NASr-21(05)

MEMORANDUM
RM-5091-NASA
AUGUST 1966

ESTIMATED SURFACE MOTIONS OF THE EARTH'S CORE

Anne B. Kahle, E. H. Vestine and R. H. Ball

PREPARED FOR:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

The RAND Corporation
SANTA MONICA • CALIFORNIA

NASr-21(05)

MEMORANDUM
RM-5091-NASA
AUGUST 1966

ESTIMATED SURFACE MOTIONS OF THE EARTH'S CORE

Anne B. Kahle, E. H. Vestine and R. H. Ball

This research is sponsored by the National Aeronautics and Space Administration under Contract No. NASr-21. This report does not necessarily represent the views of the National Aeronautics and Space Administration.

The RAND Corporation
1700 MAIN ST • SANTA MONICA • CALIFORNIA • 90406

Published by The RAND Corporation

PREFACE

Zonal or meridional shifts of the geomagnetic field at the earth's surface suggest the existence of similar motions at the surface of the central core about half-way to the earth's center, assuming that the field lines are frozen into the surface of the core. The change of the earth's magnetic field in time is used to estimate possible motions within the core boundary.

The study presented here is one of a series intended to improve predictions of the strength and patterns of the earth's magnetic field as it affects the radiation belts, and to assist in estimates of the magnetic fields likely to be encountered on other planets.

This work was supported by the National Aeronautics and Space Administration.

ABSTRACT

Our previous work on theoretically inferred surface fluid motions of the earth's core is improved and extended for epoch 1955, using secular change for the period 1912 to 1955. The surface flow is estimated using the frozen-field concepts of Alfvén, including the effect on the secular change resulting from the nonuniformity in the flow pattern. We also assume that both dipole and non-dipole terms of the main field are carried by the fluid. The flow is estimated by assuming that the horizontal surface flow can be resolved into two components, rotational and irrotational. The rotational component is composed of the well-known westward drift and flow around closed streamlines on the surface of the core. The irrotational component is flow from sources to sinks and gives us some information about the vertical flow. The westward drift does not appear to dominate the flow. In addition to the westward drift we find a strong upflow in the Southern Hemisphere, centered south of Africa, with a corresponding downflow in the Northern Hemisphere centered in the Mediterranean. Although we still obtain a downflow in the northern Pacific as we reported previously, our improved estimate has resulted in major changes in the derived flow. There are also strong rotational velocities, particularly in the region of Africa.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge our debt to Dr. John Kern for various helpful comments that assisted this work. We also wish to thank Professor P. H. Roberts for his friendly interest.

CONTENTS

I. INTRODUCTION

The outer portion of the earth's central core is usually regarded as consisting of an electrically conducting fluid in motion. The motions of this fluid can at present only be inferred from the study of temporal changes in the geomagnetic field. The changes in the earth's surface field are believed to result mainly from motion of the fluid in the core relative to the surface of the earth, with the magnetic field "frozen" into the fluid and hence moving with it. Additional changes may result from actual creation or decay of the field. Since many of these "secular" changes occur quite rapidly, within a decade or so, their source must lie in the outer few hundred km of the core. The high conductivity of the core would effectively shield from observation any such rapid variations occurring at a substantial depth within it.

Motion of the core was first estimated by Halley (1692), who noted the drift with time of the positions of isogonic lines on magnetic charts drawn for different epochs. He remarked that the core appeared to be rotating more slowly than the outer part of the earth by about 0.5°/yr. This apparent westward drift of the magnetic field was later studied by Carlheim-Gyllensköld (1896), and by Laporte at the Carnegie Institution of Washington in 1946 using charts from 1700 to 1945 (unpublished). Their results essentially agreed with Halley's. A more elaborate and careful study by Bullard, et al. (1950), using data for the period from 1907.5 to 1945, showed an estimated average drift of about 0.18°/yr for the nondipole field, and thus for the surface of the core. Results of westward drift of the eccentric dipole field were given by Vestine (1952), Yukutake (1962), and others; a typical value was about 0.28°/yr. Carlheim-Gyllensköld (1896), Bullard, et al. (1950), and Yukutake (1962) also considered the westward motion of individual harmonics. Yukutake derived drift estimates from observatory values and charts to 1955 and found that the average value of 0.2°/yr was representative. His work tended to show that the westward drift was nonuniform and varied with geographical location, a point made earlier by Whitham (1958), who studied the westward drift using data for Canada, and by Pochtarev (1964). In fact, examples of local eastward drifts in northeast Asia were noted by Yukutake (1962), and Nagata (1962) obtained a world-wide eastward drift for the spherical harmonic term P_3^2.

The various estimates of westward drift differ probably because of differences in the linear cross section of the field patterns examined, and because the shapes of field patterns can affect the accuracy of drift estimates. Estimates of the amount of horizontal drift in the charted patterns are also affected by both random and systematic errors in the data. Yukutake's work was extended by Nagata (1962), who showed that a considerable part of the secular-change field at the earth's surface could arise from the westward drift of the non-dipole field. Taking the westward drift to be 0.2°/yr, he then attributed the residual secular change to a stationary secular-change field. Such a field would represent fluid-flow patterns in the core in addition to those resulting from the time average of the westward drift.

It is the purpose of this paper to define, on the basis of geomagnetic data and hydrodynamic principles, some general features of the core-surface motions comprising the westward drift pattern and the next simplest pattern of surface flow. To do this we must know the geomagnetic field just above the surface of the core, which has been estimated using the analytic continuation of spherical harmonics. Alternative representations of field patterns have been suggested but will not be used here, although in principle they could provide alternative approaches to the problem of depicting surface fields above the core (Alldredge and Hurwitz, 1964; McNish, 1940; Lowes and Runcorn, 1951; MacDonald, 1955). We attribute the observed secular change of the earth's magnetic field to the motion of the conducting fluid in the core with the field patterns frozen in. This relationship is given approximately by the well-known expression

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) \tag{1}$$

in which the velocity $\underline{v}$ is the unknown quantity in our problem (Vestine and Kahle, 1966). The magnetic field $\underline{B}$ and the secular change field $\partial \underline{B}/\partial t$ were extrapolated downward to just above the core. Making the assumption that the horizontal velocity is of the form $\underline{v} = -\nabla_T \psi + \underline{r} \times \nabla \chi$, the quantities ψ and χ were found by a least-squares solution of Eq. (1). The velocity potential ψ (representing the curl-free part of the horizontal velocity) then also gives some information about the vertical flow in terms of sources and sinks of the horizontal flow. Also ∇_T indicates that the tangential component of the operator is used.

II. METHOD OF DETERMINING VELOCITY

Finding a method to determine the fluid velocity from the observed magnetic field and secular change field requires some approximations. Maxwell's familiar equations of electromagnetism, in emu, give

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}, \qquad \nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{B} = 4\pi \, \underline{J}$$

and

$$\underline{J} = \sigma(\underline{E} + \underline{v} \times \underline{B}), \quad \nabla \cdot \underline{J} = 0$$

where

$\underline{J}$ is the current density,

$\underline{E}$ is the electric field,

$\underline{B}$ is the magnetic field,

$\underline{v}$ is the velocity of the fluid, and

σ is the electric conductivity just beneath the surface of the core.

Combining these, we have the hydromagnetic equation of Cowling (1957):

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) + \left(\frac{1}{4\pi\sigma}\right) \nabla^2 \underline{B} \qquad (2)$$

where $\partial \underline{B}/\partial t$ in this application is the secular magnetic change. The first term on the right is associated with the transport of the field with the fluid, and the second term with the diffusion of the field through the fluid. The magnetic Reynolds number R_M is equal to

$4\pi\sigma$ LV, where L is a length comparable to those in the problem and V is a typical velocity. For our problem, if $L \approx 10^8$ cm, $V \approx 10^{-1}$ cm/sec, and $\sigma \approx 10^{-6}$ emu, then $R_M \approx 100$. When $R_M \gg 1$ the transport term dominates over the diffusion term. Hence we may replace Eq. (2) with Eq. (1):

$$\frac{\partial \underline{B}}{\partial t} \approx \nabla \times (\underline{v} \times \underline{B}) = (\underline{B} \cdot \nabla)\, \underline{v} - (\underline{v} \cdot \nabla)\, \underline{B}$$

The first term on the right is the contribution to $\partial \underline{B}/\partial t$ from the nonuniform features of the velocity, and the second term that from the translation of $\underline{B}$.

Expanding Eq. (1) in spherical coordinates r, θ, and λ,

$$\begin{aligned}
\frac{\partial \underline{B}}{\partial t} = \underline{i}_r & \left[B_r \frac{\partial v_r}{\partial r} + \frac{B_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{B_\lambda}{r \sin\theta}\frac{\partial v_r}{\partial \lambda} - v_r \frac{\partial B_r}{\partial r} - \frac{v_\theta}{r}\frac{\partial B_r}{\partial \theta} - \frac{v_\lambda}{r \sin\theta}\frac{\partial B_r}{\partial \lambda} \right] \\
+ \underline{i}_\theta & \left[B_r \frac{\partial v_\theta}{\partial r} + \frac{B_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{B_\lambda}{r \sin\theta}\frac{\partial v_\theta}{\partial \lambda} - v_r \frac{\partial B_\theta}{\partial r} - \frac{v_\theta}{r}\frac{\partial B_\theta}{\partial \theta} - \frac{v_\lambda}{r \sin\theta}\frac{\partial B_\theta}{\partial \lambda} \right. \\
& \left. + \frac{1}{r}(B_\theta v_r - B_r v_\theta) \right] \\
+ \underline{i}_\lambda & \left[B_r \frac{\partial v_\lambda}{\partial r} + \frac{B_\theta}{r}\frac{\partial v_\lambda}{\partial \theta} + \frac{B_\lambda}{r \sin\theta}\frac{\partial v_\lambda}{\partial \lambda} - v_r \frac{\partial B_\lambda}{\partial r} - \frac{v_\theta}{r}\frac{\partial B_\lambda}{\partial \theta} - \frac{v_\lambda}{r \sin\theta}\frac{\partial B_\lambda}{\partial \lambda} \right. \\
& \left. + \frac{1}{r}(B_\lambda v_r - B_r v_\lambda + B_\lambda v_\theta \cot\theta - B_\theta v_\lambda \cot\theta) \right]
\end{aligned} \qquad (3)$$

where $\underline{i}_r$, $\underline{i}_\theta$, and $\underline{i}_\lambda$ are unit vectors in the directions r, θ, and λ, and v_r, v_θ, and v_λ are the components of $\underline{v}$. At the boundary of the core $v_r = 0$; therefore, $\partial v_r/\partial\theta$ and $\partial v_r/\partial\lambda$ also equal zero at this boundary, as Roberts and Scott (1965) have pointed out.

For the radial component Eq. (3) becomes

$$\frac{\partial B_r}{\partial t} = B_r \frac{\partial v_r}{\partial r} - \frac{v_\theta}{r} \frac{\partial B_r}{\partial \theta} - \frac{v_\lambda}{r \sin \theta} \frac{\partial B_r}{\partial \lambda} \tag{4}$$

This equation requires a knowledge of the unobtainable quantity $\partial v_r/\partial r$. However, assuming that the fluid is incompressible, $\nabla \cdot \underline{v} = 0$, and

$$\frac{\partial v_r}{\partial r} = - \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta \cot \theta}{r} + \frac{1}{r \sin \theta} \frac{\partial v_\lambda}{\partial \lambda} \right) \tag{5}$$

hence, at the surface of the core

$$\begin{aligned} \frac{\partial B_r}{\partial t} = &- B_r \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta \cot \theta}{r} + \frac{1}{r \sin \theta} \frac{\partial v_\lambda}{\partial \lambda} \right) \\ &- \frac{v_\theta}{r} \frac{\partial B_r}{\partial \theta} - \frac{v_\lambda}{r \sin \theta} \frac{\partial B_r}{\partial \lambda} \end{aligned} \tag{6}$$

We see that the time change in $\underline{B}$ is caused by horizontal transport of the field $\underline{B}$ with the fluid -- the frozen-field concept of Alfvén and Fälthammer (1963) -- including the compression or dilation of flux tubes.

Equation (6) is now in terms of the horizontal components of the velocity and the horizontal derivatives of these components. We can express this two-dimensional velocity as the sum of an irrotational and a rotational velocity field in the form

$$\underline{v} = -\nabla_T \psi + \underline{r} \times \nabla \chi \tag{7}$$

where the subscript T denotes the tangential (horizontal) component of the gradient. We cannot determine the vertical velocity flow as a function of r; however, we can assume that because the three-dimensional flow is incompressible, a substantial part of the flow just beneath the surface, where Eq. (6) is applicable, will be upflow and downflow corresponding to the sources and sinks of the ψ part of the velocity.

To solve Eq. (6) for the velocity, we assumed that ψ and χ of Eq. (7) can be expressed in spherical harmonics

$$\psi = r \sum_n \sum_m (\alpha_n^m \cos m\lambda + \beta_n^m \sin m\lambda) P_n^m(\theta) \tag{8}$$

$$\chi = \sum_n \sum_m (A_n^m \cos m\lambda + B_n^m \sin m\lambda) P_n^m(\theta) \tag{9}$$

Then v_θ, v_λ, $\partial v_\theta/\partial\theta$, and $\partial v_\lambda/\partial\lambda$ can all be expressed in terms of the α_n^m, β_n^m, A_n^m, and B_n^m. These latter quantities, totaling 48 if n = 1 to 4 and m = 0 to n, become the unknowns in the problem. Equation (6) was expressed in terms of these coefficients at 612 points comprising a 10° by 10° grid over the surface of the core. Thus we obtained 612 equations in 48 unknowns. These were solved by standard least-squares and matrix methods.

To check the method of solving Eq. (6), we constructed a test velocity field consisting of $\alpha_1^1 = 15$, $\alpha_3^2 = 20$, $\beta_1^1 = -10$, $A_1^0 = 5$, $A_3^1 = -5$, and $B_3^1 = 8$. Using Eq. (6), the fictitious velocity given by this field was combined with the 1955 magnetic field to give a fictitious secular change field. This secular change field was then used in combination with the 1955 magnetic field as input to our computer programs, which were designed to find velocity by the method of least squares. The original test velocity coefficients were recovered to six figures, showing the accuracy of this phase of our calculations.

III. DATA

The magnetic data used in this study consisted of the main-field representation derived from United States world charts for 1955 (Vestine, et al., 1963), and of the secular-change field from 1912 to 1955 (Vestine, et al., 1947; Nagata and Syono, 1961). In a later paper we show that other available representations do not greatly change our results. The magnetic potential of the field of internal origin derived from observed values at the earth's surface at r = a may be written

$$V = a \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m(\theta) \qquad (10)$$

where g_n^m and h_n^m are determined from the surface data, θ is colatitude, λ the east longitude, and $P_n^m(\theta)$ the partially normalized associated Legendre polynomials of Schmidt (Chapman and Bartels, 1940). The magnetic field $\underline{B}$ is $-\nabla V$. The secular-change field $\dot{\underline{B}}$ can be expressed similarly in terms of $\dot{g}_n^m$ and $\dot{h}_n^m$. We used terms up to n = 6 and m = 6.

It is difficult to make a formal analysis of errors in spherical harmonic coefficients derived from main-field and secular-change data. For main-field harmonics, a series may be synthesized and compared with independent measurements, as has been done for data from aircraft surveys or from earth satellites (Heuring, 1965 and Cain, et al., 1965). Various studies show that the error in summed coefficients for the main field is of the order of one per cent at the earth's surface. Although the secular-change coefficients are less well determined, we have some confidence in their accuracy, since survey measurements made shortly after 1900, with the addition of secular changes from the spherical harmonic series, agree with station measurements compiled during later surveys (Vestine, et al., 1947; Al'tshuler, et al., 1955). This fairly successful use of older surveys in preparing later charts suggests that in many regions the total error may be of the order of 10 to 20 per cent, but of course may be much larger for the (smaller) high-degree harmonics, which occasionally may even be incorrect in sign. The relative importance of these high harmonics is amplified when the field is extrapolated to the region just above the surface of the core. Even so, they remain smaller than most of the better defined low-degree terms until the core interior is reached.

When secular change terms to $n = 6$ are extrapolated downward as far as the core, they will probably include substantial errors or uncertainties (Elsasser, 1950 and Roberts, 1965). It was convenient in velocity studies, therefore, to extrapolate by three stages: (1) for the earth's surface ($r = 6371$ km), (2) to a depth of 1500 km ($r = 4871$ km), and (3) to a depth of 2371 km ($r = 4000$ km), about 530 km above the surface of the core. Since the harmonics are amplified by the factor $(a/r)^{n+2}$, where a is the earth's radius and n the degree of the harmonic, we would expect oscillations to appear if the errors in the high-degree harmonics were unduly amplified. Violent oscillations of the field in latitude or longitude did not appear at any of the foregoing depths. The amplification of errors was, however, fairly pronounced at a depth of about 3000 km ($r = 3371$ km), just inside the core, and gave rise to oscillations considered improbable on physical grounds. As a compromise between the clarification of detail and the magnification of error, we selected the depth of 2371 km ($r = 4000$ km) at which to examine the patterns in detail. For velocity patterns of linear cross section 500 km or more, the values calculated for r = 4000 km can, of course, serve as well as for r = 3470 km.

Figures 1, 2, and 3 show the nondipole field for 1912 and 1955, for r = 6370, 4870, and 3370 km, respectively. The arrows show the magnitude and sense of the horizontal component of the field; contours show the isodynamic lines in Z. In Fig. 3 we see the beginnings of what are probably physically unreal oscillations caused by the amplification of errors. It is clear that the difference between the nondipole-field patterns at the two epochs is small compared with the nondipole field itself. We shall regard this difference plus the difference in the dipole terms as due to surface flow in the upper 100 km or so of the core during the 43-year period from 1912 to 1955, for which, using Eq. (6), we hope to derive a few broad-scale features.

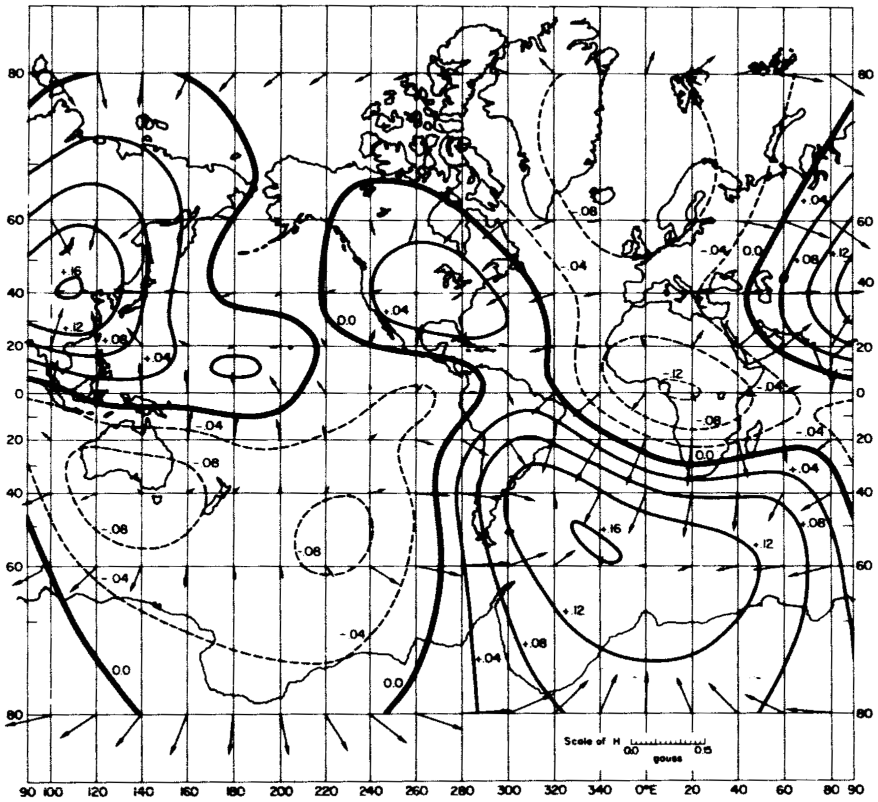

Fig. 1a

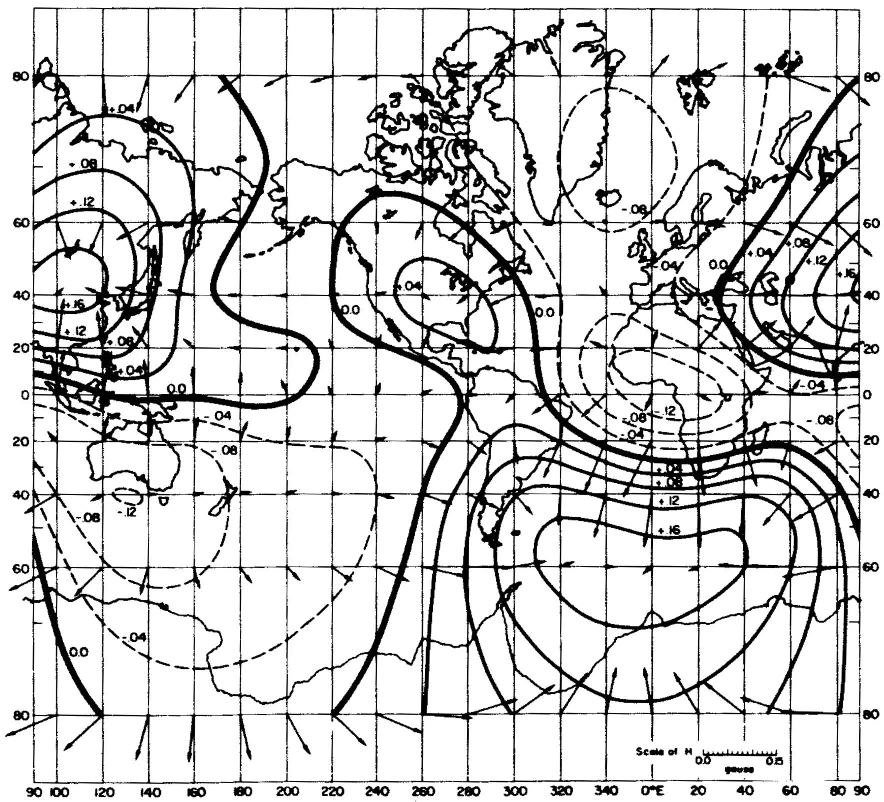

Fig. 1b

Fig. 1 -- Nondipole main field components for (a) 1912, and (b) 1955, at the earth's surface r = 6370 km. Contours are for downward component (Z) in gauss; horizontal component (H) is shown by arrows.

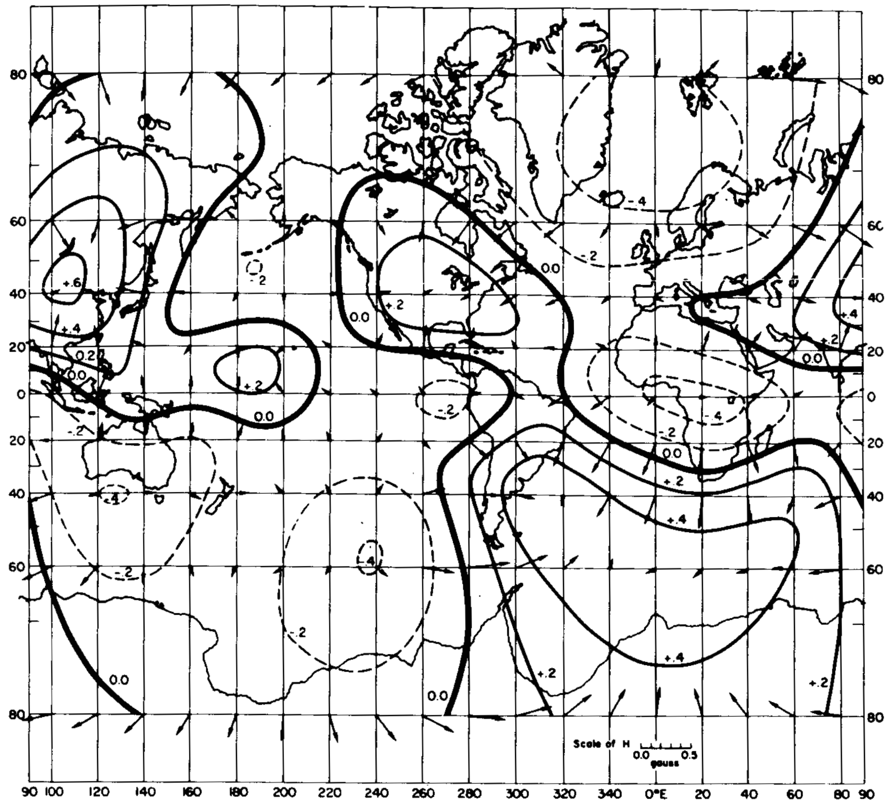

Fig. 2a

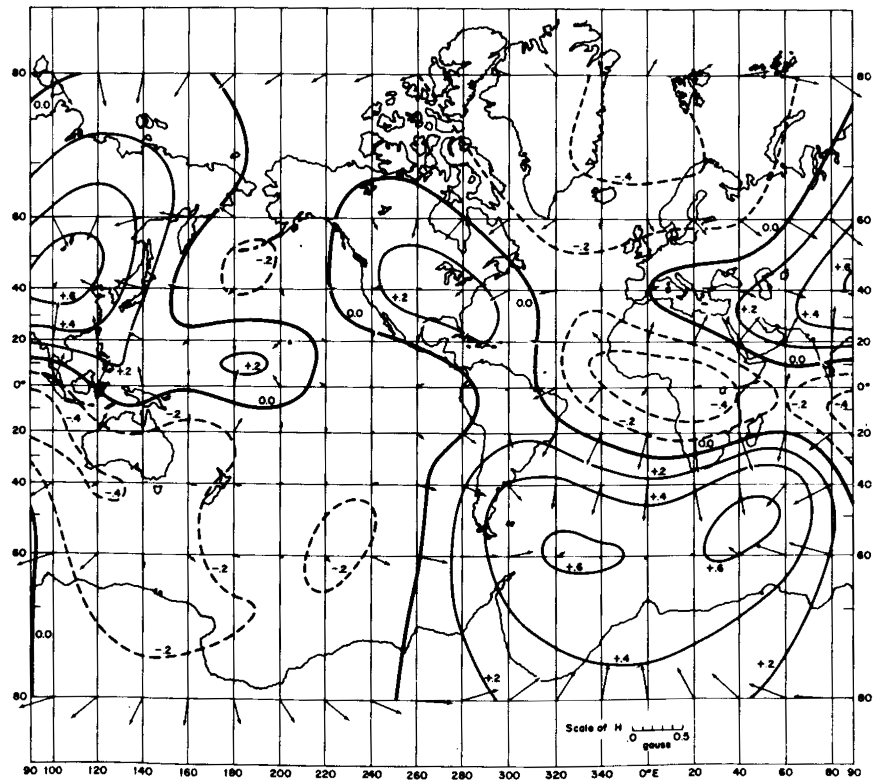

Fig. 2b

Fig. 2 -- Nondipole main field components for (a) 1912, and (b) 1955, at r = 4870 km. Contours are for downward component (Z) in gauss; horizontal component (H) is shown by arrows.

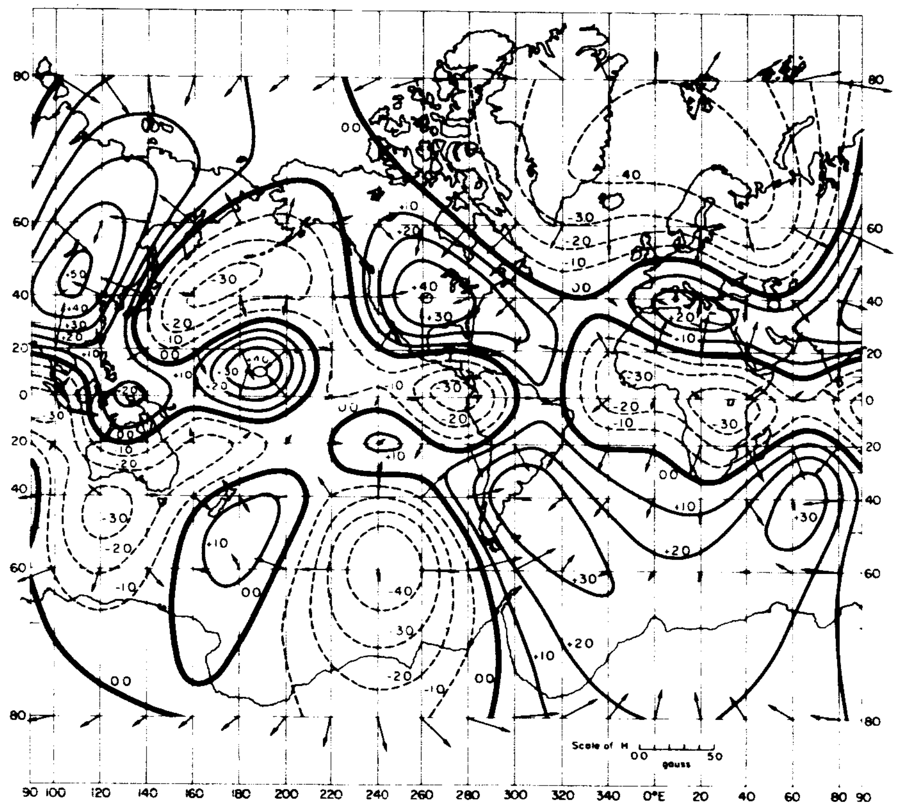

Fig. 3a

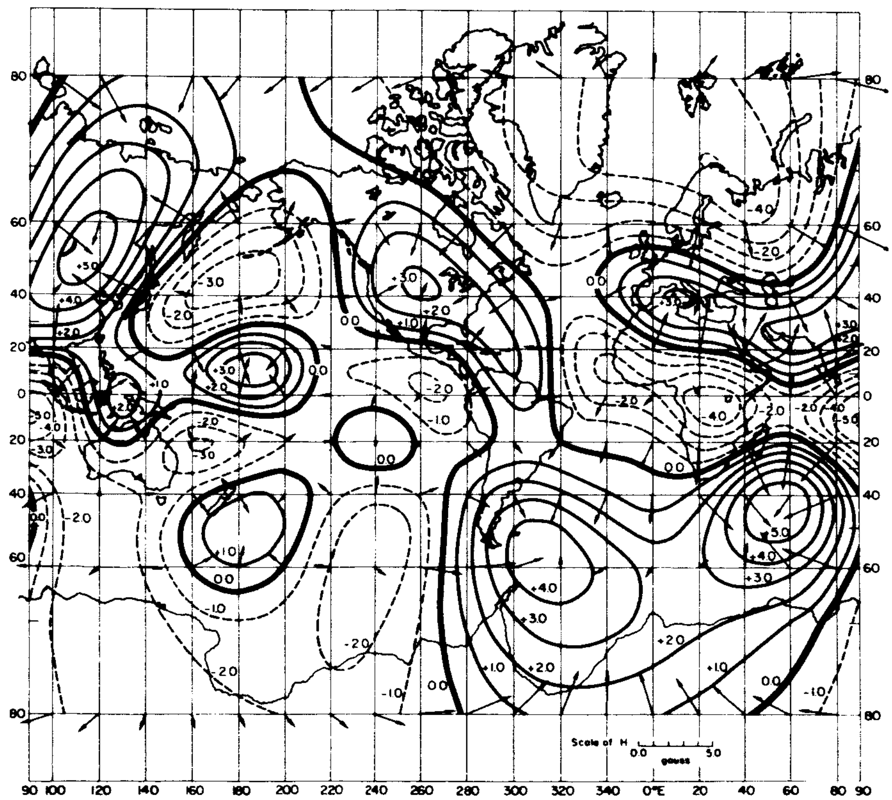

Fig. 3b

Fig. 3 -- Nondipole main field components for (a) 1912, and (b) 1955, at r = 3370 km. Contours are for downward component (Z) in gauss; horizontal component (H) is shown by arrows.

IV. RESULTS

Figure 4 shows the total horizontal velocity obtained using Eq. (6), the 1955 main magnetic field, and the secular-change field from 1912 to 1955. The arrows represent the direction and magnitude of the surface flow. The velocities of about 10 to 15 km/yr resemble magnitudes often discussed in connection with secular change and dynamo theories of the earth's field (Elsasser, 1956; Bullard and Gellman, 1954; Parker, 1955; Cowling, 1965). These velocities can be resolved into two components: the velocity potential ψ and the rotational component χ. The latter can be further resolved into the zonal or westward drift component given by the A_n^0 terms, and the remainder of the rotational flow. The spherical harmonic coefficients for ψ and χ are given in Table 1.

The westward drift, given by the terms A_n^0, is plotted in Fig. 5 as a function of latitude in both km/yr and the more conventional deg/yr. It is seen to be considerably smaller than most previous estimates. It also varies considerably with latitude, even drifting eastward in some places. Apparently some of the reduction in magnitude results from averaging the secular change over a 43-year period; some of our other results, to be reported, show significantly larger values when the velocity is determined from the secular-change field for a single epoch. The variation with latitude, including a reduced or eastward drift in the Northern Hemisphere does appear to be real. These values may differ from previous results because of the different types of contributions to the velocity from the transport term and from the distortion of the flow pattern.

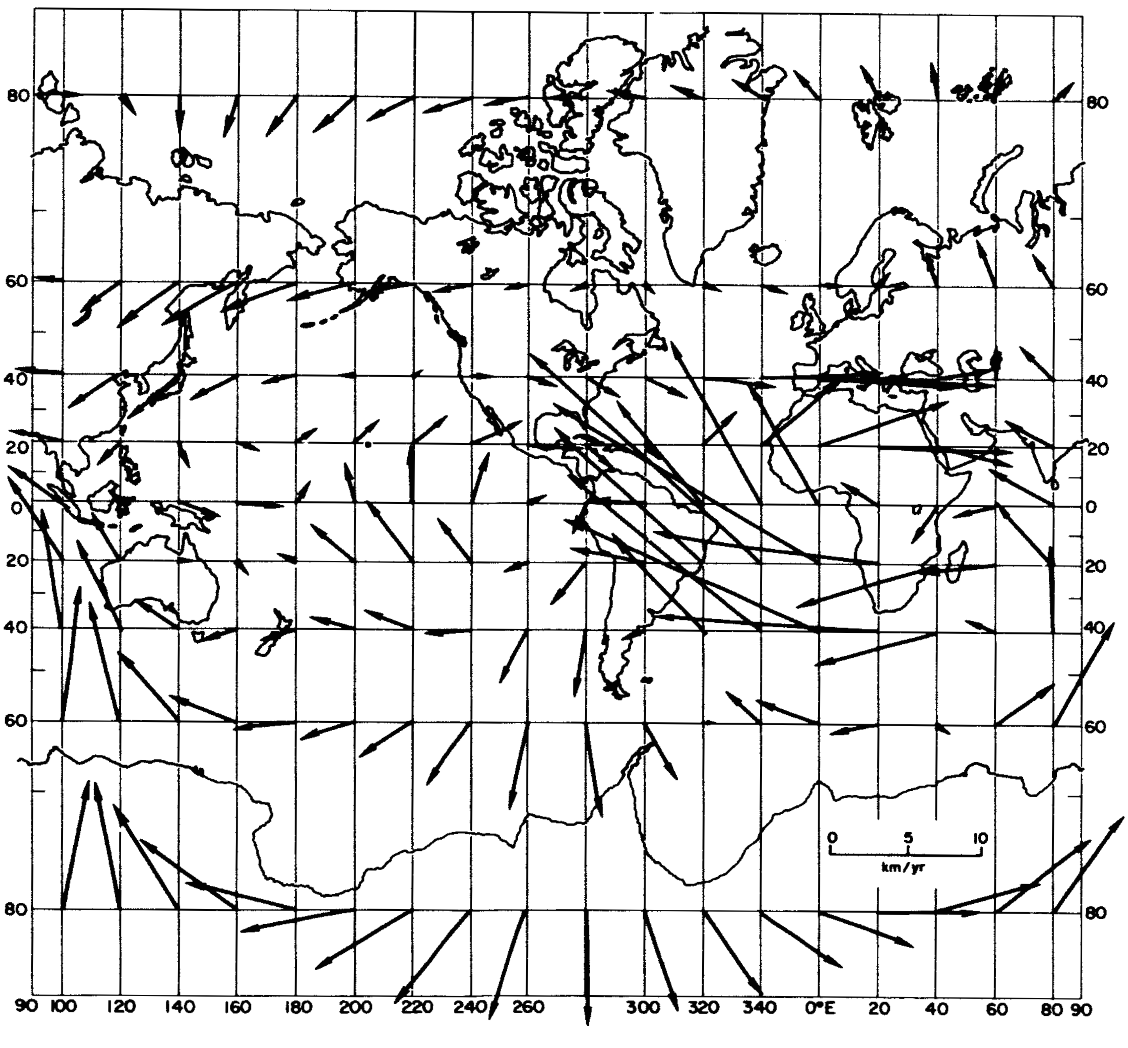

Fig. 4 -- Horizontal velocity estimated at the hypothetical core surface, r = 4000 km, for main field epoch 1955 and secular change field from 1912 to 1955.

Table 1

SPHERICAL HARMONIC COEFFICIENTS OF VELOCITY FIELD IN KM/YR

n	m	Potential, ψ α_n^m	β_n^m	Stream Function, χ A_n^m	B_n^m
1	0	-1.666	0	1.996	0
	1	-0.310	-0.054	0.365	0.654
2	0	0.804	0	-1.973	0
	1	-0.748	-0.340	1.587	0.333
	2	0.026	0.667	0.916	0.682
3	0	0.322	0	0.229	0
	1	0.393	-0.503	-2.157	0.366
	2	-0.232	0.835	-0.629	-0.253
	3	0.326	0.338	1.576	0.679
4	0	0.229	0	1.152	0
	1	-0.039	-0.048	-0.064	0.389
	2	0.090	0.070	-0.661	0.328
	3	-0.137	-0.253	0.534	-0.713
	4	-0.088	0.118	0.431	0.204

The contours of the potential ψ are shown in Fig. 6. The horizontal flow associated with this potential is everywhere perpendicular to the equipotential lines. The velocity vectors for this part of the flow are shown by the dashed arrows in Fig. 8. The contours in Fig. 6 also represent the contours of upflow (solid lines) and downflow (dotted lines) within the limitations mentioned previously. The prominent features here are an upflow in the entire Southern Hemisphere, with the strongest upflow centered south of Africa, and a downflow in the Northern Hemisphere, centered in the central Atlantic--Mediterranean--Central Europe region. If this pattern persists it could explain the smaller secular change

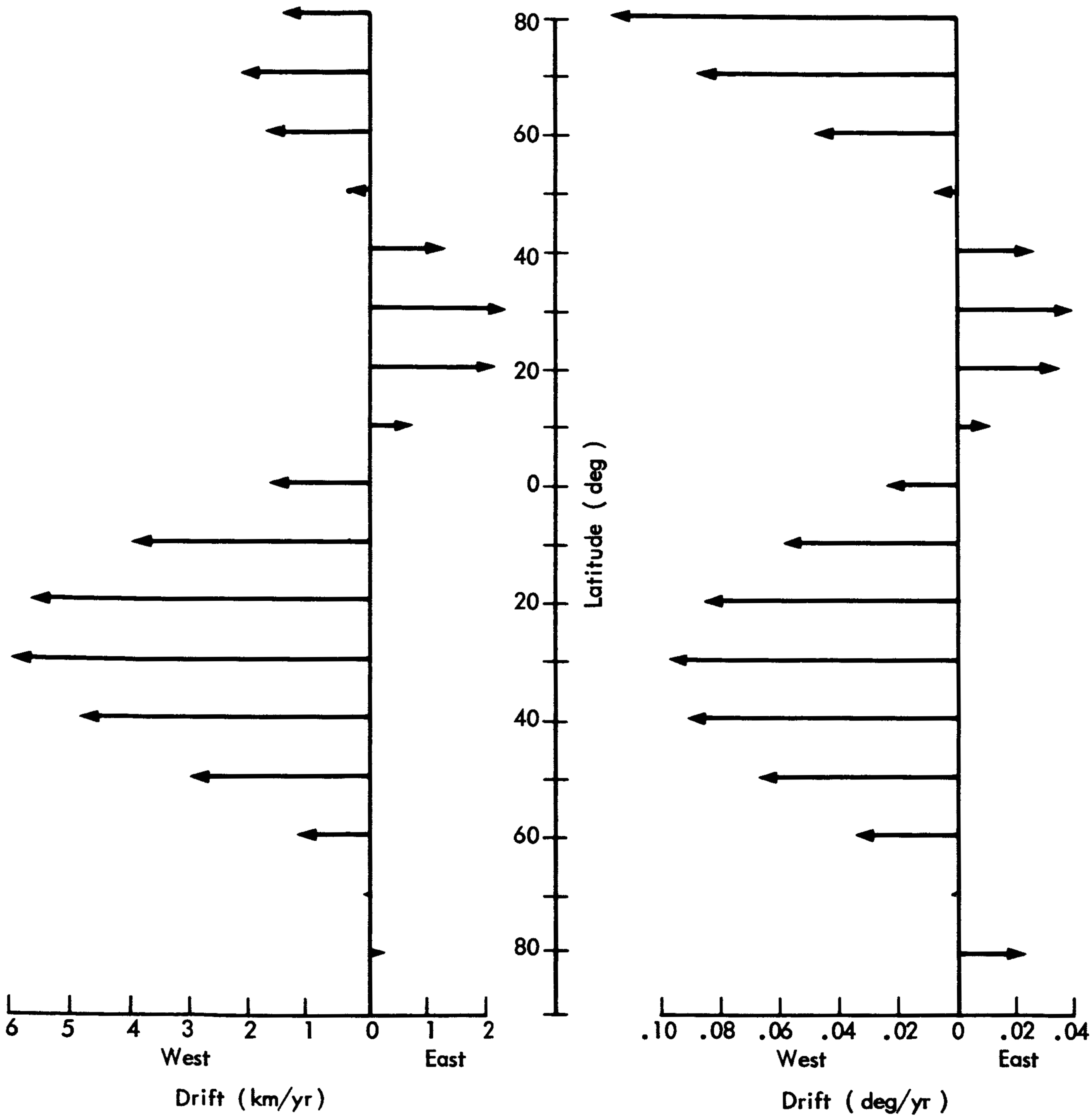

Fig. 5 -- Westward drift velocity estimated at r = 4000 km.

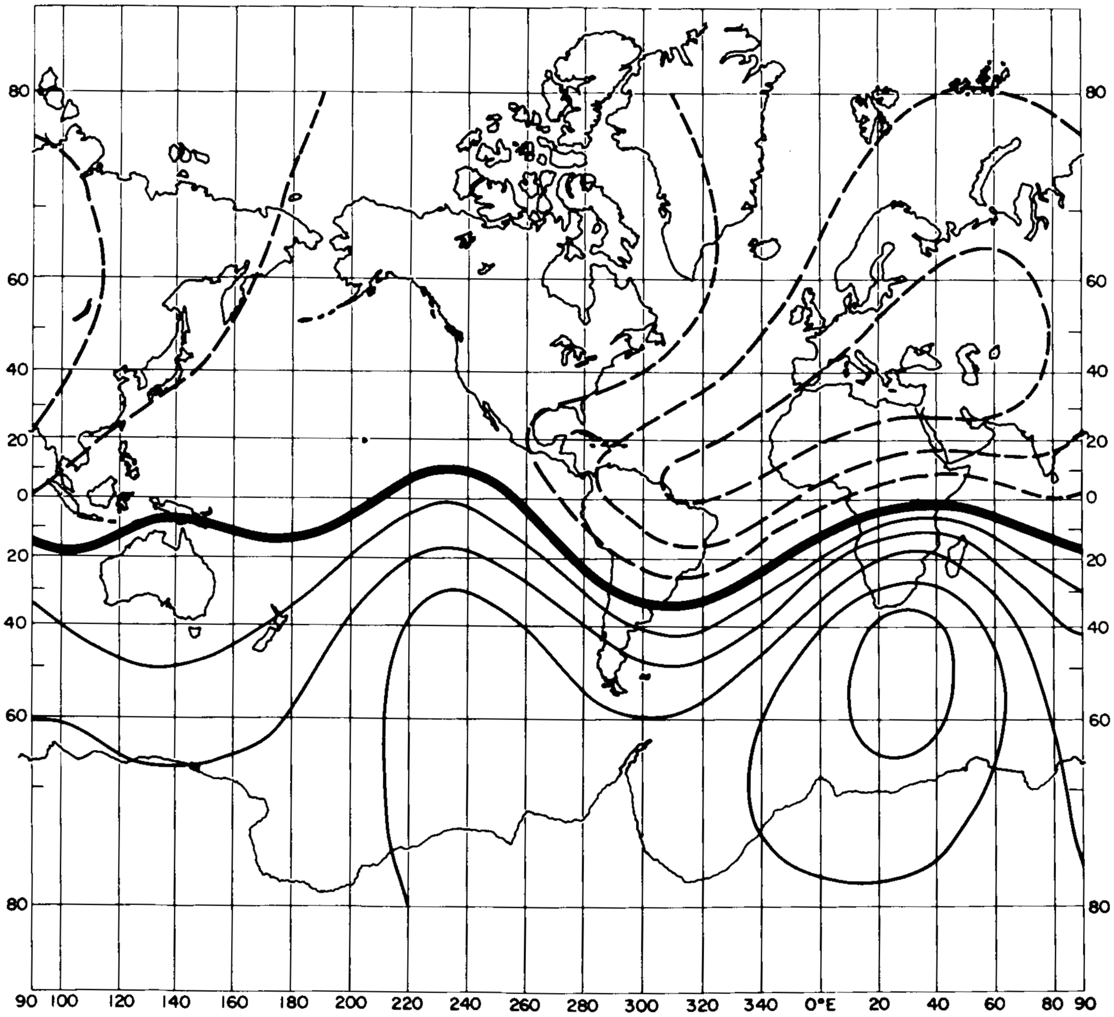

Fig. 6 -- Equipotential lines of the horizontal velocity potential ψ at r = 4000 km. Solid lines show positive potential (upflow) and dotted lines show negative potential (downflow). Contour intervals are 2325 km^2/yr.

noted in the Pacific, assuming the presence of toroidal fields increasing with depth near the surface of the core. The upflow may transport the toroidal fields upwards where they are converted to poloidal form in the more active areas of secular change, as remarked in earlier papers (Allan and Bullard, 1958, Nagata and Rikitake, 1961; Vestine, 1964; Vestine and Kahle, 1966).

The results herein, based on the complete Eq. (6), differ somewhat from those of Vestine and Kahle (1966) because the present calculations include (1) the dipole terms in the main field, and (2) the first term of the right-hand side of the expansion of Eq. (1).

The contours or streamlines of the stream function χ, with the westward drift removed, are shown in Fig. 7. This rotational component of the horizontal velocity flows <u>along</u> the streamlines rather than perpendicular to the contours as in Fig. 6. The solid lines represent the clockwise rotation and the dashed lines counterclockwise rotation. The vectors for this flow are plotted with solid arrows in Fig. 8. Some of the features suggest geostrophic flow -- the counterclockwise rotation around the slight downflow in the North Pacific, for example, and the counterclockwise rotation around the upflow south of Africa. In general, though, the patterns are too complicated and the physical processes involved too complex for such a simple interpretation.

Bullard, et al. (1950) have suggested that the westward drift of the geomagnetic field can be explained by the electromagnetic couple between the electrically conducting core and the mantle. The geophysical data relevant to the earth's rotation have been subsequently discussed in relation to Bullard's theory by others (Munk and MacDonald, 1960; Rochester, 1960; Hide and Roberts, 1961; Kern and Vestine, 1963;

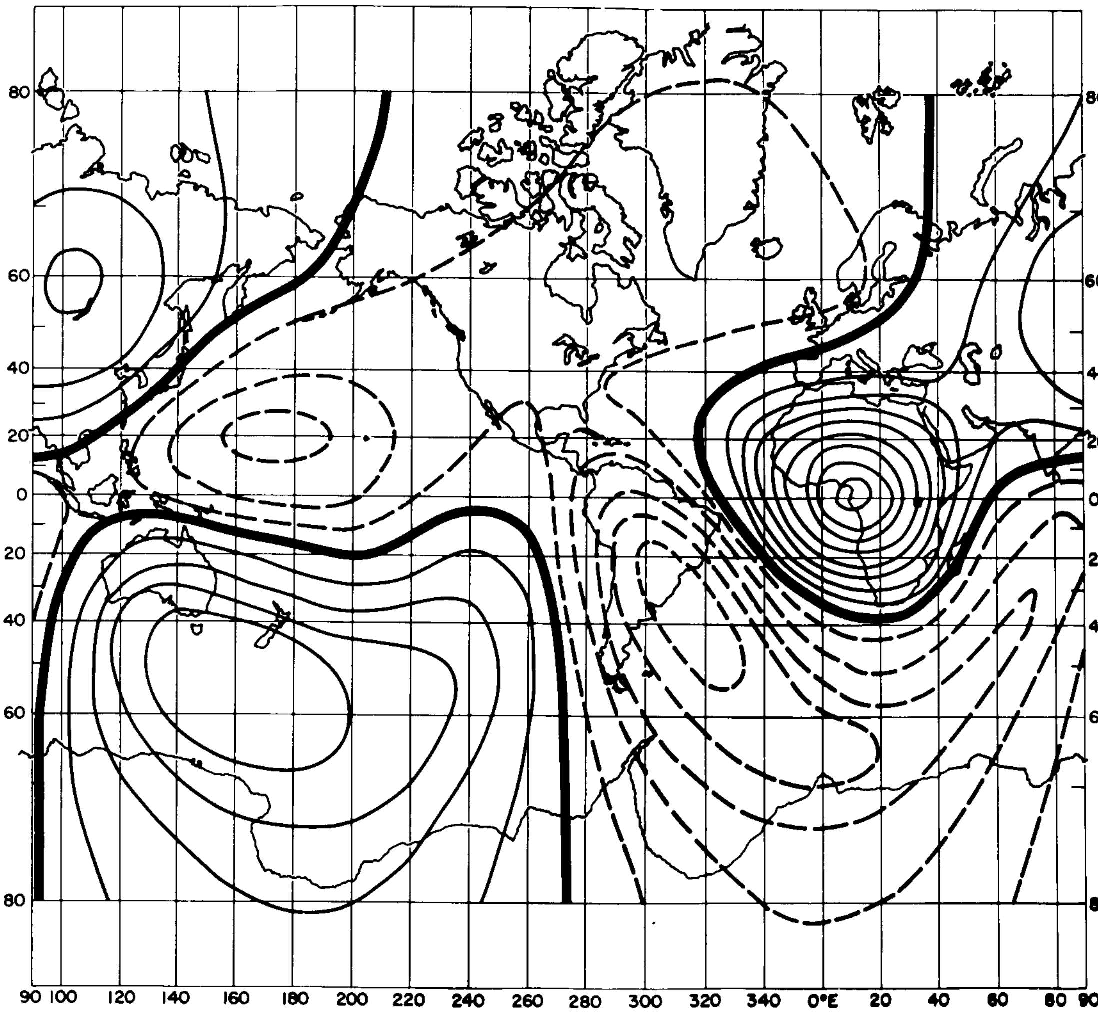

Fig. 7 -- Streamlines of the horizontal velocity stream function χ at r = 4000 km. Solid lines indicate clockwise rotation, dotted lines indicate counterclockwise rotation. Contour intervals are for $r\chi$ = 2325 km^2/yr.

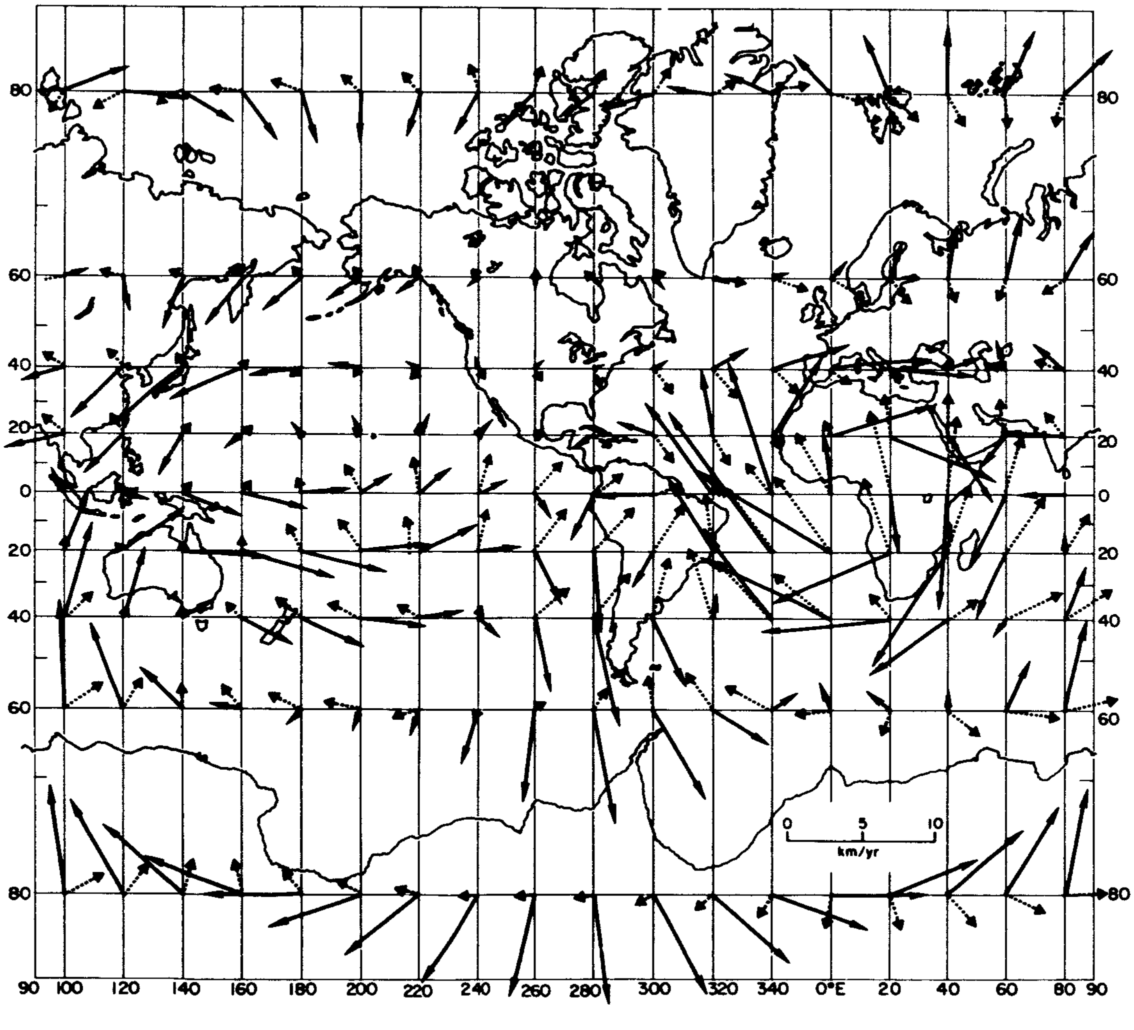

Fig. 8 -- Horizontal velocity estimated at r = 4000 km. Solid arrows show velocity from stream function χ and dotted arrows show velocity from potential ψ.

Nagata, 1965). Bullard showed that a meridional electric current $\underline{i}$, with a downflow near the core equator and an upflow in polar regions, would yield an eastward toroidal field $\underline{T}$ in middle northern latitudes of the core. In middle southern latitudes, $\underline{T}$ would be reversed. The currents are regarded as crossing the lines of force of the earth's poloidal main field $\underline{B}_s$. A mechanical force $\underline{i}_T \times \underline{B}_s$ would arise in the core, directed westward. If $\underline{B}_s$ is that of a centered dipole, the westward force would be zero at the poles and a maximum at the equator. Inspection of Figs. 5 and 6 shows that Coriolis force may modify the the westward drift.

A more detailed quantitative discussion of these and other questions is being prepared for a later paper. We expect to show results based on the main field and secular change field at several different epochs. The work done thus far gives preliminary results that seem likely to support the major features presented here, with significant changes in the velocity patterns with time. All the velocity patterns of the type we have derived so far are topologically similar; that is, they divide the earth's surface into two regions, one of upflow and one of downflow. The convection system postulated in Bullard's dynamo theory, however, has two regions of upflow and two of downflow, assuming that this convection system manifests itself in the core surface flow. We have been unable to find any evidence of these two equatorial upflows and downflows. Our result should be regarded with caution, however, since it includes only terms to P_4^4.

The uncertainty regarding systematic and random errors of spherical harmonic coefficients of secular change has been mentioned previously.

However, the velocities that we have derived do fit experimental data. Figure 9a shows the value of $\dot{Z}$ that we have extrapolated from the surface of the earth to the level r = 4000. Figure 9b shows the value of $\dot{Z} = -\partial B_r/\partial t$ computed using Eq. (1) and the velocities shown in Fig. 4. It can been seen that there is good agreement between the two.

It also appears that broad-scale features of the geoid derived from satellite studies by Kaula (1963) may be related to the flow we derived for the surface of the core. However, the relation between our results and those of Kaula is uncertain, and the subject must be studied further. Kaula (private communication, 1965) has suggested that current representations of the geoid must be revised to improve their fit with later data on satellite trajectories. It would therefore be of great interest to extend in detail those studies of heat flow, the geoid, and geomagnetism that might be related, as Lee and MacDonald (1963) and Vening-Meinez (1964) have suggested.

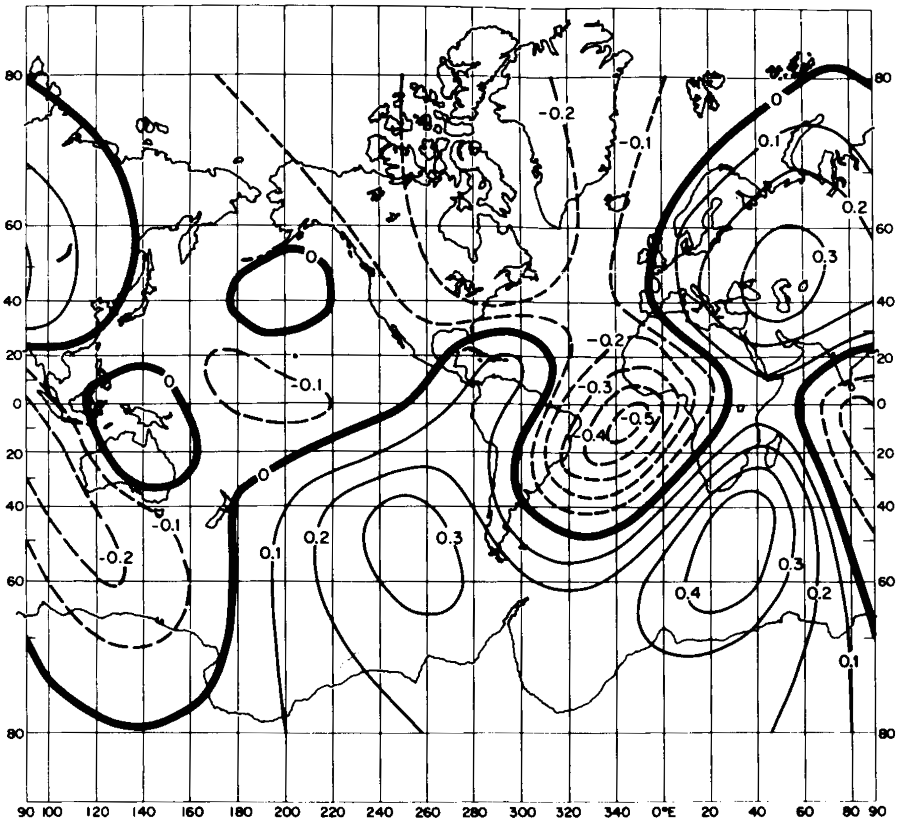

Fig. 9a

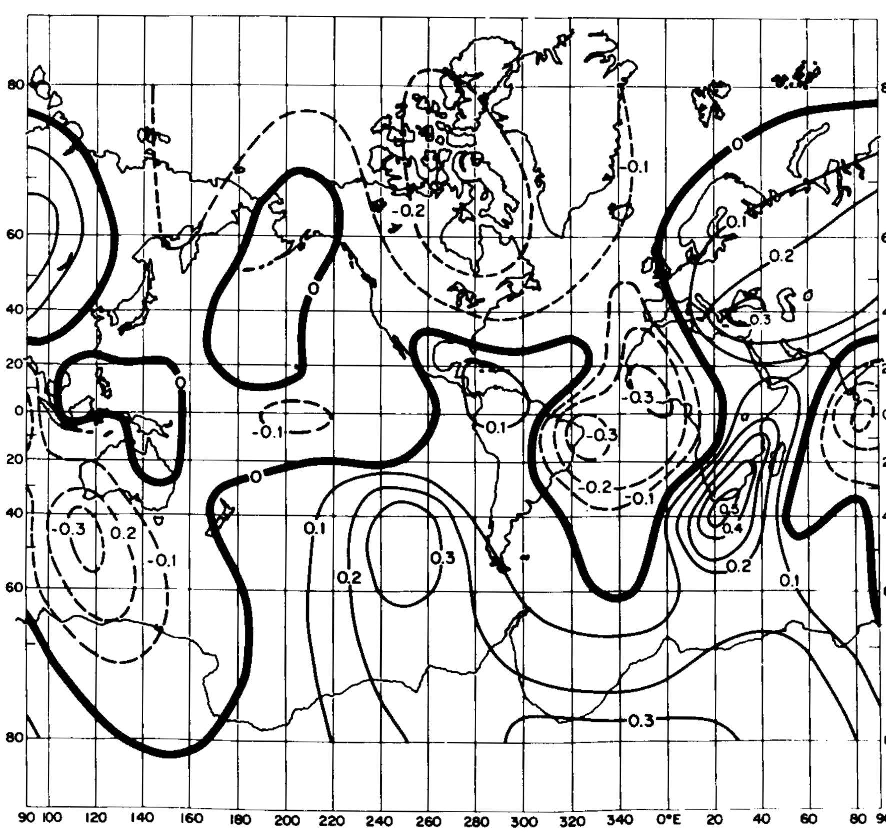

Fig. 9b

Fig. 9 -- Comparison of $\partial B_r/\partial t$ at r = 4000 in gauss/43 yrs; (a) from secular change field at earth's surface extrapolated downward, and (b) computed from derived horizontal velocity field.

BIBLIOGRAPHY

Alfvén, H., and C. Fälthammer, Cosmical Electrodynamics, Clarendon Press, Oxford, 1963.

Allan, D. W., and E. C. Bullard, "Distortion of a Toroidal Field by Convection," Rev. of Mod. Phys., 30, 1087-1088, 1958.

Alldredge, L. R., and L. Hurwitz, "Radial Dipoles as the Sources of the Earth's Main Magnetic Field," J. Geophys. Res., 69, 2631-2640, 1964.

Al'tshuler, L. I., B. D. Vints, K. A. Mal'tseva, Z. S. Churguryan, and A. P. Shlyakhtina, Memoirs of the Institute of Geomagnetism, Ionosphere and Radio Wave Propagation, 11, (21), 229-236, 1955.

Bullard, E. C., C. Freedman, H. Gellman, and J. Nixon, "The Westward Drift of the Earth's Magnetic Field," Phil. Trans. Roy. Soc., 243-A, 67-92, 1950.

Bullard, E. C., and H. Gellman, "Homogeneous Dynamos and Terrestrial Magnetism," Phil. Trans. Roy. Soc., 247A, 213-278, 1954.

Cain, J. C., W. E. Daniels, and S. J. Hendricks, "An Evaluation of the Main Geomagnetic Field, 1940-1962," J. Geophys. Res., 70, 3647-3674, 1965.

Carlheim-Gyllensköld, V., "Sur la Forme Analytique de l'Attraction Magnétique de la Terre, Exprimée en Fonction du Temps," Astron. Jaktagelser och Undersökn., Stockholms Observat., 5, Nr. 5, 36 pp., 1896.

Chapman, S., and J. Bartels, Geomagnetism, 1 and 2, Oxford University Press, Oxford, 1940.

Cowling, T. G., Magnetohydrodynamics, Interscience, New York, 1957.

Cowling, T. G., Introductory Report, Part III, "General Aspects of Stellar and Solar Magnetic Fields," in Stellar and Solar Magnetic Fields, R. Lüst (ed.), North Holland Pub. House, Amsterdam, 1965. Presented at International Astronomical Union, Symposium No. 22, Munich, Germany, September 1963.

Elsasser, W. M., "The Earth's Interior and Geomagnetism," Rev. of Mod. Phys., 22, 1-35, 1950.

Elsasser, W. M., "Hydromagnetic Dynamo Theory," Rev. of Mod. Phys., 28, 135-163, 1956.

Halley, E., "On the Cause of the Change in the Variation of the Magnetic Needle; With an Hypothesis of the Structure of the Internal Parts of the Earth," Phil. Trans. Roy. Soc., 17, 470-478, 1692.

Heuring, F. T., "Comparison of Some Recently Defined Geomagnetic Field Models to the Vanguard 3 (1959η) Data," J. Geophys. Res., 70, 4968-4971, 1965.

Hide, R., and P. H. Roberts, "The Origin of the Main Geomagnetic Field," Physics and Chemistry of the Earth, 4, 27-98, Pergamon Press, London, 1961.

Kaula, W. M., "Improved Geodetic Results from Camera Observations of Satellites," J. Geophys. Res., 68, 5183-5190, 1963.

Kern, J. W., and E. H. Vestine, "Magnetic Field of the Earth and Planets," Space Sci. Rev., 2, 136-171, 1963.

Lee, W. H. K., and G. J. F. MacDonald, "The Global Variation of Terrestrial Heat Flow," J. Geophys. Res., 68, 6481-6492, 1963.

Lowes, F. J., and S. K. Runcorn, "The Analysis of the Geomagnetic Secular Variation," Phil. Trans. Roy. Soc., 243A, 525-546, 1951.

MacDonald, K. L., "Geomagnetic Secular Variation at the Core-mantle Boundary," J. Geophys. Res., 60, 377-388, 1955.

McNish, A. G., "Physical Representations of the Geomagnetic Field," Trans. Amer. Geophys. Union, 21st Annual Meeting, II, 287-291, 1940.

Munk, W., and G. J. F. MacDonald, The Rotation of the Earth, Cambridge University Press, Cambridge, 1960.

Nagata, T., "Two Main Aspects of Geomagnetic Secular Variation: Westward Drift and Non-drifting Components," Proceedings of the Benedum Earth Magnetism Symposium, 39-55, Pittsburgh, 1962.

Nagata, T., "Main Characteristics of Recent Geomagnetic Secular Variation," J. Geomagnet. Geoelec., 17, 263-276, 1965.

Nagata, T., and T. Rikitake, "Geomagnetic Secular Variation and Poloidal Magnetic Fields Produced by Convectional Motions in the Earth's Core," J. Geomagnet. Geoelec., 13, 42-53, 1961.

Nagata, T., and Y. Syono, "Geomagnetic Secular Variation During the Period from 1955 to 1960," J. Geomagnet. Geoelec., 12, 84-98, 1961.

Parker, E. N., "Hydromagnetic Dynamo Models," Astrophys. J., 122, 293-314, 1955.

Pochtarev, V. I., "The Western Drift of the Geomagnetic Field," (English translation, American Geophysical Union), Geomagnetism and Aeronomy, 4, (2), 289-291, 1964.

Roberts, P. H., "Inferences Concerning Motions of the Core," Abstract II-2. Paper presented at the NATO Advanced Study Institute of Planetary and Stellar Magnetism, Newcastle-on-Tyne, England, April 26 to May 1, 1965.

Roberts, P. H., and S. Scott, "On Analysis of the Secular Variation; (I) A Hydromagnetic Constraint: Theory," J. Geomagnet. Geoelec., 17, 137-151, 1965.

Rochester, M. G., "Geomagnetic Westward Drift and Irregularities in the Earth's Rotation," Phil. Trans. Roy. Soc. London, A-252, 531-555, 1960.

Vening-Meinez, F. A., "A Cause of Changes of the Secular Geomagnetic Field," Proceedings Koninkl. Nederl. Akademie Van Weterschappen, 67B, (4), 341-343, Amsterdam, 1964.

Vestine, E. H., "On Variations of the Geomagnetic Fluid, Fluid Motions, and the Rate of the Earth's Rotation," Proc. Natl. Acad. Sci., 38, 1030-1038, 1952.

Vestine, E. H., "The World Magnetic Survey and the Earth's Interior," in Proceedings of The Symposium on Magnetism of the Earth's Interior, November 16-20, 1964, University of Pittsburgh, J. Geomagnet. Geoelec., 17, (34), 165-171, 1965.

Vestine, E. H., and Anne B. Kahle, "On the Small Amplitude of Magnetic Secular Change in the Pacific Area," J. Geophys. Res., 71, 527-530, 1966.

Vestine, E. H., L. Laporte, I. Lange, and W. E. Scott, "The Geomagnetic Field, Its Description and Analysis," Carnegie Inst. Wash. Pub., (580), 1947.

Vestine, E. H., W. L. Sibley, J. W. Kern, and J. L. Carlstedt, "Integral and Spherical-harmonic Analysis of the Geomagnetic Field for 1955.0, Part 1," J. Geomagnet. Geoelec., 15, 47-72, 1963.

Whitman, K., "The Relationships Between the Secular Change and the Non-dipole Fields," Can. J. Phys., 36, 1372-1396, 1958.

Yukatake, T., "The Westward Drift of the Magnetic Field of the Earth," Bull. Earthquake Res. Inst., 40, 1-65, 1962.

RAND RESEARCH
BRIEF
THE RAND CORPORATION

August 1966 RB-5091

RM-5091-NASA, Estimated Surface Motions of the Earth's Core, Anne B. Kahle, E. H. Vestine, R. H. Ball, RAND Memorandum, August 1966, 38 pp.

PURPOSE: To define, on the basis of geomagnetic data and hydrodynamic principles, some general features of the core-surface motions comprising the westward drift pattern and the next simplest pattern of surface flow. Previous work on theoretically inferred surface fluid motions of the earth's core is improved and extended for epoch 1955, using secular change for the period 1912 to 1955.

METHODOLOGY: Surface flow is estimated using the frozen-field concepts of Alfvén, including the effect on the secular change resulting from the nonuniformity in the flow pattern. It is assumed that both dipole and nondipole terms of the main field are carried by the fluid. The flow is estimated by assuming that the horizontal surface flow can be resolved into two components, rotational and irrotational. The rotational component is composed of the well-known westward drift and flow around closed streamlines on the surface of the core. The irrotational component is flow from sources to sinks and affords some information about the vertical flow.

RESULTS: The westward drift appears to be considerably smaller than previously estimated. It also varies considerably with latitude, even drifting eastward in some places. Apparently some of this reduction in magnitude results from averaging the secular change over a 43-year period. In addition to the westward drift, there seems to be a strong upflow in the Southern Hemisphere, centered south of Africa, with a corresponding downflow in the Northern Hemisphere, centered in the Mediterranean. Although a downflow was still obtained for the Northern Pacific, the improved estimate has resulted in major changes in the derived flow. There are also strong rotational velocities, particularly in the region of Africa.

BACKGROUND: This RAND study was supported by the National Aeronautics and Space Administration. It is intended to improve predictions of the strength and patterns of the earth's magnetic field as it affects the radiation belts, and to assist in estimating the magnetic fields likely to be encountered on other planets.

AG