymptotic forms of the loss factor η_a.

VI. ASYMPTOTIC BEHAVIOR OF THE LOSS FACTOR η_a

It is apparent that Eq. (4.9) is rather complex and involves many parameters of the structural system and of the gas. Although present-day computers permit us to arrive at numerical results without great difficulty, we can more readily gain some insight into the behavior of η_a in some regions of interest by examining in turn the functional factors that combine to form the expression given in Eq. (4.9) for η_a.

The factor $A_b np_o/A_p m_p \omega^2 h$ is seen to be simply the ratio of the absolute value of the stiffness impedance $A_b np_o/\omega h$ of the gas in the space between the two surfaces to the absolute value of the inertia impedance of the plate, $A_p m_p \omega$. This ratio is thus a measure of the relative energy stored in the gas and the plate under ideal conditions. The factor S/G is a measure of the influence of the rivets and of the gas in the interspace on the motion of the plate (if the two effects are assumed independent, see Section IV). Finally, the factor $(\omega_g/\omega)H_-$ is a measure of the efficiency with which the gas dissipates the energy that is imparted to it by the plate motion.

A. The asymptotic behavior of $H_-(\theta)$

1. Low ambient pressure regime, $h\Lambda^{-1}<s$

Since $\bar{c} \simeq c_o$, it is clear from Eq. (5.4) that at low ambient pressures, for which $h\Lambda^{-1}<s$, θ is substantially less than unity, by virtue of our assumption that $hk_o<<1$. In this case then:

$$H_{-}(\theta) \simeq \frac{\theta^2}{3} = \frac{h\omega s}{3\bar{c}} \quad , \tag{6.1}$$

and

$$1 - H_{+}(\theta) \simeq 0\ (\theta^4) \quad . \tag{6.2}$$

This regime is characterized by relatively large particle velocity gradients, but by small particle velocities. The efficiency of dissipation by the gas is low (see Section II).

2. High ambient pressure regime, $(h\Lambda^{-1})\ (k_o h) >> 4$
In this regime $\theta > 2$, see Eq. (5.4). The asymptotic value of $H_{-}(\theta)$ then is:

$$H_{-}(\theta) \simeq (\theta)^{-1} = h^{-1} \left(\frac{\bar{c}\Lambda}{\omega}\right)^{1/2} \quad , \tag{6.3}$$

and

$$1 - H_{+}(\theta) \simeq 1 - H_{-}(\theta) \quad . \tag{6.4}$$

This regime is characterized by relatively large particle velocities, but by low particle velocity gradients. The efficiency of dissipation by the gas is relatively low (see Section II).

The efficiency of dissipation has a maximum value when $\theta \simeq 2$; $H_{max} \approx H_{-}(2) \approx 0.4$ (see Fig. 3).

B. The asymptotic behavior of G

1. Very low ambient pressure regime, $h\Lambda^{-1} << s$

In this regime $(\omega_g/\omega)\, H_-(\theta)$ is small for the frequency range of interest. Also, the ratio $\alpha = \frac{np_o\beta}{m_p\omega^2 h}$ of the absolute value of the effective surface stiffness of the gas $(np_o\beta/\omega h)$ to the absolute value of the surface mass impedance of the plate (ωm_p) is small. From Eq. (4.10) we learn that G is unity under these conditions. In this regime the gas motion hardly affects the vibration of the plate in the strip. The mean flexural motion of the plate in the strip has its maximum value in this regime, provided that $S \simeq 1$.

2. High ambient pressure regime $(h\Lambda^{-1})\ (k_o h) >> 4$

The expression for G in this regime is

$$G \simeq \left[\left(1 - \frac{\omega_g}{\omega} \right) + \left(\frac{\alpha}{4} \right) \right]^2 + (\alpha/4)^2 \quad , \tag{4.5}$$

where

$$\alpha = \frac{np_o\beta}{\omega^2 m_p h} \quad . \tag{4.6}$$

If α is small compared with unity, in spite of the high ambient pressure, G is essentially unity in the frequency range above the critical frequency, and regime B.2 here is similar to regime B.1. Below the critical frequency, G is essentially equal to $(\omega_g/\omega)^2$, signifying that the inertial terms in the gas dominate the motion of the gas between the two surfaces. In both of these cases the function G is independent of the ambient pressure. If $(\alpha/4)$ is greater than both unity and (ω_g/ω), then G is a function of the ambient pressure. The flexural vibration of the plate strip is inhibited by the compressibility of the gas in the space between the two surfaces. This effect leads to low loss factors, which decrease with increasing ambient pressure. Note that $\alpha/4$ dominance, and thus a pressure-dependent G, can always be reached if the ambient pressure is increased sufficiently. Thus, increasing the ambient pressure in order to bring about higher loss factors has a limited range of applicability.

Some features of our discussion of this section are sketched in Fig. 6.

VII. COMPARISON BETWEEN THEORY AND EXPERIMENT

There are available some experimental data on joint damping that concern the loss factor η_a as a function of the ambient pressure p_o and the frequency $f = \omega/2\pi$. These data were obtained by Carbonell and Ungar [1,2,3], by a method described in some detail in references 1, 2 and 3. It is with these data that we compare our theoretical calculations.

The test system consists of an aluminum plate, $\frac{1''}{64}$ x 20" x 14", to which an aluminum beam, $\frac{1''}{4}$ x 1" x 17", has been attached by means of bolts with 3" spacing. The system was placed in a vacuum chamber in which the ambient pressure could be varied from atmospheric pressure (760 mm Hg) down to about 1 mm Hg. The loss factors of both the bare plate, and of the plate-beam system were measured in third-octave bands as functions of ambient pressure. The contributions to the loss factors due to the presence of the beam were computed from:

$$\bar{\eta}_a = \eta_{pb} - \eta_p \quad , \tag{7.1}$$

where η_{pb} denotes the loss factor of the plate-beam system, and η_p the loss factor of the bare plate. If the measured $\bar{\eta}_a$ is dominated by gas-pumping effects, then one would expect a close relation between $\bar{\eta}_a$ and the theoretically determined η_a. Thus, we compare our theoretical estimates of η_a with $\bar{\eta}_a$.

We have inserted the appropriate values of the experimental structural system and of the gas (air) into Eq. (4.9) and carried out the calculations with the aid of a digital computer[13/]. In these calculations we assume $n = 1.4$ [setting $n = 1$ does not alter the results significantly, see Fig. 7c.] Computations were carried out for ambient pressures between 1 mm Hg and $4x10^4$ mm Hg (the experiments were carried out in the range from 1 mm Hg to 760 mm Hg only) and frequencies between 1 kcps and 10 kcps. The lower frequency limit was chosen to be compatible with $S(k_p d) \simeq 1$, in order to avoid the complexities in $S(k_p d)$ which occur for $k_p d \lesssim 5$. The calculations are presented with $s = 30$. Changes in s in the range from 20-40 were found not to affect the results substantially in the ranges for which experimental data are available. The value for the separation h was chosen to be $7.5x10^{-3}$cm. (This value appears not to be unreasonable. Computations carried out for h in the range $3x10^{-2}$cm to $3x10^{-3}$cm indicate that the value of $7.5x10^{-2}$cm leads to loss factor values which are in better agreement with the experimental data.)

It was found empirically that the experimental data in the range of interest can be best presented by plotting the loss factor as a function of the parameter $p_c/f^{3/2}$. We have thus chosen to present the comparison between theory and experiments in terms of this parameter. The comparison on this basis is presented in graphical form in Figs. 7 (a,b,c,d and e). It is apparent that there is reasonable agreement between the predicted values and the experimental data. The agreement is satisfactory, not only in terms of "order of magnitude," but also in terms of the functional dependence of the loss factor on the frequency and the ambient-pressure variables.

The available experimental data [1,2,3] were obtained without the guidance of a working theory. Consequently, although some aspects of these data can serve to test the theory, a critical experimental validation of the theory requires additional measurements, designed specifically to bring out the most crucial aspects of the theory, such as the dependence on the separation h. Such a program of experimental studies is contemplated for the near future.

VIII. DISCUSSION AND CONCLUSIONS

We have shown that gas-pumping in the space between the plate and the beam can be made to account satisfactorily for the observed dissipation in a plate-beam system. However, the values of two parameters, s and h, both of which play significant roles in the determination of the loss factor, had to be chosen empirically. It would have been more satisfying if these parameters could have been determined directly by independent experiments or analyses. It is hoped that such an examination of these parameters will be forthcoming. Nevertheless, the analysis does permit one to estimate how to optimize the dissipation of a given system as a function of ambient pressure, and the separation distance h. Similarly, changes in the loss factor with changes in these parameters can be predicted and used in the design of structures incorporating plate-beam systems of the type we considered above.

A problem of some import is to examine the validity of the theory in the lower frequency range, where $S(k_p d)$ may differ from unity. There are two practical difficulties associated with such an examination: 1. The complex nature of $S(k_p d)$ makes calculations difficult and cumbersome. 2. Experiments would have to be made on fairly large structures, and thus would require a large vacuum chamber.

The above analysis did not account for the dissipation associated with acoustic radiation. It is important to realize that this dissipation can be relatively large in plate-beam systems even below the critical frequency[5]. In particular, if the plate-beam system is examined under conditions where the plate is unbaffled, $\bar{\eta}_a$, defined in Eq. (7.1), contains the full measure of the dissipation by radiation since the unbaffled bare plate has substantially smaller radiation efficiency than the ribbed plate[5]. In the experiments discussed in Section VII the acoustic coupling between the plate-beam system and the vacuum chamber was not accounted for. It is thus possible that at the higher ambient pressures and frequencies, where the predicted loss factors are relatively small, the acoustic coupling between the plate-beam system and the vacuum chamber could distort the observed data significantly. This coupling may explain some of the discrepancies between theory and experiments in this range, and should be considered in future studies.

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9. This procedure is closely related to Ungar's analysis[1/] of the effect of the rivets on the plate motion in the strip. While Ungar dealt with point forces, we deal here with line forces.

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13. The computations were carried out on "The Telcomp Service" time-shared computer.

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FIGURE CAPTIONS

1. Beam Riveted to Plate
   b = Beam Width, d = Rivet Spacing.

2. Coordinate System
   h = Separation between Plate and Beam Surfaces.

3. Dependence of $H\_(\theta)$ on Ratio θ of Average Beam-to-Plate Separation h to Boundary Layer Thickness δ.

4. Dependence of Particle Velocity, Particle Velocity Gradient, and Energy Dissipation in a Flowing Gas On Normalized Distance from a Solid Surface (Schematic).

5. Geometry of Infinitesimal Plate Strips, Used in Determination of Effect of Gas Pressure on Velocity in Beam-Covered Plate Region.

6. Dependence of Loss Factor on Ambient Pressure (Schematic)
   $\omega/2\pi$ = Frequency, $\omega_g/2\pi$ = Critical Frequency,
   α = Ratio of Effective Surface Stiffness of Gas to Surface Mass Impedance of Plate.

7.a Comparison between Theoretical and Experimental Values of the Loss Factor η_a.

7.b Comparison between Theoretical and Experimental Values of the Loss Factor η_a.

7.c Comparison between Theoretical and Experimental Values of the Loss Factor η_a.

FIGURE CAPTIONS (Continued)

7.d Comparison between Theoretical and Experimental Values of the Loss Factor η_a.

7.e Comparison between Theoretical and Experimental Values of the Loss Factor η_a.

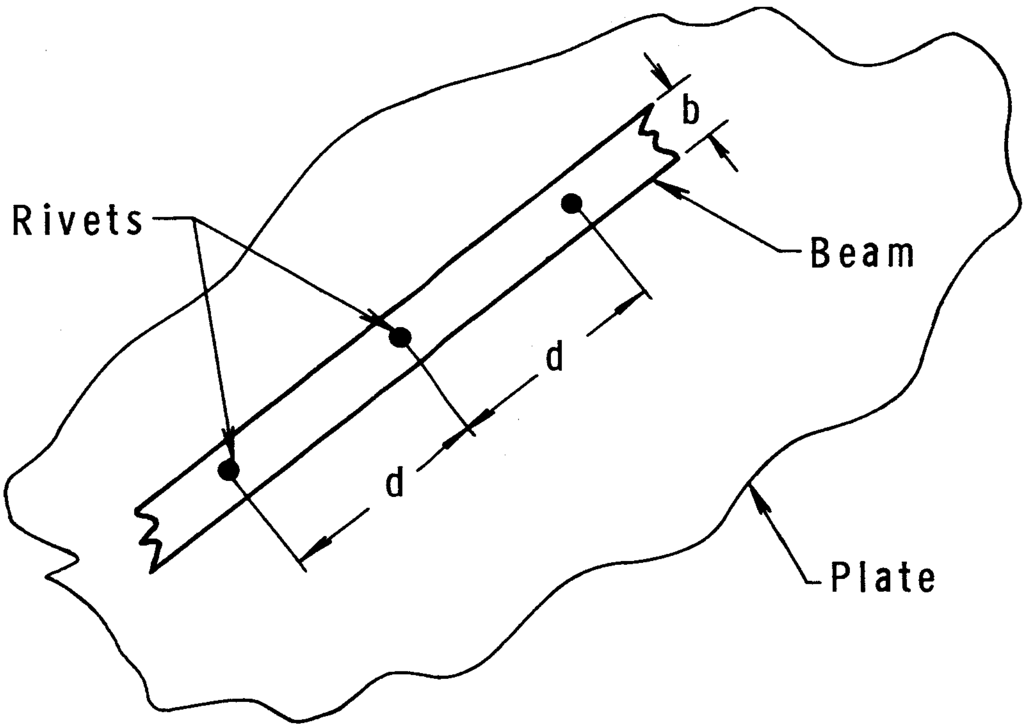

FIG. 1 A BEAM RIVITED TO PLATE
b = BEAM WIDTH, d = RIVET SPACING

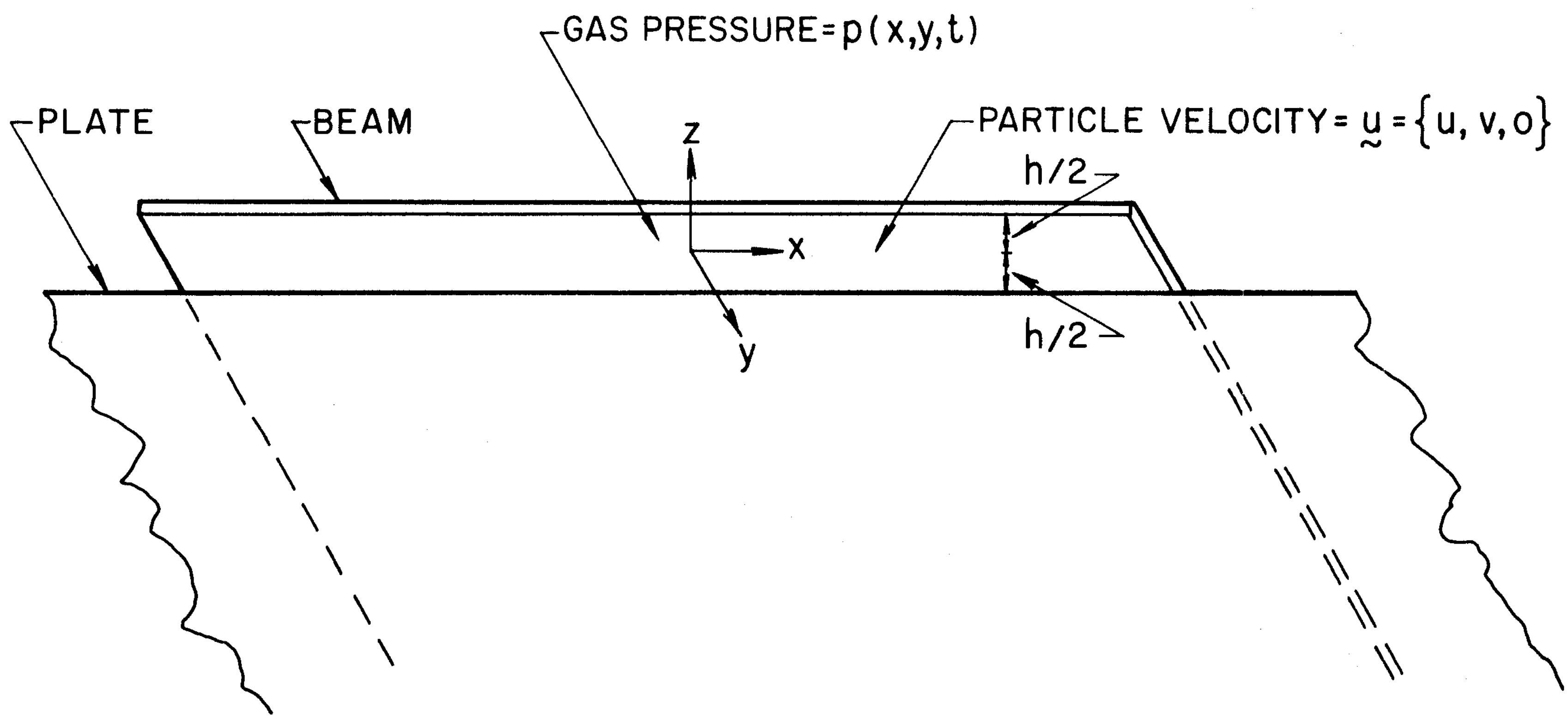

FIG. 2 COORDINATE SYSTEM
h = SEPARATION BETWEEN PLATE AND BEAM SURFACES

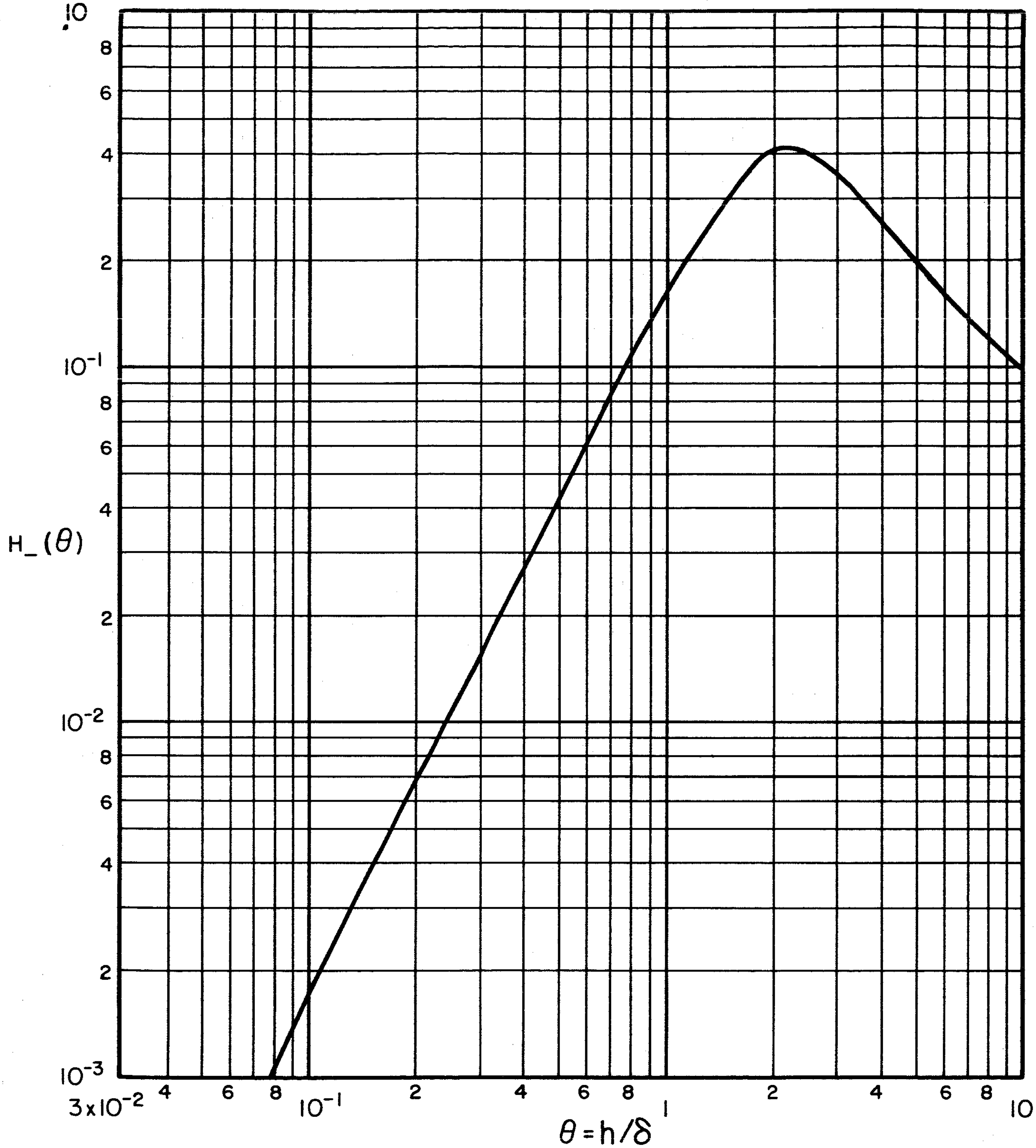

FIG. 3 DEPENDENCE OF H_(θ) ON RATIO θ OF AVERAGE BEAM-TO-PLATE SEPARATION h TO BOUNDARY LAYER THICKNESS δ.

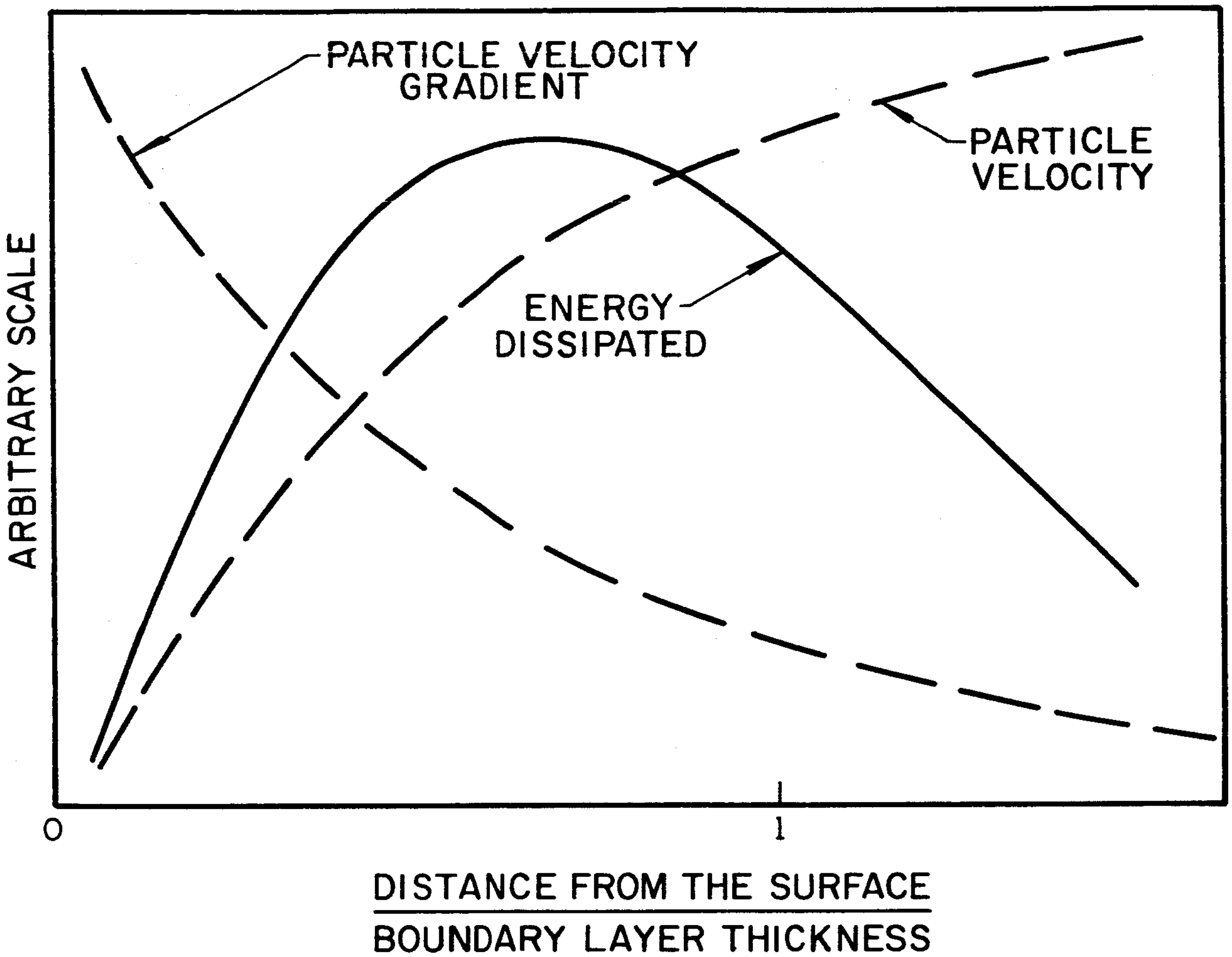

FIG. 4 DEPENDENCE OF PARTICLE VELOCITY, PARTICLE VELOCITY GRADIENT, AND ENERGY DISSIPATION IN A FLOWING GAS ON NORMALIZED DISTANCE FROM A SOLID SURFACE (SCHEMATIC)

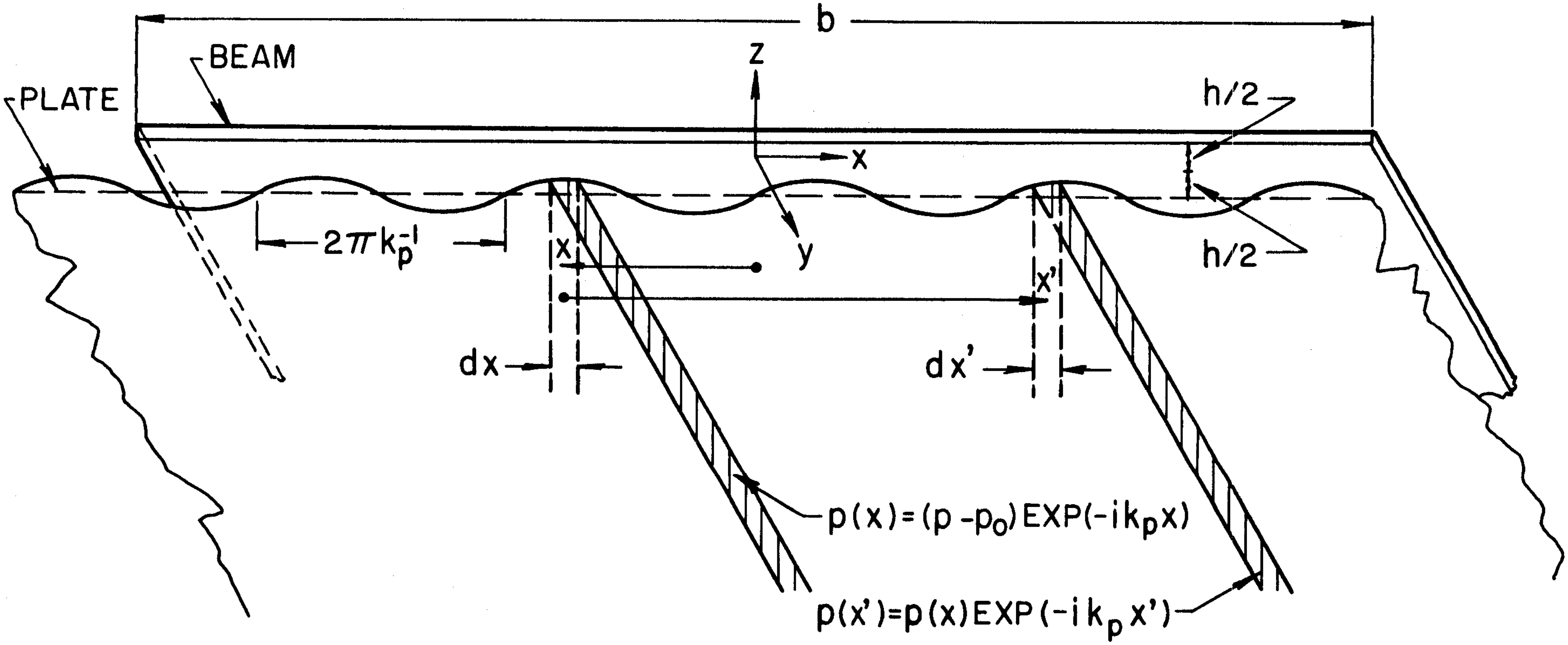

FIG. 5 GEOMETRY OF INFINITESMINAL PLATE STRIPS, USED IN DETERMINATION OF EFFECT OF GAS PRESSURE ON VELOCITY IN BEAM-COVERED PLATE REGION.

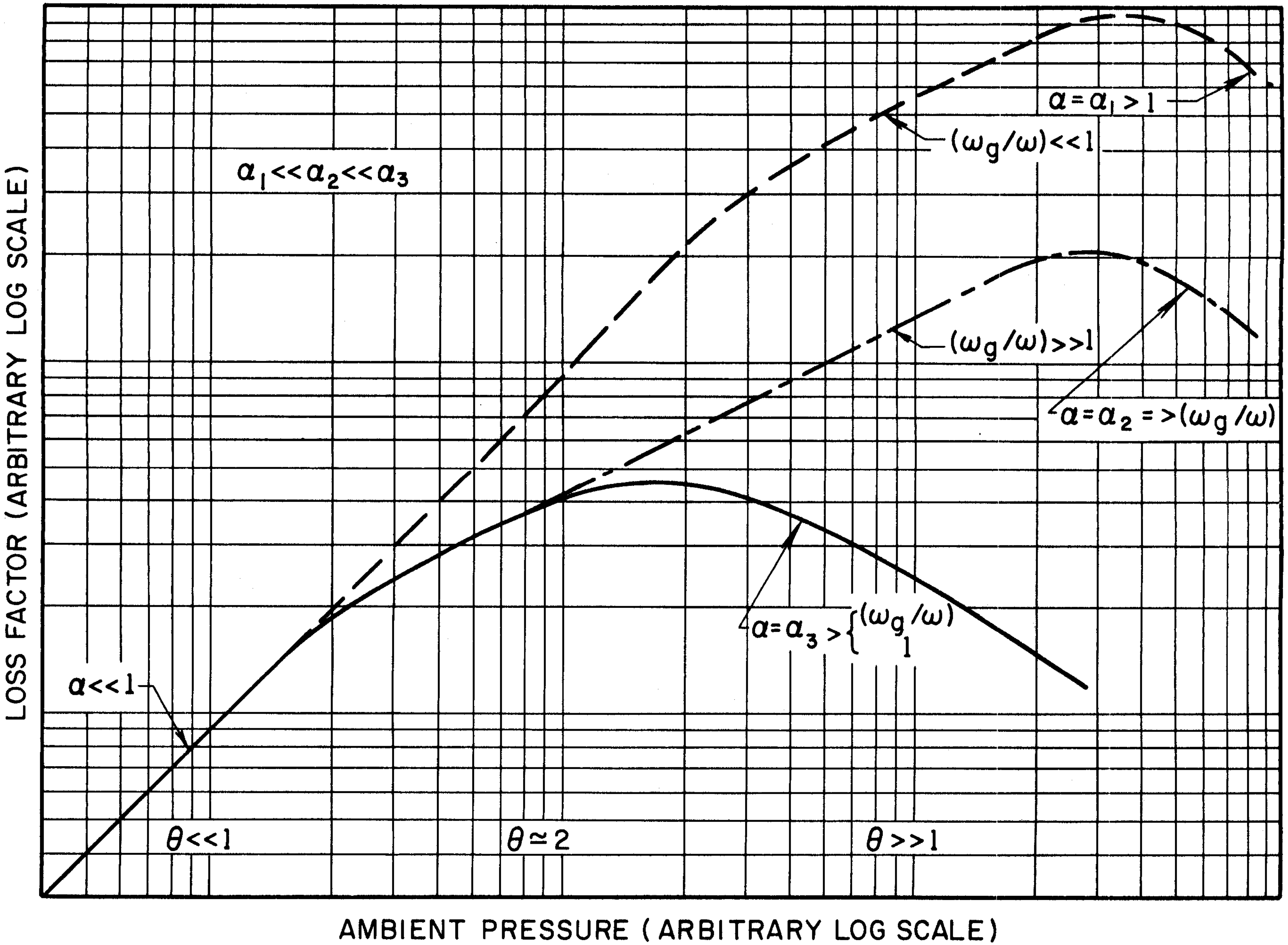

FIG. 6 DEPENDENCE OF LOSS FACTOR ON AMBIENT PRESSURE (SCHEMATIC) $\omega/2\pi$ = FREQUENCY, $\omega_g/2\pi$ = CRITICAL FREQUENCY, α = RATIO OF EFFECTIVE SURFACE STIFFNESS OF GAS TO SURFACE MASS IMPEDANCE OF PLATE

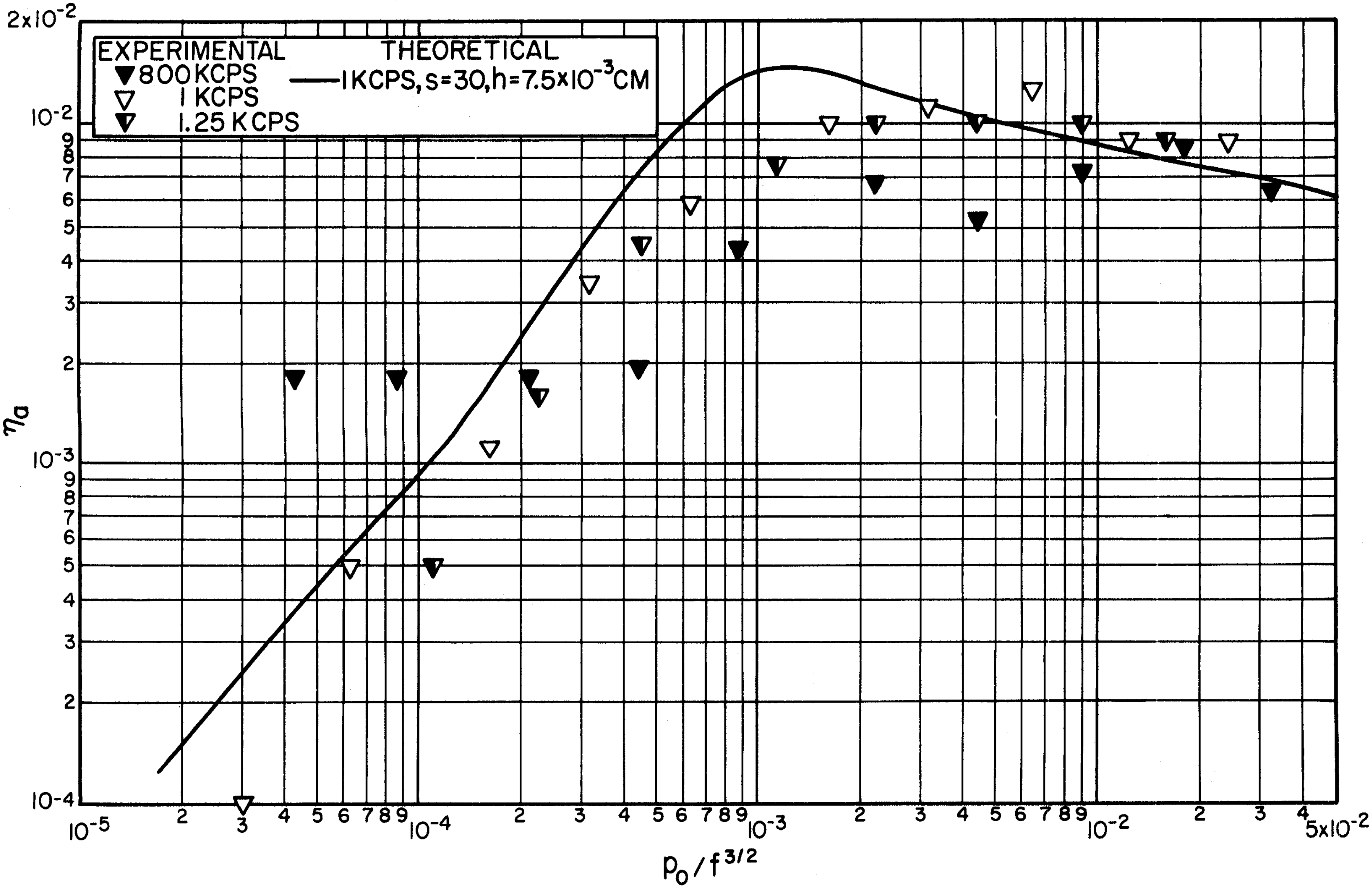

FIG. 7a COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE LOSS FACTOR η_a

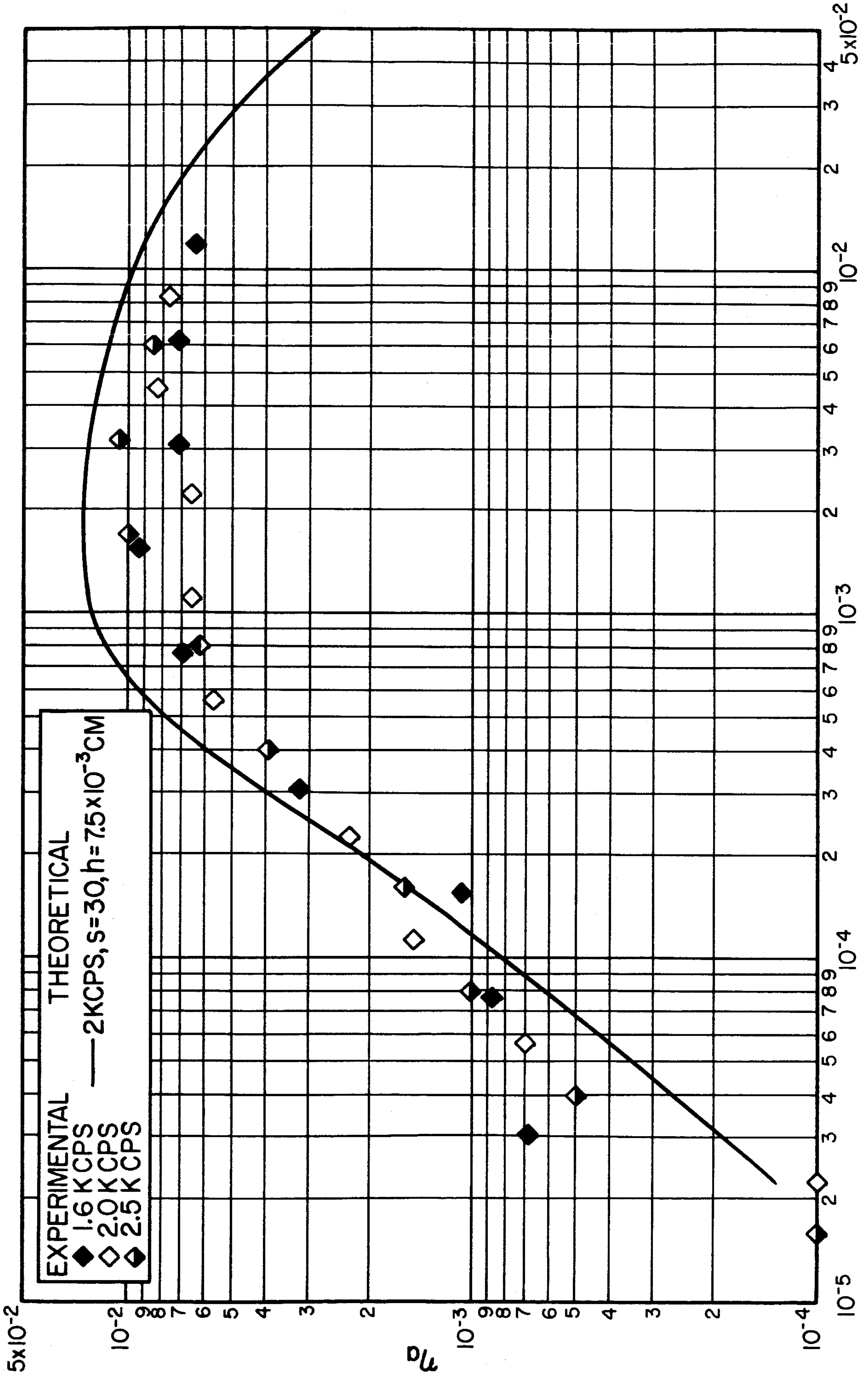

FIG. 7b COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE LOSS FACTOR η_a

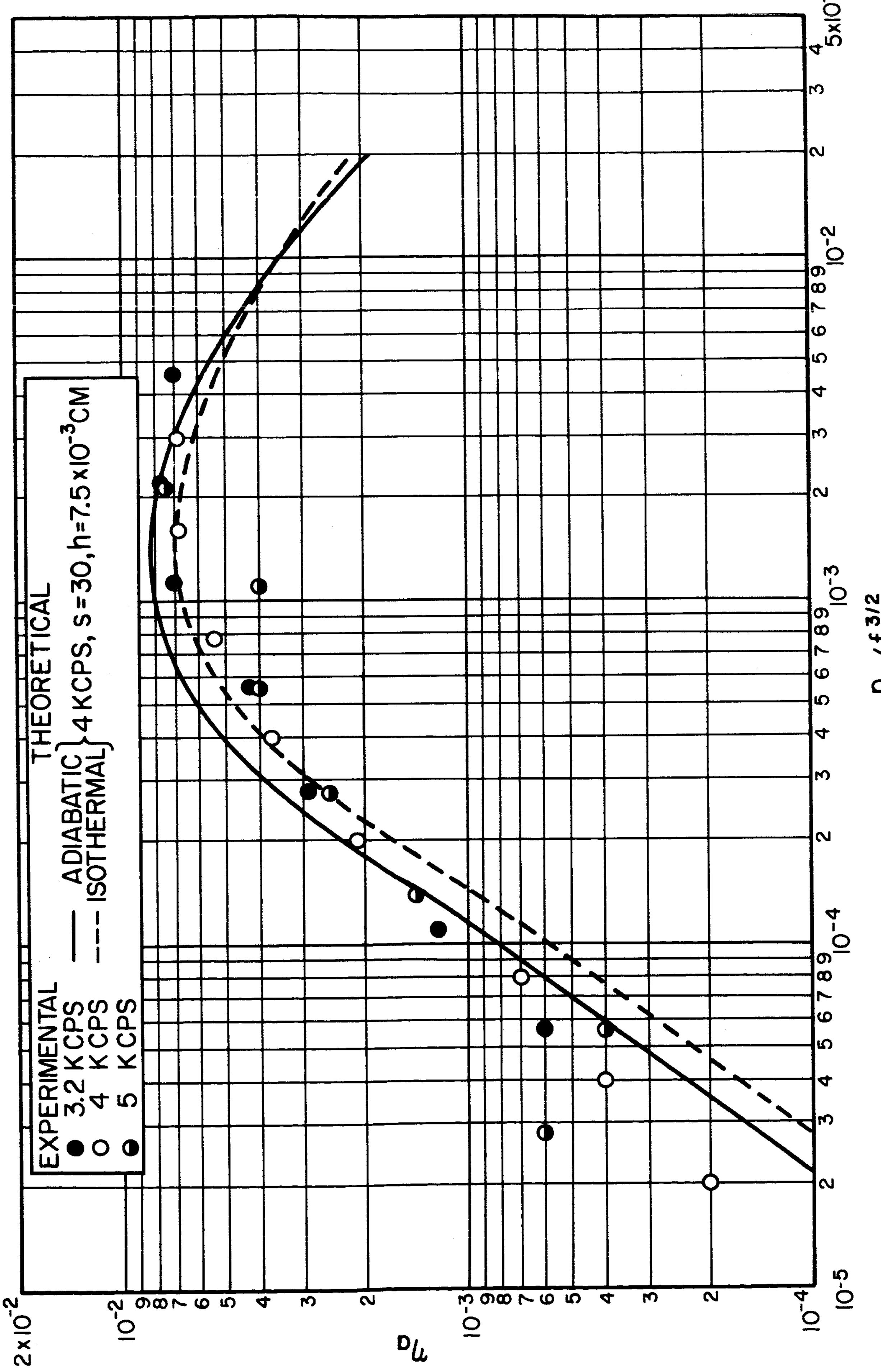

FIG. 7c COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE LOSS FACTOR η_a

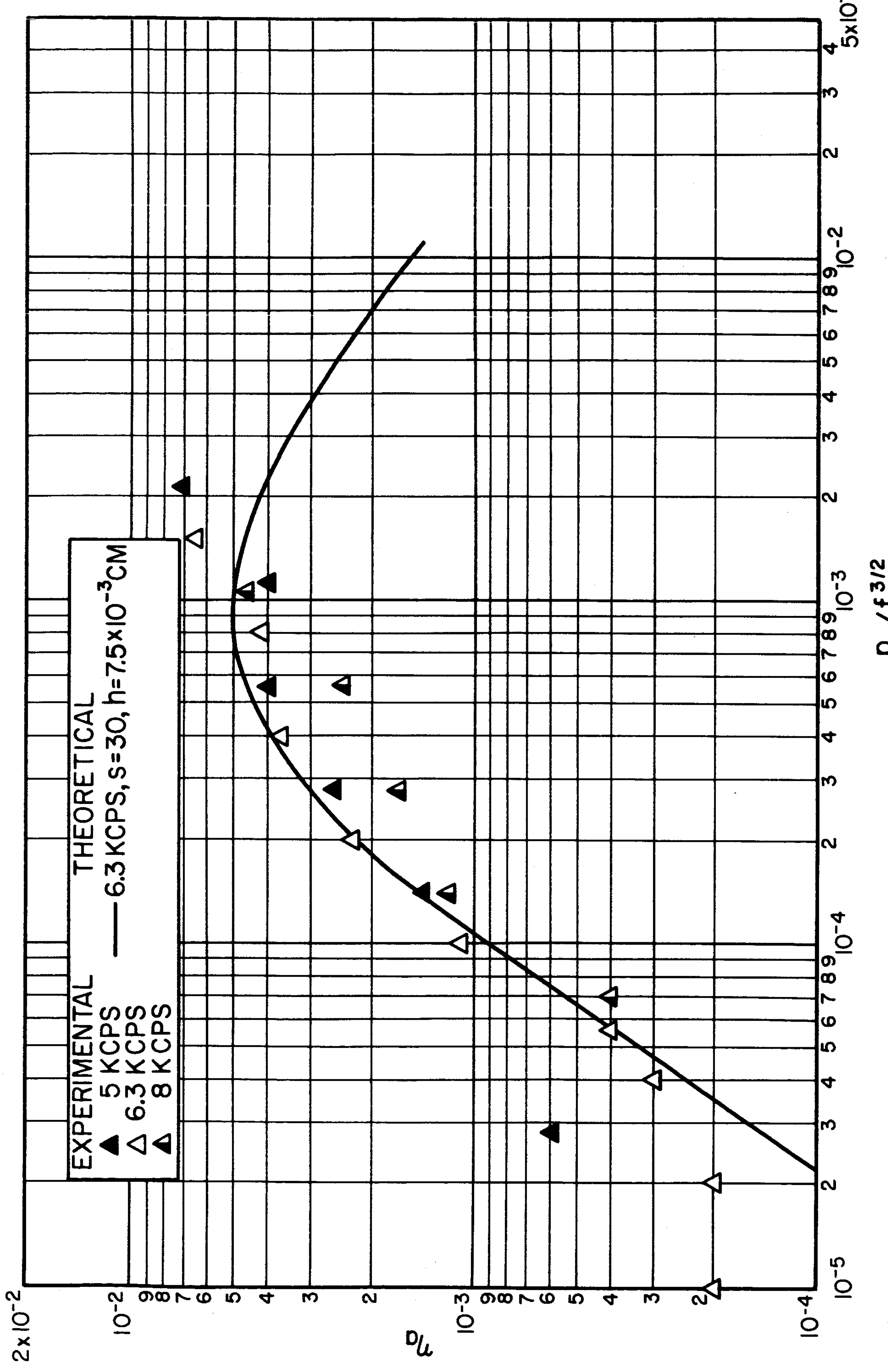

FIG. 7d COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE LOSS FACTOR η_a

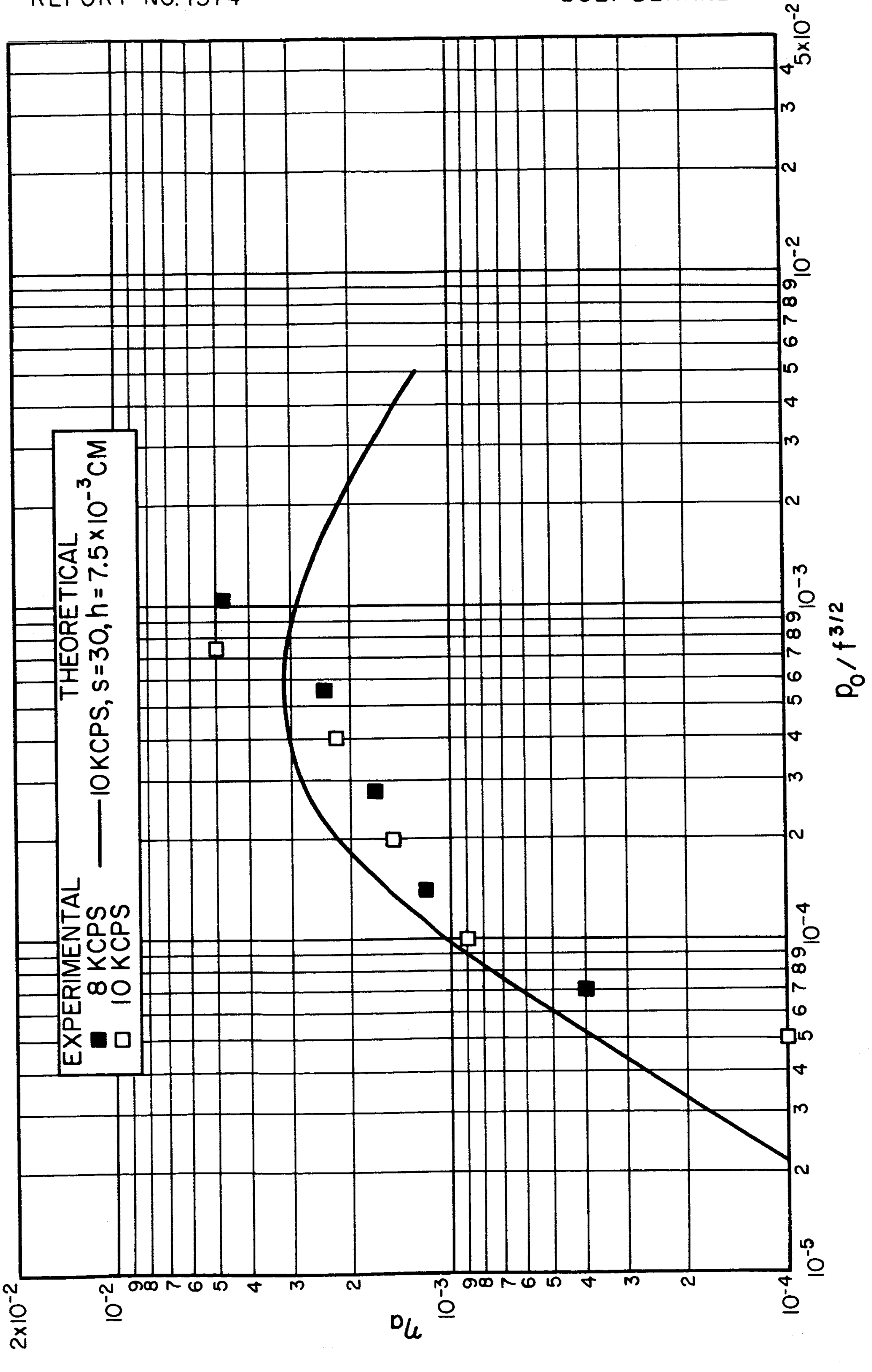

FIG. 7e COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF THE LOSS FACTOR η_a